Measuring Local Dark Matter using Normalizing Flows and Gaia DR3

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with S.H. Lim, M. Buckley, D. Shih

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THE STATE UNIVERSITY OF NEW JERSEY

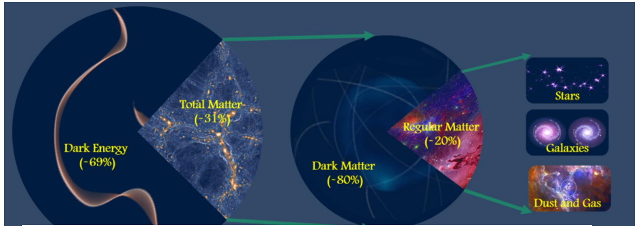
DESY.

ML4Jets 2023

arXiv:2305.13358 and arXiv:2205.01129

Introduction

Dark matter (DM) makes up 85% of matter, primordial blobs or “halos” of DM seeded formation of all baryonic structure.



Mohamed Abdullah, University of California Riverside

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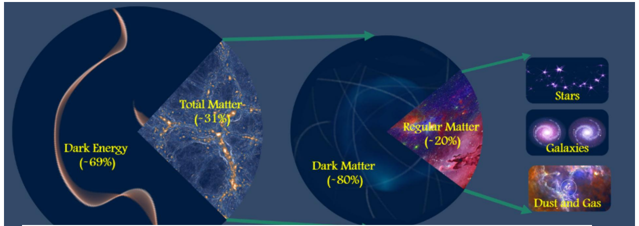
The MW is a laboratory for DM physics!

- Local density sets direct detection rate
- Density profile probes DM particle physics. Does it self-interact?



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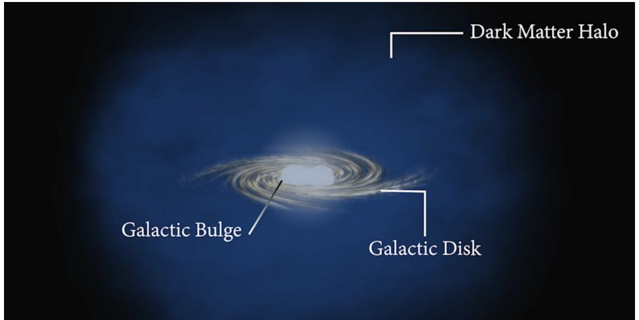

Credit: L. Jaramillo & O. Macias/Virginia Tech.

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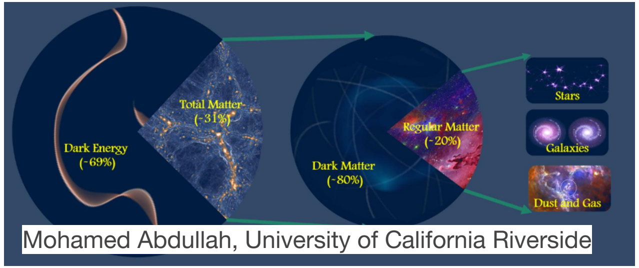

Mapping Dark Matter in the Milky Way using Normalizing Flows and Gaia DR3

Sung Hak Lim, Eric Putney, Matthew R. Buckley, David Shih

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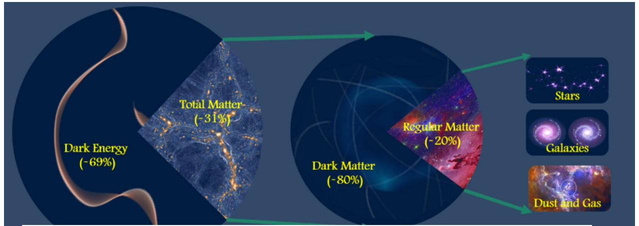
How can we measure our DM halo? Stars!

- Rotation curves trace enclosed mass
- Potential encoded in phase space f(**x**,**v**)



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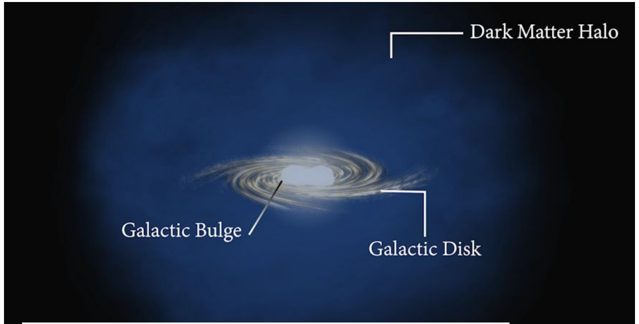

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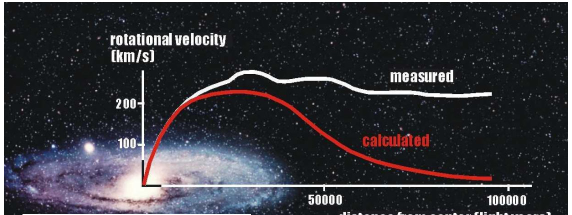


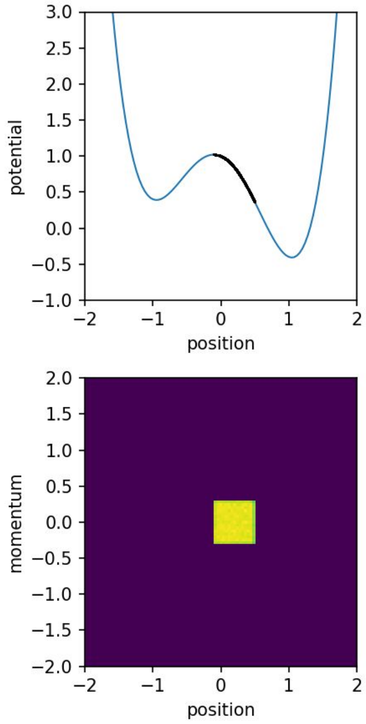
Credit: SUPERCDMS

The Collisionless Boltzmann Equation

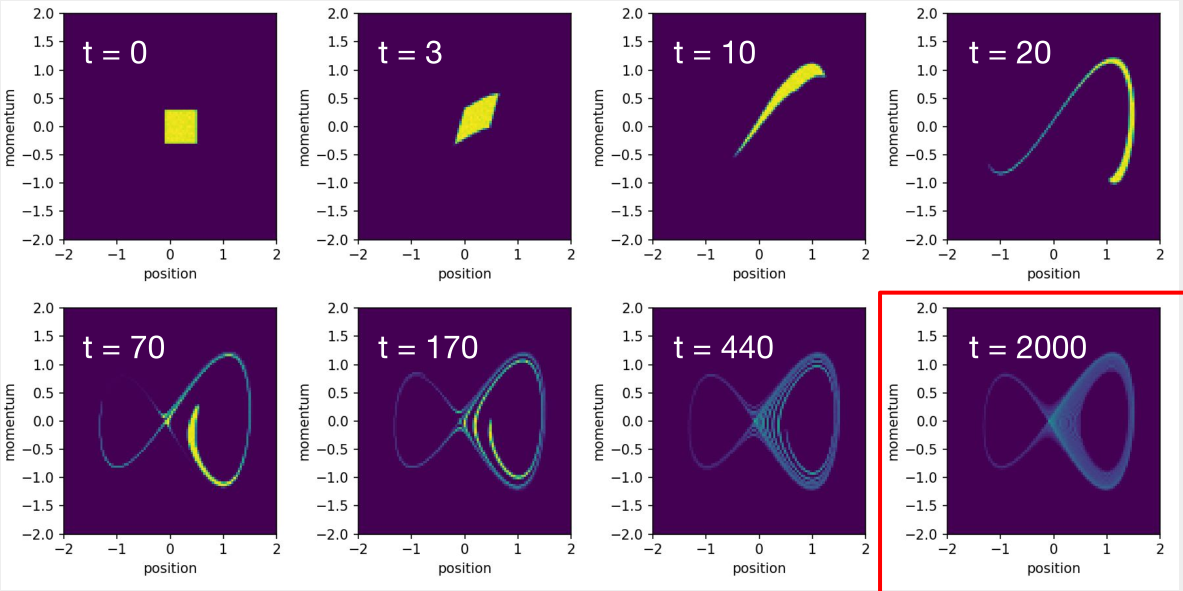
Baryons + DM source the galactic potential Φ(**x**). Gravitational tracers (stars) drawn from f(**x**,**v**,t) accelerate in response to Φ.

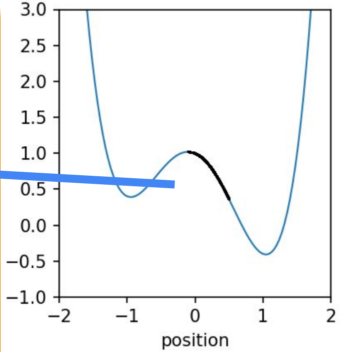
$$\frac{df}{dt} = \left[\frac{\partial}{\partial t} + v_i \frac{\partial}{\partial x_i} + a_i \frac{\partial}{\partial v_i}\right] f = 0 \qquad \text{where} \quad a_i = -\frac{\partial \Phi}{\partial x_i}$$

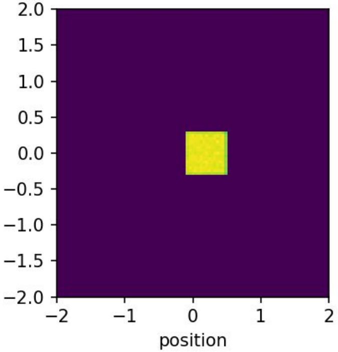
Over many dynamic timescales, f(**x**,**v**,t) equilibrates → f(**x**,**v**).



