On electroweak corrections to neutral current Drell-Yan

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Based on Chiesa, Del Pio, Piccinini, Eur. Phys. J. C 84, 539 (2024)

arXiv:2402.14659 [hep-ph]

UNIVERSITÀ DI PAVIA

LHCP Boston 2024

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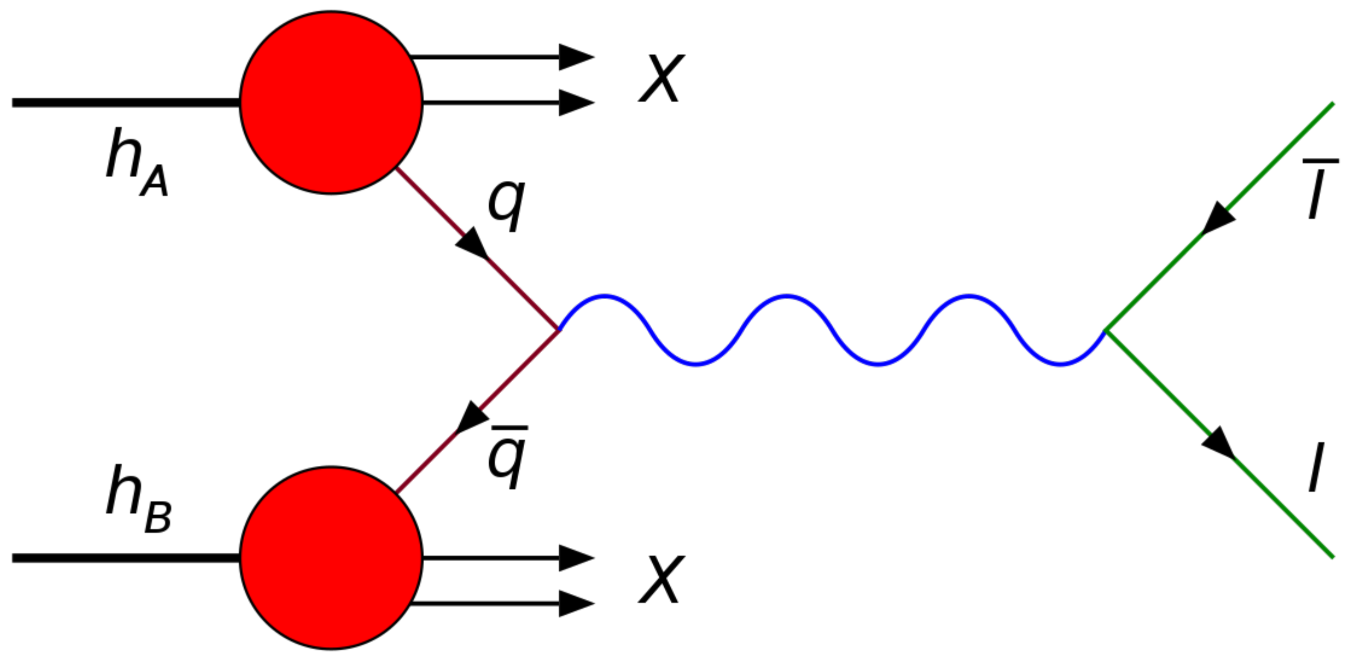
Motivation: on neutral current Drell-Yan

large cross section and clean experimental signature

detector calibration, PDFs constraining

in the high tails of $p_{T,\ell\bar{\ell}}$ and $M_{\ell\bar{\ell}}$ distributions → irreducible background for BSM searches at the LHC

precision tests of EW SM at high energy → determination e.g. $\sin^2\theta_w$



$< 10^{-2}$ experimental precision

theory state-of-art
NNLO QCD + NLO EW

Motivation: on neutral current Drell-Yan

`Z_ew-BMNNPV` package of POWHEG-BOX-V2

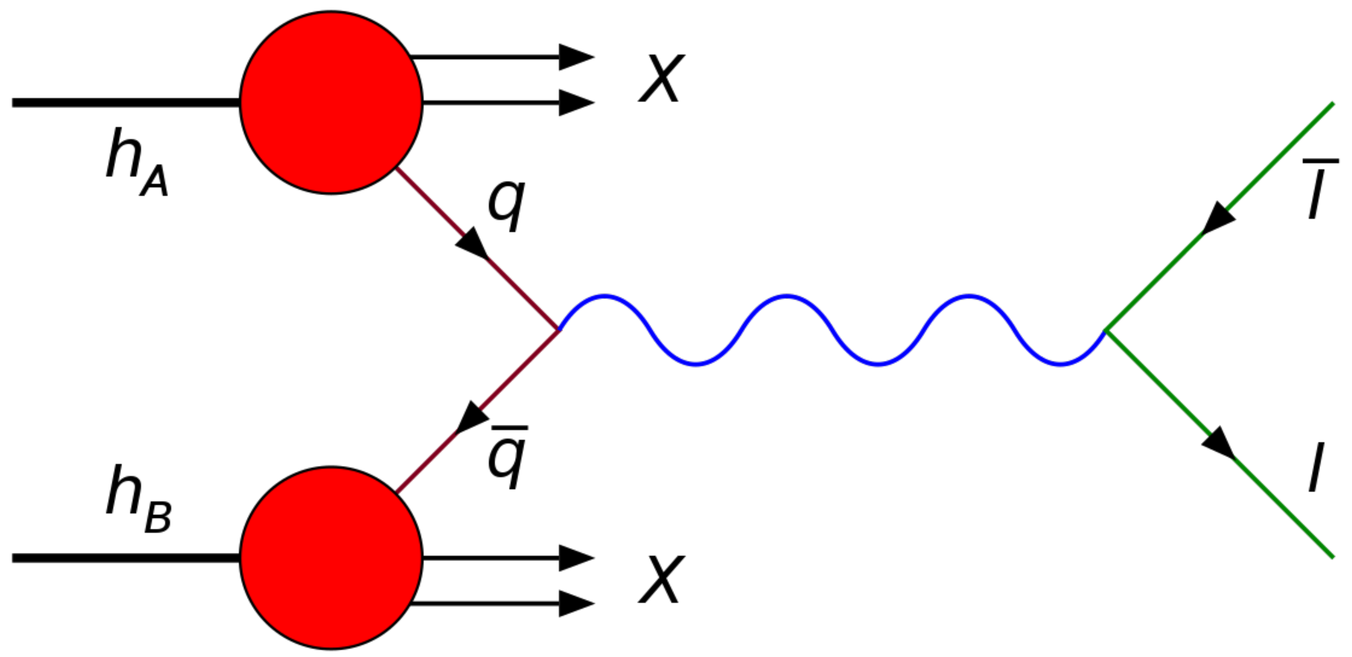
NLO QCD + NLO EW + matching to QED and QCD PS

here EW NLO + leading universal fermionic h.o. corrections

1. Input parameter and renormalization schemes

2. Handling of unstable resonance

3. Treatment of hadronic contribution to $\Delta\alpha$



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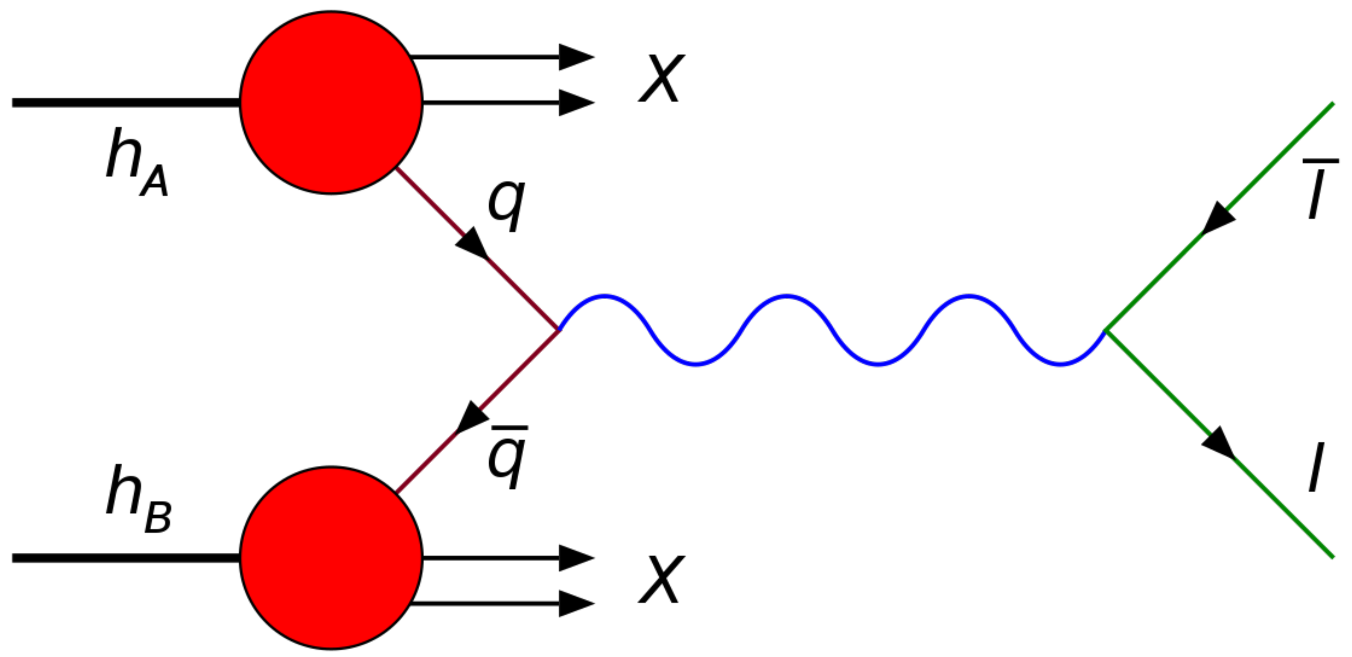

here EW NLO + leading universal fermionic h.o. corrections

1. Input parameter and renormalization schemes ⟵ this talk

2. Handling of unstable resonance

3. Treatment of hadronic contribution to $\Delta\alpha$

Renormalization procedure

$$\mathcal{L} = -\frac{1}{4}W^a_{\mu\nu}W^{\mu\nu}_a - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}G^A_{\mu\nu}G^{\mu\nu}_A$$
$$+ \mathcal{L}_{SSB}$$
$$+ \sum_i (\bar{L}'_i i\not{D} L'_i + \bar{Q}'_i i\not{D} Q'_i) + \sum_i (\bar{l}'^R_i i\not{D} l'^R_i + \bar{u}'^R_i i\not{D} u'^R_i + \bar{d}'^R_i i\not{D} d'^R_i)$$
$$+ \sum_{i,j} (\bar{L}'_i i\, h^l_{ij}\, l'^R_j \Phi + \bar{Q}'_i i\, h^u_{ij}\, u'^R_j \Phi^c + \bar{Q}'_i i\, h^d_{ij}\, d'^R_j \Phi + \mathrm{h.c.})$$
$$+ \mathcal{L}_{fix} + \mathcal{L}_{ghost}$$

EW gauge sector

Renormalization procedure

$$\mathcal{L} = -\frac{1}{4}W^a_{\mu\nu}W^{\mu\nu}_a - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}G^A_{\mu\nu}G^{\mu\nu}_A$$
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$$+ \mathcal{L}_{fix} + \mathcal{L}_{ghost}$$

EW gauge sector

- set of independent Lagrangian parameters (e, M_Z, M_W)
- $e_B = (1 + \delta Z_e)e$ $\quad M^2_{Z,B} = M^2_Z + \delta M^2_Z$ $\quad M^2_{W,B} = M^2_W + \delta M^2_W$
- renormalization conditions to fix the counterterms
- relation between renormalized parameters and input data $(\alpha, M_Z^{exp}, M_W^{exp})$

Input schemes

Chiesa, Del Pio, Piccinini, arXiv:2402.14659 [hep-ph]

Crucial for precise theoretical determinations!

