Hard probes from the viewpoint of open quantum systems

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Hard probes

Hard probes are produced in hard processes on very short time scales

$$\Delta t \sim 1/M \ll 1\mathrm{fm/c}$$

Thus, they are present in the early stages of heavy ion collisions

Two types of hard probes:

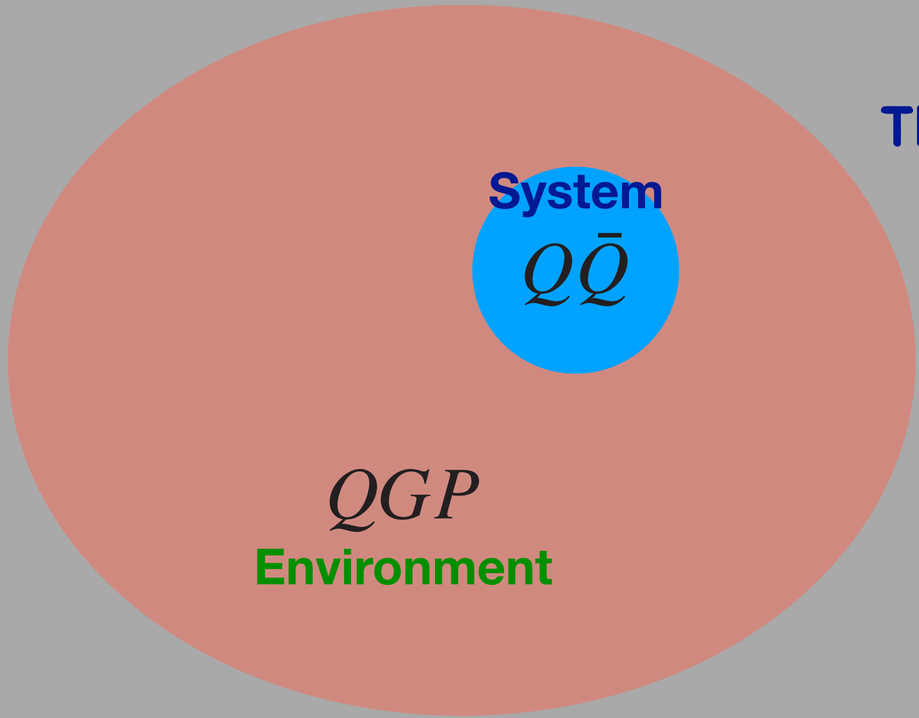
- "Elementary" HP: direct photons, Z and W bosons, etc.
- "Complex" HP: quarkonia, jets

Elementary hard probes provide information on the "initial state" (e.g. npdf): Their yield scales with the number of n-n collisions. They are weakly affected by the surrounding medium.

Complex hard probes have their own dynamics in the absence of the QGP. This dynamics can be significantly altered by the presence of the quark-gluon plasma. Understanding such modifications can yield information about the QGP properties.

Open Quantum Systems

Open quantum system (1)



The system is "small"

The environment is "big" and weakly perturbed by the presence of the system

The dynamics of the system is obtained after eliminating the degrees of freedom of the environment. This yields in general a non unitary evolution (decoherence, dissipation).

The dynamics of the system is affected by the presence of the environment via simple correlation functions characterising the environment. The system probes these correlation functions.

Open quantum system (2)

The density matrix of total system $\mathcal{D}(t)$ obeys the equation of motion

$$i\frac{\mathrm{d}\mathcal{D}}{\mathrm{d}t} = [H, \mathcal{D}].$$

We need the reduced density matrix of the system:

$$\mathcal{D}_Q(t) = \mathrm{Tr}_{\mathrm{pl}}\mathcal{D}(t)$$

Equation of motion for $\mathcal{D}_Q(t)$

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{D}_Q(t) = -i[H_Q, \mathcal{D}_Q(t)] + \int_0^{t-t_0} \mathrm{d}\tau\, \mathcal{L}(\tau)\mathcal{D}_Q(t-\tau)$$

Non hamiltonian contribution

Various strategies:

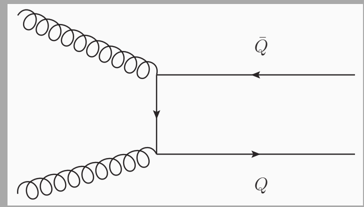
- Feynman-Vernon Influence functional
- Lindblad equation,
- Schwinger-Keldysh diagrammatic techniques,
- Etc

Heavy quarkonia

[For details, see JPB, M. Escobedo-Espinosa, 1711.10812,1803.07996]

Heavy quarks and quarkonia as 'hard probes'

Heavy quarks are produced in pairs in the early stages of URHIC. Their number remains constant.



