Quantum Information with Top Quarks at the LHC

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Quantum Theory: Quantization

- Quantum Mechanics was originally named after observation of quantized values in several systems:
  - Electromagnetic radiation (Black-body/Photoelectric effect)
  - Electron orbits (Atomic spectra)
  - Angular momentum (Stern-Gerlach)
Copenhagen interpretation of Quantum Mechanics:

- Particles described in terms of waves $\Psi \rightarrow$ Superposition of quantum states.
- Outcomes of measurements: Observable eigenvalues $\rightarrow$ Quantization.
- Probabilities of outcomes encoded in $|\Psi|^2 \rightarrow$ Quantum Interference.
- Wave-function collapses by measurement process: Quantum state projected to measured states.
Quantum vs. Classical

- Quantum Mechanics: Particles are in superposition of states → Probabilistic description of measurements.
- Classical Mechanics can also describe random outputs using classical probability distributions (noise, experimental variations...).
- Is God *just* playing dice with the Universe? →
Quantum vs. Classical

- Quantum Mechanics: Particles are in superposition of states → Probabilistic description of measurements.
- Classical Mechanics can also describe random outputs using classical probability distributions (noise, experimental variations...).
- Is God just playing dice with the Universe? → God is well beyond a mere croupier!
- Quantum Correlations=Correlations not accounted by classical probabilistic theories.
Quantum State

- **Pure state** → Wave function $|\Psi\rangle$
  1. $|\Psi\rangle = \sum_n \alpha_n \cdot |\phi_n\rangle$, $\langle \Psi | \Psi \rangle = \sum_n |\alpha_n|^2 = 1$
  2. Coherent mixture of quantum states → $\alpha_n$ are amplitudes
  3. Expectation values: $\langle A \rangle = \langle \Psi | A | \Psi \rangle = \sum_{n,m} \alpha_m^* \alpha_n \langle \phi_m | A | \phi_n \rangle$

- **Mixed state** → Generalization to density matrix $\rho$
  1. $\rho = \sum_n p_n \cdot |\phi_n\rangle \langle \phi_n|$, $\text{tr} \rho = \sum_n p_n = 1$
  2. Incoherent mixture of quantum states → $p_n$ are probabilities
  3. Expectation values: $\langle A \rangle = \text{tr}(\rho A) = \sum_n p_n \langle \phi_n | A | \phi_n \rangle$
Qubits

- Qubit: Two-level quantum system $|0\rangle, |1\rangle \rightarrow$ Most simple quantum system.
- General density matrix ($2 \times 2$) for 1 qubit $\rightarrow$ 3 parameters $B_i$:
  \[
  \rho = \frac{1 + \sum_i B_i \sigma^i}{2}
  \]
- Two qubits $\rightarrow$ Most simple example of quantum correlations.
- General density matrix ($4 \times 4$) for 2 qubits $\rightarrow$ 15 parameters $B_i^\pm, C_{ij}$
  \[
  \rho = \frac{1 + \sum_i (B_i^+ \sigma^i \otimes 1 + B_i^- 1 \otimes \sigma^i) + \sum_{i,j} C_{ij} \sigma^i \otimes \sigma^j}{4}
  \]
Quantum Discord

Classically, two equivalent expressions for mutual information of bipartite system A and B (Alice and Bob):

\[ I(A, B) = H(A) + H(B) - H(A, B) = H(A) - H(A|B) \]

\[ H(A, B) = -\sum_{x,y} p(x, y) \log_2 p(x, y) \]

\[ H(A|B) = \sum_{y} p(y)H(A|B = y) \]

Quantum mechanics can introduce a “discord” between both expressions:

\[ D(A, B) \equiv H(B) - H(A, B) + H(A|B) \neq 0 \]

Most basic form of quantum correlations!

Quantum Discord is asymmetric

\[ D(A, B) \neq D(B, A) \]
Quantum Discord: Classical states

- OK...but where is the Physics here? → Only classical states have zero discord!
  \[ \rho_{\text{class}} = \sum_{n,m} p_{n,m} |n\rangle \otimes |m\rangle \langle n| \otimes \langle m| \]
  - \( |n\rangle, |m\rangle \) form an orthonormal basis for \( A, B \)
  - \( p_{n,m} \) are purely classical probabilities of being in state \( |n\rangle \otimes |m\rangle \)!
  - Two qubits → Tails and heads with two spin directions!

\[
\begin{align*}
\rho_{\text{class}} &= p_{++} |n_A\rangle \otimes |n_B\rangle \langle n_A| \otimes \langle n_B| \\
&+ p_{+-} |n_A\rangle \otimes |-n_B\rangle \langle n_A| \otimes \langle -n_B| \\
&+ p_{-+} |-n_A\rangle \otimes |n_B\rangle \langle -n_A| \otimes \langle n_B| \\
&+ p_{--} |-n_A\rangle \otimes |-n_B\rangle \langle -n_A| \otimes \langle -n_B|
\end{align*}
\]
Entanglement

