# Moment equations for chiral particles beyond equilibrium

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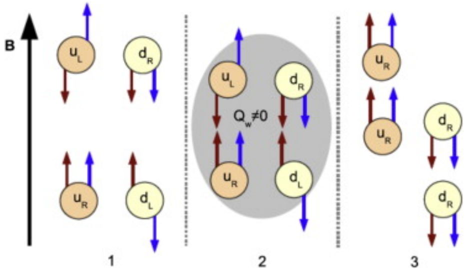
- Imbalance between left- and right-handed particles $\Longrightarrow$ axial-vector current
- Currents induced by vorticity
  $\Longrightarrow$ (axial-) chiral vortical effect
  A. Vilenkin, Phys. Rev. D 20, 1807 (1979)
  D. T. Son and P. Surowka, Phys. Rev. Lett. 103, 191601 (2009)
- Currents induced by electromagnetic fields
  $\Longrightarrow$ chiral magnetic effect/ chiral separation effect
  D. E. Kharzeev, L. D. McLerran, and H. J. Warringa, Nucl. Phys. A803, 227 (2008)




- Both are equilibrium effects
- Nonequilibrium contributions to chiral currents?

# Chiral hydrodynamics beyond local equilibrium

- Currents in chiral hydrodynamics:
  charge current $J_V^\mu$, axial-charge current $J_A^\mu$, energy-momentum tensor $T^{\mu\nu}$

- Conservation laws

$$\partial_\mu J_V^\mu = 0\,, \qquad \partial_\mu J_A^\mu = 0\,, \qquad \partial_\mu T^{\mu\nu} = 0$$

- Out of equilibrium: system of equations of motion not closed

- How to describe out-of-equilibrium dynamics of chiral currents?

# Attractors and fixed points

- Hydrodynamic-type equations of motion

  $\rightarrow$ can be derived also independently of gradient expansion

  $\rightarrow$ good description of underlying microscopic theory even far from local equilibrium

  $\Longrightarrow$ attractor solution
  **M. P. Heller, M. Spalinski, PRL 115 (2015) 7, 072501**

- Attractor related to existence of fixed points

- Modification of coefficients in hydrodynamic equations of motion
  $\Longrightarrow$ excellent description of system at any time of evolution
  **J.-P. Blaizot, L. Yan, AP 412 (2020) 167993, PLB 820 (2021) 136478, PRC 104 (2021) 5, 055201**

- Generalize for chiral currents?

# Chiral kinetic theory for Bjorken symmetry

- Kinetic theory: physics contained in distribution function $f^{\lambda}(x, \mathbf{p})$
  two chiralities $\lambda = \pm 1$

- Boost invariance: can consider only $z = 0$-slice

- Translational invariance in transverse plane: $f$ independent of $x$ and $y$

- Parity breaking in transverse plane: $f(p_x) \neq f(-p_x)$
  New compared to previous works

- Boltzmann equation describes dynamics of distribution function

$$\left(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z}\right) f^{\lambda}(\tau, \mathbf{p}) = -\frac{1}{\tau_R}\left[f^{\lambda}(\tau, \mathbf{p}) - f^{\lambda}_{\text{eq}}(\tau, \mathbf{p})\right]$$

$\tau$ proper time

relaxation time approximation

# Chiral moments

- Express currents through chiral moments

$$J_A^0 = \sum_{\lambda=\pm 1} \lambda \mathcal{J}_{00}^{\lambda} \,, \qquad J_A^x = \sum_{\lambda=\pm 1} \lambda \mathrm{Re} \mathcal{J}_{11}^{\lambda} = J_A^y \,,$$

$$J_V^0 = \sum_{\lambda=\pm 1} \mathcal{J}_{00}^{\lambda} \,, \qquad J_V^x = \sum_{\lambda=\pm 1} \mathrm{Re} \mathcal{J}_{11}^{\lambda} = J_V^y \,,$$

$$\mathcal{J}_{n\ell}^{\lambda} \equiv \int d^3p \, Y_n^{\ell}(\theta, \phi) f^{\lambda}$$

  spherical harmonics,
  needed because $f$ depends on both polar momentum angle $\theta$ and azimuthal momentum angle $\phi \rightarrow$ parity breaking

