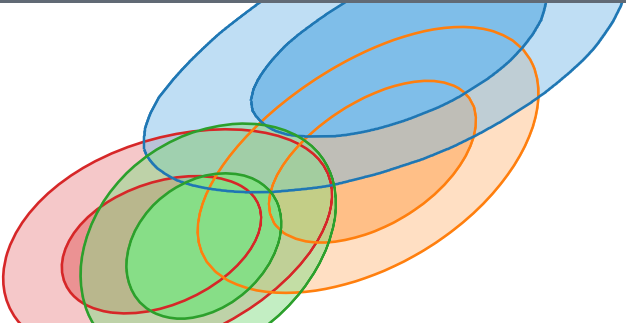
12th Large Hadron Collider Physics (LHCP) Conference, 4 Jun, 2024

Status of the global $b \to s\ell\ell$ fits

B. Capdevila University of Cambridge, DAMTP & Uni. Autònoma Barcelona, IFAE

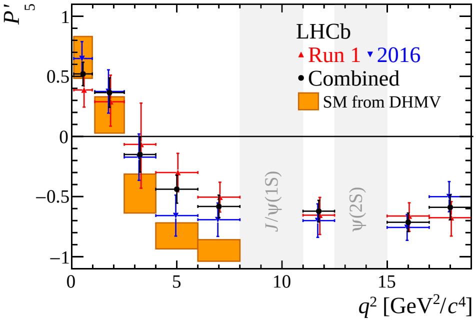


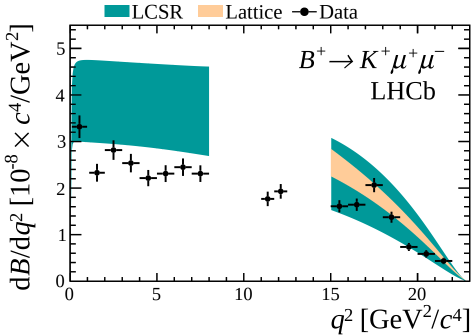
The $b \to s\ell\ell$ anomalies

$b \to s\,\mu^+\mu^-$ anomaly

Several LHCb measurements deviate from Standard model (SM) predictions* by 2-3σ:

- Angular observables in $B^{(0,+)} \to K^{*(0,+)}\mu^+\mu^-$ LHCb, arXiv:2003.04831, arXiv:2012.13241
- Branching ratios of $B \to K\mu^+\mu^-$, $B \to K^*\mu^+\mu^-$, and $B_s \to \phi\mu^+\mu^-$ LHCb, arXiv:1403.8044, arXiv:1506.08777, arXiv:1606.04731, arXiv:2105.14007





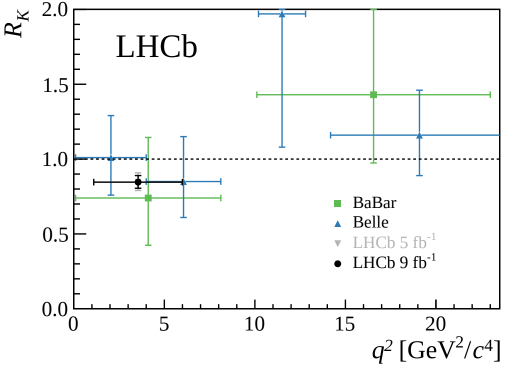
*: based on hadronic assumptions on which there is no theory consensus yet

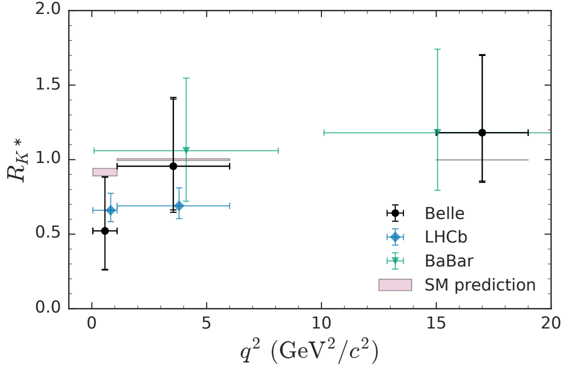
LFU violation in $b \to s\,\ell^+\ell^-$ decays (up to Dec. 2022)

Measurements of LFU ratios $R_{K^*}^{[0.045,1.1]}$, $R_{K^*}^{[1.1,6]}$, $R_K^{[1,6]}$ showed deviations from SM by 2.3, 2.5, and 3.1σ

LHCb, arXiv:1705.05802, arXiv:2103.11769
Belle, arXiv:1904.02440, arXiv:1908.01848

$$R_{K^{(*)}} = \frac{\mathcal{B}(B \to K^{(*)}\mu^+\mu^-)}{\mathcal{B}(B \to K^{(*)}e^+e^-)}$$



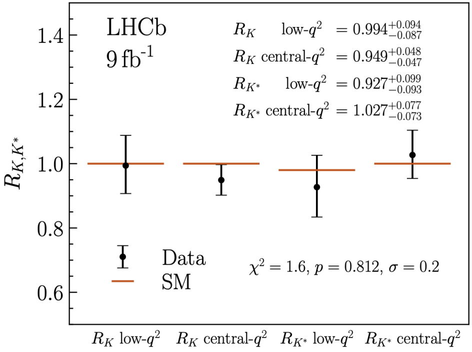


LFU violation in $b \to s\,\ell^+\ell^-$ decays

New LHCb measurement of the LFU ratios $R_K^{[0.1,1.1]}$, $R_K^{[1.1,6]}$, $R_{K^*}^{[0.1,1.1]}$, $R_{K^*}^{[1.1,6]}$

LHCb, arXiv:2212.09152, arXiv:2212.09153.

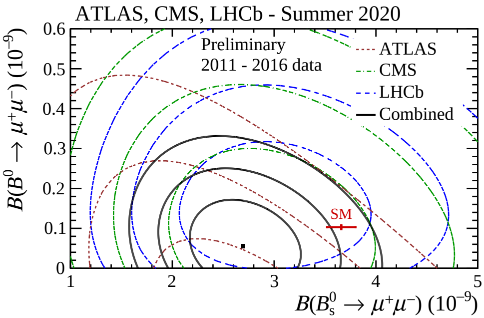
- sample of B meson decays in pp collisions collected between 2011 and 2018 (integrated luminosity of 9 fb^{-1})
- new modelling of residual backgrounds due to misidentified hadronic decays
- deviations from SM by ~ -0.0, $+1.1$, $+0.5$ and -0.4σ

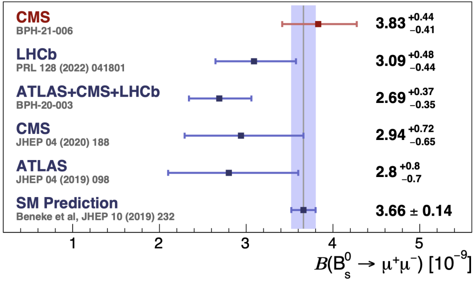


Leptonic modes $B_{s,d} \to \mu^+\mu^-$

Measurements of $\mathcal{B}(B_{s,d} \to \mu^+\mu^-)$ by LHCb, CMS, and ATLAS show deviations of only about $\sim 1\sigma$ with respect to SM predictions*

ATLAS, arXiv:1812.03017
CMS, arXiv:1910.12127,**2212.10311**
LHCb, arXiv:1703.05747,2108.09283





ATLAS update missing $\Rightarrow$ full Run 1 + Run 2 LHC combination

*: depends on parameters like V_{cb}

Bobeth, Buras, arXiv:2104.09521

Theoretical Framework

$b \to s\ell\ell$ in the Weak Effective Theory

- Effective Hamiltonian at scale m_b: $\mathcal{H}_{\text{eff}} = \mathcal{H}_{\text{eff, sl}} + \mathcal{H}_{\text{eff, had}}$
- **Semileptonic operators**: ($\mathcal{N} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} \approx (34\text{ TeV})^{-2}$)

$$\mathcal{H}_{\text{eff, sl}} = -\mathcal{N}\Big(\mathcal{C}_7 O_7 + \mathcal{C}_7' O_7' + \sum_\ell \sum_{i=9,10,P,S} \Big(\mathcal{C}_i^\ell O_i^\ell + \mathcal{C}_i'^\ell O_i'^\ell\Big)\Big) + \text{h.c.}$$

$$O_7^{(\prime)} = \frac{m_b}{e}(\bar{s}\sigma_{\mu\nu} P_{R(L)} b)\, F^{\mu\nu}\,, \quad O_9^{(\prime)\ell} = (\bar{s}\gamma_\mu P_{L(R)} b)(\bar{\ell}\gamma^\mu \ell)\,, \quad O_{10}^{(\prime)\ell} = (\bar{s}\gamma_\mu P_{L(R)} b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)\,.$$

$$\mathcal{C}_7^{\text{SM}} \simeq -0.3\,, \qquad \mathcal{C}_9^{\text{SM}} \simeq 4\,, \qquad \mathcal{C}_{10}^{\text{SM}} \simeq -4\,.$$

Not considered here: (pseudo)scalar $O_{P,S}$ vanish in SM, could appear at dim. 6 in SMEFT (and tensor O_T only at dim. 8 in SMEFT)

- **Hadronic operators**:

