PELICAN

PERMUTATION- AND LORENTZ-EQUIVARIANT NETWORKS FOR PARTICLE PHYSICS

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Overview

- Architecture Summary
- Tagging and Vector Reconstruction
- Small Parameter Limit

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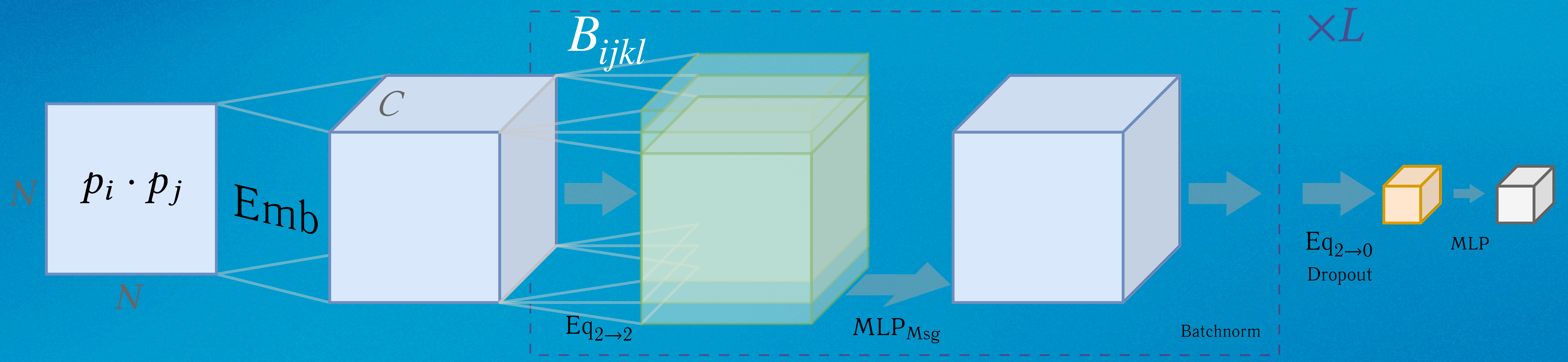
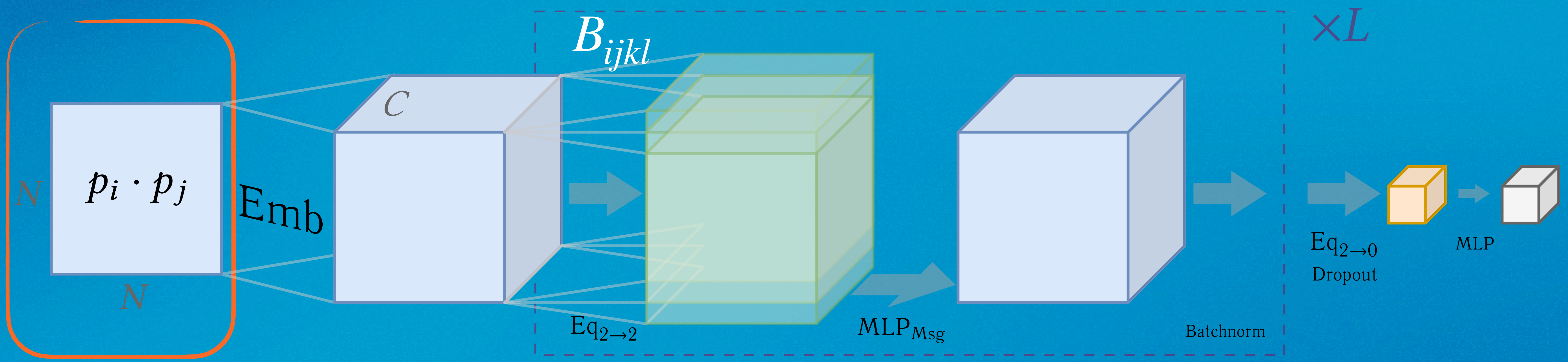
Symmetries as a Fundamental Orientation

- Lorentz and permutation symmetries are fundamentally important to jet physics
- Begin with a jet J, and its constituent four-vectors p_i
- Use the full set of natural Lorentz invariants $p_i \cdot p_j$ as building blocks
- Use particle-index permutation-equivariance tensors B_{ijkl} as fundamental network operations



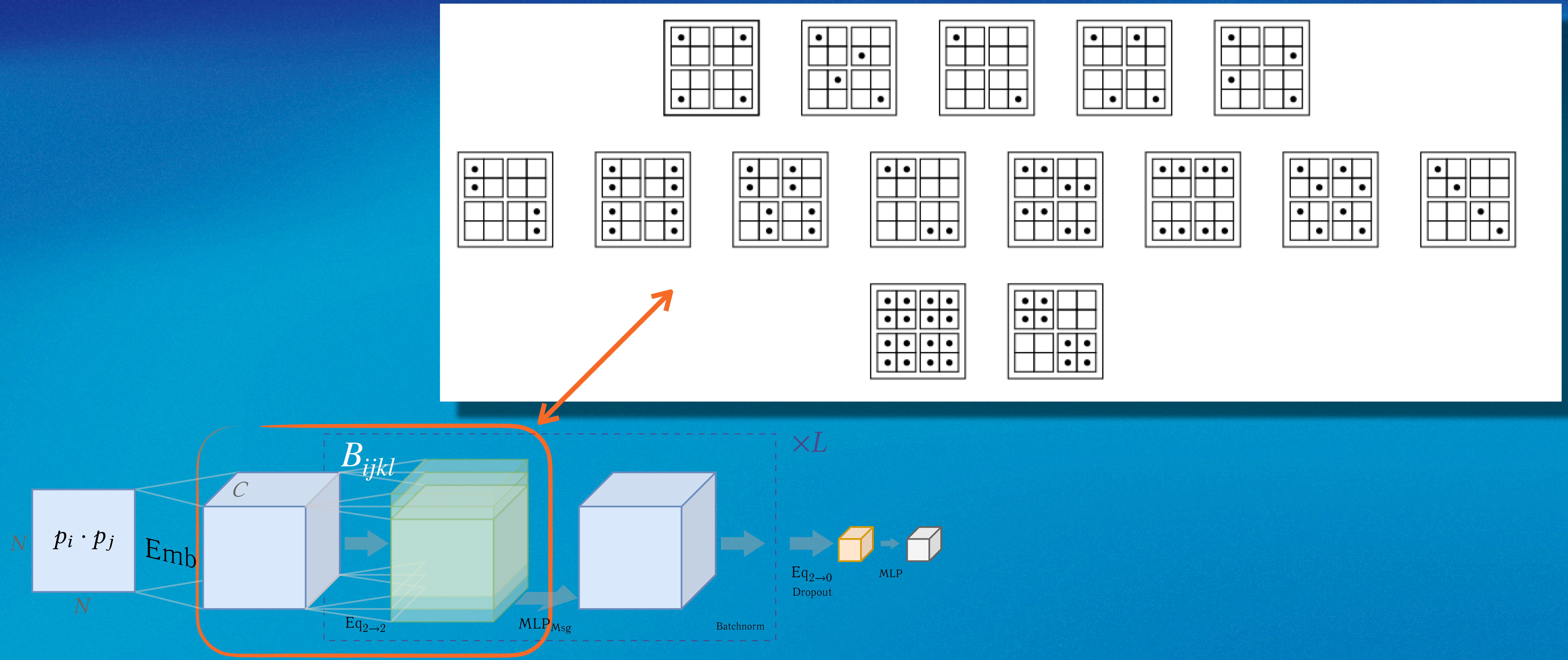
Lorentz Equivariance

- Construct Lorentz-scalars (particle ID) and vectors (intermediate particles)
- Target Lorentz-scalars: $f(\{p_i\}) = I(\{p_i \cdot p_j\})$
- Target Lorentz-vectors: $f^\mu(\{p_i\}) = \sum_k I_k(\{p_i \cdot p_j\}) p_k^\mu$



Permutation Equivariance

- 15 permutation equivariant linear maps of matrices into matrices
- 5 maps of matrices into vectors
- 2 maps of matrices into scalars



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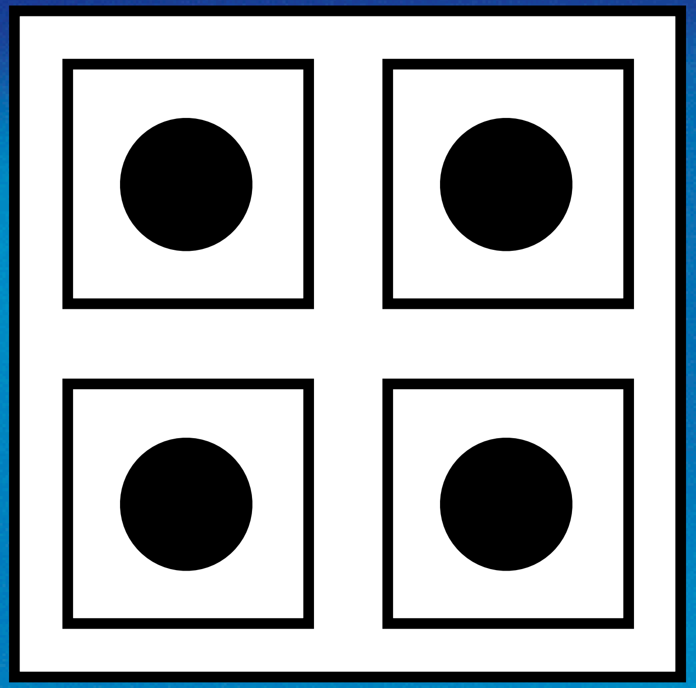
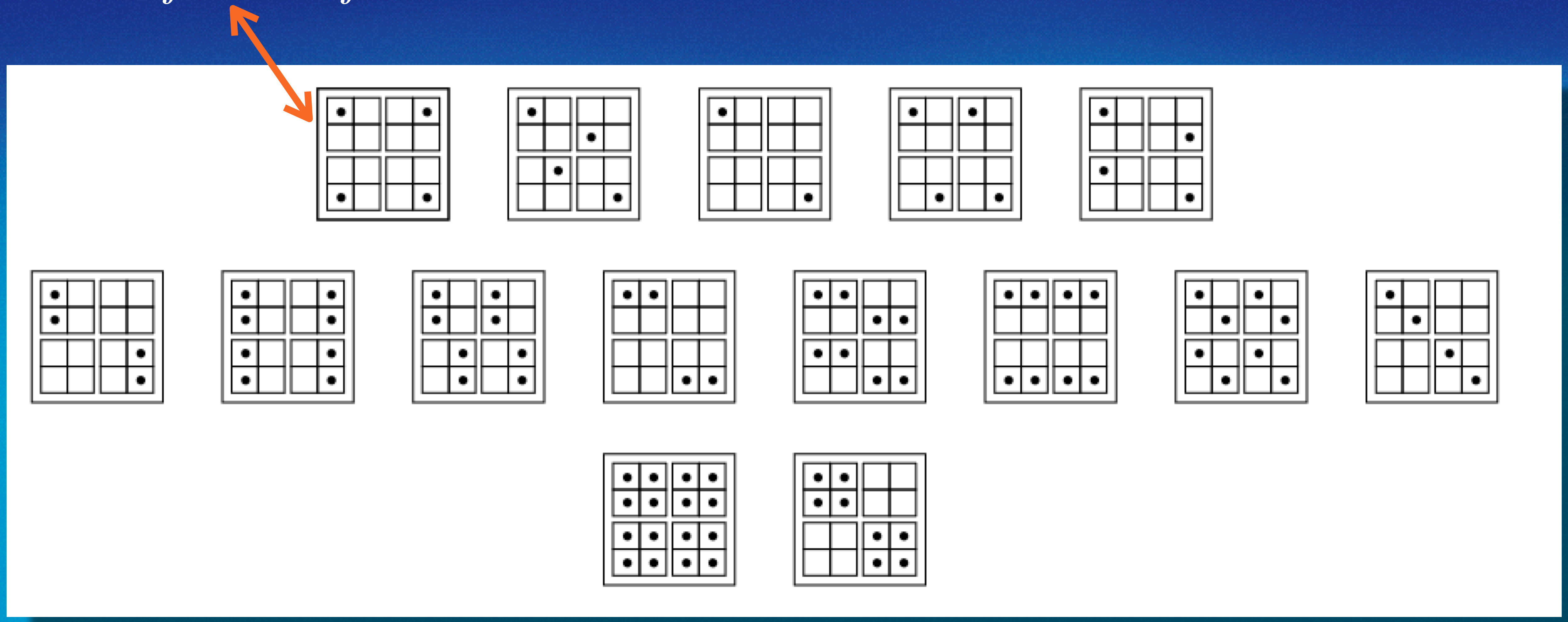
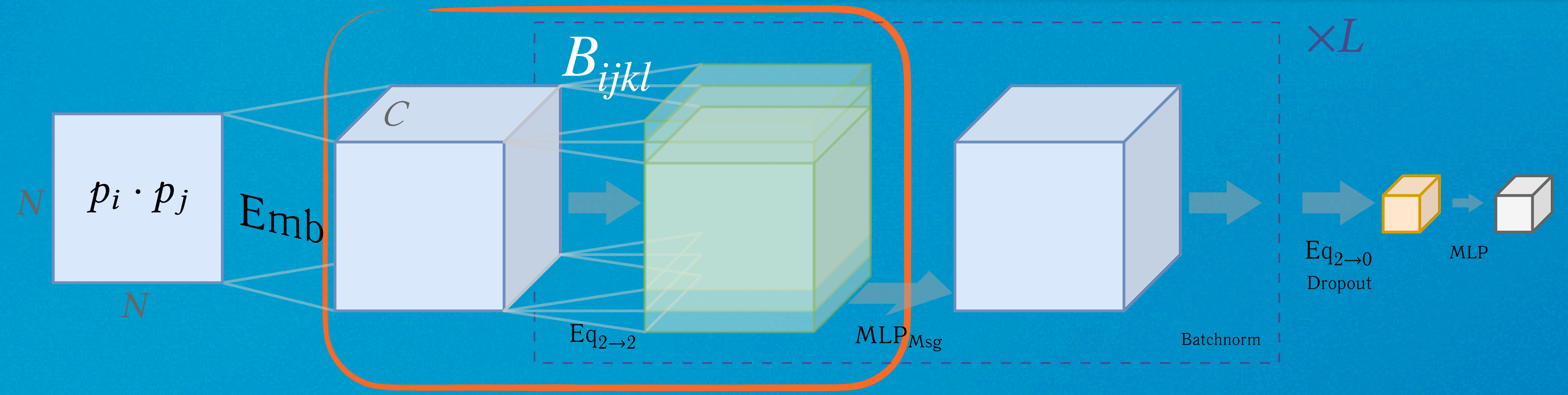
Identity