Formation time of a $Q\bar{Q}$ pair is small $\qquad \Delta t \sim \dfrac{1}{2M_Q}$

$J/\Psi \qquad M_c \simeq 1.5$ Gev $\qquad \Delta t \simeq 0.07$ fm/c

$\Upsilon \qquad M_b \simeq 4.5$ Gev $\qquad \Delta t \simeq 0.02$ fm/c

Dynamics of heavy quarks is non-relativistic

$$H = \frac{P^2}{M_Q} + V(r) \qquad \left(V(r) = \frac{\alpha_s}{r} + \sigma r \right)$$

The potential can be obtained using effective theory (pNRQCD)

[see N. Brambilla, A.Pineda, J. Soto, A. Vairo, NPB566 (2000) 275]

Heavy quark interaction at finite T

Initial suggestion (Matsui-Satz 86): screening of the potential

$$H = \frac{P^2}{M_Q} + V(r) \qquad\qquad V(r) = -\frac{\alpha}{r}\mathrm{e}^{-r\, m_D(T)} + \sigma(T) r$$

This picture predicts a "suppression" of bound states at high temperature, the most "fragile" ones (bigger, less bound) disappearing first as the temperature increases ("sequential suppression").

Hence the idea of using quarkonia to diagnose the formation of quark-gluon plasma in URHIC

However, the dynamics of the quarkonia does not reduce to a mere modification of the potential: non unitary evolution, here caused by "collisions" with plasma constituents.

Typical approximations in OQS

(i) Weak coupling between HQ and the plasma

$$H_1 = -g \int_{\boldsymbol{r}} A_0^a(\boldsymbol{r}) n^a(\boldsymbol{r}),$$

gauge potential of plasma

HQ density

$$n^a(\boldsymbol{x}) = \delta(\boldsymbol{x} - \hat{\boldsymbol{r}})\, t^a \otimes \mathbb{I} - \mathbb{I} \otimes \delta(\boldsymbol{x} - \hat{\boldsymbol{r}})\, \tilde{t}^a.$$

The presence of the heavy quarks does not modify significantly the equilibrium state of the plasma.

The influence of the plasma on the heavy quark dynamics is characterized by simple response functions (correlators)

$$\Delta(t_1, t_2) \equiv \langle A_{\rm pl}(t_1) A_{\rm pl}(t_2) \rangle_T = \mathrm{Tr}\left[A_{\rm pl}(t_1) A_{\rm pl}(t_2) \mathcal{D}_{\rm pl} \right]$$

No assumption of weak or strong coupling needs to be made concerning the plasma. The correlators can, in some cases, be obtained from lattice calculations.

(ii) The response of the plasma is "fast"

plasma response is characterized by a single energy scale, the Debye mass

$$m_D = CT \qquad (C \simeq 2)$$ In strict weak coupling $C = g$

$$m_D \ll M$$

collisions with plasma constituents involve small energy transfer

soft gluon exchanges

$$q \lesssim m_D \ll M$$

small energy transfer

$$\frac{q^2}{M} \sim \frac{m_D^2}{M} \ll m_D$$

the relevant correlator is then generically of the form

$$\Delta(\omega = 0, \mathbf{r}) = \Delta^R(\omega = 0, \mathbf{r}) + i\Delta^<(\omega = 0, \mathbf{r})$$

$$V(\boldsymbol{r}) = -\Delta^R(\omega = 0, \boldsymbol{r}), \qquad W(\boldsymbol{r}) = -\Delta^<(\omega = 0, \boldsymbol{r})$$

Screened potential Imaginary potential

from the point of view of the HQ the interactions with the plasma are nearly instantaneous ("collisions")

$$\Delta(t_x - t_y) = \int \frac{\mathrm{d}\omega}{2\pi} \mathrm{e}^{-\mathrm{i}\omega(t_x - t_y)} \left[\Delta(\omega = 0) + \omega\Delta'(\omega = 0)\right]$$

$$\simeq \delta(t_x - t_y)\Delta(\omega = 0) + \mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t_x}\delta(t_x - t_y)\Delta'(\omega = 0)$$