Example: 2D phase space in toy potential







Dynamic equilibrium: f(**x**,**v**) for old stars in our galaxy should be smooth and “phase mixed”

$$\frac{df}{dt} = 0, \quad \frac{\partial f}{\partial t} = 0$$

The *equilibrium* Collisionless Boltzmann Equation

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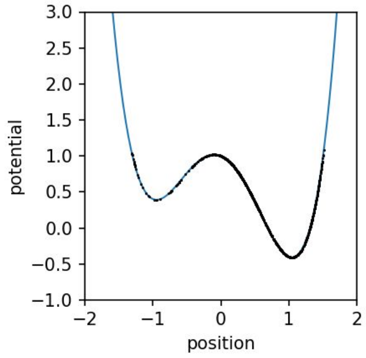

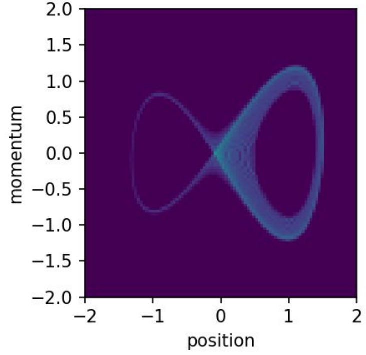

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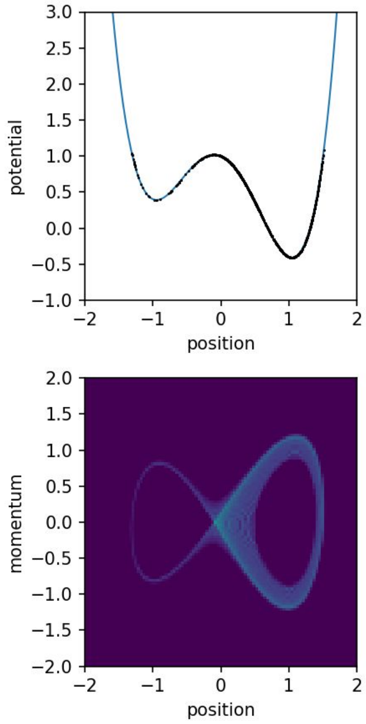
Over many dynamic timescales, f(**x**,**v**,t) equilibrates → f(**x**,**v**). **We can use a snapshot of f(x,v) today to infer the acceleration field a(x).**

Motivates the equilibrium collisionless Boltzmann equation (CBE):

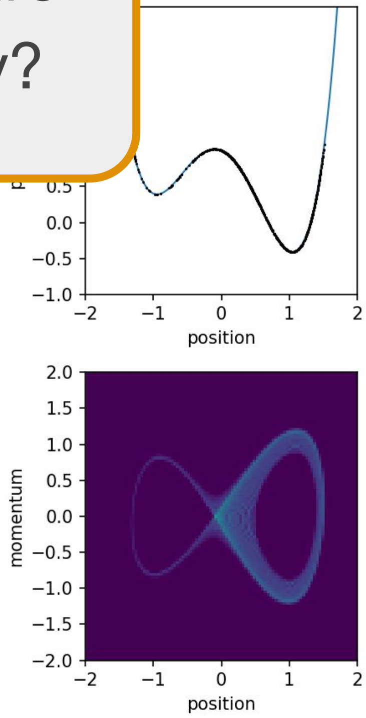
$$\boxed{-\frac{\partial f}{\partial t} = v_i \frac{\partial f}{\partial x_i} + a_i \frac{\partial f}{\partial v_i} = 0}$$

6D f(**x**,**v**) is difficult to measure (even if it's "low-dim" by our standards). Instead, astronomers solve the Jeans equations or model f(**x**,**v**).



T *equilibrium* Collisionless Boltzmann Equation

Are these approaches accurate? Can we measure the 6-dim f(**x**,**v**) in a completely data-driven way?



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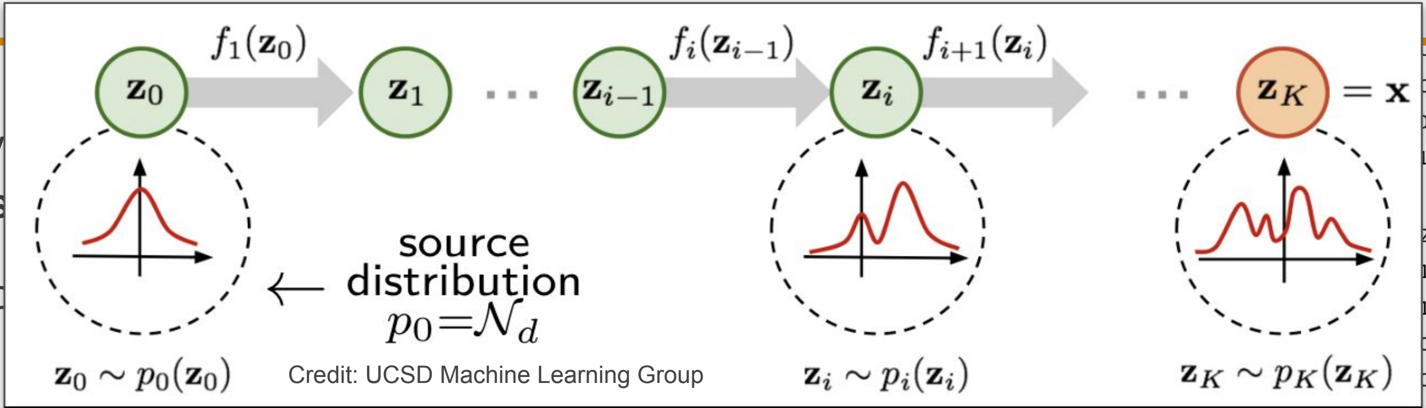
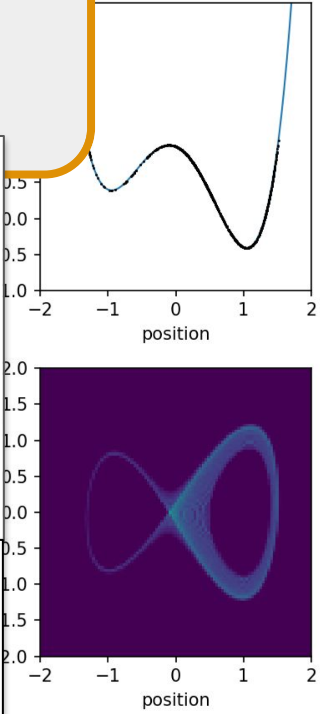


Credit: UCSD Machine Learning Group

Normalizing flows are flexible high-dimensional density estimators that can learn f(**x**,**v**) directly from the data.

6D…

Ins… astronomers solve the Jeans equations or model f(**x**,**v**).



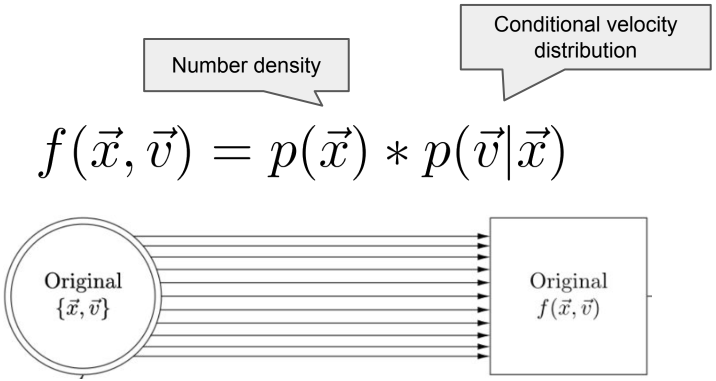
Pipeline: $\{x, v\} \rightarrow \rho(x)$

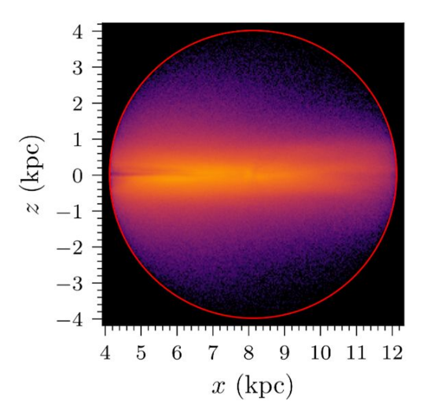
Pipeline: Training f(x,v)

Given {**x**, **v**} of a tracer population:

- Train two Masked Autoregressive Flows (MAFs)* to learn p(**x**) and p(**v**|**x**)

$$f(\vec{x},\vec{v}) = p(\vec{x}) * p(\vec{v}|\vec{x})$$





Side-on view of stars near Earth

Brighter > more stars

Lim, **EP**, Buckley, Shih (2023)

*Sequence of 20 MADE blocks w/ affine transformations and permutation layers. GELU activations. Learning rates 10^{-3} and 10^{-4} (course/fine) with patience. 80-20 training-validation split, shuffled for each training.

Pipeline: Training f(x,v)

Given {**x**, **v**} of a tracer population:

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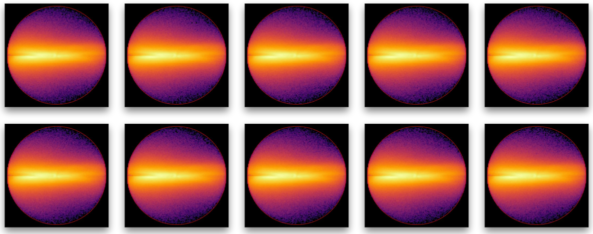
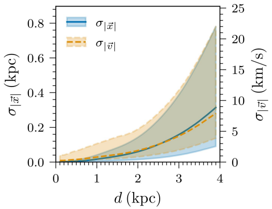
- Given {σ_x, σ_v}, draw 10 realizations of the data smeared by kinematic errors

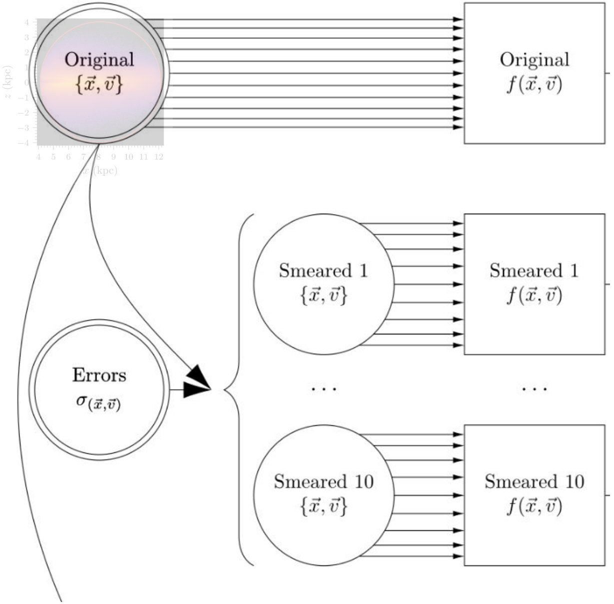




Number density

Conditional velocity distribution

$$f(\vec{x}, \vec{v}) = p(\vec{x}) * p(\vec{v}|\vec{x})$$



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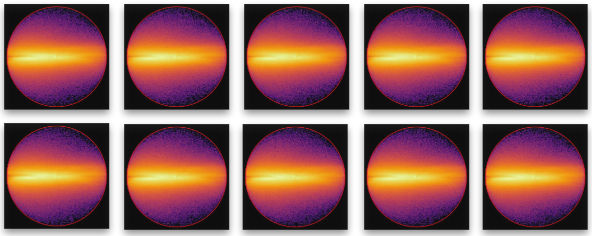
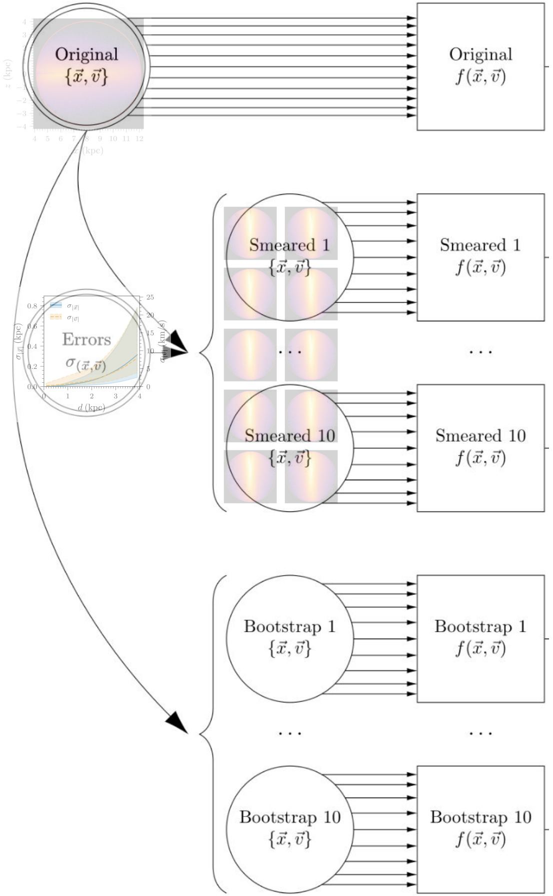
- Draw 10 realizations of the data via the non-parametric bootstrap



