A MODEL OF PENTAQUARKS

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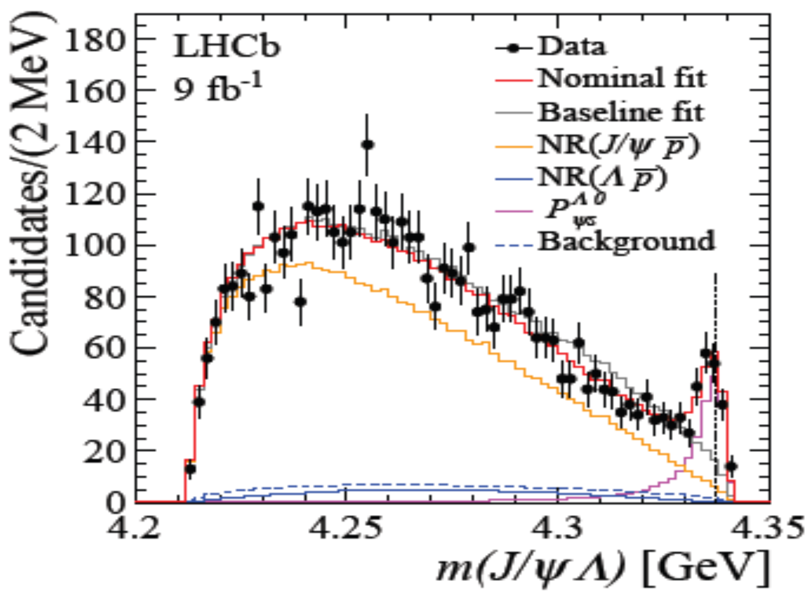
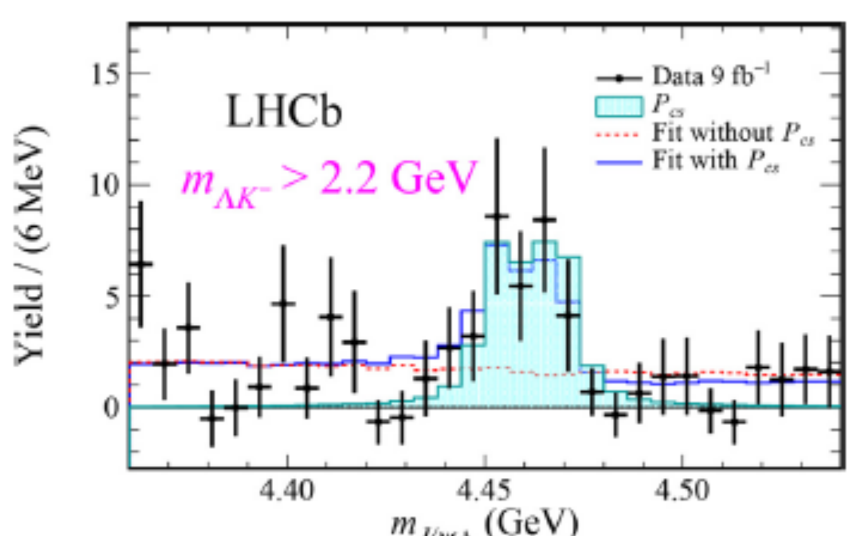
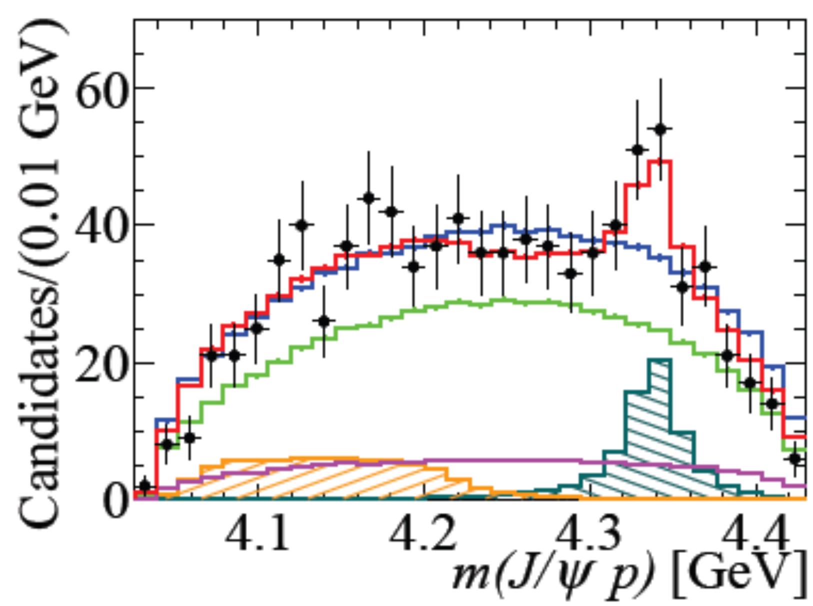
e-Print: 2403.04068 [hep-ph] with D. Germani and F. Niliani

