Uniqueness of the matching in the HEFT

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Although the Standard Model (SM) is extremely powerful, there is physics beyond it (BSM)

The Higgs sector was inaugurated in 2012, and BSM physics may be found within it

How to search for physics Beyond the Standard Model (BSM) within the Higgs sector?

The dream: direct detection! But if BSM physics is too heavy to be produced, we resort to indirect methods, by looking for deviations from the SM — in a model-independent way

The kappa formalism is often used — but it is not theoretically sound

The theoretical framework that should be used for a model-independent approach is an EFT

\[ \mathcal{L}_{\text{eff}} = \mathcal{O} (\Lambda^0) + \frac{E}{\Lambda} \mathcal{O}_1 + \left( \frac{E}{\Lambda} \right)^2 \mathcal{O}_2 + \left( \frac{E}{\Lambda} \right)^3 \mathcal{O}_3 + \ldots \]

Consistent Quantum Field Theory for heavy BSM, i.e., for small \( E/\Lambda \)

At each order in \( E/\Lambda \), all terms consistent with the symmetries are included

Renormalizable order by order; higher and higher orders become less and less relevant

It is a general description, that can later be matched to particular BSM models

Was not mature at LHC Run 1

[CMS, CMS-PAS-HIG-19-005]
Motivation

Two main EFT candidates for Higgs physics: SMEFT and HEFT

The SMEFT takes the SM before SSB and generalizes it: \[ \mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{i=1}^{n_d} \frac{C_i^{(d)}}{\Lambda^{d-4}} Q_i^{(d)} \]

The HEFT is a fusion of chiral perturbation theory (\(\chi\)PT) (in the scalar sector) with SMEFT (in the fermion and gauge sector). Just as in \(\chi\)PT:

- The 3 Goldstone bosons are independent of the Higgs, which is a gauge singlet (\(\pi^I\), imbedded into \(U = \exp(iT^I \pi^I/v)\) \(h\)) (instead of part of an SU(2) doublet)
- There is an expansion in the number of (covariant) derivatives. At LO:
  \[ \mathcal{L}_{\text{HEFT}} \supset \frac{v^2}{4} \mathcal{F}(h) \text{Tr} \left\{ D_\mu U^\dagger D_\mu U \right\} + \frac{1}{2} (\partial_\mu h)^2 - V(h) \]

with:
  \[ \mathcal{F}(h) = 1 + 2a \frac{h}{v} + b \frac{h^2}{v^2} + \ldots, \quad V(h) = \frac{1}{2} m_h^2 h^2 \left( 1 + \frac{\kappa_3}{v} h + \frac{\kappa_4}{4} \frac{h^2}{v^2} + \ldots \right) \]
  (such that the SM corresponds to \(a = b = \kappa_3 = \kappa_4 = 1\))

- Because the Higgs is a gauge singlet, it has arbitrary couplings: e.g. \(\kappa_3\) and \(\kappa_4\) are independent (whereas in the LO SMEFT they are related, since \(h\) is contained in a doublet)

The organization of HEFT is subtle, since \(\chi\)PT and SMEFT have different organizations
Ultimate goal of any EFT framework for BSM physics:

1) Find a pattern of non-zero EFT coefficients:

2) Convert (or match) them to a particular BSM model:

- BSM1: $C_{Hq}^{(l)} = 2 \sin(\alpha)$
- BSM2: $C_{Hq}^{(l)} = \frac{1}{4} \cos(\beta)$, ...

With the matching, we would convert a constraint on an EFT coefficient into a constraint on the parameters of the BSM models (all at once):

For $C_{Hq}^{(l)} \approx -0.24$, then:

- BSM1: $\sin(\alpha) \approx -0.12$
- BSM2: $\cos(\beta) \approx -0.06$, ...

The EFT, then, is just a tool, and never the ultimate answer.

The matching is thus a crucial part of the EFT framework (without it, the EFT is in vain!)

