Polarization of jets at early times in heavy-ion collisions

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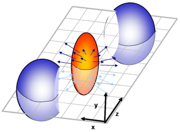
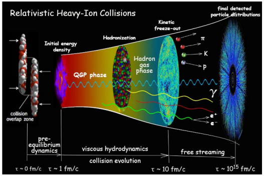
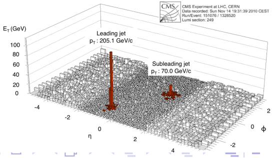
Early stages of heavy-ion collisions

- Heavy-ion collisions create a QGP fluid.
- How does far-from-equilibrium QCD medium reach hydrodynamic stage?
- Different stages of collisions:
  Glasma $\rightarrow$ Kinetic theory $\rightarrow$ Hydrodynamics
- Medium can quench jets and change jet substructure.
- How important are the initial stages for jets?
- Are there some specific signatures on jets from the initial stages?





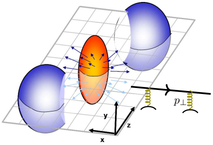
[Chun Shen, 2014]

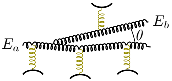


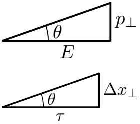
Jet physics in homogeneous, equilibrated medium

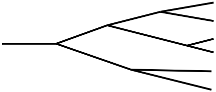
- Transverse momentum broadening: $\widehat{q} = \frac{d\langle \mathbf{p}_\perp^2 \rangle}{dt}$
- Allows for medium-induced gluon emission.
- Wavepackets overlap for a long time during emission.
- Schematic estimate:
  - $\theta \sim \frac{p_\perp}{E} \sim \frac{\Delta x_\perp}{\tau}$
  - Uncertainty principle: $p_\perp \Delta x_\perp \sim 1$
    so $\tau \sim \frac{\Delta x_\perp E}{p_\perp} \sim \frac{E}{p_\perp^2} \sim \frac{E}{\widehat{q}\tau}$
    so $\tau \sim \sqrt{E/\widehat{q}}$
- Get rate $\Gamma \sim \alpha_s P(z)/\tau \sim \alpha_s\, P(z)\, \frac{\sqrt{\widehat{q}}}{\sqrt{E}}$
- This process determines whole jet structure.

  [For vacuum-like emission see e.g. Majumder (2018); Wang, Guo (2001)]



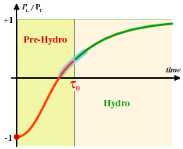


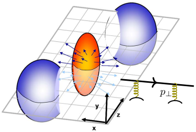


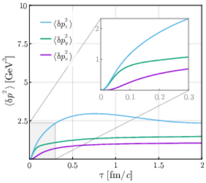


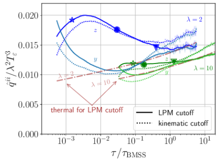
What changes out of equilibrium?

- Big pressure anisotropy at early times.
- Expect to lead to anisotropy in momentum broadening: $\hat{q}_z \neq \hat{q}_y$ with $\hat{q}_y = \frac{d\langle p_y^2 \rangle}{dt}$, $\hat{q}_z = \frac{d\langle p_z^2 \rangle}{dt}$
- Has been seen in glasma stage and kinetic theory stage calculations.
  [See also: Ipp, Muller, Schuh (2020); Carrington, Czajka, Mrowczynski (2022)]





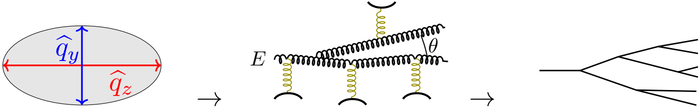




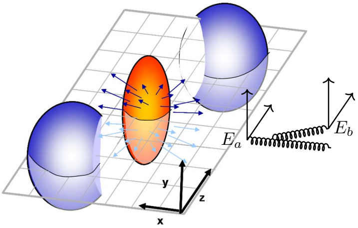
[Avramescu, Băran, Greco, Ipp, Müller, Ruggieri] [Boguslavski, Kurkela, Lappi, Lindenbauer, Peuron]

This work

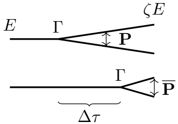
- How does anisotropy in broadening affect jet evolution?



- Jet partons become polarized.
  - Want $\frac{d\mathcal{P}_{a\to bc}}{d\zeta\, dt}$ where $abc \in \{y, z\}$, $\zeta = E_b/E_a$
  - Evaluate using BDMPS-Z formalism.