$\delta_{ik}\delta_{jl}(p_i \cdot p_j) = p_k \cdot p_l$







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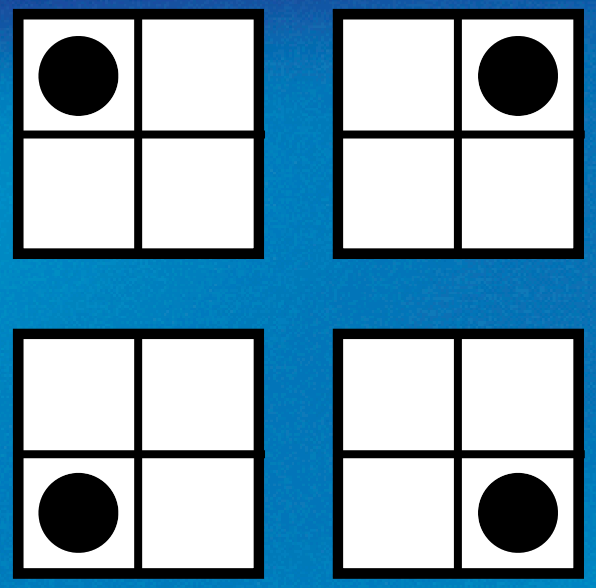

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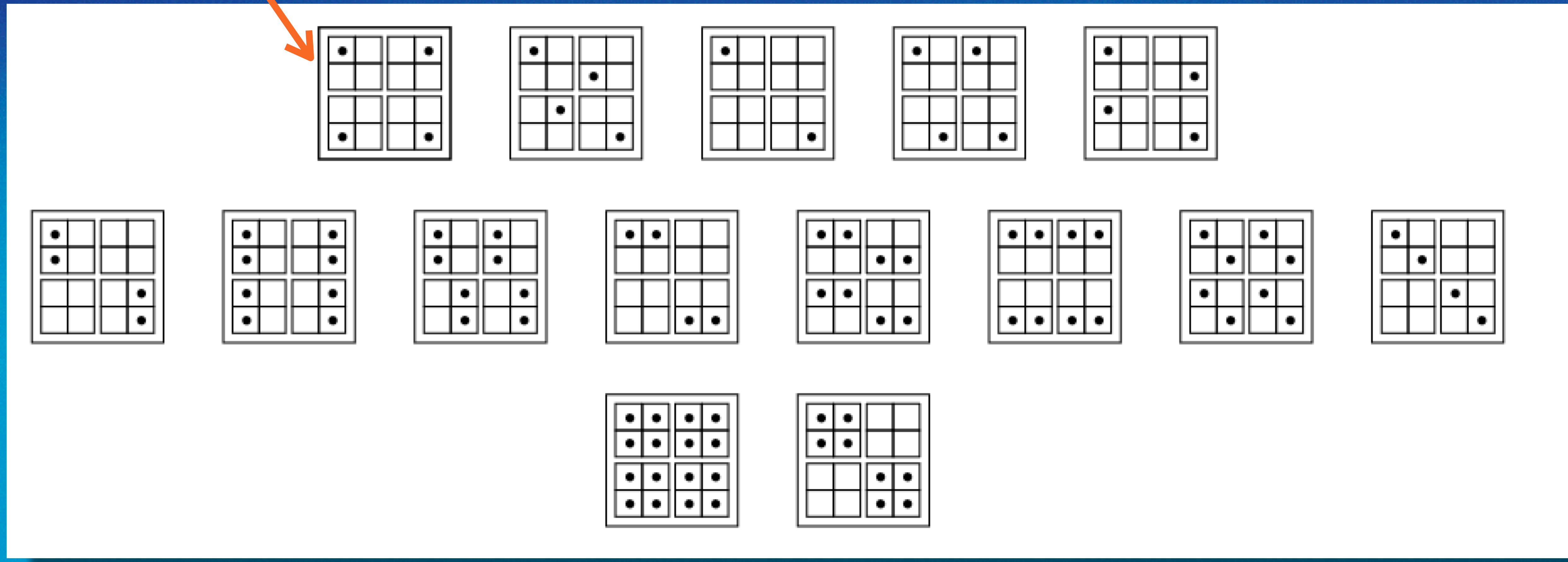

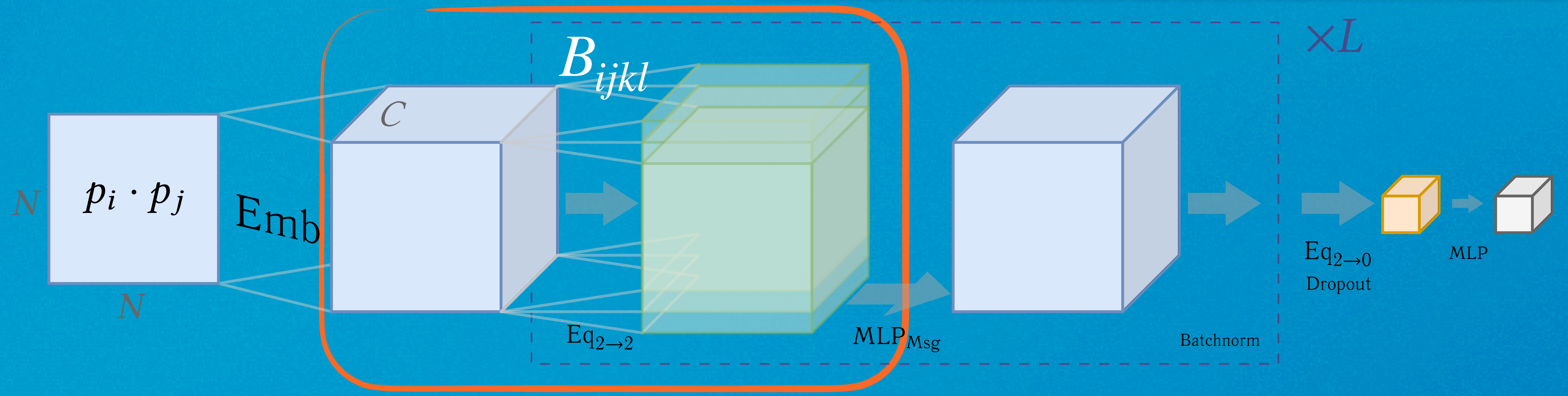

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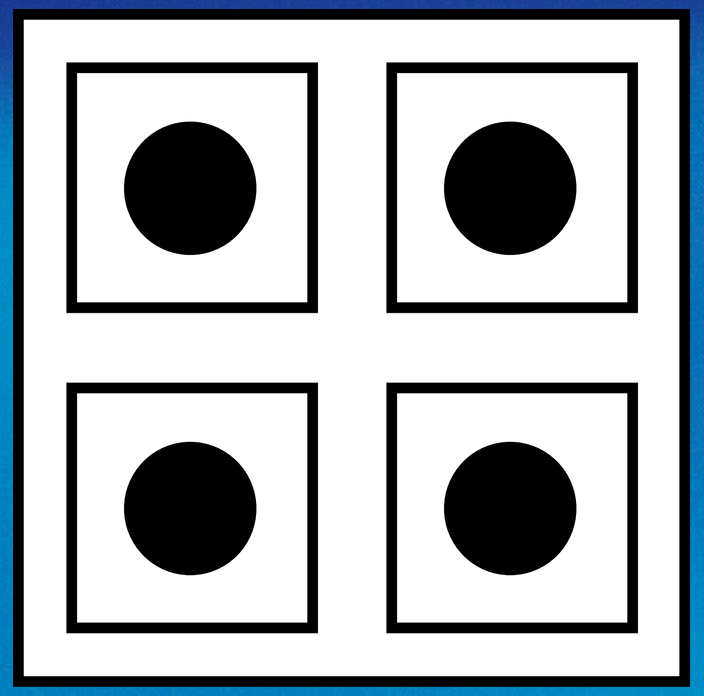

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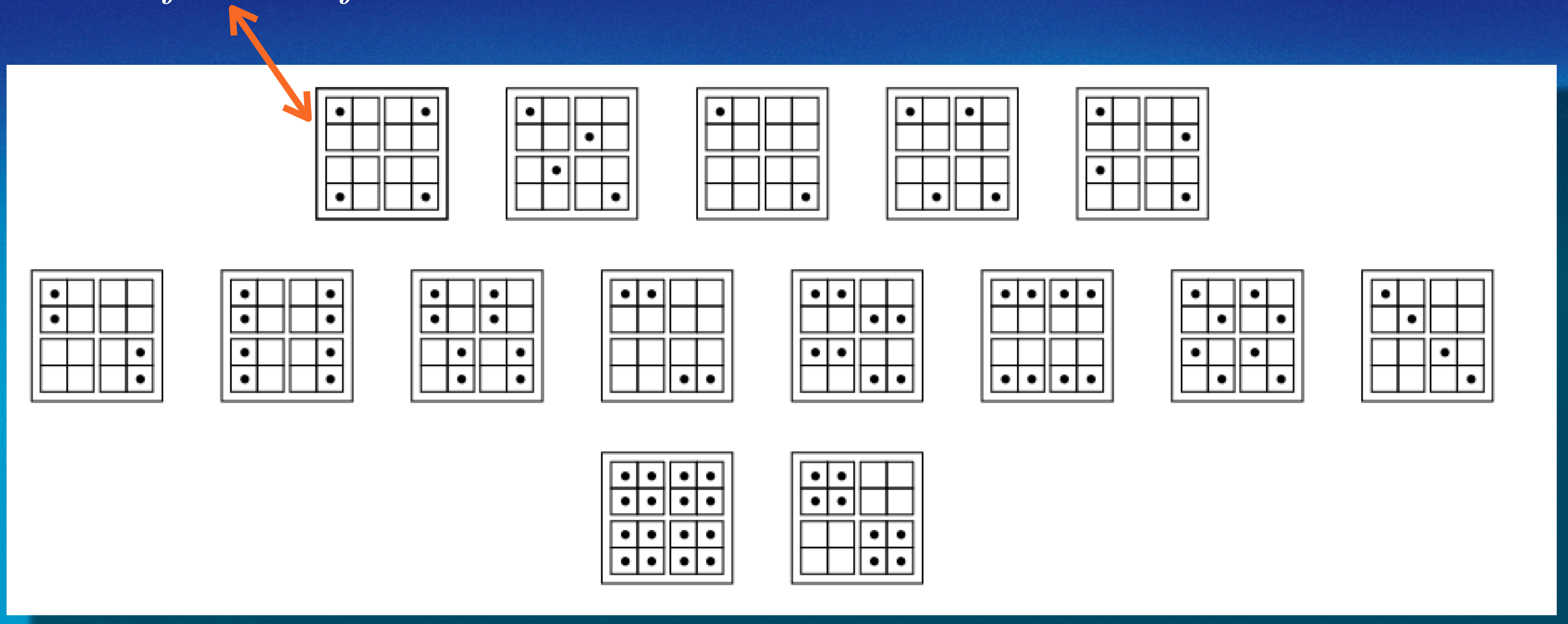