Parametric uncertainties	$\alpha_0 \quad G_\mu \quad M_Z$
Convergence of perturbative series	$\alpha_0 \,/\, \alpha(M_Z^2) \,/\, G_\mu \quad M_W \quad M_Z$
Direct determinations of parameters	$\alpha_0 \,/\, \alpha(M_Z^2) \,/\, G_\mu \quad \sin^2\theta_{eff}^{\ell} \quad M_Z$ $\alpha(\mu^2) \quad \sin^2\theta_w(\mu) \quad M_Z$

$\alpha_0 \,/\, \alpha(M_Z^2) \,/\, G_\mu,\ M_W,\ M_Z$

Böhm, Denner, Joos, Vieweg+Teubner Verlag, 2001

$\Delta\alpha$ and Δr

$$\delta M_W^2 = \mathrm{Re}\,\Sigma_T^W(M_W^2)$$

$$\delta M_Z^2 = \mathrm{Re}\,\Sigma_T^{ZZ}(M_Z^2)$$

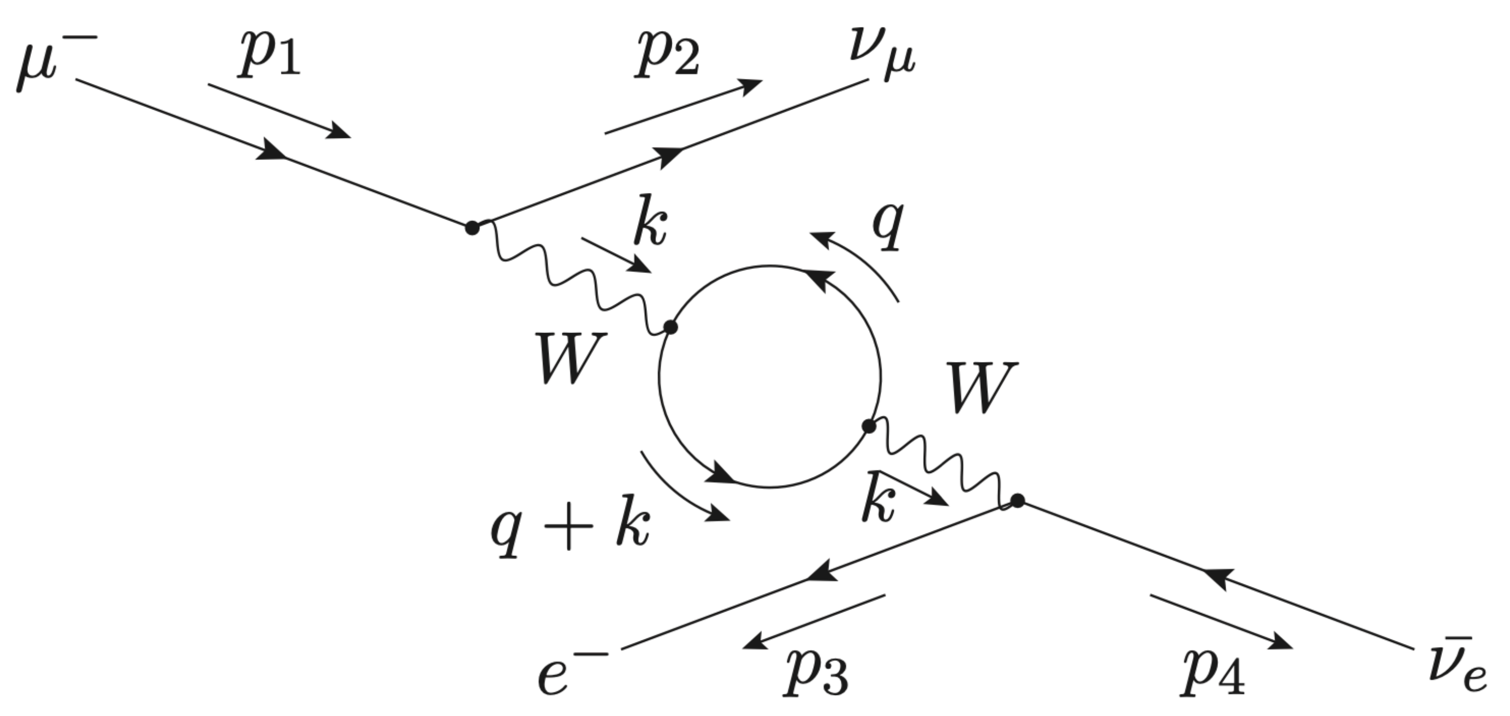
α_0
$$\delta Z_e = \frac{1}{2}\frac{\partial \Sigma_T^{AA}(k^2)}{\partial k^2}\bigg|_{k^2=0} - \frac{s_w}{c_w}\frac{\Sigma_T^{AZ}(0)}{M_Z^2}$$

$\alpha(M_Z^2)$
$$\delta Z_e \to \delta Z_e - \frac{\Delta\alpha(M_Z^2)}{2}$$

G_μ
$$\delta Z_e \to \delta Z_e - \frac{\Delta r}{2}$$

$$\Delta r = \Delta\alpha - \frac{c_w^2}{s_w^2}\Delta\rho + \Delta r_{rem}$$

$$\Delta\rho = \frac{\Sigma_T^{ZZ}(0)}{M_Z^2} - \frac{\Sigma_T^{W}(0)}{M_W^2} = \frac{\alpha}{4\pi}\frac{3}{4s_w^2}\frac{m_t^2}{M_W^2} + \ldots = \Delta\rho^{1-\mathrm{loop}} + \ldots$$



α_0 / $\alpha(M_Z^2)$ / G_μ, s^2_{eff}, M_Z

Chiesa, Piccinini, Vicini, Phys. Rev. D 100 071302, 2019

$\Delta\alpha$ and $\Delta\tilde{r}$

$$\delta M_W^2 \to \frac{\delta s^2_{eff}}{s^2_{eff}} = \mathrm{Re}\left\{ -\frac{1}{2}\frac{c^2_{eff}}{s^2_{eff}}\delta Z_{AZ} + \left(1 - \frac{Q_\ell}{I_3^\ell} s^2_{eff}\right)[\delta Z_L^\ell + \delta V^L - \delta Z_R^\ell - \delta V^R] \right\}$$

$$\delta M_Z^2 = \mathrm{Re}\, \Sigma_T^{ZZ}(M_Z^2)$$

α_0 $$\delta Z_e = \frac{1}{2}\frac{\partial \Sigma_T^{AA}(k^2)}{\partial k^2}\bigg|_{k^2=0} - \frac{s_w}{c_w}\frac{\Sigma_T^{AZ}(0)}{M_Z^2}$$

$\alpha(M_Z^2)$ $$\delta Z_e \to \delta Z_e - \frac{\Delta\alpha(M_Z^2)}{2}$$

G_μ $$\delta Z_e \to \delta Z_e - \frac{\Delta\tilde{r}}{2}$$

$$\Delta\tilde{r} = \Delta\alpha - \Delta\rho + \Delta\tilde{r}_{rem}$$

α_0, G_μ, M_Z

$$\alpha(M_Z^2) = \frac{\alpha_0}{1 - \Delta\alpha(M_Z^2)}$$

azinscheme4

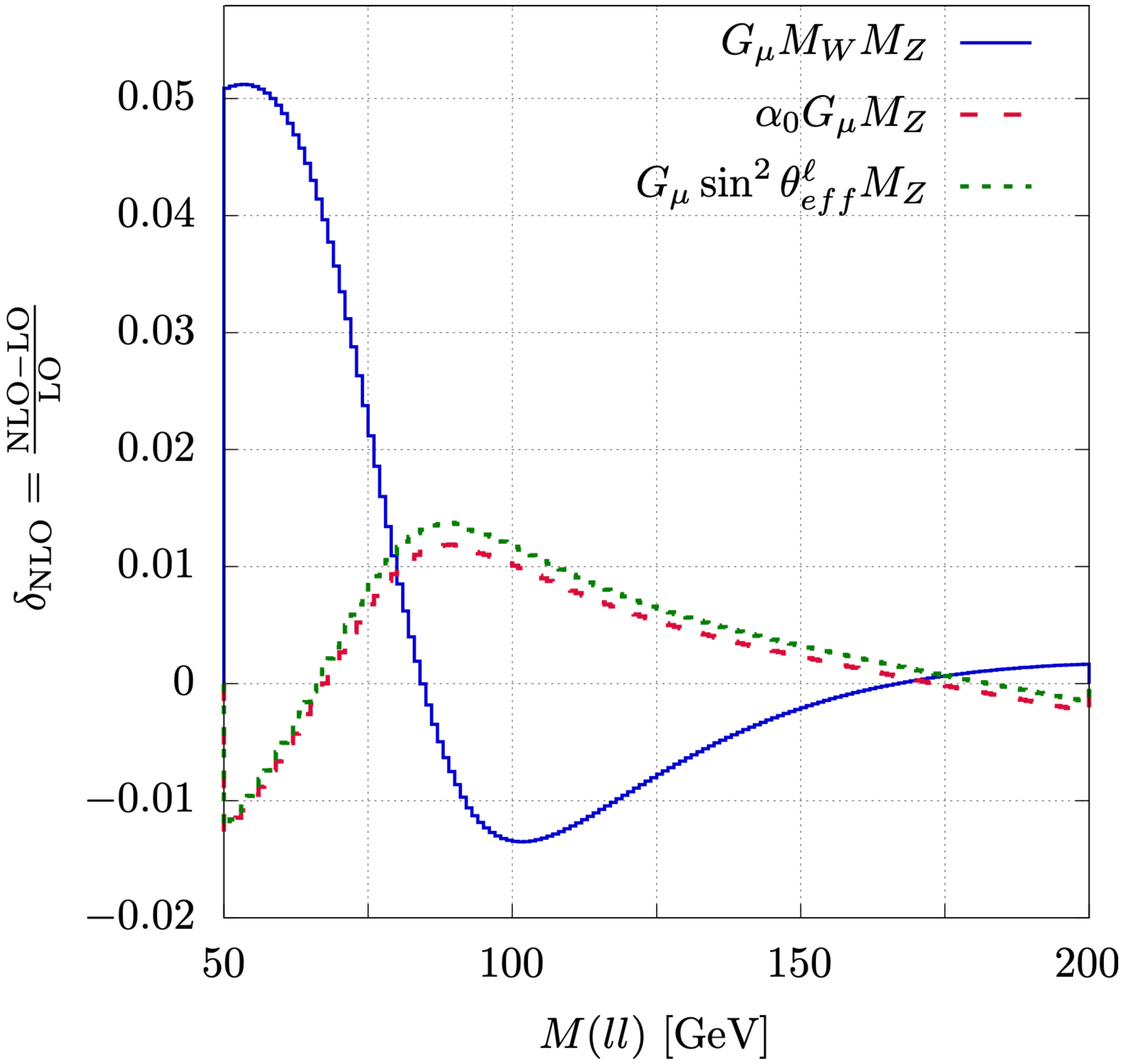
$$\delta Z_e\,, \quad \delta M_Z^2$$

$$\frac{\delta G_\mu}{G_\mu} = -\frac{2}{s_w c_w}\frac{\Sigma_T^{AZ}(0)}{M_Z^2} - \frac{\Sigma_T^W(0)}{M_W^2} - \frac{\alpha}{4\pi s_w^2}\left(6 + \frac{7-4s_w^2}{2s_w^2}\log c_w^2\right)$$

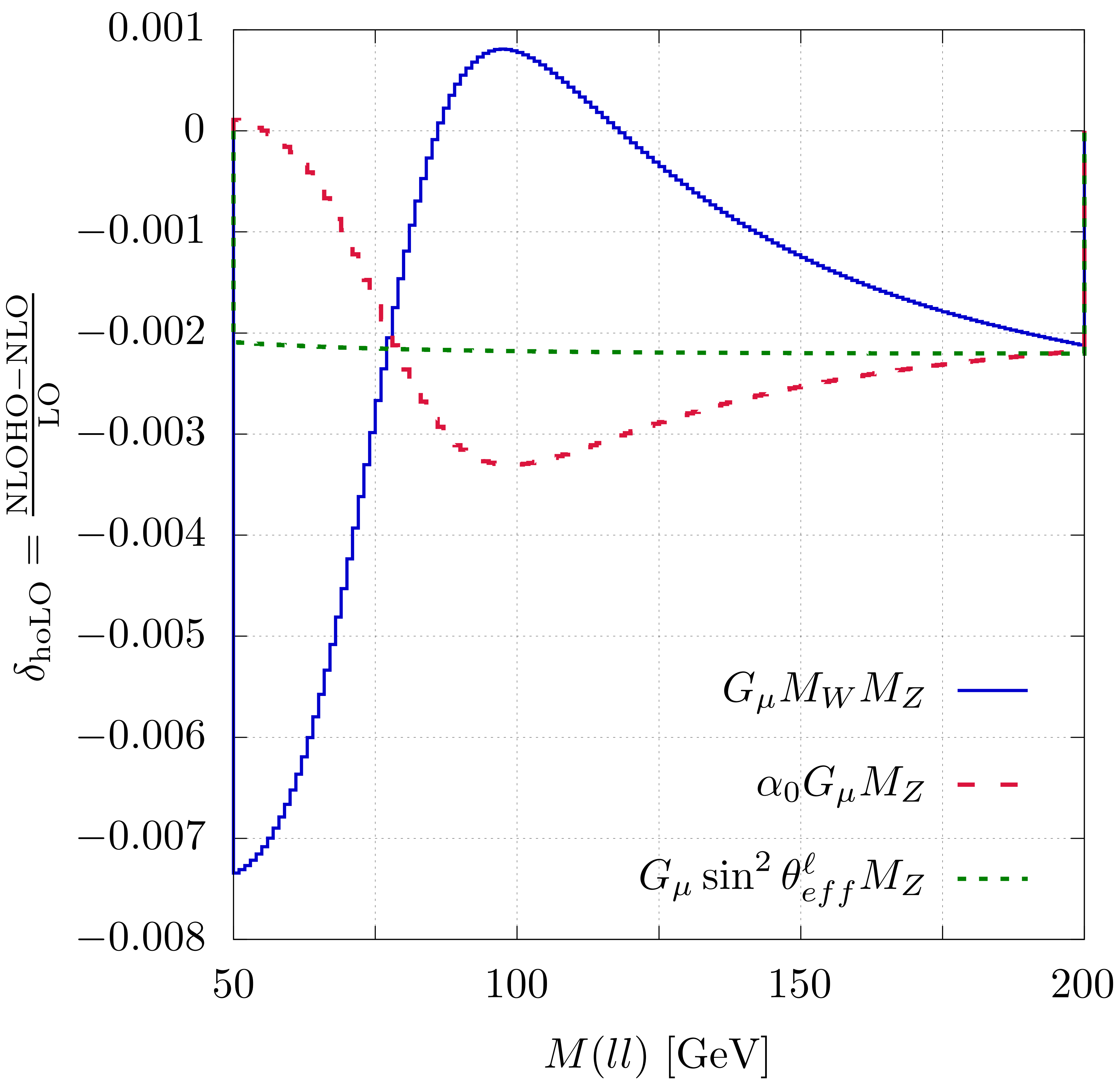
$$\sin^2\theta_{eff}^{\ell} = \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{\pi\alpha}{\sqrt{2}G_\mu M_Z^2}\left(1 + \Delta\tilde{r}_{\rm HO}\right)}$$

To calculate h.o.

