

# $v^2$ -Flows

Fast and improved neutrino reconstruction in multi-neutrino final states

ML4Jets, 6<sup>th</sup> November 2023

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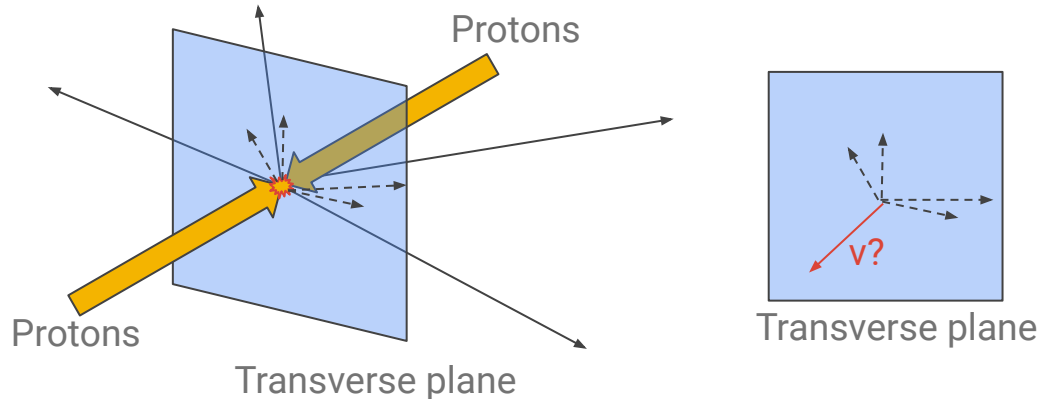


# $\nu$ -Flows - Neutrino Regression

Goal: Predicting **neutrino** momentum

Neutrinos don't interact at all with detector

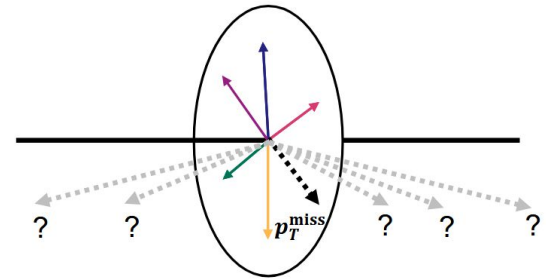
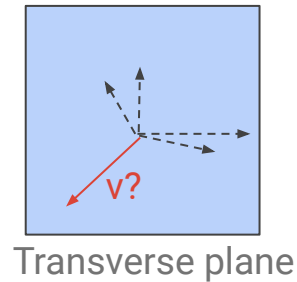
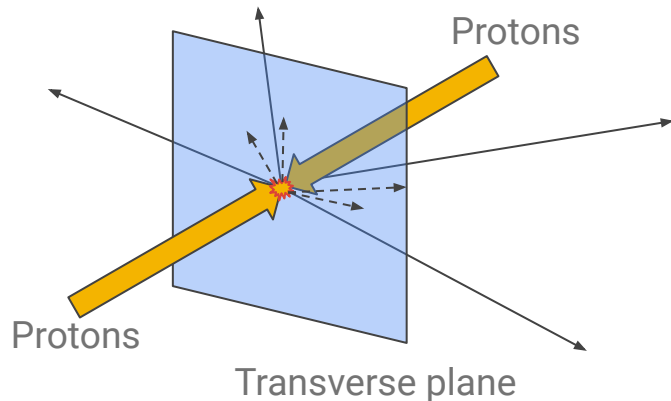
Can only infer their presence from **conservation of momentum**



# $\nu$ -Flows - Neutrino Regression

Goal: Predicting **neutrino** momentum

Momentum only conserved in transverse plane  
Only have a **2D projection** of neutrino momentum

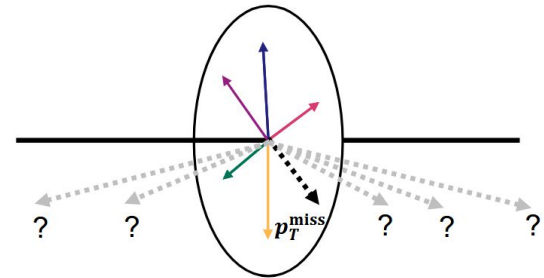
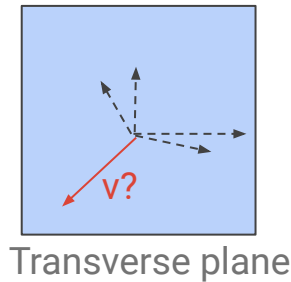
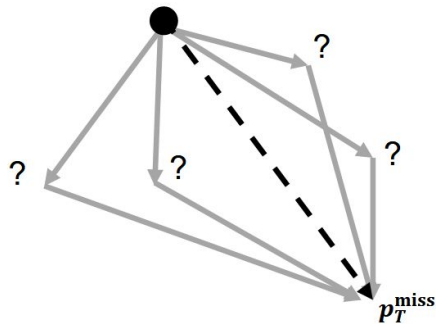


# $\nu$ -Flows - Neutrino Regression

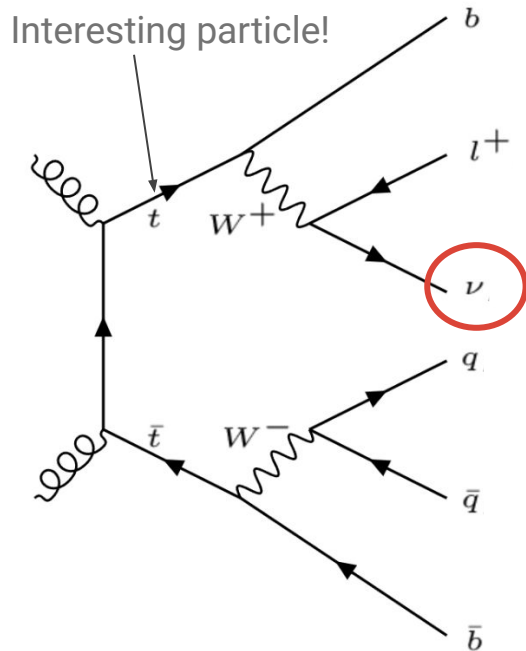
Goal: Predicting **neutrino** momentum

Momentum conservation from sum of vectors

**Under constrained** in transverse plane when multiple neutrinos



# $\nu$ -Flows - Neutrino Regression

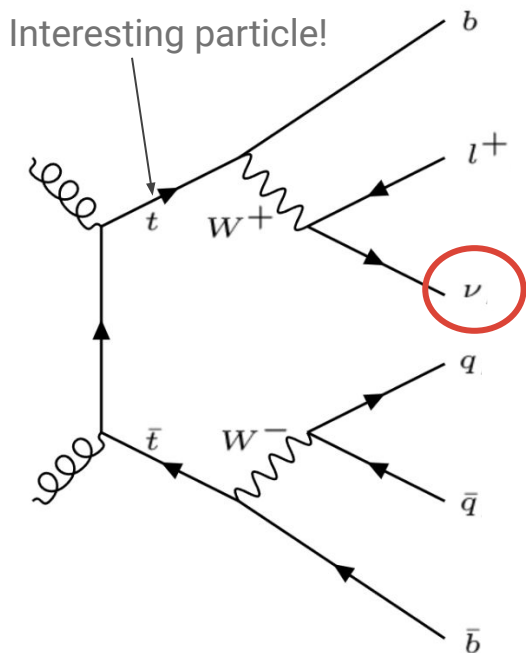


Why do we care about neutrinos?

