Determination of the $D \to \pi\pi$ penguin over tree ratio

Margarita Gavrilova, Cornell University

Based on MG, Y. Grossman and S. Schacht
Phys.Rev.D 109 (2024), arXiv: 2312.10140 [hep-ph]

LHCP, Jun 5, 2024

Is the Charm heavy?

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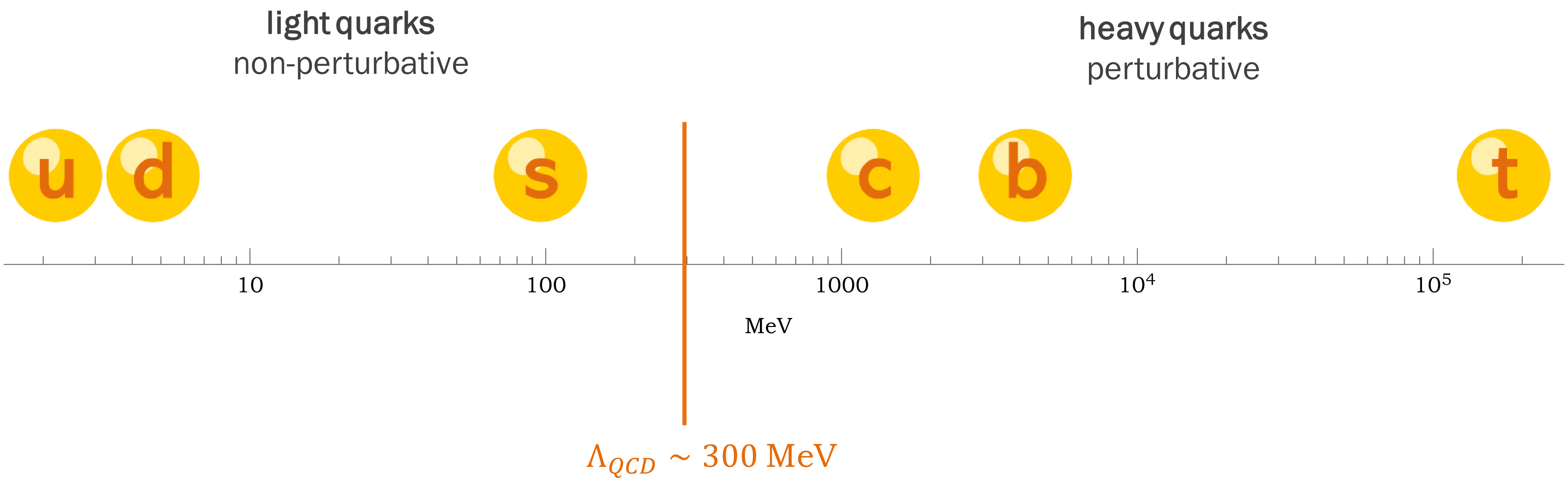
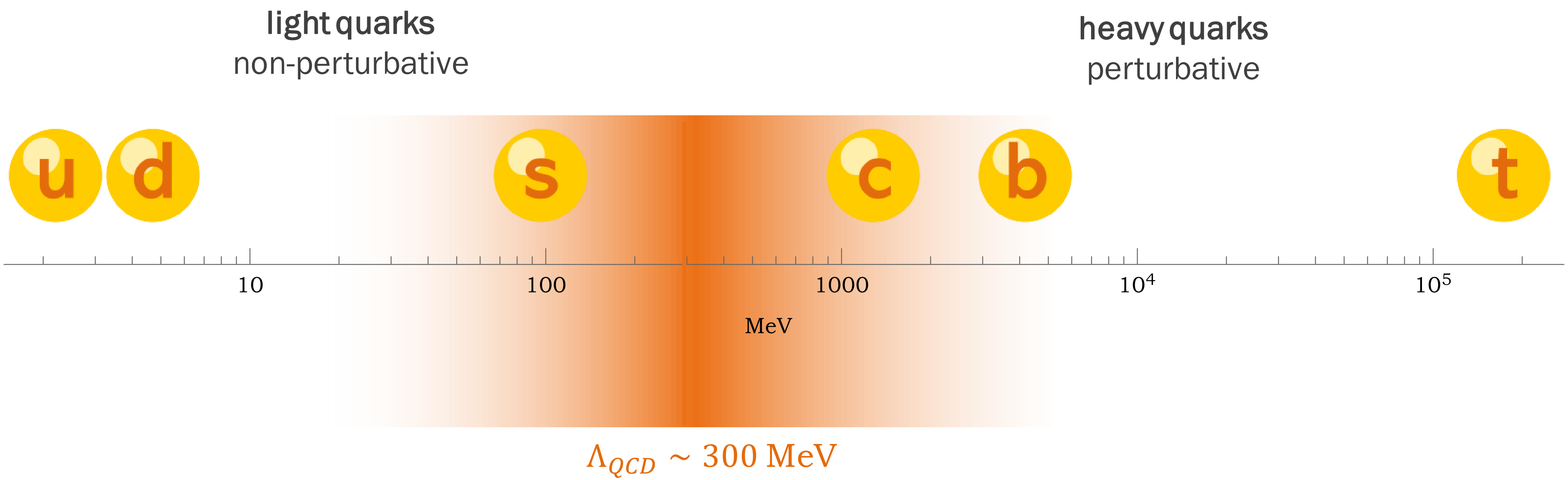
$c\bar{u}$ *