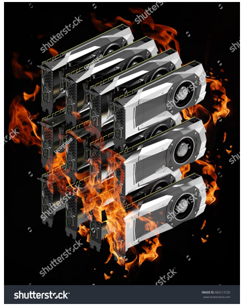


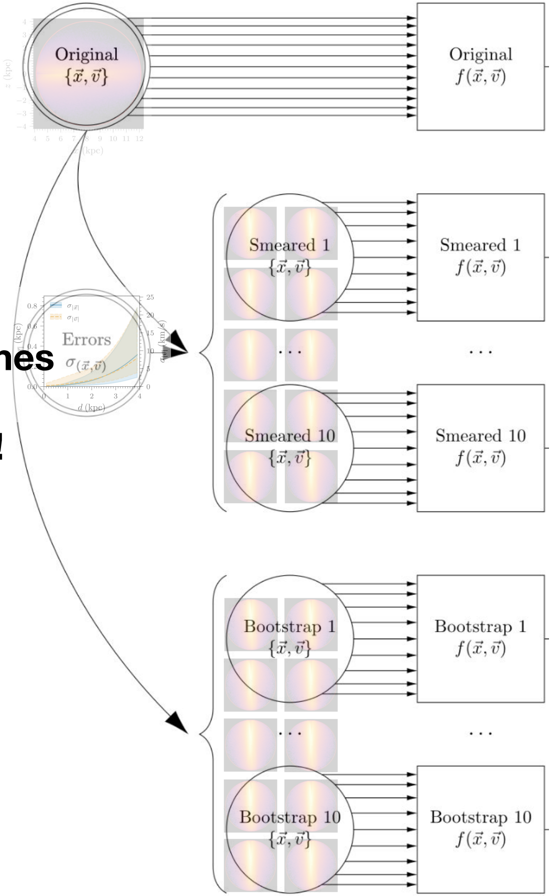
Pipeline: Training f(x,v)

Given {**x**, **v**} of a tracer population:

- Train two Masked Autoregressive Flows (MAFs)* to learn p(**x**) and p(**v**|**x**)
- Given {σ_x, σ_v}, draw 10 realizations of the data smeared by kinematic errors
- Draw 10 realizations of the data via the non-parametric bootstrap
- Uncertainties of f(**x**,**v**), and derived quantities should be robustly estimated!

Retrain many times with different initializations!



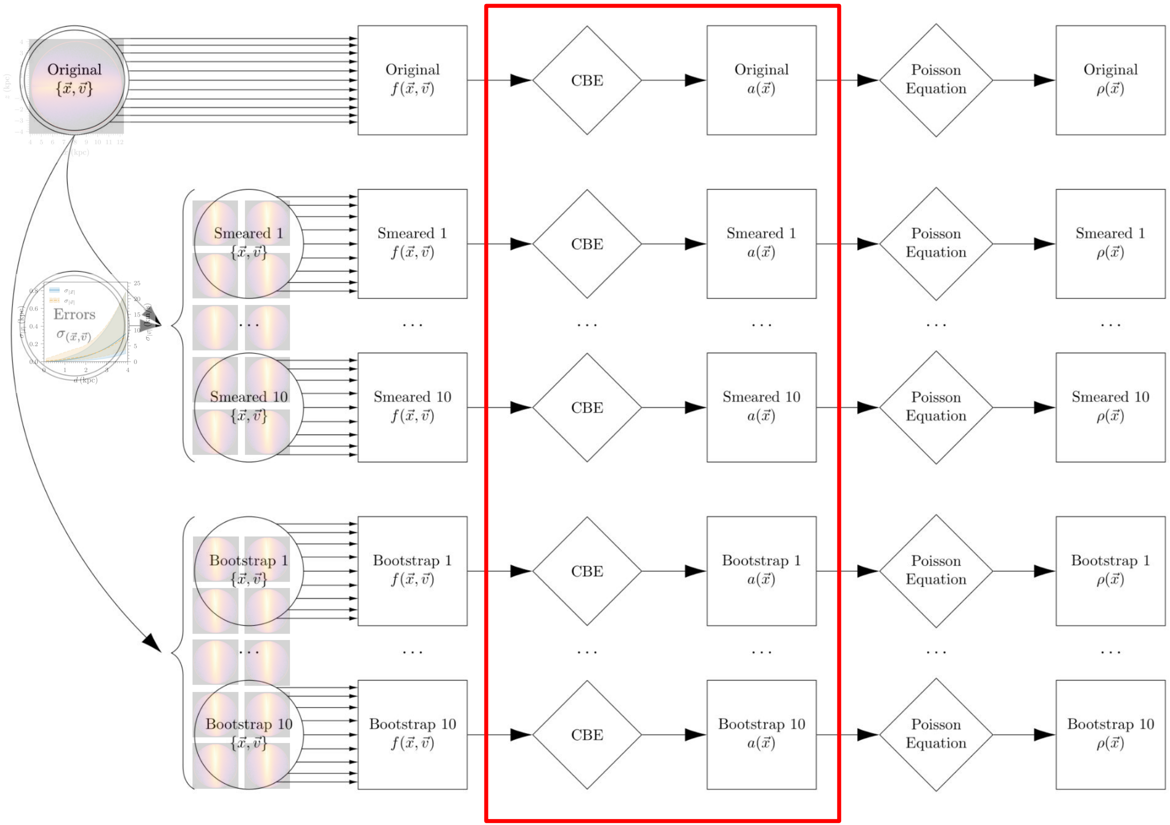


Now, let's estimate acceleration and mass density!

Pipeline: Estimating acceleration and mass density

Find **a**(**x**) such that |CBE|^2 is minimized.

$$\min(\sum_{\beta=1}^{N}\left|\vec{v}_\beta \cdot \frac{\partial f}{\partial \vec{x}} + \vec{a} \cdot \frac{\partial f}{\partial \vec{v}_\beta}\right|^2 ; f, \vec{a})$$



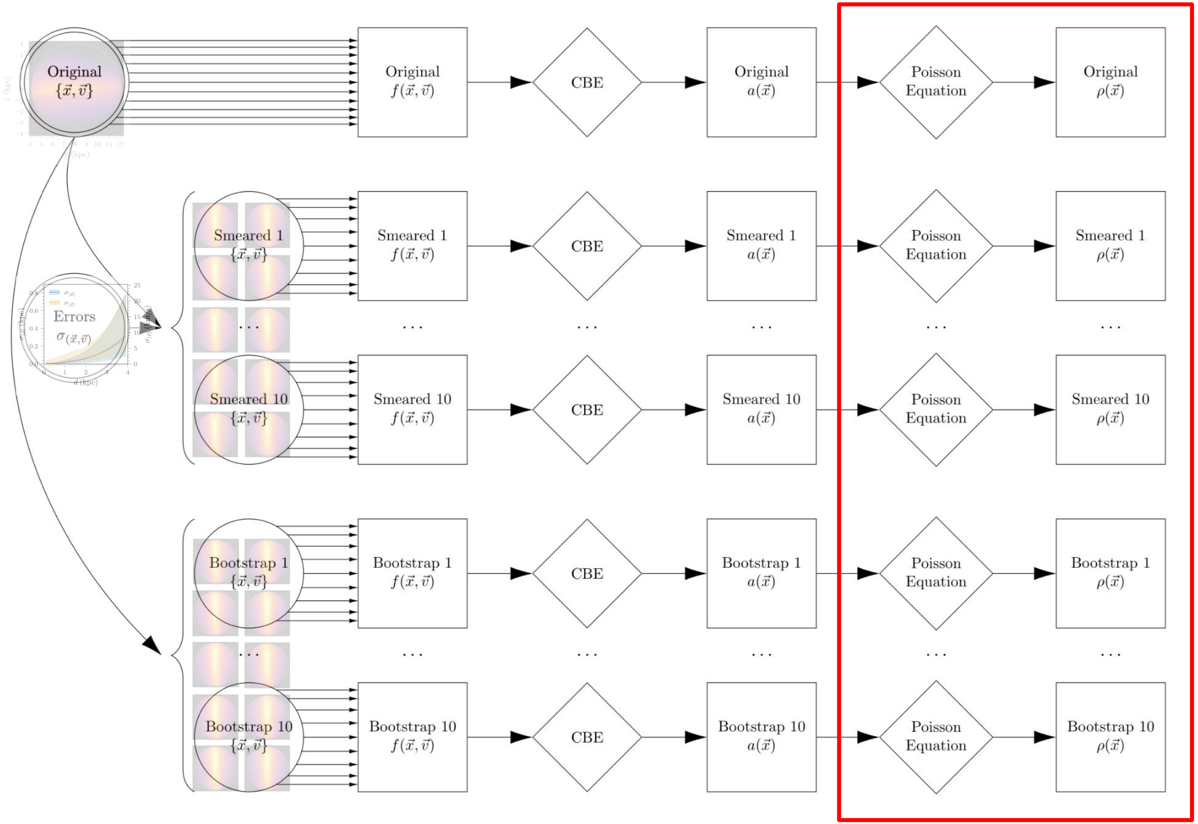
Pipeline: Estimating acceleration and mass density

Find **a**(**x**) such that $|\text{CBE}|^2$ is minimized.

$$\min\left(\sum_{\beta=1}^{N}\left|\vec{v}_\beta \cdot \frac{\partial f}{\partial \vec{x}} + \vec{a} \cdot \frac{\partial f}{\partial \vec{v}_\beta}\right|^2 ; f, \vec{a}\right)$$

