Lattice QCD perspectives on T_{bb}, T_{bc} and T_{cc} tetraquarks

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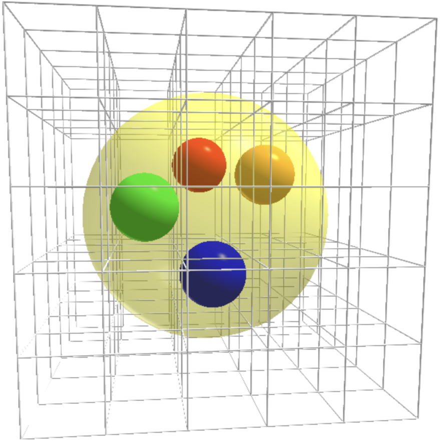

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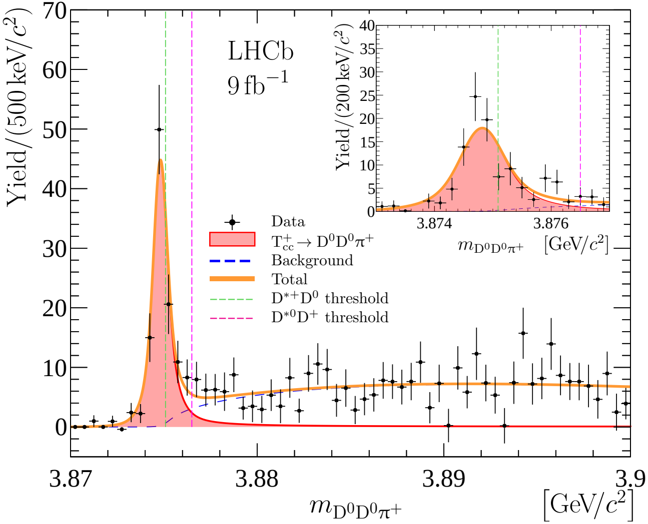
Experimental observation of T_{cc}



Aaij et al. [LHCb], Nature Phys.18,751(2022); arXiv:2109.01038

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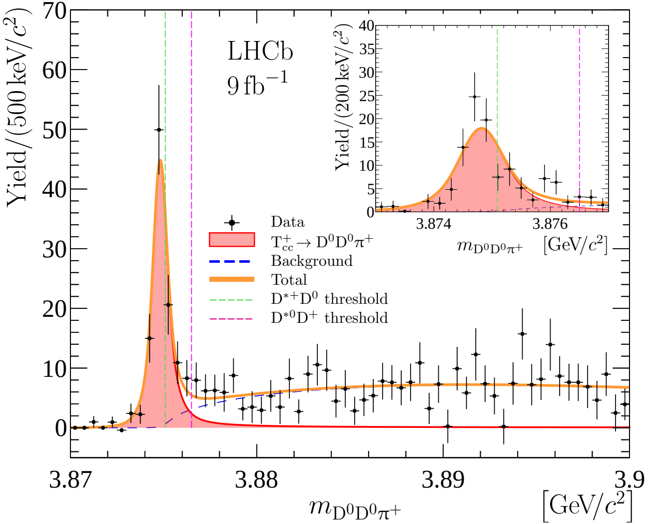

- T_{cc} is slightly below threshold:

$$\delta m \sim \frac{1}{4} \text{ MeV}$$

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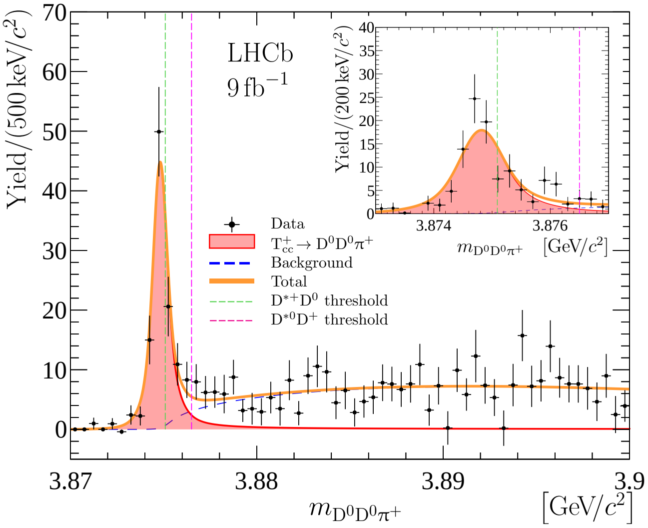