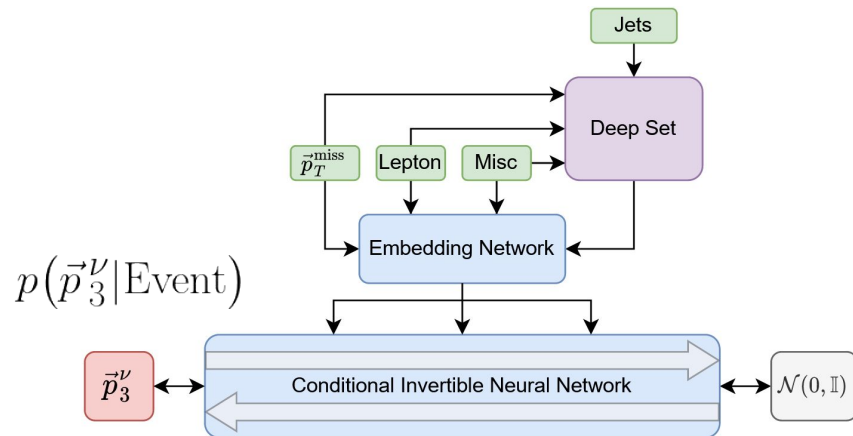
Without neutrino information no longer able to **fully reconstruct top quark**

Neutrino information helps solve combinatorics:  
Assignment of jets to partons easier if you know the W system

# $\nu$ -Flows - Neutrino Regression



Use normalizing flows to learn the conditional probability of solutions given observation from training dataset



At inference obtain distribution over all possible neutrino momenta for the observed event, sample from posterior

# $\nu$ -Flows - Neutrino regression

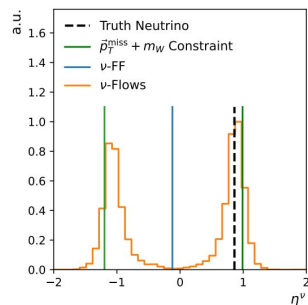
## $\nu$ -flows: Conditional neutrino regression

Matthew Leigh<sup>\*</sup>, John Andrew Raine<sup>†</sup>, Knut Zoch and Tobias Golling

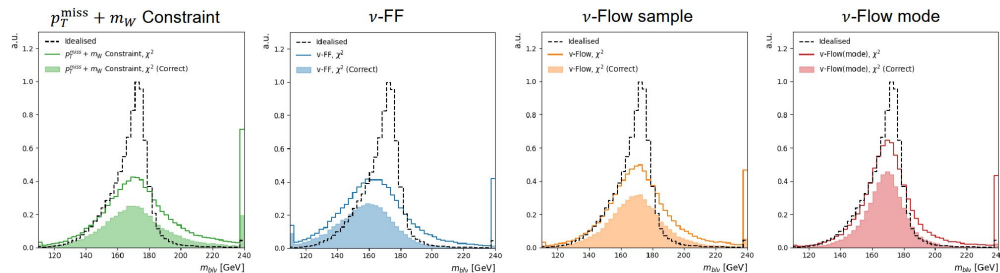
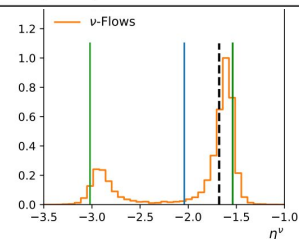
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Demonstrated substantial gains for top quark pairs with one neutrino



Neutrino Type	Number of Jets					
	4	5	6	7	8	9
Truth Neutrino	0.864	0.753	0.686	0.641	0.611	0.587
$\vec{p}_T^{\text{miss}}$ and $m_W$ Constraint	0.790	0.576	0.476	0.398	0.366	0.286
$\nu$ -FF	0.754	0.533	0.410	0.353	0.300	0.302
$\nu$ -Flows(sample)	0.803	0.624	0.515	0.457	0.391	0.357
$\nu$ -Flows(mode)	<b>0.813</b>	<b>0.664</b>	<b>0.575</b>	<b>0.508</b>	<b>0.481</b>	<b>0.405</b>

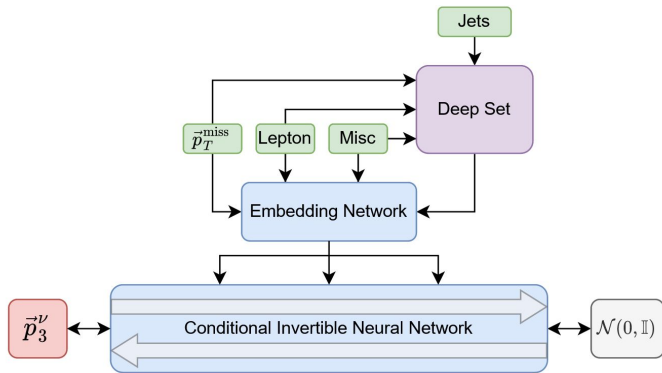


But how does it  
scale to multiple  
neutrinos?

# $\nu^2$ -Flows - Moving to multiple neutrinos

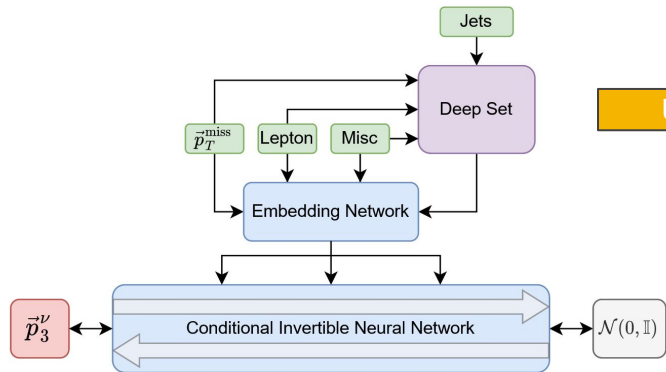
As before don't want to enforce link between objects - network learns combinatorics

Multiplicity and permutation invariant in jets



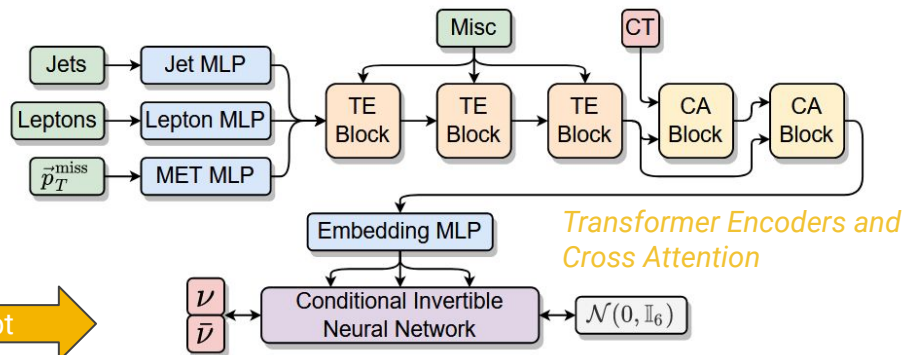
# $\nu^2$ -Flows - Moving to multiple neutrinos

As before don't want to enforce link between objects - network learns combinatorics



Update

Multiplicity and permutation invariant in jets and leptons  
Can easily add any other objects e.g. photons and taus



Adapt

Normalizing flow remains constant  
Scale dimension with number of neutrinos

# Moving to multiple neutrinos

For benchmarking consider top quark pair events with dilepton final state

Observe two leptons, at least two (b-)jets, missing momentum

Reconstructing top quark pair crucial for many measurements

- Top quark spin correlation - quantum entanglement sensitive at low  $m_{tt}$
- Top quark mass (often template fit measuring b-lepton mass)
- Top quark polarisation, charge asymmetry, differential cross sections

# Moving to multiple neutrinos

Compare to two standard approaches (relying on hard assumptions on mass)