Compute kernel-smoothed divergence of **a**(**x**)

$$\rho = -\frac{\nabla \cdot \vec{a}}{4\pi G}$$



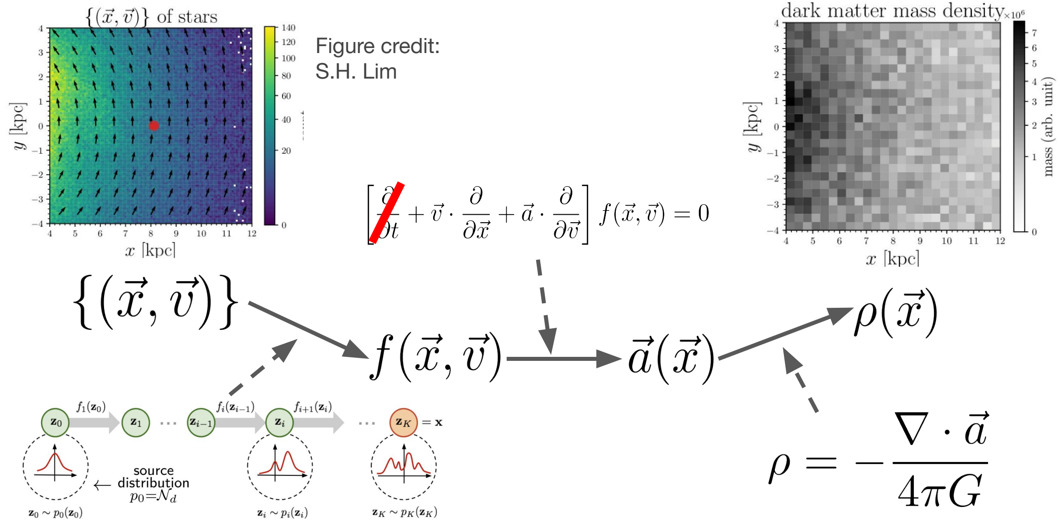
Application to Data

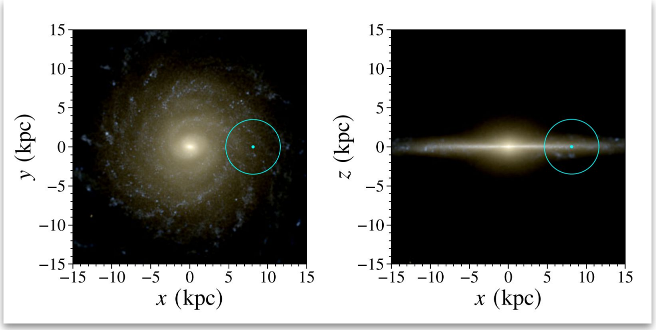
h277: Measuring Dark Matter in a Simulated Galaxy[1,2]

arXiv:2205.01129 & https://doi.org/10.1093/mnras/stad843

Buckley, Lim, **EP**, Shih (2022)

MAFs learned f(**x**,**v**) of a spherical patch of a simulated hydrodynamic galaxy. Recovered **a**(**x**) & ρ(**x**) <u>without models or symmetries</u>.





JOURNAL ARTICLE

Measuring Galactic dark matter through unsupervised machine learning Get access

Matthew R Buckley, Sung Hak Lim, Eric Putney, David Shih

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ABSTRACT

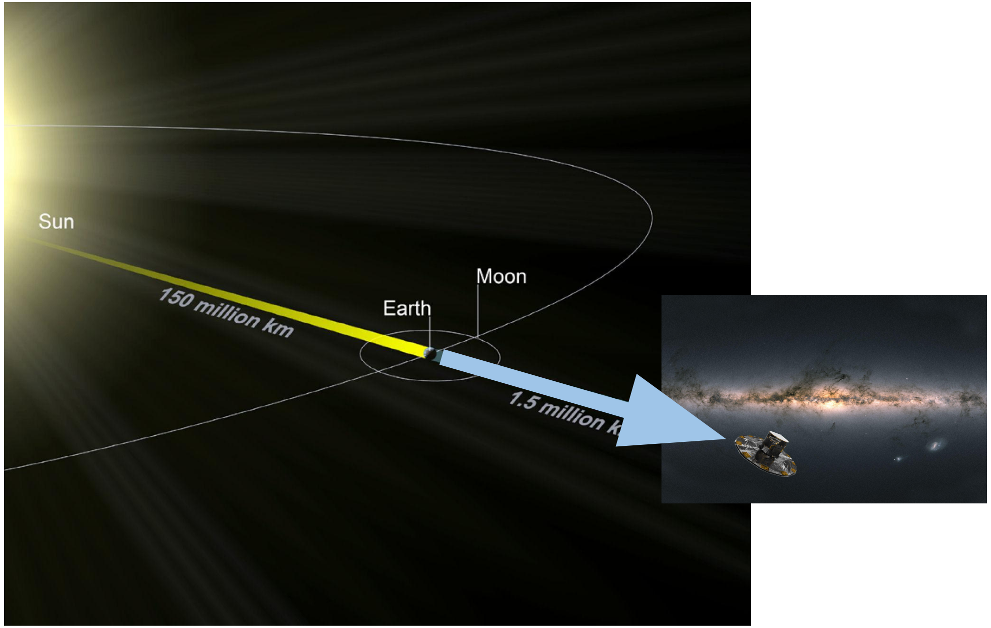
Measuring the density profile of dark matter in the Solar neighbourhood has important implications for both dark matter theory and experiment. In this work, we apply autoregressive flows to stars from a realistic simulation of a

Gaia, *the billion star surveyor*

Gaia measures **x**, **v**, and other properties of billions of sources in the MW.

Data Release 3 (DR3):

- 1.8 billion sources with 5D kinematics.



L2 diagram: © François Mignard
Gaia render: ESA/ATG medialab/Gaia/DPAC; CC BY-SA 3.0 IGO.

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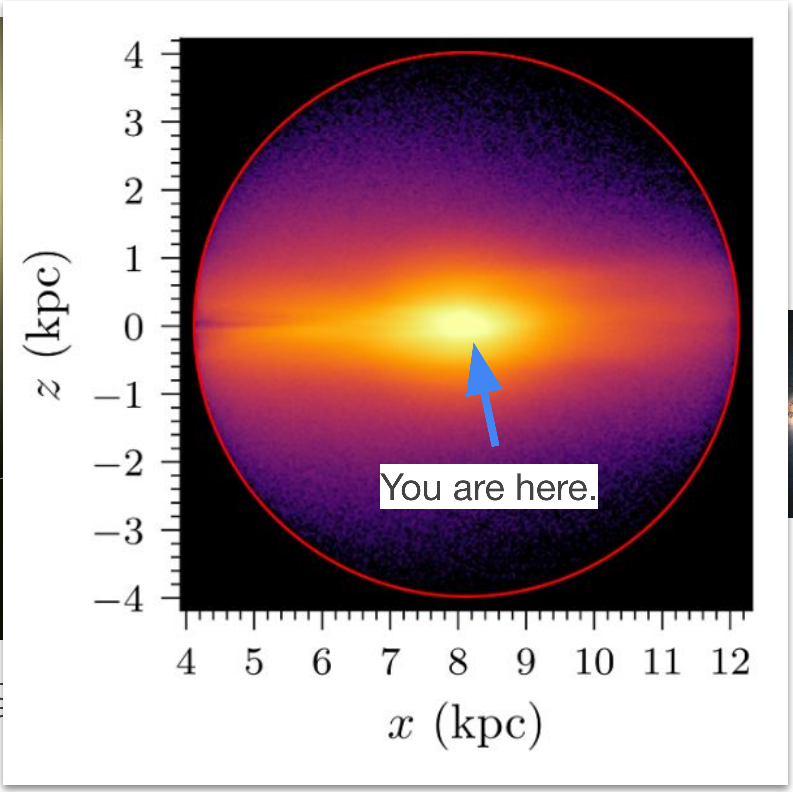
Side-on view of the stars with {**x**,**v**}
Lim, **EP**, Buckley, Shih (2023)



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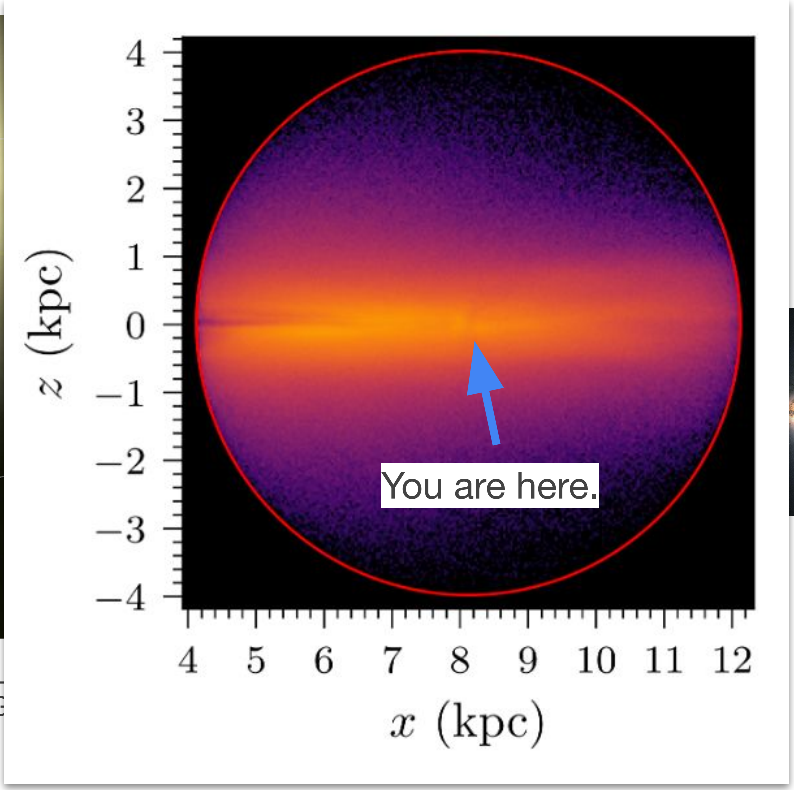
- 31.5 million with line of sight velocities. Full 6D kinematics!
- 5.8 millions stars bright enough to be *complete* within 4 kpc and old enough to be in *equilibrium*