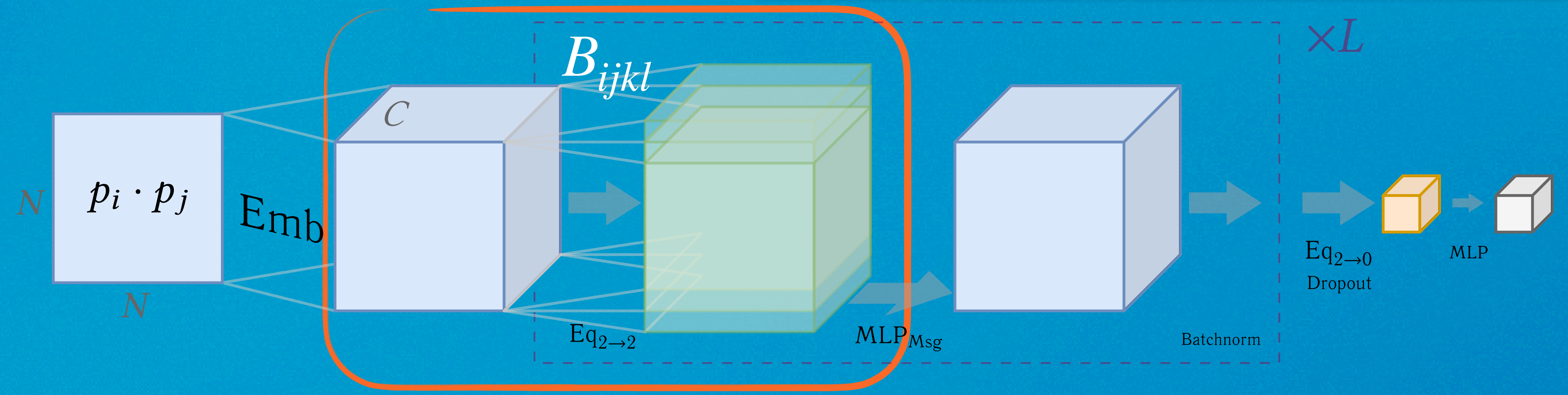

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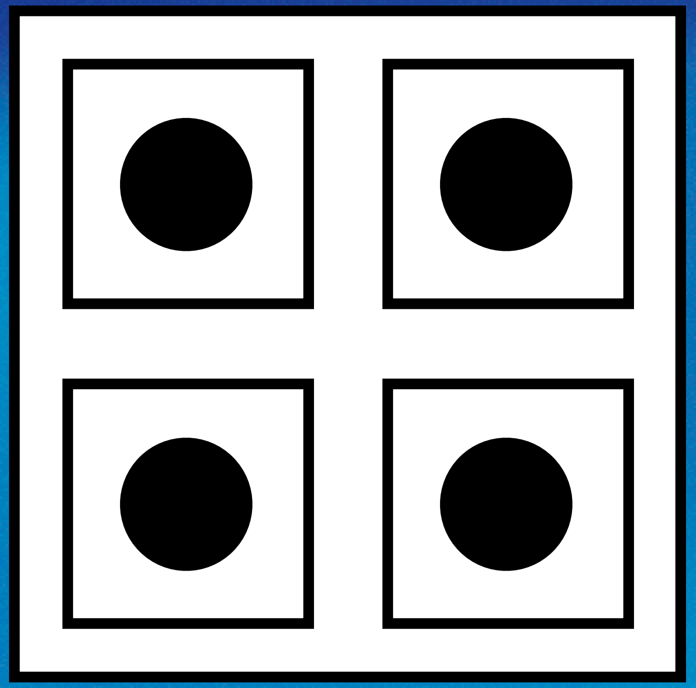
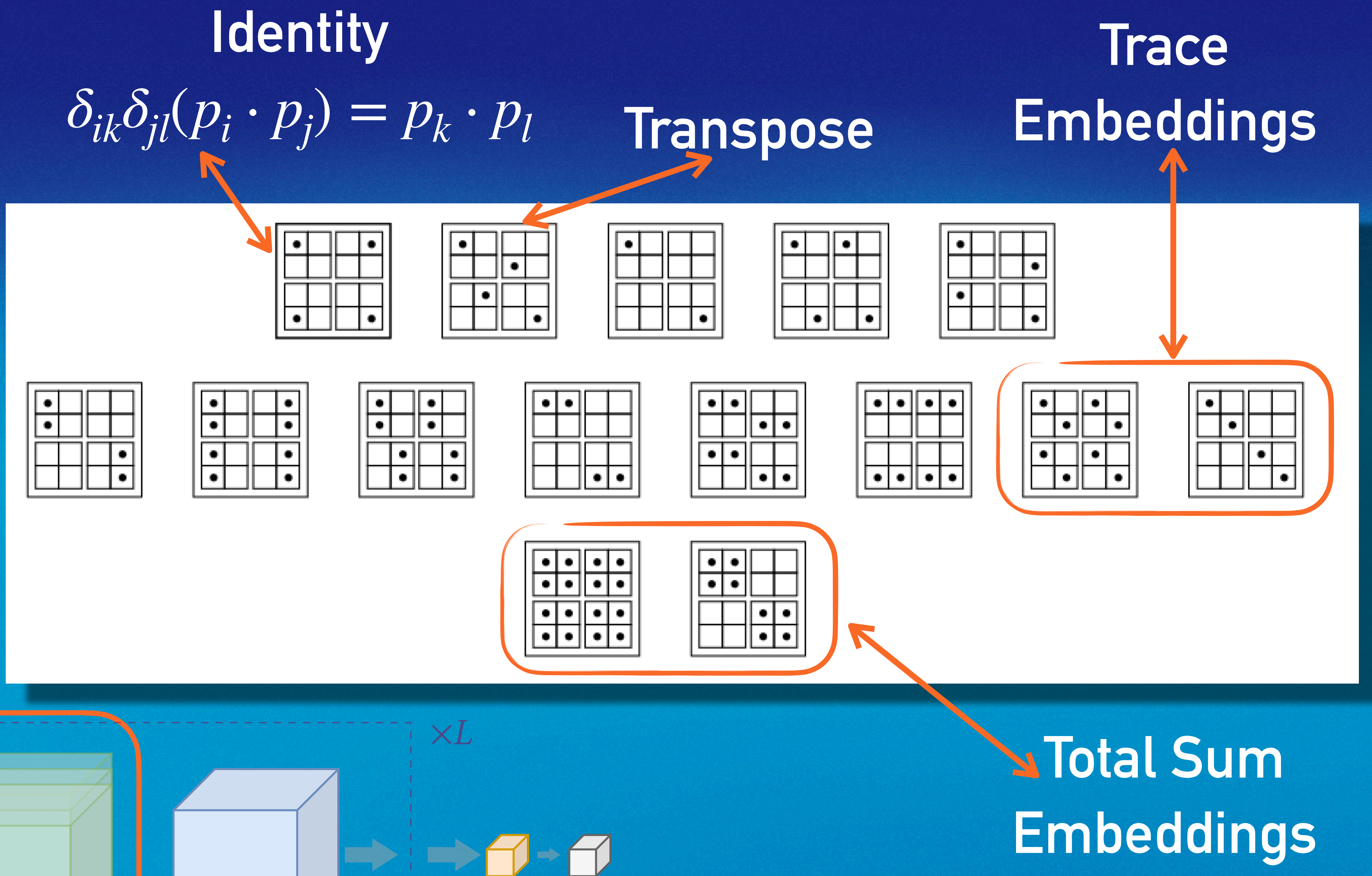
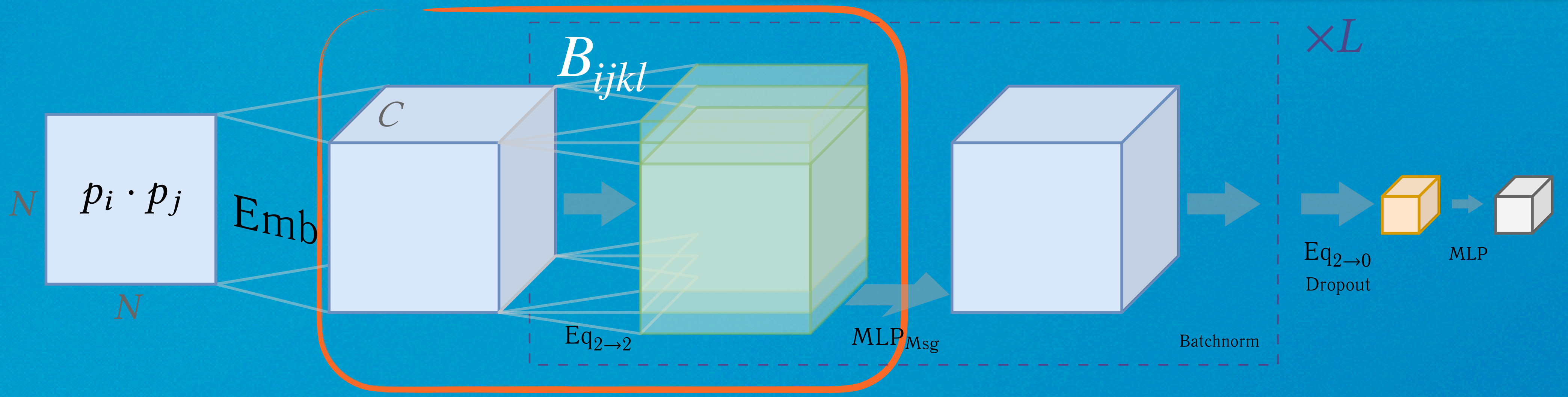


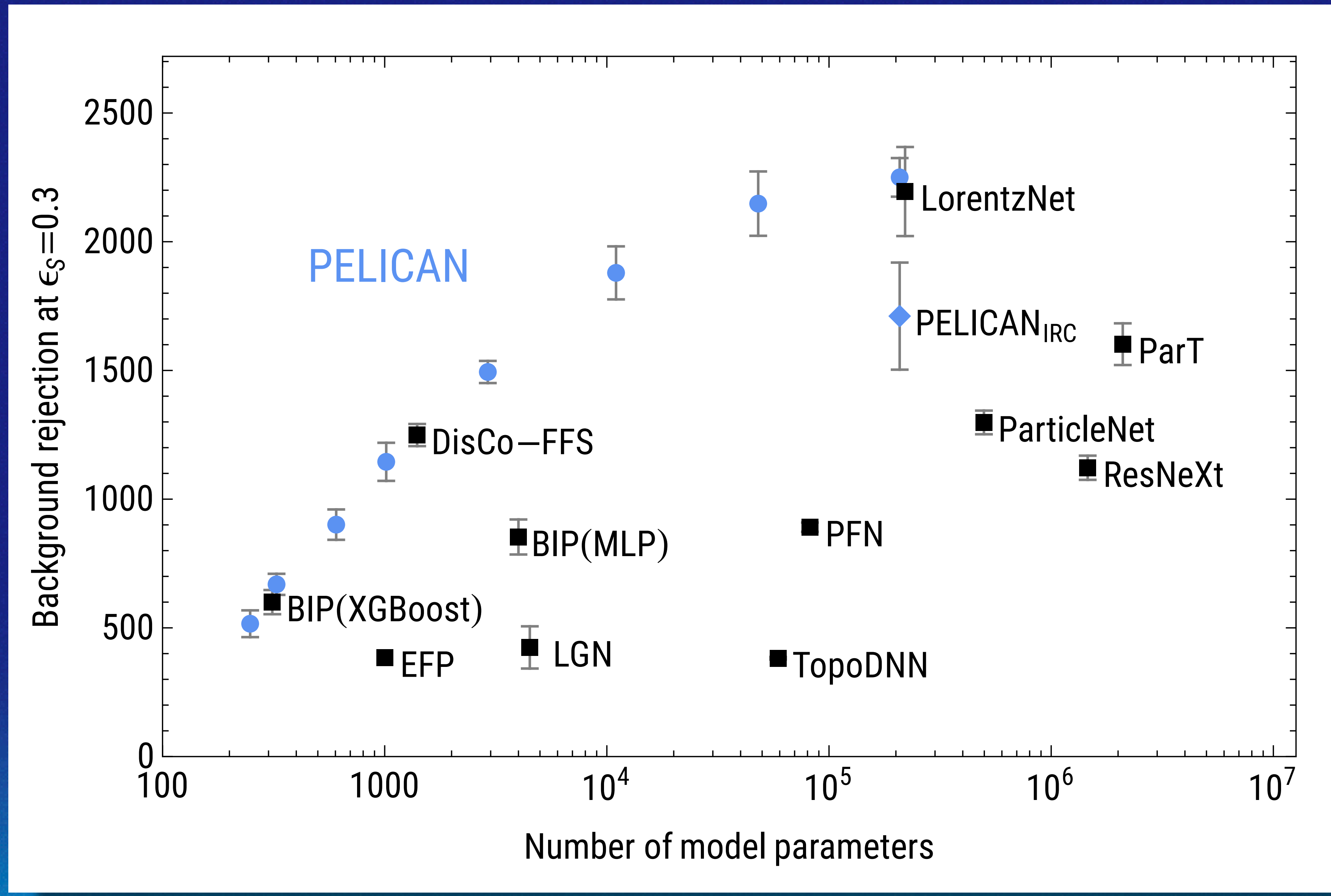




Tagging

- Target Lorentz-scalars: $f(\{p_i\}) = I(\{p_i \cdot p_j\})$
- Hadronic top-jet vs. QCD background jets
- SOTA performance at all orders of parameters!

