Quantum Information with Top Quarks at the LHC

Y. Afik, JRMdN, EPJ Plus 136, 907 (2021)
Y. Afik, JRMdN, Quantum 6, 820 (2022)
Y. Afik, JRMdN, PRL 130, 221801 (2023)
Y. Afik, Y. Kats, JRMdN, A. Soffer, D. Uzan, In progress

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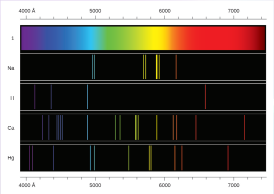
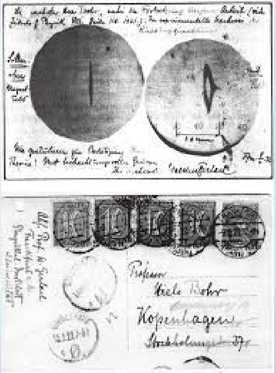
Quantum Theory: Quantization

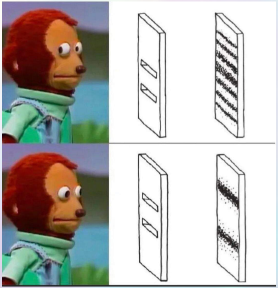
- Quantum Mechanics was originally named after observation of quantized values in several systems:
  - Electromagnetic radiation (Black-body/Photoelectric effect)
  - Electron orbits (Atomic spectra)
  - Angular momentum (Stern-Gerlach)





Quantum Theory: Copenhagen Interpretation

- Copenhagen interpretation of Quantum Mechanics:
  - Particles described in terms of waves $\Psi \to$ Superposition of quantum states.
  - Outcomes of measurements: Observable eigenvalues $\to$ Quantization.
  - Probabilities of outcomes encoded in $|\Psi|^2 \to$ Quantum Interference.
  - Wave-function collapses by measurement process: Quantum state projected to measured states.



Quantum vs. Classical

- Quantum Mechanics: Particles are in superposition of states $\rightarrow$ Probabilistic description of measurements.
- Classical Mechanics can also describe random outputs using classical probability distributions (noise, experimental variations...).
- Is God *just* playing dice with the Universe? $\rightarrow$

Quantum vs. Classical

- Quantum Mechanics: Particles are in superposition of states $\rightarrow$ Probabilistic description of measurements.
- Classical Mechanics can also describe random outputs using classical probability distributions (noise, experimental variations...).
- Is God *just* playing dice with the Universe? $\rightarrow$ God is well beyond a mere croupier!
- Quantum Correlations=Correlations not accounted by classical probabilistic theories.

Quantum State

- **Pure state** $\rightarrow$ Wave function $|\Psi\rangle$
  1. $|\Psi\rangle = \sum_n \alpha_n \cdot |\phi_n\rangle\,, \ \ \langle\Psi|\Psi\rangle = \sum_n |\alpha_n|^2 = 1$
  2. Coherent mixture of quantum states $\rightarrow$ α_n are amplitudes
  3. Expectation values: $\langle A\rangle = \langle\Psi|A|\Psi\rangle = \sum_{n,m} \alpha_m^* \alpha_n \langle\phi_m|A|\phi_n\rangle$
- **Mixed state** $\rightarrow$ Generalization to density matrix ρ
  1. $\rho = \sum_n p_n \cdot |\phi_n\rangle\langle\phi_n|\,, \ \ \mathrm{tr}\rho = \sum_n p_n = 1$
  2. Incoherent mixture of quantum states $\rightarrow$ p_n are probabilities
  3. Expectation values: $\langle A\rangle = \mathrm{tr}(\rho A) = \sum_n p_n \langle\phi_n|A|\phi_n\rangle$

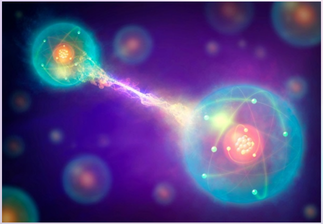
Qubits

- Qubit: Two-level quantum system $|0\rangle, |1\rangle \rightarrow$ Most simple quantum system.
- General density matrix (2×2) for 1 qubit $\rightarrow$ 3 parameters B_i:

$$\rho = \frac{1 + \sum_i B_i \sigma^i}{2}$$

- Two qubits $\rightarrow$ Most simple example of quantum correlations.
- General density matrix (4×4) for 2 qubits $\rightarrow$ 15 parameters $B_i^{\pm}$, C_{ij}

$$\rho = \frac{1 + \sum_i \left(B_i^+ \sigma^i \otimes 1 + B_i^- 1 \otimes \sigma^i\right) + \sum_{i,j} C_{ij} \sigma^i \otimes \sigma^j}{4}$$



