**Hard probes from the viewpoint of open quantum systems** 

> Journées QGP-France **Bagnoles de l'Orne June 27, 2023**

**Jean-Paul Blaizot, IPhT University Paris- Saclay**



## **Hard probes**

Hard probes are produced in hard processes on very short time scales  $\Delta t \sim 1/M \ll 1$ fm/c

Thus, they are present in the early stages of heavy ion collisions

Two types of hard probes:

- "Elementary" HP: direct photons, Z and W bosons, etc.
- "Complex" HP: quarkonia, jets

Elementary hard probes provide information on the "initial state" (e.g. npdf): Their yield scales with the number of n-n collisions. They are weakly affected by the surrounding medium.

Complex hard probes have their own dynamics in the absence of the QGP. This dynamics can be significantly altered by the presence of the quark-gluon plasma. Understanding such modifications can yield information about the QGP properties.

# **Open Quantum Systems**

# **Open quantum system (1)**



**The dynamics of the system is obtained after eliminating the degrees of freedom of the environment. This yields in general a non unitary evolution (decoherence, dissipation).**

**The dynamics of the system is affected by the presence of the environment via simple correlation functions characterising the environment. The system probes these correlation functions.**

**Open quantum system (2)** 2.1 Equation for the density matrix

The density matrix of total system  $\mathcal{D}(t)$  obeys the equation of motion The density matrix of foral system  $\mathcal{D}(t)$  obe

$$
i\frac{\mathrm{d}\mathcal{D}}{\mathrm{d}t}=[H,\mathcal{D}].
$$

**In the reduced density matrix of the system:** The interaction between the plasma and t

$$
\mathcal{D}_Q(t) = \text{Tr}_{\text{pl}}\mathcal{D}(t)
$$

turbation theory, we move to the interaction representation. We set *H* = *H*<sup>0</sup> + *H*1, with

 $\bm{E}$ quation of motion for  $\mathcal{D}_Q(t)$ 

$$
\frac{\mathrm{d}}{\mathrm{d}t} \mathcal{D}_{Q}(t) = -i[H_{Q}, \mathcal{D}_{Q}(t)] + \int_{0}^{t-t_{0}} \mathrm{d}\tau \mathcal{L}(\tau) \mathcal{D}_{Q}(t-\tau)
$$

<sup>=</sup> <sup>−</sup>*i*[*H*1(*t*)*, <sup>D</sup><sup>I</sup>* (*t*)]*, H*1(*t*) = *<sup>U</sup>*0(*t, t*0) *† H*1*U*0(*t, t*0)*.* (2.8) **Non hamiltonian contribution**

<sup>0</sup> (*t, t*0)*,* (2.7)

## **Various strategies:** d*t*

- Feynman-Vernon Influence functional Here, **Here, Here, Here,** 
	- Lindblad equation,
- ordination representation, *U*<sup>1</sup> (*the U<sub>1</sub>*), *U*<sub>1</sub> (*t)* = *U*<sub>1</sub> (*t)*, *U*<sub>1</sub>
- **expanding the Etc**

*H*<sup>0</sup> = *H<sup>Q</sup>* + *H*pl and define

**Heavy quarkonia** 

**[For details, see JPB, M. Escobedo-Espinosa, 1711.10812,1803.07996]**

# **Heavy quarks and quarkonia as 'hard probes'**

Heavy quarks are produced in pairs in the early stages of URHIC. Their number remains constant.



Formation time of a  $|Q\bar{Q}|$  pair is small



 $\Upsilon$  *M<sub>b</sub>*  $\simeq$  4.5 Gev  $\Delta t \simeq 0.02$  fm/c  $J/\Psi$  *M<sub>c</sub>*  $\simeq 1.5$  Gev  $\Delta t \simeq 0.07$  fm/c

Dynamics of heavy quarks is non-relativistic

$$
H = \frac{P^2}{M_Q} + V(r) \qquad \left(V(r) = \frac{\alpha_s}{r} + \sigma r\right)
$$

The potential can be obtained using effective theory (pNRQCD) [see N. Brambilla, A.Pineda, J. Soto, A. Vairo, NPB566 (2000) 275]

# **Heavy quark interaction at finite T**

Initial suggestion (Matsui-Satz 86): screening of the potential

$$
H = \frac{P^2}{M_Q} + V(r) \qquad V(r) = -\frac{\alpha}{r} e^{-r m_D(T)} + \sigma(T)r
$$

This picture predicts a "suppression" of bound states at high temperature, the most "fragile" ones (bigger, less bound) disappearing first as the temperature increases ("sequential suppression").

### **Hence the idea of using quarkonia to diagnose the formation of quark-gluon plasma in URHIC**

However, the dynamics of the quarkonia does not reduce to a mere modification of the potential: non unitary evolution, here caused by "collisions" with plasma constituents.

**Typical approximations in OQS (i) weak coupling between HQ and the plasma**  $\mathcal{L}^{\text{max}}$  magnetic interactions, the form takes the form takes the form takes the form takes the form  $H_1 = -g$ :<br>1 *r*  $A_0^a(\bm{r})n^a(\bm{r})$ , *H*<sup>1</sup> = −*g r Aa* <sup>0</sup>(*r*)*na*(*r*)*,* (2.2)  $H_1 = -q \int A_0^a(\mathbf{r}) n^a(\mathbf{r}).$  $\rho$  is given by  $\int r \, dr \, dr$ 

**gauge potential of plasma HQ density**

 $n^a(\bm{x}) = \delta(\bm{x}-\hat{\bm{r}})\,t^a\otimes\mathbb{I} - \mathbb{I}\otimes\delta(\bm{x}-\hat{\bm{r}})\,\tilde{t}^a,$ *,* (2.3)

 $\mathcal{A}$  the first component (for the first component) and the second component (for the second component). In

is much finally for each  $\alpha$  (24, 30). In the present paper, we shall proceed differently, by writing different

We assume that the system contains a fixed number, *NQ*, of heavy quarks (and, in

**Pair in the presence of the heavy quarks does not modify** significantly the equilibrium state of the plasma. **The components of the tensor product to the Hilbert space o** 

**The influence of the plasma on the heavy quark dynamics is characterized by simple response functions (correlators)** *n*<sup>*x*</sup>  $\frac{1}{2}$   $\$ *,* (2.3) where we use the first conduction to describe the first quantization to describe the heavy quark and antiquark, and the heavy quark and the he two components of the tensor product refer respectively. The tensor product respectively to the heavy product r<br>The heavy product refer respectively to the heavy product spaces of the heavy product respectively. The heavy of the plasma on the heavy quark and description of  $\mathbf{r}$ constant and the eigenful the heavy quark and the coupling of the coupling of the coupling of the heavy and the h the gluon is described by −t<sup>the</sup>d by −t<sup>the</sup>d by −t<sup>the</sup>d by −tt ˜*a*, with *t* ˜*<sup>a</sup>* the transpose of *t* We are looking for an effective theory for the heavy  $\alpha$  theory for the heavy  $\alpha$  dynamics, obtained by eliminating by eliminating  $\alpha$ 

 $\Delta(t_1, t_2) \equiv \langle A_{\text{pl}}(t_1) A_{\text{pl}}(t_2) \rangle_T = \text{Tr} \left[ A_{\text{pl}}(t_1) A_{\text{pl}}(t_2) \mathcal{D}_{\text{pl}} \right]$ *<sup>a</sup>* is a color matrix in the fundamental representation of SU(3) and describes the inating the plasma degrees of freedom. In previous works, this was achieved explicitly by  $\left[\left(\ell_1\right)\right]$  the Feynman-Vernon influence functional formal-version integral formal-

No assumption of weak or strong coupling needs to **be made concerning the plasma. The correlators can, in some cases, be obtained from lattice calculations. be made concerning the plasma. The corre**<br>The same cases be ebtained from lattice co We are looking the cases, be obtained from farried calculations. of the plasma. The correlators can.  $t_{\text{in}}$  are in the present section are quite common in various fields, and  $t_{\text{in}}$ to what is commonly referred to as the theory of operations (see e.g. [20]).