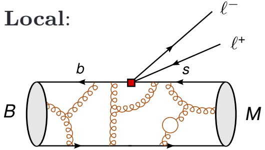
$$\mathcal{H}_{\text{eff, had}} = -\mathcal{N}\frac{16\pi^2}{e^2}\Big(\mathcal{C}_8 O_8 + \mathcal{C}_8' O_8' + \sum_{i=1,\dots,6} \mathcal{C}_i O_i\Big) + \text{h.c.}$$

$$\text{e.g.} \quad O_1 = (\bar{s}\gamma_\mu P_L T^a c)(\bar{c}\gamma^\mu P_L T^a b)\,, \quad O_2 = (\bar{s}\gamma_\mu P_L c)(\bar{c}\gamma^\mu P_L b)\,.$$

Theory of $B \to M\ell\ell$ decays ($M = K, K^*, \phi$)

$$\mathcal{M}(B \to M\ell\ell) = \langle M\ell\ell|\mathcal{H}_{\text{eff}}|B\rangle = \mathcal{N}\Big[(\mathcal{A}_V^\mu + \mathcal{H}^\mu)\, \bar{u}_\ell \gamma_\mu v_\ell + \mathcal{A}_A^\mu\, \bar{u}_\ell \gamma_\mu \gamma_5 v_\ell + \mathcal{A}_S\, \bar{u}_\ell v_\ell + \mathcal{A}_P\, \bar{u}_\ell \gamma_5 v_\ell \Big]$$

Local:

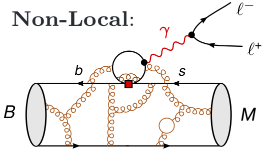


$$\mathcal{A}_V^\mu = -\frac{2im_b}{q^2}\,\mathcal{C}_7 \langle M|\bar{s}\,\sigma^{\mu\nu} q_\nu\, P_R\, b|B\rangle + \mathcal{C}_9 \langle M|\bar{s}\,\gamma^\mu\, P_L\, b|B\rangle + (P_L \leftrightarrow P_R, \mathcal{C}_i \to \mathcal{C}_i')$$

$$\mathcal{A}_A^\mu = \mathcal{C}_{10} \langle M|\bar{s}\,\gamma^\mu\, P_L\, b|B\rangle + (P_L \leftrightarrow P_R, \mathcal{C}_i \to \mathcal{C}_i')$$

$$\mathcal{A}_{S,P} = \mathcal{C}_{S,P} \langle M|\bar{s}\, P_R\, b|B\rangle + (P_L \leftrightarrow P_R, \mathcal{C}_i \to \mathcal{C}_i')$$

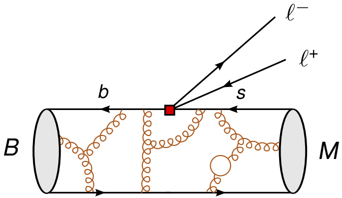
Non-Local:



$$\mathcal{H}^\mu = \frac{-16i\pi^2}{q^2} \sum_{i=1,\ldots,6,8} \mathcal{C}_i \int dx^4 e^{iq\cdot x} \langle M|T\{j^\mu_{\text{em}}(x), O_i(0)\}|B\rangle\,, \qquad j^\mu_{\text{em}} = \sum_q Q_q\, \bar{q}\gamma^\mu q$$

- **Wilson coefficients** $\mathcal{C}_i = \mathcal{C}_i^{\text{SM}} + \mathcal{C}_i^{\text{NP}}$:
  perturbative, short-distance physics (q^2 independent), well-known in SM, parameterise heavy NP
- **local** and **non-local** **hadronic matrix elements**:
  non-perturbative, long-distance physics (q^2 dependent), **main source of uncertainty**

Local matrix elements



$$\mathcal{A}_V^\mu = -\frac{2im_b}{q^2}\,\mathcal{C}_7\langle M|\bar{s}\,\sigma^{\mu\nu}q_\nu\,P_R\,b|B\rangle + \mathcal{C}_9\langle M|\bar{s}\,\gamma^\mu\,P_L\,b|B\rangle + (P_L \leftrightarrow P_R, \mathcal{C}_i \to \mathcal{C}_i')$$

$$\mathcal{A}_A^\mu = \mathcal{C}_{10}\langle M|\bar{s}\,\gamma^\mu\,P_L\,b|B\rangle + (P_L \leftrightarrow P_R, \mathcal{C}_i \to \mathcal{C}_i')$$

$$\mathcal{A}_{S,P} = \mathcal{C}_{S,P}\langle M|\bar{s}\,P_R\,b|B\rangle + (P_L \leftrightarrow P_R, \mathcal{C}_i \to \mathcal{C}_i')$$

- $\langle M|\bar{s}\,\Gamma_i\,b|B\rangle$ matrix elements are parameterised by:
  - **3 form factors** for each **spin zero** final state $M = \boldsymbol{K}$
  - **7 form factors** for each **spin one** final state $M = \boldsymbol{K^*}, \boldsymbol{\phi}$
- Determination of form factors
  - high q^2: **Lattice QCD**

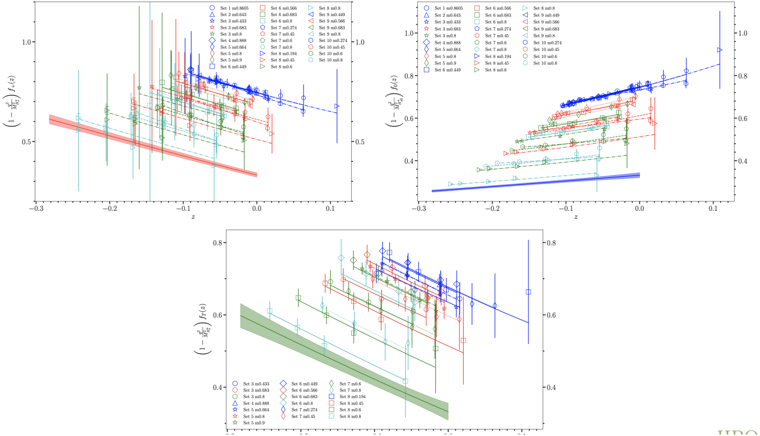
    HPQCD, arXiv:1306.2384,2207.12468
    Fermilab, MILC, arXiv:1509.06235
    Horgan, Liu, Meinel, Wingate, arXiv:1310.3722, arXiv:1501.00367
  - low q^2: **Continuum methods**
    e.g. Light-cone sum rules (LCSR)

    Ball, Zwicky, arXiv:hep-ph/0406232
    Khodjamirian, Mannel, Pivovarov, Wang, arXiv:1006.4945
    Bharucha, Straub, Zwicky, arXiv:1503.05534
    Gubernari, Kokulu, van Dyk, arXiv:1811.00983
  - low + high q^2: Combined fit to **continuum methods** + **lattice** / **lattice**

    Altmannshofer, Straub, arXiv:1411.3161
    Bharucha, Straub, Zwicky, arXiv:1503.05534
    Gubernari, Kokulu, van Dyk, arXiv:1811.00983

Theory Update: $B \to K$ lattice form factors at all q^2

- Lattice QCD calculation of the $B \to K$ form factors **across the full physical** q^2 range
  - $\Rightarrow$ highly improved staggered quark (HISQ) formalism (valence quarks)
  - $\Rightarrow$ gluon field configurations by MILC
  - $\Rightarrow$ first fully relativistic calculation, using the heavy-HISQ method

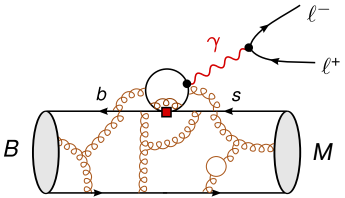


Non-negligible impact on $B \to K\ell\ell$ observables

Predictions with HPQCD'22 Form Factors			
$10^7 \times \mathcal{B}(B^+ \to K^+\mu^+\mu^-)$	Standard Model	Experiment	Pull
$[0.1, 0.98]$	0.320 ± 0.025	0.29 ± 0.02	$+0.8$
$[1.1, 2]$	0.329 ± 0.025	0.21 ± 0.02	$+3.9$
$[2, 3]$	0.365 ± 0.027	0.28 ± 0.02	$+2.4$
$[3, 4]$	0.366 ± 0.027	0.25 ± 0.02	$+3.3$
$[4, 5]$	0.366 ± 0.028	0.22 ± 0.02	$+4.4$
$[5, 6]$	0.366 ± 0.029	0.23 ± 0.02	$+3.9$
$[6, 7]$	0.367 ± 0.032	0.25 ± 0.02	$+3.3$
$[7, 8]$	0.371 ± 0.042	0.23 ± 0.02	$+3.1$
$[15, 22]$	1.150 ± 0.159	0.85 ± 0.05	$+1.8$

HPQCD, arXiv:2207.12468
LHCb, arXiv:1403.8044

Non-local matrix elements



$$\mathcal{H}^\mu = \frac{-16i\pi^2}{q^2} \sum_{i=1..6,8} \mathcal{C}_i \int dx^4 e^{iq\cdot x} \langle M | T\{j^\mu_{\rm em}(x), O_i(0)\} | B \rangle$$

$$j^\mu_{\rm em} = \sum_q Q_q\, \bar{q}\gamma^\mu q$$

- Contributions at low q^2 from QCD factorization (QCDF) Beneke, Feldmann, Seidel, arXiv:hep-ph/0106067
- **Beyond-QCDF** contributions **the main source of uncertainty**
- Non-local contributions can mimic New Physics in $\mathcal{C}_9$
- Several approaches to estimate beyond-QCDF contributions at low q^2
  - fit of sum of resonances to data Blake, Egede, Owen, Pomery, Petridis, arXiv:1709.03921
  - direct fit to angular data Ciuchini, Fedele, Franco, Mishima, Paul, Silvestrini, Valli, arXiv:1512.07157
  - Light-Cone Sum Rules estimates Khodjamirian, Mannel, Pivovarov, Wang, arXiv:1006.4945; Gubernari, van Dyk, Virto, arXiv:2011.09813
  - analyticity + experimental data on $b \to sc\bar{c}$ Bobeth, Chrzaszcz, van Dyk, Virto, arXiv:1707.07305; Gubernari, van Dyk, Virto, arXiv:2011.09813