- Analogously for energy-momentum tensor

# Equations of motion

- Derive equations of motion for chiral moments from Boltzmann equation

$$\partial_\tau \mathcal{J}^\lambda_{n\ell} = -\frac{1}{\tau}\left(a_{n\ell}\mathcal{J}^\lambda_{n\ell} + b_{n\ell}\mathcal{J}^\lambda_{(n-2)\ell} + c_{n\ell}\mathcal{J}^\lambda_{(n+2)\ell}\right) - \frac{\mathcal{J}^\lambda_{n\ell} - \mathcal{J}^\lambda_{n\ell,\mathbf{eq}}}{\tau_R} ,$$

transport coefficients

- For (axial) charge density, $n = 0 = \ell$

$$\partial_\tau \mathcal{J}^\lambda_{00} = -\frac{1}{\tau}\mathcal{J}^\lambda_{00}$$

ideal equation of motion

- For (axial) charge current in transverse plane, $n = 1 = \ell$

$$\partial_\tau \mathcal{J}^\lambda_{11} = -\frac{1}{\tau}\left(a_{11}\mathcal{J}^\lambda_{11} + c_{11}\mathcal{J}^\lambda_{31}\right) - \frac{\mathcal{J}^\lambda_{11}}{\tau_R} ,$$

not known

- Two limits of equation of motion

$$\partial_\tau \mathcal{J}_{11}^\lambda = -\frac{1}{\tau}\left(a_{11}\mathcal{J}_{11}^\lambda + c_{11}\mathcal{J}_{31}^\lambda\right) - \frac{\mathcal{J}_{11}^\lambda}{\tau_R}$$

free streaming: $\tau \ll \tau_R$ collision dominated: $\tau \gg \tau_R$

- Late time: collisions $\Longrightarrow$ exponential decay

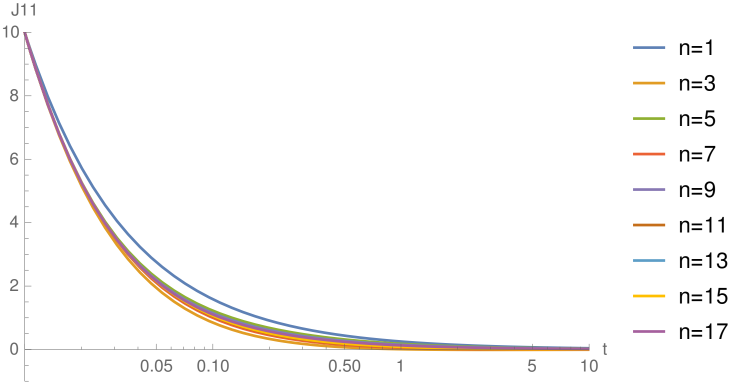
$$\mathcal{J}_{11}^\lambda \sim e^{-\tau/\tau_R}$$

- Early time: free streaming

$$\partial_\tau \mathcal{J}_{11}^\lambda = -\frac{1}{\tau}\left(a_{11}\mathcal{J}_{11}^\lambda + c_{11}\mathcal{J}_{31}^\lambda\right)$$

# Can we ignore $\mathcal{J}_{31}^{\lambda}$?

- First attempt: set $\mathcal{J}_{31}^{\lambda} = 0$
- Free-streaming solution for $\mathcal{J}_{11}^{\lambda}$, setting $\mathcal{J}_{(n+2)1}^{\lambda} = 0$




- This does not look so great
  $\Longrightarrow$ Can we do better?