*quark content of the charm meson, $D^0 = (c\bar{u})$

Is the Charm Heavy?



Is the Charm Heavy?



The fundamental question:

How to treat QCD in charm?*

(or in other words "Is the Charm heavy?")

This talk's way to answer:

experiment

*everywhere in this talk we assume the SM; I'll comment on the possibility of the BSM interpretation of the result at the end

The fundamental problem

number of observables $<$ number of theory parameters

The fundamental problem

number of observables < number of theory parameters 😭

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number of observables $<$ number of theory parameters 😭

approximate $SU(3)_F$ of QCD,
the symmetry between u, d and s quarks

The fundamental challenge

number of observables $<$ number of theory parameters 😭

approximate $SU(3)_F$ of QCD,
the symmetry between u, d and s quarks

number of observables $\geq$ number of theory parameters 😊

the values of the theory parameters can be extracted from experiment!

The plan of action

Step 1: Find theory parameters sensitive to non-perturbative QCD in charm

↓

Step 2: Use flavor symmetries to reduce the number of independent theory parameters

↓

Step 3: Extract the values of the parameters of interest from experimental data

$D \to \pi\pi$

- We consider the following system of 6 **charm decays**:

$$D^0 \to \pi^+\pi^- \qquad D^0 \to \pi^0\pi^0 \qquad D^+ \to \pi^+\pi^0$$
$$\overline{D}^0 \to \pi^+\pi^- \qquad \overline{D}^0 \to \pi^0\pi^0 \qquad D^- \to \pi^-\pi^0$$

3 decays and their CP-conjugates

- These decays are related by **isospin**, $SU(2)$ subgroup of $SU(3)_F$ that relates $\boldsymbol{u}$ and $\boldsymbol{d}$
- The particles in the initial and final state form **isospin multiples**:

$$\begin{bmatrix} D^+ \\ D^0 \end{bmatrix} = \begin{bmatrix} c\bar{d} \\ c\bar{u} \end{bmatrix}, \qquad \begin{bmatrix} \overline{D}^0 \\ D^- \end{bmatrix} = \begin{bmatrix} \bar{c}u \\ \bar{c}d \end{bmatrix}, \qquad \begin{bmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{bmatrix} = \begin{bmatrix} u\bar{d} \\ \frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d}) \\ d\bar{u} \end{bmatrix}$$

isospin doublets

isospin triplet

Observables

- We consider the following system of 6 **charm decays**:

$$D^0 \to \pi^+\pi^- \qquad D^0 \to \pi^0\pi^0 \qquad D^+ \to \pi^+\pi^0$$
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3 decays and their CP-conjugates

1. Averaged branching ratios

phase space — decay amplitude — CP-conjugate decay amplitude

$$\mathcal{B}^{+-},\, \mathcal{B}^{00},\, \mathcal{B}^{+0}: \quad \mathcal{B}^f = \frac{1}{2}\mathcal{P}^f \left(|A^f|^2 + |\overline{A}^f|^2\right), \quad f = +-,\, 00,\, +0$$