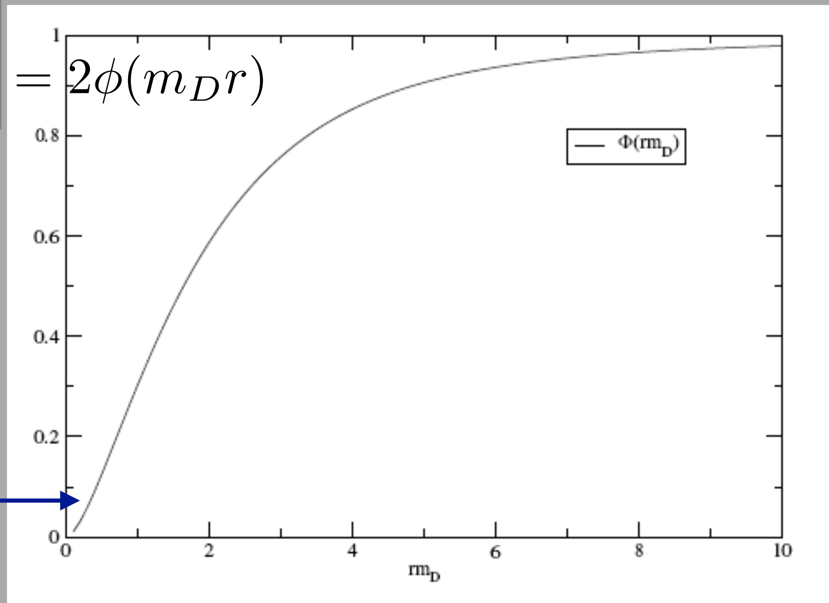
Static response and "optical potential"

(*first obtained by M. Laine et al hep-ph/ 0611300)

$$\mathcal{V}(r) = V(r) + iW(r)$$

$$\Delta^R(\omega = 0, r) = -V(r) \qquad \Delta^<(\omega = 0, r) = -W(r)$$

$$\boxed{\Gamma(\boldsymbol{r}) = W(\boldsymbol{r}) - W(0)} = 2\phi(m_D r)$$

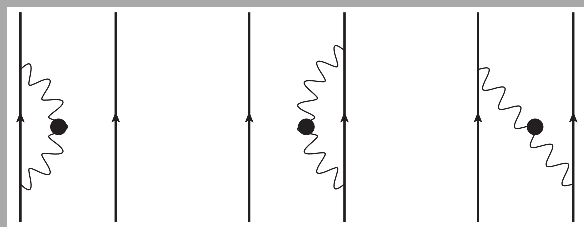


At large distance the imaginary part is twice the damping rate of the heavy quark

$$\Phi(x) = \frac{x^2}{3}\left(-\ln x + \frac{4}{3} - \gamma_E\right)$$

makes density matrix nearly diagonal in coordinate space

At short distance, interference produces cancellation: a small dipole does not “see” the electric field fluctuations.



For one heavy quark

$$\partial_t \langle \boldsymbol{r}|\mathcal{D}_Q|\boldsymbol{r}'\rangle = \cdots - \Gamma(\boldsymbol{r} - \boldsymbol{r}')\langle \boldsymbol{r}|\mathcal{D}_Q|\boldsymbol{r}'\rangle$$

(iii) semi-classical approximation

$$M \gg T$$

HQ thermal wavelength $\lambda_{\rm th} \sim \dfrac{1}{\sqrt{MT}} \ll \dfrac{1}{T}$

Density matrix becomes nearly diagonal

$$\langle \mathbf{r} | \mathcal{D}_Q | \mathbf{r}' \rangle \simeq 0 \quad \text{when} \quad |\mathbf{r} - \mathbf{r}'| \gtrsim \lambda_{\rm th}$$

Expansion in $|\mathbf{r} - \mathbf{r}'|$ $\longrightarrow$ Fokker-Planck and Langevin equations

Semi-classical expansion for heavy quark motion

- Equation for the density matrix $\longrightarrow$ Langevin equation

- Langevin equation for the relative motion

$$\frac{M}{2}\ddot{\boldsymbol{r}}^i = -\gamma_{ij}\boldsymbol{v}^j - \boldsymbol{\nabla}^i V(\boldsymbol{r}) + \xi^i(\boldsymbol{r}, t)$$

$$\gamma_{ij}(\boldsymbol{r}) = \frac{1}{2T}\eta_{ij}(\boldsymbol{r}) \qquad \langle \xi^i(\mathbf{r}, t)\xi^j(\mathbf{r}, t')\rangle = \eta_{ij}(\mathbf{r})\delta(t - t')$$