Fit setup

$b \to s\ell\ell$ global analyses

Results presented here by:

- ABCDMN (M. Algueró, A. Biswas, B. Capdevila, S. Descotes-Genon, J. Matias, M. Novoa-Brunet)
  Statistical framework: χ^2-fit, based on private code arXiv:2304.07330
- AS / GSSS (W. Altmannshofer, P. Stangl / A. Greljo, J. Salko, A. Smolkovic, P. Stangl)
  Statistical framework: χ^2-fit, based on public code `flavio` arXiv:2212.10497.
- CFFPSV (M. Ciuchini, M. Fedele, E. Franco, A. Paul, L. Silvestrini, M. Valli)
  Statistical framework: Bayesian MCMC fit, based on public code `HEPfit` arXiv:2212.10516
- HMMN (T. Hurth, F. Mahmoudi, D. Martínez-Santos, S. Neshatpour)
  Statistical framework: χ^2-fit, based on public code `SuperIso` arXiv:23xx.xxxxx

See also similar fits by other groups:

- N. Gubernari, M. Reboud, D. van Dyk, J. Virto
  Statistical framework: Bayesian fit with improved parameterisation of non-local matrix elements, based on public code EOS (see Nico Gubernari & Javier Virto's talks) arXiv:2206.03797

Geng et al., arXiv:2103.12738, Alok et al., arXiv:1903.09617, Datta et al., arXiv:1903.10086, Kowalska et al., arXiv:1903.10932, D'Amico et al., arXiv:1704.05438, Hiller et al., arXiv:1704.05444, ...

Observables in $b \to s\ell\ell$ global analyses

- Inclusive decays
  - $B \to X_s \gamma$ ($\mathcal{B}$)
  - $B \to X_s \ell^+ \ell^-$ ($\mathcal{B}$)
- Exclusive leptonic decays
  - $B_{s,d} \to \ell^+ \ell^-$ ($\mathcal{B}$)
- Exclusive radiative/semileptonic decays
  - $B \to K^* \gamma$ ($\mathcal{B}$, $S_{K^*\gamma}$, A_I)
  - $B^{(0,+)} \to K^{(0,+)} \ell^+ \ell^-$ ($\mathcal{B}_\mu$, R_K, angular observables)
  - $B^{(0,+)} \to K^{*(0,+)} \ell^+ \ell^-$ ($\mathcal{B}_\mu$, $R_{K^{*0}}$, angular observables)
  - $B_s \to \phi \mu^+ \mu^-$ ($\mathcal{B}$, angular observables)
  - $\Lambda_b \to \Lambda \mu^+ \mu^-$ ($\mathcal{B}$, angular observables)
- Fits might include $150 \sim 250$ observables $\Rightarrow$ **global** $b \to s\ell\ell$ analyses

Comparison between the groups

- Different experimental inputs, e.g.
  - $q^2 \in [6, 8]$ GeV^2 data (ABCDMN, CFFPSV, HMMN)
  - High-q^2 data (AS / GSSS, ABCDMN, HMMN)
  - Radiative decays (ABCDMN, CFFPSV, HMMN)
  - $\Lambda_b \to \Lambda\mu^+\mu^-$ (AS / GSSS, HMMN)
- Different form factor inputs
  - Low-q^2: form factors from LCSR, reduced with heavy-quark & large-energy symmetries + (uncorrelated) power corrections. High-q^2: lattice form factors ($B \to V\ell\ell$ ABCDMN)
  - Full q^2 region: form factors from HPQCD lattice fit across all q^2, with full correlations ($B \to P\ell\ell$ ABCDMN)
  - Full q^2 region: form factors from combined LCSR + lattice fit, with full correlations (AS / GSSS, HMMN)
  - Low q^2 region: form factors from combined LCSR + lattice fit, with full correlations (CFFPSV)
- Different assumptions about non-local matrix elements
  - Order of magnitude estimates based on theory calculations from continuum methods, with different parameterisations (ABCDMN, AS / GSSS, HMMN)
  - Direct fit to data in each scenario, relying on continuum methods only for $q^2 \leq 1$ GeV^2 while allowing them to freely grow for larger q^2 (CFFPSV)
- Different statistical frameworks

New Physics interpretation

General remarks about global fits

Most important Wilson coefficients:

- $\mathcal{C}_{9\mu}$: dominant contributions to angular observables, LFU observables
- $\mathcal{C}_{10\mu}$: dominant contributions to $B_s \to \mu\mu$, LFU observables

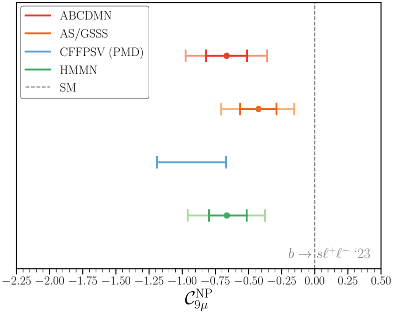
"Uninteresting" NP scenarios:

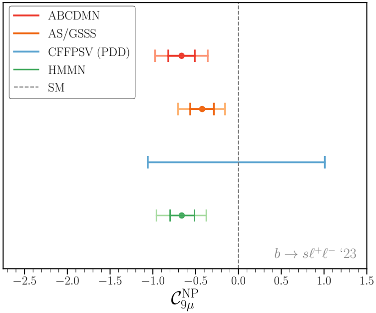
- $\mathcal{C}_{7(')}$: strongly constrained by radiative decays and very low-q^2 bin of $B \to K^* e^+ e^-$
- $\mathcal{C}_{10\mu}$: new $\mathcal{B}(B_s \to \mu^+\mu^-)$ combination greatly constraints $\mathcal{C}_{10\mu}^{\rm NP} \approx 0$
- $\mathcal{C}_{9'\ell,10'\ell}$: dominant contribution from coefficients with right-handed quarks disfavoured by $R_K \approx R_{K^*}$

Interesting NP scenarios:

- 1D scenarios: $\mathcal{C}_{9\mu}^{\rm NP}$, $\mathcal{C}_{9\mu}^{\rm NP} = \mathcal{C}_{9e}^{\rm NP} = \mathcal{C}_9^{\rm U}$
- 2D scenario: $(\mathcal{C}_{9\mu}^{\rm NP}, \mathcal{C}_{10\mu}^{\rm NP})$, $(\mathcal{C}_{9\mu}^{\rm NP}, \mathcal{C}_{9e}^{\rm NP})$, $(\mathcal{C}_9^{\rm U}, \mathcal{C}_{i=10(\mu),9'(\mu),10'(\mu)}^{\rm V,U})$ (since $R_K \approx R_{K^*} \approx 1$)

1-dimensional global fits

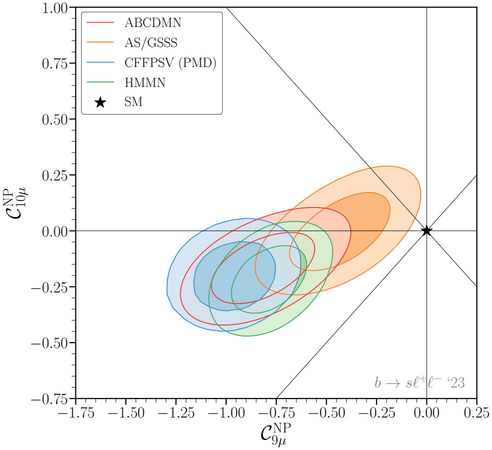


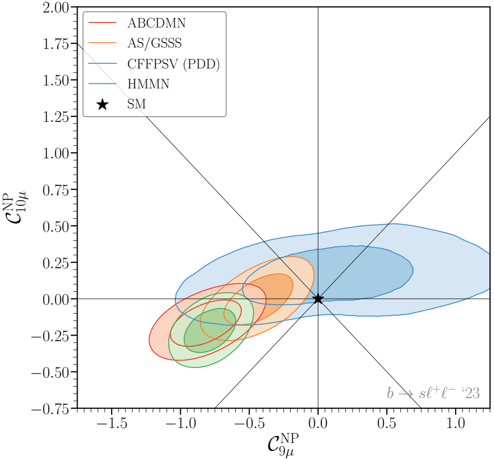


- NP scenarios preferred over SM with $\mathrm{Pull_{SM}}^* \sim 4\sigma$
- Different results due to different assumptions about non-local matrix elements, different choices of form factors and observables, etc.
- Remarkable agreement between fits of different groups despite different approaches
  $\Rightarrow$ **$b \to s\ell\ell$ global analyses are robust**

*: $\mathrm{Pull_{SM}} \neq$ global significance; conservative global significance (2021) $\simeq 4.3\sigma$ determined in Isidori, Lancierini, Owen, Serra, arXiv:2104.05631