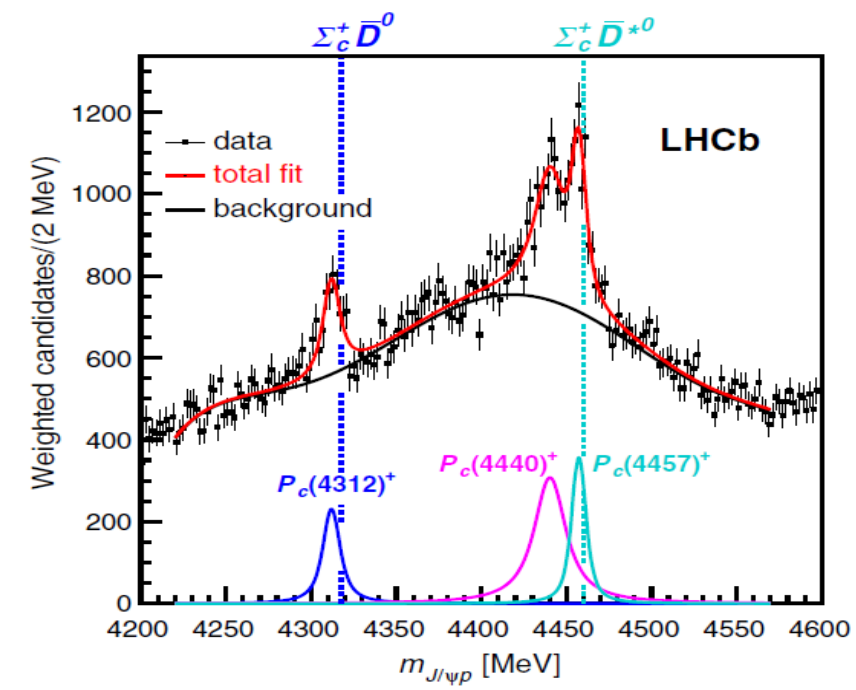
Current Experimental Situation



State	Mass [MeV]	Width [MeV]	Observed Process	Year
$P_c(4312)$	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+3.7}_{-4.5}$	$\Lambda^0_b \to (J/\psi\, p)\, K^-$	2019
$\widetilde{P}_c(4337)$	$4337^{+7}_{-4}{}^{+2}_{-2}$	$29^{+26}_{-12}{}^{+14}_{-14}$	$B^0_s \to (J/\psi p)\, \overline{p}$	2022
$P_c(4440)$	$4440.3 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+8.7}_{-10.1}$	$\Lambda^0_b \to (J/\psi\, p)\, K^-$	2019
$P_c(4457)$	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+5.7}_{-1.9}$	$\Lambda^0_b \to (J/\psi\, p)\, K^-$	2019
$\widetilde{P}_{cs}(4338)^{\frac{1}{2}^-}$	$4338.2 \pm 0.7 \pm 0.4$	$7.0 \pm 1.2 \pm 1.3$	$B^- \to (J/\psi\, \Lambda)\, \overline{p}$	2022
$P_{cs}(4459)$	$4458.9 \pm 2.9^{+4.7}_{-1.1}$	$17.3 \pm 6.5^{8.0}_{-5.7}$	$\Xi^-_b \to (J/\psi\, \Lambda)\, K^-$	2021







R. Aaij et al. arXiv:1904.03947 [hep-ex] (2019). R. Aaij et al. arXiv:2012.10380v2 [hep-ex] (2021). R. Aaij et al. arXiv:2210.10346 [hep-ex] (2021). R. Aaij et