To tune s_{eff}^2 in LEP-like scheme



Leading universal higher orders



Results here with NLO $\Delta\alpha$ (`dalpha_lep_2loop 0`)

$$\Delta\rho = 3x_t(\Delta\rho^{(1)} + x_t\Delta\rho^{(2)})\left(1 + \frac{\alpha_S}{\pi}\delta^{(2)}_{QCD} + \frac{\alpha_S^2}{\pi^2}\delta^{(3)}_{QCD}\right) + x_t^3\Delta\rho^{x_t^3} + \frac{\alpha_S}{\pi}x_t^2\Delta\rho^{x_t^2\alpha_S} - 3x_t^2\Delta\rho^{(2)}\frac{\alpha_S^2}{\pi^2}\delta^{(2)}_{QCD}$$

Veltman, Nucl. Phys. B 123, 89–99, 1977 - Fleischer, Tarasov, Jegerlehner, Phys. Lett. B 319, 249–256, 1993
Fleischer, Tarasov, Jegerlehner, Phys. Rev. D 51, 3820–3837, 1995
Djouadi, Verzegnassi, Phys. Lett. B 195, 265–271, 1987 - Djouadi, Nuovo Cim. A 100, 357, 1988
Kniehl, Nucl. Phys. B 347, 86–104, 1996 - Avdeev, Fleischer, Mikhailov, Tarasov, Phys. Lett. B 336, 560–566, 1994
Faisst, Kuhn, Seidensticker, Veretin, Nucl. Phys. B 665, 649–662, 2003

Contributions from

$$\delta Z_e\,, \quad \delta s_w^2\,, \quad \Delta r\,/\,\Delta\tilde{r}\,, \quad \delta G_\mu/G_\mu + \delta M_Z^2/M_Z^2$$

Hybrid $\overline{MS}$ scheme $\alpha(\mu^2)$, $s_w^2(\mu^2)$, M_Z

Erler, Ramsey-Musolf, Phys. Rev. D 72 073003, 2005
Erler, Ferro-Hernández, JHEP 03 196, 2018

$$\delta Z_{e\,\overline{MS}}(\mu^2) = -\frac{\alpha}{4\pi}\Bigg\{\sum_{f=l,q}\frac{N_C^f 2Q_f^2}{3}\left[-\Delta_{\rm UV}+\log\frac{\mu^2}{\mu^2_{\rm Dim}}\right]+\frac{7}{2}\left(\Delta_{\rm UV}-\log\frac{\mu^2}{\mu^2_{\rm Dim}}\right)$$
$$+\delta_{\rm D,\,top}\frac{8}{9}\log\frac{M^2_{\rm top}}{\mu^2}\theta(M^2_{\rm top}-\mu^2)+\delta_{\rm D,\,W}\left[-\frac{7}{2}\log\frac{M^2_{\rm W,\,thr.}}{\mu^2}+\frac{1}{3}\right]\theta(M^2_{\rm W,\,thr.}-\mu^2)\Bigg\}$$

$$\frac{\delta s^2_{W\,\overline{\rm MS}}}{s^2_{W\,\overline{\rm MS}}}(\mu^2) = \frac{c_{W\,\overline{\rm MS}}}{2s_{W\,\overline{\rm MS}}}\left(\delta Z_{ZA\,\overline{\rm MS}}-\delta Z_{AZ\,\overline{\rm MS}}\right)+\delta_{\rm D,\,W}\frac{\alpha}{6\pi}\frac{c^2_{W\,\overline{\rm MS}}}{s^2_{W\,\overline{\rm MS}}}\theta(M^2_{W,\,\rm thr.}-\mu^2)$$

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decouplemtOFF, decouplemwOFF, OFFthreshcorrs
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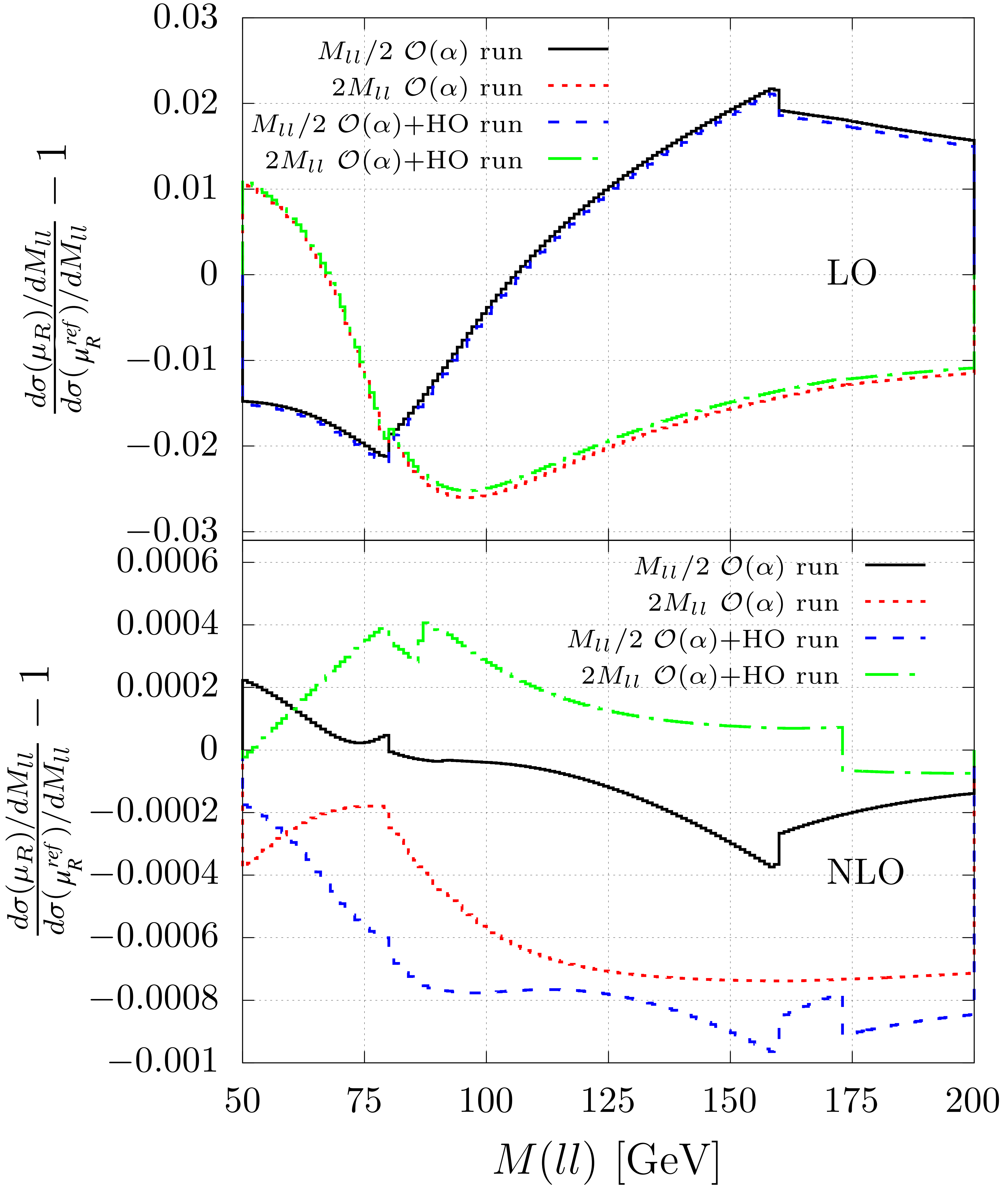
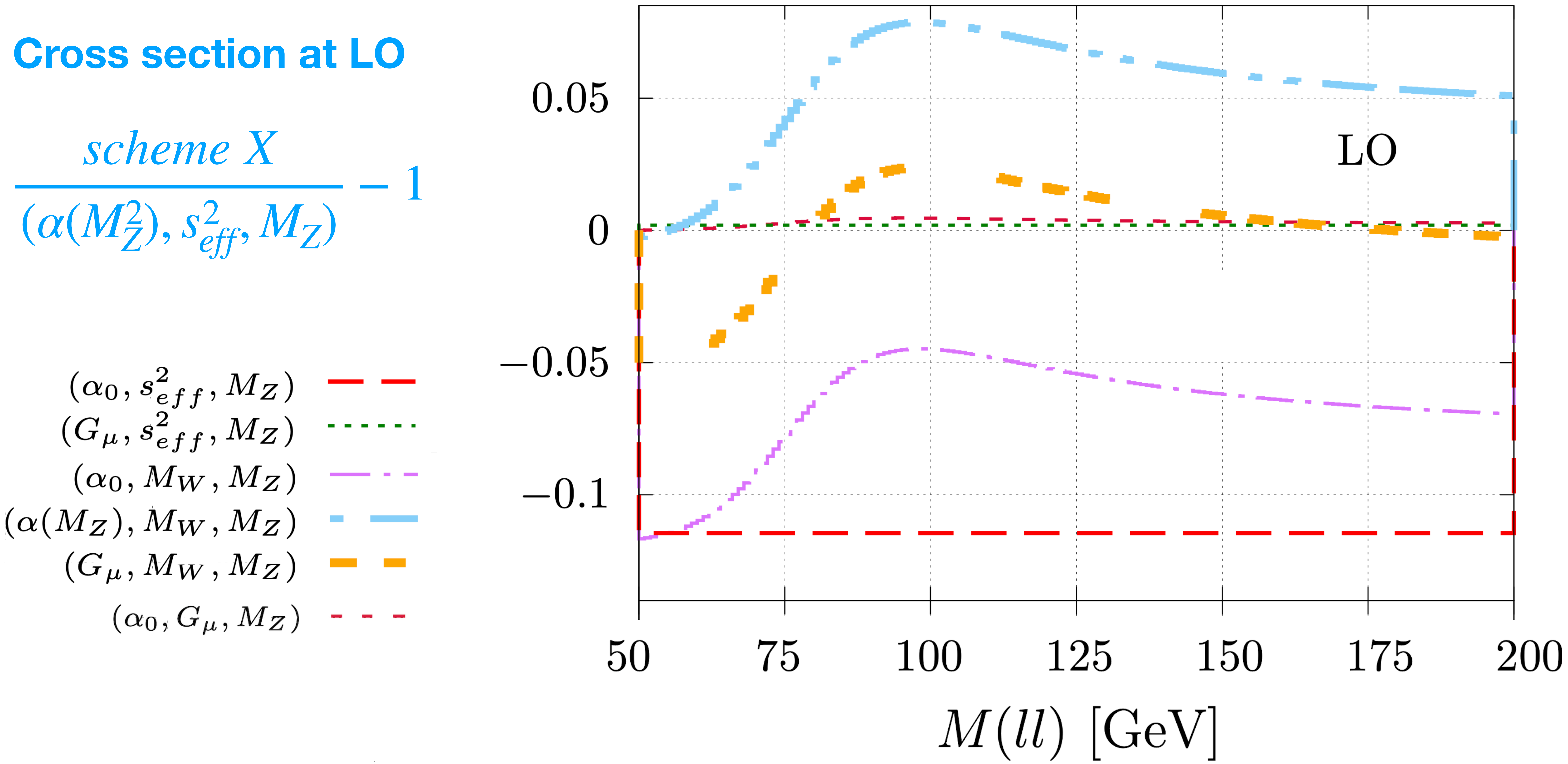