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- Predictions varied widely:

$$-300\ \text{MeV} \lesssim \delta m \lesssim 300\ \text{MeV}$$

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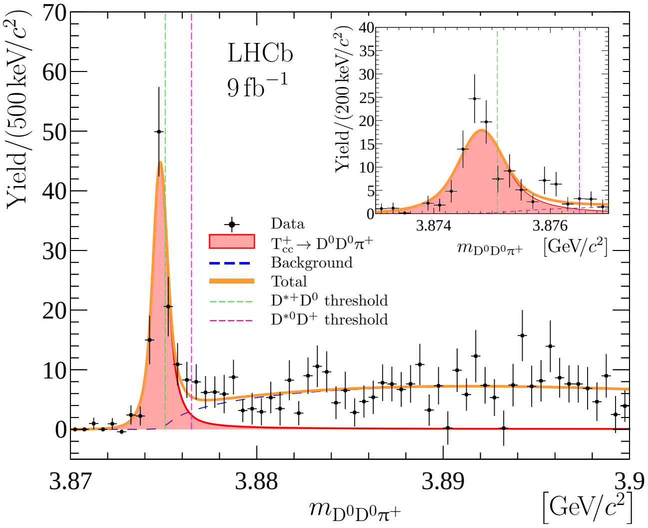

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- Directly from lattice QCD?

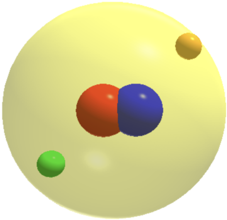
T_{cc} is an extreme challenge

T_{bb} and T_{bc} are more accessible

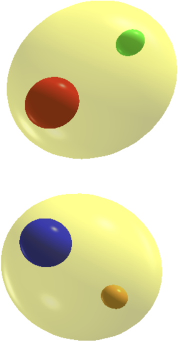
Aaij et al. [LHCb], Nature Phys.18,751(2022); arXiv:2109.01038

Intuition for a very heavy diquark

tetraquark



pair of mesons



Binding energies involving light quarks are $\sim \Lambda_{QCD}$.

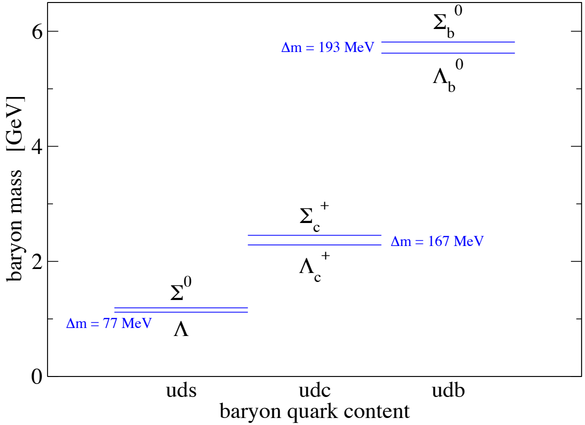
The heavy diquark is compact and has binding energy $\sim \alpha_s m_Q \sim \dfrac{m_Q}{\ln(m_Q)}$.

Therefore, for $m_Q \to \infty$, the tetraquark is a stable particle in QCD.

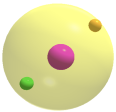
Intuition for a good light diquark

Recall some standard heavy baryons.

Each Λ_Q is more deeply bound than its Σ_Q partner, especially as $m_Q \to \infty$.



baryon

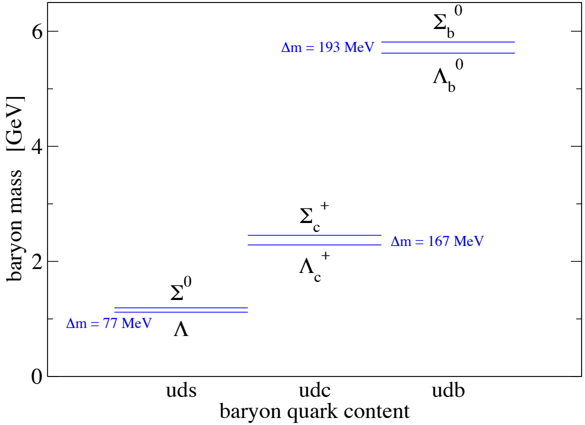


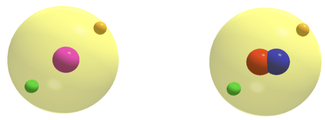
Intuition for a good light diquark

Recall some standard heavy baryons.