Quantum Discord

- Classically, two equivalent expressions for mutual information of bipartite system A and B (Alice and Bob):

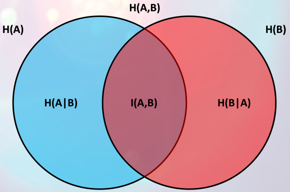
$$I(A,B) = H(A) + H(B) - H(A,B) = H(A) - H(A|B)$$
$$H(A,B) = -\sum_{x,y} p(x,y)\log_2 p(x,y)$$
$$H(A|B) = \sum_{y} p(y) H(A|B=y)$$

- Quantum mechanics can introduce a "discord" between both expressions:

$$\mathcal{D}(A,B) \equiv H(B) - H(A,B) + H(A|B) \neq 0$$

- Most basic form of quantum correlations!
- Quantum Discord is asymmetric $\mathcal{D}(A,B) \neq \mathcal{D}(B,A)$

PRL 88, 017901 (2001)



Quantum Discord: Classical states

- OK...but where is the Physics here? $\rightarrow$ Only classical states have zero discord!

$$\rho_{\text{class}} = \sum_{n,m} p_{n,m} \left|n\right\rangle \otimes \left|m\right\rangle \left\langle n\right| \otimes \left\langle m\right|$$

- $\left|n\right\rangle, \left|m\right\rangle$ form an *orthonormal* basis for A, B
- $p_{n,m}$ are purely classical probabilities of being in state $\left|n\right\rangle \otimes \left|m\right\rangle$!
- Two qubits $\rightarrow$ Tails and heads with two spin directions!

$$\begin{aligned}
\rho_{\text{class}} &= p_{++} \left|\mathbf{n}_A\right\rangle \otimes \left|\mathbf{n}_B\right\rangle \left\langle \mathbf{n}_A\right| \otimes \left\langle \mathbf{n}_B\right| \\
&+ p_{+-} \left|\mathbf{n}_A\right\rangle \otimes \left|-\mathbf{n}_B\right\rangle \left\langle \mathbf{n}_A\right| \otimes \left\langle -\mathbf{n}_B\right| \\
&+ p_{-+} \left|-\mathbf{n}_A\right\rangle \otimes \left|\mathbf{n}_B\right\rangle \left\langle -\mathbf{n}_A\right| \otimes \left\langle \mathbf{n}_B\right| \\
&+ p_{--} \left|-\mathbf{n}_A\right\rangle \otimes \left|-\mathbf{n}_B\right\rangle \left\langle -\mathbf{n}_A\right| \otimes \left\langle -\mathbf{n}_B\right|
\end{aligned}$$

Entanglement

- What if we generalize the previous idea? $\rightarrow$ Separability:

$$\rho_{\text{sep}} = \sum_{n,m} p_{n,m} \left|n\right\rangle \otimes \left|m\right\rangle \left\langle n\right| \otimes \left\langle m\right| = \sum_k p_k \rho_k^{(A)} \otimes \rho_k^{(B)}$$

- $\left|n\right\rangle, \left|m\right\rangle$ not necessarily *orthonormal* now $\rightarrow$ $p_{n,m}$ are quasi-probabilities (not disjoint events)
- Any classically correlated state (classical probability) is separable.
- **Entanglement**: Non-separability of a bipartite quantum state.