### **(ii) The response of the plasma is "fast''**  *<sup>x</sup>*′(−<sup>τ</sup> ) = *<sup>n</sup><sup>a</sup> <sup>x</sup>*′ <sup>−</sup> <sup>τ</sup>*n*˙ *<sup>a</sup> <sup>x</sup>*′] *,* (2.21) the time-dependence of *nx*′(*t*) being given by the Heisenberg representation, *n<sup>a</sup>*

 $P$  plasma response is characterized by a single energy scale, the Debye mass JHEP<br>John Constant<br>Danmark Constant<br>Danmark Constant

 $m_D = CT$  (*C*  $\simeq$  2) **In strict weak coupling**  $C = g$  $\approx$  2)  $\equiv$  11  $\sim$  $\lambda$ ct weak coupling  $\breve{\le}$  $m_D \ll M$ 

*x* **x**<sup>2</sup> **x**<sup>2</sup> **x**<sup>2</sup> **x**<sup>2</sup> t١ [*n<sup>a</sup> <sup>x</sup>, n*˙ *<sup>a</sup>* e s de *x x* − *x* **collisions with plasma constituents involve small energy transfer** 



the relevant correlator is then generically of the form 0*, r*), which we identify with the real and imaginary part of a "potential". More precisely,

 $\Delta(\omega = 0, r) = \Delta^R(\omega = 0, r) + i\Delta^<(\omega = 0, r)$ 

 $V(\bm{r}) = -\Delta^R(\omega=0,\bm{r}), \qquad W(\bm{r}) = -\Delta^<(\omega=0,\bm{r}).$ This terminology stems from the fact that *V* (*r*) + *iW*(*r*) plays the role of a complex **Imaginary potential imaginary potential**  $\overline{\mathbf{a}}$  . In the calculation of the influence of

prom the point of view of the rig the interactions with the plasma **nearly instantaneous ("collisions")** *Functional, we have to accept the to allow for two spacessive spaces of the HQ the interactions with the plasma are* **E** point of view of the five interactions with the plasma are<br>A about the movie was **from the point of view of the HQ the interactions with the plasma are** 

$$
\Delta(t_x - t_y) = \int \frac{d\omega}{2\pi} e^{-i\omega(t_x - t_y)} \left[ \Delta(\omega = 0) + \omega \Delta'(\omega = 0) \right]
$$

$$
\approx \delta(t_x - t_y) \Delta(\omega = 0) + i \frac{d}{dt_x} \delta(t_x - t_y) \Delta'(\omega = 0)
$$

### **Static response and "optical potential"** strategies to solve numerically the resulting equations in the case of a quarkantiquark pairs in the involvement of the involvement of the internal random color in the internal random colo *Department of Physics, P.O. Box 35, FI-40014 University of Jyv¨askyl¨a, Finland*

of the semi-classical approximation more involved. We explore the semi-classical approximate  $\mathcal{L}$ 

Carlo techniques.

1. Introduction

 $f*first obtained by M1aine et al. hen-hb1$ urations as collisions as collisions in a Boltzmann equation with  $\omega$ **(\*first obtained by M. Laine et al hep-ph/ 0611300)**

collection of heavy quarks and antiquarks immersed in a quark gluon plasma.

$$
\mathcal{V}(r) = V(r) + iW(r)
$$

$$
\Delta^{R}(\omega = 0, r) = -V(r) \qquad \Delta^{<}(\omega = 0, r) = -W(r)
$$



## **(iii) semi-classical approximation**

 $M \gg T$ 

 $\lambda_{\rm th} \sim \frac{1}{\sqrt{h}}$ *MT*  $\ll$ 1 HQ thermal wavelength  $\quad \lambda_{\rm th} \sim \frac{1}{\sqrt{MT}} \ll \frac{1}{T}$ 

**Density matrix becomes nearly diagonal** 

$$
\langle \mathbf{r} | \mathcal{D}_{Q} | \mathbf{r}' \rangle \simeq 0
$$
 when  $|\mathbf{r} - \mathbf{r}'| \ge \lambda_{\text{th}}$ 

Expansion in  $|\mathbf{r} - \mathbf{r}'|$  **Fokker-Planck and Langevin equations** 

- Semi-classical expansion for heavy quark motion
	- $L$ angevin equation  $\blacksquare$  for the density matrix  $\longrightarrow$  Lang *i*  $\frac{1}{2}$  (*r*) ( 1<br>1  $\frac{1}{2}$  for the density matrix .<br>اس  $\mathbf{p}$  the density matrix *Langevin equation* ⌘*ij* (*r*) = <sup>1</sup> • Equation for the density matrix **and the part (2)**
- The Langevin equation for the relative motion is the formulation of the formula  $\mathbb{R}^n$  $\overline{a}$   $\overline{b}$   $\overline{c}$   $\overline{$ *<sup>L</sup>*rel <sup>3</sup> = .<br>ا <sup>2</sup>*MT* (*Hij* (0) + *<sup>H</sup>ij* (*r*))r*<sup>i</sup> <sup>p</sup>p<sup>j</sup>* =  $\mathbf{d}$ arive mori *<sup>v</sup>v<sup>j</sup> .* (73) • Langevin equation for the relative motion

$$
\frac{M}{2}\ddot{\boldsymbol{r}}^i = -\gamma_{ij}\boldsymbol{v}^j - \boldsymbol{\nabla}^i V(\boldsymbol{r}) + \xi^i(\boldsymbol{r},t)
$$
\n
$$
\gamma_{ij}(\boldsymbol{r}) = \frac{1}{2T}\eta_{ij}(\boldsymbol{r}) \qquad \langle \xi^i(\boldsymbol{r},t)\xi^j(\boldsymbol{r},t') \rangle = \eta_{ij}(\boldsymbol{r})\delta(t-t')
$$
\nNon trivial noise

⌘*ij* (*r*) = *ij*⌘(*r*)*,* ⌘(*r*) = <sup>1</sup> • For an isotropic plasma, we have  $\bullet$  so have  $\bullet$ 

$$
\eta_{ij}(\boldsymbol{r}) = \delta_{ij}\eta(\boldsymbol{r}) \qquad \quad \eta(\boldsymbol{r}) = \frac{1}{6}\left(\nabla^2 W(0) + \nabla^2 W(\boldsymbol{r})\right)
$$

*v* = *P /*(2*M*). We get  $\overline{\text{orrelation}}$  functio = *v ·* r*<sup>R</sup>*  $\overline{\phantom{a}}$  All the inemation to of the dynamics of the llOs and selected. *v i like ingred* <sup>0</sup> <sup>=</sup> *<sup>P</sup> ·* <sup>r</sup>*<sup>R</sup>*  $\Omega$ ushed ingradiante of the dynamice of the  $\Omega$ e are calculated. *v i m m mg* calents of the a *phames* of *m* <sup>0</sup> <sup>=</sup> *<sup>P</sup> ·* <sup>r</sup>*<sup>R</sup>* **i**<sub>*from*</sub> the ingredients of 6 from the plasma correlation functions • All the ingredients of the dynamics of the HQs are calculated

# **Jet momentum broadening**

**[Based on J. Barata, JPB and Y. Mehtar-Tani, 2305.10476]**

## Momentum broadening studied as the motion of an effective non-relativistic particle in

**Consider a high energy quark propagating in the positive z-direction** 



**The dynamics reduce to a two-dimensional non-**<br> **evolution of the motion of the motion of the can be shown to be relativistic problem in the transverse plane with E**  $\overline{\phantom{a}}$ **playing the role of a mass**  $(x^+ \mapsto t, p^+ \mapsto E)$ 

to the hyperplane *t*

$$
\left[i\partial_t + \frac{\partial_\perp^2}{2E} + gA(\mathbf{r}, t)\right]\psi(\mathbf{r}, t) = 0
$$
  

$$
E \gg T \qquad A \mapsto A_a^- t^a
$$

### strained to the transverse plane at fixed light-cone en-**EXECUTE: The reduced density matrix ences if you feel they should be there, but they do not** (*<sup>x</sup> <sup>y</sup>*) = *<sup>g</sup>*2*<sup>n</sup> z reduced density matrix*

 $\frac{1}{\sqrt{2}}$ *x* the go The gauge potential of the plasma is a fluctuating field with Gaussian correlation function

$$
\langle A_a^-(x^+, x) A_b^-(y^+, y) \rangle = \delta_{ab} \delta(x^+ - y^+) \gamma(x^+, x - y)
$$

$$
\gamma(0) - \gamma(r) = g^2 n \int \frac{\mathrm{d}^2 q}{(2\pi)^2} \frac{1 - \mathrm{e}^{i\bm{q} \cdot \bm{r}}}{(q^2 + m_D^2)^2} \qquad g^2(\gamma(0) - \gamma(\bm{r})) = \frac{n}{2} \sigma(\bm{r})
$$