Each Λ_Q is more deeply bound than its Σ_Q partner, especially as $m_Q \to \infty$.

A tetraquark can have this same Λ-type light diquark.





In both hadrons, the light diquark sees a heavy color triplet.

Tetraquark quantum numbers

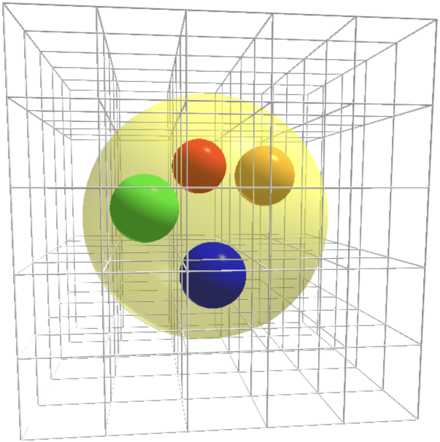
The good light diquark, as found in Λ_Q, has $J^P = 0^-$ and $I = 0$.
Any pair from u, d, s can be considered.

For bb or cc, the heavy diquark needs $J^P = 1^-$.
For bc, the heavy diquark can have $J^P = 0^-$ or 1^-.

Tetraquark candidates for deepest binding are therefore

- $J^P = 1^+$ for T_{bb} and T_{cc},
- $J^P = 0^+$ or 1^+ for T_{bc}.

Roles for lattice QCD



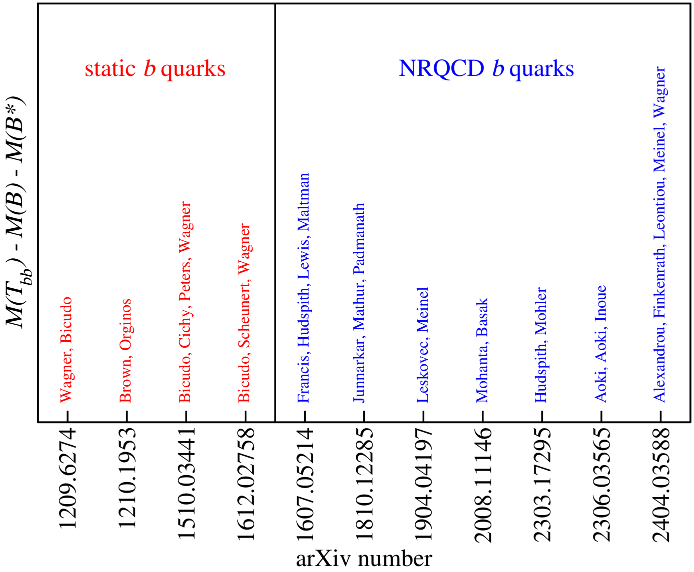
Computing directly from QCD is the goal.

Extrapolations are required in lattice spacing, volume and some quark masses.

Choices made by different authors give valuable insight into systematic effects.

A consensus from the lattice community will provide confidence for tetraquark physics.

T_{bb} binding energy for $ud\bar{b}\bar{b}$ from lattice QCD



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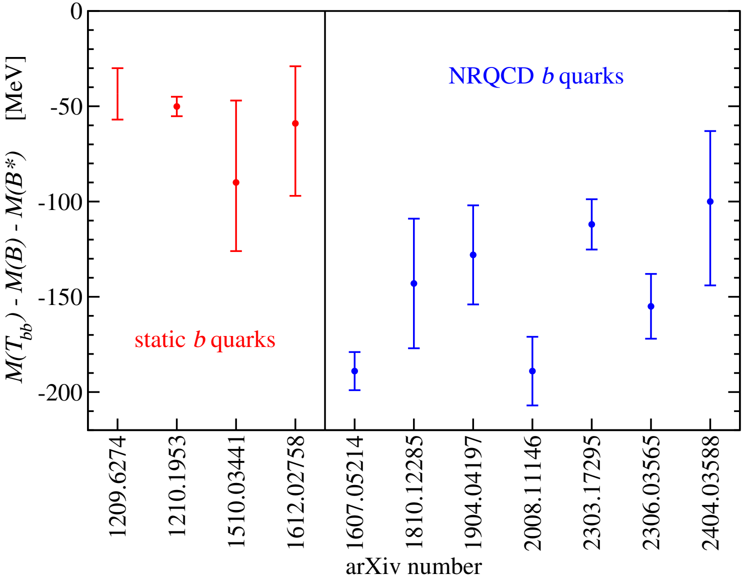