Entanglement: Two qubits

- Two qubits: Separability=Positive P-representation $P(\mathbf{n}_A, \mathbf{n}_B) \geq 0$:

$$\rho = \int \mathrm{d}\Omega_A \mathrm{d}\Omega_B \, P(\mathbf{n}_A, \mathbf{n}_B) \left|\mathbf{n}_A \mathbf{n}_B\right\rangle \left\langle \mathbf{n}_A \mathbf{n}_B\right|, \quad \int \mathrm{d}\Omega_A \mathrm{d}\Omega_B \, P(\mathbf{n}_A, \mathbf{n}_B) = 1$$

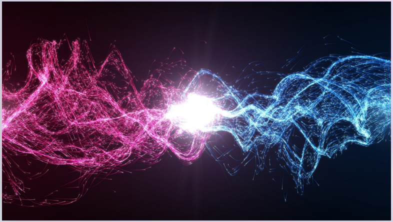
- $P(\mathbf{n}_A, \mathbf{n}_B)$ is a quasi-probability: Overlap $|\langle \mathbf{n}_A|\mathbf{n}_B\rangle|^2 \neq 0$
- Separability=Purely classical spins pointing at directions $\mathbf{n}_A, \mathbf{n}_B$

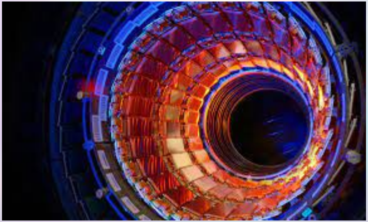
$$C_{ij} = \langle \sigma^i \otimes \sigma^j \rangle = \int \mathrm{d}\Omega_A \mathrm{d}\Omega_B \; P(\mathbf{n}_A, \mathbf{n}_B) n_A^i n_B^j$$

- Entanglement=NO positive P-representation $\rightarrow$ Genuine non-classical!

Quantum High-Energy Colliders?

- Standard Model is a Relativistic Quantum Field Theory = Special Relativity + Quantum Mechanics.
- Naively, Quantum Correlations should be easily studied in colliders...right? → Not so fast!
  - Momentum measurement → Decoherence
  - Lack of control of internal d.o.f. in initial state → Decoherence
  - Most relevant observables in colliders: cross-sections, lifetimes. . . → Classical probabilistic objects
  - Even measuring quantum interference in colliders is challenging: PRD 105, 096012 (2022)



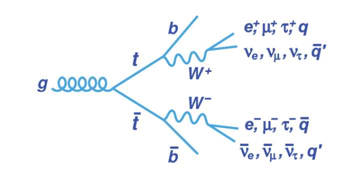


Quantum Tomography: Two qubits, two tops

- **Quantum Tomography**: Reconstruction of quantum state from measurement of a set of observables.
- Two-qubits $\rightarrow$ Quantum tomography $=$ Measurement of spin polarizations and spin correlations.
- Top quarks: Spin polarizations $\mathbf{B}^{\pm}$ and spin correlation matrix $\mathbf{C}$ extracted from cross-section $\sigma_{\ell\bar{\ell}}$ of dileptonic decay

$$\frac{1}{\sigma_{\ell\bar{\ell}}}\frac{d\sigma_{\ell\bar{\ell}}}{d\Omega_{+}d\Omega_{-}}=\frac{1}{(4\pi)^2}\left[1+\mathbf{B}^{+}\cdot\hat{\ell}_{+}-\mathbf{B}^{-}\cdot\hat{\ell}_{-}-\hat{\ell}_{+}\cdot\mathbf{C}\cdot\hat{\ell}_{-}\right]$$

- $\hat{\ell}_{\pm}$: lepton directions in each top (antitop) rest frames.



$t\bar{t}$ Quantum state

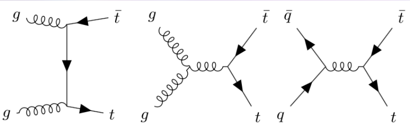
- $t\bar{t}$ production from most elementary QCD processes:

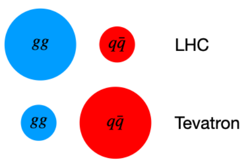
$$\begin{aligned} q+\bar{q} &\rightarrow t+\bar{t},\ q=u,d\ldots \\ g+g &\rightarrow t+\bar{t} \end{aligned}$$

- Each initial state $I = q\bar{q}, gg \rightarrow t\bar{t}$ quantum state $\rho^I(M_{t\bar{t}}, \hat{k})$
- LHC $\rightarrow$ Total quantum state: *Incoherent* mixture of $I = q\bar{q}, gg$ processes with probability w_I

$$\rho(M_{t\bar{t}}, \hat{k}) = \sum_{I=q\bar{q},gg} w_I(M_{t\bar{t}})\rho^I(M_{t\bar{t}}, \hat{k})$$

- QCD Input: $w_I(M_{t\bar{t}}), \rho^I(M_{t\bar{t}}, \hat{k}) \rightarrow$ QI Output: Textbook problem of *convex sum* of quantum states!