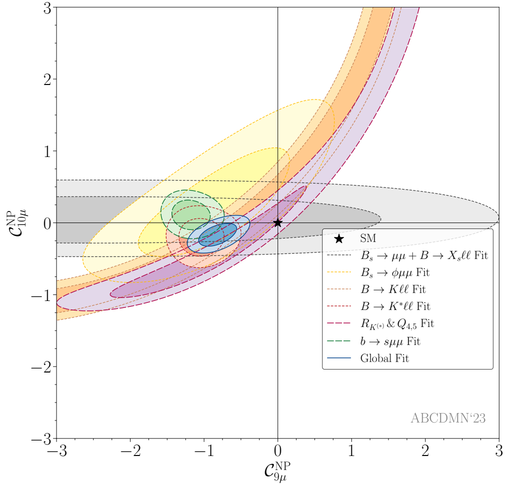
2-dimensional global fits

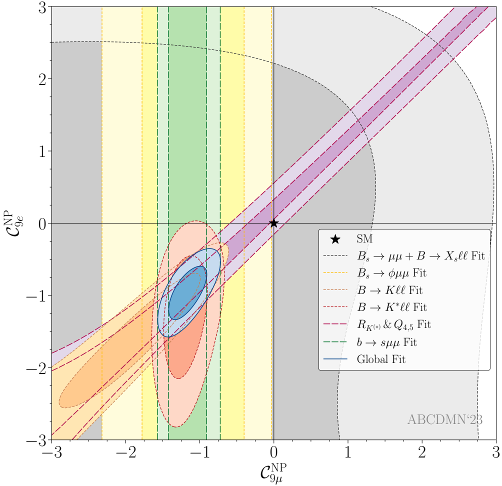




- Again, 2D NP scenarios preferred over SM with $\text{Pull}_{\text{SM}} \sim 4\sigma$
- Impressive agreement between fits of different groups despite different approaches (PMD vs PDD & bin $[6., 8.]$)

Structure of the multidimensional fits

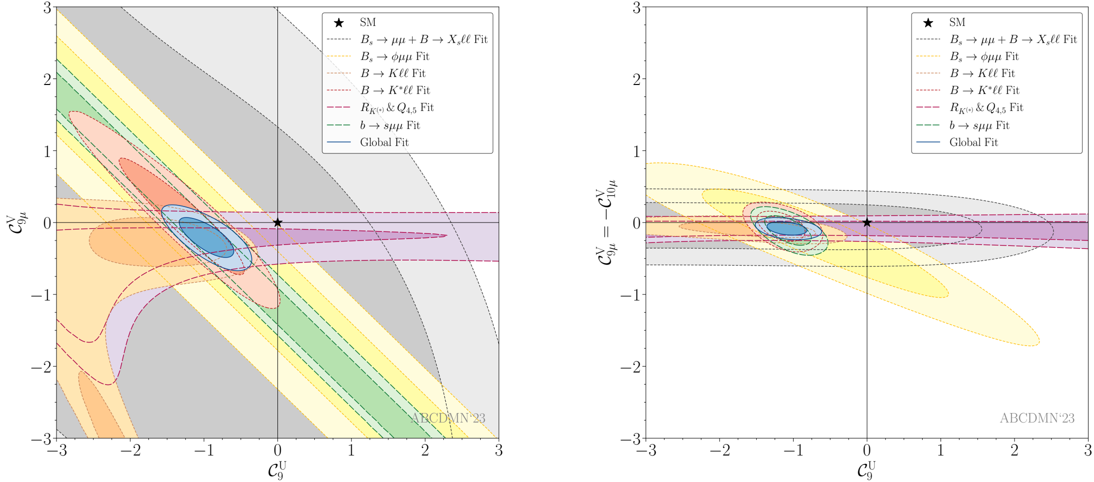




- NP hypothesis that do not allow for LFU show important internal tensions among fit components
- NP hypothesis with LFU embedded are very competitive describing all data

Algueró, Biswas, Capdevila, Descotes-Genon, Matias, Novoa-Brunet; arxiv:2304.07330

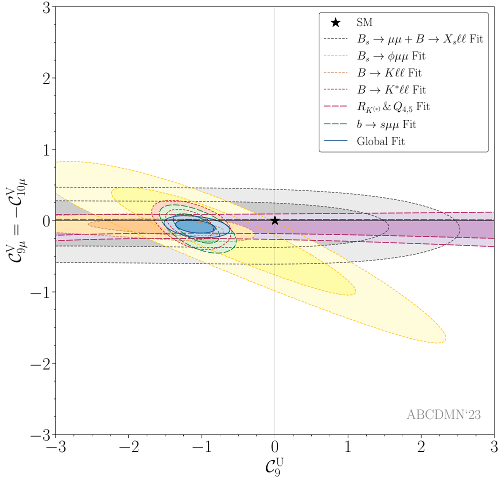
Internal coherence of fits including LFU NP to C_9



- NP hypothesis with $\mathcal{C}_9^{U}$ are very competitive in explaining the data ($\text{Pull}_{\text{SM}} \sim 5.5\sigma$)

Algueró, Biswas, Capdevila, Descotes-Genon, Matias, Novoa-Brunet; arxiv:2304.07330

NP fits with LFU contributions



WET at 4.8 GeV

- Two-parameter fit in space of $C^{\rm V}_{9\mu} = -C^{\rm V}_{10\mu}$ and $C^{\rm U}_9$

  scenario first considered in
  Algueró et al., arXiv:1809.08447

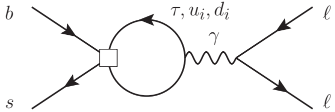
- Large **non-zero** $C^{\rm U}_9$ but LFUV compatible with 0

- This scenario is one of the most successful NP solutions to solve the $b \to s\ell\ell$ anomalies

  ⇒ $\text{Pull}_{\rm SM} = 5.6\sigma$

  ⇒ It successfully describes, with optimal internal consistency, $b \to s\mu\mu$ angular data + LFU ratios

  ⇒ Can arise from RG effects:

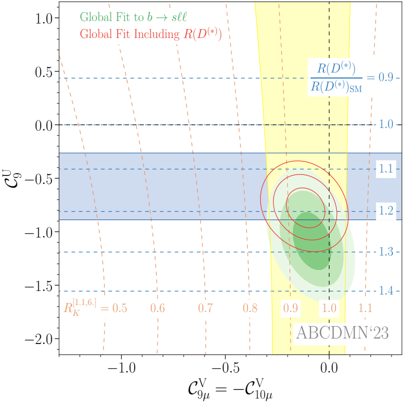


Bobeth, Haisch, arXiv:1109.1826
Crivellin, Greub, Müller, Saturnino, arXiv:1807.02068

Model independent connection $b \to s\mu\mu$ & $b \to c\ell\nu$ (with LFU NP)

- NP scenario ($\mathcal{C}_9^{\rm U}, \mathcal{C}_{9\mu}^{\rm V} = -\mathcal{C}_{10\mu}^{\rm V}$) allows for connections between $b \to s\ell\ell$ and $b \to c\tau\nu$ ($R_{D^{(*)}}$)
- SMEFT condition: $\mathcal{C}^{(1)} = \mathcal{C}^{(3)}$
- $\mathcal{O}_{2322} \Rightarrow$ LFUV NP $\mathcal{C}_{9\mu}^{\rm V} = -\mathcal{C}_{10\mu}^{\rm V}$ & $\mathcal{O}_{2333} \Rightarrow$ LFU NP $\mathcal{C}_9^{\rm U}$

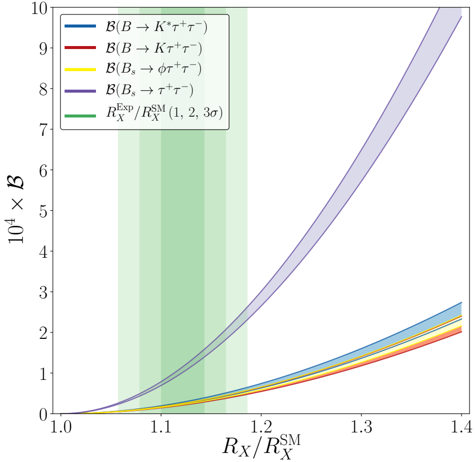
Crivellin, Greub, Muller, Saturnino; arxiv:1807.02068



$\Rightarrow$ $\text{Pull}_{\rm SM} = 6.3\sigma$

Algueró, Biswas, Capdevila, Descotes-Genon, Matias, Novoa-Brunet; arxiv:2304.07330