Non trivial noise

- For an isotropic plasma

$$\eta_{ij}(\boldsymbol{r}) = \delta_{ij}\eta(\boldsymbol{r}) \qquad \eta(\boldsymbol{r}) = \frac{1}{6}\left(\nabla^2 W(0) + \nabla^2 W(\boldsymbol{r})\right)$$

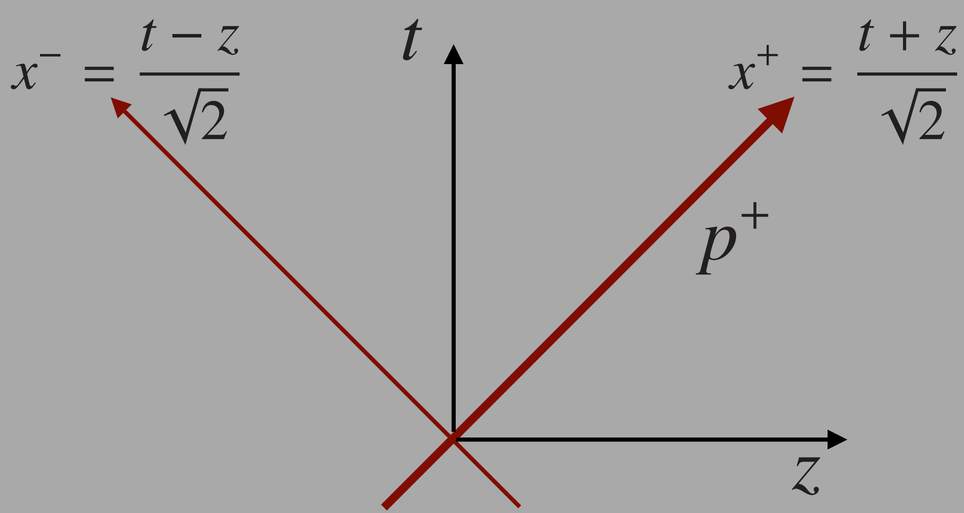
- All the ingredients of the dynamics of the HQs are calculated from the plasma correlation functions

Jet momentum broadening

[Based on J. Barata, JPB and Y. Mehtar-Tani, 2305.10476]

Momentum broadening

Consider a high energy quark propagating in the positive z-direction



The dynamics reduce to a two-dimensional non-relativistic problem in the transverse plane with E playing the role of a mass $(x^+ \mapsto t,\ p^+ \mapsto E)$

$$\left[i\partial_t + \frac{\partial_\perp^2}{2E} + gA(\boldsymbol{r},t) \right] \psi(\boldsymbol{r},t) = 0$$

$$E \gg T \qquad A \mapsto A_a^- t^a$$

The reduced density matrix

The gauge potential of the plasma is a fluctuating field with Gaussian correlation function

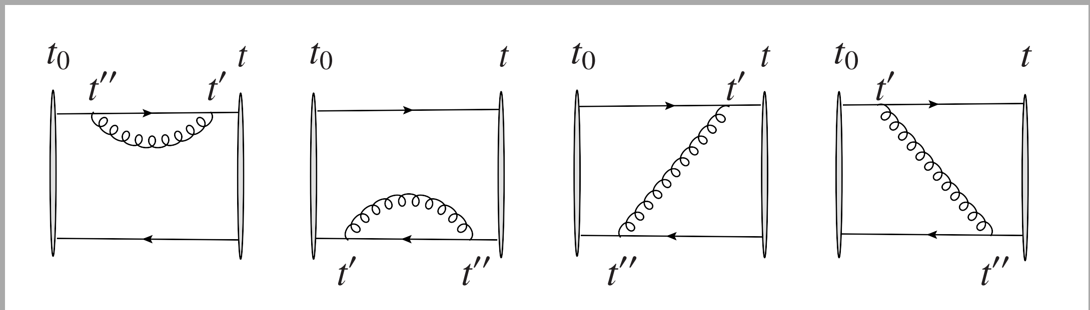
$$\langle A_a^-(x^+,\boldsymbol{x})A_b^-(y^+,\boldsymbol{y})\rangle = \delta_{ab}\delta(x^+-y^+)\gamma(x^+,\boldsymbol{x}-\boldsymbol{y})$$

$$\gamma(0)-\gamma(\boldsymbol{r}) = g^2 n\int\frac{\mathrm{d}^2\boldsymbol{q}}{(2\pi)^2}\frac{1-\mathrm{e}^{i\boldsymbol{q}\cdot\boldsymbol{r}}}{(\boldsymbol{q}^2+m_D^2)^2} \qquad g^2(\gamma(0)-\gamma(\boldsymbol{r})) = \frac{n}{2}\sigma(\boldsymbol{r})$$