Single gluon emission in an anisotropic medium



- Hard splitting sensitive to different components of $\mathbf{P}$. E.g. for $\zeta \ll 1$
  - Opposite polarization: $|\Gamma_{z\to z,y}|^2 + |\Gamma_{z\to y,z}|^2 = 4P_y\overline{P}_y/\zeta^2$
  - Same polarization: $|\Gamma_{z\to z,z}|^2 + |\Gamma_{z\to y,y}|^2 = 4P_z\overline{P}_z/\zeta^2$
- An anisotropic medium picks out certain polarization states.
- $\tilde{S}^{(3)}$ describes the momentum broadening of three partons with potential $\widehat{q}_z r_z^2 + \widehat{q}_y r_y^2$.

$$\frac{d\mathcal{P}_{a\to bc}}{d\zeta dt} = \frac{g^2 N_c}{8\pi\omega^2\zeta(1-\zeta)} \mathrm{Re} \int_0^L d\Delta t \int \frac{d^2\boldsymbol{P}}{(2\pi)^2} \int \frac{d^2\bar{\boldsymbol{P}}}{(2\pi)^2} \times \Gamma_{a\to bc}(\boldsymbol{P},\zeta)\, \Gamma_{a\to bc}(\bar{\boldsymbol{P}},\zeta)\, \tilde{S}^{(3)}(\Delta t, \boldsymbol{P}, \bar{\boldsymbol{P}})$$

$$\sim \overline{\alpha}_s \frac{(\widehat{q}_y\widehat{q}_z)^{1/4}}{\sqrt{2E}} A_{a\to bc}(\zeta, \widehat{q}_z/\widehat{q}_y)$$

[E.g. Baier, Dokshitzer, Mueller, Peigne, Schiff (1996); Zakharov (1996); Wiedemann, Gyulassy (1999); Blaizot, Dominguez, Iancu, Mehtar-Tani (2013)]

Single gluon emission in an anisotropic medium

- Ensemble of gluons: Probability p of polarization in beam direction.

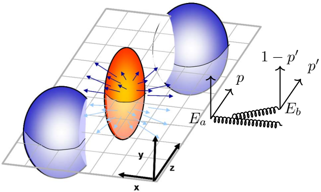
$$p' = \frac{p\,\mathcal{P}_{z\to z} + (1-p)\mathcal{P}_{y\to z}}{p\,(\mathcal{P}_{z\to z} + \mathcal{P}_{z\to y}) + (1-p)\,(\mathcal{P}_{y\to z} + \mathcal{P}_{y\to y})}$$

- Daughter parton has ($\zeta = E_b/E_a$)

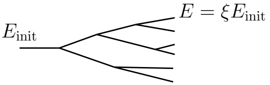
$$p' - \tfrac{1}{2} = f(\zeta)\left(p - \tfrac{1}{2}\right) + g(\zeta)\,G(\widehat{q}_z/\widehat{q}_y)$$

$$f(\zeta) = \frac{\zeta^2}{(1-\zeta)^2 + \zeta^2 + \zeta^2(1-\zeta)^2}, \quad g(\zeta) = \frac{(1-\zeta)^2}{(1-\zeta)^2 + \zeta^2(1-\zeta)^2 + \zeta^2}$$

- Isotropic medium:
  Polarization reduced at each splitting.
- Anisotropic:
  Unpolarized mother radiates polarized daughter! $G \sim 0.3$ maximally.
- Two competing effects. Which one wins out?



Evolution of polarization



- Consider total evolution of jet in glasma brick with constant $G(\widehat{q}_z/\widehat{q}_y)$.
    - $\tau = \frac{\alpha_s N_c}{\pi}\sqrt{\frac{\widehat{q}}{E}}t$
-

$$\frac{dD_{\text{tot}}(\xi,\tau)}{d\tau} = \int_\xi^1 dz\, \mathcal{K}_0(\zeta)\sqrt{\frac{\zeta}{\xi}}\, D_{\text{tot}}\left(\frac{\xi}{\zeta},\tau\right) - \int_0^1 d\zeta\, \mathcal{K}_0(\zeta)\,\frac{\zeta}{\sqrt{\xi}}\, D_{\text{tot}}(\xi,\tau)$$

$$\frac{d\widetilde{D}(\xi,\tau)}{d\tau} = \int_\xi^1 d\zeta\, \mathcal{M}_0(\zeta)\sqrt{\frac{\zeta}{\xi}}\, \widetilde{D}\left(\frac{\xi}{\zeta},\tau\right) - \int_0^1 d\zeta\, \mathcal{K}_0(\zeta)\,\frac{\zeta}{\sqrt{\xi}}\, \widetilde{D}(\xi,\tau) + \int_\xi^1 d\zeta\, \mathcal{L}_0(\zeta)\sqrt{\frac{\zeta}{\xi}}\, D_{\text{tot}}\left(\frac{\xi}{\zeta},\tau\right).$$