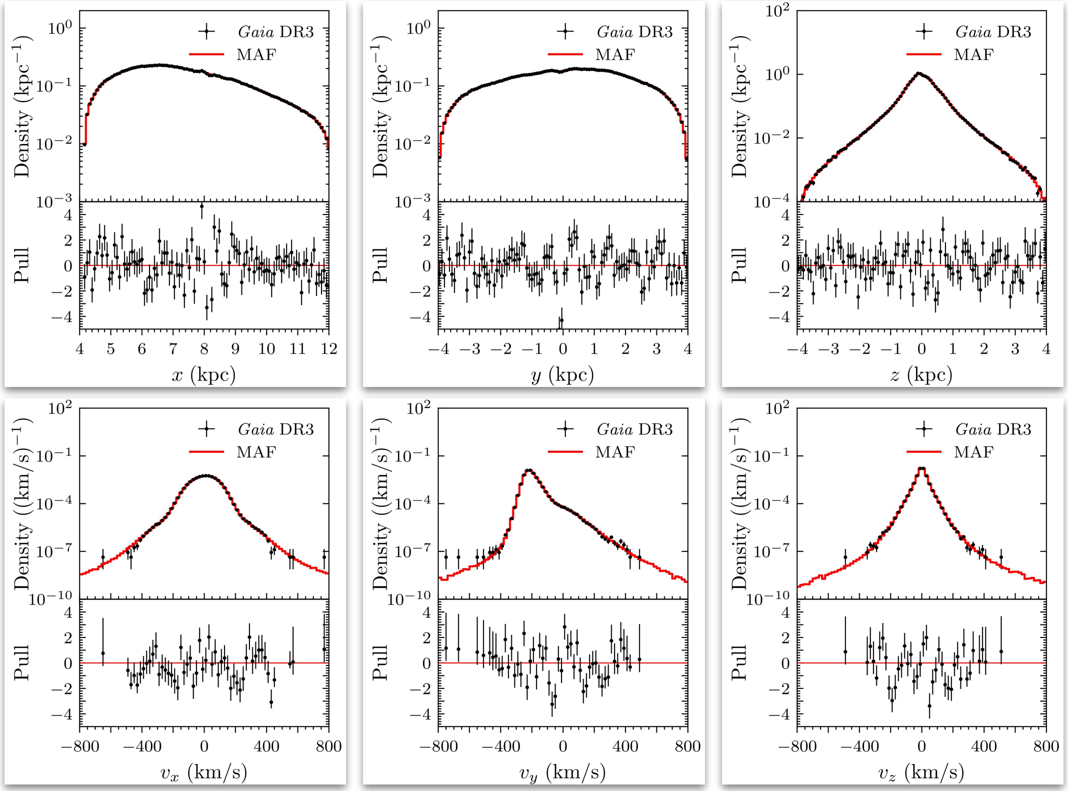


Training Results

Lim, **EP**, Buckley, Shih (2023)

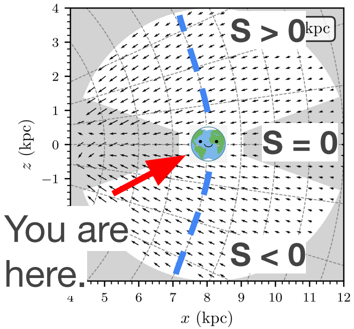
Flows precisely map out phase space. **Key takeaways:**

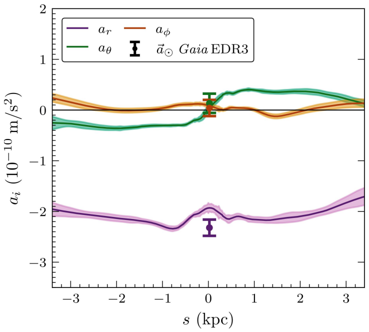
1. Pulls distributed around 0. Deviation at the tails consistent with Poisson noise
2. Small deviation near the solar location due to dust
3. Some periodicity to pull distribution



Local Acceleration and Mass Density Field

Solar acceleration consistent
with previous studies!





	Gaia EDR3	**This work**
a_x $(10^{-10}\mathrm{m/s^2})$	-2.32 ± 0.16	-1.94 ± 0.22
a_y $(10^{-10}\mathrm{m/s^2})$	0.04 ± 0.16	0.08 ± 0.08
a_z $(10^{-10}\mathrm{m/s^2})$	-0.14 ± 0.19	-0.06 ± 0.08
$\lvert\vec{a}\rvert$ $(10^{-10}\mathrm{m/s^2})$	2.32 ± 0.16	1.94 ± 0.22

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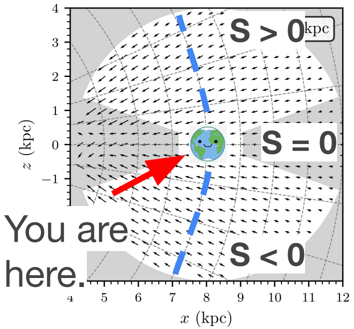

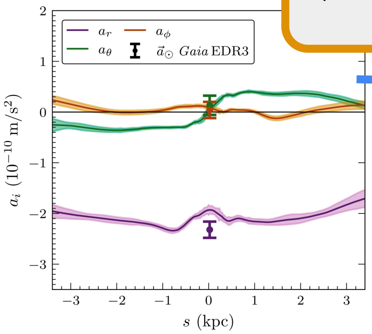

$$\rho = -\frac{\nabla \cdot \vec{a}}{4\pi G}$$

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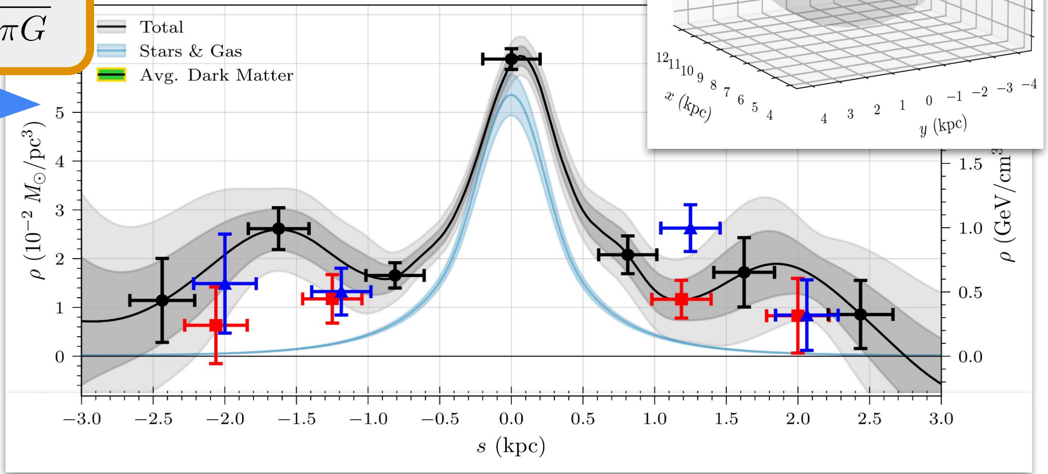
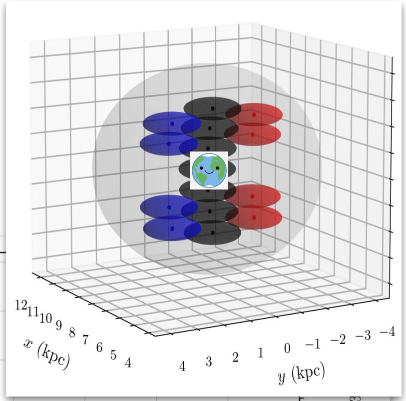
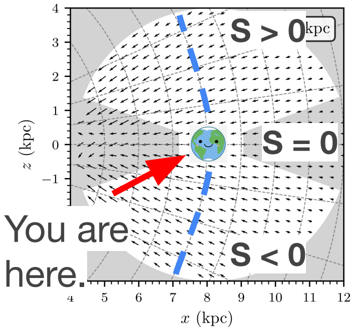


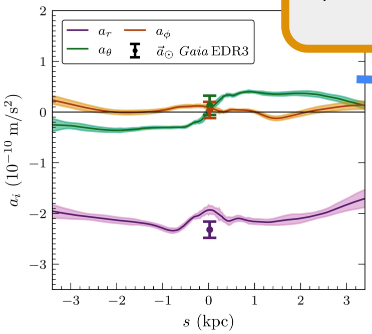


ρ along polar arc length *s* peaks in the disk, levels off for |s| > 1 kpc. Estimate ρ_b via parametric models from McKee et al. (2015)[3] and Ou et al. (2023)[4]

Local Acceleration and Mass Density Field

Solar acceleration consistent with previous studies!





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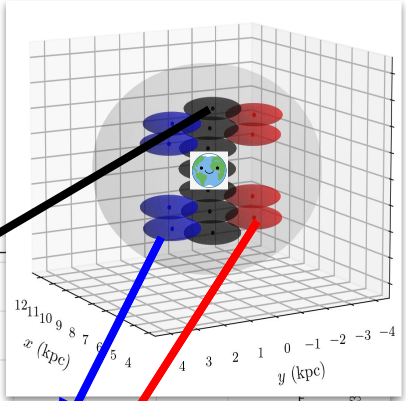
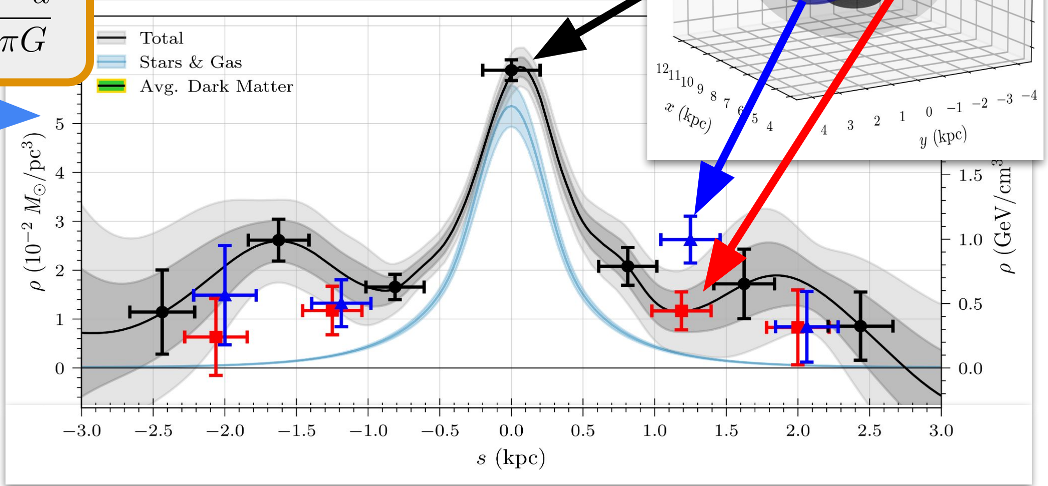




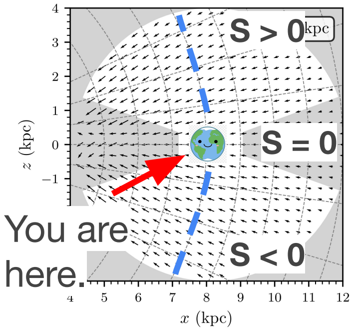
ρ along polar arc length *s* peaks in the disk, levels off for |s| > 1 kpc. Estimate ρ_b via parametric models from McKee et al. (2015)[3] and Ou et al. (2023)[4]

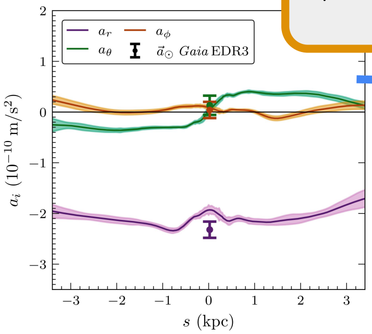
Local Acceleration and Mass Density Fi

Subtracting the baryons off the total density gives us…

Solar acceleration consistent with previous studies!

