A MODEL OF PENTAQUARKS

AD POLOSA, SAPIENZA UNIVERSITY OF ROME

e-Print: 2403.04068 [hep-ph] with D. Germani and F. Niliani
Current Experimental Situation

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<tbody>
<tr>
<td>$P_c(4312)$</td>
<td>$4311.9 \pm 0.7^{+6.8}_{-0.6}$</td>
<td>$9.8 \pm 2.7^{+3.7}_{-4.5}$</td>
<td>$\Lambda_b^0 \rightarrow (J/\psi p) K^-$</td>
<td>2019</td>
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<tr>
<td>$\bar{P}_c(4337)$</td>
<td>$4337^{+7}<em>{-4} +^{2}</em>{-2}$</td>
<td>$29^{+26}<em>{-12}^{+14}</em>{-14}$</td>
<td>$B_s^0 \rightarrow (J/\psi p) \bar{p}$</td>
<td>2022</td>
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<tr>
<td>$P_c(4440)$</td>
<td>$4440.3 \pm 1.3^{+4.7}_{-5.1}$</td>
<td>$20.6 \pm 4.9^{+8.7}_{-10.1}$</td>
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<td>2019</td>
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<tr>
<td>$P_c(4457)$</td>
<td>$4457.3 \pm 0.6^{+1.7}_{-1.1}$</td>
<td>$6.4 \pm 2.0^{+5.7}_{-1.9}$</td>
<td>$\Lambda_b^0 \rightarrow (J/\psi p) K^-$</td>
<td>2019</td>
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<tr>
<td>$P_{cs}(4338)\frac{1}{2}^-$</td>
<td>$4338.2 \pm 0.7 \pm 0.4$</td>
<td>$7.0 \pm 1.2 \pm 1.3$</td>
<td>$B^- \rightarrow (J/\psi \Lambda) \bar{p}$</td>
<td>2022</td>
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<tr>
<td>$P_{cs}(4459)$</td>
<td>$4458.9 \pm 2.9^{+4.7}_{-1.1}$</td>
<td>$17.3 \pm 6.5^{+8.0}_{-5.7}$</td>
<td>$\Xi^- \rightarrow (J/\psi \Lambda) K^-$</td>
<td>2021</td>
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$K^{-}$ and $\bar{p}$ associated production

- We divide the spectrum according to strangeness content.
- Data suggests two different types of production: in association with a $K^{-}$ or with an $\bar{p}$.
- Pentaquark seems to appear in triplet

Can we build a model to account for these properties?
BARYO-CHARMONIUM

Require Fermi Statistics for the three identical light quarks. The energy values of these three quarks depend on their total spin: this is exchange interaction (not there for spins higher than 1/2).

Under exchange the spin wf $\psi_{\text{spin}}$ of two particles gets $(-1)^{2s+S}$. Statistics requires $\Psi( = \psi_{\text{spin}} \psi_{\text{coord}})$ to get $(-1)^{2s}$ so that $\psi_{\text{coord}} \sim (-1)^S \Rightarrow L$ is even(odd) if $S$ is even(odd).
If we include color-flavor: \( \Psi = \psi_{\text{col-flav}} \psi_{\text{spin}} \psi_{\text{coord}} \)

<table>
<thead>
<tr>
<th>( \Psi )</th>
<th>( \psi_{\text{col-flav}} )</th>
<th>( \psi_{\text{coord}} )</th>
<th>( \psi_{\text{spin}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>S</td>
<td>S</td>
<td>A</td>
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<td>A</td>
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<td>A</td>
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On the use of Fermi statistics for pentaquarks see
**EXCHANGE INTERACTION**

\[ \psi_{\text{coord}}(\mathbf{r}_1, \mathbf{r}_2) = \frac{1}{\sqrt{2}} \left( \phi_1(\mathbf{r}_1)\phi_2(\mathbf{r}_2) \pm \phi_1(\mathbf{r}_2)\phi_2(\mathbf{r}_1) \right) \]

\[ \langle U \rangle_{\psi_{\text{coord}}} = \int \psi_{\text{coord}}^*(\mathbf{r}_1, \mathbf{r}_2) U(\mathbf{r}_1 - \mathbf{r}_2) \psi_{\text{coord}}(\mathbf{r}_1, \mathbf{r}_2) \, d^3r_1 \, d^3r_2 \]

\[ \langle U \rangle_{\psi_{\text{coord}}} = C \pm J \]