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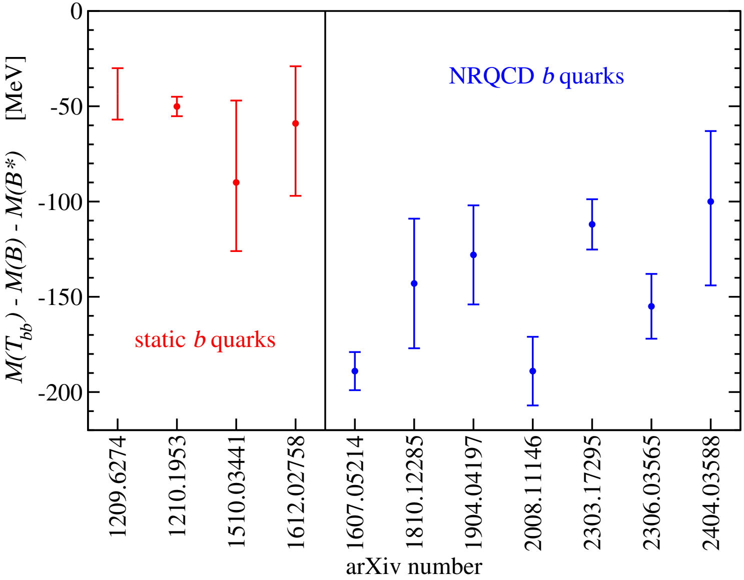
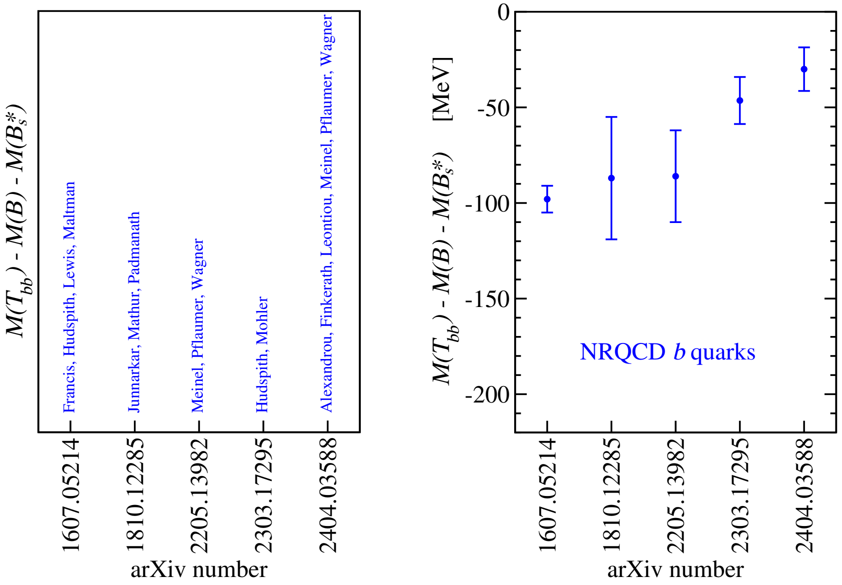


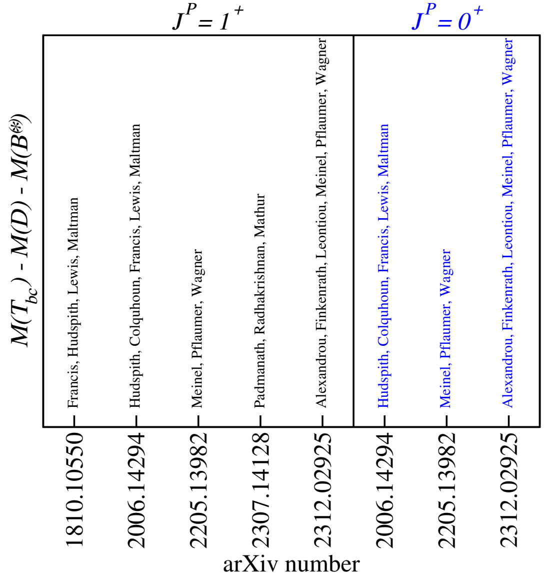
Systematic effects:

- chosen operator set (local, nonlocal, smeared, meson pair, diquark pair, scattering states)
- tuning of NRQCD
- extrapolations (lattice spacing, volume, quark masses)
- chosen lattice action (Wilson, HISQ, overlap, domain wall)