Enhancement of $b \to s\tau\tau$



⇒ Typical **enhancement by** 10^2**-**10^3 compared to SM value

Capdevila, Crivellin, Descotes-Genon, Hofer, Matias; arxiv:1712.01919

Conclusions

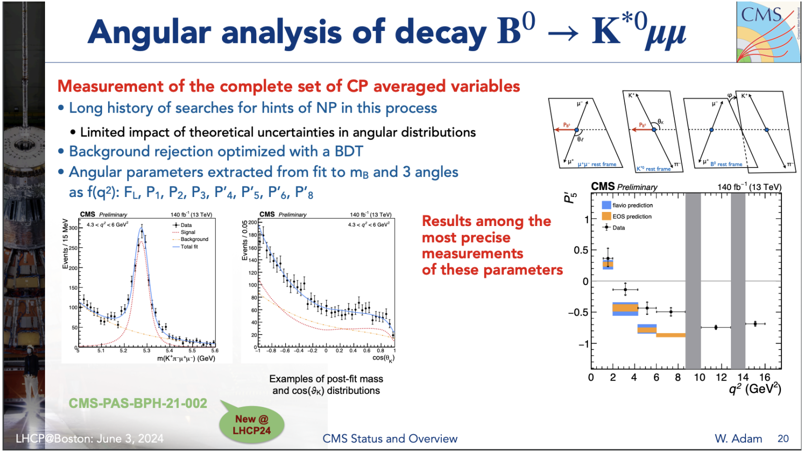
Conclusions

- Substantial reduction on the significance of the most preferred NP scenarios
  - ⇒ $\mathcal{C}_9$ continues to be the WC where most of the NP signal is encapsulated
  - ⇒ LFUV components are mostly suppressed
  - ⇒ High significances for scenarios with universal NP $\mathcal{C}_9^{\mathrm{U}}$
- Important tensions in the inner structure of the fit:
  - ⇒ LFU ratios are SM-like
  - ⇒ $B \to K^{(*)}\mu\mu$ and branching ratios for $B \to K\mu\mu$ continue to deviate with high significance
- $R_{D^{(*)}}$ and $b \to s\tau^+\tau^-$ can be correlated from fairly general assumptions:
  - ⇒ $b \to s\tau^+\tau^-$ processes dominated by NP approximately three orders of magnitude larger than SM
- Exploit the correlations among $b \to s\ell\ell$ and $b \to c\ell\nu$ and $b \to s\tau\tau$ to test the nature of $\mathcal{C}_9^{\mathrm{U}}$: either NP or hadronic effects (or a combination)

Wishlist

- Explicit numerical experimental likelihoods, e.g. to avoid digitisation of $B_{s,d} \to \mu\mu$ contour plots
- Measurements of other LFU observables, like e.g. R_ϕ or $Q_{4,5}/D_{P'_{4,5}}$
- $B \to K^* e^+ e^-$ angular analysis
- CP asymmetries to constrain imaginary parts of Wilson coefficients
- **Experimental updates** and **new measurements**, not only from **LHCb** but also from **ATLAS** and **CMS**, and eventually from **Belle II**

CMS $B^0 \to K^{*0}\mu^+\mu^-$ angular analysis



⇒ CMS measurement **highly in agreement** with LHCb's result for P_5'

⇒ Exciting new experimental result largely poised to **increase the significance** for $\mathcal{C}_9^{\mathrm{U}}$ solutions

See Wolfgang Adam talk

Thank you!

Backup slides

p-value SM fit

For the frequentist fits, the p-value of goodness-of-fit can be computed from Wilks' theorem

$$p - value_{SM} = 1 - F(\chi^2_{SM}; n_{obs})$$

with $F(\chi^2; n_{obs})$ the χ^2 CDF and n_{obs} the number of independent observables in the fit (measurements of a given observable by different experiments are counted as different observables).

- ABCDMN

$$Global\ fit:\ n_{\rm dof} = 256 \quad \Rightarrow \quad p - value = 5.13\%$$
$$LFU\ fit^*:\ n_{\rm dof} = 26 \quad \Rightarrow \quad p - value = 92.45\%$$

- AS

$$Global\ fit:\ n_{\rm dof} = ??? \quad \Rightarrow \quad p - value = ?.?\%$$
$$LFU\ fit^*:\ n_{\rm dof} = ?? \quad \Rightarrow \quad p - value = ?.?\%$$

- HMMN

$$Global\ fit:\ n_{\rm dof} = ??? \quad \Rightarrow \quad p - value = ?.?\%$$
$$LFU\ fit^*:\ n_{\rm dof} = ?? \quad \Rightarrow \quad p - value = ?.?\%$$

*LFU fit: all the measured LFU observables + $\mathcal{B}(B_s \to \mu^+\mu^-)$ (all groups)
+ effective $B_s \to \mu\mu$ lifetime + radiative decays + $\mathcal{B}(B_s \to X_s\mu^+\mu^-)$ (depending on the group)

ABCDMN: Improved QCDF

Improved QCDF (iQCDF) approach: $m_b \to \infty$ and $E_{V,P} \to \infty$ ($V = K^*$, ϕ, $P = K$)
decomposition of full form factors (FF)

$$F^{\text{Full}}(q^2) = F^{\infty}(\xi_\perp(q^2), \xi_\parallel(q^2)) + \Delta F^{\alpha_s}(q^2) + \Delta F^{\Lambda}(q^2)$$

where F stands for any FF (either helicity or transversity basis)

Charles et al; hep-ph/9901378
Beneke, Feldman; hep-ph/0008255
Descotes-Genon, Hofer, Matias, Virto; arXiv:1407.8526

- $m_b \to \infty$ and $E_{V,P} \to \infty$ symmetries: low-q^2 and LO in α_s and Λ/m_b
  $\Rightarrow$ **Dominant correlations** automatically taken into account
  (important for a maximal cancellation of errors)

Capdevila, Descotes-Genon, Hofer, Matias; arXiv:1701.08672

- $\mathcal{O}(\alpha_s)$ corrections $\Rightarrow$ QCDF

$$\langle \ell^+ \ell^- \bar{K}_i^* | \mathcal{H}_{\text{eff}} | \bar{B} \rangle = \sum_{a,\pm} \mathcal{C}_{i,a} \xi_a + \Phi_{B,\pm} \otimes T_{i,a,\pm} \otimes \Phi_{K^*,a} \quad (i = \perp, \parallel, 0)$$

Beneke, Feldman; hep-ph/0008255
Beneke, Feldman, Seidel; hep-ph/0106067

- $\mathcal{O}(\Lambda/m_b)$ corrections $\Rightarrow$ $\Delta F^{\Lambda}(q^2) = a_F + b_F \dfrac{q^2}{m_B^2} + c_F \dfrac{q^4}{m_B^4}$

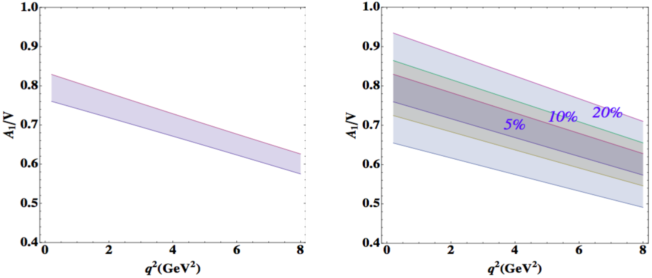
Jäger, Camalich; arXiv:1212.2263
Descotes-Genon, Hofer, Matias, Virto; arXiv:1407.8526

ABCDMN: Improved QCDF (vs full FF approach)

- How to estimate ΔF^{Λ}?
  - ⇒ Central values for a_F, b_F, c_F from **fit to full FF** (continuum calculation)
  - ⇒ Error estimate: assign **uncorrelated** errors to $\Delta a_F, \Delta b_F, \Delta c_F = \mathcal{O}(\Lambda/m_b) \times F$

Descotes-Genon, Hofer, Matias, Virto; arXiv:1407.8526

- Is this a conservative estimation of errors?



⇒ iQCDF with a 5% power corrections (right) reproduces the full FF (BSZ param.) approach errors (left)

Capdevila, Descotes-Genon, Hofer, Matias; arXiv:1701.08672

ABCDMN: Estimating beyond QCDF contribution at low-q^2

- LO (factorisable) charm-loop contribution accounted for in the $Y(q^2)$ (perturbative) function,

$$\mathcal{C}_9^{\rm eff}(q^2) = \mathcal{C}_9^{\rm SM} + Y(q^2)$$

Buras, Münz; hep-ph/9501281
Krüger, Lunghi; hep-ph/0008210

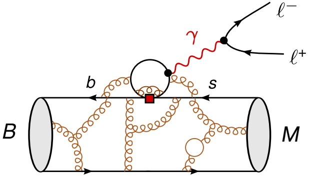
- Estimate of the soft-gluon emission contribution at low q^2:
  - $\Rightarrow$ Calculations based on continuum methods

Khodjamirian, Mannel, Pivovarov, Wang; arxiv:1006.4945
Gubernari, van Dyk, Virto; arxiv:2011.09813

  - $\Rightarrow$ Shift in $\mathcal{C}_9^{\rm eff}$. Order of magnitude for the shift estimated from theory calculations

$$\mathcal{C}_{9\,i}^{\rm eff}(q^2) = \mathcal{C}_9^{\rm eff}(q^2) + \mathcal{C}_9^{\rm NP} + s_i \delta\mathcal{C}_9^{{\rm LD},i}(q^2) \quad (i = \perp, \parallel, 0)$$

Descotes-Genon, Hofer, Matias, Virto; arxiv:1407.8526
Descotes-Genon, Hofer, Matias, Virto; arxiv:1510.04239



ABCDMN: Estimating beyond QCDF contribution at low-q^2

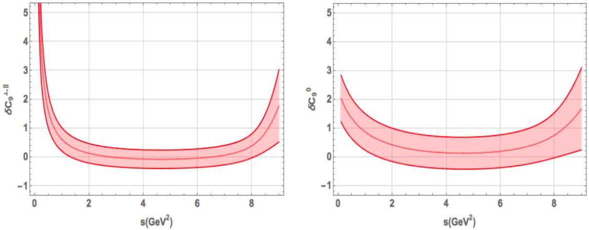
- Parameterisation for the long-distance contribution

$$\delta C_9^{\mathrm{LD},\perp}(q^2) = \frac{a^\perp + b^\perp q^2 (c^\perp - q^2)}{q^2 (c^\perp - q^2)} \qquad \delta C_9^{\mathrm{LD},\|}(q^2) = \frac{a^{||} + b^{||} q^2 (c^{||} - q^2)}{q^2 (c^{||} - q^2)}$$

$$\delta C_9^{\mathrm{LD},0}(q^2) = \frac{a^0 + b^0 (q^2 + s_0)(c^0 - q^2)}{(q^2 + s_0)(c^0 - q^2)}$$