Even without non-zero EFT coefficients, we should understand how matching works
**Motivation**

- **Understanding matching:**
  - **Recipe:**
    1. Choose a set of independent parameters in the full theory
    2. Define a small quantity ($\xi$) to organize the to-be-built EFT expansion
    3. Decide how each of the independent parameters scales with $\xi$
    4. Equate specific amplitudes in the full theory and EFT order by order in $\xi$
  - If we knew the values of the parameters, we would know how to scale them \( \text{e.g. } \sin(\alpha) \sim O(\xi^0) \)
  - But since we do not, we may consider multiple possibilities \( \sin(\alpha) \sim O(\xi^0) \text{ or } \sim O(\xi^1) \text{ or ...} \)
  - Each possibility will lead to different expansions or **power countings (PCs)**
  - I will consider two particular BSM models to be matched to the HEFT:
    - The real singlet extension of the SM with a Z2 symmetry (Z2RSE)
    - The 2 Higgs Doublet Model (2HDM)
  - For each of them, we will consider 3 different PCs, which differ in how they scale the parameters
  - The goal is to find the best PC — the fastest to converge to the BSM model

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**Z2RSE**

**2HDM**

**Discussion**
The Z2RSE in a nutshell:

- Add a scalar singlet $S$ to the SM, subject to a Z2 symmetry: $S \rightarrow -S$. The potential reads:

$$V = -\frac{\mu_1^2}{2} \phi^\dagger \phi - \frac{\mu_2^2}{2} S^2 + \frac{\lambda_1}{4} (\phi^\dagger \phi)^2 + \frac{\lambda_2}{4} S^4 + \frac{\lambda_3}{2} \phi^\dagger \phi S^2$$

- The parameters $\mu_1^2, \mu_2^2, \lambda_1, \lambda_2, \lambda_3$ are all real, and the fields can be written as:

$$\phi = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}} (v + h_1 + iG_0) \end{pmatrix}, \quad S = \frac{v_s + h_2}{\sqrt{2}}$$

- $h_1, h_2$ are not yet mass states; they can be diagonalized via a mixing angle $\chi$

$$\begin{pmatrix} h \\ H \end{pmatrix} = \begin{pmatrix} c_\chi & -s_\chi \\ s_\chi & c_\chi \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

- The fields $h, H$ are then mass states, with masses $m, M$, respectively

- We can use the relations of the theory to choose two different sets of independent parameters:

  - SET_R1: $v, m, v_s, M, s_\chi$
  - SET_R2: $v, m, \mu_2^2, M, s_\chi$
For a HEFT approach, we assume $M \gg m$, so that we integrate out $H$ (and we are left with the d.o.f. of the SM after SSB).

It is then clear that $m/M$ should be small — say, $\left(\frac{m}{M}\right)^2 \sim \mathcal{O}(\xi)$.

But what about the other parameters? We consider 3 different PCs:

- **$\text{PC}_1^R$** takes SET_R1 as the set of independent parameters, and imposes:
  \[
  \left(\frac{m}{M}\right)^2 \sim \mathcal{O}(\xi), \quad \left(\frac{m}{v_s}\right)^2 \sim \mathcal{O}(\xi), \quad s^2_\chi \sim \mathcal{O}(\xi)
  \]  
  (and all the others are $\mathcal{O}(\xi^0)$)

- **$\text{PC}_2^R$** takes SET_R1 as the set of independent parameters, and imposes:
  \[
  \left(\frac{m}{M}\right)^2 \sim \mathcal{O}(\xi)
  \]  
  (and all the others are $\mathcal{O}(\xi^0)$)

- **$\text{PC}_3^R$** takes SET_R2 as the set of independent parameters, and imposes:
  \[
  \left(\frac{m}{M}\right)^2 \sim \mathcal{O}(\xi)
  \]  
  (and all the others are $\mathcal{O}(\xi^0)$)

Example of matching for $\mathcal{L}_{\text{HEFT}} \ni -\kappa_3 m_h^2 \frac{h^3}{2v}$. Defining $\Delta \kappa_3 \equiv \kappa_3 - 1$, we find:

<table>
<thead>
<tr>
<th>$\Delta \kappa_3$</th>
<th>$\text{PC}_1^T$</th>
<th>$\text{PC}_2^T$</th>
<th>$\text{PC}_3^T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-\xi \frac{3 s^2_\chi}{2}$</td>
<td>$c^3_\chi - \frac{s^3_\chi v_s}{v_s} - 1$</td>
<td>$-1 + c_\chi - \xi \frac{s^2_\chi}{M^2 c_\chi} (m^2 - \mu_2^2)$</td>
<td>$-\xi^2 \frac{m^2 s^2_\chi}{M^4 c_\chi} (m^2 - \mu_2^2)$</td>
</tr>
<tr>
<td>$+\xi^2 \frac{s^3_\chi}{8 v_s} (3 s_\chi v_s - 8 v)$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Numerical results for the differential cross section of** $hh \rightarrow hh$

- Close to $s_x = 0$, everything works. Elsewhere, there are significant differences.
- In particular, even if $PC_2^R$ and $PC_3^R$ only differ by $\mu_2^2$ vs. $\nu_s$, $PC_2^R$ is much more accurate.
- $PC_2^R$ is the best (quickest to converge) $PC$, for the entire range of $s_x$.
- In the right panel, the EFT assumption starts to break down.

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**Motivation**

- Z2RSE

**Discussion**

- 2HDM
Now, the 2HDM. The model in a nutshell:

- Add a second Higgs doublet to the SM. This leads to:

\[
\mathcal{L}_{\text{2HDM}} \ni -V + \mathcal{L}_Y:
\]

\[
V = Y_1 H_1^+ H_1 + Y_2 H_2^+ H_2 + \left( Y_3 H_1^+ H_2 + \text{h.c.} \right) + \frac{Z_1}{2} \left( H_1^+ H_1 \right)^2 + \frac{Z_2}{2} \left( H_2^+ H_2 \right)^2 + Z_3 \left( H_1^+ H_1 \right) \left( H_2^+ H_2 \right) + Z_4 \left( H_1^+ H_2 \right) \left( H_2^+ H_1 \right) + \left\{ \frac{Z_5}{2} \left( H_1^+ H_2 \right)^2 + Z_6 \left( H_1^+ H_1 \right) \left( H_1^+ H_2 \right) + Z_7 \left( H_2^+ H_2 \right) \left( H_1^+ H_2 \right) + \text{h.c.} \right\},
\]

\[
\mathcal{L}_Y = -\lambda_u^{(1)} H_1^+ \tilde{q}_L u_R - \lambda_u^{(2)} H_2^+ \tilde{q}_L u_R - \lambda_d^{(1)} H_2^+ u_R - \lambda_d^{(2)} \tilde{u}_R H_2 q_L - \lambda_i^{(1)} e_R H_1 H_2 l_L - \lambda_i^{(2)} e_R H_2 H_2 l_L + \text{h.c.} \quad (\tilde{q}_L \equiv -i\sigma_2(q_L)^T)
\]

where \( Y_i, Z_j \) and \( \lambda_z \) are real parameters, and where the doublets can be written as:

\[
H_1 = \left( \begin{array}{c}
\frac{1}{\sqrt{2}} (v + s_{\beta-\alpha} h + c_{\beta-\alpha} H + iG_0)
\end{array} \right), \quad H_2 = \left( \begin{array}{c}
\frac{1}{\sqrt{2}} (c_{\beta-\alpha} h - s_{\beta-\alpha} H + iA)
\end{array} \right)
\]

with \( h \) the scalar found at the LHC, while \( H, A, H^+ \) are BSM scalars, assumed to be heavy.

- Avoid FCNC via a \( Z_2 \) symmetry → 4 types of 2HDM: Type-I, Type-II, Type-L, Type-F

- Take some of the parameters as independent:
  - SET_T1: \( c_{\beta-\alpha}, Y_2, m_h, m_H, m_A, m_{H^\pm}, \beta, m_f \)
  - SET_T2: \( c_{\beta-\alpha}, m_{12}^2, m_h, m_H, m_A, m_{H^\pm}, \beta, m_f \)
Let us consider the degenerate case, and define \( M \equiv m_H = m_A = m_{H^\pm} \)

The PCs to be studied are: (dimensional terms are normalized to \( m_h \))