 $\sigma(r)$  is called the dipole cross section. It is the analog of the imaginary **rations is approximately, that is, it i** 

 $\overline{\phantom{a}}$  The re quark is infinitely boosted along the positive *z* direction. Also, in The reduced density matrix is obtained by averaging over the gauge field *m<sup>D</sup>* ⌧ *q* ⌧ 1*/r*. To leading logarithmic accuracy, it can be evaluated by expanding the  $\blacksquare$  The reduced density matrix is obt

with the jet  $q$  (evaluated at the typical scale  $q$  (evaluated at the typical scale 1 $\sigma$ ) given by

speed of light in the positive *z* direction. This ener-

getic parton couples to the long wavelength modes of

the fluctuating gauge potential *Aa, µ*(*r, t*) describing the

direction. Because the light-cone energy of the incident

<sup>2</sup> In this work, we use light-cone coordinates such that the light-cone

coordinate and *z* the longitudinal coordinate, see Fig. 1. The last

˜ the usual time

fluctuations:

factors), and 1*/*(*q*<sup>2</sup> +*m*<sup>2</sup>

the screening length (inverse Debye mass) *m*<sup>1</sup>

˜ + *z*)*/*

the logarithm, and set

$$
\rho \equiv \text{tr}_A(\rho[A]) = \left\langle |\psi_A(t)\rangle\langle\psi_A(t)| \right\rangle_A.
$$



#### operators in transverse space. Their matrix elements in *Ai denotes the Schrodinger solution of the Schrodinger space. The Schrodinger spac* Eq. (2) for a given field configuration *A*(*r, t*), and the **Various representations** into singlet ( $\ldots$ octet ( $\ldots$ o) components: h*r|* ⇢s*,*o(*t*)*|r*¯i = h*b* + *x/*2*|* ⇢s*,*o(*t*)*|b x/*2i *,* (6) ⇢ = ⇢*†* is hermitian, and normalized to unity (Tr⇢ = 1).  $V$ urious representations

**Color struc** Eq. (2) for a given field configuration *A*(*r, t*), and the  $\alpha(t) = \alpha + t^a e^a$  $\mu(v) = \rho_s + v \rho_o$  $\Gamma$   $\alpha$   $\alpha$ field average is performed over a gaussian distribution ture  $c(t) = e^{-t} t^{a} e^{a}$ ⇢<sup>s</sup> or ⇢*<sup>a</sup>*  $\overline{\phantom{a}}$ field average is performed over a gaussian distribution  $\rho(t)$  $\rho(t) \equiv \rho_{\rm s} + t^a \rho_{\rm o}^a$  $\alpha$ <sup> $\pm$ </sup>  $x^2 + t^2 \rho_0^2$ .  $\frac{1}{\sqrt{2}}$ *g***<sub>4</sub>**  $\alpha$  is the color charge ructure  $\rho(t) = \rho_0 + t^a \rho^a$  $p(y) = \rho s + r \rho_0$ 

Eq. (2) for a given field configuration *A*(*r, t*), and the field average is performed over a gaussian distribution and average in the set of the set the coordinate representation read (with ⇢ being either  $\frac{1}{2}$  or  $\frac{1}{2}$   $\frac{1}{2}$ h*r|* ⇢s*,*o(*t*)*|r*¯i = h*b* + *x/*2*|* ⇢s*,*o(*t*)*|b x/*2i *,* (6) *A<sup>a</sup>*(*q, t*)*A†<sup>b</sup>* (*q*<sup>0</sup> *, t*<sup>0</sup> )  $\alpha$  ordinate space representation *N<sup>c</sup>* **Coordinate space representation** Tr*c*(⇢)+2 *t*  $\sim$   $\sim$ in *Loo*rd  $\mathbf{r}$  , the function given in Eq. (6) to the space representation  $\mathbf{a}^T \mathbf{b} = \mathbf{a}^T \mathbf{b}$  with the momentum  $\mathbf{b}^T \mathbf{b}$  $\overline{C}$  $\overline{\phantom{a}}$  between the treatment of  $\overline{\phantom{a}}$ 

where *| <sup>A</sup>*i denotes the solution of the Schrodinger ¨

where *| <sup>A</sup>*i denotes the solution of the Schrodinger ¨

sions of the high energy quark with the medium con-

stituents, and is thus related to high energy limit of the

in-medium elastic scattering rate, see e.g. Refs. [3, 29]. In

in a quark-gluon plasma [20], and that presented here

for the motion of a jet in the transverse plane, with the

energy jet *E* playing the role of the large heavy quark

mass. In both cases, it is convenient to treat the effect of

field average is performed over a gaussian distribution

a thermalized plasma. When the quark energy is large, because of the time dilation, the quarks sees these correlations as if they were "instantaneous" [8]. Similar considerations are involved in order to obtain Markovian

 $+$ 

tion *| <sup>A</sup>*(*t*)i of Eq. (2) for a given *A*(*r, t*). The quantity encodes the form of the effective interaction between the quark and the medium. More precisely, the aver-

sions of the high energy quark with the medium constituents, and is thus related to high energy limit of the in-medium elastic scattering rate, see e.g. Refs. [3, 29]. In

the limit of high momentum transfer it scales as (*q*) ⇡ *g*<sup>4</sup>*n/q*<sup>4</sup>, where *n* is the density of color charges in the

that of a pure state, i.e., ⇢ is the projector on the solu-

in *x*+, of the order of the inverse of the Debye mass for

a thermalized plasma. When the quark energy is large,

because of the time dilation, the quarks sees these corre-

tion *| <sup>A</sup>*(*t*)i of Eq. (2) for a given *A*(*r, t*). The quantity encodes the form of the effective interaction between

We already emphasized in the introduction the analogy between the treatment of heavy quarks propagating in a quark-gluon plasma [20], and that presented here for the motion of a jet in the transverse plane, with the energy jet *E* playing the role of the large heavy quark

the quark and the medium. More precisely, the aver-

age over the gauge field fluctuations is to be seen as a

simple and efficient way to take into account the colli-

in *x*+, of the order of the inverse of the Debye mass for

in *x*+, of the order of the inverse of the Debye mass for

used in [30] to treat the effect of soft photons or gluons

on the propagation of a hard fermion. There, a corre-

sponding averaging is made with an approximate quan-

a thermalized plasma. When the quark energy is large,

siderations are involved in order to obtain Markovian

a thermalized plasma. When the quark energy is large,

lations as if they were "instantaneous" [8]. Similar con-

We are now equipped to compute the quark density

siderations are involved in order to obtain Markovian

equations for the heavy quark problem [20].

In writing Eq. (3), we have assumed that, prior to the

averaging over the gauge field, the density matrix is

In writing Eq. (3), we have assumed that, prior to the

matrix. As a matrix in color space, it can be decomposed

averaging over the gauge field, the density matrix is

 $\cdots$ 

tion *| <sup>A</sup>*(*t*)i of Eq. (2) for a given *A*(*r, t*). The quantity

encodes the form of the effective interaction between

where Tr*<sup>c</sup>* denotes the trace over the fundamental color

the quark and the medium. More precisely, the aver-

that of a pure state, i.e., ⇢ is the projector on the solu-

medium and *g* is the strong coupling constant.

tion *| <sup>A</sup>*(*t*)i of Eq. (2) for a given *A*(*r, t*). The quantity encodes the form of the effective interaction between

the quark and the medium. More precisely, the aver-

age over the gauge field fluctuations is to be seen as a

**Coorainare space representation**

\n
$$
\langle r | \rho_{s,o}(t) | \bar{r} \rangle = \langle b + x/2 | \rho_{s,o}(t) | b - x/2 \rangle \quad \mapsto \rho_{s,o}(b, x)
$$
\n
$$
b \equiv \frac{r + \bar{r}}{2} \qquad x \equiv r - \bar{r}
$$