Toptag dataset



Multiclass jet dataset (q, g, W, Z, t)

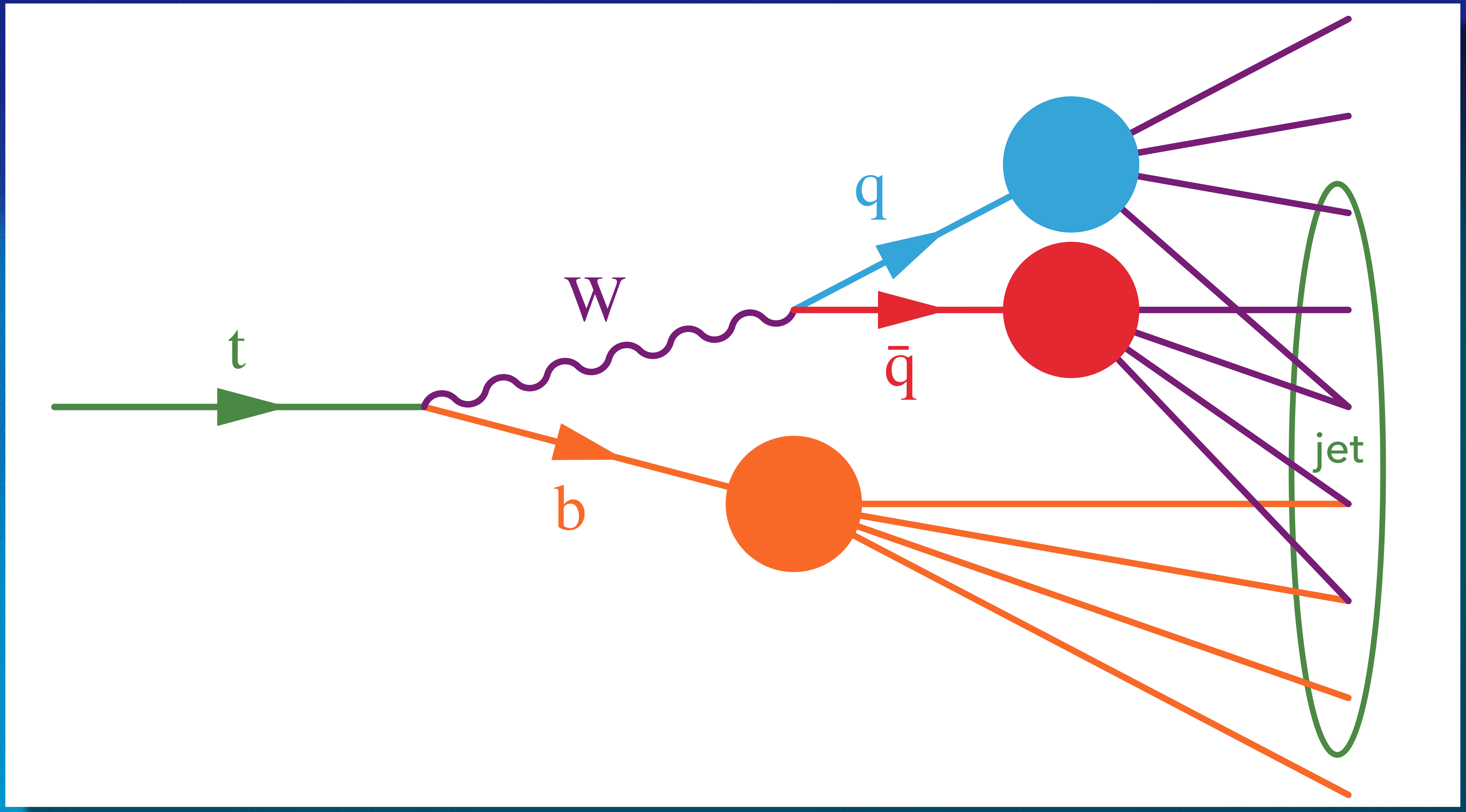
Architecture	Gluon	Light quark	W-boson	Z-boson	Top quark	# Params
			AUC			
JEDI-net	0.9529	0.9301	0.9739	0.9679	0.9683	34k
PCT	0.9623	0.9414	0.9789	0.9814	0.9757	193k
LorentzNet	0.9681(3)	0.9479(4)	0.9837(2)	0.9813(3)	0.9793(3)	224k
PELICAN	**0.9693(1)**	**0.9493(1)**	**0.9840(1)**	**0.9816(1)**	**0.9803(1)**	208k

Quark-gluon dataset

Architecture	Accuracy	AUC	$1/\epsilon_B$ ($\epsilon_S = 0.3$)	$1/\epsilon_B$ ($\epsilon_S = 0.5$)	# Params
Not IRC-safe, w/ PID					
PFN-ID[24]	–	0.9052(7)	–	37.4 ± 0.7	82k
ParticleNet-ID[50]	0.840	0.9116	98.6 ± 1.3	39.8 ± 0.2	498k
ABCNet[26]	0.840	0.9126	118.2 ± 1.5	–	230k
LorentzNet[26]	0.844	0.9156	110.2 ± 1.3	42.4 ± 0.4	220k
ParT$_{full}$ [35]	0.849	0.9203	129.5 ± 0.9	47.9 ± 0.5	2.1M
PELICAN$_{PID}$	0.8555(2)	0.9247(3)	134.8 ± 1.8	51.3 ± 0.7	211k

Four-Vector Reconstruction

- Target Lorentz-vectors: $f^{\mu}(\{p_i\}) = \sum_k I_k(\{p_i \cdot p_j\}) p_k^{\mu}$
- Consider hadronically decaying top-quarks