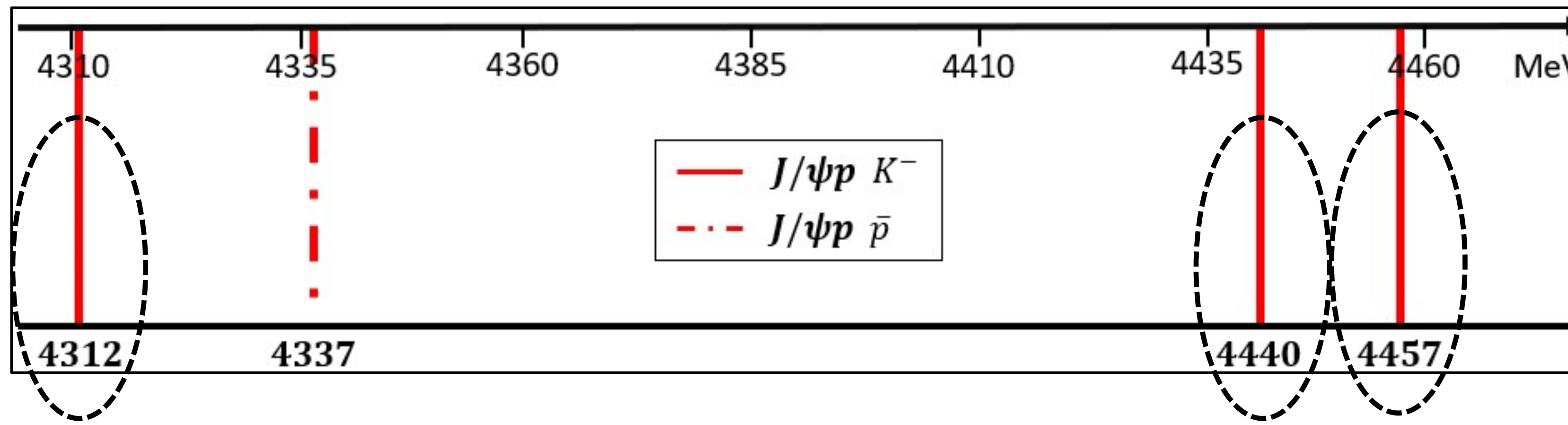
Slide by D. Germani

K^- and $\bar{p}$ associated production

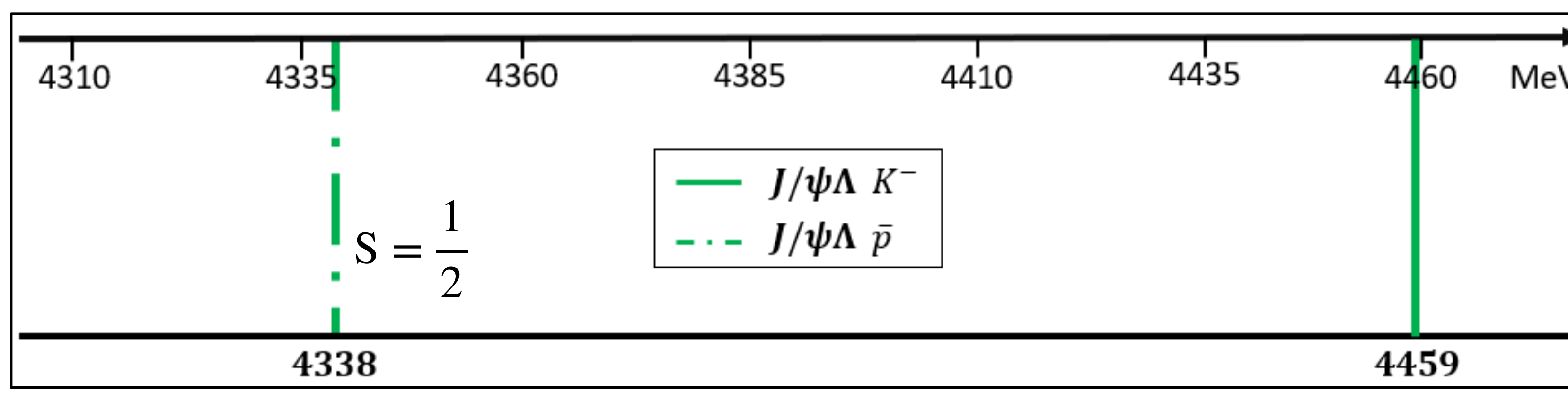
- We divide the spectrum according to strangeness content.
- Data suggests two different type of production: in association with a K^- or with an $\bar{p}$.
- Pentaquark seems to appear in triplet

Can we build a model to account for these properties?