$$\rho = -\frac{\nabla \cdot \vec{a}}{4\pi G}$$





	Gaia EDR3	**This work**
a_x $(10^{-10}\mathrm{m/s^2})$	-2.32 ± 0.16	-1.94 ± 0.22
a_y $(10^{-10}\mathrm{m/s^2})$	0.04 ± 0.16	0.08 ± 0.08
a_z $(10^{-10}\mathrm{m/s^2})$	-0.14 ± 0.19	-0.06 ± 0.08
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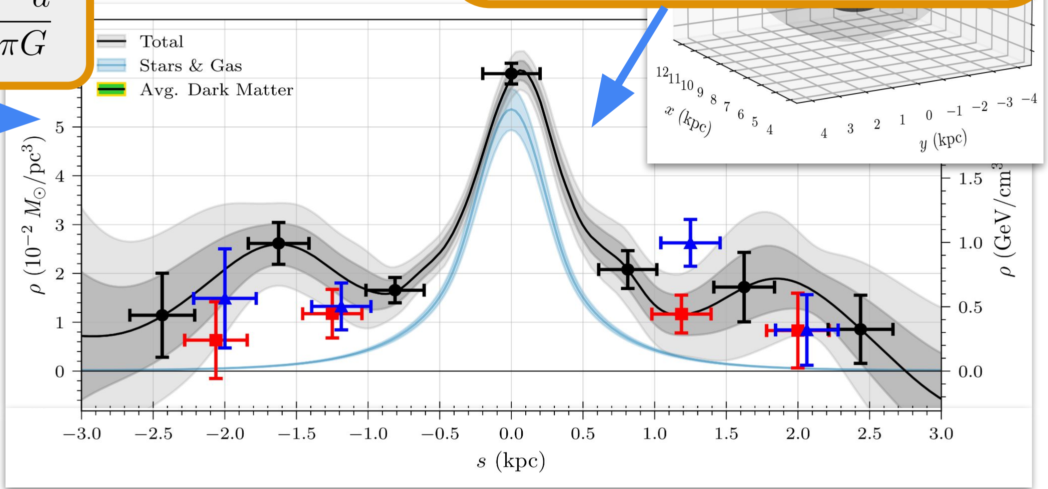
Lim, **EP**, Buckley, Shih (2023)

Lim, **EP**, Buckley



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Local Acceleration and Mass Density Fi

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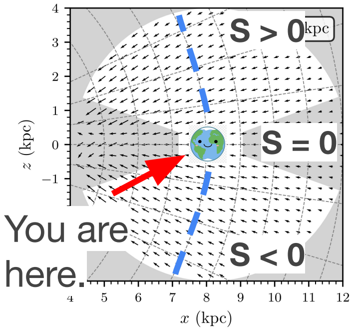
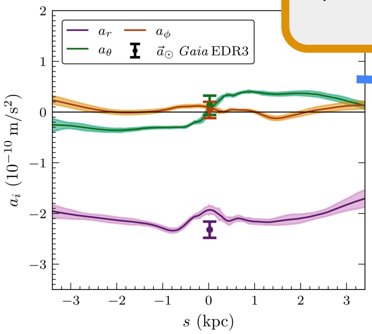
Dark Matter!

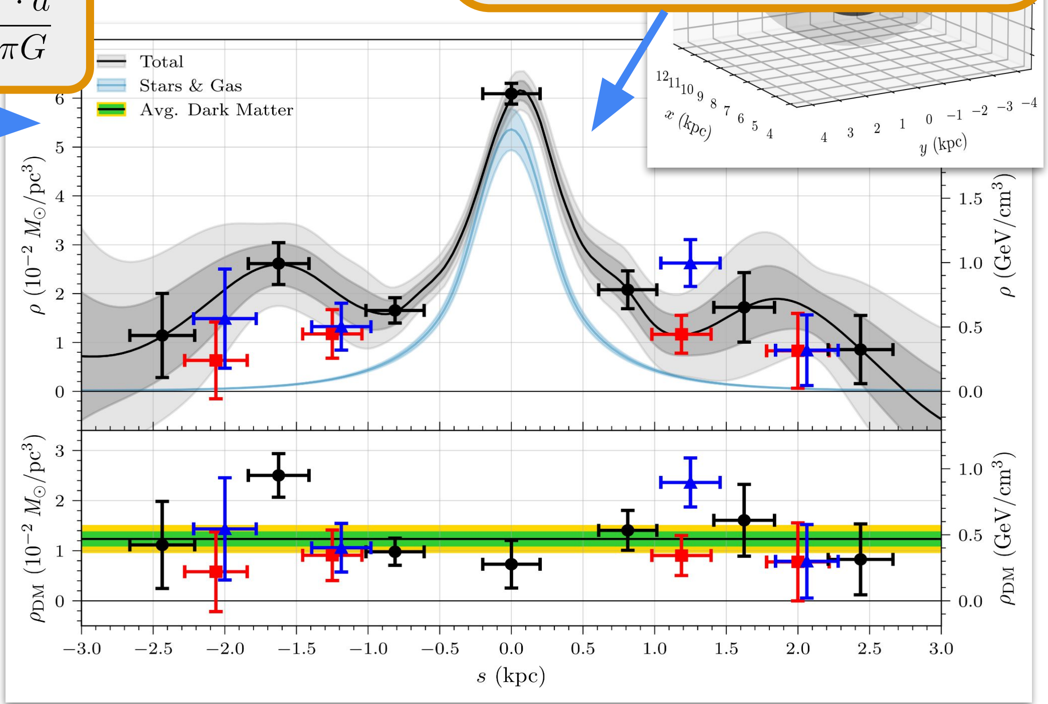
Solar acceleration consistent with previous studies!

$$\rho = -\frac{\nabla \cdot \vec{a}}{4\pi G}$$

Lim, **EP**, Buckley







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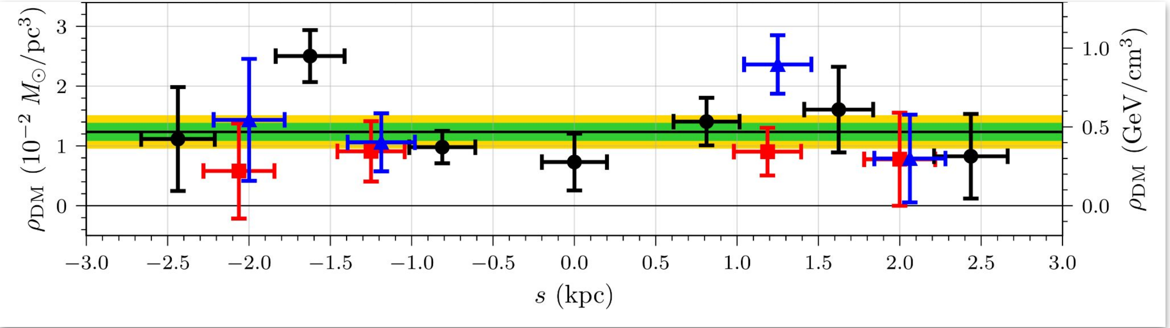
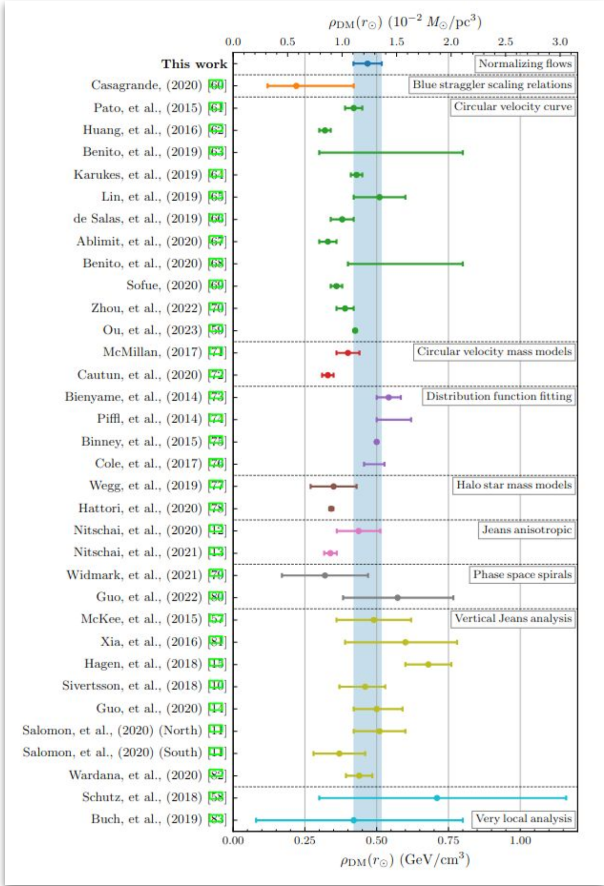
Lim, **EP**, Buckley, Shih (2023)

Local Density of Dark Matter

Each of these 15 points probes ρ_{DM} at the same solar radius of ~8.1 kpc. Assuming a spherical DM halo:

Density	$(10^{-2}\ M_\odot/\mathrm{pc}^3)$	$(\mathrm{GeV}/\mathrm{cm}^3)$	χ^2_ν
$\rho_\odot$	6.17 ± 0.20	2.34 ± 0.08	
$\rho_{b,\odot}$	5.34 ± 0.42	2.03 ± 0.16	
$\rho_{\mathrm{DM},\odot}$	0.83 ± 0.47	0.32 ± 0.18	
$\overline{\rho}_{\mathrm{DM}}(r = r_\odot)$	1.18 ± 0.14	0.47 ± 0.05	1.38