$t\bar{t}$ Quantum Correlations

- Quantum state $\rho(M_{t\bar{t}}, \hat{k})$: Function of scattering angle Θ and $M_{t\bar{t}}$.
- Two main regions of quantumness:
  - Ultrarelativistic high-p_T for both $q\bar{q}$ and gg (spin triplet)
  - Threshold for gg (spin singlet).

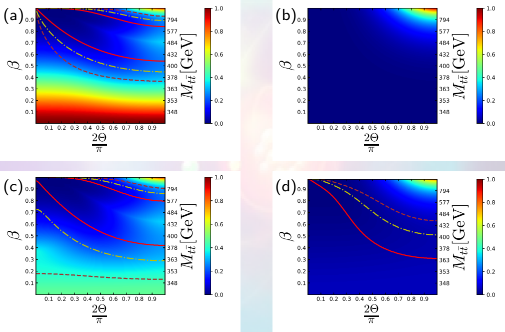
- Colorbar: Discord.
- Solid, dashed-dotted, dashed: Boundaries of Entanglement, Steering, Bell Nonlocality $\rightarrow$ Hierarchy!

a) $gg \rightarrow t\bar{t}$

b) $q\bar{q} \rightarrow t\bar{t}$

c) Run 2 LHC $\sqrt{s} = 13$ TeV

d) Tevatron $\sqrt{s} = 1.96$ TeV



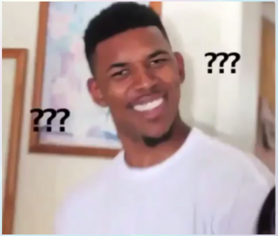
Cauchy-Schwarz violation

- Simple criterion of entanglement: Cauchy-Schwarz violation

$$|\mathrm{tr}\ \mathbf{C}| = |\langle \boldsymbol{\sigma}_+ \cdot \boldsymbol{\sigma}_- \rangle| = \left| \int \mathrm{d}\Omega_A \mathrm{d}\Omega_B \ P(\mathbf{n}_A, \mathbf{n}_B) \mathbf{n}_A \cdot \mathbf{n}_B \right|$$

$$\leq \int \mathrm{d}\Omega_A \mathrm{d}\Omega_B \ P(\mathbf{n}_A, \mathbf{n}_B) \left| \mathbf{n}_A \cdot \mathbf{n}_B \right| \leq \int \mathrm{d}\Omega_A \mathrm{d}\Omega_B \ P(\mathbf{n}_A, \mathbf{n}_B) = 1$$

- $D = \frac{\mathrm{tr}\ \mathbf{C}}{3} < -1/3 \rightarrow$ Violation of Cauchy-Schwarz inequality = Entanglement
- Wait a minute...Average of a cosine larger than one???
- D=Quantum observable with a genuine quantum range of values $-1 < D < -1/3$



Steering ellipsoid

- Normalized dileptonic cross-section $\rightarrow$ Angular distribution:

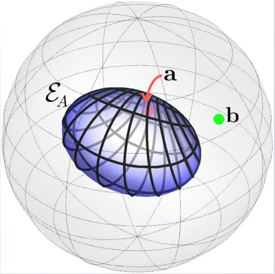
$$p(\hat{\ell}_+,\hat{\ell}_-)=\frac{1}{\sigma_{\ell\bar{\ell}}}\frac{\mathrm{d}\sigma_{\ell\bar{\ell}}}{\mathrm{d}\Omega_+\mathrm{d}\Omega_-}=\frac{1+\mathbf{B}^+\cdot\hat{\ell}_+-\mathbf{B}^-\cdot\hat{\ell}_--\hat{\ell}_+\cdot\mathbf{C}\cdot\hat{\ell}_-}{(4\pi)^2}$$