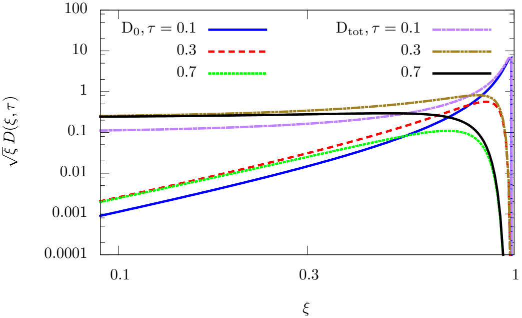
$$\mathcal{K}_0(\zeta) \approx \frac{1}{\zeta^{3/2}(1-\zeta)^{3/2}}, \qquad \mathcal{M}_0(\zeta) \approx \zeta^2\mathcal{K}_0(\zeta), \qquad \mathcal{L}_0(\zeta) \approx G(\widehat{q}_z/\widehat{q}_y)(1-\zeta)^2\,\mathcal{K}_0(\zeta)$$

- $D_{\text{tot}} = \xi\frac{d(N_z+N_y)}{d\xi}$ is energy spectrum, $\widetilde{D} = \xi\frac{d(N_z-N_y)}{d\xi}$ is polarization.

  [Equation for D_{tot}: Blaizot, Iancu, Mehtar-Tani (2013); Blaizot, Mehtar-Tani (2015); Fister, Iancu (2014); Iancu, Wu (2015); Escobedo, Iancu (2016). See also e.g. Mehtar-Tani, Schlichting (2018), Adhya, Salgado, Spousta, Tywoniuk (2021).]

Propagation of polarization in isotropic medium

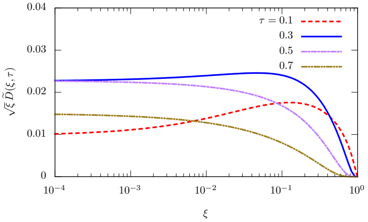
- Have solved new equation numerically (and analytically for small ξ).
- In an isotropic medium, polarization quickly goes away.
- Use $\widetilde{D}(\xi, \tau = 0) = 1$: get $\widetilde{D}(\xi, \tau) \sim \xi^{3/2}$ for small ξ.

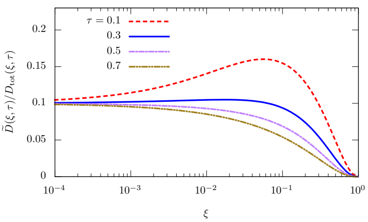


- Since $\mathcal{M}_0(\zeta) = \zeta^2 \mathcal{K}_0(\zeta)$ get $\widetilde{D} = \xi^2 D_{\text{tot}} \sim \xi^{3/2}$

Propagation of polarization in anisotropic medium

- Anisotropic medium: Use $D_{\rm tot}(\xi,\tau=0)=\delta(1-\xi) \quad \widetilde{D}(\xi,\tau=0)=0.$





- Get that $\widetilde{D}\sim 1/\sqrt{\xi}$ and thus $\widetilde{D}/D_{\rm tot}$ is constant at small ξ.
- Confirm by analytical solution at $\xi \ll \tau$:
  - Get

$$\widetilde{D}(\xi,\tau)\approx \frac{G(\widehat{q}_z/\widehat{q}_y)}{3}\,\frac{\tau e^{-\pi\tau^2}}{\sqrt{\xi}}$$

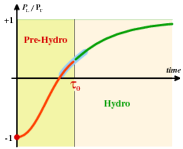
- Nearly constant fraction of jet partons is polarized across different energies.

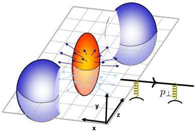
Link to phenomenology

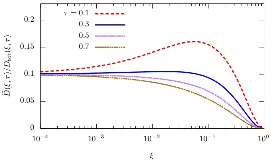
- Polarization of jet partons is determined by anisotropy when jet escapes medium.
- How to deal with hadronization?
- Polarization of partons leads to preferred direction of hadrons.
  - For quarks the fragmentation is extracted from experiments (Collin's function).
- Thus polarization leads to anisotropy in distribution of hadrons inside jet cone.
  - Other contributions: Anistropic broadening, gradients...