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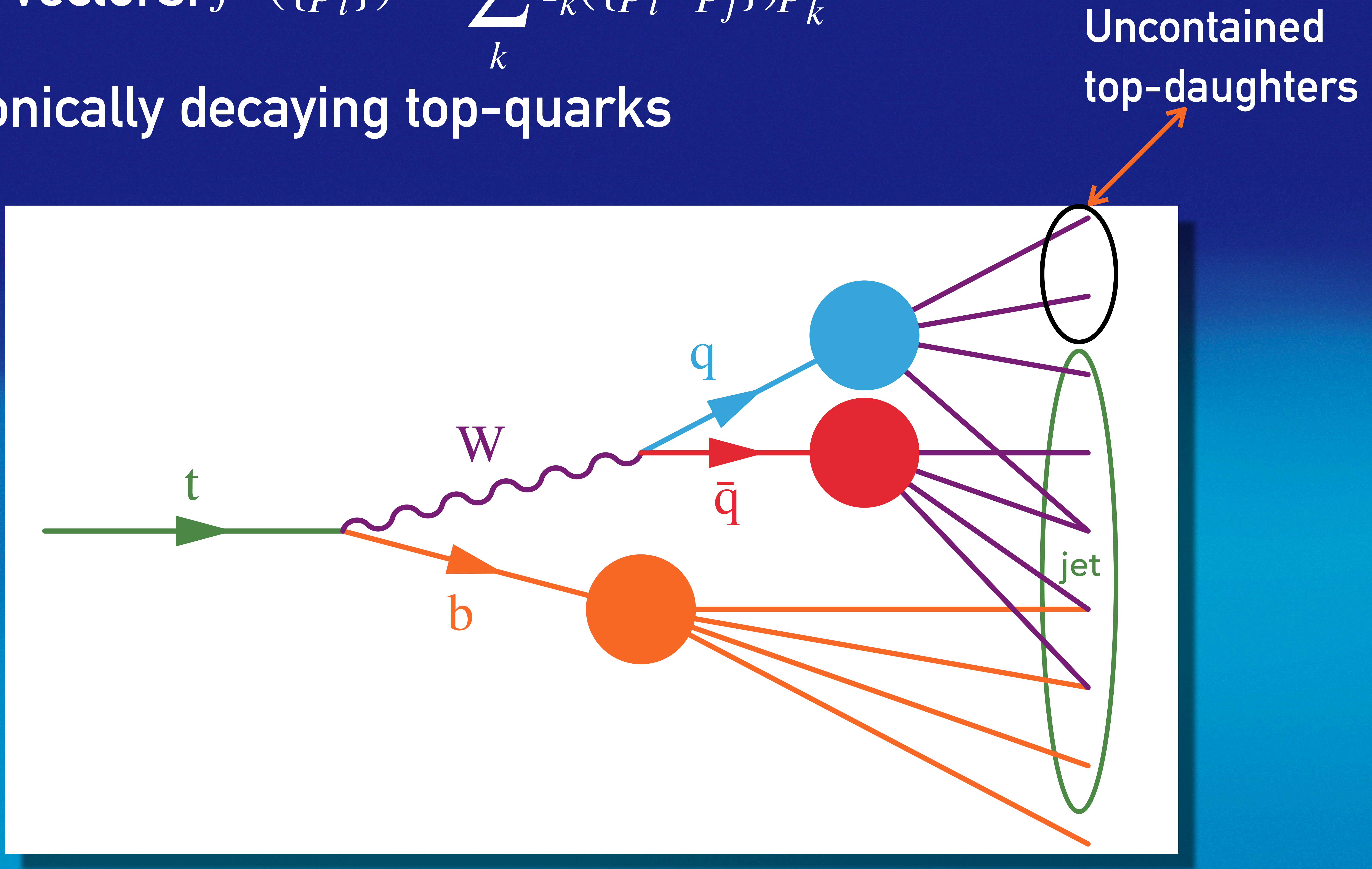
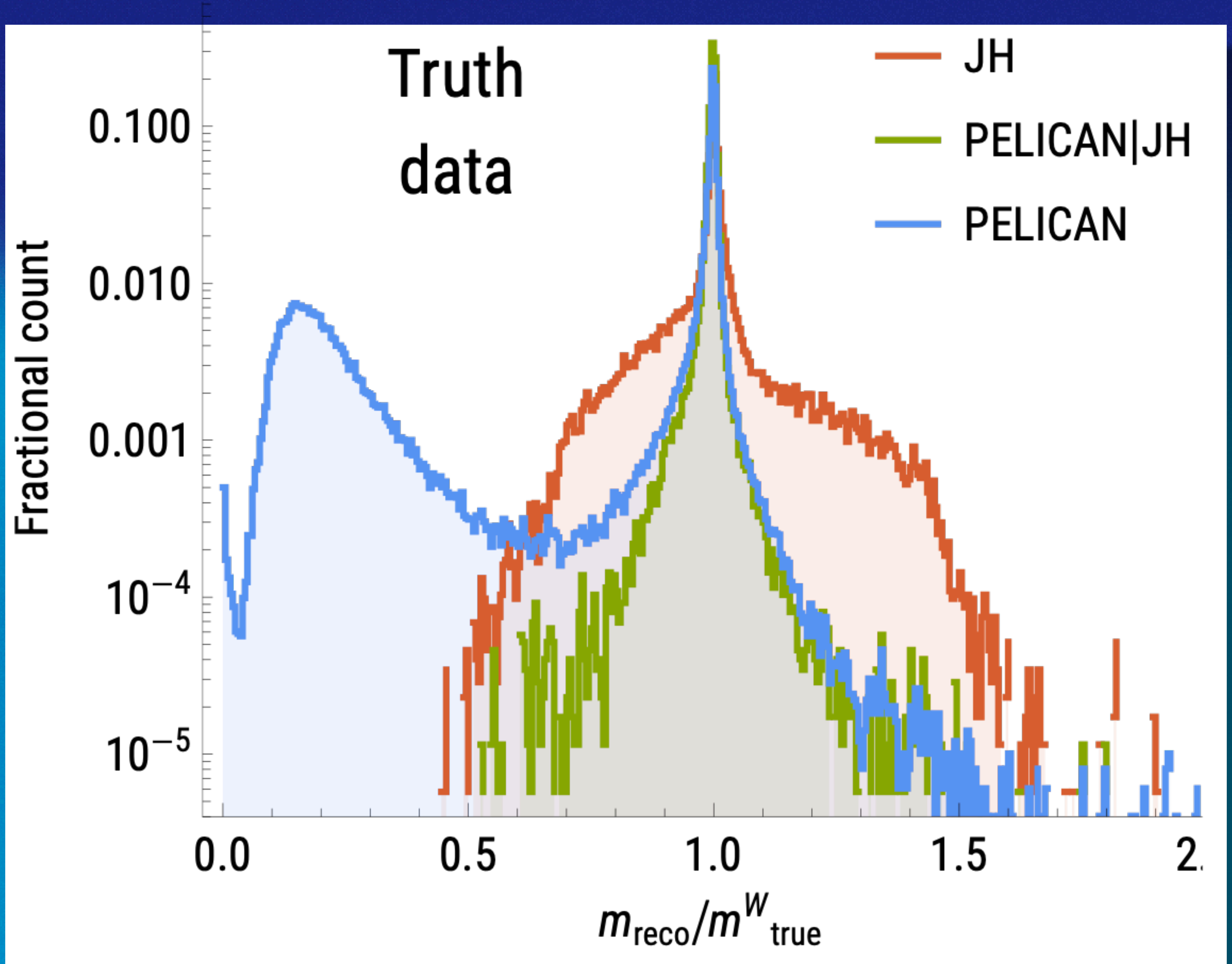
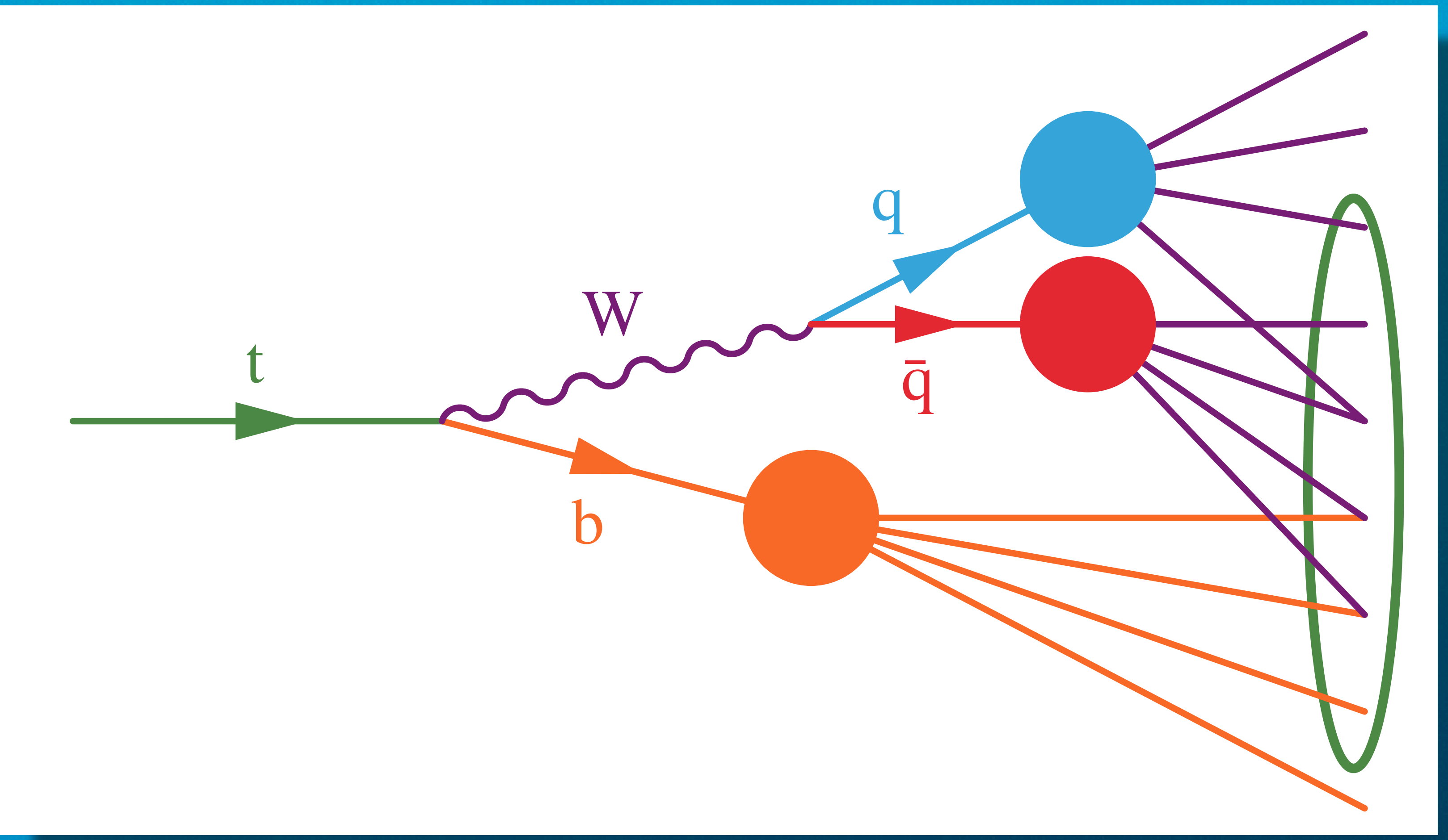


Four-Vector Reconstruction

- Target Lorentz-vectors: $f^{\mu}(\{p_i\}) = \sum_k I_k(\{p_i \cdot p_j\}) p_k^{\mu}$
- Compare to John-Hopkins W-boson reconstruction



	Method	σ_{p_T} (%)	σ_m (%)	$\sigma_{\Delta R}$ (centirad)
DELPHES	JH	9.8 %	8.3 %	9.6
	PELICAN|JH	3.6 %	2.8 %	3.1
	PELICAN	6.2 %	39.6 %	5.6



Reconstruction as a Tagger

- Start with mixed bag of top-quark and QCD jets: classify → vector reconstruction

	True Signal	True Background
Predicted Signal	88.7%	11.2%
Predicted Background	11.3%	88.8%

Reconstruction as a Tagger

- Start with mixed bag of top-quark and QCD jets: classify → vector reconstruction
- Can cut on assumed reconstructed "W-boson" as a tagger!

	True Signal	True Background
Predicted Signal	88.7%	11.2%
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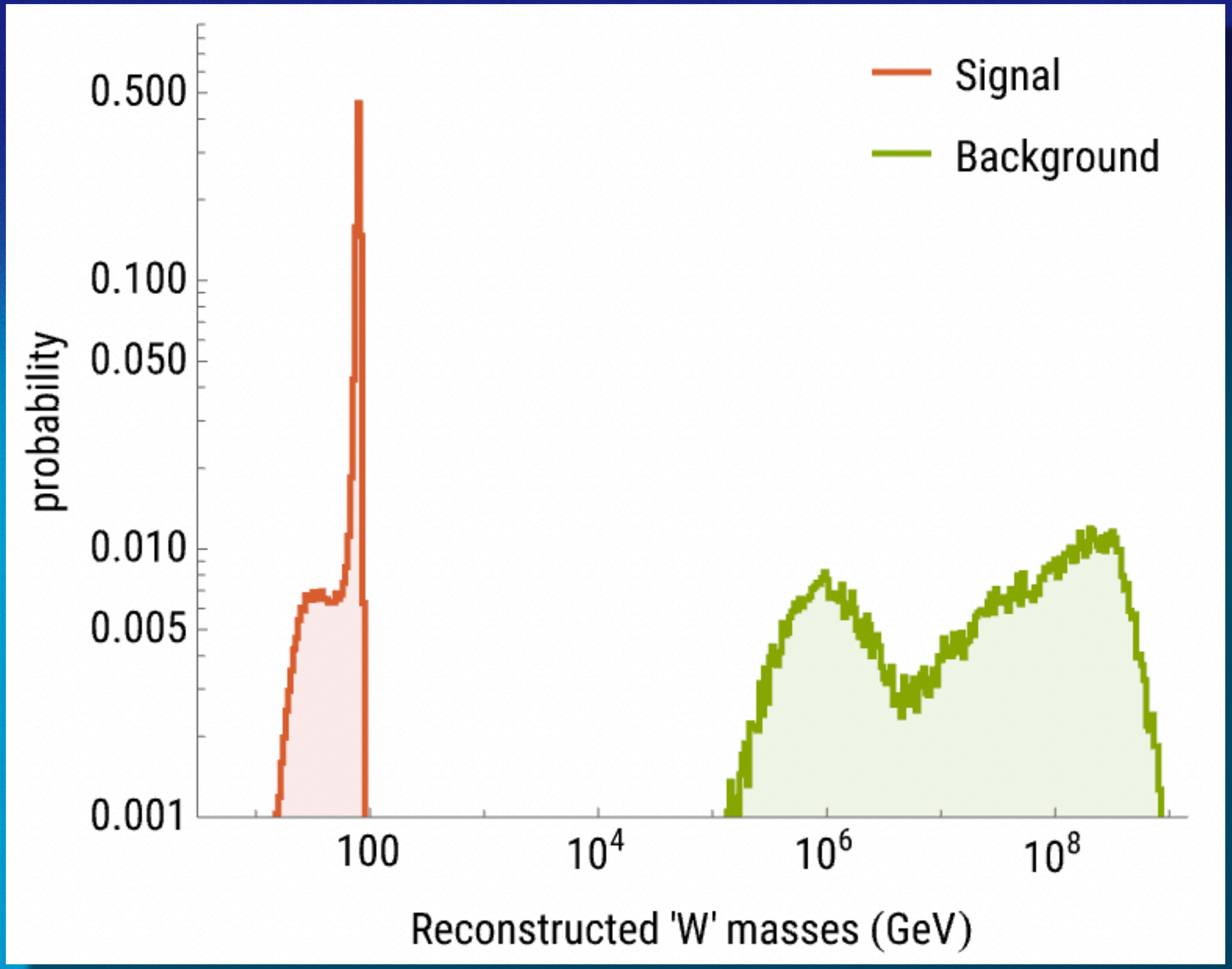
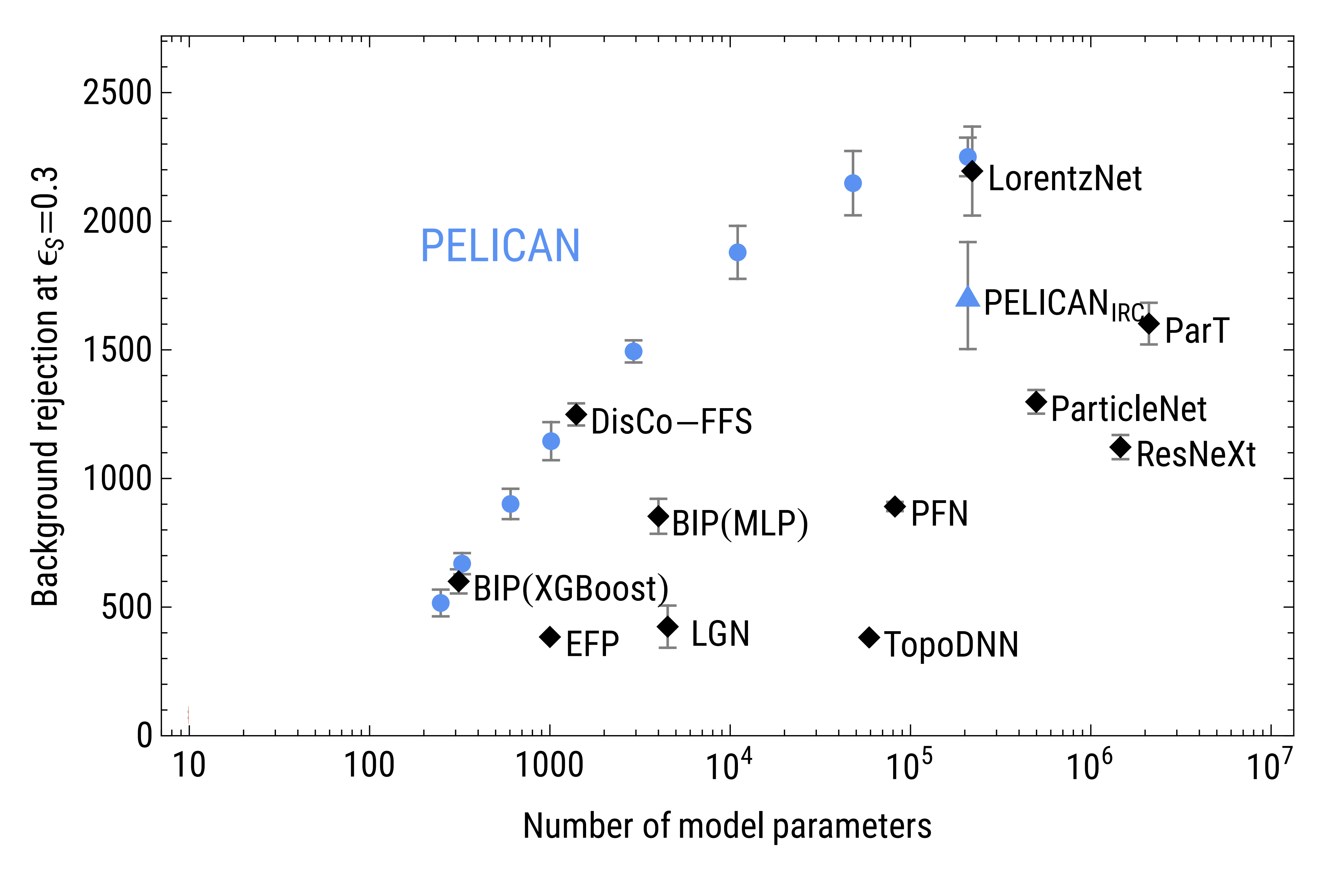