# Stable free-streaming fixed point

- Relation between coefficients

$$a_{n\ell}\mathcal{P}_n^\ell(0) + b_{n\ell}\mathcal{P}_{n-2}^\ell(0) + c_{n\ell}\mathcal{P}_{n+2}^\ell(0) = \mathcal{P}_n^\ell(0)$$

$\mathcal{P}_n^\ell(x)$ associated Legendre polynomials

- Solution for equations of motion:

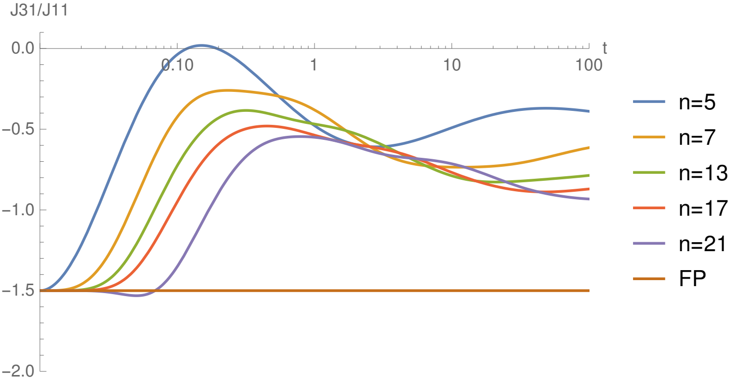
$$\mathcal{J}_{n1}^\lambda = [\mathcal{P}_n^1(0)/\mathcal{P}_1^1(0)]\mathcal{J}_{11}^\lambda(0)$$

Stable free-streaming fixed point

$\Longrightarrow$ solutions with any initial conditions will approach this solution at long time for free streaming

# Fixed points for naive truncation

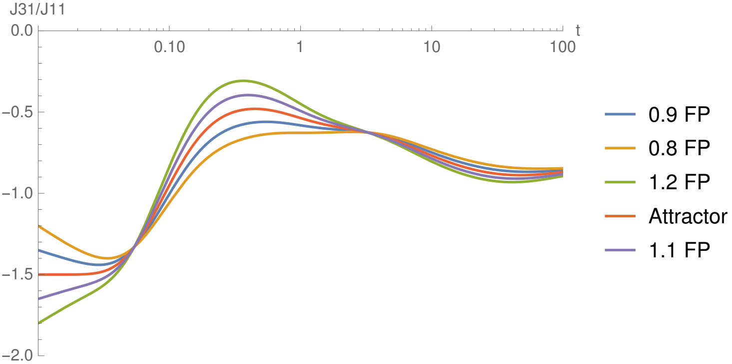
- Truncation shifts fixed point
- Convergence to true fixed point very slow




$\mathcal{J}^\lambda_{31}/\mathcal{J}^\lambda_{11}$, setting $\mathcal{J}^\lambda_{1(n+2)} = 0$, initial condition $\mathcal{J}^\lambda_{31} = [\mathcal{P}^1_3(0)/\mathcal{P}^1_1(0)]\mathcal{J}^\lambda_{11} = -1.5\mathcal{J}^\lambda_{11}$

# Solutions for naive truncation

- Consequence: Attractor solution shifted

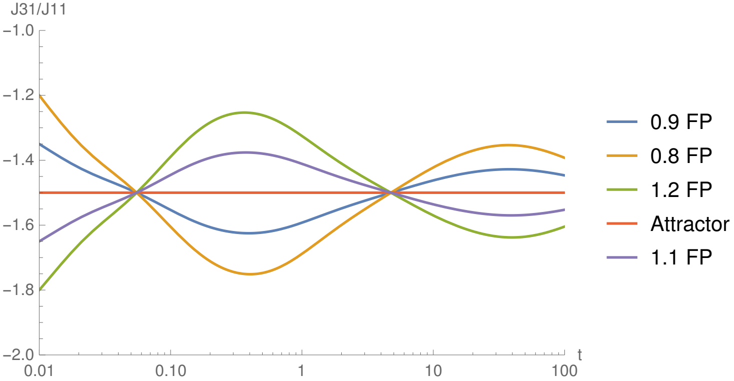



$\mathcal{J}^{\lambda}_{31}/\mathcal{J}^{\lambda}_{11}$, setting $\mathcal{J}^{\lambda}_{1(19)} = 0$, different initial conditions