) in Eq. (4) stands for **b in Eq. (4) Momentum space represent** <sup>2</sup> *, <sup>x</sup>* ⌘ *<sup>r</sup> <sup>r</sup>*¯ *.* (7)  $\blacksquare$ . The medium correlations have a finite extent with  $\blacksquare$ We shall denote by  $\alpha$  *rating in Eq. (6)* the function given in Eq. (6) the function given in Eq. (6 in Eq. (4) standard planet planet<br>Standard planet pla<br>  $+$ We shall denote by  $\alpha$  *r*<sup>*, t*</sup>, the function given in Eq. (6) the function given in Eq. (6) the function given in Eq. (6) Momentum space representation and the contract of the contract <sup>3</sup> In the formulation of [20], the averaging over the gauge field yields  $\mathbb{E}[\mathbf{S}^{\text{max}}]$ averaging the matrix is not am space represe collisions as an averaging over a fluctuating as an averaging over a fluctuating as an averaging background ba<br>Collisions as a fluctuating background background background background background background background backg  $\overline{\phantom{a}}$  $\tan \theta$  **|**  $\tan \theta$  |<br> $\tan \theta$  |  $\tan \$ 

$$
\rho(\ell, K, t) \equiv \int_{b,x} e^{-i\ell \cdot b} e^{-iK \cdot x} \rho(b, x, t) \qquad K \equiv \frac{k + \bar{k}}{2} \qquad \ell \equiv k - \bar{k}
$$

the coordinate representation representation read (with  $\alpha$  being either  $\alpha$  being either  $\alpha$ 

are the variables conjugate respectively to *x* and *b* in the

⇢ = ⇢*†* is hermitian, and normalized to unity (Tr⇢ = 1).

h*r|* ⇢s*,*o(*t*)*|r*¯i = h*b* + *x/*2*|* ⇢s*,*o(*t*)*|b x/*2i *,* (6)

With these definitions, we can definition  $\mathcal{C}$  and  $\mathcal{C}$  are equations of equations of equations of equations of  $\mathcal{C}$ 

motion for ⇢s*,*o(`*, <sup>K</sup>*) ⌘ h*k|*⇢s(*t*)*|k*¯<sup>i</sup> (see Appendix <sup>A</sup> for

<sup>4</sup> In this work we denote two dimensional momentum integrals as

<sup>5</sup> Note the abuse of notation: we denote by the same symbol ⇢ differ-

ent functions. The arguments of the function should suffice to lift

considered later in this paper is positive definite, as follows from

*<sup>q</sup>*, while for position integrals we use <sup>R</sup> *<sup>d</sup>*2*<sup>x</sup>* ⌘ <sup>R</sup>

because of the time discriming the time discriming  $\mathsf{w}$ **Space matrix element, where they were that they were the<br>Instantaneous con**siderations are involved in order to obtain Markovian  $h = \frac{1}{2}$  $\mathbf{w}$  is the similar constant of the  $\mathbf{w}$  $\blacksquare$  wigher representation **In writing Eq. (3), we have assumed that, prior to the shall also use mixed representations, such as the second representation** age over the gauge field field fluctuations in the gauge field fluctuations is to be seen as a set of the gauge field fluctuations in the gauge field fluctuations in the gauge field fluctuations is to be seen as a set of t simple and efficient way to take into account the colli-Wigner transform ⇢*W*(*b, K, t*)<sup>5</sup> which is the Fourier **Wigner representation**

*g*<sup>4</sup>*n/q*<sup>4</sup>, where *n* is the density of color charges in the

) (*q*)*.* (4)

) (*q*)*.* (4)

Eq. (2) for a given field configuration *A*(*r, t*), and the

<sup>2</sup>(2)(*<sup>q</sup> <sup>q</sup>*<sup>0</sup>

*<sup>a</sup>*⇢)*,* (5)

**c** 1, and 1

Recall that the function (*t t*

whose two-point function is given by

*, t*<sup>0</sup>

) in Eq. (4) stands for

Wigner representation  
\n
$$
\rho_W(b, K, t) \equiv \int_x e^{-iK \cdot x} \rho(b, x, t)
$$
\nThe Wiener function has many features of a classical phase

are the variables conjugate respectively to  $\alpha$  and  $\alpha$  and  $\alpha$  and  $\alpha$  and  $\alpha$  and  $\alpha$  and  $\alpha$  in the variable  $\alpha$ In writing Eq. (3), we have assumed that, prior to the averaging over the gauge field, the density matrix is that of a pure state, i.e.,  $\mathsf{sp}_\mathsf{f}$ igner rundrion nuo r<br>dietributian function **Expanding Service Ser** the medium. More precisely, the medium. More precisely, the average precisely, the aver*b*,*x*<br> **definition in the Wigner function has many features of a classical ph** encodes the vight interaction has no  $\int_a^b x$ <br>The Wigner function has many features of a classica e*i*`*·<sup>b</sup>*e*iK·<sup>x</sup>* ⇢(*b, x, t*)*.* (9) The Wigner function has many features of a classical phase

$$
\int_{\mathbf{b}} \rho_{\scriptscriptstyle{W}}(\mathbf{b}, \mathbf{K}, t) = \mathcal{P}(\mathbf{K}, t)
$$
\n(momentum distribution)

\n
$$
\int_{\mathbf{K}} \rho_{\scriptscriptstyle{W}}(\mathbf{b}, \mathbf{K}, t) = \rho(\mathbf{b}, t)
$$
\n(density)

#### **Equations for the reduced density matrix**  $\Gamma$  and  $\alpha$  is  $\alpha$  fact the simplified version of the  $\alpha$ Liquations for the reduced by @*t*⇢s*,*o(`*, x, t*) = there the dissipative terms. It reads, in the position repr the rec for the singlet and octet components, respectively. The singlet components, respectively. The singlet components,  $\mathcal{C}$  $t$  to the requirements for the re Fig. 2. The equations iterate the elementary processes  $t \cdot \theta$ dencitus matrix in  $\theta$ *toda* diagrams den d for the singlet and octet components, respectively. The Figure 2. Graphs contributing to the transverse space evolusolution is ⇢*<sup>W</sup>* (*b tK/E, K, t* = 0), showing in partiquallows for line realiced demslig mal

there terms. It reads, in the position representative terms. It reads, in the position rep-Using the same approximations as in the HQ case, one gets<br>and the same of proximations as in the HQ case, one gets The (`*, x*) mixed representation is convenient because the interaction term is local in  $\mathcal{O}(\mathcal{X})$ insertions. Remarkably, the evolutions for  $\mathcal{C}$ theirs the come opposited ions of in the Fig. 2. The same approximations as in the **to case one gets** zero momentum exchange in  $\alpha$  cattering single scattering diagram denotes real model  $\frac{1}{2}$  and the same approximations as in the HQ case, one gets i<br>S **Using the same approximations as in the HQ case, one gets**

$$
\frac{\partial}{\partial t} \langle \mathbf{r} | \rho_{\mathbf{s},\mathbf{o}}(t) | \bar{\mathbf{r}} \rangle = -\frac{i}{2E} \left( \frac{\partial^2}{\partial \bar{\mathbf{r}}^2} - \frac{\partial^2}{\partial \mathbf{r}^2} \right) \langle \mathbf{r} | \rho_{\mathbf{s},\mathbf{o}}(t) | \bar{\mathbf{r}} \rangle - \Gamma_{\mathbf{s},\mathbf{o}}(\bar{\mathbf{r}} - \mathbf{r}) \langle \mathbf{r} | \rho_{\mathbf{s}}(t) | \bar{\mathbf{r}} \rangle
$$

$$
\Gamma_{\mathbf{s}}(\mathbf{x}) = C_F \int_{\mathbf{q}} \left( 1 - e^{i\mathbf{q} \cdot \mathbf{x}} \right) \gamma(\mathbf{q})
$$

$$
\Gamma_{\mathbf{o}}(\mathbf{x}) = \int_{\mathbf{q}} \left( C_F + \frac{1}{2N_c} e^{i\mathbf{q} \cdot \mathbf{x}} \right) \gamma(\mathbf{q})
$$

### to the strict eikonal approximation and, as we shall be shall been see, is directly connected to decoherence in coordinate see, is directed to decoherence in connected to decoherence in coordinate in coordinate in coordinate in coordinate  $\alpha$ space.8 In Eq. (12), s*,*<sup>o</sup> stand for the singlet and octet Strict eikonal approximation

The first term on the first term on the right-hand side represents unitary  $\mathcal{L}_\text{max}$ 

The derivation of Eq. (10) presented in Appendix A

derstood by calling on the identities *t*

tively.