$P_c^+\ [c\bar{c}uud]$



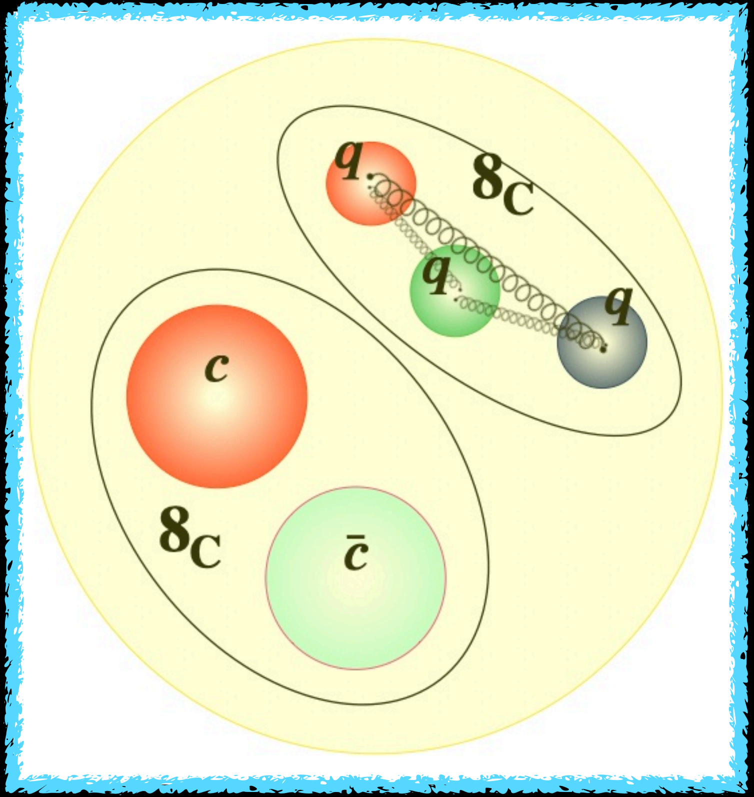
$P_{cs}^0\ [c\bar{c}uds]$



Slide by D. Germani

BARYO-CHARMONIUM

Require Fermi Statistics for the three identical light quarks.
The *energy values* of these three quarks depend on their *total spin:* this is *exchange interaction* (not there for spins higher than $1/2$).



Under *exchange* the spin wf $\psi_{\rm spin}$ of two particles gets $(-1)^{2s+S}$.
Statistics requires $\Psi(=\psi_{\rm spin}\,\psi_{\rm coord})$ to get $(-1)^{2s}$ so that
$\psi_{\rm coord.} \sim (-1)^S \Rightarrow L$ is even(odd) if S is even(odd).

FERMI STATISTICS

If we include color-flavor: $\Psi = \psi_{\text{col}-\text{flav}}\, \psi_{\text{spin}}\, \psi_{\text{coord}}$

Ψ	$\psi_{\text{col}-\text{flav}}$	ψ_{coord}	ψ_{spin}
A	S	S	A
A	S	A	S
A	A	A	A
A	A	S	S

On the use of Fermi statistics for pentaquarks see
L. Maiani, ADP, V. Riquer Eur. Phys. J. C83 (2023) 5, 378

EXCHANGE INTERACTION

$$\psi_{\text{coord}}(\mathbf{r}_1, \mathbf{r}_2) = \frac{1}{\sqrt{2}} \left(\phi_1(\mathbf{r}_1)\phi_2(\mathbf{r}_2) \pm \phi_1(\mathbf{r}_2)\phi_2(\mathbf{r}_1) \right)$$

$$\langle U \rangle_{\psi_{\text{coord}}} = \int \psi^*_{\text{coord}}(\mathbf{r}_1, \mathbf{r}_2)\, U(\mathbf{r}_1 - \mathbf{r}_2)\, \psi_{\text{coord}}(\mathbf{r}_1, \mathbf{r}_2)\, d^3 r_1\, d^3 r_2$$

$$\langle U \rangle_{\psi_{\text{coord}}} = C \pm J$$

Color-flavor symmetric case:

Symmetric in coordinates i.e. $+ \Rightarrow S = 0$ in a pair of fermions ($S = 0{,}1$)
$\Rightarrow S(S+1) - 1 = -1 \Rightarrow$

$$+J = \langle V \rangle_{\psi_{\text{spin}}} = \left\langle -\sum_{\text{pairs}} J_{ab} \underbrace{\left(\frac{1}{2} + 2\mathbf{S}_a \cdot \mathbf{S}_b \right)}_{S(S+1)-1} \right\rangle$$

EXCHANGE INTERACTION

Coordinates anti-symm. i.e. $-$ $\Rightarrow \boldsymbol{S} = \mathbf{1}$ in a pair of fermions ($\boldsymbol{S} = \mathbf{0,1}$)

$\Rightarrow S(S+1) - 1 = +1 \Rightarrow$

$$-J = \langle V \rangle_{\psi_{\text{spin}}} = \left\langle -\sum_{\text{pairs}} J_{ab} \underbrace{\left(\frac{1}{2} + 2\mathbf{S}_a \cdot \mathbf{S}_b\right)}_{S(S+1)-1} \right\rangle$$

$$J_{ab} = \int (\phi_a(\mathbf{r}_a)\phi_b(\mathbf{r}_b))^* \, U(\mathbf{r}_a - \mathbf{r}_b) \, (\phi_a(\mathbf{r}_b)\phi_b(\mathbf{r}_a)) \, d^3 r_a \, d^3 r_b$$

Determine the exchange splitting of the energy levels of a system of three quarks (treating the interaction as a perturbation)

$$\Delta E_{1/2} = \pm \sqrt{J_{12}^2 + J_{13}^2 + J_{23}^2 - J_{12}J_{13} - J_{12}J_{23} - J_{13}J_{23}}$$

$$\Delta E_{3/2} = -J_{12} - J_{13} - J_{23}$$

COLOR-FLAVOR

In the case the color-flavor is anti-symmetric, change the sign in the definition of the exchange potential V.

Let a, b, c be flavor indices and α, β, γ color indices. Requiring both color and flavor to be in the adjoint representation we can form the tensor

$$A^{abc}_{\alpha\beta\gamma} = \psi^{[a}_{\alpha}\psi^{b]}_{\beta}\psi^{c}_{\gamma} - \psi^{[a}_{\beta}\psi^{b]}_{\gamma}\psi^{c}_{\alpha} = \psi^{a}_{(\alpha}\psi^{b}_{\beta)}\psi^{c}_{\gamma} - \psi^{a}_{(\beta}\psi^{b}_{\gamma)}\psi^{c}_{\alpha}$$

anti-symmetric under the exchange of any two quarks provided that

$$\psi^{a}_{\alpha}\psi^{b}_{\beta} = -\psi^{b}_{\beta}\psi^{a}_{\alpha}$$

COLOR-FLAVOR

Similarly we can form the symmetric color-flavor tensor

$$S^{abc}_{\alpha\beta\gamma} = \eta^{[a}_{\alpha}\eta^{b]}_{\beta}\eta^{c}_{\gamma} - \eta^{[a}_{\beta}\eta^{b]}_{\gamma}\eta^{c}_{\alpha} = \eta^{a}_{[\alpha}\eta^{b}_{\beta]}\eta^{c}_{\gamma} - \eta^{a}_{[\beta}\eta^{b}_{\gamma]}\eta^{c}_{\alpha}$$

This is symmetric under the exchange of any two quarks, provided that

$$\eta^{a}_{\alpha}\eta^{b}_{\beta} = +\,\eta^{b}_{\beta}\eta^{a}_{\alpha}$$

This allows to make the baryon octet: assume that tensor $\boldsymbol{S}$ represents the flavor octet with spin indices $\boldsymbol{\alpha, \beta, \gamma}$. Assign additional color indices and anti-symmetrize in color

$$\eta^{ai}_{\alpha}\eta^{bj}_{\beta} = \eta^{bi}_{\beta}\eta^{aj}_{\alpha} = -\,\eta^{bj}_{\beta}\eta^{ai}_{\alpha}$$

so that we can use $\boldsymbol{\eta \to \psi}$. Spin-flavor (S) color (A)

$$\mathcal{S}^{abc}_{\alpha\beta\gamma} = \epsilon_{ijk}(\psi^{ai}_{[\alpha}\psi^{bj}_{\beta]}\psi^{ck}_{\gamma} + \psi^{ai}_{[\gamma}\psi^{bj}_{\beta]}\psi^{ck}_{\alpha}) = B^{acb}_{\alpha\gamma\beta}$$