## Neutrino Weighting

$$(\ell_{1,2} + \nu_{1,2})^2 = m_w^2 = (80.38 \text{ GeV})^2,$$

$$(\ell_{1,2} + \nu_{1,2} + b_{1,2})^2 = m_t^2 = (172.5 \text{ GeV})^2,$$

Scan eta values for both neutrinos

Choose solution which maximises a weight

$$w = \exp\left(-\frac{\|\vec{p}_T^{\text{miss}} - \vec{p}_T^{\nu\bar{\nu}}\|_2^2}{2\sigma^2}\right)$$

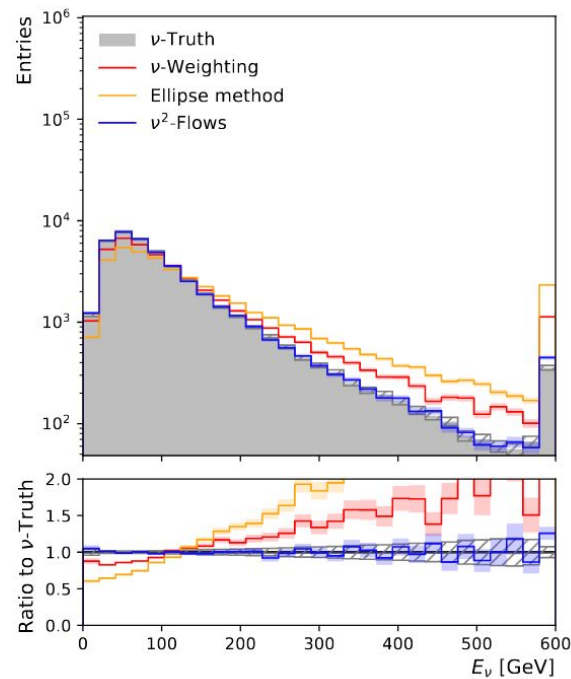
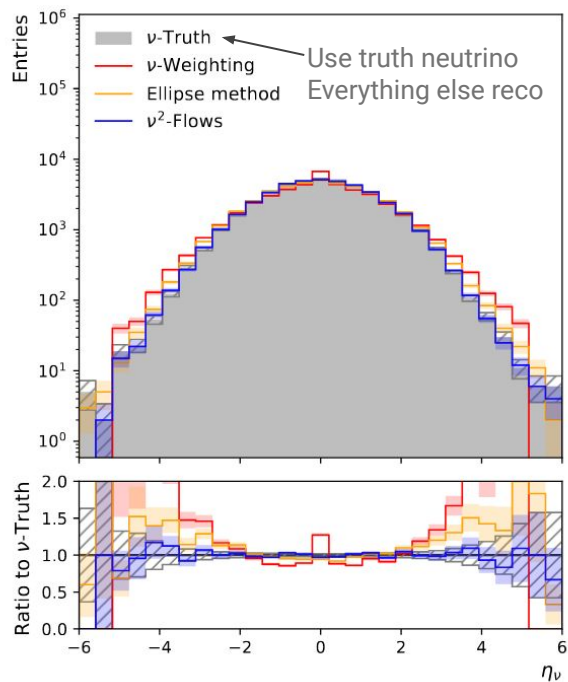
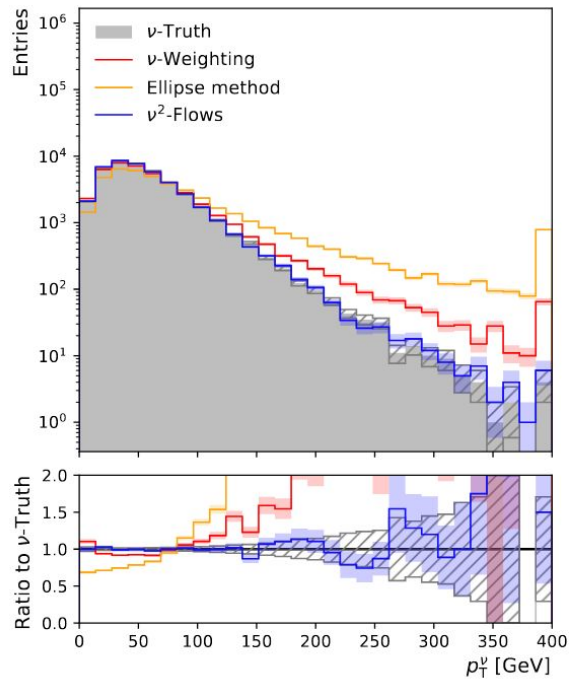
Additionally scan top quark mass values to improve acceptance

## Ellipse method

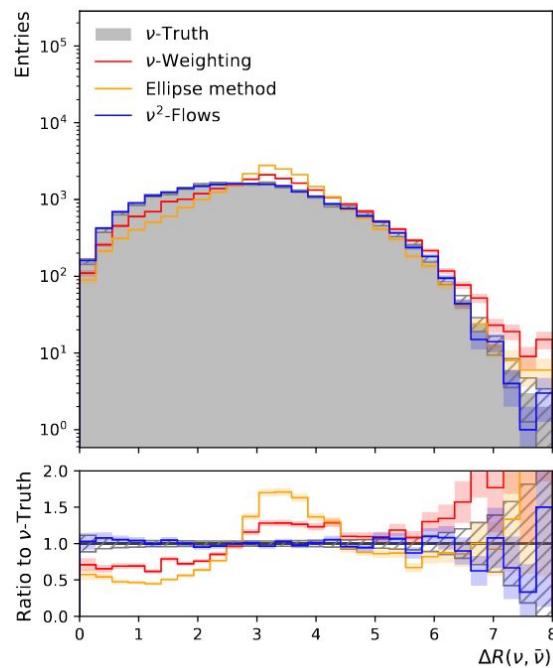
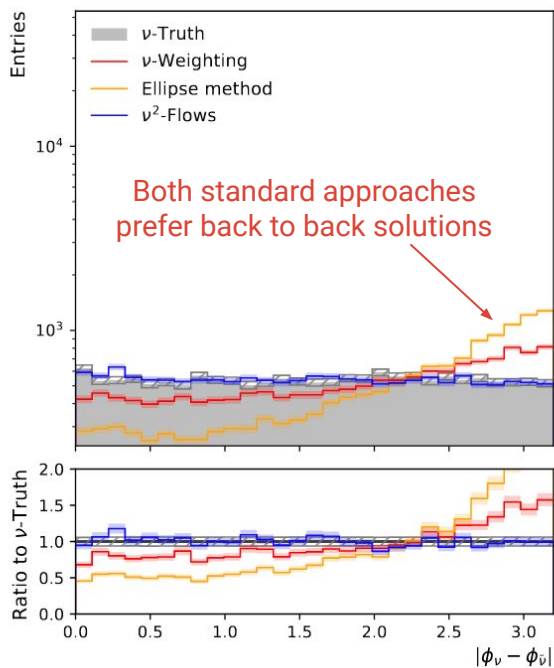
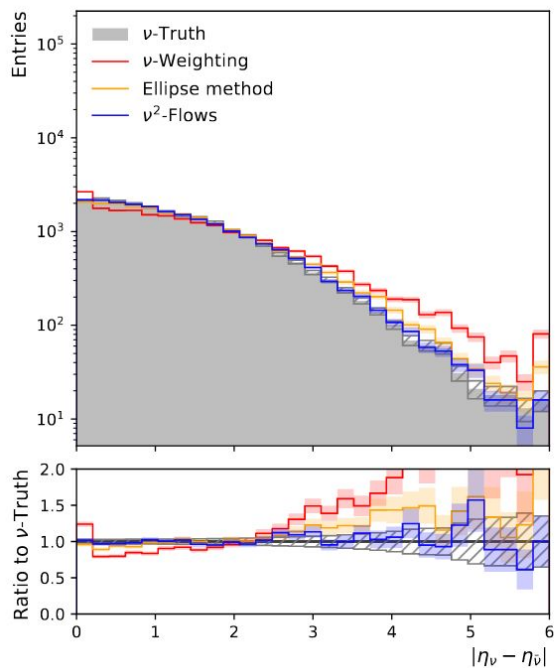
Use observed missing momentum to constrain solution further