- Conditional quantum states:

$$\rho^{(\pm)}_{\hat{\mathbf{n}}}=\frac{1+\mathbf{B}^{\pm}_{\hat{\mathbf{n}}}\cdot\boldsymbol{\sigma}_{\pm}}{2},\quad \mathbf{B}^{\pm}_{\hat{\mathbf{n}}}=\frac{\mathbf{B}^{\pm}+\mathbf{C}^{\pm}\cdot\hat{\mathbf{n}}}{1+\mathbf{B}^{\mp}\cdot\hat{\mathbf{n}}},\quad \mathbf{C}^+=\mathbf{C},\ \mathbf{C}^-=\mathbf{C}^T$$

- Direct conditional quantum tomography:

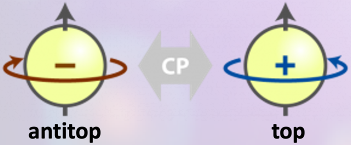
$$p(\hat{\ell}_\pm|\hat{\ell}_\mp=\mp\hat{\mathbf{n}})=\frac{p(\hat{\ell}_\pm,\hat{\ell}_\mp=\mp\hat{\mathbf{n}})}{p(\hat{\ell}_\mp=\mp\hat{\mathbf{n}})}=\frac{1\pm\mathbf{B}^{\pm}_{\hat{\mathbf{n}}}\cdot\hat{\ell}_\pm}{4\pi}$$



- Discord $\rightarrow$ Minimization over conditional entropies.
- $\mathbf{B}^{\pm}_{\hat{\mathbf{n}}}$ $\rightarrow$ Steering ellipsoid. PRL 113, 020402 (2014)
- Highly-challenging measurements in conventional setups $\rightarrow$ Natural implementation in colliders!

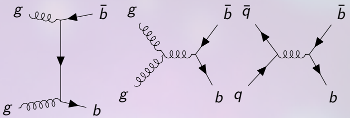
New Physics Witnesses

- Approximate *CP*-invariance of Standard Model $\rightarrow$ $\mathbf{C} = \mathbf{C}^{\mathrm{T}}$, $\mathbf{B}^{+} = \mathbf{B}^{-}$ $\rightarrow$ Symmetric discord and steering ellipsoids!
- Therefore: Discord and/or Steering asymmetry $\rightarrow$ New Physics!
- New physics witnesses: Symmetry protected observables by SM, only non-zero for New Physics:
  - $\Delta\mathcal{D}_{t\bar{t}} \equiv \mathcal{D}_t - \mathcal{D}_{\bar{t}}$
  - Asymmetries in ellipsoid centers and/or semiaxes.
- No SM contribution to New Physics witnesses!



Quantum Information with $b\bar{b}$

- *Mutatis mutandis*: Quantum information with $b\bar{b}$!
- $b\bar{b}$ quantum tomography: $\Lambda_b(udb), \bar{\Lambda}_b(\bar{u}\bar{d}\bar{b})$ decays retain $b\bar{b}$ spin information Y. Kats, D. Uzan, JHEP 03 (2024) 063.
- Experimentally challenging $\longleftrightarrow$ Theoretically interesting:
  - Spin correlations in $b\bar{b}$ not measured yet $\rightarrow$ Uncharted territory!
  - Ultrarelativistic $b\bar{b}$ at LHC
  - ATLAS, CMS and also LHCb can play the game!
  - Paves the way to study quantum correlations in hadronizing systems $\rightarrow$ Quark-Gluon Plasma STAR Collaboration, Nature 548, 62 (2017).



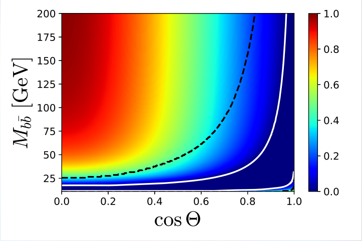
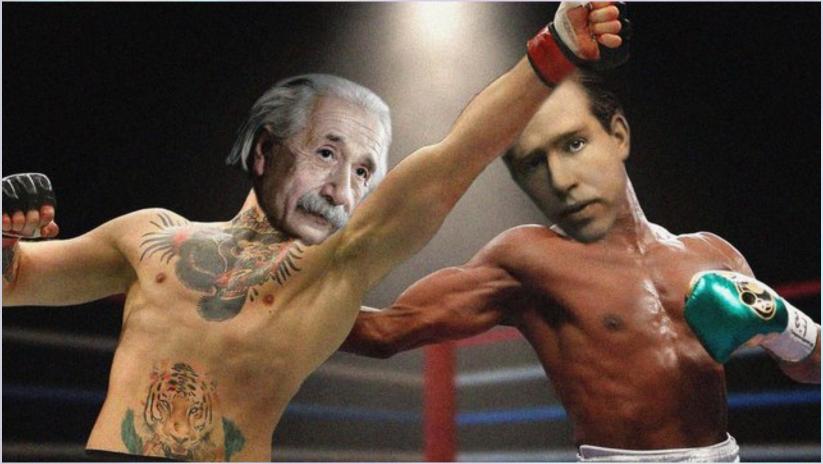