- **PC\(_1^T\)** takes SET_T1 as the set of independent parameters, and imposes:
  \[
  Y_2 \sim \mathcal{O}(\xi^{-1}), \quad M^2 = Y_2 + \mathcal{O}(\xi^0) \sim \mathcal{O}(\xi^{-1}), \quad c_{\beta-\alpha} \sim \mathcal{O}(\xi)
  \]
  (and all the others are \( \mathcal{O}(\xi^0) \))

- **PC\(_2^T\)** takes SET_T1 as the set of independent parameters, and imposes:
  \[
  Y_2 \sim \mathcal{O}(\xi^{-2}), \quad M^2 \sim \mathcal{O}(\xi^{-2}), \quad c_{\beta-\alpha} \sim \mathcal{O}(\xi)
  \]
  (and all the others are \( \mathcal{O}(\xi^0) \))

- **PC\(_3^T\)** takes SET_T2 as the set of independent parameters, and imposes:
  \[
  M^2 \sim \mathcal{O}(\xi^{-1})
  \]
  (and all the others are \( \mathcal{O}(\xi^0) \))

**Example:**

<table>
<thead>
<tr>
<th>( \Delta \kappa_3 )</th>
<th>( {\text{PC}}_1^T )</th>
<th>( {\text{PC}}_2^T )</th>
<th>( {\text{PC}}_3^T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(- \xi \frac{2 c_{\beta-\alpha}^2}{m_h^2} \frac{Y_2}{m_h^2} + \xi^2 \frac{1}{2} c_{\beta-\alpha}^2 )</td>
<td>(- \frac{2 Y_2 c_{\beta-\alpha}^2}{m_h^2} + \xi \frac{c_{\beta-\alpha}^3}{m_h^2 t_\beta} (t_\beta^2 - 1)(Y_2 - M^2) + \xi^2 \frac{c_{\beta-\alpha}^2}{2 m_h^2 t_\beta} \left( c_{\beta-\alpha} \left[ M^2 (t_\beta^4 - 4 t_\beta^2 + 1) + 2 Y_2 t_\beta^2 \right] + m_h^2 t_\beta^2 \right) )</td>
<td>(-1 + s_{\beta-\alpha} (1 + 2 c_{\beta-\alpha}^2) + c_{\beta-\alpha}^2 \left[ -2 s_{\beta-\alpha} m_{12}^2 \right] + 2 c_{\beta-\alpha} \cot 2 \beta \left( 1 - m_{12}^2 \right) )</td>
<td></td>
</tr>
</tbody>
</table>
**Numerical results for two different processes**

For an observable like $h \rightarrow b\bar{b}$, $PC_3^T$ is clearly the most convenient, as it is identical to the full model.

$PC_1^T$ and $PC_2^T$ are identical in this case, and they only provide an adequate description at $O(\xi^1)$.

However, for an observable like $hh \rightarrow hh$, $PC_2^T$ is by far the best choice, providing an excellent replication of the 2HDM result immediately at $O(\xi^0)$.

$PC_3^T$ can only get closer to that at $O(\xi^2)$.

Conclusion: according to the process, different PCs should be chosen.
What happens in different regions of parameter space of a certain process?

- In these cases, $Y_2$ starts to diminish; we thus expect $PC^T_1$ and $PC^T_2$ to fail.
- $PC^T_1$ indeed fails. However, in $PC^T_2$, the scaling of $M^2$ partially compensates that.
- Whereas $PC^T_2$ is still the most adequate PC in the left plot, $PC^T_3$ passes it in the right one.
- In sum: the most adequate PC depends not only on the process, but also on the region.
- **EFTs** are a consistent and general approach to BSM models

- Yet, they are but a tool, that requires matching the EFT coefficients to particular BSM models

- The matching is built based on assumptions about the size of the parameters of the model

- Since we do not know the sizes, we can consider multiple possibilities or power countings (PCs)

- I considered the HEFT, and discussed its matchings of both of the Z2RSE and the 2HDM

- In both cases, we presented three different PCs — leading to three different matchings (the HEFT matching is not unique!)

- In the Z2RSE, one of the PCs was always preferred

- In the 2HDM, by contrast, the most adequate PC depended on the process and region

- This complicates the interpretation of HEFT coefficients in terms of parameters of UV models