Less flexible to resolution effects but computationally more efficient

# Comparing kinematics

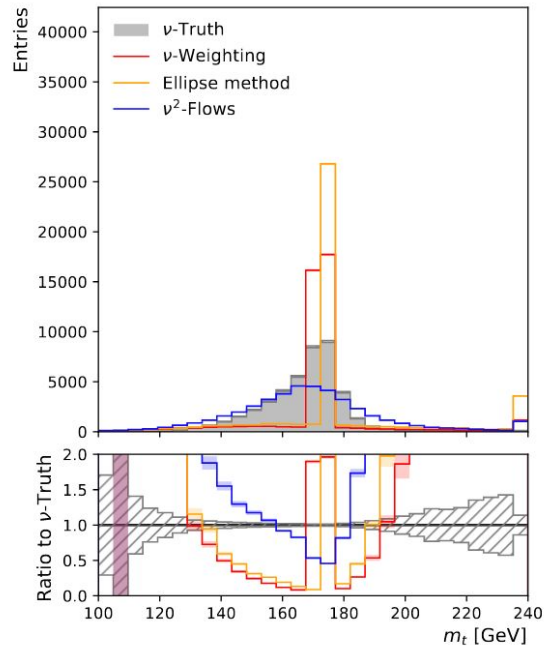
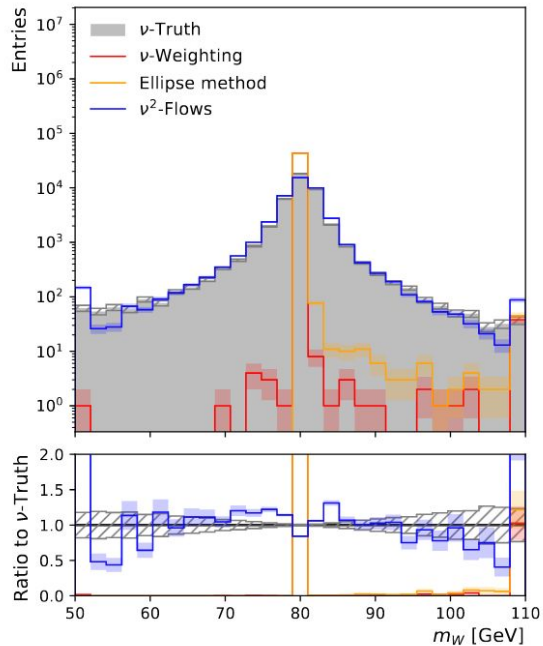


# Comparing correlations



# Compare impact of mass constraints

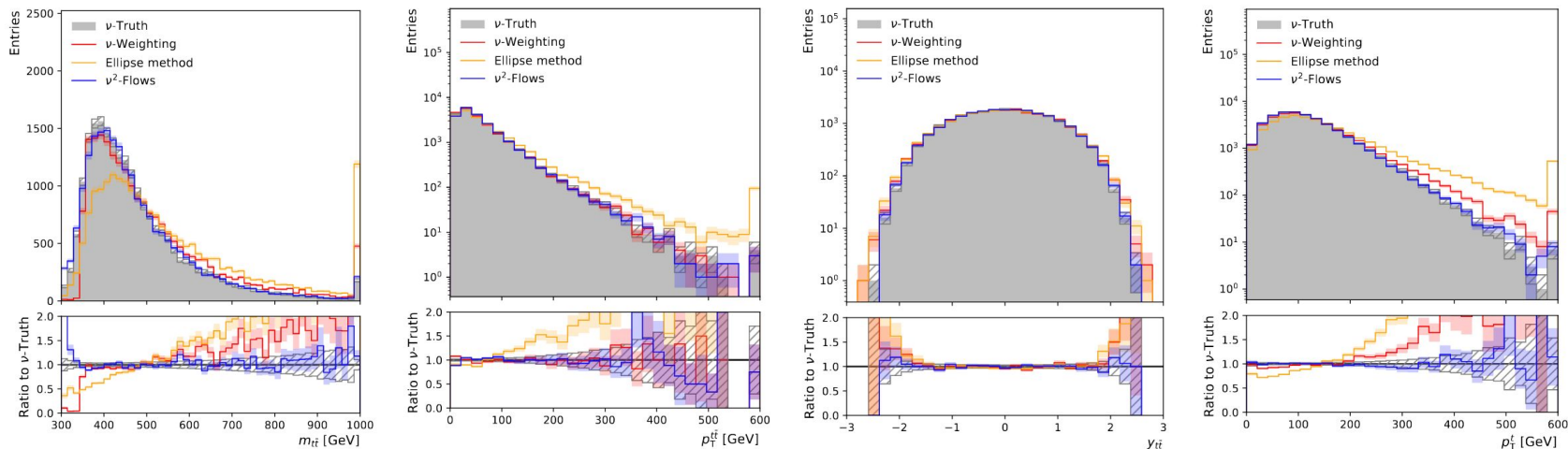
*Don't see a perfect peak due to wrong top association for standard methods*



# Comparing top quark kinematics

Capturing complex distributions much more closely with  $\nu^2$ -Flows

In particular  $m_{t\bar{t}}$  at threshold and high values is much closer to optimum



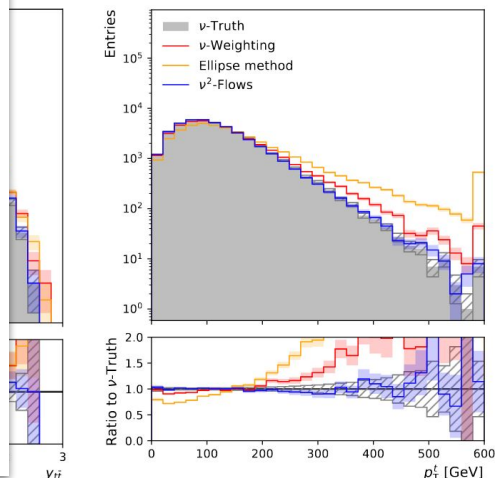
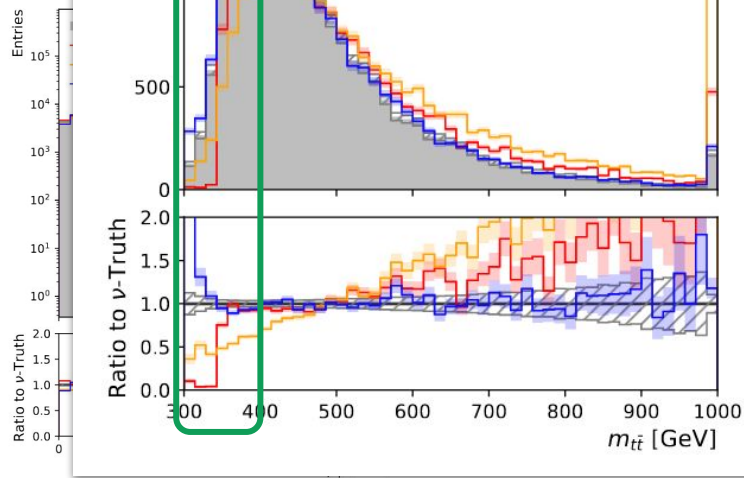
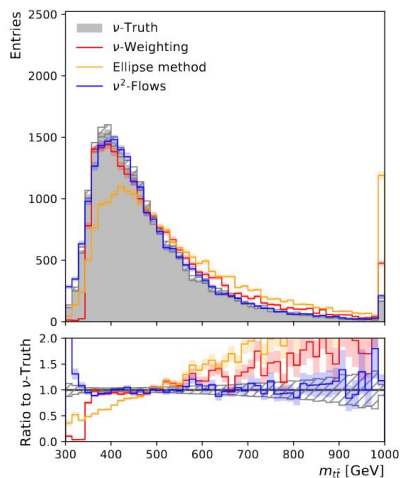
# Comparing to $\nu^2$ -Flows