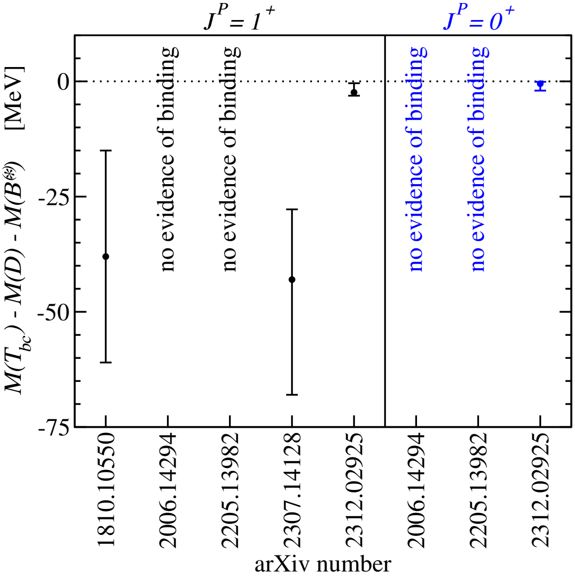
T_{bb} binding energy for $us\bar{b}\bar{b}$ from lattice QCD



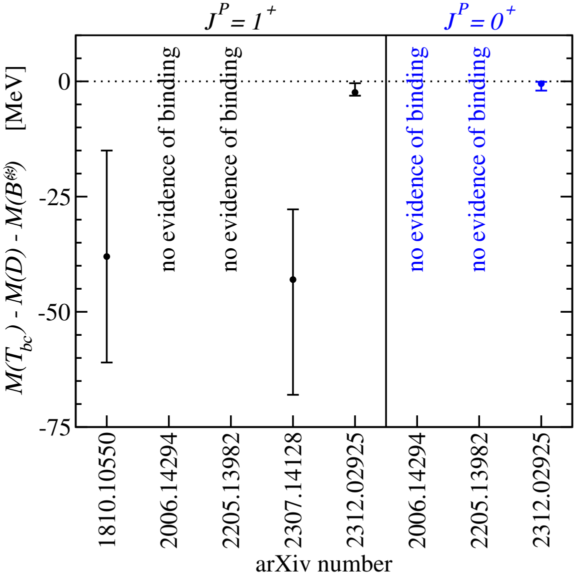
T_{bc} binding energies for $ud\bar{c}\bar{b}$ from lattice QCD



T_{bc} binding energies for $ud\bar{c}\bar{b}$ from lattice QCD



T_{bc} binding energies for $ud\bar{c}\bar{b}$ from lattice QCD



Additional systematics:

- larger threshold effects
- relativistic charm quark

Operators used for the most recent T_{bc} lattice study

Alexandrou, Finkenrath, Leontiou, Meinel, Pflaumer, Wagner, PhysRevLett132,151902 = 2312.02925

For $J^P = 0^+$

$$\sum_{\vec{x}} \left[\bar{b}^a(\vec{x})\gamma_5 C\bar{c}^{b,T}(\vec{x})\right] \left[u^{a,T}(\vec{x})C\gamma_5 d^b(\vec{x})\right] - (d \leftrightarrow u)$$

$$\sum_{\vec{x}} B^+(\vec{x})D^-(\vec{x}) - \sum_{\vec{x}} B^0(\vec{x})\bar{D}^0(\vec{x})$$

$$\sum_{\vec{x},j} B_j^{*+}(\vec{x})D_j^{*-}(\vec{x}) - \sum_{\vec{x},j} B_j^{*0}(\vec{x})\bar{D}_j^{*0}(\vec{x})$$

$$B^+(\vec{q})D^-(-\vec{q}) - B^0(\vec{q})\bar{D}^0(-\vec{q})$$

for $\frac{|\vec{q}|L}{2\pi} \in \{0, 1, \sqrt{2}, \sqrt{3}\}$

For $J^P = 1^+$

$$\sum_{\vec{x}} \left[\bar{b}^a(\vec{x})\gamma_j C\bar{c}^{b,T}(\vec{x})\right] \left[u^{a,T}(\vec{x})C\gamma_5 d^b(\vec{x})\right] - (d \leftrightarrow u)$$

$$\sum_{\vec{x}} B^{*+}(\vec{x})D^-(\vec{x}) - \sum_{\vec{x}} B^{*0}(\vec{x})\bar{D}^0(\vec{x})$$

$$\sum_{\vec{x}} B^+(\vec{x})D^{*-}(\vec{x}) - \sum_{\vec{x}} B^0(\vec{x})\bar{D}^{*0}(\vec{x})$$