resentation

@*t*

$$
\rho_{\text{s,o}}(\boldsymbol{b}, \boldsymbol{x}, t) = \, \rho_{\text{s,o}}^{(0)}\left(\boldsymbol{b}, \boldsymbol{x}\right) \mathrm{e}^{-t \, \Gamma_{\text{s,o}}(\boldsymbol{x})}
$$

h*r|*⇢s*,*o(*t*)*|r*¯i

<sup>8</sup> Eq. (12) is similar to that introduced in [32] to study collisional de- $(colar$  *transparency*) *<sup>E</sup> t* and ⇢ is the initial condition. the interaction term is local in *x* while the partial the interaction term is local in *x* while the partial matrix is damped at rates s*,*o(*x*) that depend solely on **singlet is not damped at short distance (color transparency)** en de la provincia de la provi<br>La provincia de la provincia d [20]. Equation (10) is in fact a simplified version of the there there terms. It reads, in the dissipative terms. It reads, in the position rep-@*r*¯<sup>2</sup> @<sup>2</sup> m The (`*, x*) mixed representation is convenient because Fourier transform diagonalizes part of the kinetic endamping affects non diagonal matrix elements: collisional decoherence there there terms. It reads, in the dissipative terms. It reads, in the position rep- $\mathbf{A}$ h*r|*⇢s*,*o(*t*)*|r*¯i The (`*, x*) mixed representation is convenient because  $\mathcal{F}_{\mathcal{F}}$  fourier transform diagonalizes part of the kinetic energy part of the kinetic energy  $\mathcal{F}_{\mathcal{F}}$ ergy. Eqs. (14) can be found to formally solved by an above the formal solved by an action of the main of the<br>International can be found to the main of the main with the translation operator experience operator  $\mathcal{L}(\mathcal{L})$ *x***. These rates exhibit interesting features in the limit interesting features in th** of small *xx damping affects non diagonal matrix elements: collisional decoherence***<br>
<b>damping affects non diagonal matrix elements: collisional decoherence** Using the harmonic approximation one can write Eq. (14) as the following (frictionless) Fokker-Planck equation for the Wigner function

*<sup>E</sup>* <sup>+</sup> s*,*o(*x*)

s(*x*) ⇡ <sup>4</sup>⇡↵<sup>2</sup>

o(*x*) ⇡

Figure 2. Graphs contributing to the transverse space evolu-

actions, while the single scattering diagram denotes real mo-

tion of the quark density matrix in an infinitesimal time step

In order to solve  $E$  , the solve Eqs. (10) and (11) or equivalent lying  $E$  and (11) or equivalently lying  $E$ 

Eqs. (12), it is convenient to work in the mixed repre-

In order to solve  $E$  ,  $\mathcal{L}_{\mathcal{A}}$  and (11) or equivalently define the solve  $E$  $\mathcal{L}_{\mathcal{A}}$  , it is convenient to work in the mixed representation of  $\mathcal{L}_{\mathcal{A}}$ 

with the translation operator exp [(` *·* @*x*)*t/E*], leading to

@*t*⇢s*,*o(`*, y*(*t*)*, t*) = s*,*o(*y*(*t*))⇢s*,*o(`*, y*(*t*)*, t*)*,* (15)

@*t*⇢s*,*o(`*, y*(*t*)*, t*) = s*,*o(*y*(*t*))⇢s*,*o(`*, y*(*t*)*, t*)*,* (15)

*<sup>r</sup>* the

`*, x* +

@*t*⇢*W*(*b, <sup>K</sup>, t*) =

frared cut-off chosen to be the medium's Debye mass

*m<sup>D</sup>* ⇠ *gT*, with *T* the medium's temperature. Notice

that since / *<sup>n</sup>* ⇠ *<sup>T</sup>*<sup>3</sup>, then *<sup>q</sup>*<sup>ˆ</sup> ⇠ *<sup>T</sup>*<sup>3</sup>. We can also under-

stand <sup>o</sup> ⇠ <sup>1</sup>*/*`fmp ⇠ *<sup>g</sup>*<sup>2</sup>*T*, as defining the inverse of a

mean free path `fmp for gluons in the medium.

<sup>s</sup>*,*<sup>o</sup> (`*, <sup>X</sup>*(*t*)) e <sup>R</sup> *<sup>t</sup>*

⇢s*,*o(`*, x, t*)*,* (14)

⇢s*,*o(`*, x, t*)*,* (14)

### **Harmonic approximation** 38, 39]. It corresponds to the approximation = *g*<sup>4</sup>*n/q*<sup>4</sup> of (*q*), with which we get frarmonic approximation *m<sup>D</sup>* ⇠ *gT*, with *T* the medium's temperature. Notice  $t$ (2)<sup>2</sup> komputer is simply

$$
\Gamma_{\rm s}(\boldsymbol{x}) \approx 4\pi \alpha_s^2 C_F n \log\left(\frac{Q^2}{m_D^2}\right) \frac{\boldsymbol{x}^2}{4} \equiv \frac{\hat{q}}{4} \boldsymbol{x}^2
$$

**The other simple solution corresponds to the corresponding simple solution** proximation vields a Fokker-Planck equation for the Wigner **Matrician is dependent** *x*. These rates exhibit interesting features in the limit equation for the Wigner function  $\mathcal{E}_{\mathcal{A}}$ **components Harmor** singlet density matrix where all color states are equally Harmonic approximation yields a Fokker-Planck equation for the Wigner transform

$$
\partial_t \rho_W(\boldsymbol{b}, \boldsymbol{K}, t) = \left[ -\frac{\boldsymbol{K}}{E} \frac{\partial}{\partial \boldsymbol{b}} + \frac{\hat{q}}{4} \frac{\partial^2}{\partial \boldsymbol{K}^2} \right] \rho_W(\boldsymbol{b}, \boldsymbol{K}, t)
$$

Note the absence of dissination in this equation *m*<sup>2</sup> *m*<sup>2</sup> *m*<sup>2</sup> *m*<sup>2</sup> *g*<sup>*g*</sup>, with *T* the medium is the equation. be equivalently obtained by solving an associated Langevin equation, which is not the present case takes the present case of dissipation in this equation in the p<br>Note the absence of dissipation in this equation

<sup>s</sup>*,*<sup>o</sup> (*b, x*) e*<sup>t</sup>* s*,*o(*x*) *.* (18) Equivalent to a simple Langevin equation and the contract of the contract of the contract of the contract of the stand of  $\frac{1}{2}$  as defining the inverse of a  $\frac{1}{2}$  model the inverse of a set as defining the inverse of a set of a the upper and lower lines of the diagrams in Fig. 2 as creases as *|x|* increases, it suppresses the non diagonal matrix elements of ⇢(*b, x*). This suppression of non di-

ily interpreted. The first case corresponds to the absence

of interactions with the medium. Then, ⇢s*,*o(`*, x, t*) =

tures to emerge in the coordinate space description of

tion does not take place. In fact, in the limit of vanishing

size, the octet behaves as a gluon, and the corresponding

damping factor involves the so-called gluon damping

rate [35, 36]. This suppression of the color octet compo-

nent of the density matrix results in the equilibration of

colors: the density matrix of a quark initially in a given

color state (generally containing both singlet and octet

populated [20]. A similar color equilibration was re-

cently illustrated for the case of color antenna, with fixed

the dynamics. In particular, in this limit *x* and *b* are in-

This formula shows that the coordinate space density

of small *x*. Indeed, when *x* ! 0, <sup>s</sup> ! 0 while <sup>o</sup> !

0, is related to the phenomenon commonly referred to as

color transparency [33, 34]. Technically, it results from

a destructive interference between the three diagrams

There is another effect of the damping s: since it in-

agonal matrix elements due to collisions is commonly

referred to as decoherence. It will be discussed in more

A useful approximation for the rates s*,*o(*x*) is the so-

*q*ˆ is the jet quenching parameter which will be specified

the two members of a fictitious color dipole propagating

into the medium. A color singlet dipole of small size is

seen as a neutral object by the medium, which results in

In the case of the octet the aforementioned cancella-

tion does not take place. In fact, in the limit of vanishing

size, the octet behaves as a gluon, and the corresponding

<sup>s</sup>*,*<sup>o</sup> (`*, X*(*t*)) describes free streaming, where the rela-

tive coordinate *X*(*t*) evolves in time at constant veloc-

ity `*/E*. The Wigner transform corresponding to this

solution is ⇢*<sup>W</sup>* (*b tK/E, K, t* = 0), showing in particular that, as expected, the momentum distribution is

<sup>s</sup>*,*<sup>o</sup> (*b, x*) e*<sup>t</sup>* s*,*o(*x*) *.* (18)

where the drift term can be ignored, which occurs ei-

ther for ` = 0 or *E* ! 1, i.e. the exact eikonal limit. This corresponds to the infinite mass limit in the heavy

quark problem, a limit where one expects classical fea-

*<sup>q</sup>* (*q*), see Eq. (19) below. The first property, <sup>s</sup> !

tures to emerge in the coordinate space description of

the dynamics. In particular, in this limit *x* and *b* are in-

This formula shows that the coordinate space density

matrix is damped at rates s*,*o(*x*) that depend solely on

*x*. These rates exhibit interesting features in the limit

of small *x*. Indeed, when *x* ! 0, <sup>s</sup> ! 0 while <sup>o</sup> !