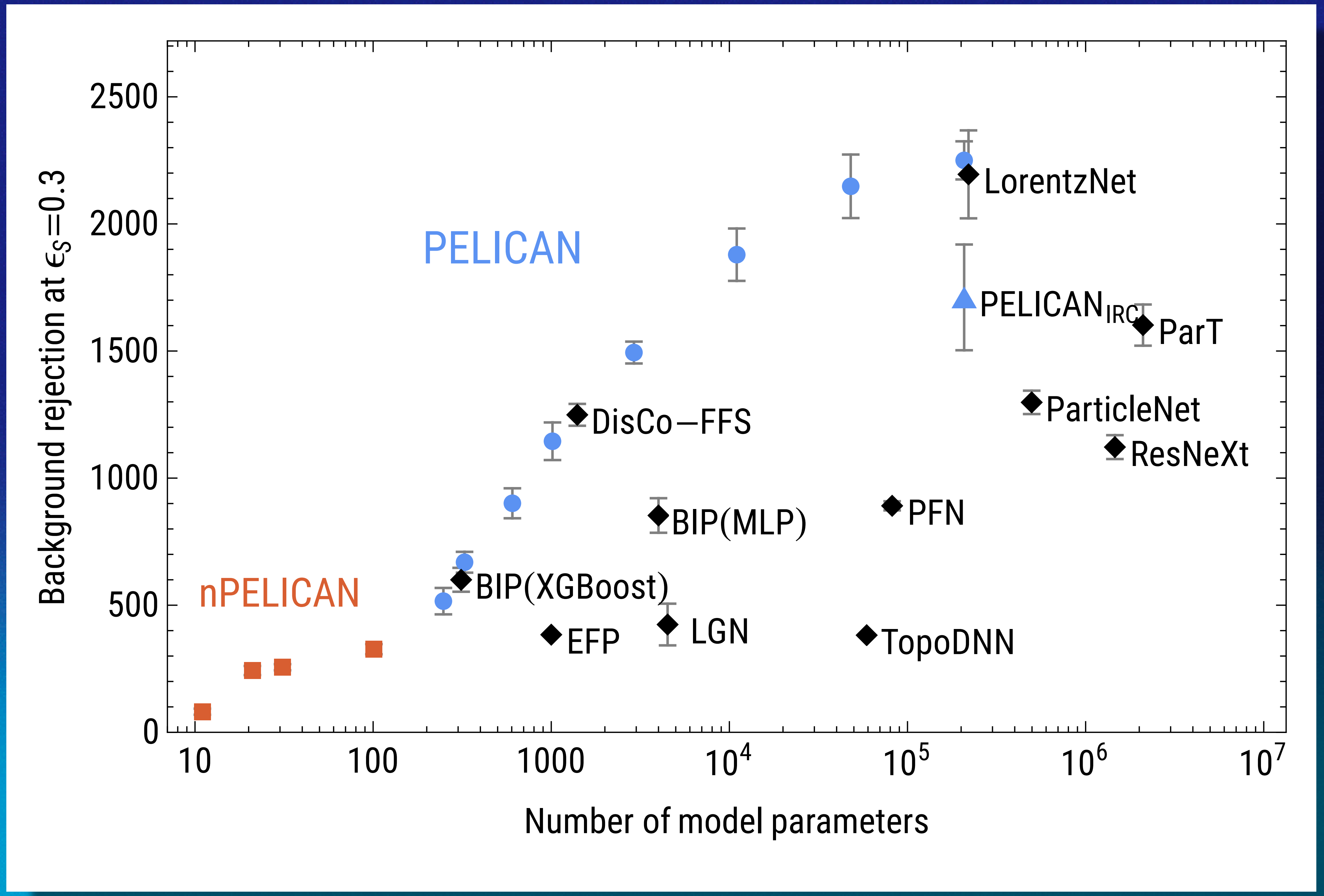
Small Model Limit: nanoPELICAN

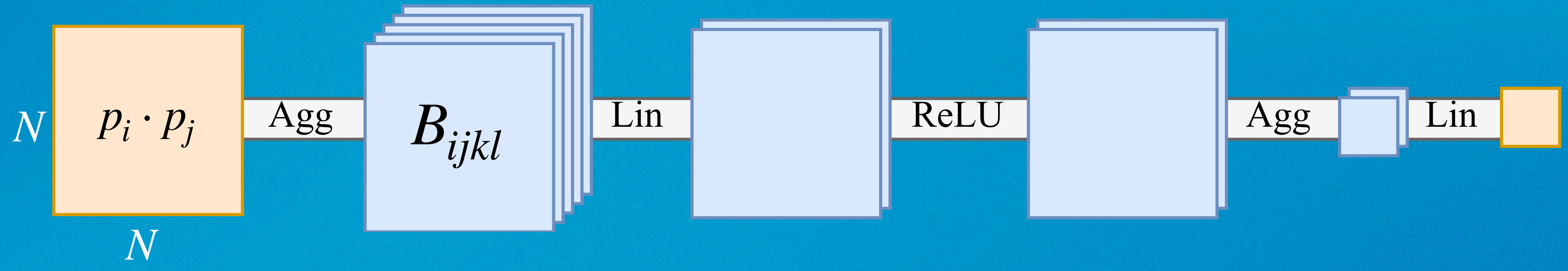
- Interpretability is crucial in particle physics
- Construct smallest viable models



Small Model Limit: nanoPELICAN

- Interpretability is crucial in particle physics
- Construct smallest viable models
- Reduce to a single hidden layer
- $p_i \cdot p_j$ is traceless and symmetric
- Only 6 aggregators remain

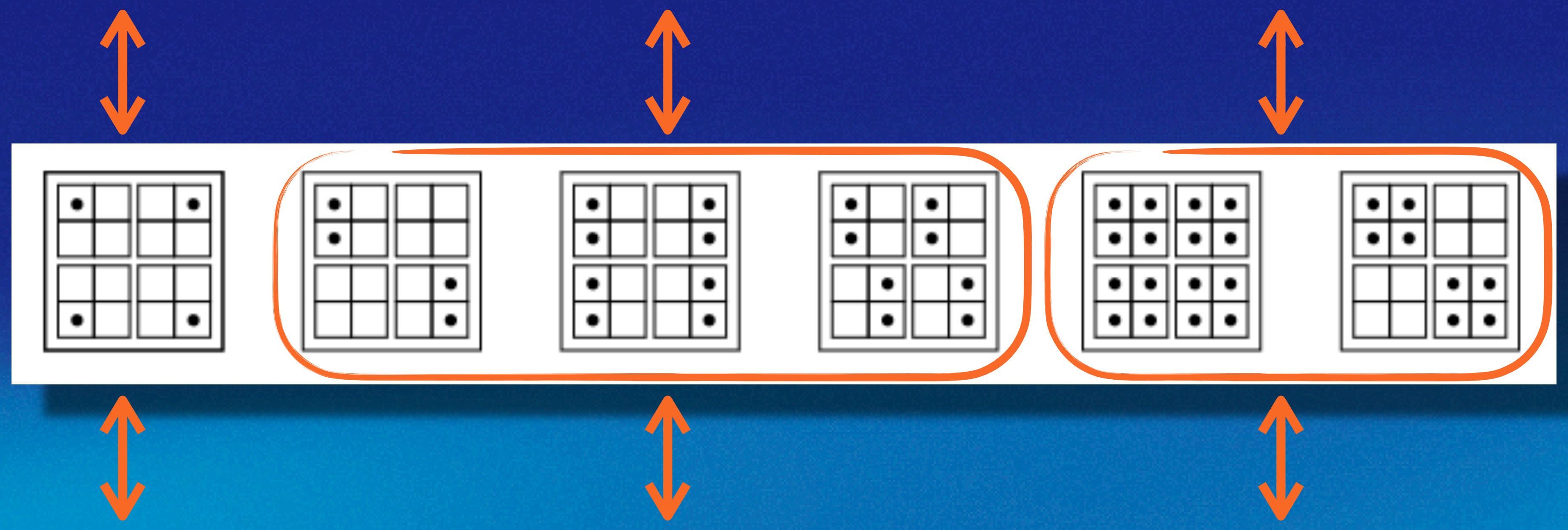




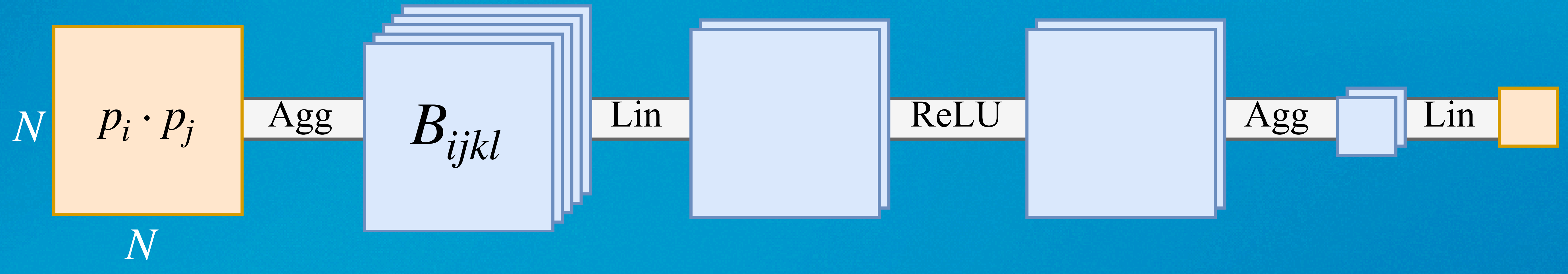
nanoPELICAN

- Only 6 aggregators remain

Identity | Row Aggregation | Total Aggregation



Energy Correlator $\approx E_i E_j \theta_{ij}^2$ | Jet-frame Energy $\sim J \cdot p_j$ | Jet Mass M_J