⇒ We vary s_i in the range $[-1, 1]$

⇒ a^i, b^i, c^i parameters floated according to KMPW calculation

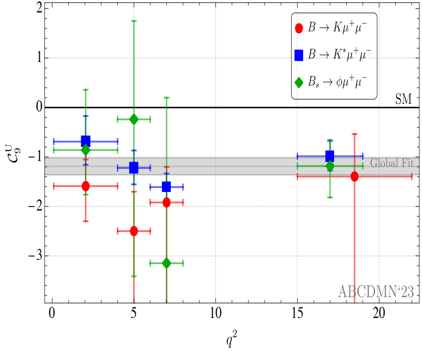


Khodjamirian, Mannel, Pivovarov, Wang, arXiv:1006.4945
Descotes-Genon, Hofer, Matias, Virto; arxiv:1407.8526
Descotes-Genon, Hofer, Matias, Virto; arxiv:1510.04239

ABCDMN: Consistency over q^2

Testing the q^2 dependence of $C_9^{\rm NP}$ by means of data:

- Fit to $B \to K^*\mu^+\mu^-$ ($\mathcal{B}$'s + Ang. obs) + $B_s \to \mu^+\mu^-$ + $B \to X_s\mu^+\mu^-$ + $b \to s\gamma$
- $C_9^{\rm NP}$ bin-by-bin fit (assuming KMPW-like $\delta C_9^{{\rm LD},i}(q^2)$)
- Good agreement with global fit (1σ range)
- No indication of a strong q^2 dependence
- Consistency large and low recoil (different theo. treatments)



Algueró, Biswas, Capdevila, Descotes-Genon, Matias, Novoa-Brunet: arxiv:2303.xxxxx

ABCDMN: Statistical framework

We parametrise the Wilson coefficients as,

$$C_i = C_i^{\rm SM} + C_i^{\rm NP} \quad (i = 7_\mu^{(\prime)}, 9_\mu^{(\prime)}, 10_\mu^{(\prime)},\ C_i^{\rm NP} \in \mathbb{R} \Rightarrow \text{no CPV})$$

Standard frequentist fit to the NP contributions to the Wilson coefficients,

$$\chi^2(C_i^{\rm NP}) = \left(\mathcal{O}^{\rm th}(C_i^{\rm NP}) - \mathcal{O}^{\rm exp}\right)_i Cov_{ij}^{-1} \left(\mathcal{O}^{\rm th}(C_i^{\rm NP}) - \mathcal{O}^{\rm exp}\right)_j$$

- Both **theory and experiment** contribute to the covariance matrix

  $\Rightarrow Cov = Cov^{\rm th} + Cov^{\rm exp}$
- Experimental covariance

  $\Rightarrow$ **Experimental correlations** between observables (if not provided, assumed uncorrelated). Assume gaussian errors (symmetrize if needed)
- Theoretical covariance

  $\Rightarrow$ Compute the **theoretical correlations** by performing a multivariate gaussian scan over all nuisance parameters
- $Cov = Cov(C_i)$

  $\Rightarrow$ **Mild** dependency $\Rightarrow Cov = Cov_{\rm SM} \equiv Cov(C_i = 0)$. Descotes-Genon, Hofer, Matias, Virto; arxiv:1510.04239
  Capdevila, Crivellin, Descotes-Genon, Matias, Virto; arxiv:1704.05340

ABCDMN: Statistical framework

- Fit procedure:
  - ⇒ **Best fit points** (bfp): $\chi^2(C_i^{\rm NP}) \to \chi^2_{\rm min} = \chi^2(C_{i^{\rm NP}})$
  - ⇒ **Confidence intervals**: $\chi^2(C_i^{\rm NP}) - \chi^2_{\rm min} \leq Q^2$
    $(1\sigma \to Q^2 = 1,\ 2\sigma \to Q^2 = 4,\ ...)$
  - ⇒ Compute **pulls** (σ): $\text{Pull}_{\rm SM} = \sqrt{\chi^2_{\rm SM} - \chi^2_{\rm min,Sc}}$
- Two types of fits
  - ⇒ *Canonical* (or *All*) fit: fit to **all data** (246 data points)
  - ⇒ LFUV fit: R_K, R_{K^*}, $P'^{\,e\mu}_{4,5}(B \to K^* \ell\ell)$ and $b \to s\gamma$ (22 data points)
- Testing different **hypotheses**
  - ⇒ Hypotheses with NP only in one Wilson coefficient (**1D fits**)
  - ⇒ Hypotheses with NP in two Wilson coefficients (**2D fits**)
  - ⇒ Hypotheses with NP in the six Wilson coefficients (**6D fits**)

Descotes-Genon, Hofer, Matias, Virto; arxiv:1510.04239
Capdevila, Crivellin, Descotes-Genon, Matias, Virto; arxiv:1704.05340

ABCDMN: 1D NP fits

	Global (before exp. updates 2022)			
1D Hyp.	bfp	1σ	Pull_{SM}	p-value (%)
$C_{9\mu}^{\text{NP}}$	-0.67 (-1.01)	[-0.82,-0.52] ($[-1.15, -0.87]$)	4.5 (7.0)	20.2 (24.0)
$C_{9\mu}^{\text{NP}} = -C_{10\mu}^{\text{NP}}$	-0.19 (-0.45)	[-0.25, -0.13] ($[-0.52, -0.37]$)	3.1 (6.5)	9.9 (16.9)
$C_{9\mu}^{\text{NP}} = -C'_{9\mu}$	-0.47 (-0.92)	[-0.66, -0.30] ($[-1.07, -0.75]$)	3.0 (5.7)	9.5 (8.2)
	LFUV			
1D Hyp.	bfp	1σ	Pull_{SM}	p-value (%)
$C_{9\mu}^{\text{NP}}$	-0.21 (-0.87)	$[-0.38, -0.04]$ ($[-1.11, -0.65]$)	1.2 (4.4)	92.4 (40.7)
$C_{9\mu}^{\text{NP}} = -C_{10\mu}^{\text{NP}}$	-0.08 (-0.39)	$[-0.15, -0.01]$ ($[-0.48, -0.31]$)	1.1 (5.0)	91.6 (73.5)
$C_{9\mu}^{\text{NP}} = -C'_{9\mu}$	-0.04 (-1.60)	$[-0.26, 0.15]$ ($[-2.10, -0.98]$)	0.2 (3.2)	87.5 (8.4)

⇒ Substantial drop in significances

⇒ $C_{9\mu}^{\text{NP}}$ is the strongest signal for the Global fit

⇒ p-value for $C_{9\mu}^{\text{NP}} = -C_{10\mu}^{\text{NP}}$ reduces significantly (less fit coherence: ang. obs. vs LFU ratios)

⇒ NP contributions to the WC compatible with SM values for the LFUV fit

Algueró, Capdevila, Descotes-Genon, Matias, Novoa-Brunet; arxiv:2104.08921
Algueró, Biswas, Capdevila, Descotes-Genon, Matias, Novoa-Brunet; arxiv:2304.07330

ABCDMN: Are we overlooking LFU NP?

⇒ Rotation of the basis of operators with a **LFU-LFUV alignment** (instead of flavour)

$$C_{i\ell}^{\rm NP} = C_{i\ell}^{\rm V} + C_i^{\rm U} \quad (C_i^{\rm U} \text{ the same } \forall \ell)$$

where $i = 9, 10, 9', 10'$ and $\ell = e, \mu$ (trivial extension to $\ell = \tau$)

⇒ The NP parameter space can be equally described with $\{C_{i\mu}^{\rm NP}, C_{ie}^{\rm NP}\}$ or $\{C_{i\mu}^{\rm V}, C_i^{\rm U}\}$ $(C_{ie}^{\rm V} = 0)$

⇒ The LFU vs LFUV language generates non-obvious NP directions in the μ vs e language

$$\begin{cases} C_{9\mu}^{\rm V} = -C_{10\mu}^{\rm V} \\ C_9^{\rm U} \end{cases} \Rightarrow \begin{cases} C_{9\mu}^{\rm NP} = -C_{10\mu}^{\rm NP} + C_{9e}^{\rm NP} \\ C_{9e}^{\rm NP} \end{cases}$$

Algueró, BC, Descotes-Genon, Masjuan, Matias; arxiv:1809.08447

AS: Setup

- Quantify agreement between theory and experiment by χ^2 function

$$\chi^2(\vec{C}) = \left(\vec{O}_{\rm exp} - \vec{O}_{\rm th}(\vec{C})\right)^{\rm T} \left(C_{\rm exp} + C_{\rm th}(\vec{C})\right)^{-1} \left(\vec{O}_{\rm exp} - \vec{O}_{\rm th}(\vec{C})\right) .$$

- **theory errors** and **correlations** in covariance matrix $\boldsymbol{C}_{\rm th}$
- **experimental errors** and available **correlations** in covariance matrix $\boldsymbol{C}_{\rm exp}$
- Theory errors depend on new physics Wilson coefficients $\boldsymbol{C}_{\rm th}(\vec{C})$ *NEW*