$\sigma(\boldsymbol{r})$ is called the dipole cross section. It is the analog of the imaginary part of the potential in the HQ problem

The reduced density matrix is obtained by averaging over the gauge field

$$\rho\equiv \mathrm{tr}_A(\rho[A]) = \Big\langle |\psi_A(t)\rangle\langle\psi_A(t)| \Big\rangle_A .$$



Various representations

Color structure $\rho(t) \equiv \rho_{\rm s} + t^a \rho_{\rm o}^a$

Coordinate space representation

$$\langle \boldsymbol{r}|\, \rho_{\rm s,o}(t)\, |\bar{\boldsymbol{r}}\rangle = \langle \boldsymbol{b} + \boldsymbol{x}/2|\, \rho_{\rm s,o}(t)\, |\boldsymbol{b} - \boldsymbol{x}/2\rangle \qquad \mapsto \rho_{\rm s,o}(\boldsymbol{b}, \boldsymbol{x})$$

$$\boldsymbol{b} \equiv \frac{\boldsymbol{r} + \bar{\boldsymbol{r}}}{2} \qquad \boldsymbol{x} \equiv \boldsymbol{r} - \bar{\boldsymbol{r}}$$

Momentum space representation

$$\rho(\boldsymbol{\ell}, \boldsymbol{K}, t) \equiv \int_{\boldsymbol{b},\boldsymbol{x}} {\rm e}^{-i\boldsymbol{\ell}\cdot\boldsymbol{b}} {\rm e}^{-i\boldsymbol{K}\cdot\boldsymbol{x}}\, \rho(\boldsymbol{b}, \boldsymbol{x}, t) \qquad \boldsymbol{K} \equiv \frac{\boldsymbol{k} + \bar{\boldsymbol{k}}}{2} \qquad \boldsymbol{\ell} \equiv \boldsymbol{k} - \bar{\boldsymbol{k}}$$

Wigner representation

$$\rho_{_W}(\boldsymbol{b}, \boldsymbol{K}, t) \equiv \int_{\boldsymbol{x}} {\rm e}^{-i\boldsymbol{K}\cdot\boldsymbol{x}}\, \rho(\boldsymbol{b}, \boldsymbol{x}, t)$$

The Wigner function has many features of a classical phase space distribution function.

$$\int_{\boldsymbol{b}} \rho_{_W}(\boldsymbol{b}, \boldsymbol{K}, t) = \mathcal{P}(\boldsymbol{K}, t)$$ **(momentum distribution)**

$$\int_{\boldsymbol{K}} \rho_{_W}(\boldsymbol{b}, \boldsymbol{K}, t) = \rho(\boldsymbol{b}, t)$$ **(density)**

Equations for the reduced density matrix

Using the same approximations as in the HQ case, one gets

$$\frac{\partial}{\partial t}\langle \boldsymbol{r}|\rho_{\mathrm{s,o}}(t)|\bar{\boldsymbol{r}}\rangle = -\frac{i}{2E}\left(\frac{\partial^2}{\partial \bar{\boldsymbol{r}}^2} - \frac{\partial^2}{\partial \boldsymbol{r}^2}\right)\langle \boldsymbol{r}|\rho_{\mathrm{s,o}}(t)|\bar{\boldsymbol{r}}\rangle \; - \Gamma_{\mathrm{s,o}}(\bar{\boldsymbol{r}} - \boldsymbol{r})\,\langle \boldsymbol{r}|\rho_{\mathrm{s}}(t)|\bar{\boldsymbol{r}}\rangle$$

$$\Gamma_{\mathrm{s}}(\boldsymbol{x}) = C_F \int_{\boldsymbol{q}} \left(1 - e^{i\boldsymbol{q}\cdot\boldsymbol{x}}\right)\gamma(\boldsymbol{q})$$

$$\Gamma_{\mathrm{o}}(\boldsymbol{x}) = \int_{\boldsymbol{q}} \left(C_F + \frac{1}{2N_c} e^{i\boldsymbol{q}\cdot\boldsymbol{x}}\right)\gamma(\boldsymbol{q})$$