Scheme comparison

Cross section at LO

$$\frac{scheme\ X}{(\alpha(M_Z^2), s_{eff}^2, M_Z)} - 1$$



Scheme comparison

Cross section at NLO

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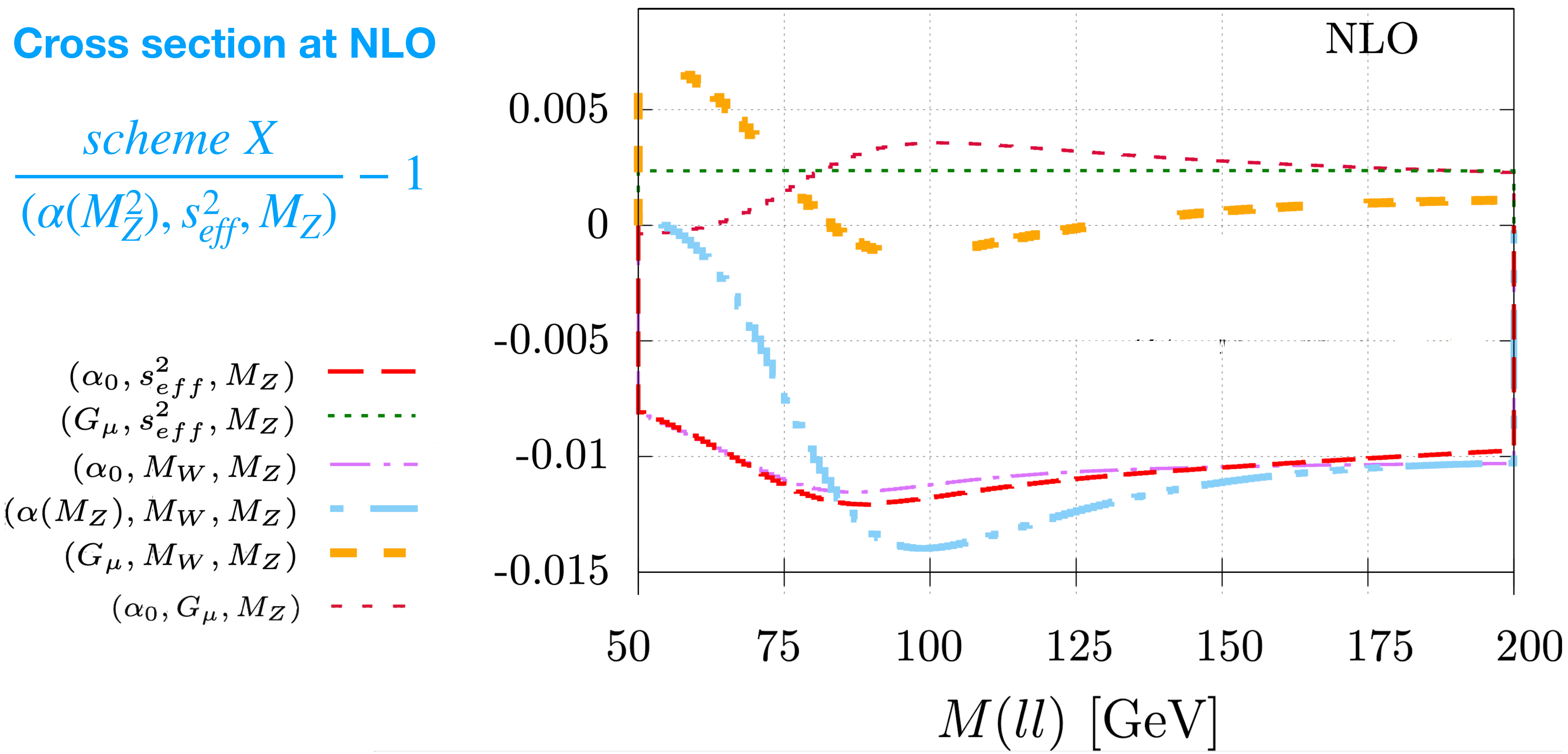
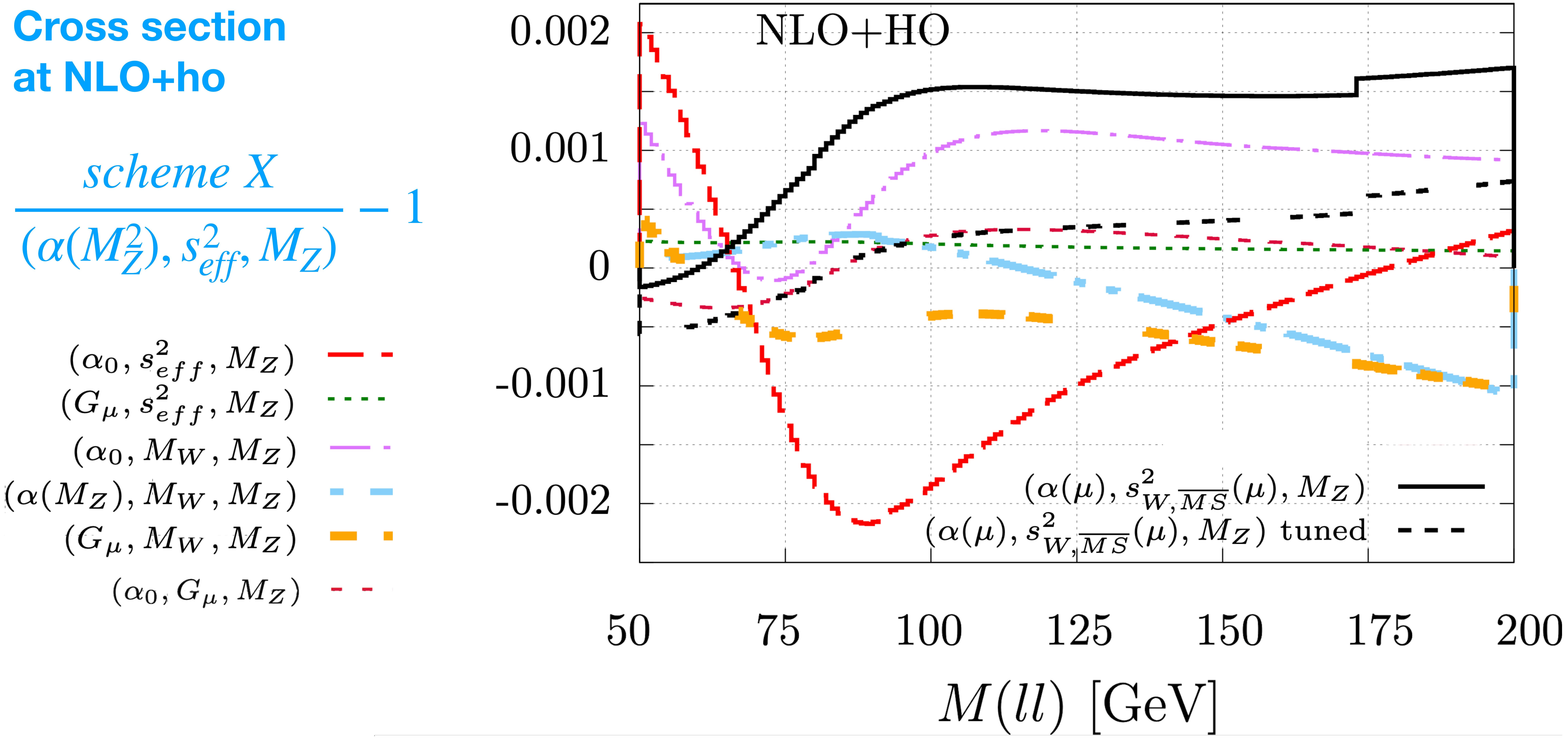


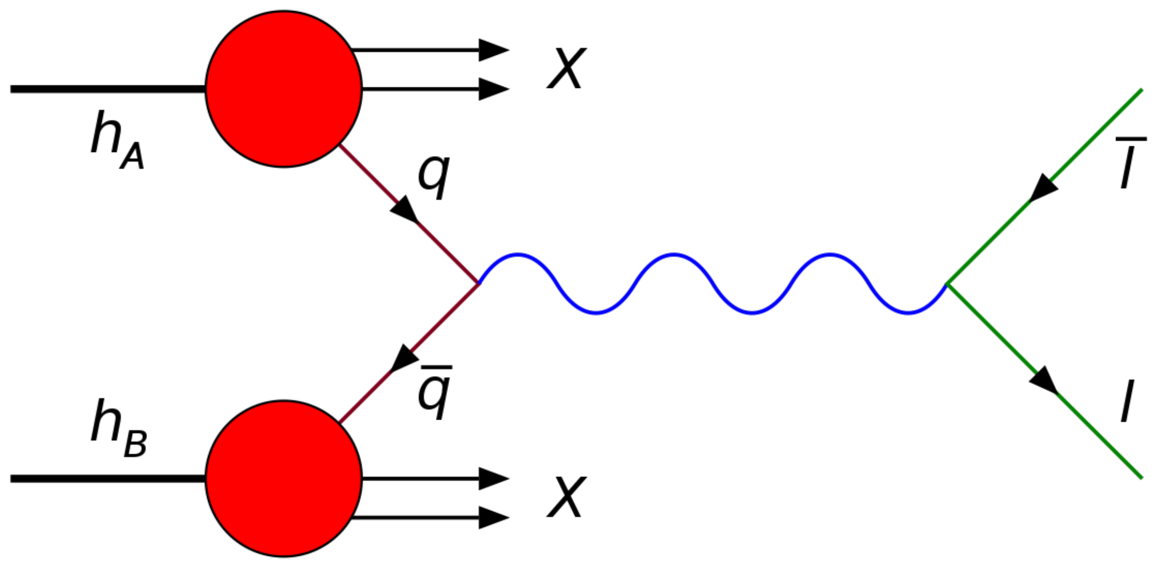
Scheme comparison

Cross section at NLO+ho

$$\frac{scheme\ X}{(\alpha(M_Z^2), s_{eff}^2, M_Z)} - 1$$



Comparison with LEP1

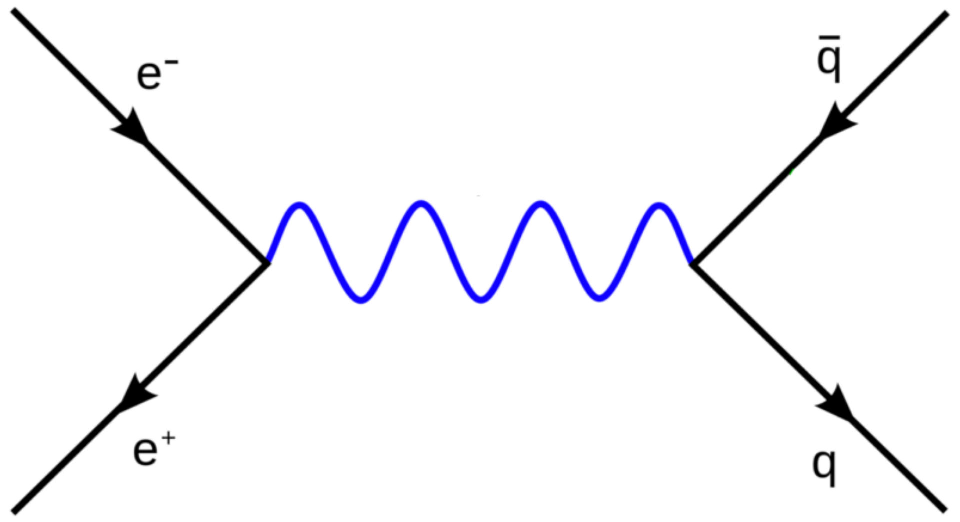


LHC: hadronic machine

Determine s^2_{eff} in Collins-Soper frame with template fit

MC code: different input schemes

$\sim 10^{-3}$ agreement on x-section



LEP: leptonic machine at $\sqrt{s} \sim M_Z$

Extract s^2_{eff} from pseudo-observables

MC codes: all schemes are “tuned realisations” of α_0, G_μ, M_Z

10^{-4} agreement on $\Gamma_{Z\ell\bar{\ell}}$ and x-section

Comparison with LEP1

1. **Tuning** from reference scheme (α_0, G_μ, M_Z)

$$\alpha(M_Z^2) = \frac{\alpha_0}{1-\Delta\alpha}$$

$(\alpha_0, s^2_{eff}, M_Z)$

$$s^2_{eff}\big|_{G_\mu} = \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{\pi}{\sqrt{2}G_\mu M_Z^2}\,\alpha(M_Z^2)\left(1 + \Delta\tilde{r}^{(1)} - \Delta\alpha + \Delta\rho^{(1)} - \Delta\rho\right)}$$

(α_0, M_W, M_Z)

$$M_W\big|_{G_\mu} = \frac{M_Z^2}{2}\left(1 + \sqrt{1 - \frac{\sqrt{8}\pi}{G_\mu M_Z^2}\,\alpha(M_Z^2)\,\frac{1 + \Delta r^{(1)} - \Delta\alpha + \frac{c_W^2}{s_W^2}\Delta\rho^{(1,X)}}{1 + \frac{c_W^2}{s_W^2}\Delta\rho^{(X)}}}\right)$$

$$\Delta\rho^{(1,X)} = \frac{\Sigma_T^{ZZ}(M_Z)}{M_Z^2} - \frac{\Sigma_T^{W}(M_W)}{M_W^2}\bigg|_{fin,\,\mu_{dim}=M_Z}$$

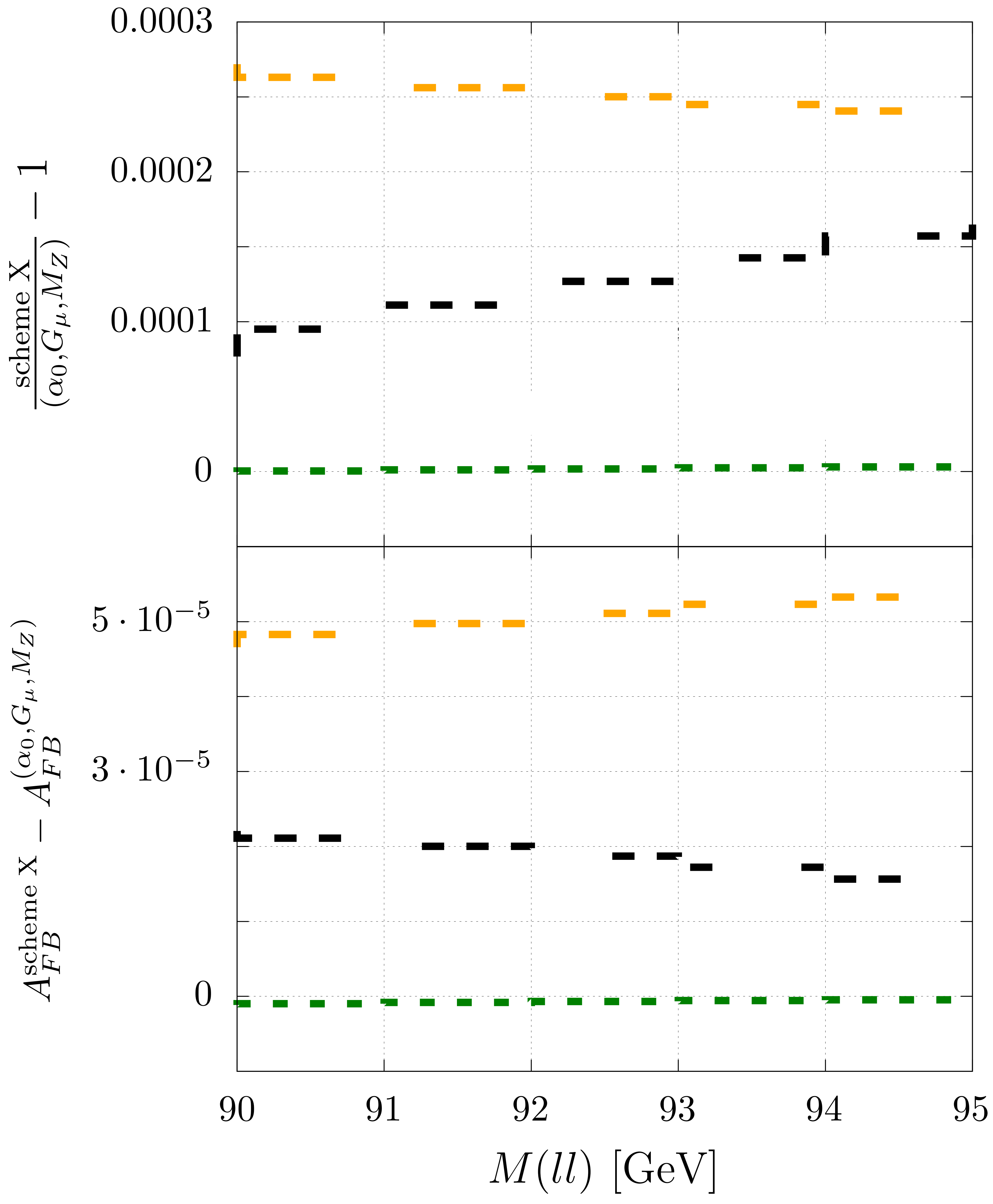
Comparison with LEP1

Tuning from (α_0, G_μ, M_Z)

$(G_\mu, \sin^2\theta^l_{eff}, M_Z)$ (green dashes)

(G_μ, M_W, M_Z) (orange dashes)

$(\alpha(\mu), s^2_{W,\overline{MS}}(\mu), M_Z)$ tuned (black dashes)



Outlook

- Particle physics at the next frontier at LHC and HL-LHC: **high-precision determinations** of EW SM parameters from NC DY within `Z_ew-BMNNPV`

- **Input scheme** featuring the parameter to be measured and scheme comparison for $d\sigma/dM_{ll}$ and A_{FB}

✦ Interfacing the code to **existing tools at NNLO accuracy** at the Z peak

✦ EW corrections to DIS at future EIC and LHeC

Thank you!