0, is related to the phenomenon commonly referred to as

color transparency [33, 34]. Technically, it results from

a destructive interference between the three diagrams

*<sup>q</sup>* (*q*), see Eq. (19) below. The first property, <sup>s</sup> !

<sup>4</sup> *q*ˆ*x*<sup>2</sup>, where

*<sup>b</sup>* ⇢*<sup>W</sup>* (*b, K, t*) =

$$
E\frac{\mathrm{d}^2\boldsymbol{b}}{\mathrm{d}t} = \boldsymbol{\xi}(t), \quad \langle \xi_i(t_1)\xi_j(t_2)\rangle = \frac{\hat{q}}{2}\delta_{ij}\delta(t_1 - t_2)
$$

#### *b*(*t*) = 2*µ*<sup>2</sup> 2*µ*<sup>2</sup> 2*t*<sup>1</sup> *t ,* Solution for a simple initial condition  $\int_{\Omega}$  for  $\alpha$  curve extending  $\int_{\Omega}$  or corresponding  $\int_{\Omega}$  $\overline{\phantom{a}}$   $\overline{\phantom{a}}$   $\overline{\phantom{a}}$   $\overline{\phantom{a}}$   $\overline{\phantom{a}}$  $SO$ *<i>DLUC* <u>íon</u>  $\sim$  ( **P** *µ*2 i<br>L  $\boldsymbol{\mathcal{U}}$ *t* 2 *t*2 0 +  $\overline{1}$ 3 *t* 3 *t*3 2 l, .e iv *<sup>c</sup>*(*t*) = *<sup>E</sup>*<sup>2</sup> *<sup>µ</sup>*<sup>2</sup> <sup>+</sup> *<sup>µ</sup>*<sup>2</sup>*<sup>t</sup>* on.  $\frac{1}{2}$ *E*<sup>2</sup>  $\overline{L}$  $\mathbf{1}$  + *t* 2  $\blacksquare$  $\overline{\mathbf{t}}$ *t* 3 *t*3 I *.* (31) *<sup>b</sup>*(*t*) = 2*µ*<sup>2</sup>*<sup>t</sup> qt*<sup>ˆ</sup> <sup>2</sup> ⌘ 2*µ*<sup>2</sup> 2*t*<sup>1</sup> *<sup>c</sup>*(*t*) = *<sup>E</sup>*<sup>2</sup> *<sup>µ</sup>*<sup>2</sup> <sup>+</sup> *<sup>µ</sup>*<sup>2</sup>*<sup>t</sup>* n foi *E*<sup>2</sup> 1 + *t* 2 <u>ኮ</u>ር *.* (31)  $\int$ sounder tor a simple think *plution for*  $\overline{\phantom{a}}$ *t t*1  $\delta$ *lution for a simple ini*  $\sim$   $\sim$   $\sim$   $\sim$   $\sim$   $\sim$ was *Cover* a for a serveped erected corrected Fig. **??** we plot the real part of <sup>h</sup>*k<sup>|</sup>* ⇢ *<sup>|</sup>k*¯<sup>i</sup> in terms of the solution for a simple i lution times and taking *<sup>k</sup>* <sup>k</sup> *<sup>k</sup>*¯. <sup>11</sup> In addition, we provide

density ⇢(*b*), obtained from ⇢(*b, x*) in Eq. (39) by setting

 $\blacksquare$  Thitial G  $\overline{a}$  and  $\overline{a}$ *n*<br>3<br>2<br>2 sian wave packet  $\rho_w(b, K, 0) = 4e^{-\mu^2 b^2} e^{-\frac{K^2}{\mu^2}}$  $-\frac{K^2}{\mu^2}$  $\mu^2$  . (26) and coordinate space, with an accuracy specified by the single parameter *µ*. Thus the dispersion in position and momentum are correlated and when *µ* ! 0,  $\boldsymbol{\mathsf{a}}$ l Gaussian wave packet  $\rho_{\bm{w}}(\boldsymbol{b},\boldsymbol{K},0)=4$ tation which is best seen on the Wigner transform, given  $\mathcal{L}_{\mathbf{W}}$ and coordinate space, with an accuracy specified by the single parameter *µ*. Thus the dispersion in posi-The time scales *t*<sup>1</sup> and *t*<sup>2</sup> will be defined shortly. The co- $\bm{\mu}$  Gaussian wave packet  $\bm{\rho}_{\bm{w}}(\bm{b}, \bm{K}, 0) = 4$ where the particle is localized in both momentum space  $T<sub>min</sub>$ the single parameter *µ*. Thus the dispersion in posi- $Gaussian$  wave packet  $\alpha$   $(k$   $K$   $0) = 4\alpha$ **Duussiuit wuve putnet**  $p_W(\boldsymbol{\theta}, \mathbf{\Lambda}, 0) = 4e$ nitial Gaussian wave packet  $\quad$   $\alpha$  (b  $\boldsymbol{K}$  ())  $=4\mathrm{e}^{-\mu^{2}}$ of the density matrix. It seems the density matrix  $\mathbf{r} \cdot \mathbf{w}$ **nitial Gauss** performing averages over both the noise (as in Eq. (24)) and the initial condition ( large extent, the time evolution is governed by the clas*a*  $(1 + x^2) = 4 - u^2b^2$ (40) **Initial Gaussian v**  $2 \cdot 2 \cdot 2$ **Initial Gaussian wave packet** ✓` ⌘ *<sup>|</sup><sup>k</sup> <sup>k</sup>*¯*|/E*) Initial Gaussian wave packet  $\rho_m(b, K, 0) =$  $P_W(0,11,0)$ 

played in Eq. (22) with *qt*ˆ substituted by

#### **tion are correlated as a linguist Solution for the Wigner transform remains Gaussian at a** ⇢(*K*) ! (2⇡)<sup>2</sup>(*K*), while when *<sup>µ</sup>* ! 1, ⇢(*b*) ! (*b*). by This Wigner transform is akin to that of a coherent state, where the particle is localized in both momentum space ⇢*W*(*b, <sup>K</sup>*) = <sup>4</sup> *<sup>D</sup>* exp ⇢ 1 ✓  $\mathcal{L}$  that, due to our specific choice of initial condition,  $\mathcal{L}$ the Wigner transform is initially positive and remains so  $rac{1}{2}$  *frans <sup>E</sup> <sup>b</sup> · <sup>K</sup>* <sup>+</sup> *c <sup>E</sup>*<sup>2</sup> *<sup>K</sup>*<sup>2</sup> on for the Wigner transform remains G  $S_{0}$ the Wigner transform is initially positive and remains so *rransform remain* **, and Solution for the Wigner transform remains G** ⇢(*K*) ! (2⇡)<sup>2</sup>(*K*), while when *<sup>µ</sup>* ! 1, ⇢(*b*) ! (*b*). the Wigner transform is initially positive and remains so *i* for the wigher fransform remains Ga **Solution for the Wigner transform remains Gau** *<sup>t</sup>*<sup>1</sup> <sup>=</sup> *<sup>µ</sup>*<sup>2</sup>  $F = \frac{1}{2}$  the momentum distribution, the momentum distribution is ob- $\mathsf r$  the wigher fransform remains Gaussian at **also reflects the Solution for the Wigner transform remains Golution for the Wigner transform remains Go**  $t_{\rm eff}$  the product of a free product of and and a free product of a free pr **Solution for the Wigner transform remains Gaussian at all times** Solution for the Wigner transform remains Gaussian at all times **The condition for the condition of all above are clearly visited above and condition**

$$
\rho_W(\mathbf{b}, \mathbf{K}) = \frac{4}{D} \exp \left\{ -\frac{1}{D} \left( a \, \mathbf{b}^2 + \frac{b}{E} \, \mathbf{b} \cdot \mathbf{K} + \frac{c}{E^2} \mathbf{K}^2 \right) \right\}
$$
\n
$$
a = \langle \mathbf{K}^2 \rangle_t \qquad \frac{c}{E^2} = \langle \mathbf{b}^2 \rangle_t \qquad \frac{b}{E} = -2 \langle \mathbf{b} \cdot \mathbf{K} \rangle_t \qquad D = \langle \mathbf{b}^2 \rangle \langle \mathbf{K}^2 \rangle - \langle \mathbf{K} \cdot \mathbf{b} \rangle = \frac{ac}{E^2} - \frac{b}{2E}
$$