Capturing complex dist

In particular  $m_{t\bar{t}}$  at thresh

$\nu^2$ -Flows

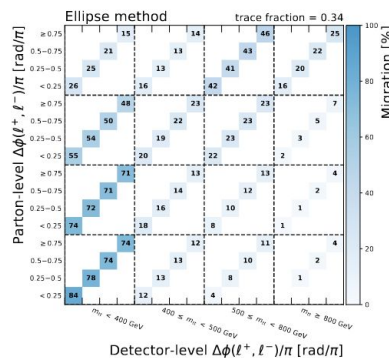
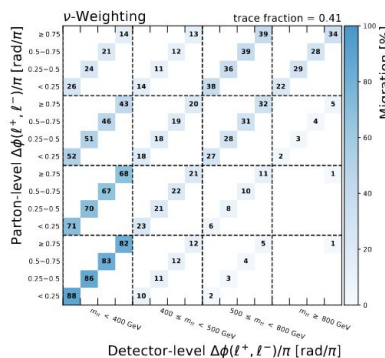
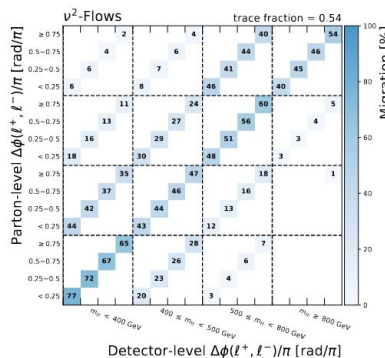
user to optimum



# Quantifying improvement

Substitute in each method into a double-differential measurement

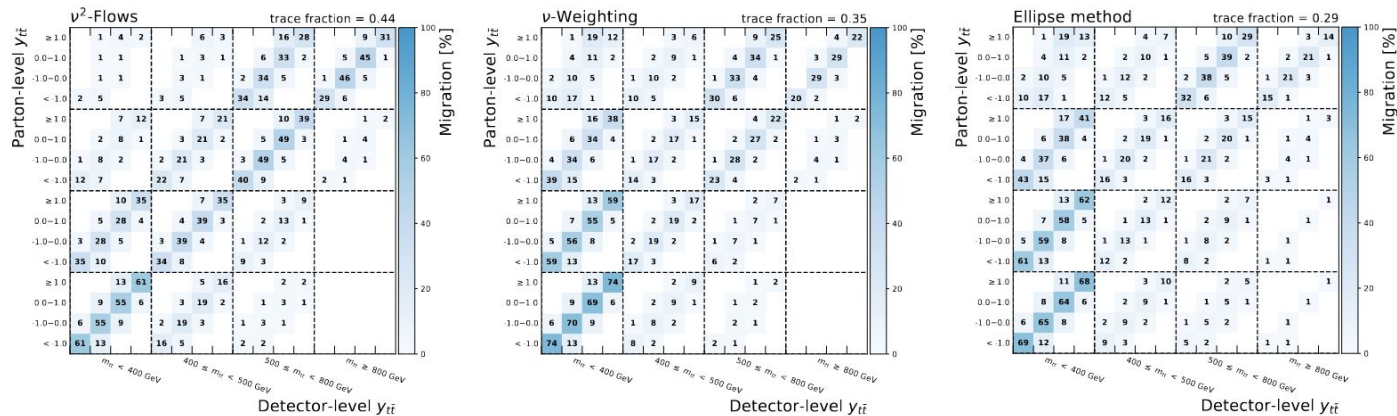
- Look at response matrix - is it diagonal?
- Look at statistical precision in each bin after unfolding
- Look at efficiency of method in each truth level bin



Observable	Bin edges
$m_{t\bar{t}}$	[0, 400, 500, 800, inf] GeV
$\Delta\phi(\ell^+\ell^-)$	[0.0, 0.25, 0.5, 0.75, 1.0] rad/ $\pi$
$p_T^t$	[0, 75, 125, 175, inf] GeV
$p_T^{\bar{t}}$	[0, 70, 140, 200, inf] GeV
$y_{t\bar{t}}$	[-inf, -1.0, 0.0, 1.0, inf]

# $\nu^2$ -Flows - Extending to multiple neutrinos

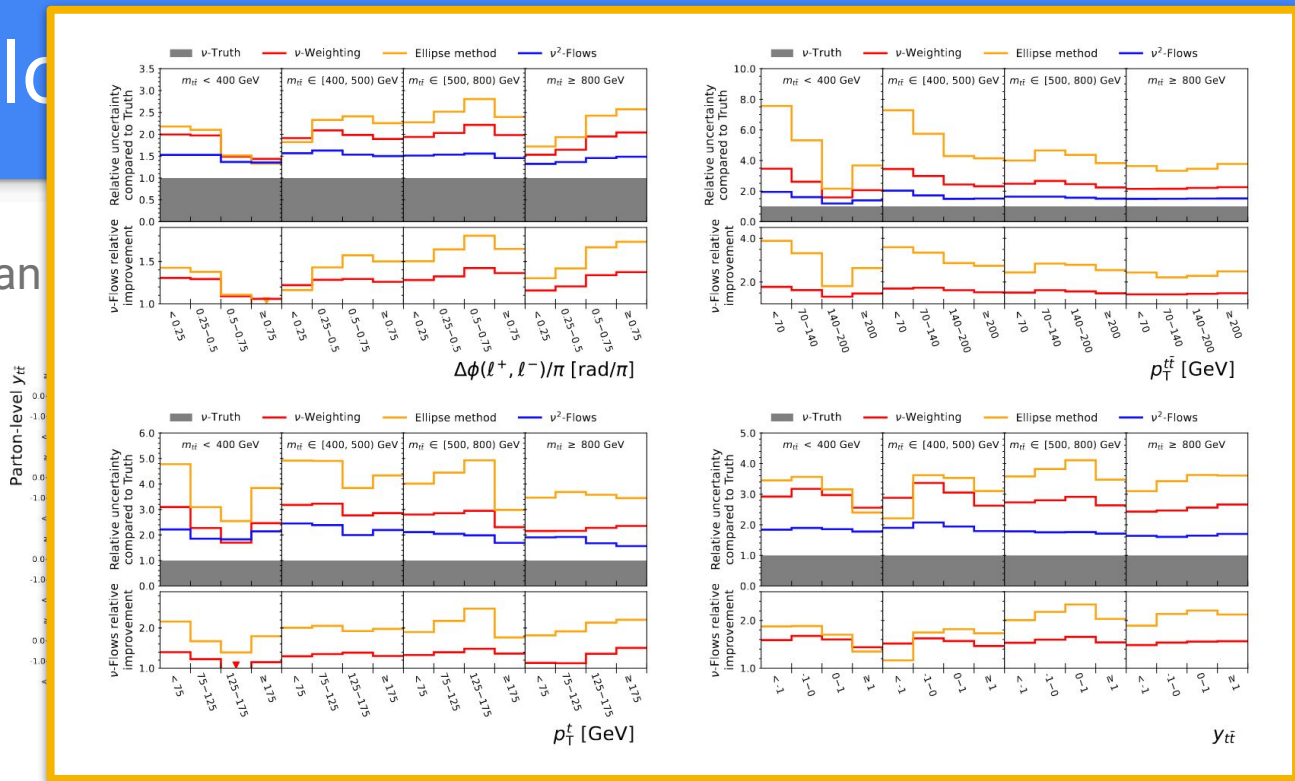
Use in an unfolding measurement - compare migration of truth to reconstructed



Much improved diagonal response! Relationship between truth and reco is well captured

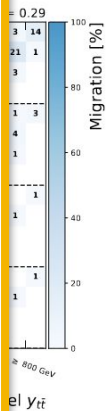
# $\nu^2$ -Flows

Use in an



mos

structured



Improvements in all double-differential distributions with  $\nu^2$ -Flows -> direct statistical benefit in analysis!