Back-up

Sudakov regime

Channel with d-quarks only: no PDFs dependence
→ no large unphysical distortions at high energies

Sudakov logs

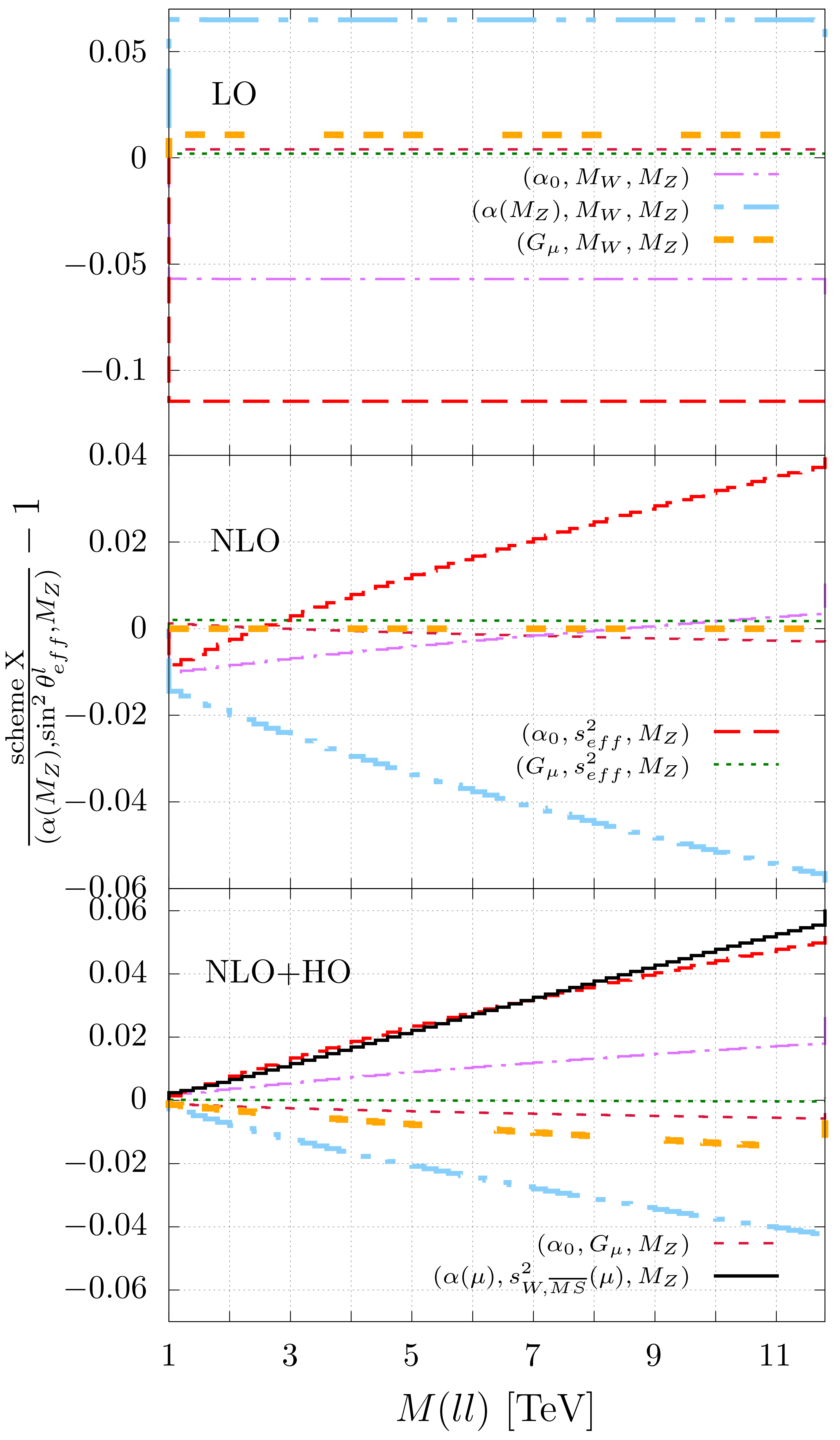
$$A(\alpha, s_w^2)\ \ln^2 \frac{s}{M_Z^2} + B(\alpha, s_w^2)\ \ln \frac{s}{M_Z^2}$$

Parameter renormalization logs

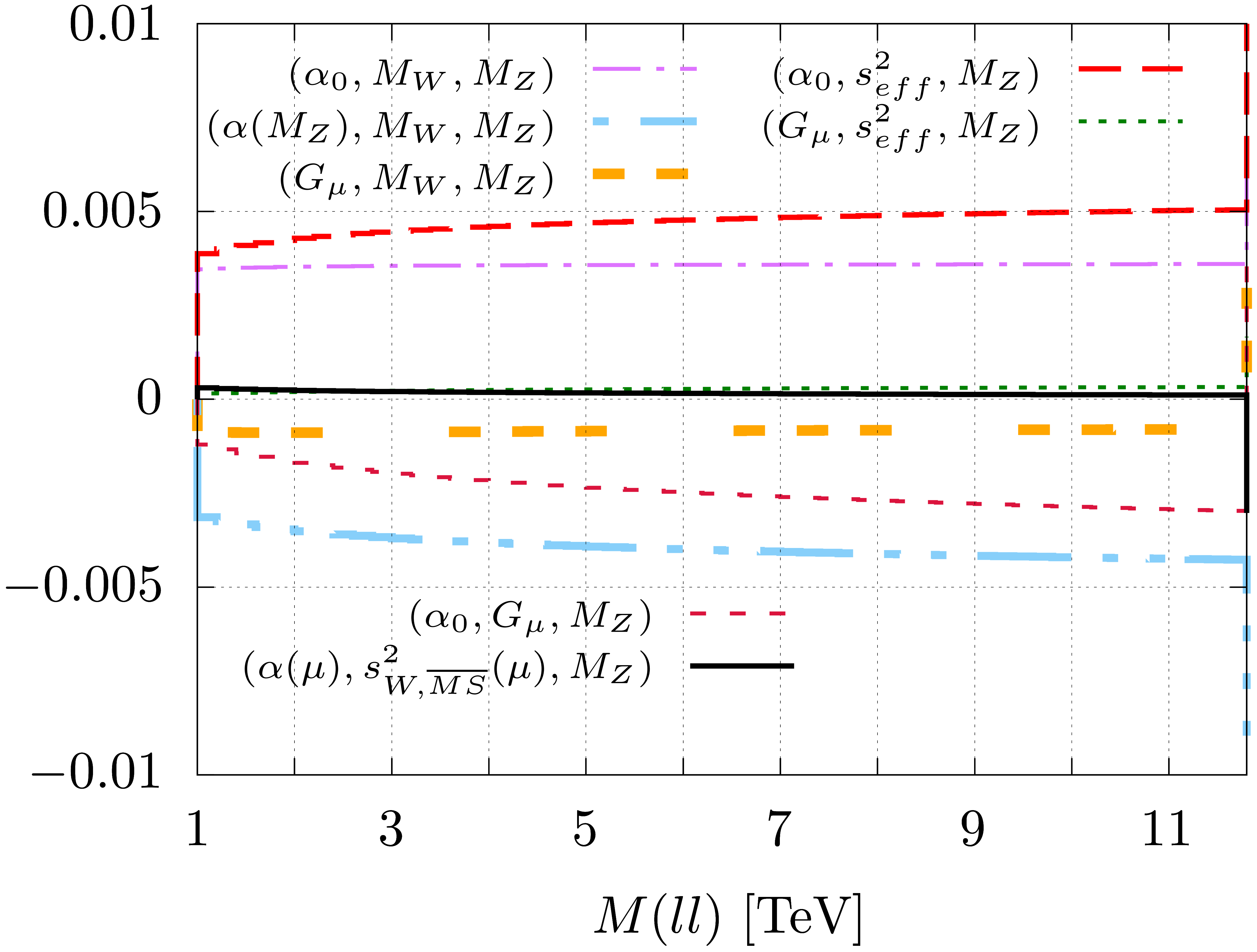
$$\frac{1}{\epsilon} - \ln \frac{r_{ct}^2}{\mu_{dim}^2} - \frac{1}{\epsilon} + \ln \frac{r_{bare}^2}{\mu_{dim}^2} = \ln \frac{r_{bare}^2}{r_{ct}^2} \sim \ln \frac{M_{ll}^2}{m^2}$$

Counterterms from parameter renorm.

Bare diagrams



Sudakov regime



Cross section at NLO+ho - (Sudakov + param. renorm. logs)

$$\frac{scheme\ X}{(\alpha(M_Z^2), s^2_{eff}, M_Z)} - 1$$

$\overline{MS}$ no-running: logs from running already subtracted at denominator

Hybrid $\overline{MS}$ scheme $\alpha(\mu^2)$, $s_w^2(\mu^2)$, M_Z

Erler, Ramsey-Musolf, Phys. Rev. D 72 073003, 2005
Erler, Ferro-Hernández, JHEP 03 196, 2018

$$\delta Z_{e\,\overline{MS}}(\mu^2) = -\frac{\alpha}{4\pi}\Bigg\{\sum_{f=l,q}\frac{N_C^f 2Q_f^2}{3}\left[-\Delta_{\rm UV} + \log\frac{\mu^2}{\mu^2_{\rm Dim}}\right] + \frac{7}{2}\left(\Delta_{\rm UV} - \log\frac{\mu^2}{\mu^2_{\rm Dim}}\right)$$
$$+\delta_{\rm D,\,top}\frac{8}{9}\log\frac{M^2_{\rm top}}{\mu^2}\theta(M^2_{\rm top}-\mu^2) + \delta_{\rm D,\,W}\left[-\frac{7}{2}\log\frac{M^2_{W,\,\rm thr.}}{\mu^2} + \frac{1}{3}\right]\theta(M^2_{W,\,\rm thr.}-\mu^2)\Bigg\}$$

$$\frac{\delta s^2_{W\,\overline{\rm MS}}}{s^2_{W\,\overline{\rm MS}}}(\mu^2) = \frac{c_{W\,\overline{\rm MS}}}{2s_{W\,\overline{\rm MS}}}\left(\delta Z_{ZA\,\overline{\rm MS}} - \delta Z_{AZ\,\overline{\rm MS}}\right) + \delta_{\rm D,\,W}\frac{\alpha}{6\pi}\frac{c^2_{W\,\overline{\rm MS}}}{s^2_{W\,\overline{\rm MS}}}\theta(M^2_{W,\,\rm thr.}-\mu^2)$$