Conclusions

- Have considered anisotropy in momentum broadening at early times, $\widehat{q}_y \neq \widehat{q}_z$.
- Leads to polarization of emitted gluons.
  - Have evaluated in BDMPS-Z framework.
- Competing effect: Abundant sourcing of polarization vs. washing out.
  - Derived and solved evolution equations for jets.
  - Polarization is sensitive to instantaneous anisotropy in medium.
- Affects distribution of hadrons in jet cone.

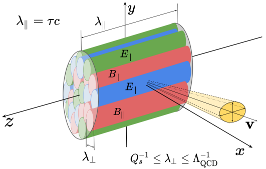




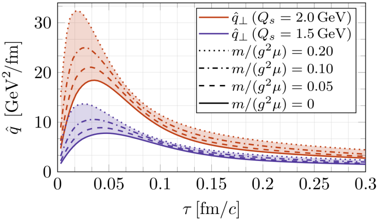


Jet broadening in glasma

- Jet partons traverse heavily occupied gluon fields. [Ipp, Muller, Schuh (2020); Carrington, Czajka, Mrowczynski (2022); Avramescu, Băran, Greco, Ipp, Müller, Ruggieri (2023)]
- Deflected by chromomagnetic and chromoelectric forces.
- Claim that as much broadening as during hydro stage.
  - $\Delta p_\perp^2\big|_{\text{glasma}} / \Delta p_\perp^2\big|_{\text{hydro}} \approx 0.9$ [Carrington, Czajka, Mrowczynski (2022)]
- Heavily anisotropic broadening, $\widehat{q}_z \approx 2\widehat{q}_y$

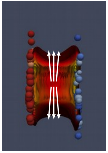


[Carrington, Czajka, Mrowczynski (2022)]

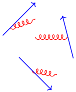


[Ipp, Muller, Schuh (2020)]

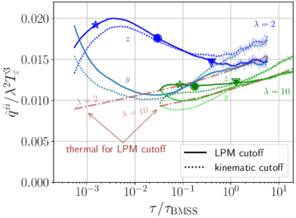
Jet broadening in kinetic theory



- Anisotropic broadening recently evaluated in kinetic theory.
  Boguslavski, Kurkela, Lappi, Lindenbauer, Peuron (2023)

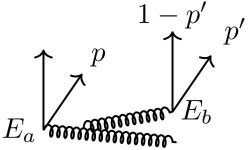


- Longitudinal expansion squeezes momentum distribution.
  - Get $\widehat{q}_z > \widehat{q}_y$



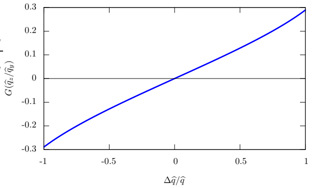
- Calculation uncertainties: Cutoff dependence, matching with hydro $\widehat{q}$, strength of coupling.

Single gluon emission in an anisotropic medium



- Two intuitive limits:
  - $\zeta \to 0$:
    $p' - \frac{1}{2} = \zeta^2\,(p - \frac{1}{2}) + G(\widehat{q}_z/\widehat{q}_y)$
  - $\zeta \to 1$: $p' - \frac{1}{2} = (p - \frac{1}{2}) + (1-\zeta)^2\,G(\widehat{q}_z/\widehat{q}_y)$
- Size of polarization given by $G(\widehat{q}_z/\widehat{q}_y)$.

$$G(\widehat{q}_z/\widehat{q}_y) = \frac{f\left(\sqrt{\widehat{q}_y/\widehat{q}_z}\right) - f\left(\sqrt{\widehat{q}_z/\widehat{q}_y}\right)}{f\left(\sqrt{\widehat{q}_y/\widehat{q}_z}\right) + f\left(\sqrt{\widehat{q}_z/\widehat{q}_y}\right)}; \qquad \frac{\Delta\widehat{q}}{\widehat{q}} = \frac{\widehat{q}_z - \widehat{q}_y}{\widehat{q}_z - \widehat{q}_y}$$



  - Maximal value is around $G \sim 0.3$.
- Expected branching is democratic ($\zeta \sim \frac{1}{2}$).
  - Not clear which wins out in the end.
  - Need evolution of jet as a whole