Observables	Method	Efficiency at finding a solution (per bin) [%]															
$y_{t\bar{t}}$	$\nu$ -Weighting	97	98	97	97	96	96	97	96	93	94	94	94	89	90	89	89
	Ellipse	91	92	92	91	84	82	84	82	67	68	68	70	58	54	51	50
$m_{t\bar{t}}$	$\nu$ -Weighting	99	95	91	89	98	95	90	84	97	92	89	82	94	88	81	81
	Ellipse	96	86	74	59	88	79	66	50	75	66	52	38	60	52	34	37
$p_{\text{T}}^t$	$\nu$ -Weighting	98	96	92	86	97	97	96	91	95	95	95	93	96	89	90	89
	Ellipse	95	88	72	51	89	86	80	63	77	74	72	63	61	56	55	51
$\Delta\phi(\ell^+\ell^-)$	$\nu$ -Weighting	96	97	98	99	96	96	97	97	93	94	94	94	90	90	91	88
	Ellipse	90	92	92	94	84	84	83	82	76	74	68	64	57	57	57	48

In many bins even more **statistical gains** due to  $\nu^2$ -Flows always having a solution!

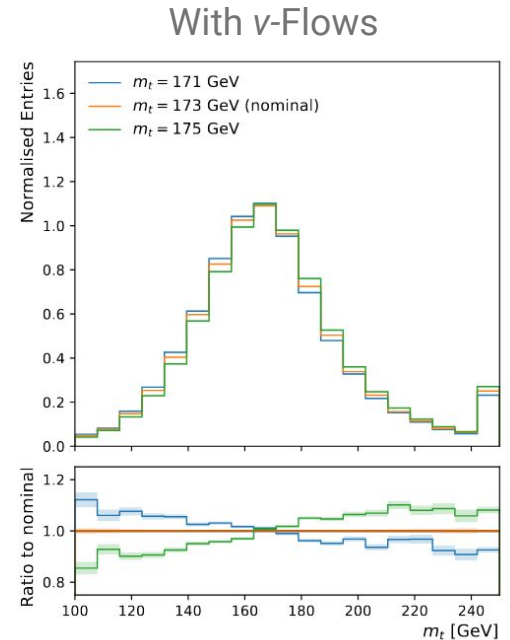
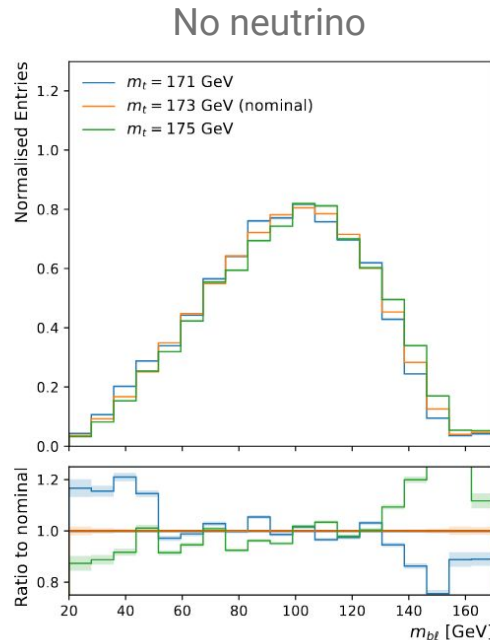
# And what about the top quark mass?

Can use neutrino solutions in top quark mass reconstruction

**When using  $\nu^2$ -Flows retain sensitivity to top mass!**

*Train on one value of mass*

*Evaluate on samples with different truth mass values*



# Conclusions

## $\nu^2$ -Flows...

outperforms standard approaches still used in current ttbar dilepton analyses

can accommodate any neutrino and event object multiplicities

- Takes advantage of modern transformer architectures with conditional normalizing flows
- Shown on ttbar but framework designed to be adaptable to any final state

has fast inference and is competitive with fast analytical methods

can even be used in a mass analysis

# Backup

# Links

$\nu$ -Flows paper: <https://arxiv.org/abs/2207.00664>

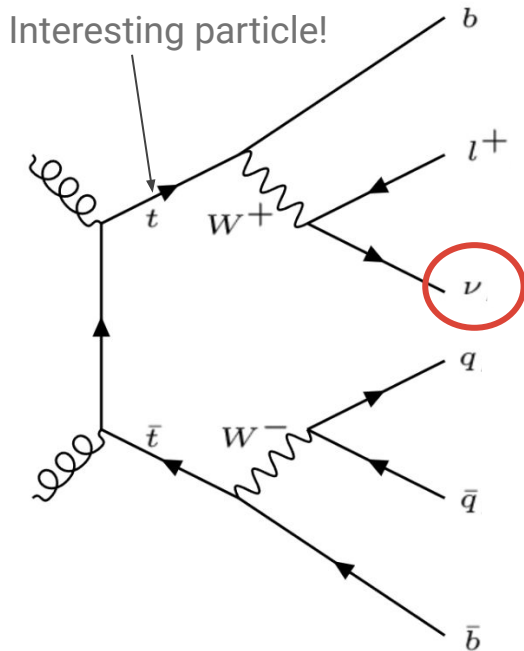
$\nu^2$ -Flows implementation: <https://github.com/rodem-hep/nu2flows>

Dataset: <https://zenodo.org/record/8113516>

Neutrino Weighting implementation from: <https://arxiv.org/abs/1903.07570>

Ellipse implementation from: <https://github.com/betchart/analytic-nu>

# $\nu$ -Flows - Neutrino Regression



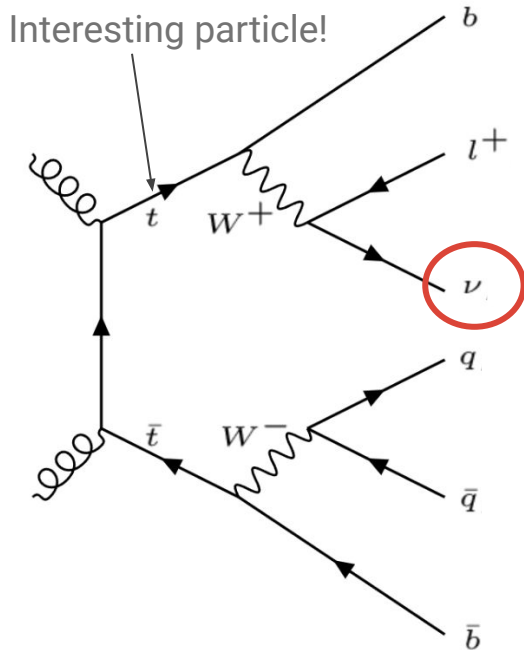
Standard approach in HEP for  $1\ell$

Assume mass of  $W$  exactly 80.46 GeV

$$\text{Given } p_4^W = p_4^\nu + p_4^\ell \quad p_4 = (p_x, p_y, p_z, m)$$

- Use missing transverse momentum for  $p_T^\nu$
- Solve quadratic equation for  $p_z^\nu$

# $\nu$ -Flows - Neutrino Regression



Standard approach in HEP for  $1\ell$  Ignore width of resonance  
Ignores detector resolution