- Attractor: initial condition at true fixed point
- Different initial conditions: damped oscillation around attractor
- But attractor solution should be constant

# Fixed-point truncation

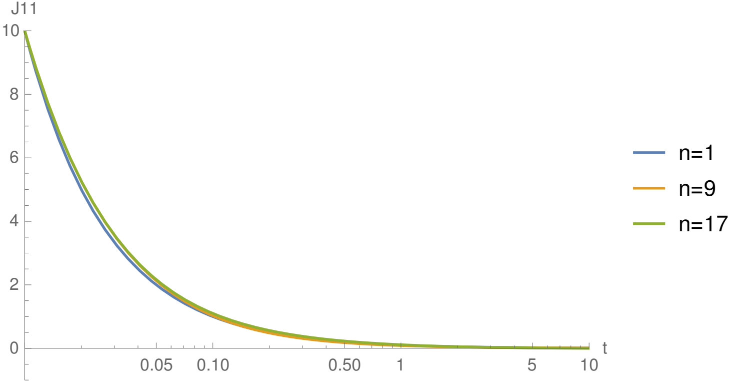
- Idea: Approximate $\mathcal{J}^{\lambda}_{1(n+2)} \simeq [\mathcal{P}^1_{n+2}(0)/\mathcal{P}^1_1(0)]\mathcal{J}^{\lambda}_{11}$
- Enforces correct behavior of attractor solution




$\mathcal{J}^{\lambda}_{31}/\mathcal{J}^{\lambda}_{11}$, setting $\mathcal{J}^{\lambda}_{1(19)} = [\mathcal{P}^1_{19}(0)/\mathcal{P}^1_1(0)]\mathcal{J}^{\lambda}_{11}$, different initial conditions

# Can we approximate $\mathcal{J}_{31}^{\lambda}$?

- Fixed-point truncation: very good agreement with exact solution even at lowest order




Free-streaming solution for $\mathcal{J}_{11}^{\lambda}$, setting $\mathcal{J}_{1(n+2)}^{\lambda} = [\mathcal{P}_{n+2}^{1}(0)/\mathcal{P}_{1}^{1}(0)]\mathcal{J}_{11}^{\lambda}$

# Full equations of motion

- So far: free streaming
- Reminder: late-time dynamics governed by exponential decay

  $\Longrightarrow$ Fixing free-streaming fixed point sufficient to describe system during full evolution

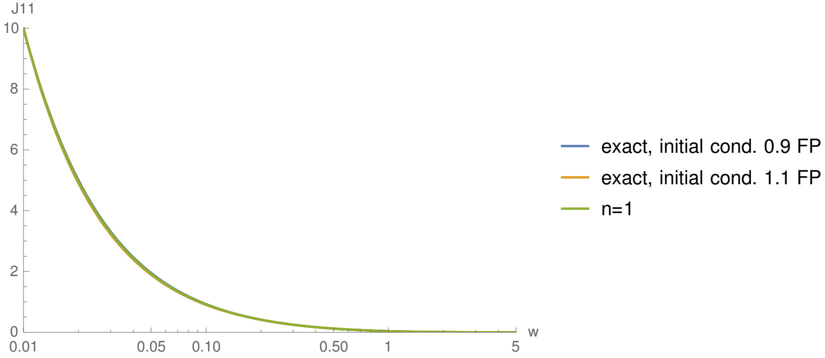
$$\partial_\tau \mathcal{J}_{11}^{\lambda} = -\frac{1}{\tau}\left(a_{11}\mathcal{J}_{11}^{\lambda} + c_{11}\mathcal{J}_{31}^{\lambda}\right) - \frac{\mathcal{J}_{11}^{\lambda}}{\tau_R}$$