Strict eikonal approximation

$$\rho_{\mathrm{s,o}}(\boldsymbol{b},\boldsymbol{x},t) = \; \rho^{(0)}_{\mathrm{s,o}}\,(\boldsymbol{b},\boldsymbol{x})\,\mathrm{e}^{-t\,\Gamma_{\mathrm{s,o}}(\boldsymbol{x})}$$

octet is damped (gluon damping rate)

singlet is not damped at short distance (color transparency)

damping affects non diagonal matrix elements: collisional decoherence

Harmonic approximation

$$\Gamma_{\mathrm{s}}(\boldsymbol{x}) \approx 4\pi\alpha_s^2 C_F n \log\left(\frac{Q^2}{m_D^2}\right)\frac{\boldsymbol{x}^2}{4} \equiv \frac{\hat{q}}{4}\boldsymbol{x}^2$$

Harmonic approximation yields a Fokker-Planck equation for the Wigner transform

$$\partial_t \rho_W(\boldsymbol{b}, \boldsymbol{K}, t) = \left[-\frac{\boldsymbol{K}}{E}\frac{\partial}{\partial \boldsymbol{b}} + \frac{\hat{q}}{4}\frac{\partial^2}{\partial \boldsymbol{K}^2}\right]\rho_W(\boldsymbol{b}, \boldsymbol{K}, t)$$

Note the absence of dissipation in this equation

Equivalent to a simple Langevin equation

$$E\frac{\mathrm{d}^2\boldsymbol{b}}{\mathrm{d}t} = \boldsymbol{\xi}(t), \quad \langle \xi_i(t_1)\xi_j(t_2)\rangle = \frac{\hat{q}}{2}\delta_{ij}\delta(t_1 - t_2)$$

Solution for a simple initial condition

Initial Gaussian wave packet $\rho_W(\boldsymbol{b},\boldsymbol{K},0) = 4\mathrm{e}^{-\mu^2\boldsymbol{b}^2}\mathrm{e}^{-\frac{\boldsymbol{K}^2}{\mu^2}}$

Solution for the Wigner transform remains Gaussian at all times

$$\rho_W(\boldsymbol{b},\boldsymbol{K}) = \frac{4}{D}\exp\left\{-\frac{1}{D}\left(a\,\boldsymbol{b}^2 + \frac{b}{E}\,\boldsymbol{b}\cdot\boldsymbol{K} + \frac{c}{E^2}\boldsymbol{K}^2\right)\right\}$$

$$a = \langle \boldsymbol{K}^2\rangle_t \qquad \frac{c}{E^2} = \langle \boldsymbol{b}^2\rangle_t \qquad \frac{b}{E} = -2\langle \boldsymbol{b}\cdot\boldsymbol{K}\rangle_t \qquad D = \langle \boldsymbol{b}^2\rangle\langle \boldsymbol{K}^2\rangle - \langle \boldsymbol{K}\cdot\boldsymbol{b}\rangle = \frac{ac}{E^2} - \frac{b}{2E}$$

Three time scales $t_0 \equiv \frac{E}{\mu^2} \qquad t_1 = \frac{\mu^2}{\hat{q}} \qquad t_2^3 \equiv \frac{E^2}{\hat{q}\mu^2}$

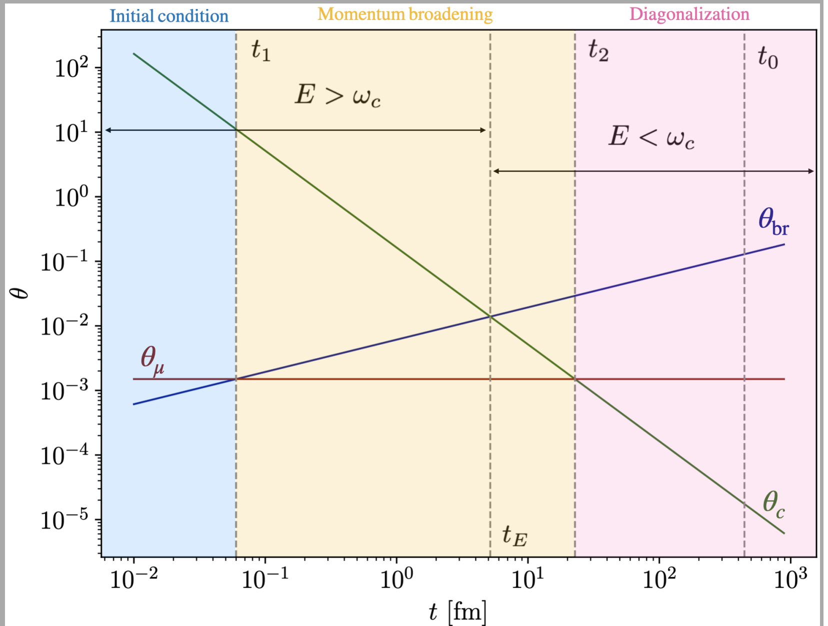
$(t_0 < t_1 < t_2)$

$$\langle \boldsymbol{k}^2\rangle_t = \mu^2 + \hat{q}t = \mu^2\left(1+\frac{t}{t_1}\right)$$

$$\langle \boldsymbol{b}^2\rangle_t = \frac{c(t)}{E^2} = \frac{1}{\mu^2}\left[1 + \frac{t^2}{t_0^2} + \frac{1}{3}\frac{t^3}{t_2^3}\right]$$