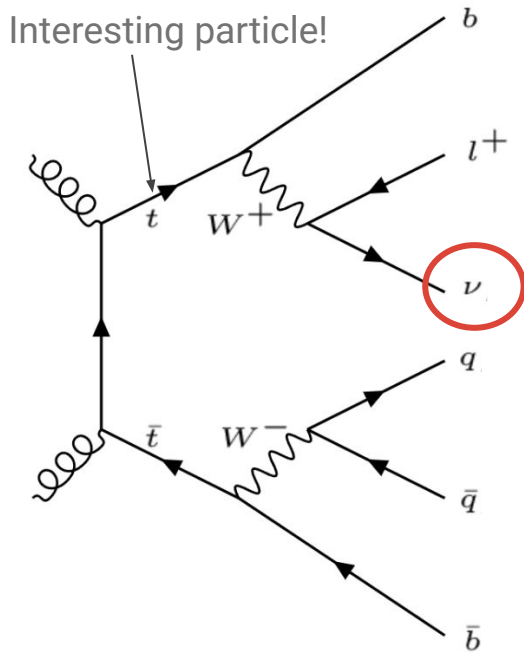
Assume mass of  $W$  exactly 80.46 GeV

Given  $p_4^W = p_4^{\nu} + p_4^{\ell}$        $p_4 = (p_x, p_y, p_z, m)$

- Use missing transverse momentum for  $p_T^{\nu}$
- Solve quadratic equation for  $p_z^{\nu}$

Can give two solutions with no preference  
Can yield no real solutions

# $\nu$ -Flows - Neutrino Regression



Standard approach in HEP for  $1\ell$

Assume mass of  $W$  exactly 80.46 GeV

$$\text{Given } p_4^W = p_4^\nu + p_4^\ell \quad p_4 = (p_x, p_y, p_z, m)$$

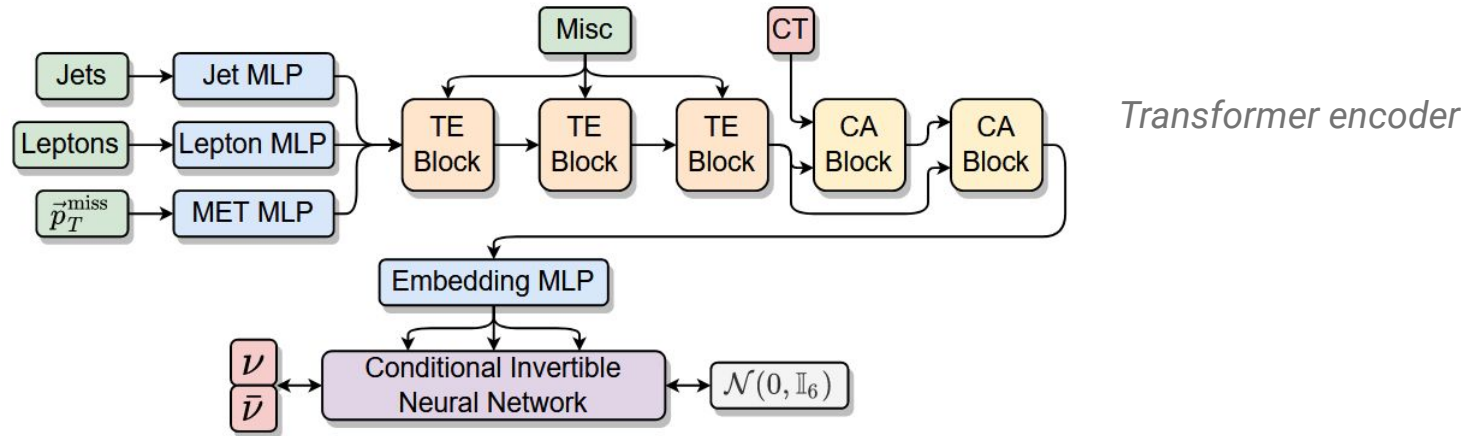
- Use missing transverse momentum for  $p_T^\nu$
- Solve quadratic equation for  $p_z^\nu$

For  $2\ell$  also assume top quark mass

- Improve efficiency by smearing  $m_{\text{top}}$  and scanning  $\eta^\nu$
- Use  $p_T^{\nu\nu} / p_T^{\text{miss}}$  to select best solution

# $\nu^2$ -Flows - Extending to multiple neutrinos

Move to 2 neutrinos - permutation invariant architecture for all objects



Truth level information, can enforce an order as long as consistent in training

# Inputs

Category	Variables	Description
	$\vec{p}_T^{\text{miss}}$	Missing transverse momentum 2-vector
	$p_x^{\text{miss}}, p_y^{\text{miss}}$	
	$p_x^\ell, p_y^\ell, p_z^\ell, \log E^\ell$	Lepton momentum 4-vector
Leptons	$q^\ell$	Lepton charge
	$\ell^{\text{flav}}$	Whether lepton is an electron or muon
	$p_x^j, p_y^j, p_z^j, \log E^j$	Jet momentum 4-vector
Jets	$isB$	Whether jet passes $b$ -tagging criteria
Misc	$N_{\text{jets}}, N_{b\text{jets}}$	Jet and $b$ -jet multiplicities in the event

# Observables

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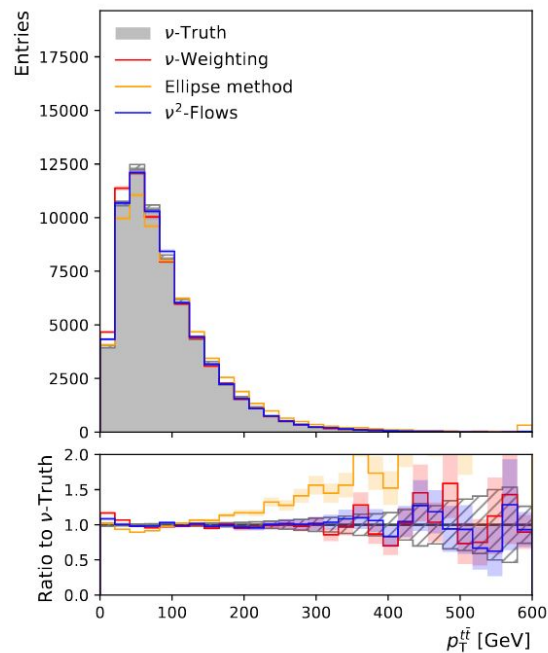
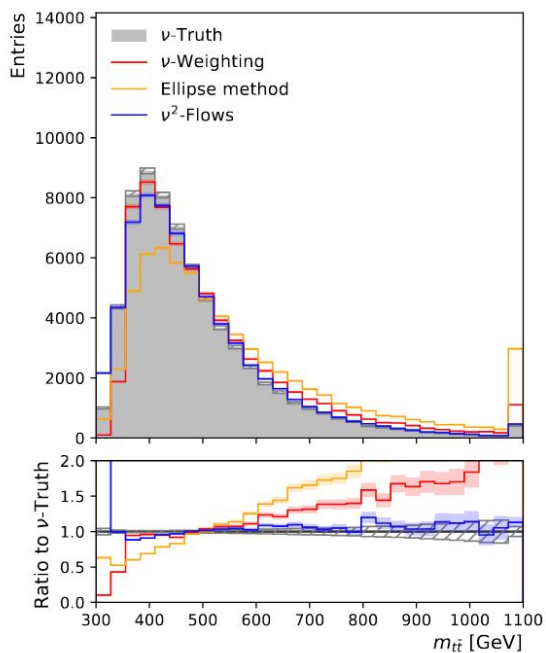
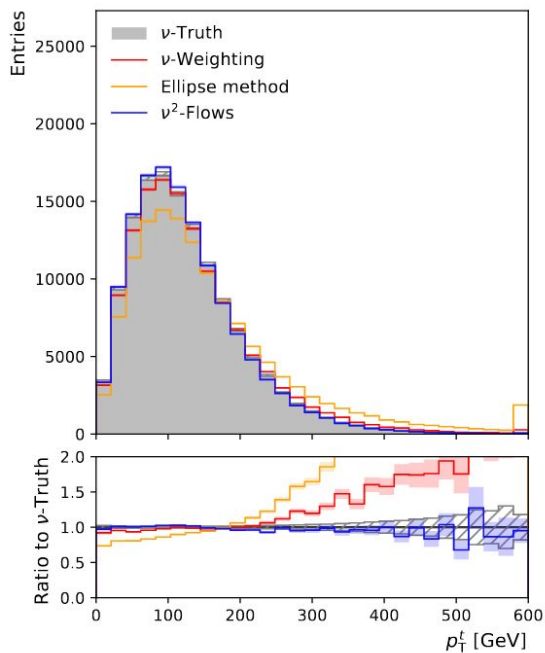
## Observables