P AND $\widetilde{P}$ PENTAQUARKS

We assign

$$S^{abc}_{\alpha\beta\gamma} \longrightarrow P$$

for $(J/\psi\, p)$ or $(J/\psi\, \Lambda)$ pentaquarks produced in association with a kaon K^- and

$$A^{abc}_{\alpha\beta\gamma} \longrightarrow \widetilde{P}$$

$(J/\psi\, p)$ or $(J/\psi\, \Lambda)$ pentaquarks produced in association with an anti-proton $\bar{p}$; in this case change the sign in the definition of the exchange potential V.

P PENTAQUARKS OF THE $(J/\psi\, p)$ KIND

Let us consider the case $qqq = uud$ in the color-flavor configuration S.

$$S^{121}_{\alpha\beta\gamma} = u_{[\alpha}d_{\beta]}u_\gamma + u_{[\gamma}d_{\beta]}u_\alpha$$

since the uu cannot be anti-symmetric in flavor space.
The pentaquark contains the light quarks u_α, d_β and u_γ with the u quarks, u_α and $u_{\gamma'}$ in a color symmetric (repulsive) representation and ud in color attractive, anti-symmetric pairings.
Therefore we require

$$J^{uu}_S = -\frac{1}{2}J^{ud}_A \qquad (C_6 - 2C_3) = -1/2(C_3 - 2C_3)$$

$$J_{12} = J^{uu}_S > 0 \qquad J_{13} = J_{23} = J^{ud}_A < 0$$

P PENTAQUARKS OF THE $(J/\psi\, p)$ KIND

Assume that the ordering in mass corresponds to the lower one being spin $1/2$ and the higher two being $3/2$ and $1/2$ respectively. Then we have to solve the simultaneous equations

$$M_{P_c}(4457) - M_{P_c}(4312) = 2\,|\,J_S^{uu} - J_A^{ud}\,|$$

$$M_{P_c}(4440) - \frac{1}{2}(M_{P_c}(4312) + M_{P_c}(4457)) = -\,J_S^{uu} - 2J_A^{ud}$$