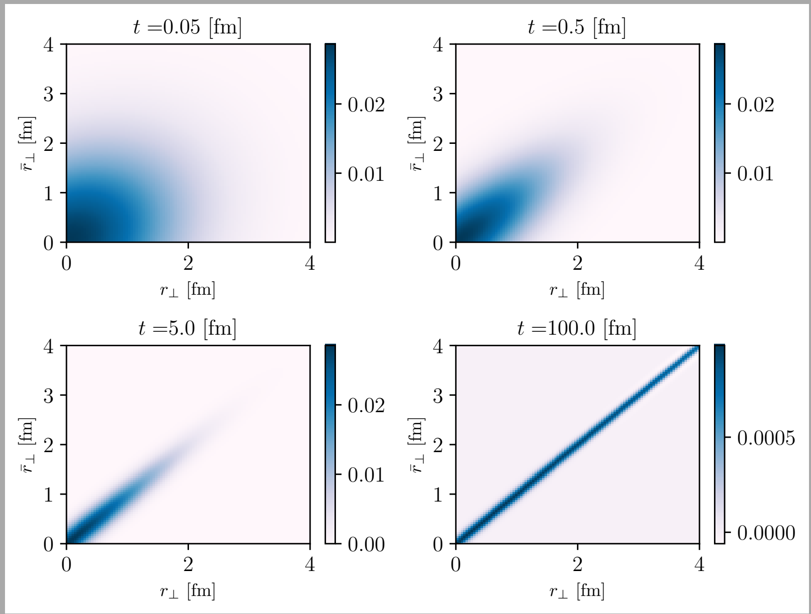
Angular variables

$$\theta_\mu^2 = \frac{\mu^2}{E^2}\,, \quad \theta_c^2(t) = \frac{1}{\hat{q}t^3}\,, \quad \theta_{\rm br}^2(t) = \frac{\hat{q}t}{E^2}$$

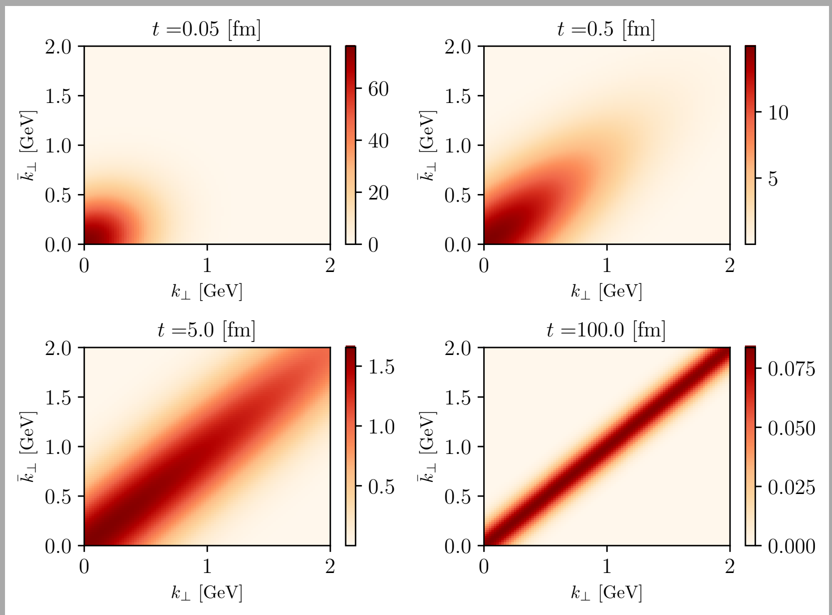


The reduced density matrix becomes diagonal

….. in both coordinate space and in momentum space !