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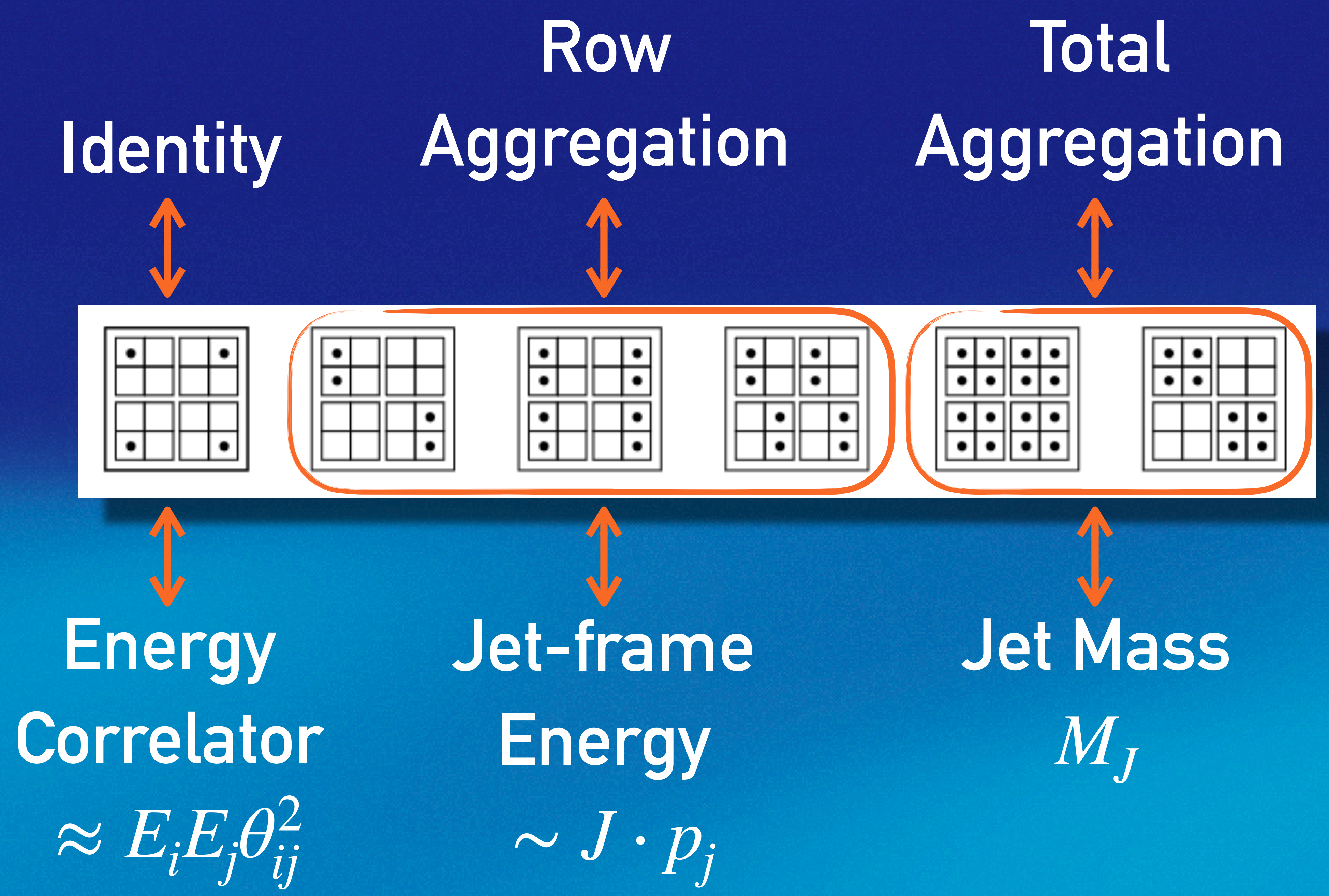
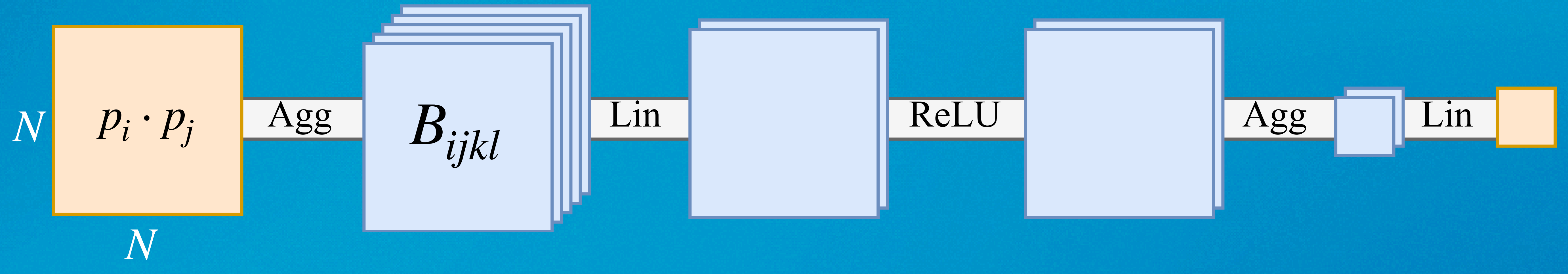
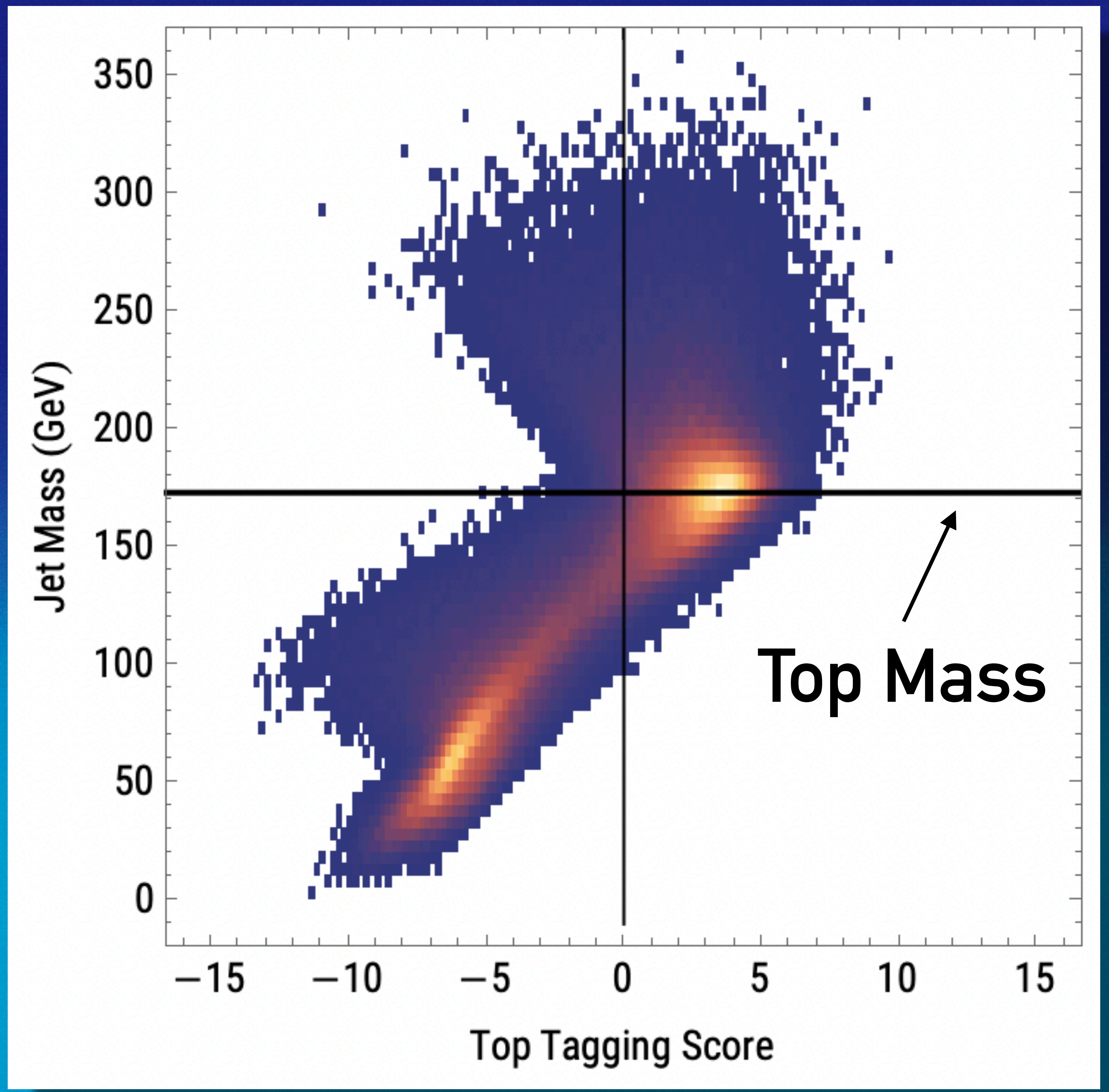




Accuracy: 91.8%

AUC: 97.2%

Parameters: 19



Conclusions

- PELICAN achieves SOTA performance with explainable outputs on Lorentz-scalar and Lorentz-vector targets
- Relatively performant even with O(10) parameters!
    - Gives hope for full PELICAN interpretability
- Many prospects for future applications!
    - Fast tagging and other online analyses
    - Explainable energy calibration
    - Particle helicity measurements
    - High-precision jet-containment measurements for offline analysis
    - Track reconstruction
    - Astrophysics applications

Thanks!

PELICAN paper

NanoPELICAN paper

PELICAN codebase

nanoPELICAN codebase

Data generation codebase

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Backup

More Classification

Multiclass

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PELICAN	**0.9693(1)**	**0.9493(1)**	**0.9840(1)**	**0.9816(1)**	**0.9803(1)**	208k
			TPR at FPR=0.10			
JEDI-net	0.878(1)	0.822(1)	0.938(1)	0.910(1)	0.930(1)	34k
PCT	0.891(1)	0.833(1)	0.932(1)	**0.946(1)**	0.941(1)	193k
LorentzNet	0.912(1)	0.855(1)	0.952(1)	0.939(1)	0.949(1)	224k
PELICAN	**0.916(1)**	**0.860(1)**	**0.953(1)**	0.940(1)	**0.951(1)**	208k
			TPR at FPR=0.01			
JEDI-net	0.485(1)	0.302(1)	0.704(1)	0.769(1)	0.633(1)	34k
PCT	0.513(2)	0.298(2)	**0.834(1)**	0.781(1)	0.700(3)	193k
LorentzNet	0.557(4)	0.319(2)	0.800(3)	0.850(3)	0.753(3)	224k
PELICAN	**0.567(1)**	**0.320(1)**	0.804(1)	**0.850(1)**	**0.761(1)**	208k

Quark-gluon

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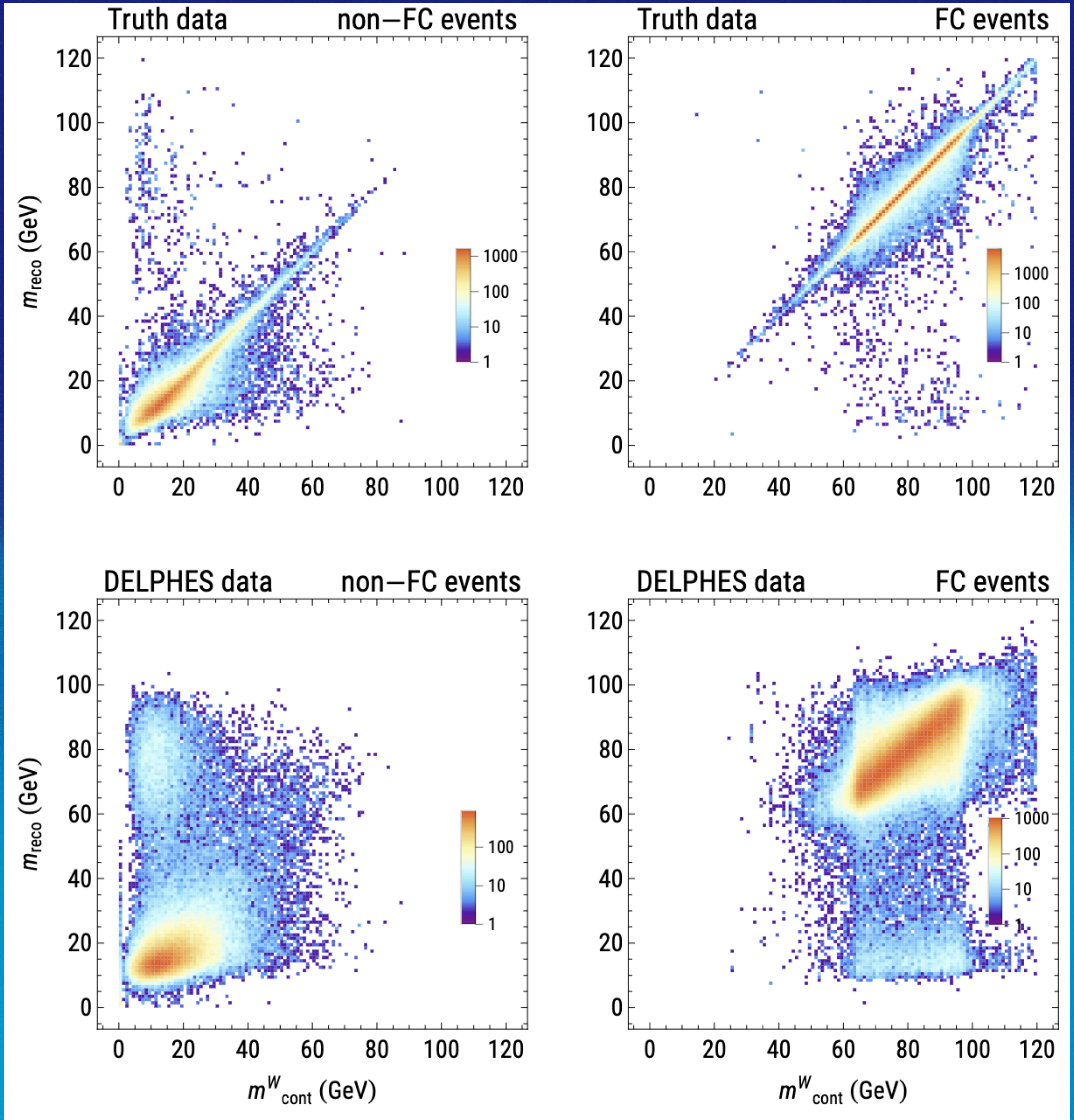
ParticleNet[50]	0.828	0.9014	85.4	33.7	498k
PELICAN	0.8342(2)	0.9059(8)	88.9 ± 0.5	36.0 ± 0.2	209k
IRC-safe					
EFN[24]	–	0.8824(5)	–	28.6 ± 0.3	82k
EFP[18]	–	0.8919	–	29.7	1k
EMPN[55]	–	0.8932(6)	–	30.8 ± 0.2	~110k
$\mathrm{PELICAN_{IRC}}$	0.8299(3)	0.8955(18)	85.7 ± 1.2	33.8 ± 0.2	209k