2. CP-asymmetries

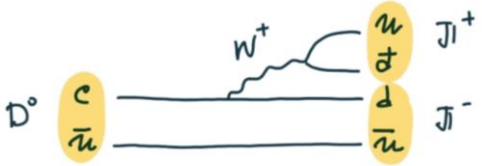
$$a_{CP}^{+-},\, a_{CP}^{00},\, a_{CP}^{+0}: \quad a_{CP}^f = \frac{|A^f|^2 - |\overline{A}^f|^2}{|A^f|^2 + |\overline{A}^f|^2}, \quad f = +-,\, 00,\, +0$$

Why CP-violation?

ϕ_d, ϕ_s: weak phases
δ_d, δ_s: strong phases

CP-violation is an interference effect!

$$D^0 \xrightarrow{V_{cd}^* V_{ud}} \pi^+\pi^-$$



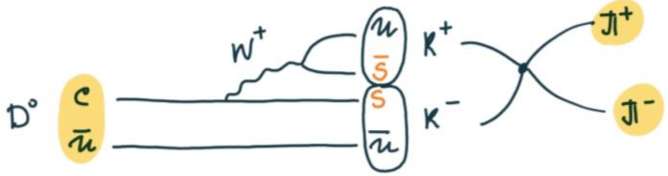
matrix elements:

$$V_{cd}^* V_{ud} \langle O_d \rangle^f = |V_{cd}^* V_{ud}| \times \left| \langle O_d \rangle^f \right| \times e^{i(\phi_d + \delta_d)}$$

CP-conjugate matrix elements:

$$V_{cd} V_{ud}^* \langle O_d \rangle^f = |V_{cd}^* V_{ud}| \times \left| \langle O_d \rangle^f \right| \times e^{i(-\phi_d + \delta_d)}$$

$$D^0 \xrightarrow{V_{cs}^* V_{us}} K^+K^-, \ldots \xrightarrow{\text{QCD}} \pi^+\pi^-$$



$$V_{cs}^* V_{us} \langle O_s \rangle^f = |V_{cs}^* V_{us}| \times \left| \langle O_s \rangle^f \right| \times e^{i(\phi_s + \delta_s)}$$

$$V_{cs} V_{us}^* \langle O_s \rangle^f = |V_{cs}^* V_{us}| \times \left| \langle O_s \rangle^f \right| \times e^{i(-\phi_s + \delta_s)}$$

RESCATTERING – pure QCD effect!

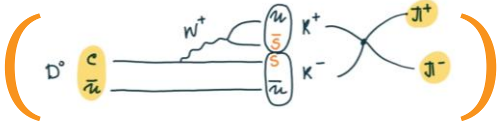
NOTE: different CKM-factors → different weak phase → CPV

Theoretical parametrization

Theoretical parametrization of the amplitudes of $D \to \pi\pi$:

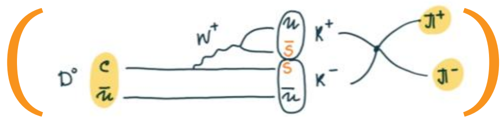
$$A^f = (-V_{cd}^* V_{ud}) \times T^f - \left(\frac{V_{cb}^* V_{ub}}{2}\right) \times P^f, \quad f = +-,\, 00,\, +0$$

$$P^f = 2\langle O_s \rangle^f$$ $\sim$



RESCATTERING – pure QCD effect!