Altmannshofer, PS, arXiv:2103.13370

- $\Delta\chi^2$ and pull

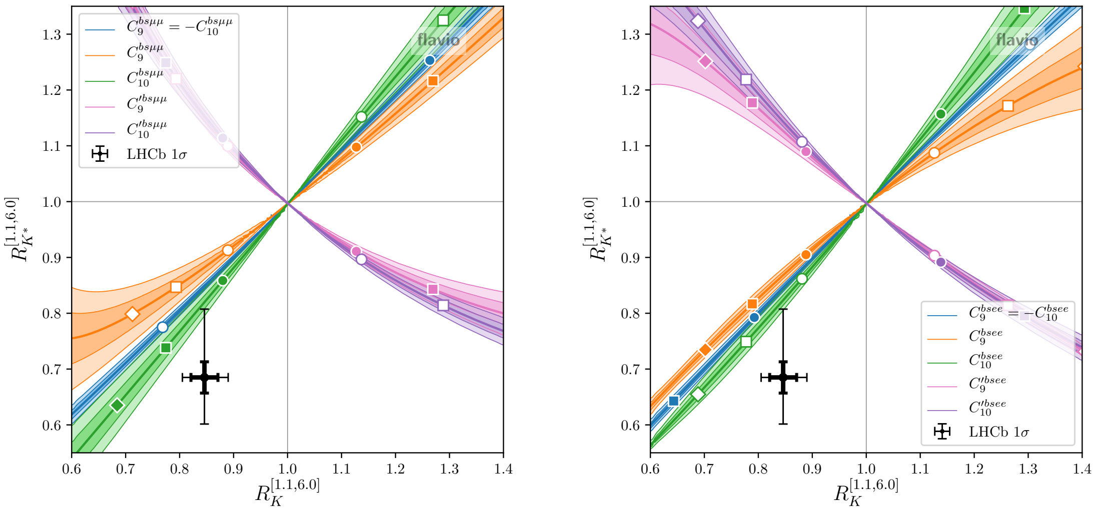
$$\text{pull}_{\rm 1D} = 1\sigma \cdot \sqrt{\Delta\chi^2}\,, \qquad \text{where } \Delta\chi^2 = \chi^2(\vec{0}) - \chi^2(\vec{}$$

C$_{\text{best fit}}$) .

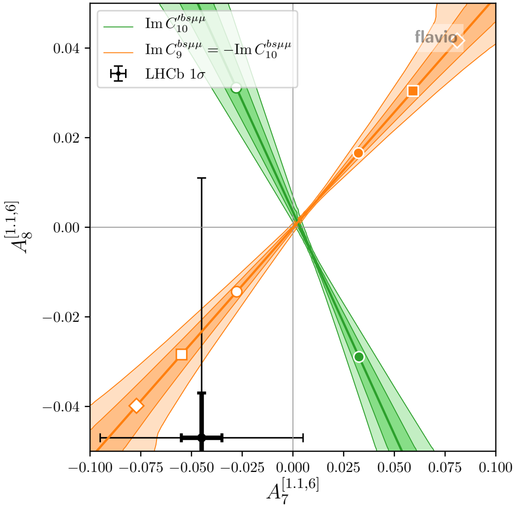
$$\text{pull}_{\rm 2D} = 1\sigma, 2\sigma, 3\sigma, \ldots \quad \text{for} \quad \Delta\chi^2 \approx 2.3, 6.2, 11.8, \ldots$$

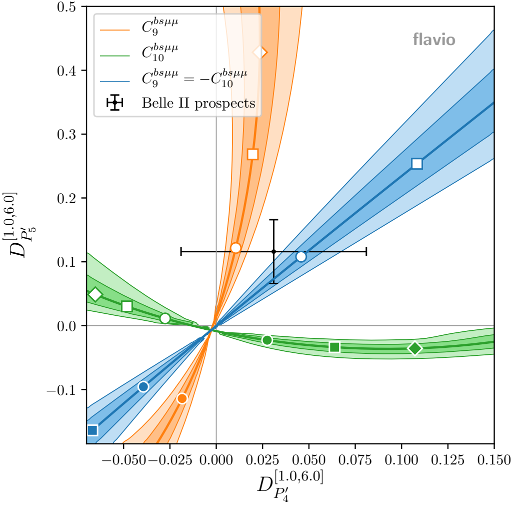
- New physics scenarios in **Weak Effective Theory (WET)** at scale 4.8 GeV

AS: Theory uncertainties in presence of NP



AS: Theory uncertainties in presence of NP





AS: Parameterisation of beyond-QCDF contributions for $B \to K$

$$\mathcal{C}_9^{\text{eff}}(q^2) \to \mathcal{C}_9^{\text{eff}}(q^2) + a_K + b_K(q^2/\,\text{GeV}^2) \quad \text{at low } q^2\,,$$

$$\mathcal{C}_9^{\text{eff}}(q^2) \to \mathcal{C}_9^{\text{eff}}(q^2) + \mathcal{C}_K \quad \text{at high } q^2\,,$$

$$\text{Re}(a_K) = 0.0 \pm 0.08\,, \qquad \text{Re}(b_K) = 0.0 \pm 0.03\,, \qquad \text{Re}(c_K) = 0.0 \pm 0.2\,,$$
$$\text{Im}(a_K) = 0.0 \pm 0.08\,, \qquad \text{Im}(b_K) = 0.0 \pm 0.03\,, \qquad \text{Im}(c_K) = 0.0 \pm 0.2\,.$$

1σ uncertainties enclose the effects considered in Khodjamirian et al. arXiv:1006.4945, Beylich et al. arXiv:1101.5118, Khodjamirian et al. arXiv:1211.0234

AS: Parameterisation of beyond-QCDF contributions for $B \to K^*$ and $B_s \to \phi$

$$\begin{aligned} \mathcal{C}_7^{\rm eff}(q^2) &\to \mathcal{C}_7^{\rm eff}(q^2) + a_{0,-} + b_{0,-}(q^2/\,{\rm GeV}^2) \\ \mathcal{C}_7' &\to \mathcal{C}_7' + a_+ + b_+(q^2/\,{\rm GeV}^2) \end{aligned} \qquad \text{at low } q^2\,,$$

$$\mathcal{C}_9^{\rm eff}(q^2) \to \mathcal{C}_9^{\rm eff}(q^2) + c_\lambda \qquad \text{at high } q^2\,,$$

$$\begin{aligned}
&{\rm Re}(a_+) = 0.0 \pm 0.004\,, && {\rm Re}(b_+) = 0.0 \pm 0.005\,, && {\rm Re}(c_+) = 0.0 \pm 0.3\,,\\
&{\rm Im}(a_+) = 0.0 \pm 0.004\,, && {\rm Im}(b_+) = 0.0 \pm 0.005\,, && {\rm Im}(c_+) = 0.0 \pm 0.3\,,\\
&{\rm Re}(a_-) = 0.0 \pm 0.015\,, && {\rm Re}(b_-) = 0.0 \pm 0.01\,, && {\rm Re}(c_-) = 0.0 \pm 0.3\,,\\
&{\rm Im}(a_-) = 0.0 \pm 0.015\,, && {\rm Im}(b_-) = 0.0 \pm 0.01\,, && {\rm Im}(c_-) = 0.0 \pm 0.3\,,\\
&{\rm Re}(a_0) = 0.0 \pm 0.12\,, && {\rm Re}(b_0) = 0.0 \pm 0.05\,, && {\rm Re}(c_0) = 0.0 \pm 0.3\,,\\
&{\rm Im}(a_0) = 0.0 \pm 0.12\,, && {\rm Im}(b_0) = 0.0 \pm 0.05\,, && {\rm Im}(c_0) = 0.0 \pm 0.3\,.
\end{aligned}$$

1σ uncertainties enclose the effects considered in Khodjamirian et al. arXiv:1006.4945, Beylich et al. arXiv:1101.5118

CFFPSV: Parameterisation of non-local hadronic matrix elements

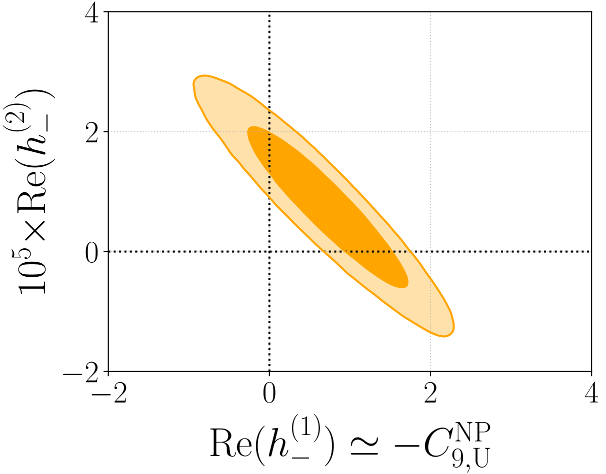
$$H_V^- \propto \left\{ \left(\mathcal{C}_9^{\rm SM} + h_-^{(1)}\right) \widetilde{V}_{L-} + \frac{m_B^2}{q^2} \left[\frac{2m_b}{m_B} \left(\mathcal{C}_7^{\rm SM} + h_-^{(0)}\right) \widetilde{T}_{L-} - 16\pi^2 h_-^{(2)}\, q^4\right] \right\}$$

$$H_V^+ \propto \left\{ \left(\mathcal{C}_9^{\rm SM} + h_-^{(1)}\right) \widetilde{V}_{L+} + \frac{m_B^2}{q^2} \left[\frac{2m_b}{m_B} \left(\mathcal{C}_7^{\rm SM} + h_-^{(0)}\right) \widetilde{T}_{L+} - 16\pi^2 \left(h_+^{(0)} + h_+^{(1)}\, q^2 + h_+^{(2)}\, q^4\right)\right] \right\}$$

$$H_V^0 \propto \left\{ \left(\mathcal{C}_9^{\rm SM} + h_-^{(1)}\right) \widetilde{V}_{L0} + \frac{m_B^2}{q^2} \left[\frac{2m_b}{m_B} \left(\mathcal{C}_7^{\rm SM} + h_-^{(0)}\right) \widetilde{T}_{L0} - 16\pi^2 \sqrt{q^2} \left(h_0^{(0)} + h_0^{(1)}\, q^2\right)\right] \right\}$$