Vector Reconstruction



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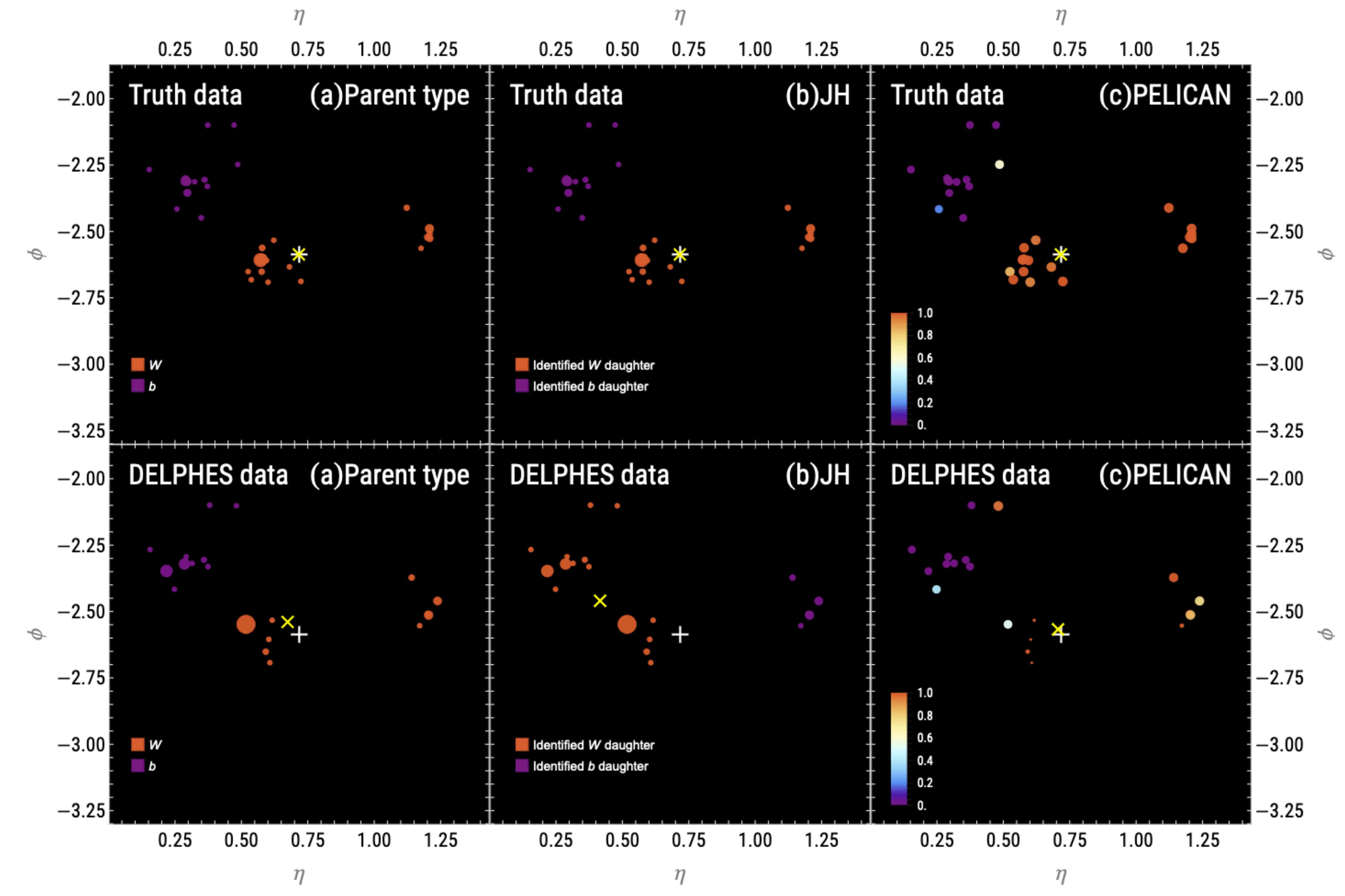

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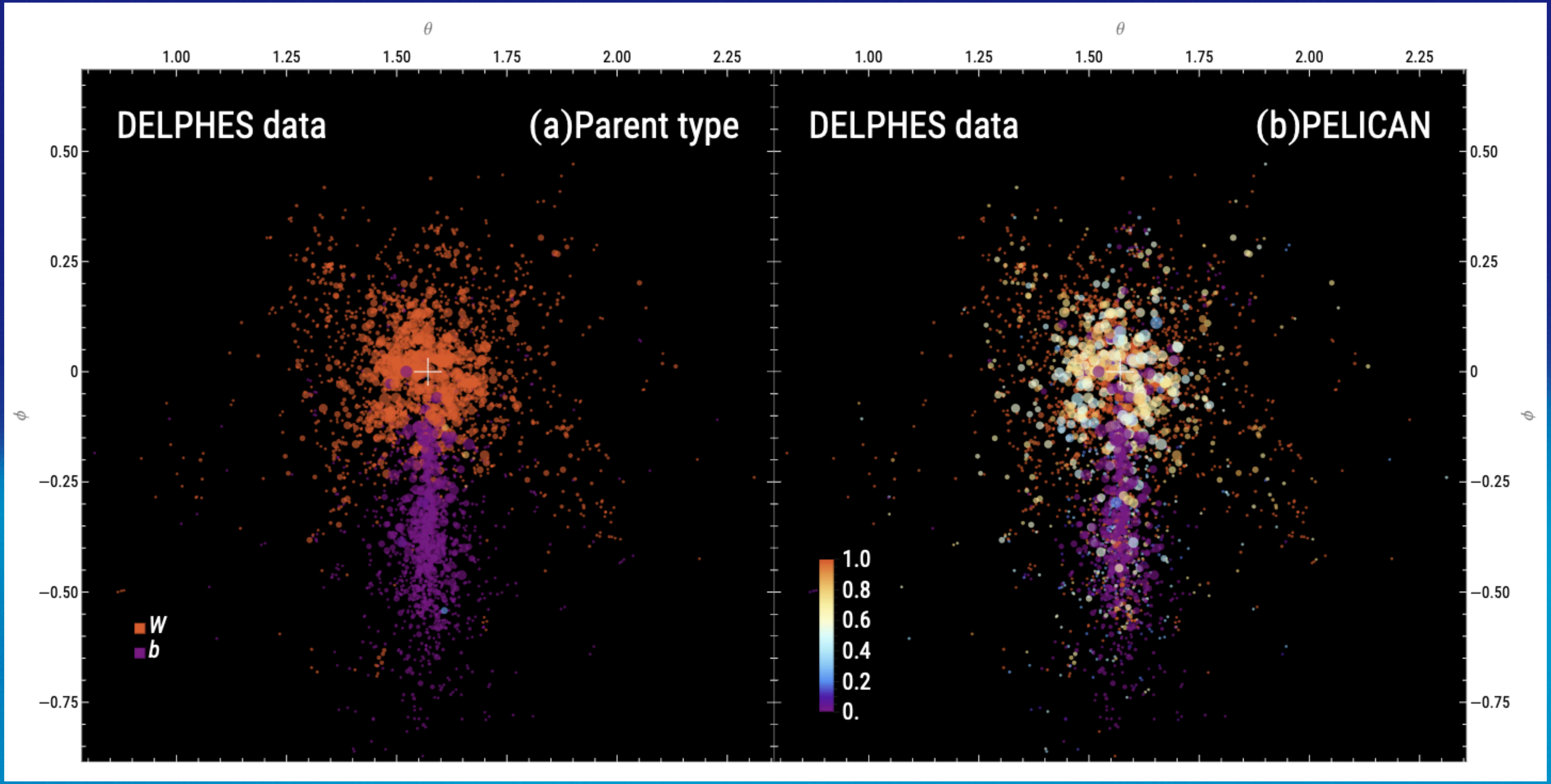
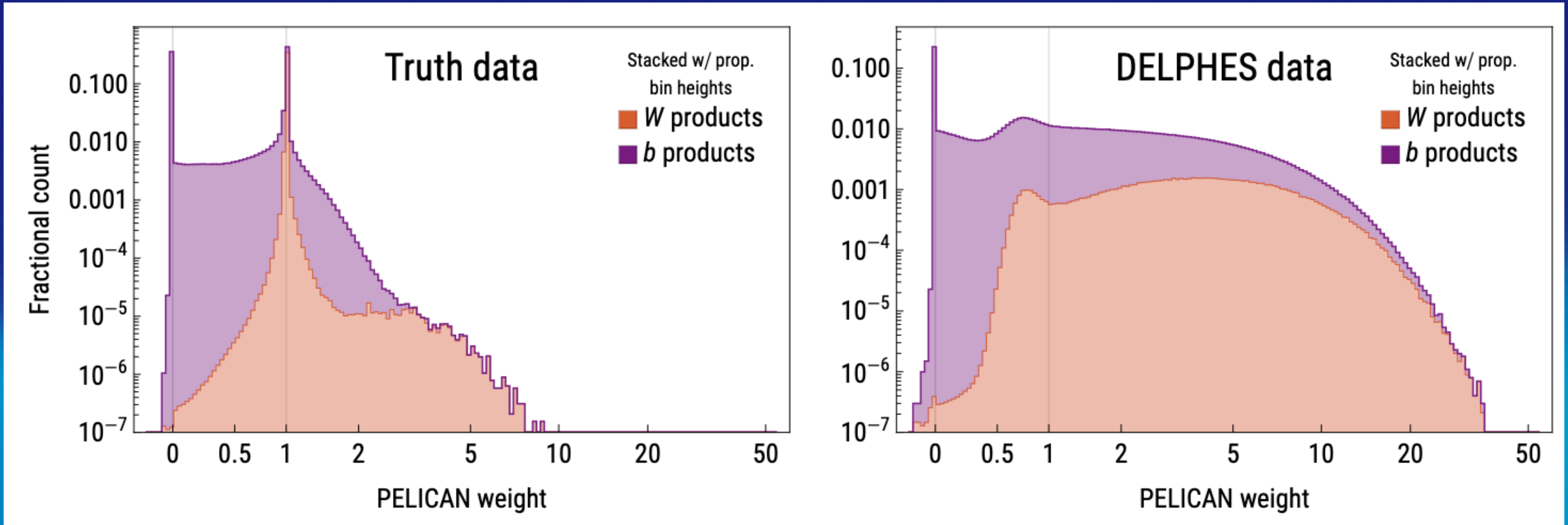


Vector Reconstruction Weights



Dataset Details