$$T^f = \langle O_s \rangle^f - \langle O_d \rangle^f$$ $\sim$



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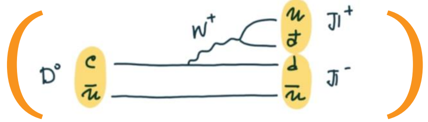

$$a_{CP}^f = \text{CKM} \times \left|\frac{P^f}{T^f}\right| \times \sin \delta^f$$

phase of P/T = strong phase

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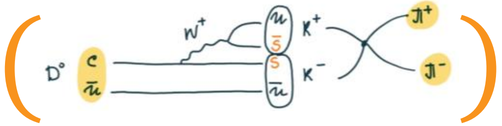
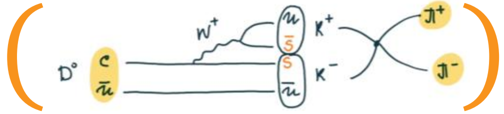
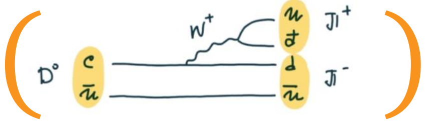
 $= 2\langle O_s\rangle^f \quad \sim$ 

RESCATTERING – pure QCD effect!

$= \langle O_s\rangle^f - \langle O_d\rangle^f \quad \sim$ 

$+$ 

$a_{CP}^f = \mathrm{CKM} \times$  $\times \sin\delta^f$

phase of P/T = strong phase

Penguin vs Tree

- The P/T ratio is a measure of rescattering in $D \to \pi\pi$

$$\left|\frac{P^f}{T^f}\right|_{\text{no rescatt.}} \ll 1$$

perturbative

$$\left|\frac{P^f}{T^f}\right|_{\text{rescatt.}} \gtrsim 1$$

non-perturbative

- Measuring P/T

$$a_{CP}^f = \text{CKM} \times \left|\frac{P^f}{T^f}\right| \times \sin\delta^f$$

without isospin, we can only measure the product of P/T and the strong phase

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with isospin and the BR measurements, we can separate the two contributions!

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Summary of the result

- Assuming isospin, we can extract the magnitude of P/T and its phase from the measurements of branching fractions and CP-asymmetries (don't need any assumptions about the strong phase!)

$$\left|\frac{P^f}{T^f}\right| = F(\mathcal{B}^{+-}, \mathcal{B}^{00}, \mathcal{B}^{+0}, a_{CP}^{+-}, a_{CP}^{00})$$

$$\sin\delta^f = f(\mathcal{B}^{+-}, \mathcal{B}^{00}, \mathcal{B}^{+0}, a_{CP}^{+-}, a_{CP}^{00})$$

- Isospin is expected to hold at order 1%, thus the relations have theoretical **precision of order few percent**

What does the data say?

Parameter	Current data (LHCb, Belle, CLEO)	Future data (LHCb Upgrade II, Belle-II)
$\|P/T\|^{00}$	$5.2^{+13.3}_{-2.4}$	$5.2^{+1.6}_{-1.2}$
$\|P/T\|^{+-}$	$5.5^{+14.2}_{-2.7}$	$5.5^{+1.8}_{-1.3}$

- $|P/T|$ **is large**, driven by sizable $a_{CP}(\mathrm{D}^0 \to \pi^+\pi^-)$
- future data will significantly reduce the errors!
- $|P/T|^f = 0$ hypothesis is at $\sim 3.8\sigma$ (no rescattering)
- $|P/T|^f = 0.1$ hypothesis is at $\sim 3.7\sigma$ (small rescattering)
- in the future data scenario, all the listed hypothesis are rejected at $> 5\sigma$

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Soo… Is the Charm Heavy? – the data hints that its not!

Backup

Some caveats

- **Numerical results:** we performed a "theorist's" numerical estimates for the current and future data, a dedicated experimental analysis is called for!
- **The SM prediction for $|P/T|^f$**
  - Light Cone Sum Rules (LCSR): $|P/T|^{+-} \sim O\ (0.1)$

  [Petrov Khodjamirian 1706.07780, Chala Lenz Rusov Scholtz 1903.10490, Lenz Piscopo Rusov 2312.13245]

$$|P/T|^{+-} = 0.089^{+0.042}_{-0.037}$$

  - Using dispersion relations and $\pi\pi/KK$ rescattering data: under debate

  [Franco Mishima Silvestrini 1203.3131, Bediaga Frederico Magalhaes 2203.04056, Pich Solomonidi Vale Silva 2305.11951]
  - Lattice: first conceptual ideas

  [Hansen Sharpe 1204.0826]