Figure: $b\bar{b}$ concurrence. Work in progress: Y. Afik, Y. Kats, JRMdN, A. Soffer, D. Uzan.

Conclusions and outlook

- Quantum Information theory $\longleftrightarrow$ High-Energy Physics. Interdisciplinary, huge potential and great interest!
- QI perspective:
  1. Highest-energy observation of entanglement ever!
  2. Genuinely relativistic, exotic symmetries and interactions, fundamental nature $\rightarrow$ Frontier of known Physics!
  3. Highly-demanding measurements naturally implemented at LHC.
- HEP perspective:
  1. Quantum Tomography: Novel experimental tool.
  2. QI techniques can inspire new approaches for searching New Physics: PRD 106, 055007 (2022), JHEP 148, (2023), EPJC 83, 162 (2023)
- Already first measurements of $t\bar{t}$ entanglement by ATLAS and CMS (see previous talks). Highest-energy entanglement ever!
- Many more to come!
  1. Qubits: b (In progress), τ (EPJC 83, 162 (2023))
  2. Qutrits: $W^{\pm}$(PLB 825, 136866 (2022)), Z^0(PRD 107, 016012 (2023)).
- Extension to e^+e^- colliders: Spin of initial state can be controlled!
  $\rightarrow$ Manipulation of qubits? Quantum gates?

Thank You



Backup

Quantum Discord: Two qubits

- How do we translate classical into quantum?

Quantum Discord: Two qubits

- How do we translate classical into quantum?
- Shannon entropy $\rightarrow$ Von Neumann entropy ($p_n \geq 0$, ρ eigenvalues)

$$H(A,B) \quad \rightarrow \quad H(\rho) = -\sum_n p_n \log_2 p_n$$

$$H(A) \quad \rightarrow \quad H(\rho_A),\ H(B) \rightarrow H(\rho_B),\ \rho_{A,B} = \mathrm{Tr}_{B,A}\rho$$

- Conditional probability $\rightarrow$ Conditional state $\rho_{A|B}$ = One-qubit state after Bob's spin measurement along $\hat{\mathbf{n}}$:

$$H(A|B) = p_{\hat{\mathbf{n}}} H(\rho_{\hat{\mathbf{n}}}) + p_{-\hat{\mathbf{n}}} H(\rho_{-\hat{\mathbf{n}}})$$

$$\rho_{\hat{\mathbf{n}}} = \frac{\Pi^B_{\hat{\mathbf{n}}} \rho \Pi^B_{\hat{\mathbf{n}}}}{p_{\hat{\mathbf{n}}}} = \frac{1 + \mathbf{B}^+_{\hat{\mathbf{n}}} \cdot \sigma}{2},\ \mathbf{B}^+_{\hat{\mathbf{n}}} = \frac{\mathbf{B}^+ + \mathbf{C} \cdot \hat{\mathbf{n}}}{1 + \hat{\mathbf{n}} \cdot \mathbf{B}^-},\ p_{\hat{\mathbf{n}}} = \frac{1 + \hat{\mathbf{n}} \cdot \mathbf{B}^-}{2}$$

- Genuine quantumness $\rightarrow$ Minimization over all spin directions to exclude quantization effects:

$$\mathcal{D}(A,B) = H(\rho_B) - H(\rho) + \min_{\hat{\mathbf{n}}} p_{\hat{\mathbf{n}}} H(\rho_{\hat{\mathbf{n}}}) + p_{-\hat{\mathbf{n}}} H(\rho_{-\hat{\mathbf{n}}}) \neq 0$$