Three time scales		
$t_0 < t_1 < t_2$	$t_1 = \frac{\mu^2}{\hat{q}}$	$t_2^3 \equiv \frac{E^2}{\hat{q}\mu^2}$
$(t_0 < t_1 < t_2)$	$(t_2^2)_t = \mu^2 + \hat{q}t = \mu^2 \left(1 + \frac{t}{t_1}\right)$	
$\langle \mathbf{k}^2 \rangle_t = \frac{c(t)}{E^2} = \frac{1}{\mu^2} \left[1 + \frac{t^2}{t_0^2} + \frac{1}{3} \frac{t^3}{t_2^3}\right]$		

and the new time scale *t*<sup>2</sup> is given by (see Eqs. (31) and *x* = 0. One gets In this formula, *s*(*u*) is a linear function of *u* such that the density variables of the following argument that the following argument that the following angles of the following angles exploits the underlying classical dynamics. Indeed, the more closely related to jet observables. To that aim, we

*<sup>µ</sup>*<sup>2</sup> *.* (26)

*<sup>µ</sup>*<sup>2</sup> *.* (26)

the density matrix is similar to that in Eq. (17), and it

*<sup>µ</sup>*<sup>2</sup> *.* (26)

Z *<sup>t</sup>*

*K*<sup>2</sup>⇢*<sup>w</sup>* (*b, K, t*)*.* (35)

г

*µ*2

⇣

*.* (37)

)*.* (24)

*,* (38)

*du* (*s*(*u*))<sup>2</sup>

. This expression of

*<sup>µ</sup>*<sup>2</sup> *.* (26)

This Wigner transform is akin to that of a coherent state,

Alternatively, the same expectation values can be obtained by solving the Langevin equation (Eq. (23)), and

This Wigner transform is akin to that of a coherent state,

This Wigner transform is akin to that of a coherent state, where the particle is localized in both momentum space

 $\sim$  this point, it is useful to make further contact with  $\sim$ 

the formalism used in [8] (see also Appendix A). There,

we introduced a two-point function *S*(2) which plays a

role similar to that of the Liouville operator that propagates the density matrix from the initial time to the final

 $\mathbb{R}^n$ 

parameter *q*ˆ. This equation is easily solved. As an illustration, consider the momentum *K* = *E* d*b/*d*t*. By

In this formula, *s*(*u*) is a linear function of *u* such that

This solution can be used to evaluate the expectation

value <sup>h</sup>*K*<sup>2</sup>(*t*)<sup>i</sup> by averaging over the noise. Taking, for simplicity, as initial condition *K*(*t* = 0) = 0, one gets

<sup>h</sup>*K*<sup>2</sup>(*t*)<sup>i</sup> = ˆ*qt*, in agreement with the result obtained ear-

In the following we shall analyze in detail various fea-

tures of the solution of the equation of motion for the quark density matrix for a specific initial condition and using the harmonic approximation. Since, as we have argued above, the color octet component of the density matrix is damped when the energetic quark propagates in a color neutral medium, we now focus on the singlet component and study in more detail its time evolution. As we only consider singlet quantities, we drop from

the density matrix is similar to that in Eq. (17), and it also reflects the structure of Eq. (12). The first line in this expression is the product of a free propagator and a complex conjugate one, and represents free motion. The second line is the exponential representing the effects of the collisions, as in Eq. (17). The two lines are not independent since the trajectory *s*(*u*) depends on the end

This Wigner transform is akin to that of a coherent state, where the particle is localized in both momentum space

tion and momentum are correlated and when *µ* ! 0, ⇢(*K*) ! (2⇡)<sup>2</sup>(*K*), while when *<sup>µ</sup>* ! 1, ⇢(*b*) ! (*b*).

⇢(*K*) ! (2⇡)<sup>2</sup>(*K*), while when *<sup>µ</sup>* ! 1, ⇢(*b*) ! (*b*).

In the absence of interactions, the initial wave packet spreads freely and, as observed earlier, its Wigner trans-

time (see Appendix B1 in [8]) . We have indeed

 $\sim$  Given an initial density matrix  $\sim$ 

this expression is the product of a free propagator and a

ate to get only the momentum distribution.

In the absence of interactions, the initial wave packet spreads freely and, as observed earlier, its Wigner transform evolves as ⇢*W*(*b* (*K/E*)*t, K,* 0). The momentum distribution remains unchanged, but the density ⇢(*b, t*), obtained by integrating the Wigner transform over *K*,

In the absence of interactions, the initial wave packet spreads freely and, as observed earlier, its Wigner transform evolves as ⇢*W*(*b* (*K/E*)*t, K,* 0). The momentum distribution remains unchanged, but the density ⇢(*b, t*), obtained by integrating the Wigner transform over *K*,

form evolves as ⇢*W*(*b* (*K/E*)*t, K,* 0). The momentum distribution remains unchanged, but the density ⇢(*b, t*), obtained by integrating the Wigner transform over *K*,

the single parameter *µ*. Thus the dispersion in posi-

*<sup>r</sup>*0*,r*¯0;*r,r*¯(*t*)h*r|* ⇢(*t* = 0)*|r*¯i*,* (36)

sical Langevin equation, quantum mechanics entering

*s*(0) = *r r*¯ and *s*(*t*) = *r*<sup>0</sup> *r*¯<sup>0</sup>

tion and momentum are correlated and when *µ* ! 0,

$$
\theta_{\mu}^{2} = \frac{\mu^{2}}{E^{2}}, \quad \theta_{c}^{2}(t) = \frac{1}{\hat{q}t^{3}}, \quad \theta_{\rm br}^{2}(t) = \frac{\hat{q}t}{E^{2}}
$$



and gradually identifies to the momentum distribution  $\mathcal{L}_\mathcal{A}$ 

This corresponds to the time where the time where the energy of the quark satisfies *<sup>E</sup>* = ˆ*qt*<sup>2</sup> ⌘ !*c*. However, neither the time scale *t<sup>E</sup>* nor the energy !*<sup>c</sup>* seem to play any significant

Figure 3. Time evolution of the real part of the quark density matrix, in momentum (top, red) and position (bottom, blue) space. We have used *q*ˆ = 0*.*3 GeV<sup>3</sup>, *µ* = 0*.*3 GeV, and *E* =

*<sup>a</sup>*(*t*) = *<sup>µ</sup>*<sup>2</sup> + ˆ*qt* ⌘ *<sup>µ</sup>*<sup>2</sup>

where we have set *b* = (*r* + *r*<sup>0</sup>

Fourier transform of the variable *b*.

*<sup>c</sup>*(*t*) = *<sup>E</sup>*<sup>2</sup>

e*itK·*`*/*(2*E*) *,* (56)

⇢*W*(*b, <sup>K</sup>*) = <sup>4</sup>

spect to *b*. It reads

*<sup>b</sup>*(*t*) = 2*µ*<sup>2</sup>*<sup>t</sup> qt*<sup>ˆ</sup> <sup>2</sup> ⌘ 2*µ*<sup>2</sup>

is easily verified that this is the same as Eq. (30 ) after

We return now to the analysis of the physical content of the density matrix. Its momentum representation is obtained by a Fourier transform of ⇢*<sup>W</sup>* (*b, K*) with re-

The time scales *t*<sup>1</sup> and *t*<sup>2</sup> will be defined shortly. The co-

efficients *a*(*t*)*, b*(*t*)*, c*(*t*) have a simple physical interpre-

tation which is best seen on the Wigner transform, given

### time. a ty matrix becomes alagonal tum and position spaces, so we shall consider both rep-The reduced density matrix becomes diagonal **compatibility**

space. We have a space the discussion of the discussion of the discussion of the discussion of the discussion o<br>Space representation of the discussion mare opace and in momentum **….. in both coordinate space and in momentum space !**



 $0.0 -$ 

 $\mathsf{L}_0$ 

 $0.0 + 0.0 + 0.0$ 

0*.*5 1*.*0 1*.*5 2*.*0

¯ *k*? [GeV]

0*.*5

 $\vert \cdot \vert 1.0$ 

1*.*5

0 1 2  $k_{\perp}$  [GeV]