Flavor symmetry

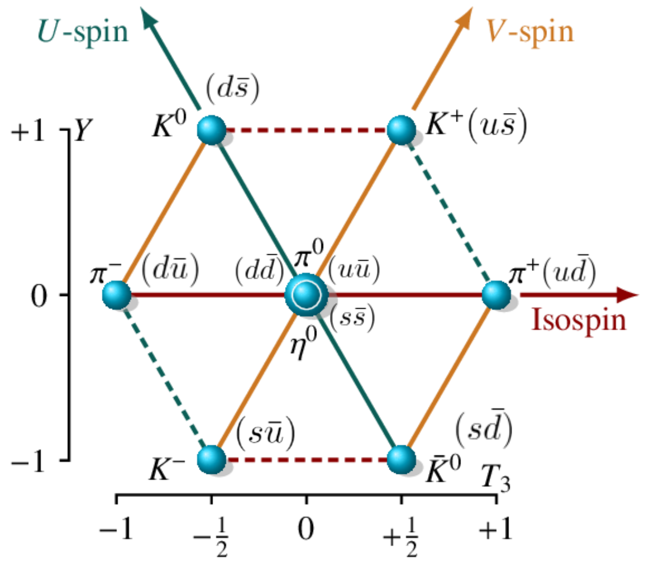
- QCD has an approximate $SU(3)$ flavor symmetry of light quarks u, d, s
- $SU(3)$ flavor contains an $SU(2)$ subgroup

  Isospin (u, d): λ_1 λ_2 λ_3

- $SU(3)$ flavor is broken by quark masses $m_u \neq m_d \neq m_s$
- The breaking of the isospin can be parametrized by $\varepsilon \sim \Delta m / \Lambda_{QCD} \sim 1\%$
- This means that predictions based on isospin have theoretical uncertainty of the order few %

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$



Effective Hamiltonian

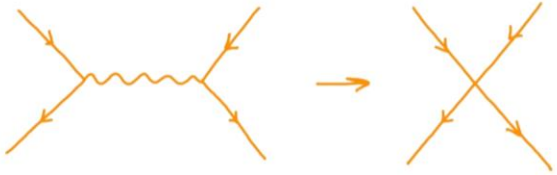
$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left(\sum_{q=d,s} \lambda_q \left(C_1 Q_1^q + C_2 Q_2^q \right) \right) \equiv \sum_{q=d,s} \lambda_q \mathcal{O}^q \,,$$

$$Q_1^q \equiv (\bar{u}\gamma_\mu(1-\gamma_5)q)\,(\bar{q}\gamma_\mu(1-\gamma_5)c)\,,$$

$$Q_2^q \equiv (\bar{q}\gamma_\mu(1-\gamma_5)q)\,(\bar{u}\gamma_\mu(1-\gamma_5)c)\,,$$

$$\langle \mathcal{O}^q \rangle^f \equiv \langle f | \mathcal{O}^q \left| D^0 \right\rangle \,,$$

CKM: $V_{cq}^* V_{uq}$



- integrated out EW bosons
- integrated out b
- neglected E&M interactions

then the decay amplitudes can be written as

$$A(D^0 \to f) = \lambda_d \langle \mathcal{O}^d \rangle^f + \lambda_s \langle \mathcal{O}^s \rangle^f = -\lambda_d \left(\langle \mathcal{O}^s \rangle^f - \langle \mathcal{O}^d \rangle^f \right) - \frac{\lambda_b}{2} \left(2 \langle \mathcal{O}^s \rangle^f \right)$$

unitarity of the CKM: $\lambda_d + \lambda_s + \lambda_b = 0$

hierarchy of the CKM is such that $\left| \frac{\lambda_b}{\lambda_d} \right| \sim 10^{-3}$ and thus the observables can be written as series in λ_b / λ_d