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decouplemtOFF, decouplemwOFF, OFFthreshcorrs
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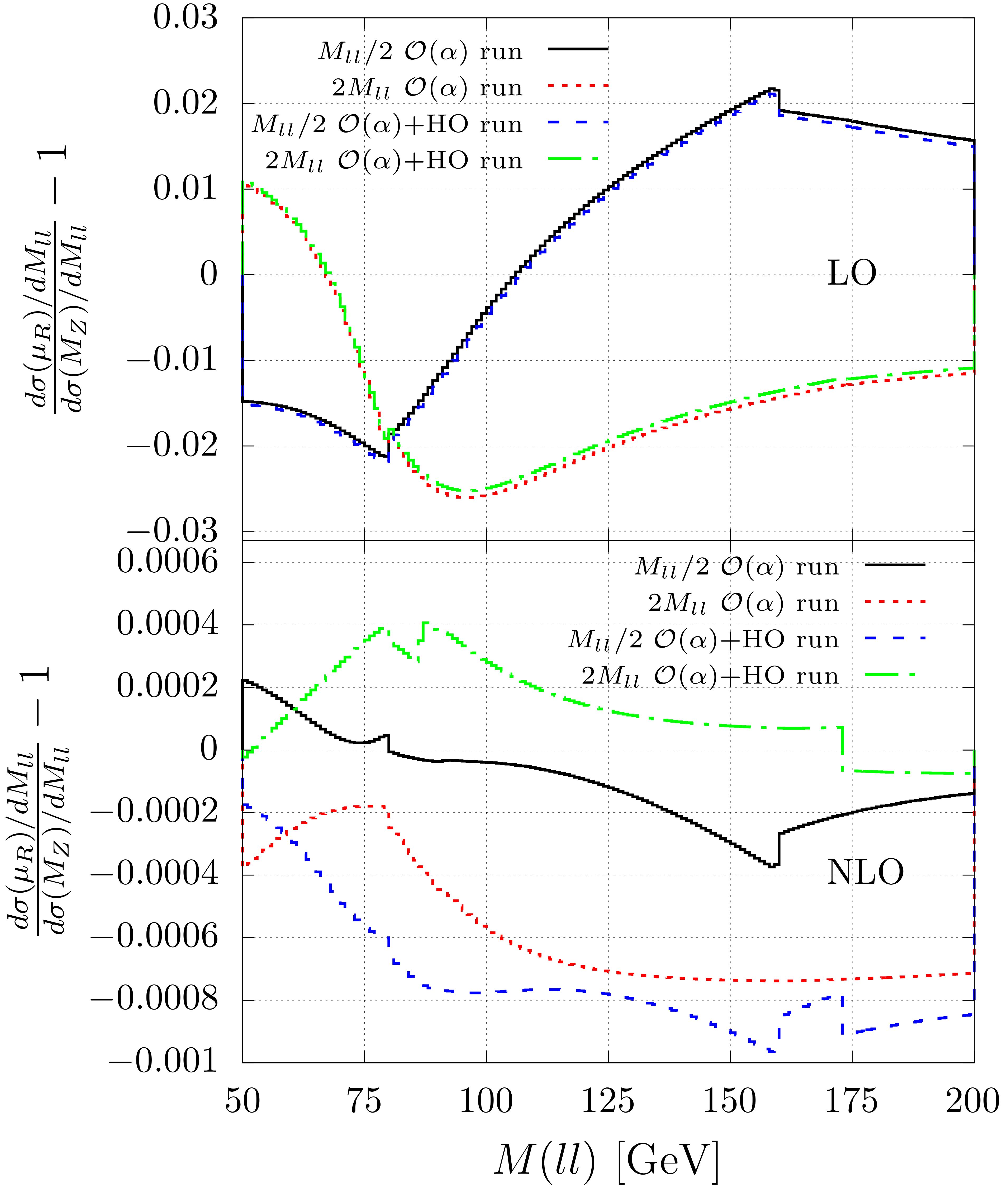
Precise prediction of $\overline{MS}$ parameters

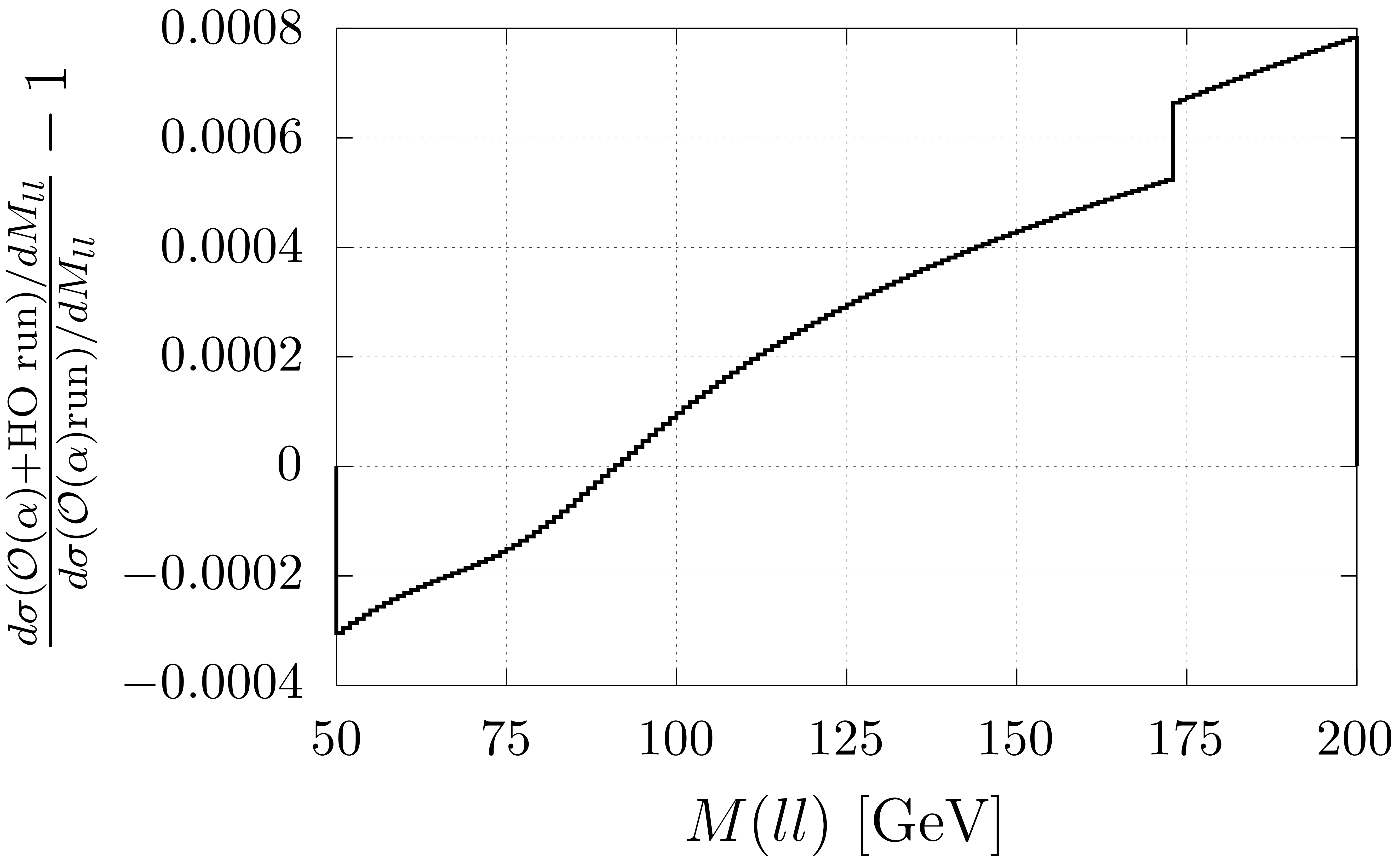
$$\alpha_{\overline{\rm MS}}(M_Z^2) = \frac{\alpha_0}{1-\Delta\hat{\alpha}(M_Z^2)}$$

$$s^2_{W\,\overline{\rm MS}}(M_Z^2) = \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{\pi\alpha_0}{\sqrt{2}G_\mu M_Z^2(1-\Delta\hat{\alpha}(M_Z^2))}\left(1+\Delta r_{\overline{\rm MS},\,\rm HO}\right)}$$



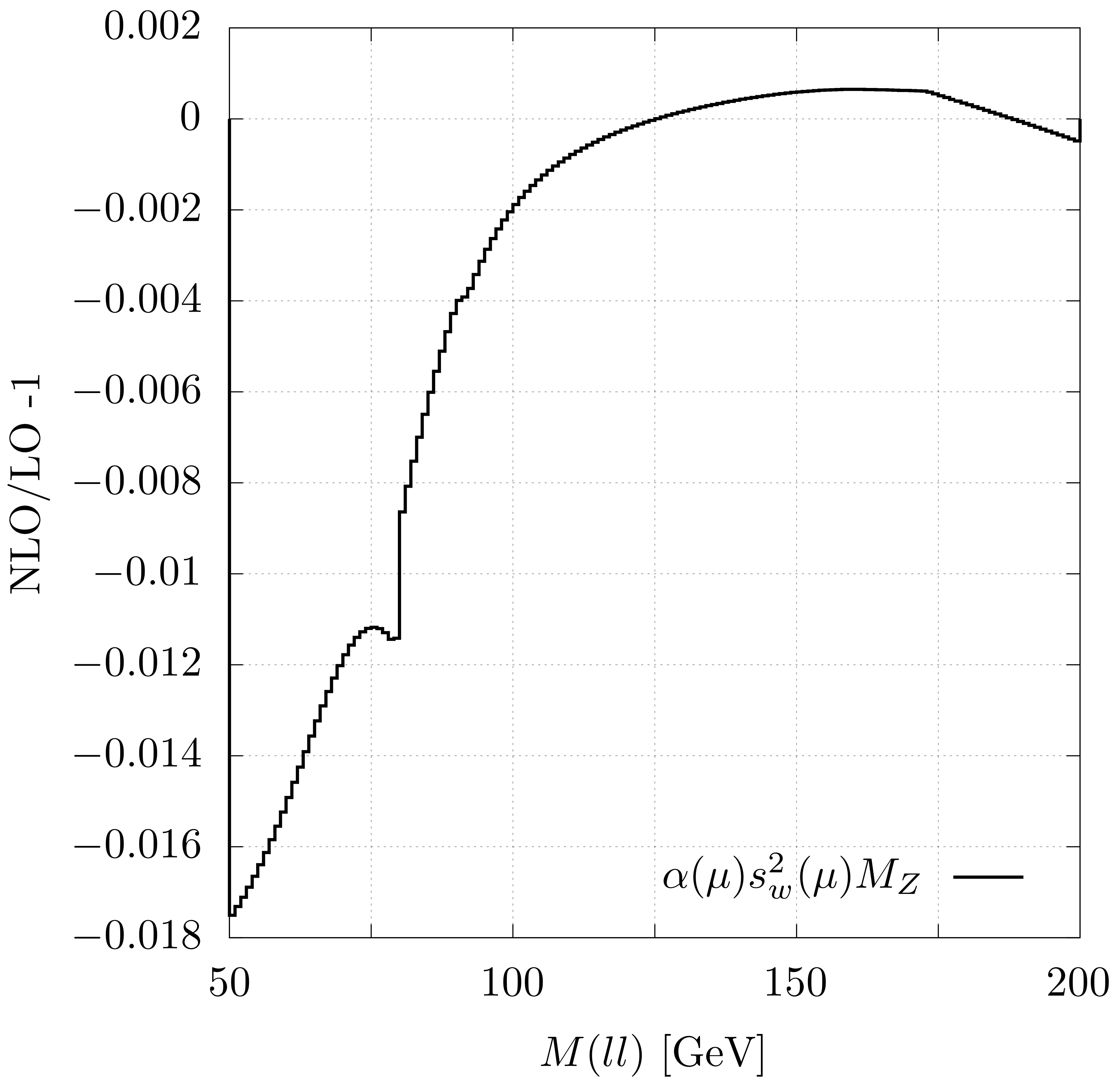
Hybrid $\overline{MS}$ scheme $\alpha(\mu^2)$, $s_w^2(\mu^2)$, M_Z

Higher order effects in running - NLO cross section



Hybrid $\overline{MS}$ scheme $\alpha(\mu^2)$, $s_w^2(\mu^2)$, M_Z

NLO effects on cross section - NLO+ho running

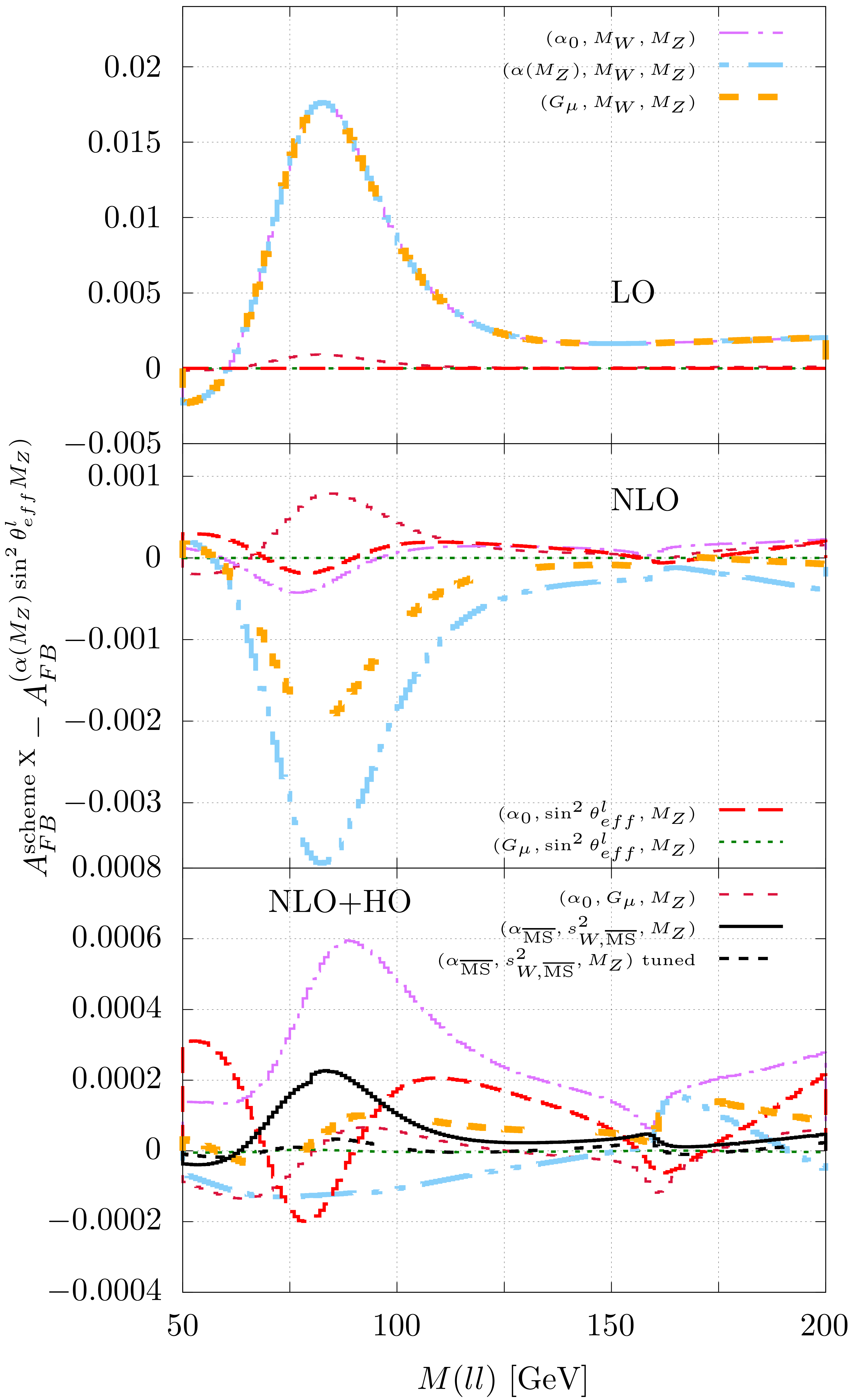


Scheme comparison

Asymmetry

scheme X $- (\alpha(M_Z^2), s_{eff}^2, M_Z)$

Overall effects cancelled out!