What if we generalize the previous idea? → Separability:

\[ \rho_{\text{sep}} = \sum_{n,m} p_{n,m} |n\rangle \otimes |m\rangle \langle n| \otimes \langle m| = \sum_k p_k \rho_k^{(A)} \otimes \rho_k^{(B)} \]

- \(|n\rangle, |m\rangle\) not necessarily orthonormal now → \(p_{n,m}\) are quasi-probabilities (not disjoint events)
- Any classically correlated state (classical probability) is separable.
- **Entanglement**: Non-separability of a bipartite quantum state.
Two qubits: Separability = Positive $P$-representation $P(n_A, n_B) \geq 0$:

$$
\rho = \int d\Omega_A d\Omega_B P(n_A, n_B) |n_A n_B\rangle \langle n_A n_B|,
\int d\Omega_A d\Omega_B P(n_A, n_B) = 1
$$

$P(n_A, n_B)$ is a quasi-probability: Overlap $|\langle n_A |n_B\rangle|^2 \neq 0$

Separability = Purely classical spins pointing at directions $n_A, n_B$

$$
C_{ij} = \langle \sigma^i \otimes \sigma^j \rangle = \int d\Omega_A d\Omega_B P(n_A, n_B) n_A^i n_B^j
$$

Entanglement = NO positive $P$-representation $\rightarrow$ Genuine non-classical!
Quantum High-Energy Colliders?

- Standard Model is a Relativistic Quantum Field Theory = Special Relativity + Quantum Mechanics.
- Naively, Quantum Correlations should be easily studied in colliders...right? → Not so fast!
  - Momentum measurement → Decoherence
  - Lack of control of internal d.o.f. in initial state → Decoherence
  - Most relevant observables in colliders: cross-sections, lifetimes... → Classical probabilistic objects
  - Even measuring quantum interference in colliders is challenging: PRD 105, 096012 (2022)
Quantum Tomography: Two qubits, two tops

- **Quantum Tomography**: Reconstruction of quantum state from measurement of a set of observables.
- Two-qubits $\rightarrow$ Quantum tomography $=$ Measurement of spin polarizations and spin correlations.
- Top quarks: Spin polarizations $B^\pm$ and spin correlation matrix $C$ extracted from cross-section $\sigma_{\ell\bar{\ell}}$ of dileptonic decay

$$\frac{1}{\sigma_{\ell\bar{\ell}}} \frac{d\sigma_{\ell\bar{\ell}}}{d\Omega_+ d\Omega_-} = \frac{1}{(4\pi)^2} \left[ 1 + B^+ \cdot \hat{l}_+ - B^- \cdot \hat{l}_- - \hat{l}_+ \cdot C \cdot \hat{l}_- \right]$$

- $\hat{l}_{\pm}$: lepton directions in each top (antitop) rest frames.
$t\bar{t}$ Quantum state

- $t\bar{t}$ production from most elementary QCD processes:
  
  $q + \bar{q} \rightarrow t + \bar{t}, \ q = u, d...$

  $g + g \rightarrow t + \bar{t}$

- Each initial state $I = q\bar{q}, gg \rightarrow t\bar{t}$ quantum state $\rho^I(M_{t\bar{t}}, \hat{k})$

- LHC → Total quantum state: *Incoherent* mixture of $I = q\bar{q}, gg$ processes with probability $w_I$

  $$\rho(M_{t\bar{t}}, \hat{k}) = \sum_{I=q\bar{q},gg} w_I(M_{t\bar{t}}) \rho^I(M_{t\bar{t}}, \hat{k})$$

- QCD Input: $w_I(M_{t\bar{t}}), \rho^I(M_{t\bar{t}}, \hat{k}) \rightarrow$ QI Output: Textbook problem of *convex sum* of quantum states!
Quantum state $\rho(M_{t\bar{t}}, \hat{k})$: Function of scattering angle $\Theta$ and $M_{t\bar{t}}$.

Two main regions of quantumness:
- Ultrarelativistic high-$p_T$ for both $q\bar{q}$ and $gg$ (spin triplet)
- Threshold for $gg$ (spin singlet).

Colorbar: Discord.