Single gluon emission in an ansiotropic medium

- $\tilde{S}^{(3)}$ describes the momentum broadening of the three partons during emission.
- $\tilde{S}^{(3)} \sim \langle GGG \rangle$ with propagator

$$G \sim \int \mathcal{D}\mathbf{r}\; e^{i\frac{E}{2}\int dt \dot{\mathbf{r}}^2} P e^{ig\int dt\; A^-}$$

- Medium average gives

$$\langle A^-(x^+, \mathbf{x}) A^-(y^+, \mathbf{y}) \rangle \sim \delta(x^+ - y^+)\gamma(\mathbf{x} - \mathbf{y})$$

- Use harmonic oscillator approximation but in anisotropic medium

$$\gamma(0) - \gamma(\mathbf{r}) \sim \widehat{q}_z r_z^2 + \widehat{q}_y r_y^2$$

- Get that

$$S^{(3)}(\Delta t, \mathbf{u}, \mathbf{v}) = \int_{\mathbf{r}(t=0)=\mathbf{u}}^{\mathbf{r}(t=\Delta t)=\mathbf{v}} \mathcal{D}\mathbf{r} \exp\left\{ iM \int_0^{\Delta t} dt \;\left[\frac{\dot{\mathbf{r}}^2}{2} + \frac{\Omega_z^2 r_z^2 + \Omega_y^2 r_y^2}{2} \right] \right\}$$

Single gluon emission in an anisotropic medium

Get e.g. for $\frac{d\mathcal{P}_{z\to z}}{d\zeta dt} = \sum_c \frac{d\mathcal{P}_{z\to zc}}{d\zeta dt}$

$$\frac{d\mathcal{P}_{z\to z}}{d\zeta dt} = \overline{\alpha}_s \frac{(\widehat{q}_y \widehat{q}_z)^{1/4}}{\sqrt{2\omega}} A(\widehat{q}_z/\widehat{q}_y)\,\gamma(\zeta)\left[\mathcal{F}^0_{z\to z}(\zeta) + G(\widehat{q}_z/\widehat{q}_y)\mathcal{F}^1_{z\to z}(\zeta)\right].$$

where

$$A(\widehat{q}_z/\widehat{q}_y) \equiv \frac{1}{2}\left[f(a) + f(1/a)\right], \qquad G(\widehat{q}_z/\widehat{q}_y) \equiv \frac{f(1/a) - f(a)}{f(a) + f(1/a)}$$

$$f(a) \equiv \int_0^\infty dx \left[\frac{1}{a^{1/2}x^2} - \frac{1}{\sinh^{1/2} ax \, \sinh^{3/2} x}\right].$$

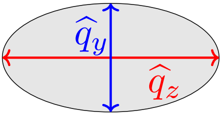
and

$$\gamma(\zeta) = \frac{[1-\zeta(1-\zeta)]^{1/2}}{\zeta^{1/2}(1-\zeta)^{1/2}},$$

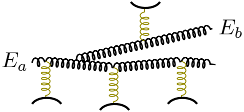
$$\mathcal{F}^0_{z\to z}(\zeta) = \frac{1}{2}\left(\frac{1-\zeta}{\zeta} + \frac{2\zeta}{1-\zeta} + \zeta(1-\zeta),\right), \qquad \mathcal{F}^1_{z\to z}(\zeta) = \frac{1}{2}\left(\frac{1-\zeta}{\zeta} + \zeta(1-\zeta)\right)$$

Single gluon emission in an anisotropic medium

- Total unpolarized rate is $\frac{d\mathcal{P}}{d\zeta dt} = \frac{1}{2}\sum_{a,b,c} \frac{d\mathcal{P}_{a\to bc}}{d\zeta dt}$

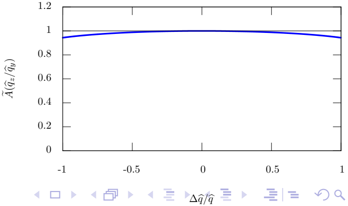


- It is nearly unaffected by anisotropy
  ($\zeta = E_b/E_a$; $\widehat{q} = \widehat{q}_z + \widehat{q}_y$)



$$\frac{d\mathcal{P}}{d\zeta dt} = \frac{\alpha_s}{2\pi} P_{g\to g}(\zeta) \frac{\sqrt{1-\zeta(1-\zeta)}}{\sqrt{\zeta(1-\zeta)E_a}} \left(4\widehat{q}_z\widehat{q}_y\right)^{1/4} \times \frac{1}{2}\left[f\left(\sqrt{\frac{\widehat{q}_z}{\widehat{q}_y}}\right) + f\left(\sqrt{\frac{\widehat{q}_y}{\widehat{q}_z}}\right)\right]$$

- Plot $(d\mathcal{P})_{\text{aniso}}/(d\mathcal{P})_{\text{iso}}$ at fixed $\widehat{q}$ with $\frac{\widehat{q}_z - \widehat{q}_y}{\widehat{q}_z + \widehat{q}_y}$ varying.