$$\sum_{\vec{x}} \epsilon_{jkl} B_j^{*+}(\vec{x})D_k^{*-}(\vec{x}) - \sum_{\vec{x}} B_j^{*0}(\vec{x})\bar{D}_k^{*0}(\vec{x})$$

$$B_j^{*+}(\vec{q})D^-(-\vec{q}) - B_j^{*0}(\vec{q})\bar{D}^0(-\vec{q})$$

for $\frac{|\vec{q}|L}{2\pi} \in \{0, 1, 1_\perp, \sqrt{2}\}$

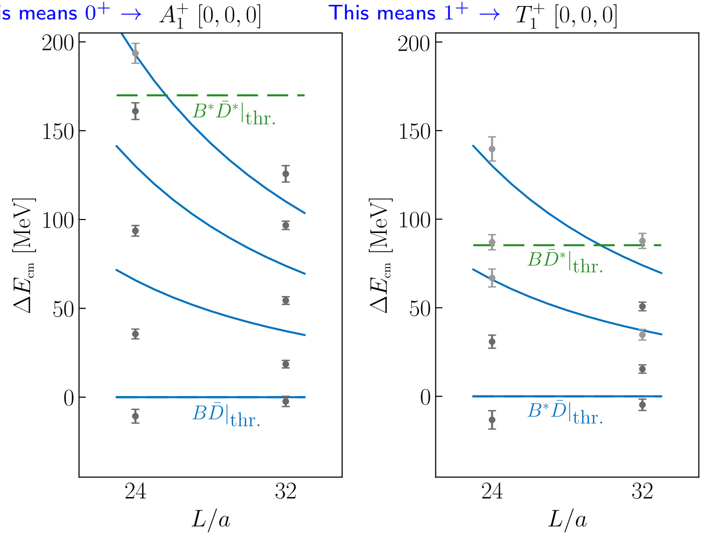
where

$$B^+(x) = \bar{b}(\vec{x})\gamma_5 u(\vec{x}), \qquad B^+(\vec{q}) = \frac{1}{\sqrt{V}} \sum_{\vec{x}} B^+(x) e^{2\pi i \vec{q}\cdot\vec{x}/L}, \qquad \text{etc.}$$

Finite-volume energies from the most recent T_{bc} lattice study

Alexandrou, Finkenrath, Leontiou, Meinel, Pflaumer, Wagner, PhysRevLett132,151902 = 2312.02925

This means $0^+ \to$ This means $1^+ \to$



Curves are free $B\bar{D}$ energy levels.

Lüscher's method converts volume dependence into scattering amplitudes (to handle resonances properly).

This study finds T_{bc}:

0^+ at $-0.5^{+0.4}_{-1.5}$ MeV,

1^+ at $-2.4^{+2.0}_{-0.7}$ MeV.

Toward the T_{cc} binding energy from lattice QCD

An early attempt:

1810.12285	Junnarkar, Mathur, Padmanath	-23 ± 11 MeV

Recent studies:

(DD^* scattering via Luscher's method or the HALQCD method)

2202.10110	Padmanath, Prelovsek	$-9.9^{+3.6}_{-7.1}$ MeV
2206.06185	Chen, Shi, Chen, Gong, Liu, Sun, Zhang	$I = 0$ attractive but $I = 1$ repulsive
2302.04505	Lyu, Aoki, Doi, Hatsuda, Ikeda, Meng	$-59 \left(^{+53}_{-99}\right) \left(^{+2}_{-67}\right)$ keV
2402.14715	Collins, Nefediev, Padmanath, Prelovsek	"a very delicate fine tuning" is observed
2405.15741	Whyte, Wilson, Thomas	-41 ± 31 MeV (virtual bound state)

Top priority now:

Physical pion masses will be necessary because pion exchange contributions are significant.

Note: All existing lattice studies of T_{cc} used $m_\pi > m_{D^*} - m_D$, meaning D^* mesons are stable.

Conclusions

Lattice methods provide systematically improvable results directly from QCD.

There is clearly a bound T_{bb}. Lattice results exist for $ud\bar{b}\bar{b}$ and $us\bar{b}\bar{b}$.
The various systematic errors are being determined through multiple studies.

T_{bc} does not yet have a consensus among lattice groups.
Several studies exist and real progress has been made.

T_{cc} is a challenge for lattice QCD because binding energy $\sim \frac{1}{4}$ MeV $\ll \Lambda_{QCD}$.
Nevertheless, impressive efforts have been reported.