# Full equations of motion

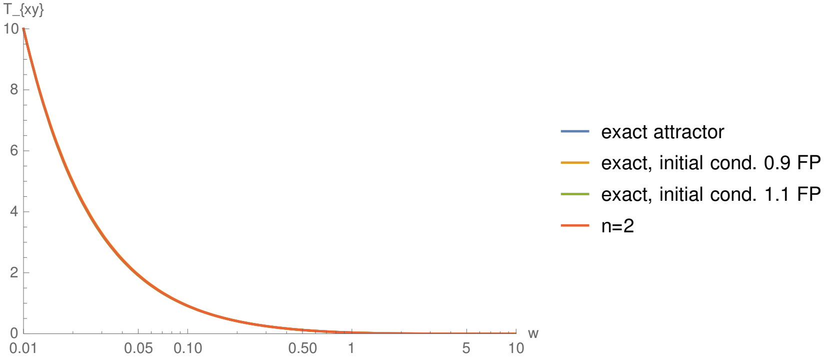
- So far: free streaming
- Reminder: late-time dynamics governed by exponential decay

  $\Longrightarrow$ Fixing free-streaming fixed point sufficient to describe system during full evolution

$$\partial_\tau \mathcal{J}_{11}^\lambda = -\frac{1}{\tau}\left(a_{11} - 1.5\, c_{11}\right) \mathcal{J}_{11}^\lambda - \frac{\mathcal{J}_{11}^\lambda}{\tau_R}$$

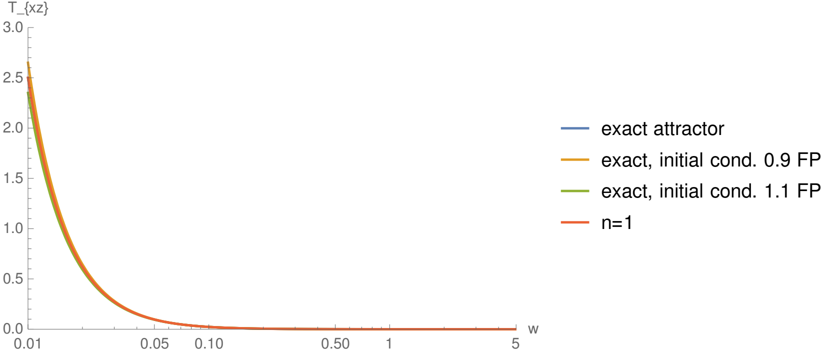


- Similar procedure for components of energy-momentum tensor
- Also here: excellent agreement with exact solution




lowest-order truncation vs. exact solution for $T^{xy}$

- Also here: excellent agreement with exact solution




lowest-order truncation vs. exact solution for $T^{xz}$

- $T^{xz}$ decays even faster than other components

# Comparison to 14-moment approximation

- Compare to 14-moment approximation (Israel-Stewart)
- Consider lowest order truncation

| | **14-moment approximation** | **fixed-point truncation** |
|---|---|---|
| **basis** | $1,\ p_z/p,\ (p_z/p)^2,\ \ldots$ | $\mathcal{P}_n^\ell(p_z/p)$ |
| **idea** | neglect $\mathcal{O}(p_z/E_p)$ | approximate $\mathcal{P}_n^\ell(p_z/E_p) \to \mathcal{P}_n^\ell(0)$ |
| **effect** | $p_z/E_p \to 0$ | $p_z/E_p \to 0$ |

$\Longrightarrow$ Our results are identical to 14-moment approximation
for all moments which vanish in equilibrium

$\Longrightarrow$ Bjorken expansion $\to$ distribution function peaked around $p_z = 0$
By chance, 14-moment approximation is consistent with this property

# Conclusions and outlook

- All currents are well described by lowest-order fixed-point truncation at any time of evolution
- Parity-violating moments decay exponentially at late time
  $\Longrightarrow$ contributions from initial conditions have disappeared at freeze out
  $\Longrightarrow$ measurement of spatial chiral currents signature for vorticity or magnetic field
- Charge and axial-charge densities follow ideal equations of motion at any time of evolution
  $\Longrightarrow$ local-equilibrium description sufficient to describe full dynamics
- Here: translational invariance $\Longrightarrow$ no vorticity

  Future: include vorticity $\Longrightarrow$ equilibrium contributions to parity-violating moments
- Here: massless particles, chiral degrees of freedom

  Future: massive particles, spin