Comparison with LEP1

2. **Higher orders** in Born Improved Approximation with $\alpha(M_Z^2)$ and s^2_{eff}

(α_0, G_μ, M_Z)

$$\tilde{s}^2_{w,\,\mathrm{LO}} = \frac{1}{2}\frac{-g_R}{g_L - g_R}$$

$$\tilde{s}^2_{w,\,\mathrm{NLO}} = \frac{1}{2}\frac{-g_R}{g_L - g_R} + \frac{1}{2}\frac{g_L g_R}{(g_L - g_R)^2}\mathrm{Re}\left(\frac{\delta g_L}{g_L} - \frac{\delta g_R}{g_R}\right)$$

(α_0, M_W, M_Z)

$(\alpha_0, s^2_{eff}, M_Z)$

Comparison with LEP1

2. **Higher orders** in Born Improved Approximation with $\alpha(M_Z^2)$ and s_{eff}^2

(α_0, G_μ, M_Z)

$$\tilde{s}^2_{w,\,\mathrm{NLO+HO}} = \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{\pi\alpha}{\sqrt{2}G_\mu M_Z^2}(1+\Delta\tilde{r}|_{s_w^2})}$$

(α_0, M_W, M_Z)

$$\tilde{s}^2_{w,\,\mathrm{NLO+HO}} = s_w^2\left(1+\frac{c_w^2}{s_w^2}\Delta\rho^{(X)}\right)\left[1-\frac{c_w^2}{s_w^2}\Delta\rho^{(1,X)}+\frac{1}{s_w^2}\frac{1}{2}\frac{g_L g_R}{(g_L-g_R)^2}\mathrm{Re}\left(\frac{\delta g_L}{g_L}-\frac{\delta g_R}{g_R}\right)\right]$$

$(\alpha_0, s_{eff}^2, M_Z)$ already ok

(α_0, G_μ, M_Z)

$$\overline{\sin}^2\theta^\ell_{eff} = s_W^2\left(1 + \frac{\delta s_w^2}{s_w^2} - \frac{\delta s_{eff}^2}{s_w^2}\right) = s_w^2 + \frac{s_w^2 c_w^2}{c_w^2 - s_w^2}\Delta\tilde{r} = \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{\pi\alpha}{\sqrt{2}G_\mu M_Z^2}} + \frac{1}{2}\frac{\frac{\pi\alpha}{\sqrt{2}G_\mu M_Z^2}}{\sqrt{\frac{1}{4} - \frac{\pi\alpha}{\sqrt{2}G_\mu M_Z^2}}}\Delta\tilde{r}$$

Comparison with LEP1

$\Gamma_{Z\ell\bar{\ell}}$

Observable	Exp.	Theor. Predictions	Average
Γ_l (MeV)	83.96 ± 0.18	BHM $83.919^{+0.020}_{-0.013}$ LEPTOP $83.930^{+0.023}_{-0.023}$ TOPAZ0 $83.931^{+0.015}_{-0.012}$ WOH $83.943^{+0.022}_{-0.022}$ ZFITTER $83.941^{+0.013}_{-0.021}$	83.933

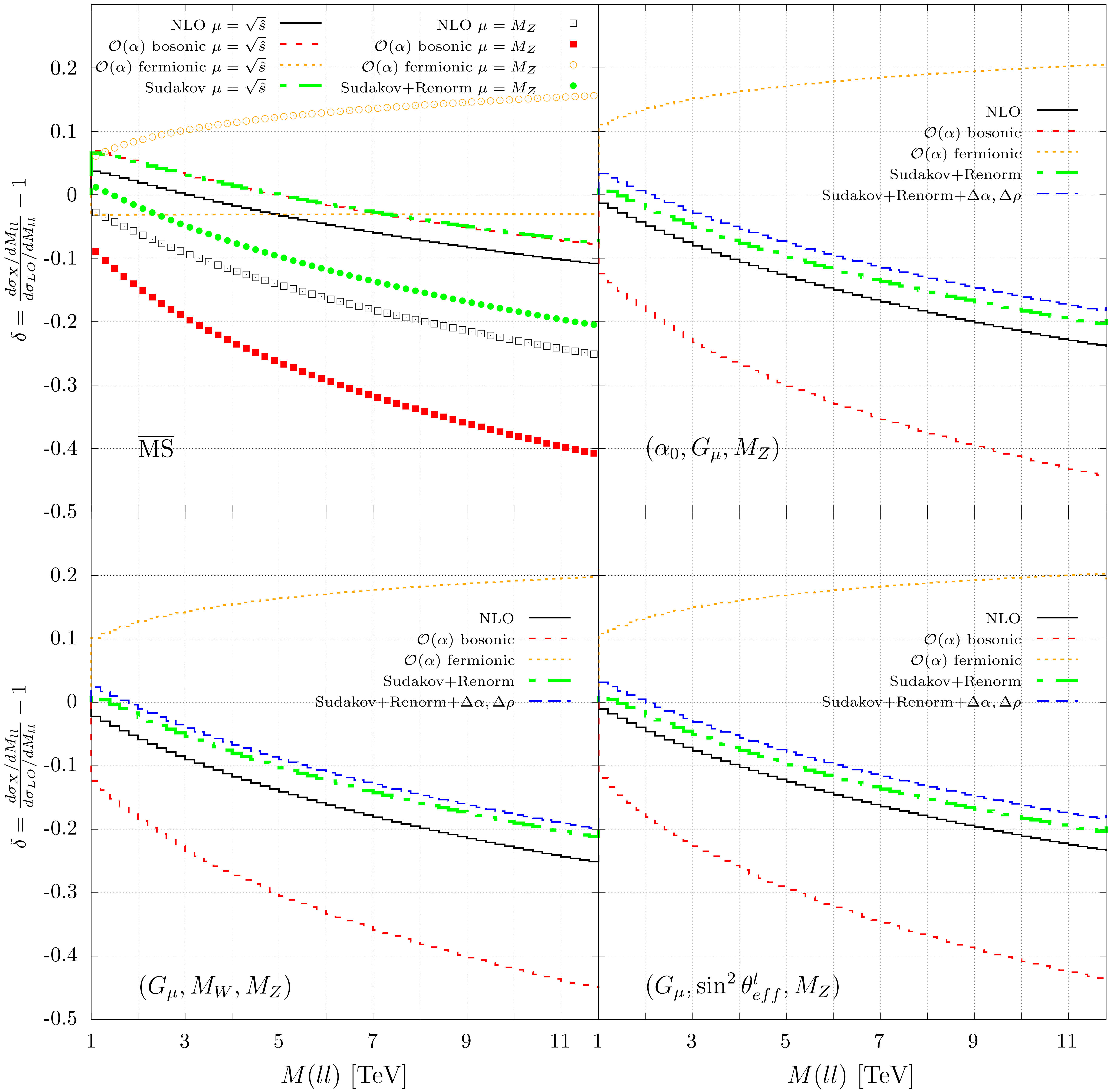
Bardin *et al.*, CERN 95-03, 1995

Agreement within 10^{-4}

	$(\alpha(M_Z^2), M_W\|_{G_\mu}, M_Z)$	$(\alpha(M_Z^2), s^2_{eff}\|_{G_\mu}, M_Z)$	$(\alpha(M_Z^2), G_\mu, M_Z)$
$\tilde{s}^2_{w,\,\mathrm{NLO+HO}}$	0.2316749	0.2315919	0.2315965
Γ_e LO	$8.58920545 \cdot 10^{-2}$	$8.3203418 \cdot 10^{-2}$	$8.3315838 \cdot 10^{-2}$
Γ_e NLO+HO	$8.3697741 \cdot 10^{-2}$	$8.3717562 \cdot 10^{-2}$	$8.3717744 \cdot 10^{-2}$