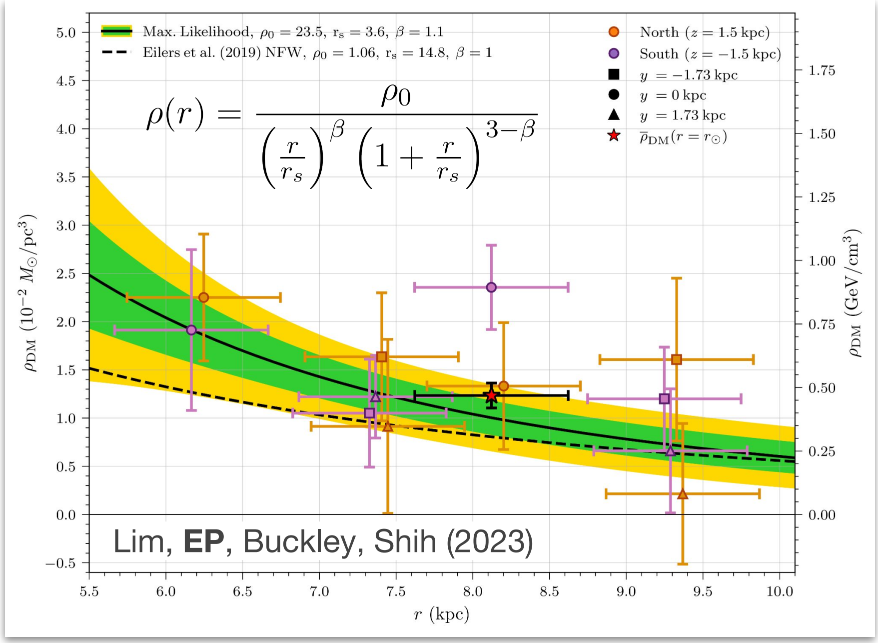


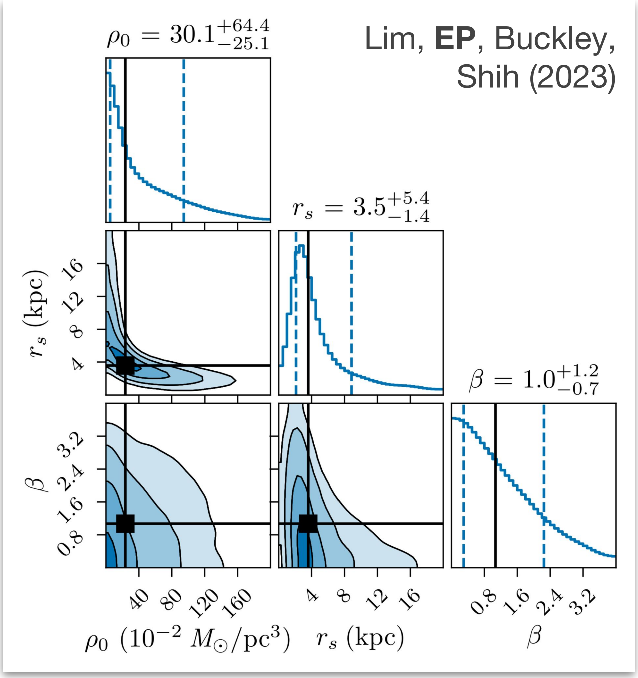


Lim, **EP**, Buckley, Shih (2023)

Radial Profile of the Milky Way's Dark Matter Halo

Can place loose constraints on the MW halo. Radial profile consistent with NFW (β=1) when fit to a generalized NFW (gNFW) profile.





In Summary

- We present a model of the stellar phase space f of the local Milky Way using MAFs
- We can use f to solve many problems in galactic dynamics
- Measured local ρ_{DM} and the dark matter density profile with minimal assumptions and comprehensive errors

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Future Directions:

- Replace MAF with SOTA flow models: (CNFs, flow-matching)
- Parameterizing $\Phi(\mathbf{x})$ with a NN, conservative force constraint[5]
- Disk phase space suppressed by dust, can we infer the true phase space density?

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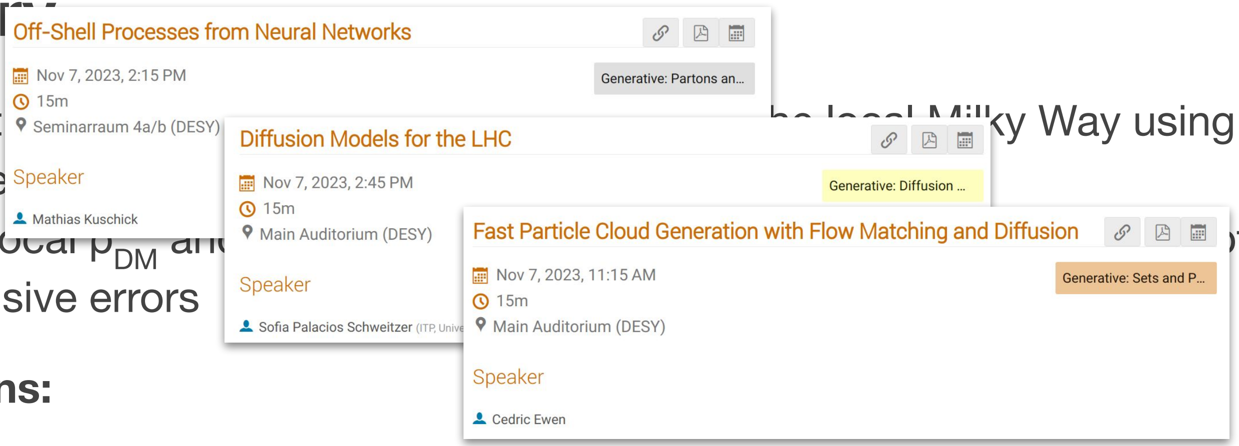

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References:

1. M. R. Buckley, S. H. Lim, E. Putney, and D. Shih, arXiv e-prints arXiv:2205.01129 (2022), 2205.01129
2. A. Zolotov, A. M. Brooks, B. Willman, F. Governato, A. Pontzen, C. Christensen, A. Dekel, T. Quinn, S. Shen, and J. Wadsley, Astrophys. J. 761, 71 (2012), 1207.0007
3. C. F. McKee, A. Parravano, and D. J. Hollenbach, Astrophys. J. 814, 13 (2015), 1509.05334.
4. X. Ou, A.-C. Eilers, L. Necib, and A. Frebel, arXiv eprints arXiv:2303.12838 (2023), 2303.12838.
5. G. Green, Y. Ting, H. Kamdar, Astrophys. J. 942, 26 (2023), 10.3847/1538-4357.

Additional Slides

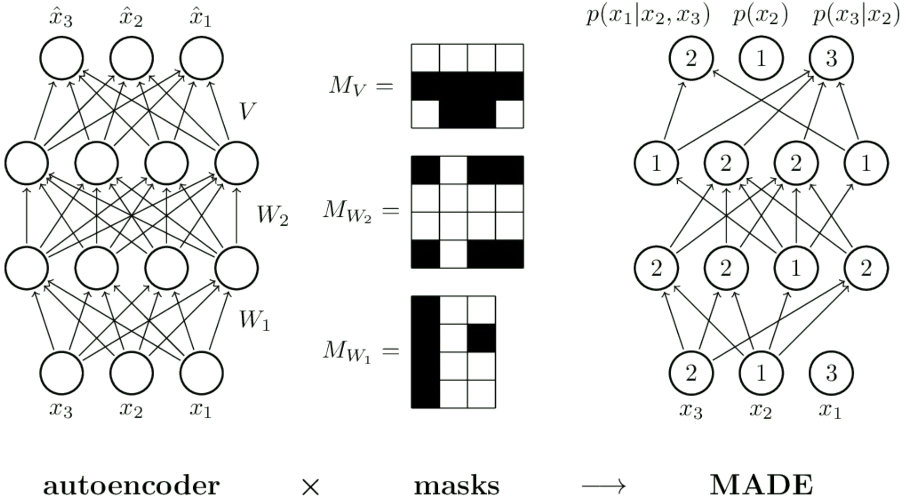
MAF Visualization

Visualizing the flow: Base distribution (3D Gaussian) mapped to Gaia DR3's p(**x**). (Viewed along 3 axes)

Masked Autoregressive Flows

Autoregressive model: transformation modeled as a product of conditionals. MAFs enforce autoregressive property by masking network connections, allowing for complex non-linear transformations.

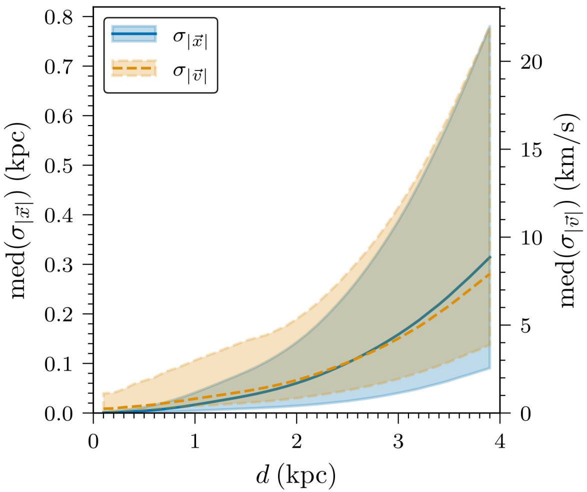
$$p(x_1, ..., x_n) = p(x_1)p(x_2|x_1)p(x_3|x_2, x_1)...p(x_n|x_{n-1}, ..., x_1)$$

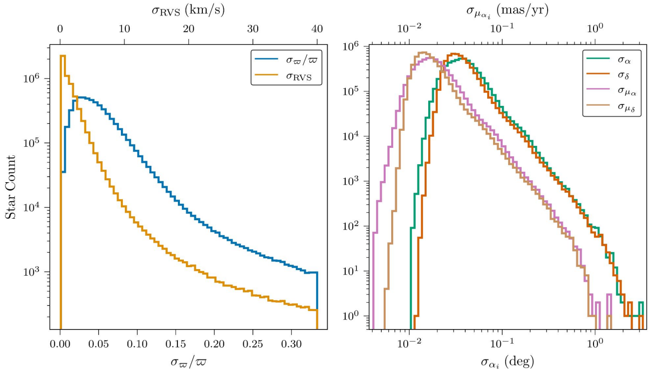


Gaia Measurement Errors

Astrometric errors and propagated from ICRS to Cartesian coordinates.

Primary uncertainties for stars are parallax and radial velocities.





Jeans Analyses

Moments of the CBE yield the Jeans equations:

$$\int d^3p(p_z * \text{CBE}) \longrightarrow \quad a_z = \frac{1}{\nu}\frac{\partial(\nu\overline{v_z^2})}{\partial z} + \frac{1}{\nu R}\frac{\partial(\nu R\overline{v_R v_z})}{\partial R}$$

$$\int d^3p(p_R * \text{CBE}) \longrightarrow a_R = \frac{1}{\nu}\frac{\partial(\nu\overline{v_R^2})}{\partial R} + \frac{1}{\nu}\frac{\partial(\nu\overline{v_R v_z})}{\partial z} + \frac{\overline{v_R^2} - \overline{v_\phi^2}}{R}$$

Key notes:

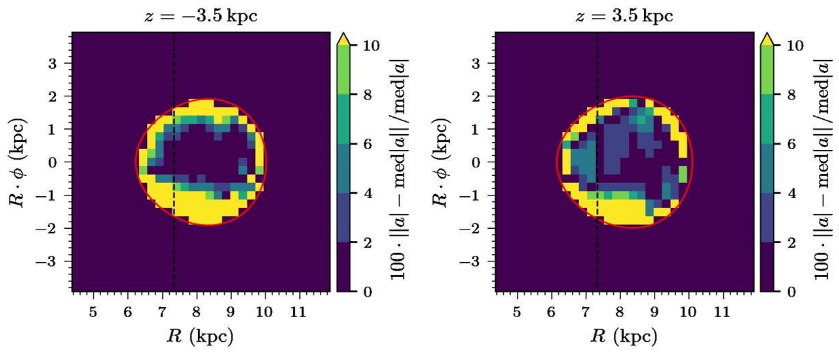
- Axisymmetry implicitly assumed when $a_\phi = 0$.
- Must model each velocity moment and number density
- Many terms are noisy. Calculation of ρ can involve high order derivatives of fits to noisy data.