**Color-flavor symmetric case:**

Symmetric in coordinates i.e. \( + \Rightarrow S = 0 \) in a pair of fermions \( (S = 0,1) \)

\( \Rightarrow S(S + 1) - 1 = -1 \Rightarrow \)

\[ +J = \langle V \rangle_{\psi_{\text{spin}}} = \left\langle - \sum_{\text{pairs}} J_{ab} \left( \frac{1}{2} + 2S_a \cdot S_b \right) \right\rangle_{S(S+1)-1} \]
EXCHANGE INTERACTION

Coordinates anti-symm. i.e. \(-\Rightarrow S = 1\) in a pair of fermions \(S = 0,1\)

\[ S(S + 1) - 1 = +1 \Rightarrow \]

\[ -J = \langle V \rangle_{\psi_{\text{spin}}} = \left\langle - \sum_{\text{pairs}} J_{ab} \left( \frac{1}{2} + 2S_a \cdot S_b \right) \right\rangle_{S(S+1)-1} \]

\[ J_{ab} = \int (\phi_a(r_a)\phi_b(r_b))^* U(r_a - r_b) (\phi_a(r_b)\phi_b(r_a)) \, d^3r_a \, d^3r_b \]

Determine the exchange splitting of the energy levels of a system of three quarks (treating the interaction as a perturbation)

\[ \Delta E_{1/2} = \pm \sqrt{J_{12}^2 + J_{13}^2 + J_{23}^2 - J_{12}J_{13} - J_{12}J_{23} - J_{13}J_{23}} \]

\[ \Delta E_{3/2} = -J_{12} - J_{13} - J_{23} \]
COLOR-FLAVOR

In the case the color-flavor is anti-symmetric, change the sign in the definition of the exchange potential $V$.

Let $a, b, c$ be flavor indices and $\alpha, \beta, \gamma$ color indices. Requiring both color and flavor to be in the adjoint representation we can form the tensor

$$A_{\alpha \beta \gamma}^{abc} = \psi^a_{\alpha} \psi^b_{\beta} \psi^c_{\gamma} - \psi^a_{\beta} \psi^b_{\gamma} \psi^c_{\alpha} = \psi^{a}_{(\alpha} \psi^{b}_{\beta)} \psi^{c}_{\gamma} - \psi^{a}_{(\beta} \psi^{b}_{\gamma)} \psi^{c}_{\alpha}$$

anti-symmetric under the exchange of any two quarks provided that

$$\psi^a_{\alpha} \psi^b_{\beta} = - \psi^b_{\beta} \psi^a_{\alpha}$$
Similarly we can form the symmetric color-flavor tensor

\[ S^{abc}_{\alpha\beta\gamma} = \eta_{[a}^{\alpha} \eta_{\beta]}^{b} \eta_{\gamma]}^{c} - \eta_{[\alpha}^{a} \eta_{\beta]}^{b} \eta_{\gamma]}^{c} = \eta_{[\alpha}^{a} \eta_{\beta]}^{b} \eta_{\gamma]}^{c} - \eta_{[\beta}^{a} \eta_{\gamma]}^{b} \eta_{\alpha]}^{c} \]

This is symmetric under the exchange of any two quarks, provided that

\[ \eta_{\alpha}^{a} \eta_{\beta}^{b} = + \eta_{\beta}^{b} \eta_{\alpha}^{a} \]