 $t = 100.0$  [fm]

0 1 2  $k_{\perp}$  [GeV]

 $\Box$ <sub>0.000</sub>

0*.*025

0*.*050

0*.*075

0 1 2  $k_{\perp}$  [GeV]

 $t = 5.0$  [fm]

0 1 2  $k_{\perp}$  [GeV]

 $0.0 + 0.0 + 0.0$ 

 $0.0 + 0.0 + 0.0$ 

0*.*5 1*.*0 1*.*5 2*.*0

¯ *k*? [GeV]

$$
\rho(\boldsymbol{b}=0,\boldsymbol{x},t) \approx \frac{1}{\pi \langle \boldsymbol{b}^2 \rangle_t} \exp \left\{-\frac{\langle \boldsymbol{k}^2 \rangle_t \boldsymbol{x}^2}{4}\right\}
$$

*.* (50)

*,* (51)

$$
\rho(\ell, \mathbf{K} = 0, t) \approx \frac{4\pi}{\mu^2 + \hat{q}t} \exp\left\{-\frac{\ell^2 \hat{q}t^3}{48E^2}\right\}
$$

$$
= \frac{4\pi}{\langle \mathbf{k}^2 \rangle_t} \exp\left\{-\frac{\ell^2 \langle \mathbf{b}^2 \rangle_t}{16}\right\}
$$

 $\frac{1}{1000}$ 

tum distribution *P*(*k* = 0*, t*) when *t* & *t*1. The other

factor is a Gaussian distribution in ` <sup>=</sup> *<sup>k</sup> <sup>k</sup>*¯, whose

#### **Entropy growth** from the pure state  $\alpha$  pure states that  $\alpha$ = ln <sup>h</sup>*K*<sup>2</sup>i*<sup>t</sup>* **Entropy growth**, in the protocontext, carries essentially a protocontext, Transfer tropy, the Renyi-2 entropy and the Renyi-2 entropy and the Wigner entropy and the Wigner entropy carry entropy To illustrate all the above points, we display in  $\mathcal{L}_\mathcal{D}$  in  $\mathcal{L}_\mathcal{D}$  in  $\mathcal{L}_\mathcal{D}$  in  $\mathcal{L}_\mathcal{D}$ the density matrix. We shall also mention the Renyi-2 the same information as the von Neumann entropy. 1, and *p* = 1. When the density matrix represent a sta-In the present case, the calculation of *S*vN reduces to = ln <sup>h</sup>*K*<sup>2</sup>i*<sup>t</sup> µ µ µ <i>n <i>n n*

tistical mixture, *p =* 1. In takes the form of *p + 1. In t*erms of *p + 1. In t* It increases logarithmically with <sup>h</sup>*K*<sup>2</sup>i*t*. Note the  $\sum_{i=1}^{n}$ **von Neumann entropy Svan Neumann entropy** still sensitive to the initial configuration of the system. In positive to the system of multidimensional Gaussian in<br>The calculation of multipliers in position space, this region is characterized by the suppression tistical mixture, *p <* 1. In terms of *p*, *S*vN takes the form It is explicitly called the control of the shown of t *t*1⌧*t*⌧*t*<sup>2</sup> *t*1

entropy associated with the Wigner representation of the Wigner representation of the Wigner representation of

(shaded blue, yellow and pink regions), related to the evolu- $\cdot$  in  $\cdot$ condition sets the form of the density matrix. This is followed  $\sim$  1  $\mathcal{L}_{\mathcal{A}}$ still sensitive to the initial configuration of the system. In position space, this region is characterized by the suppression  $\sim$  0 same form as the initial condition. Finally, at later times the system loses all knowledge of the initial condition (pink re-

 $t$  in momentum space. In the first (blue), the initial space. In the initial space, the initial space, the initial space.

condition sets the form of the density matrix. This is followed

by a region (yellow) where momentum broadening drives dif-

fusion in momentum space, but the off-diagonal elements are

Interestingly, the angle ✓*<sup>c</sup>* has been previously identi-

*E* = 200 GeV. We also identify the scale when the quark energy matches the scale !*c*, i.e. when *t* = *tE*. This divides the

Interestingly, the angle ✓*<sup>c</sup>* has been previously identi-

fied in the context of color coherence of QCD antennas

in the medium [41–46]. It is also related to the character-

istic angle for medium induced radiation [5, 6]. In the

present case, ✓*<sup>c</sup>* controls the loss of coherence between

the two legs of the effective dipole in the third diagram

in Fig. 2. As discussed earlier, this loss of coherence, that

manifests itself in the vanishing of the off-diagonal ma-

trix elements of <sup>h</sup>*k<sup>|</sup>* ⇢ *<sup>|</sup>k*¯i, can be related to the random

, *µ* = 0*.*3 GeV, and

$$
S_{\rm vN}[\rho]=-\text{Tr}\rho\ln\rho\ =\log\left(\frac{1-p}{4p}\right)+\frac{1}{\sqrt{p}}\,\ln\frac{1+p+2\sqrt{p}}{(1-p)}
$$

"purity" 
$$
p \equiv \text{Tr}\rho^2
$$
  
\n
$$
\frac{1}{p} = \left(1 + \frac{t}{t_1}\right) \left(1 + \frac{t^3}{12t_2^3} \frac{t + 4t_1}{t + t_1}\right)
$$

#### regime: **time: behavior** three regimes  $\frac{25}{100}$ *<sup>S</sup>*vN ' ln <sup>1</sup> *r*<br>Le time behavior **accurate time behavior** the form of  $25$ fied in the color coherence of  $\mathcal{L}$ in the medium is also related to the character set of the character set of the character set of the character<br>The character set of the c **late time behavior** the entropy behavior

$$
S_{\rm vN}\simeq \ln \frac{1}{p}\simeq \ln \frac{\hat{q}^2 t^4}{E^2}\sim \ln \langle \bm{k}^2 \rangle_t \langle \bm{b}^2 \rangle_t \qquad \qquad \sum_{\substack{\bm{\hat{\beta}} = 15 \\ \bm{\hat{\beta}} = 15 \\ \bm{\hat{\beta}}}} \, .
$$

#### **the classical Wigner entropy with the classical feature, let us consider the consider the consider**  $10^{-1}$ following Wigner entropy explicit time of **p and the formulation**  $\mathbb{P}$  in the formulation of  $\mathbb{P}$  in the coefficients of the coefficients o  $\blacksquare$   $\blacksquare$  $\mathbf{r}$  is the vanishing of the off-diagonal mass  $\mathbf{r}$ **and** *classical Wigner entropy*  $\alpha$  is increased increase as the logarithm of the total phase  $\alpha$ space measured by his whom is what one could be a space of countries of could be a space of could be a space of could be a space of countries of

$$
S_{_{\text{W}}} \equiv -\int_{\boldsymbol{K},\boldsymbol{b}} \rho_{_{\text{W}}}(\boldsymbol{b},\boldsymbol{K}) \log \rho_{_{\text{W}}}(\boldsymbol{b},\boldsymbol{K}) \qquad \qquad \frac{5}{0}
$$



At late time all three entropies, the von Neumann en-

Thus, to within a factor 1*/*2 and the constant 2 ln 4, it is identical to the Wigner entropy introduced in Eq. (68).

At late time all three entropies, the von Neumann en-

tropy, the Renyi-2 entropy and the Wigner entropy carry

To illustrate all the above points, we display in  $\mathcal{L}_\mathcal{D}$  in  $\mathcal{L}_\mathcal{D}$  in  $\mathcal{L}_\mathcal{D}$  in  $\mathcal{L}_\mathcal{D}$ 

As a final remark, we note that the apparent un-

bounded growth of the entropy at late times, is a con-

sequence of the fact that the master equation for the re-

duced density matrix that we have used accounts only

the time evolutions of both the von Neumann and the Wigner entropies. This figure confirms that *S*vN ' *S<sup>W</sup>*

bounded growth of the entropy at late times, is a consequence of the fact that the master equation for the reduced density matrix that we have used accounts only for collisional decoherence, but ignore friction and dissipation. We could expect indeed friction to alter the late time evolution and drive the transverse motion of

# **Summary**

- **• The theory of open quantum systems offers a useful framework to calculate the interactions of complex hard probes with their environment.**
- **• It also provides interesting perspectives and reveals connections between seemingly unrelated problems (examples discussed in the talk: quarkonia and (simplified) jets).**
- **• It allows to derive many different approaches from a common starting point.**