Disequilibrium

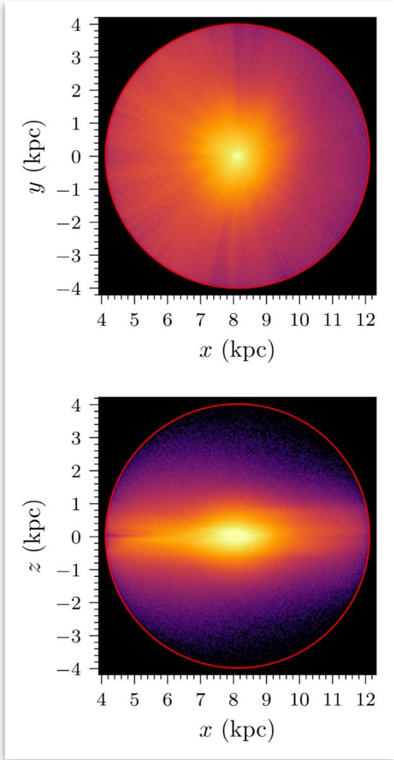
Rotating system in dynamic equilibrium should be axisymmetric. We solve the equilibrium CBE, but we do not enforce equilibrium in our model of f(**x**,**v**).

We can perform a closure test of this assumption by measuring deviations from axisymmetry in **a**(**x**) and ρ(**x**)

In the future, we can use f(**x**,**v**) with independent measurements of **a**(**x**) to estimate df/dt.



Gaia Dataset: The Red Clump (and friends)



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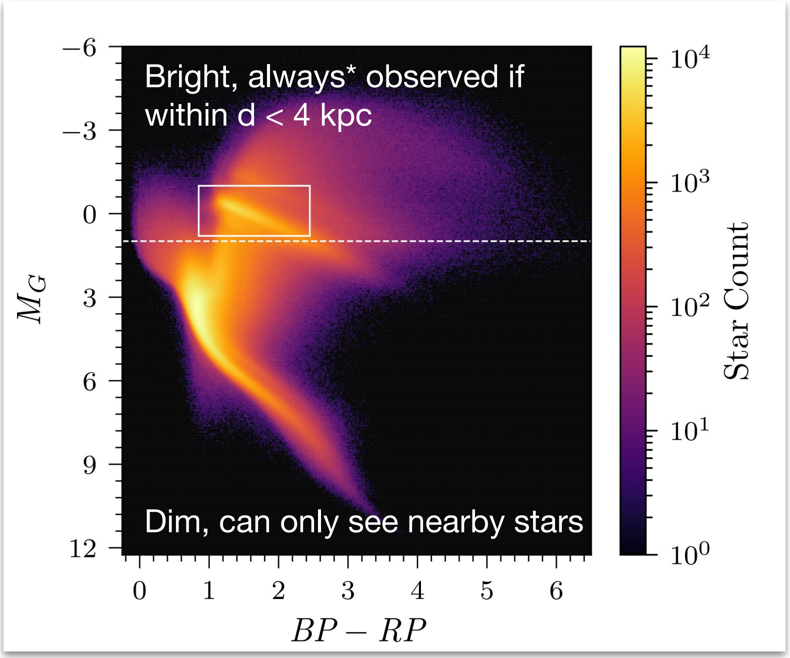

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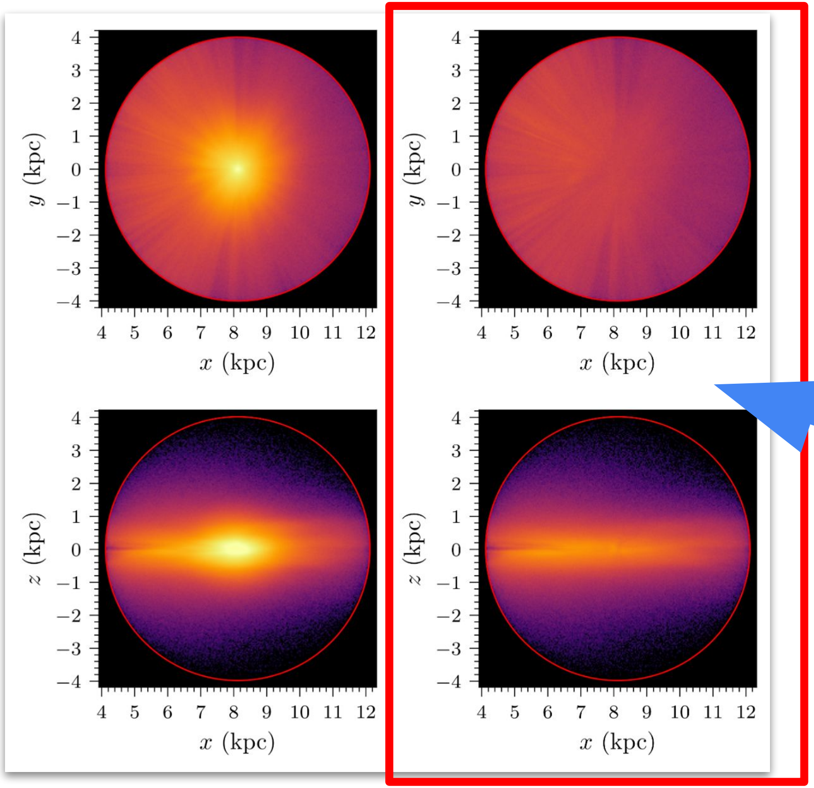

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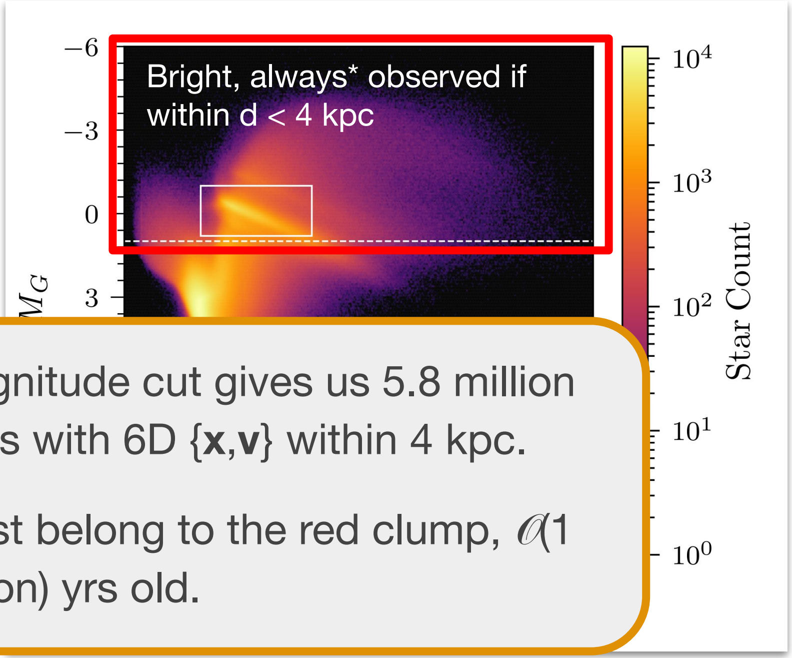
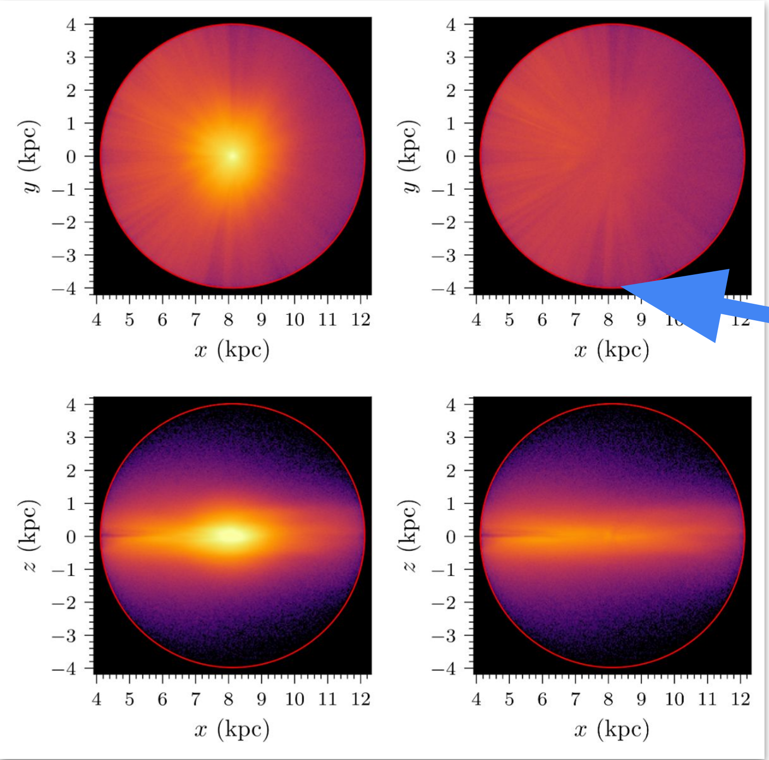


Magnitude cut gives us 5.8 million stars with 6D {**x**,**v**} within 4 kpc.

Most belong to the red clump, $\mathcal{O}$(1 billion) yrs old.

Gaia Dataset: The Red Clump (and friends)



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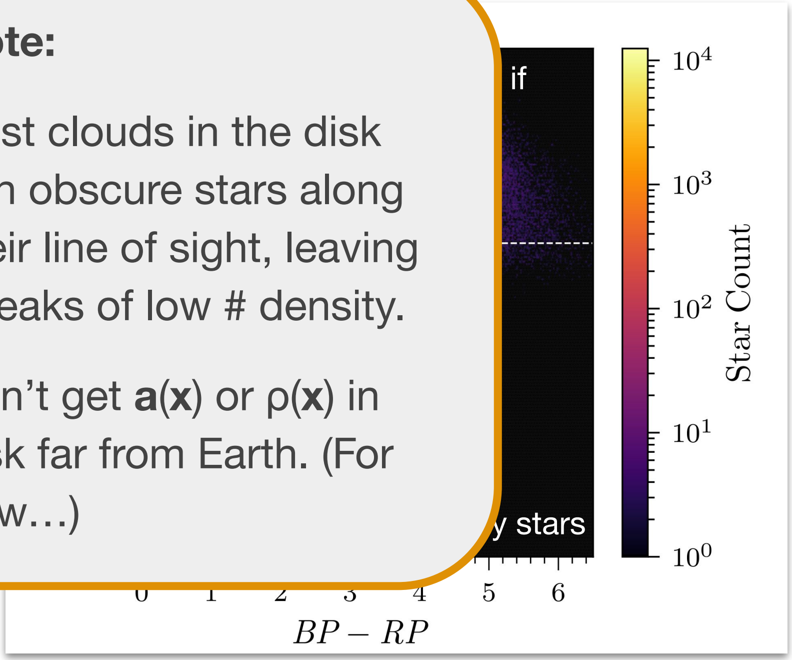

Note:

Dust clouds in the disk can obscure stars along their line of sight, leaving streaks of low # density.

Can't get **a**(**x**) or ρ(**x**) in disk far from Earth. (For now…)