Steering: Two qubits

- Steering: Original conception of Schrödinger of EPR paradox=Quantum Mechanics+Locality → Only well-defined in 2007! (PRL 98, 140402 (2007))
- Alice post-measurement state described by local-hidden states:

$$\tilde{\rho}_{\hat{\mathbf{n}}} = \Pi^B_{\hat{\mathbf{n}}} \rho \Pi^B_{\hat{\mathbf{n}}} = \int d\lambda \; p(1|\hat{\mathbf{n}}\lambda) p(\lambda) \rho_B(\lambda)$$

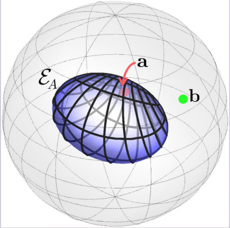
- If not → Bob can "steer" quantum state of Alice → **Steering**.

Steering Ellipsoid

- Alice post-measurement state: same as for quantum discord.

$$\rho_{\hat{\mathbf{n}}} = \frac{\tilde{\rho}_{\hat{\mathbf{n}}}}{\mathrm{Tr}\tilde{\rho}_{\hat{\mathbf{n}}}} = \frac{1 + \mathbf{B}^{+}_{\hat{\mathbf{n}}} \cdot \sigma}{2}, \quad \mathbf{B}^{+}_{\hat{\mathbf{n}}} = \frac{\mathbf{B}^{+} + \mathbf{C} \cdot \hat{\mathbf{n}}}{1 + \hat{\mathbf{n}} \cdot \mathbf{B}^{-}}$$

- Set of conditional polarizations $\mathbf{B}^{+}_{\hat{\mathbf{n}}}$ describes an ellipsoid.
- Steering ellipsoid: Fundamental QI object, containing all information about the system.
- Similar for Bob $\rightarrow$ Steering: also asymmetric between Alice and Bob.



PRL 113, 020402 (2014)

Bell inequality: Two qubits

- Local realism: Joint Alice and Bob measurements M_A, M_B accounted by local hidden-variable model

$$p(a, b|M_A M_B) = \int d\lambda \; p(a|M_A\lambda)p(b|M_B\lambda)p(\lambda)$$

- Local realism holds if Bell inequality is satisfied. Two qubits → **CHSH inequality** ($\mathbf{a}_i, \mathbf{b}_i$ spin axes of measurements M_A, M_B)

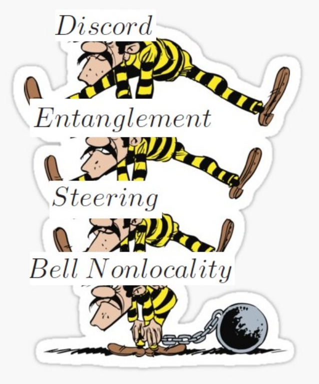
$$|\mathbf{a}_1^{\mathrm{T}}\mathbf{C}\,(\mathbf{b}_1 - \mathbf{b}_2) + \mathbf{a}_2^{\mathrm{T}}\mathbf{C}\,(\mathbf{b}_1 + \mathbf{b}_2)\,| \leq 2$$

- Stronger condition than entanglement → "Spooky action at distance"

Hierarchy of Quantum Correlations

- Steering and Discord can be asymmetric between Alice and Bob.
- Bell Nonlocality and Entanglement are always symmetric.
- Quantum Hierarchy:

$$\textit{Bell Nonlocality} \subset \textit{Steering} \subset \textit{Entanglement} \subset \textit{Discord}$$



Top pair Quantum State

- How to translate HEP features to Quantum Information language?
- $t\bar{t}$ spins described by production spin density matrix $R(M_{t\bar{t}}, \hat{k})$:

$$R = \tilde{A} + \sum_i \left(\tilde{B}_i^+ \sigma^i + \tilde{B}_i^- \bar{\sigma}^i \right) + \sum_{i,j} \tilde{C}_{ij} \sigma^i \bar{\sigma}^j$$

- Quantum state in experiment: Momentum measurements+Average over events $\rightarrow$ Genuine density-matrix description!
- Proper spin density matrix $\rho(M_{t\bar{t}}, \hat{k}) = \dfrac{R(M_{t\bar{t}}, \hat{k})}{\mathrm{tr}\left[R(M_{t\bar{t}}, \hat{k})\right]}$