Solid, dashed-dotted, dashed: Boundaries of Entanglement, Steering, Bell Nonlocality $\rightarrow$ Hierarchy!

a) $gg \rightarrow t\bar{t}$
b) $q\bar{q} \rightarrow t\bar{t}$
c) Run 2 LHC $\sqrt{s} = 13$ TeV
d) Tevatron $\sqrt{s} = 1.96$ TeV
Cauchy-Schwarz violation

- Simple criterion of entanglement: Cauchy-Schwarz violation

\[ |\text{tr } C| = |\langle \sigma_+ \cdot \sigma_- \rangle| = \left| \int d\Omega_A d\Omega_B \ P(n_A, n_B) n_A \cdot n_B \right| \]

\[ \leq \int d\Omega_A d\Omega_B \ P(n_A, n_B) |n_A \cdot n_B| \leq \int d\Omega_A d\Omega_B \ P(n_A, n_B) = 1 \]

- \( D = \frac{\text{tr } C}{3} < -1/3 \rightarrow \text{Violation of Cauchy-Schwarz inequality} = \text{Entanglement} \)

- Wait a minute...Average of a cosine larger than one???

- \( D = \text{Quantum observable with a genuine quantum range of values} \)

\( -1 < D < -1/3 \)
Steering ellipsoid

- Normalized dileptonic cross-section → Angular distribution:
  \[
p(\hat{l}_+, \hat{l}_-) = \frac{1}{\sigma_{\ell\ell}} \frac{d\sigma_{\ell\ell}}{d\Omega_+ d\Omega_-} = \frac{1 + B^+ \cdot \hat{l}_+ - B^- \cdot \hat{l}_- - \hat{l}_+ \cdot C \cdot \hat{l}_-}{(4\pi)^2}
\]

- Conditional quantum states:
  \[
  \rho_{\hat{n}}^{(\pm)} = \frac{1 + B^\pm_{\hat{n}} \cdot \sigma^\pm}{2}, \quad B^\pm_{\hat{n}} = \frac{B^\pm + C^\pm \cdot \hat{n}}{1 + B^\mp \cdot \hat{n}}, \quad C^+ = C, \quad C^- = C^T
  \]

- Direct conditional quantum tomography:
  \[
p(\hat{l}_\pm | \hat{l}_\mp = \mp \hat{n}) = \frac{p(\hat{l}_\pm, \hat{l}_\mp = \mp \hat{n})}{p(\hat{l}_\mp = \mp \hat{n})} = \frac{1 \pm B^\pm_{\hat{n}} \cdot \hat{l}_\pm}{4\pi}
  \]

- Discord → Minimization over conditional entropies.
- \( B^\pm_{\hat{n}} \) → Steering ellipsoid. PRL 113, 020402 (2014)

- Highly-challenging measurements in conventional setups → Natural implementation in colliders!
New Physics Witnesses

- Approximate $CP$-invariance of Standard Model $\rightarrow C = C^T$, $B^+ = B^-$ $\rightarrow$ Symmetric discord and steering ellipsoids!
- Therefore: Discord and/or Steering asymmetry $\rightarrow$ New Physics!
- New physics witnesses: Symmetry protected observables by SM, only non-zero for New Physics:
  - $\Delta D_{t\bar{t}} \equiv D_t - D_{\bar{t}}$
  - Asymmetries in ellipsoid centers and/or semi-axes.
- No SM contribution to New Physics witnesses!
Quantum Information with $b\bar{b}$

- *Mutatis mutandis*: Quantum information with $b\bar{b}$!
- $b\bar{b}$ quantum tomography: $\Lambda_b(udb), \bar{\Lambda}_b(\bar{u}\bar{d}\bar{b})$ decays retain $b\bar{b}$ spin information Y. Kats, D. Uzan, JHEP 03 (2024) 063.

Experimentally challenging $\leftrightarrow$ Theoretically interesting:
- Spin correlations in $b\bar{b}$ not measured yet $\rightarrow$ Uncharted territory!
- Ultrarelativistic $b\bar{b}$ at LHC
- ATLAS, CMS and also LHCb can play the game!

**Figure**: $b\bar{b}$ concurrence. Work in progress: Y. Afik, Y. Kats, JRMdN, A. Soffer, D. Uzan.
Conclusions and outlook

- Quantum Information theory $\leftrightarrow$ High-Energy Physics. Interdisciplinary, huge potential and great interest!
- QI perspective:
  1. Highest-energy observation of entanglement ever!
  2. Genuinely relativistic, exotic symmetries and interactions, fundamental nature $\rightarrow$ Frontier of known Physics!
  3. Highly-demanding measurements naturally implemented at LHC.
- HEP perspective:
  1. Quantum Tomography: Novel experimental tool.
  2. QI techniques can inspire new approaches for searching New Physics:
  3. Already first measurements of $t\bar{t}$ entanglement by ATLAS and CMS (see previous talks). Highest-energy entanglement ever!
  4. Many more to come!
     1. Qubits: $b$ (In progress), $\tau$ (EPJC 83, 162 (2023))
     2. Qutrits: $W^{\pm}$ (PLB 825, 136866 (2022)), $Z^0$ (PRD 107, 016012 (2023)).
- Extension to $e^+e^-$ colliders: Spin of initial state can be controlled! $\rightarrow$ Manipulation of qubits? Quantum gates?
Thank You
Backup
Quantum Discord: Two qubits