Sudakov regime

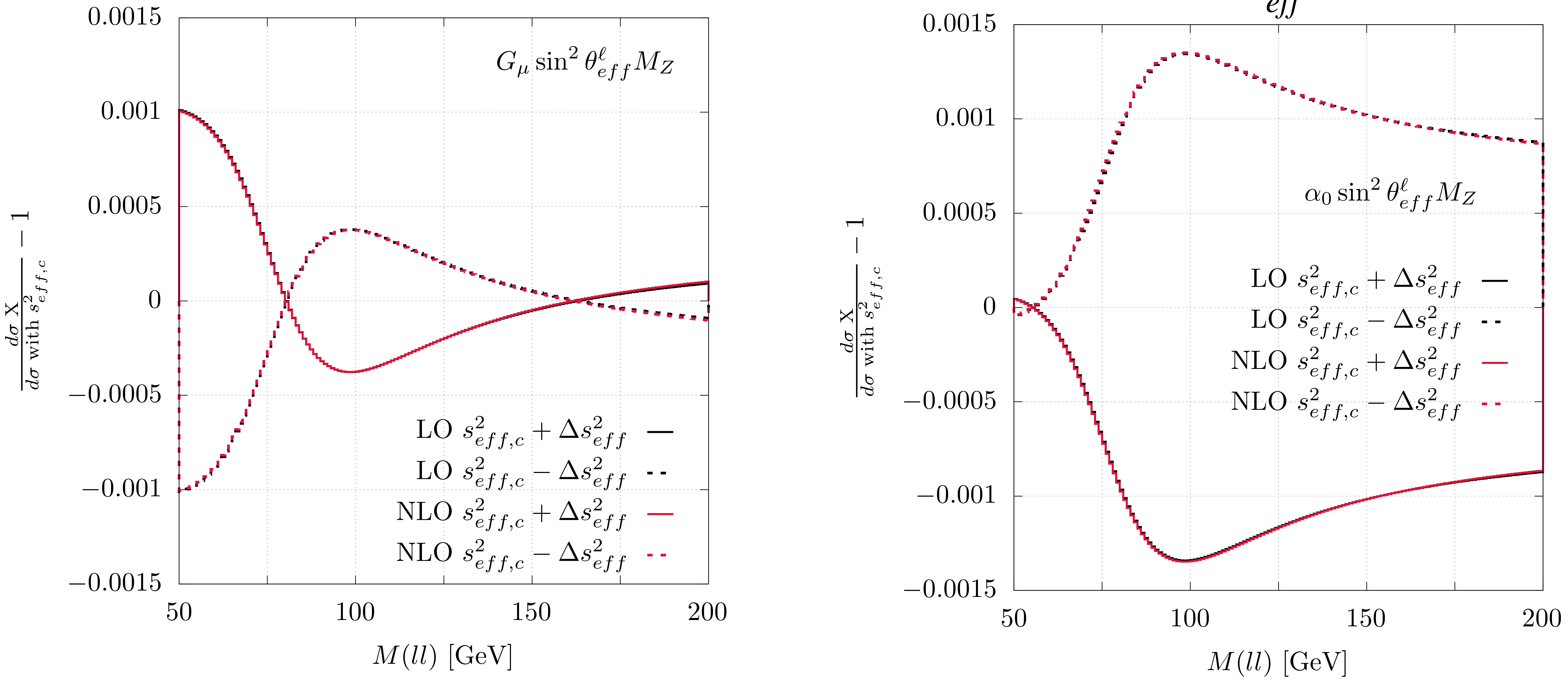
True NLO - approx. = constant 5 % shift



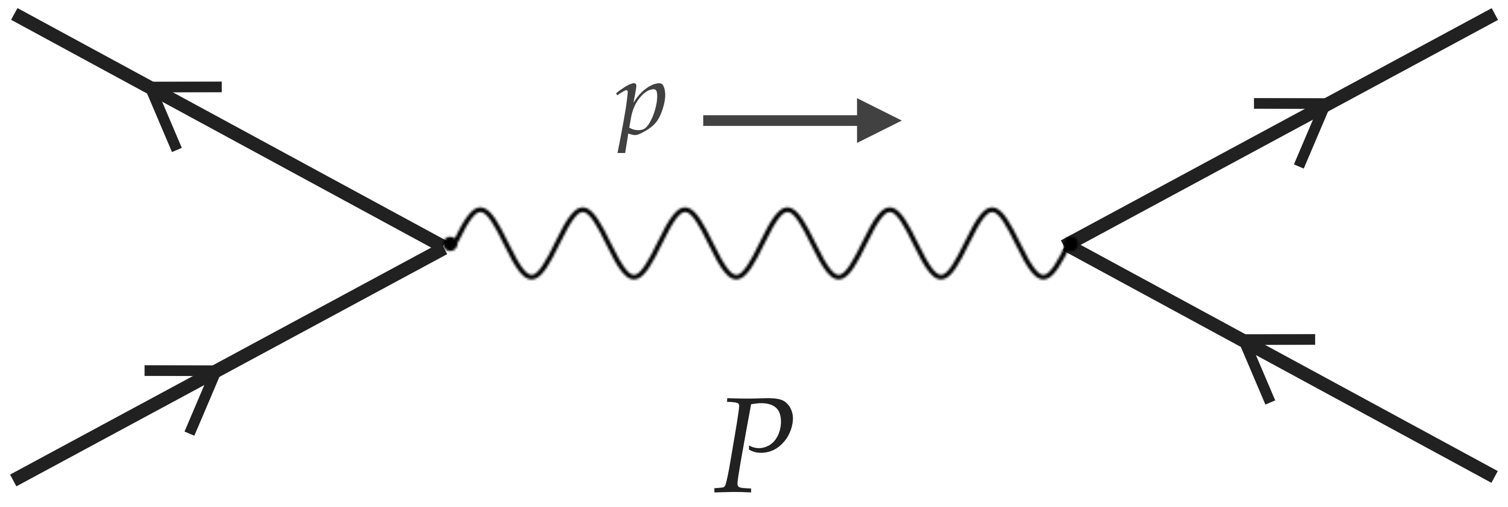
Parametric dependence on s^2_{eff}

Cross section in G_μ , s^2_{eff} , M_Z and α_0 , s^2_{eff} , M_Z

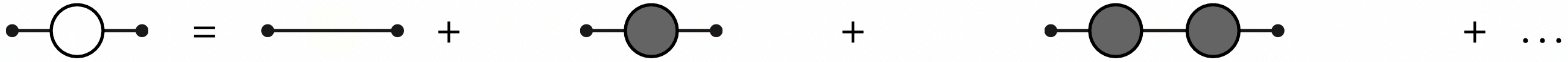
$\Delta s^2_{eff} = \pm\, 0.00016$



Resonances and gauge invariance



$$\sim \frac{1}{p^2 - M_P^2} \xrightarrow{p^2 \to M_P^2} \infty$$



$$G_P(p^2) = \frac{\mathrm{i}}{p^2-M_P^2} + \frac{\mathrm{i}}{p^2-M_P^2}\mathrm{i}\Sigma_\mathrm{R}(p^2)\frac{\mathrm{i}}{p^2-M_P^2} + \frac{\mathrm{i}}{p^2-M_P^2}\mathrm{i}\Sigma_\mathrm{R}(p^2)\frac{\mathrm{i}}{p^2-M_P^2}\mathrm{i}\Sigma_\mathrm{R}(p^2)\frac{\mathrm{i}}{p^2-M_P^2} + \ldots$$

$$= \frac{\mathrm{i}}{p^2 - M_P^2 + \Sigma_\mathrm{R}(p^2)}.$$

OS scheme $\quad \mathrm{Re}\,\Sigma_R(M_P^2) = 0 \qquad -G_P(p^2 \simeq M_p^2)g^{\mu\nu} = \dfrac{-ig^{\mu\nu}}{p^2 - M_P^2 + i\mathrm{Im}\,\Sigma_R(M_P^2)}$

through the optical theorem $\quad M_P\Gamma_P$

Complex-mass scheme

Denner, Dittmaier, Roth, Wackeroth, Nucl. Phys. B 560 no. 1-3, 33–65, 1999
Denner, Dittmaier, Roth, Wieders, Nucl. Phys. B 724 no. 1-2, 247–294, 2005
Denner, Dittmaier, Nucl. Phys. B - Proceedings Supplements 160, 22–26, 2006

$$\mu_Z = M_Z - i\Gamma_Z M_Z \qquad \mu_W = M_W - i\Gamma_W M_W$$

Pole scheme

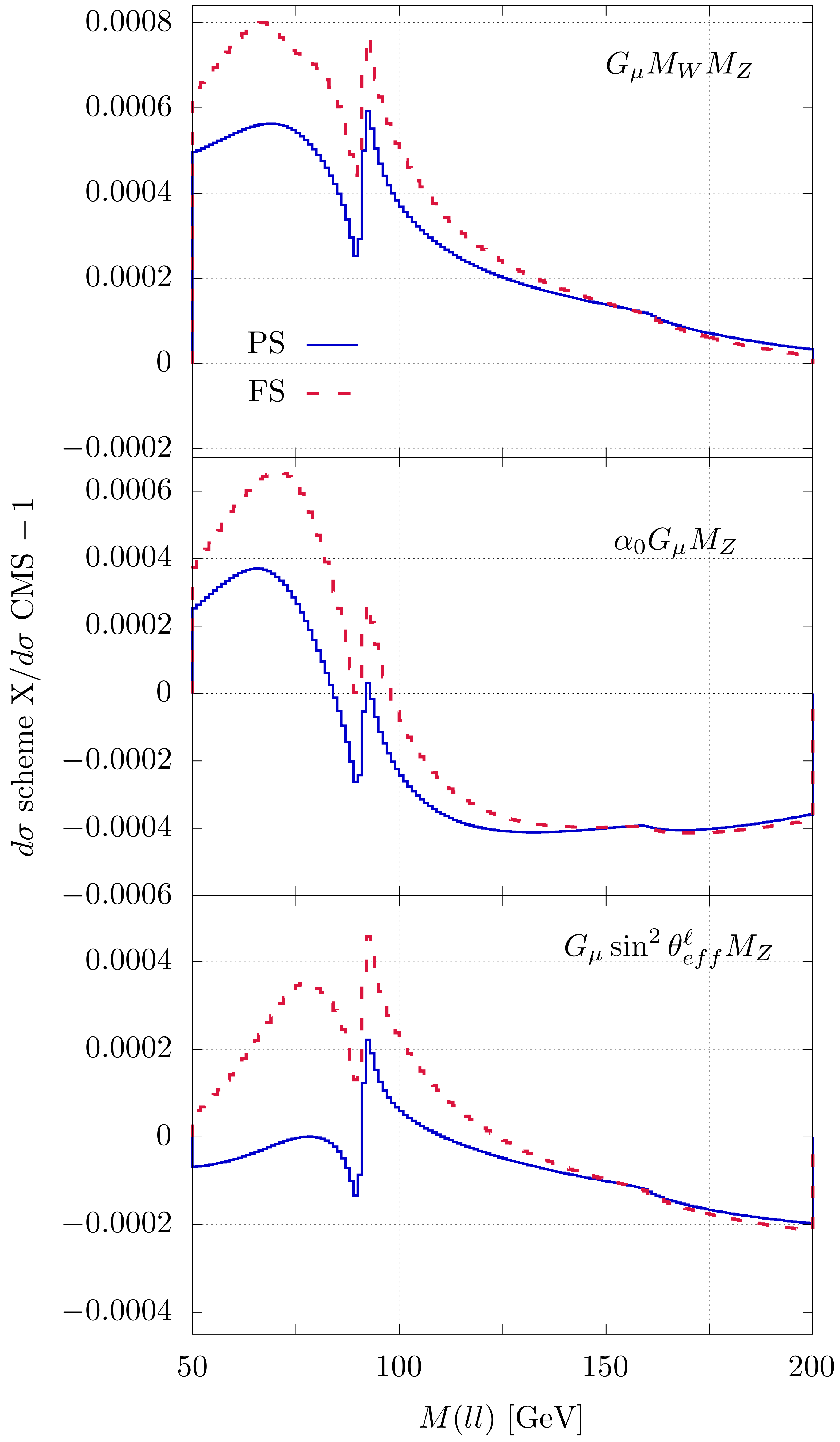
Stuart, Phys. Lett. B 262 no. 1, 113–119, 1991 - Sirlin, Phys. Lett. B 267 no. 2, 240–242, 1991
Gambino, Grassi, Phys. Rev. D 62 no. 7, 2000 - Grassi, Kniehl, Sirlin, Phys. Rev. D 65 no. 8, 2002
Stuart, Phys. Rev. Lett. 70, 3193–3196, 1993 - Dittmaier, Huber, JHEP 2010 no. 1, 2010)

$$\mathscr{M} = \frac{\tilde{R}(\mu_P^2)}{p^2 - \mu_P^2} + \frac{R(p^2) - R(M_P^2)}{p^2 - M_P^2} + \tilde{N}(p^2)$$

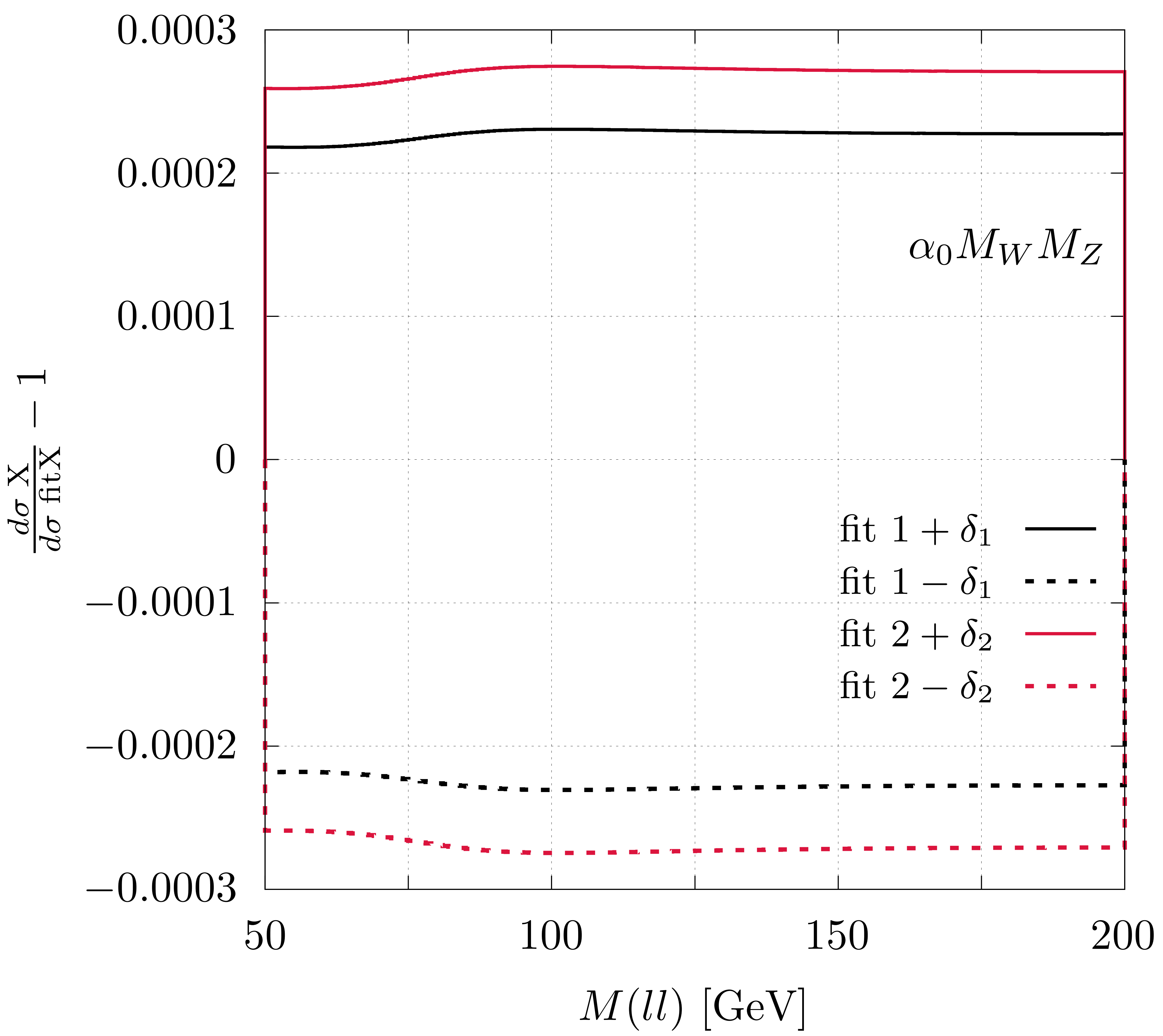
Factorization scheme

Argyres et al., Phys. Lett. B 358 no. 3-4, 339–346, 1995
Kurihara, Perret-Gallix, Shimizu, Phys. Lett. B 349 no. 3, 367–374, 1995
S. Dittmaier and M. Krämer, Phys. Rev. D 65 no. 7, 2002

$$f_P(p^2) = \frac{p^2 - M_P^2}{p^2 - \mu_P^2}$$



Treatment of $\Delta\alpha^{had}$



$$\Delta\alpha^{\text{had pert.}}(q^2) = -\frac{\mathrm{Re}\Sigma^{\text{had}}_{AA}(q^2)}{q^2} + \frac{\partial\Sigma^{\text{had}}_{AA}(q^2)}{\partial q^2}\bigg|_{q^2=0}$$

To fix light-quark masses

$$\Delta\alpha^{\text{had pert.}}(M_Z^2) = \Delta\alpha^{\text{had fit}}(M_Z^2)$$

experimental results from e^+e^- collisions using dispersion relations

$$\Delta\alpha^{\text{had fit}}(q^2) = -\frac{\mathrm{Re}\Sigma^{\text{had}}_{AA}(q^2)}{q^2} + \frac{\partial\Sigma^{\text{had fit}}_{AA}(q^2)}{\partial q^2}\bigg|_{q^2=0}$$

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da_had_from_fit=1
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`fit=1  HADR5X19.F` F. Jegerlehner, Z. Phys. C 32 195, 1986

`fit=2  KNT v3.0.1` Hagiwara, *et al.*, Phys. Rev. D 69 093003, 2004

Parametric dependence on s^2_{eff}

Asymmetry in G_μ , s^2_{eff} , M_Z