- $h_-^{(0)}$ and $h_-^{(1)}$ can be considered as constant shifts to the WCs $\mathcal{C}_{7,9}^{\rm SM}$, hence indistinguishable from universal NP contributions to $O_{7,9}$
- remaining parameters describing purely hadronic contributions

CFFPSV: Parameterisation of non-local hadronic matrix elements



HMMN: New Physics vs hadronic fit (Status 2021)

Non-local matrix element contributions can mimic $\mathcal{C}_9^{\rm NP}$ since both appear in the vectorial helicity amplitude

$$H_V^\mu \propto \left\{ \mathcal{C}_9^{\rm eff} \tilde{V}_\lambda(q^2) + \frac{m_B^2}{q^2} \left[\frac{2m_b}{m_B} \mathcal{C}_7^{\rm eff} \tilde{T}_\lambda(q^2) - 16\pi^2 \left(\text{LO in QCDf} + h_\lambda(q^2) \right) \right] \right\}$$

Instead of *guesstimating* $h_\lambda(q^2)$, can be parameterised by a general ansatz:

Jäger, Camalich, arXiv:1412.3183
Ciuchini et al., arXiv:1512.07157
Chobanova, Hurth, Mahmoudi, Martinez-Santos, SN, arXiv:1702.02234

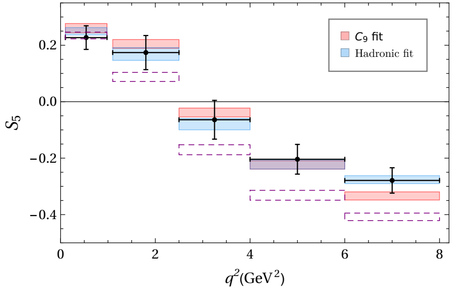
$$h_{\pm,[0]} = \left[\sqrt{q^2} \times \right] \left(h_{\pm,[0]}^{(0)} + q^2 h_{\pm,[0]}^{(1)} + q^4 h_{\pm,[0]}^{(2)} \right)$$

A. Arbey, T. Hurth, F. Mahmoudi, SN: 2006.04213

NP effect in $\mathcal{C}_9$ are embedded in the hadronic contributions
$\Rightarrow$ Wilks' test can be used to compare separate fits to:

- Hadronic quantities $h_{+,-,0}^{(0,1,2)}$ (18 parameters)
- Wilson coefficient $\mathcal{C}_9^{\rm NP}$ (1 parameter)

$B \to K^* \gamma/\mu\mu$ observables		
	Real $\mathcal{C}_9^{\rm NP}$ (1)	Hadronic fit h_λ (18)
Plain SM	6.0σ	4.7σ
Real $\mathcal{C}_9^{\rm NP}$	–	1.5σ



■ Hadronic fit describes the data well, however adding 17 more param. to NP doesn't significantly improve the fit (1.5σ)

HMMN: New Physics vs hadronic fit (minimal description, Status 2021)

18-parameter description of the hadronic contributions cannot get strongly constrained with current data

A (minimal) description of hadronic contributions with fewer parameters

$$h_\lambda(q^2) = -\frac{\tilde{V}_\lambda(q^2)}{16\pi^2}\frac{q^2}{m_B^2}\Delta\mathcal{C}_9^{\lambda,\mathrm{PC}}$$

a different Δ_9^{PC} for each helicity $\lambda = +,-,0$
$\rightarrow$ 3 (6) free parameters if assumed real (complex)

If NP in $\mathcal{C}_9$ is the favoured scenario, the different fitted helicities should give the same value
$\Rightarrow$ Can work as a null test for NP

	best fit value
$\Delta\mathcal{C}_9^{+,\mathrm{PC}}$	$(3.39 \pm 6.44) + i(-14.98 \pm 8.40)$
$\Delta\mathcal{C}_9^{-,\mathrm{PC}}$	$(-1.02 \pm 0.22) + i(-0.68 \pm 0.79)$
$\Delta\mathcal{C}_9^{0,\mathrm{PC}}$	$(-0.83 \pm 0.53) + i(-0.89 \pm 0.69)$

Fitted parameters not the same for different helicities but in agreement with each other within 1σ

$B \to K^*\gamma/\mu\mu$ observables		
	Real $\mathcal{C}_9^{\mathrm{NP}}$ (1)	Hadronic fit $\Delta\mathcal{C}_9^{\lambda,\mathrm{PC}}$ (6)
Plain SM	6.0σ	5.5σ
Real $\mathcal{C}_9^{\mathrm{NP}}$	–	1.8σ

■ Adding the hadronic parameters improve the fit with less than 2σ significance
$\Rightarrow$ Strong indication that the NP interpretation is a valid option, although the situation remains inconclusive

HMMN: Multi-dimensional global fit (Status 2021)

Considering only one or two Wilson coefficients may not give the full picture

A generic set of Wilson coefficients: $\mathcal{C}_7, \mathcal{C}_8, \mathcal{C}_9^\ell, \mathcal{C}_{10}^\ell, \mathcal{C}_S^\ell, \mathcal{C}_P^\ell$ + primed coefficients

All observables with $\chi^2_{\rm SM} = 225.8$ $\chi^2_{\rm min} = 151.6$; Pull$_{\rm SM} = 5.5(5.6)\sigma$			
$\delta\mathcal{C}_7$ 0.05 ± 0.03		$\delta\mathcal{C}_8$ -0.70 ± 0.40	
$\delta\mathcal{C}_7'$ -0.01 ± 0.02		$\delta\mathcal{C}_8'$ 0.00 ± 0.80	
$\delta\mathcal{C}_9^\mu$ -1.16 ± 0.17	$\delta\mathcal{C}_9^e$ -6.70 ± 1.20	$\delta\mathcal{C}_{10}^\mu$ 0.20 ± 0.21	$\delta\mathcal{C}_{10}^e$ degenerate w/ $\mathcal{C}_{10}'^e$
$\delta\mathcal{C}_9'^\mu$ 0.09 ± 0.34	$\delta\mathcal{C}_9'^e$ 1.90 ± 1.50	$\delta\mathcal{C}_{10}'^\mu$ -0.12 ± 0.20	$\delta\mathcal{C}_{10}'^e$ degenerate w/ $\mathcal{C}_{10}^e$
$\mathcal{C}_{Q_1}^\mu$ 0.04 ± 0.10	$\mathcal{C}_{Q_1}^e$ -1.50 ± 1.50	$\mathcal{C}_{Q_2}^\mu$ -0.09 ± 0.10	$\mathcal{C}_{Q_2}^e$ -4.10 ± 1.5
$\mathcal{C}_{Q_1}'^\mu$ 0.15 ± 0.10	$\mathcal{C}_{Q_1}'^e$ -1.70 ± 1.20	$\mathcal{C}_{Q_2}'^\mu$ -0.14 ± 0.11	$\mathcal{C}_{Q_2}'^e$ -4.20 ± 1.2

- Considering most general NP description and eliminating insensitive params. and flat directions based on the fit and not based on data, look-elsewhere effect is avoided
- Many parameters are weekly constrained at the moment
- Effective degree of freedom is (19)
- **Effective degrees of freedom**: degrees of freedom minus the spurious degrees of as realised from likelihood profiles, correlations and $\mathcal{C}_i^{\rm NP}$ only weakly affecting the χ^2 such that $|\chi^2(\mathcal{C}_i^{\rm NP} \neq 0) - \chi^2(\mathcal{C}_i^{\rm NP} = 0)| \lesssim 1$

HMMN: Comparison of different multi-dimensional global fits (Status 2021)

Pull_{SM} of 1, 2, 6, 10 and 20 dimensional fit:

Set of WC	param.	χ^2_{min}	Pull_{SM}	Improvement
SM	0	225.8	-	-
$\mathcal{C}_9^\mu$	1	168.6	7.6σ	7.6σ
$\mathcal{C}_9^\mu, \mathcal{C}_{10}^\mu$	2	167.5	7.3σ	1.0σ
$\mathcal{C}_7, \mathcal{C}_8, \mathcal{C}_9^{(e,\mu)}, \mathcal{C}_{10}^{(e,\mu)}$	6	158.0	7.1σ	2.0σ
All non-primed WC	10	157.2	6.5σ	0.1σ
All WC (incl. primed)	20 (19)	151.6	$5.5\,(5.6)\sigma$	$0.2\,(0.3)\sigma$

- In the last column the significance of improvement of the fit compared to the scenario of the previous row is given
- The "All non-primed WC" includes in addition to the previous row, the scalar and pseudoscalar Wilson coefficients
- The last row also includes the chirality-flipped counterparts of the Wilson coefficients
- The number in parentheses corresponds to the effective degrees of freedom (19)

HMMN: Evolution in the $(\mathcal{C}_{9\mu}^{\rm NP}, \mathcal{C}_{10\mu}^{\rm NP})$ plane