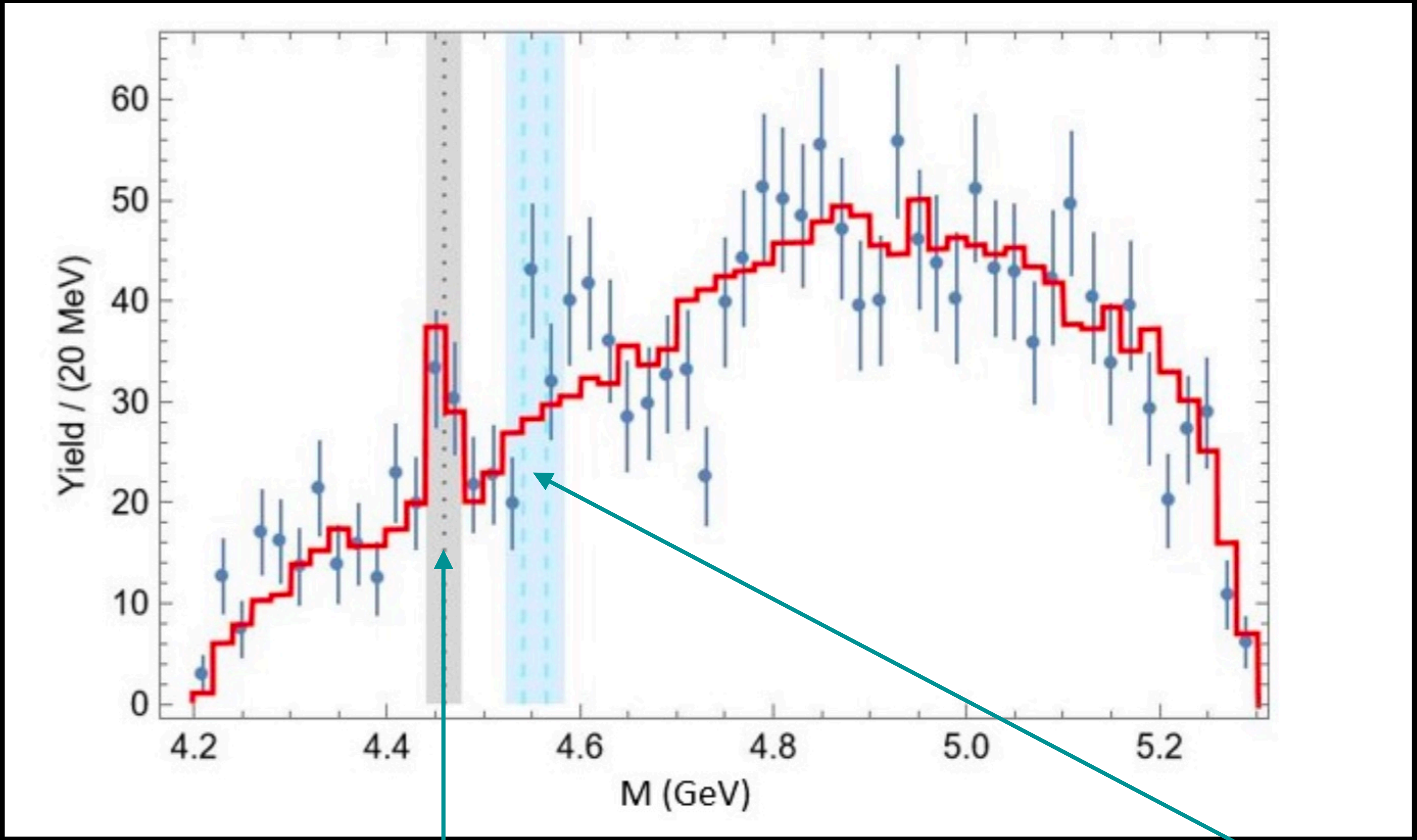
A solution can be found

$J_S^{uu} = 29.9^{+2.5}_{-2.8}$ MeV $\qquad$ $J_A^{ud} = -\,42.8^{+2.4}_{-1.6}$ MeV