- How do we translate classical into quantum?
Quantum Discord: Two qubits

- How do we translate classical into quantum?
- Shannon entropy $\rightarrow$ Von Neumann entropy ($p_n \geq 0$, $\rho$ eigenvalues)
  
  $$H(A, B) \rightarrow H(\rho) = -\sum_{n} p_n \log_2 p_n$$

  $$H(A) \rightarrow H(\rho_A), \ H(B) \rightarrow H(\rho_B), \ \rho_{A,B} = \text{Tr}_{B,A}\rho$$

- Conditional probability $\rightarrow$ Conditional state $\rho_{A|B} = \text{One-qubit state after Bob's spin measurement along } \hat{n}$:
  
  $$H(A|B) = p_{\hat{n}} H(\rho_{\hat{n}}) + p_{-\hat{n}} H(\rho_{-\hat{n}})$$

  $$\rho_{\hat{n}} = \frac{\prod_{\hat{n}}^B \rho \prod_{\hat{n}}^B}{p_{\hat{n}}} = \frac{1 + \hat{B}_{\hat{n}} \cdot \sigma}{2}, \ \hat{B}_{\hat{n}} = \frac{B^+ + C \cdot \hat{n}}{1 + \hat{n} \cdot B^-}, \ p_{\hat{n}} = \frac{1 + \hat{n} \cdot B^-}{2}$$

- Genuine quantumness $\rightarrow$ Minimization over all spin directions to exclude quantization effects:

  $$\mathcal{D}(A, B) = H(\rho_B) - H(\rho) + \min_{\hat{n}} p_{\hat{n}} H(\rho_{\hat{n}}) + p_{-\hat{n}} H(\rho_{-\hat{n}}) \neq 0$$
Steering: Two qubits

- Steering: Original conception of Schrödinger of EPR paradox = Quantum Mechanics + Locality $\rightarrow$ Only well-defined in 2007! (PRL 98, 140402 (2007))

- Alice post-measurement state described by local-hidden states:

$$\tilde{\rho}_n = \prod^B_n \rho \prod^B_n = \int d\lambda \ p(1|\hat{n}\lambda)p(\lambda)\rho_B(\lambda)$$

- If not $\rightarrow$ Bob can “steer” quantum state of Alice $\rightarrow$ **Steering**.
Steering Ellipsoid

- Alice post-measurement state: same as for quantum discord.
  \[ \rho_n = \frac{\tilde{\rho}_n}{\text{Tr}\tilde{\rho}_n} = \frac{1 + B^+_n \cdot \sigma}{2}, \quad B^+_n = \frac{B^+ + C \cdot \hat{n}}{1 + \hat{n} \cdot B^-} \]

- Set of conditional polarizations \( B^+_n \) describes an ellipsoid.

- Steering ellipsoid: Fundamental QI object, containing all information about the system.

- Similar for Bob → Steering: also asymmetric between Alice and Bob.

PRL 113, 020402 (2014)
Bell inequality: Two qubits

- Local realism: Joint Alice and Bob measurements $M_A, M_B$ accounted by local hidden-variable model

$$p(a, b | M_A M_B) = \int d\lambda \ p(a | M_A \lambda)p(b | M_B \lambda)p(\lambda)$$

- Local realism holds if Bell inequality is satisfied. Two qubits $\rightarrow$ **CHSH inequality** ($a_i, b_i$ spin axes of measurements $M_A, M_B$)

$$|a_1^T C (b_1 - b_2) + a_2^T C (b_1 + b_2)| \leq 2$$

- Stronger condition than entanglement $\rightarrow$ "Spooky action at distance"
Hierarchy of Quantum Correlations

- Steering and Discord can be asymmetric between Alice and Bob.
- Bell Nonlocality and Entanglement are always symmetric.
- Quantum Hierarchy:

\[
\text{Bell Nonlocality} \subset \text{Steering} \subset \text{Entanglement} \subset \text{Discord}
\]
How to translate HEP features to Quantum Information language?

$t\bar{t}$ spins described by production spin density matrix $R(M_{t\bar{t}}, \hat{k})$:

$$R = \tilde{A} + \sum_i \left( \tilde{B}_i^+ \sigma^i + \tilde{B}_i^- \bar{\sigma}^i \right) + \sum_{i,j} \tilde{C}_{ij} \sigma^i \bar{\sigma}^j$$

Quantum state in experiment: Momentum measurements + Average over events → Genuine density-matrix description!

Proper spin density matrix $\rho(M_{t\bar{t}}, \hat{k}) = \frac{R(M_{t\bar{t}}, \hat{k})}{\text{tr} \left[ R(M_{t\bar{t}}, \hat{k}) \right]}$