$$\rho(\boldsymbol{b}=0,\boldsymbol{x},t) \approx \frac{1}{\pi\langle \boldsymbol{b}^2\rangle_t}\exp\left\{-\frac{\langle \boldsymbol{k}^2\rangle_t\,\boldsymbol{x}^2}{4}\right\}$$



$$\rho(\boldsymbol{\ell},\boldsymbol{K}=0,t) \approx \frac{4\pi}{\mu^2+\hat{q}t}\exp\left\{-\frac{\boldsymbol{\ell}^2\,\hat{q}t^3}{48E^2}\right\}$$

$$= \frac{4\pi}{\langle \boldsymbol{k}^2\rangle_t}\exp\left\{-\frac{\boldsymbol{\ell}^2\langle \boldsymbol{b}^2\rangle_t}{16}\right\}$$

Entropy growth

von Neumann entropy

$$S_{\rm vN}[\rho] = -{\rm Tr}\rho \ln \rho \quad = \log\left(\frac{1-p}{4p}\right) + \frac{1}{\sqrt{p}} \ln \frac{1+p+2\sqrt{p}}{(1-p)}$$

"purity" $p \equiv {\rm Tr}\rho^2$

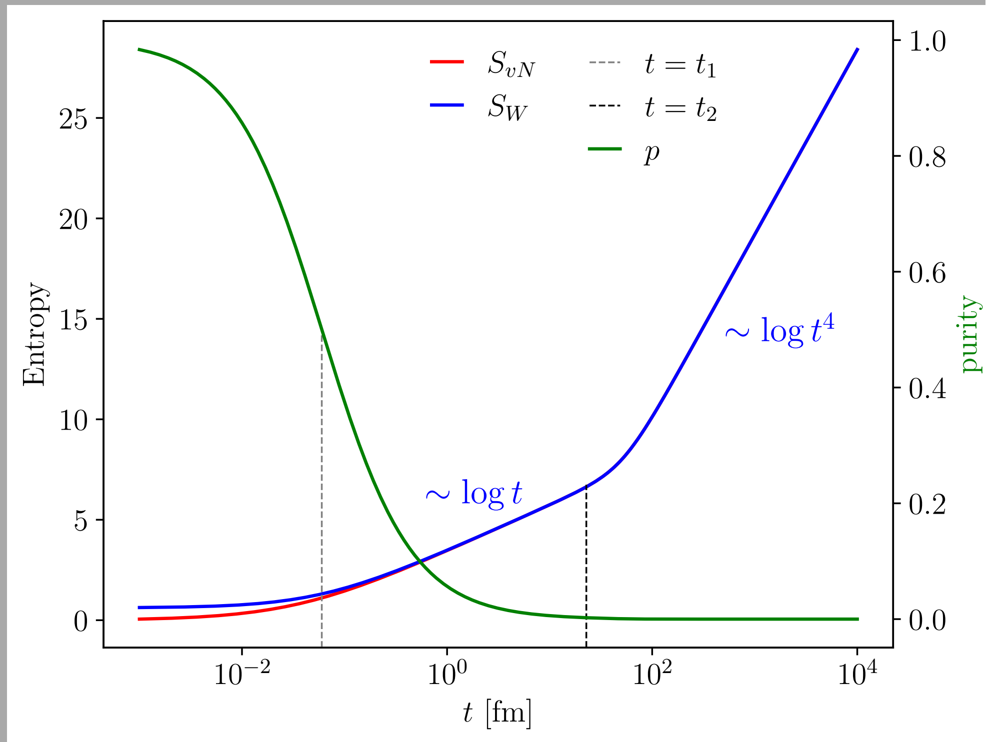
$$\frac{1}{p} = \left(1+\frac{t}{t_1}\right)\left(1+\frac{t^3}{12t_2^3}\,\frac{t+4t_1}{t+t_1}\right)$$

late time behavior

$$S_{\rm vN} \simeq \ln\frac{1}{p} \simeq \ln\frac{\hat{q}^2 t^4}{E^2} \sim \ln\langle \boldsymbol{k}^2\rangle_t \langle \boldsymbol{b}^2\rangle_t$$

classical Wigner entropy

$$S_{\rm w} \equiv -\int_{\boldsymbol{K},\boldsymbol{b}} \rho_{\rm w}(\boldsymbol{b},\boldsymbol{K}) \log \rho_{\rm w}(\boldsymbol{b},\boldsymbol{K})$$



Summary

- The theory of open quantum systems offers a useful framework to calculate the interactions of complex hard probes with their environment.
- It also provides interesting perspectives and reveals connections between seemingly unrelated problems (examples discussed in the talk: quarkonia and (simplified) jets).
- It allows to derive many different approaches from a common starting point.