$$\frac{J_S^{uu}}{J_A^{ud}} = -\,0.7 \pm 0.1$$

P PENTAQUARKS OF THE $(J/\psi\Lambda)$ KIND

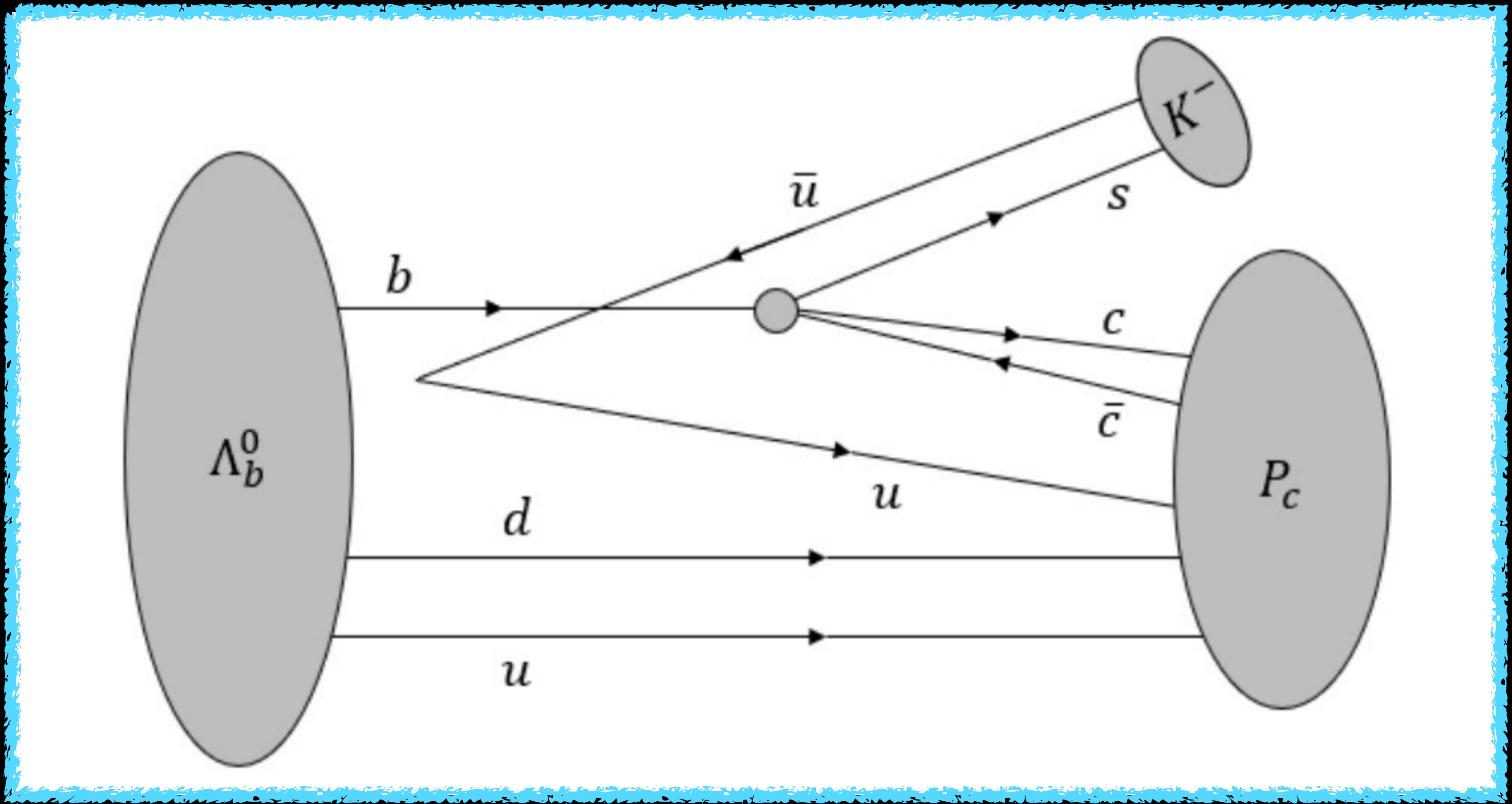
Using the same methods we can study the strange pentaquarks.
One has been observed and two more are predicted.



$P_{cs}(4459)$

Predictions: P'_{cs}, P''_{cs}

$\widetilde{P}$ PENTAQUARKS OF THE $(J/\psi\Lambda, p)$ KIND



Assuming that the color-flavor of the spectator ***ud*** pair is kept in the final state, the process $\Lambda_b^0 \to \tilde{P}_c K^-$ is expected to be suppressed, as suggested by observation: the $[ud]_0$ in Λ is in a CF-symmetric configuration as opposite to $\tilde{P}$ as described by

$$A^{udu}_{\alpha\beta\gamma} = u_{(\alpha} d_{\beta)} u_\gamma - u_{(\gamma} d_{\beta)} u_\alpha$$

P AND $\widetilde{P}$ PENTAQUARKS & CONCLUSIONS

The spin ordering **(1/2, 3/2, 1/2)** is a main prediction.

	Mass [MeV]		Mass [MeV]
$P_c(4312)$	$(4311.9^{+7}_{-0.9})$	$P_{cs}(4459)$	$(4458.8^{+6}_{-3.1})$
$P_c(4440)$	(4440.0^{+4}_{-5})	$\mathbf{P}'_{cs}$	4541 ± 6
$P_c(4457)$	$(4457.3^{+7}_{-1.8})$	$\mathbf{P}''_{cs}$	4565 ± 6
$\widetilde{\mathbf{P}}''_{\mathbf{c}}$	4187 ± 7	$\widetilde{P}_{cs}(4338)$	(4338.2 ± 0.8)
$\widetilde{\mathbf{P}}'_{\mathbf{c}}$	4276 ± 12	$\widetilde{\mathbf{P}}'_{\mathbf{cs}}$	4387 ± 4
$\widetilde{P}_c(4337)$	4332 ± 7 $(4337^{+7}_{-4}{}^{+2}_{-2})$	$\widetilde{\mathbf{P}}''_{\mathbf{cs}}$	4435 ± 4