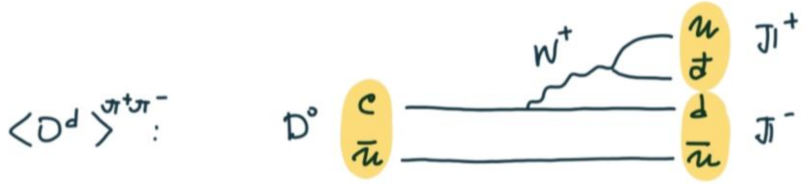
$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left(\sum_{q=d,s} \lambda_q \left(C_1 Q_1^q + C_2 Q_2^q \right) \right) \equiv \sum_{q=d,s} \lambda_q \mathcal{O}^q ,$$
$$Q_1^q \equiv (\bar{u}\gamma_\mu(1-\gamma_5)q)\,(\bar{q}\gamma_\mu(1-\gamma_5)c)\,,$$
$$Q_2^q \equiv (\bar{q}\gamma_\mu(1-\gamma_5)q)\,(\bar{u}\gamma_\mu(1-\gamma_5)c)\,,$$
$$\langle \mathcal{O}^q \rangle^f \equiv \langle f | \mathcal{O}^q \left| D^0 \right\rangle ,$$

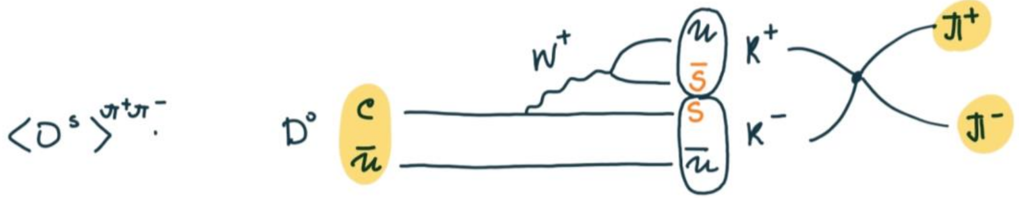
Penguin over tree ratio

What is the physical significance of r^f?

$$r^f \equiv \left| \frac{2\langle \mathcal{O}^s \rangle^f}{\langle \mathcal{O}^s \rangle^f - \langle \mathcal{O}^d \rangle^f} \right|$$

For concreteness, let us consider $D^0 \to \pi^+\pi^-$





rescattering

=

Closed-form expressions

$$\sin\arg(P/T)^{00} = \frac{-\mathsf{sign}\,(a_{CP}^{00})}{\sqrt{1+\frac{1}{\sin^2\delta_d}\left(\frac{a_{CP}^{+-}}{a_{CP}^{00}}\sqrt{\frac{1}{2}\frac{\mathcal{B}^{+-}}{\mathcal{P}^{+-}}\frac{\mathcal{P}^{00}}{\mathcal{B}^{00}}}+\cos\delta_d\right)^2}}\,,$$

$$\frac{r^{00}}{r^{+-}} = \sqrt{\frac{1}{2}\frac{\mathcal{B}^{+-}}{\mathcal{P}^{+-}}\frac{\mathcal{P}^{00}}{\mathcal{B}^{00}}}, \qquad r^f \equiv \left|\frac{P^f}{T^f}\right|$$

$$\sin\arg(P/T)^{+-} = \frac{-\mathsf{sign}\,(a_{CP}^{+-})}{\sqrt{1+\frac{1}{\sin^2\delta_d}\left(\frac{a_{CP}^{00}}{a_{CP}^{+-}}\sqrt{2\frac{\mathcal{P}^{+-}}{\mathcal{B}^{+-}}\frac{\mathcal{B}^{00}}{\mathcal{P}^{00}}}+\cos\delta_d\right)^2}}\,,$$

$$|P/T|^{00} = \frac{1}{|\mathrm{Im}\,(-\lambda_b/\lambda_d)|}\sqrt{(a_{CP}^{00})^2+\frac{(a_{CP}^{+-}\sqrt{\mathcal{B}^{+-}\mathcal{P}^{00}}+a_{CP}^{00}\sqrt{2\mathcal{B}^{00}\mathcal{P}^{+-}}\cos\delta_d)^2}{2\mathcal{B}^{00}\mathcal{P}^{+-}\sin^2\delta_d}}\,,$$

$$|P/T|^{+-} = \frac{1}{|\mathrm{Im}\,(-\lambda_d/\lambda_d)|}\sqrt{\left(a_{CP}^{+-}\right)^2+\frac{\left(a_{CP}^{00}\sqrt{2\mathcal{B}^{00}\mathcal{P}^{+-}}+a_{CP}^{+-}\sqrt{\mathcal{B}^{+-}\mathcal{P}^{00}}\cos\delta_d\right)^2}{\mathcal{B}^{+-}\mathcal{P}^{00}\sin^2\delta_d}}\,.$$