Interpretation

- Green's function is

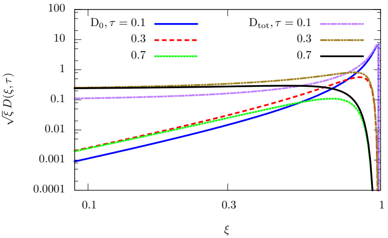
$$G(\xi,\xi_1,\tau) \;=\; \frac{1}{\xi_1}\,\widetilde{D}_0\left(\frac{\xi}{\xi_1},\frac{\Delta\tau}{\sqrt{\xi_1}}\right) \approx \left(\frac{\xi}{\xi_1(\xi_1-\xi)}\right)^{3/2}\Delta\tau\, e^{-\pi\Delta\tau^2/(\xi_1-\xi)}$$

- Get $\xi_1-\xi\sim\Delta\tau^2$
- In general also have $\Delta\tau\sim\sqrt{\xi}$
- Thus
  - Democratic branching $\xi_1\sim 2\xi$, quasilocal in energy.
  - Also quasilocal in time $\Delta\tau\sim\sqrt{\xi}\ll 1$.
- Polarization is produced abundantly.
- Quickly goes away after a few branchings.
- Thus a good approximation is

$$\widetilde{D}(\xi,\tau)\propto G(\widehat{q}_z/\widehat{q}_y)D_{\rm tot}(\xi,\tau)$$

Propagation of polarization in isotropic medium

- Have solved new equation numerically (and analytically for small ξ).
- In an isotropic medium, polarization quickly goes away.
- Use $\widetilde{D}(\xi, \tau = 0) = 1$: get $\widetilde{D}(\xi, \tau) \sim \xi^{3/2}$ for small ξ after time $\tau \sim \sqrt{\xi}$.



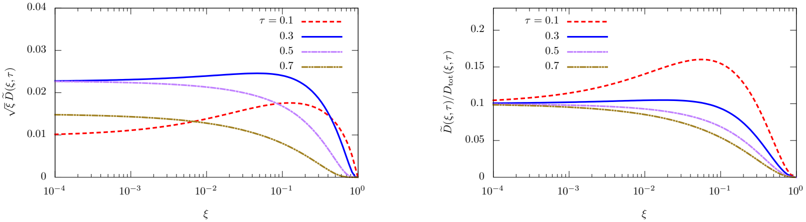
- Well known that for $D_{tot}(\xi, \tau = 0) = \delta(1 - \xi)$

$$D_{tot}(\xi, \tau) = \frac{\tau}{\sqrt{\xi}(1-\xi)^{3/2}} e^{-\pi\tau^2/(1-\xi)} \sim \frac{\tau e^{-\pi\tau^2}}{\sqrt{\xi}}$$

- Physics of democratic branching and turbulence.
- Since $\mathcal{M}_0(\zeta) = \zeta^2 \mathcal{K}_0(\zeta)$ get $\widetilde{D} = \xi^2 D_{tot}$

Propagation of polarization in anisotropic medium

- Anisotropic medium: Use $D_{\rm tot}(\xi, \tau = 0) = \delta(1-\xi) \quad \widetilde{D}(\xi, \tau = 0) = 0.$



- Get that $\widetilde{D} \sim 1/\sqrt{\xi}$ and thus $\widetilde{D}/D_{\rm tot}$ is constant at small ξ.
- Confirm by analytical solution at $\xi \ll \tau$:
  - Use method of Green's functions:

$$\widetilde{D}(\xi,\tau) = \int_\xi^1 d\xi_1 \int_0^\tau d\tau_1\, G(\xi,\xi_1,\tau-\tau_1) I(\xi_1,\tau_1),$$

  - Get

$$\widetilde{D}(\xi,\tau) \approx \frac{G(\widehat{q}_z/\widehat{q}_y)}{3}\,\frac{\tau e^{-\pi\tau^2}}{\sqrt{\xi}}$$