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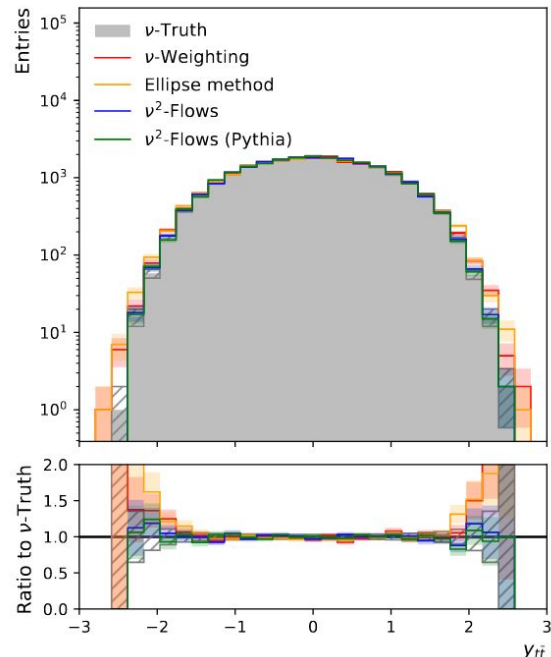
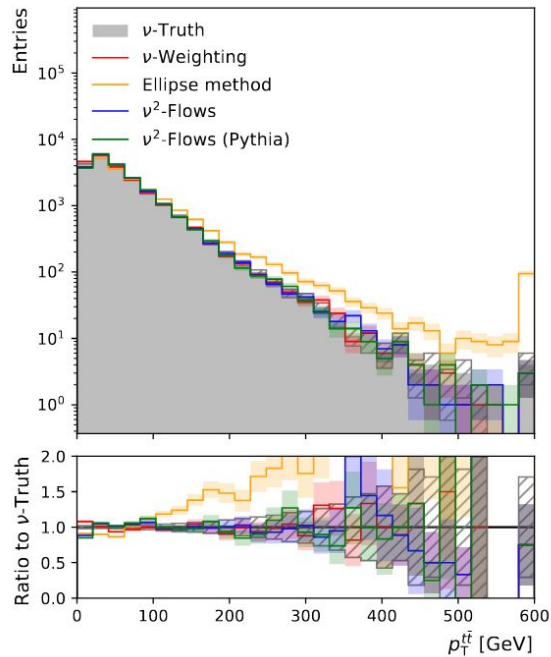
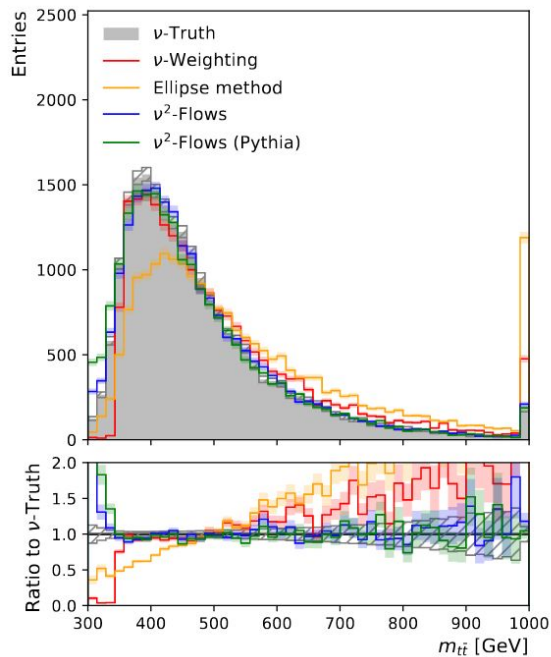
$m_{t\bar{t}}$	Invariant mass of $t\bar{t}$ system
$\Delta\phi(\ell^+\ell^-)$	Separation in $\phi$ between the two leptons
$p_{\text{T}}^t$	Transverse momentum of the top quark
$p_{\text{T}}^{t\bar{t}}$	Transverse momentum of the $t\bar{t}$ system
$y_{t\bar{t}}$	Rapidity of the $t\bar{t}$ system

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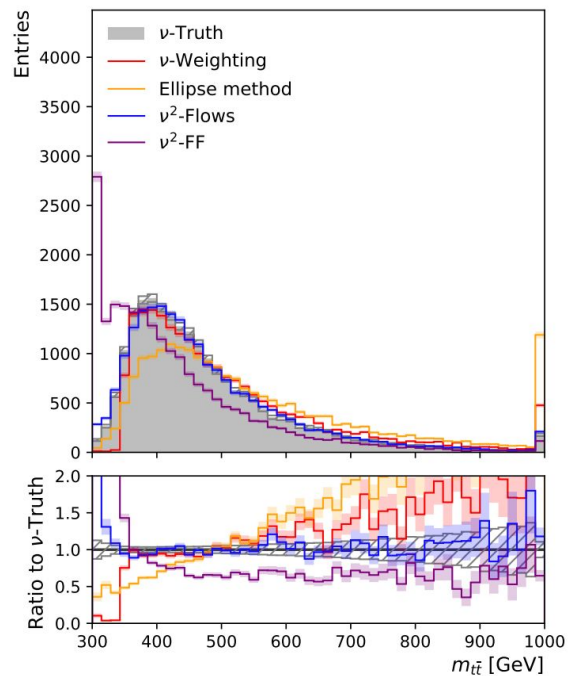
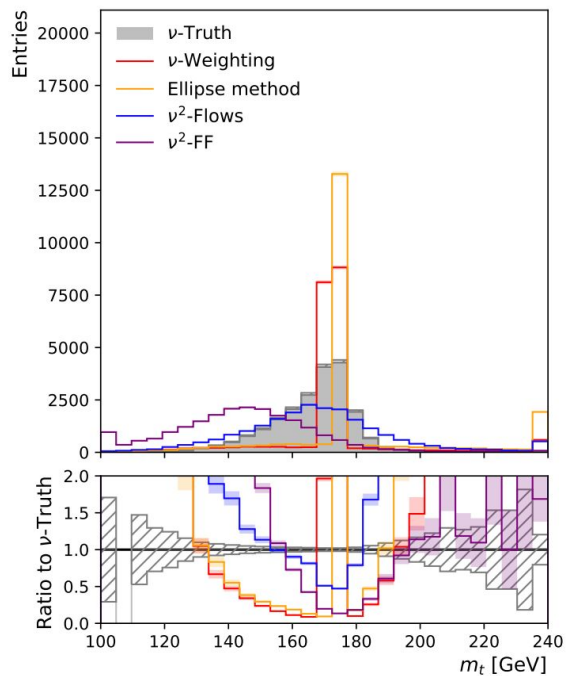
# Evaluating on extra ISR/FSR sample



# Training on Pythia LO instead of MG5



# Comparison to a feed-forward network



# Origin of improvement in distributions

Better distributions everywhere

Not just improving due to low scoring  
NW events

Where NW has large  $w$  weight

Where NW has small  $w$  weight

Performance seems independent of  
NW weight