This allows to make the baryon octet: assume that tensor \( S \) represents the flavor octet with spin indices \( \alpha, \beta, \gamma \). Assign additional color indices and anti-symmetrize in color

\[ \eta_{\alpha}^{ai} \eta_{\beta}^{bj} = \eta_{\beta}^{bi} \eta_{\alpha}^{aj} = - \eta_{\beta}^{bj} \eta_{\alpha}^{ai} \]

so that we can use \( \eta \rightarrow \psi \). Spin-flavor (S) color (A)

\[ S^{abc}_{\alpha\beta\gamma} = \epsilon_{ijk} (\psi_{[\alpha}^{ai} \psi_{\beta]}^{bj} \psi_{\gamma]}^{ck} + \psi_{[\gamma}^{ai} \psi_{\beta]}^{bj} \psi_{\alpha]}^{ck} ) = B^{acb}_{\alpha\beta\gamma} \]
We assign

\[ S_{\alpha\beta\gamma}^{abc} \rightarrow P \]

for \((J/\psi p)\) or \((J/\psi \Lambda)\) pentaquarks produced in association with a kaon \(K^-\) and

\[ A_{\alpha\beta\gamma}^{abc} \rightarrow \bar{P} \]

\((J/\psi p)\) or \((J/\psi \Lambda)\) pentaquarks produced in association with an anti-proton \(\bar{p}\); in this case change the sign in the definition of the exchange potential \(V\).
Let us consider the case $qqq = uud$ in the color-flavor configuration $S$.

$$S_{\alpha\beta\gamma}^{121} = u_{[\alpha d\beta]} u_\gamma + u_{[\gamma d\beta]} u_\alpha$$

since the $uu$ cannot be anti-symmetric in flavor space. The pentaquark contains the light quarks $u_\alpha, d_\beta$ and $u_\gamma$ with the $u$ quarks, $u_\alpha$ and $u_\gamma$, in a color symmetric (repulsive) representation and $ud$ in color attractive, anti-symmetric pairings. Therefore we require

$$J_{uu}^S = -\frac{1}{2} J_{ud}^A \quad (C_6 - 2C_3) = -1/2(C_3 - 2C_3)$$

$$J_{12} = J_{uu}^S > 0 \quad J_{13} = J_{23} = J_{ud}^A < 0$$
Assume that the ordering in mass corresponds to the lower one being spin $1/2$ and the higher two being $3/2$ and $1/2$ respectively. Then we have to solve the simultaneous equations

\[ M_P(4457) - M_P(4312) = 2 |J_{uu} - J_{ud}| \]

\[ M_P(4440) - \frac{1}{2}(M_P(4312) + M_P(4457)) = - J_{uS} - 2J_{A}^{ud} \]

A solution can be found

\[ J_{uS}^{uu} = 29.9^{+2.5}_{-2.8} \text{ MeV} \quad J_{A}^{ud} = - 42.8^{+2.4}_{-1.6} \text{ MeV} \]

\[ \frac{J_{uS}^{uu}}{J_{A}^{ud}} = - 0.7 \pm 0.1 \]
Pentaquarks of the \((J/\psi\Lambda)\) Kind

Using the same methods we can study the strange pentaquarks. One has been observed and two more are predicted.

\[ P_{cs}(4459) \]

Predictions: \( P'_{cs}, P''_{cs} \)
Assuming that the color-flavor of the spectator $ud$ pair is kept in the final state, the process $\Lambda_b^0 \rightarrow \tilde{P}_c K^-$ is expected to be suppressed, as suggested by observation: the $[ud]_0$ in $\Lambda$ is in a CF-symmetric configuration as opposite to $\tilde{P}$ as described by

$$A^{udu}_{\alpha\beta\gamma} = u(\alpha d_\beta)u_\gamma - u(\gamma d_\beta)u_\alpha$$
The spin ordering \((1/2, 3/2, 1/2)\) is a main prediction.

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<td>((4440.0^{+4}_{-5}))</td>
<td>(P'_{cs})</td>
<td>(4541 \pm 6)</td>
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<td>(P''_{cs})</td>
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<tr>
<td>(\tilde{P}'_{c})</td>
<td>(4187 \pm 7)</td>
<td>(\tilde{P}_{cs}(4338))</td>
<td>((4338.2 \pm 0.8))</td>
</tr>
<tr>
<td>(\tilde{P}''_{c})</td>
<td>(4276 \pm 12)</td>
<td>(\tilde{P}'_{cs})</td>
<td>(4387 \pm 4)</td>
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<td>(\tilde{P}_c(4337))</td>
<td>(4332 \pm 7 ) ((4337^{+7}<em>{-4}^{+2}</em>{-2}))</td>
<td>(\tilde{P}''_{cs})</td>
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