Numerical results

Direct CP Asymmetries		
a_{CP}^{+0}	$+0.004 \pm 0.008$	[79–82]
a_{CP}^{00}	-0.0002 ± 0.0064	[a][79, 83, 84]
a_{CP}^{+-}	0.00232 ± 0.00061	[2]
Branching Ratios		
$\mathcal{B}(D^0 \to \pi^+\pi^0)$	$(1.247 \pm 0.033) \cdot 10^{-3}$	[85]
$\mathcal{B}(D^0 \to \pi^+\pi^-)$	$(1.454 \pm 0.024) \cdot 10^{-3}$	[85]
$\mathcal{B}(D^0 \to \pi^0\pi^0)$	$(8.26 \pm 0.25) \cdot 10^{-4}$	[85]
Further Numerical Inputs		
$\mathrm{Im}\,(\lambda_b/(-\lambda_d))$	$(-6.1 \pm 0.3) \cdot 10^{-4}$	[85]

TABLE I. Experimental input data. We use the decay times and masses from Ref. [85]. [a]Our extraction from $A_{CP}(D^0 \to \pi^0\pi^0) = -0.0003 \pm 0.0064$ [79] and $\Delta Y = (-1.0 \pm 1.1 \pm 0.3) \cdot 10^{-4}$ [52].

a_{CP}^{+-}	$(2.32 \pm 0.07) \cdot 10^{-3}$
a_{CP}^{00}	$(-2 \pm 9) \cdot 10^{-4}$

TABLE II. Future data scenario employing the current central values and using prospects for the errors from Table 6.5 of Ref. [86] (300 fb^{-1}) and Table 122 of Ref. [87] (50 ab^{-1}) for $D^0 \to \pi^+\pi^-$ and $D^0 \to \pi^0\pi^0$, respectively. All other input data is left as specified in Table I.

Parameter	Current data	Future data scenario
r_t	3.43 ± 0.06	3.43 ± 0.06
$\cos\delta_t$	0.06 ± 0.02	0.06 ± 0.02
$\cos\delta_d$	-0.68 ± 0.01	-0.68 ± 0.01
$\|\sin\delta^{00}\|$	0^{+1}_{-0}	$0.06^{+0.20}_{-0.06}$
$\|\sin\delta^{+-}\|$	$0.7^{+0.3}_{-0.5}$	$0.69^{+0.21}_{-0.16}$
r^{00}	$5.2^{+13.3}_{-2.4}$	$5.2^{+1.6}_{-1.2}$
r^{+-}	$5.5^{+14.2}_{-2.7}$	$5.5^{+1.8}_{-1.3}$

TABLE III. Numerical results for current and hypothetical future data. In the future data scenario, the results for r_t, $\cos\delta_t$ and $\cos\delta_d$ are identical to the ones with current data, as these depend only on the branching ratio data which is not modified in the future data scenario compared to current data. Furthermore, in the future data scenario $\sin\delta^{+-} < 0$. The overall additional relative systematic uncertainty of $\mathcal{O}(10\%)$ due to the universality assumption of ΔY for the extraction of the direct CP asymmetries comes on top of the errors shown here, see text for details.

Hypothesis	Current data
$r^{+-} = 1.0$	2.7σ
$r^{+-} = 0.1$	3.7σ
$r^{00} = 1.0$	2.6σ
$r^{00} = 0.1$	3.7σ
$p_{1/2} = 0$	3.8σ

TABLE IV. Test of benchmark hypotheses and significance of their rejection for current data. In the considered future data scenario all hypotheses listed here are rejected at $> 5\sigma$. In order to account for the overall $\mathcal{O}(10\%)$ relative systematic uncertainty due to the assumption of a universal ΔY for the extraction of the direct CP asymmetries, we multiply the hypotheses for r^f by a factor 1.10, resulting in a more conservative (lower) significance of rejection, see text for details.