Binary Discrimination at Next-to-Leading Order

Andrew Larkoski

2309.14417

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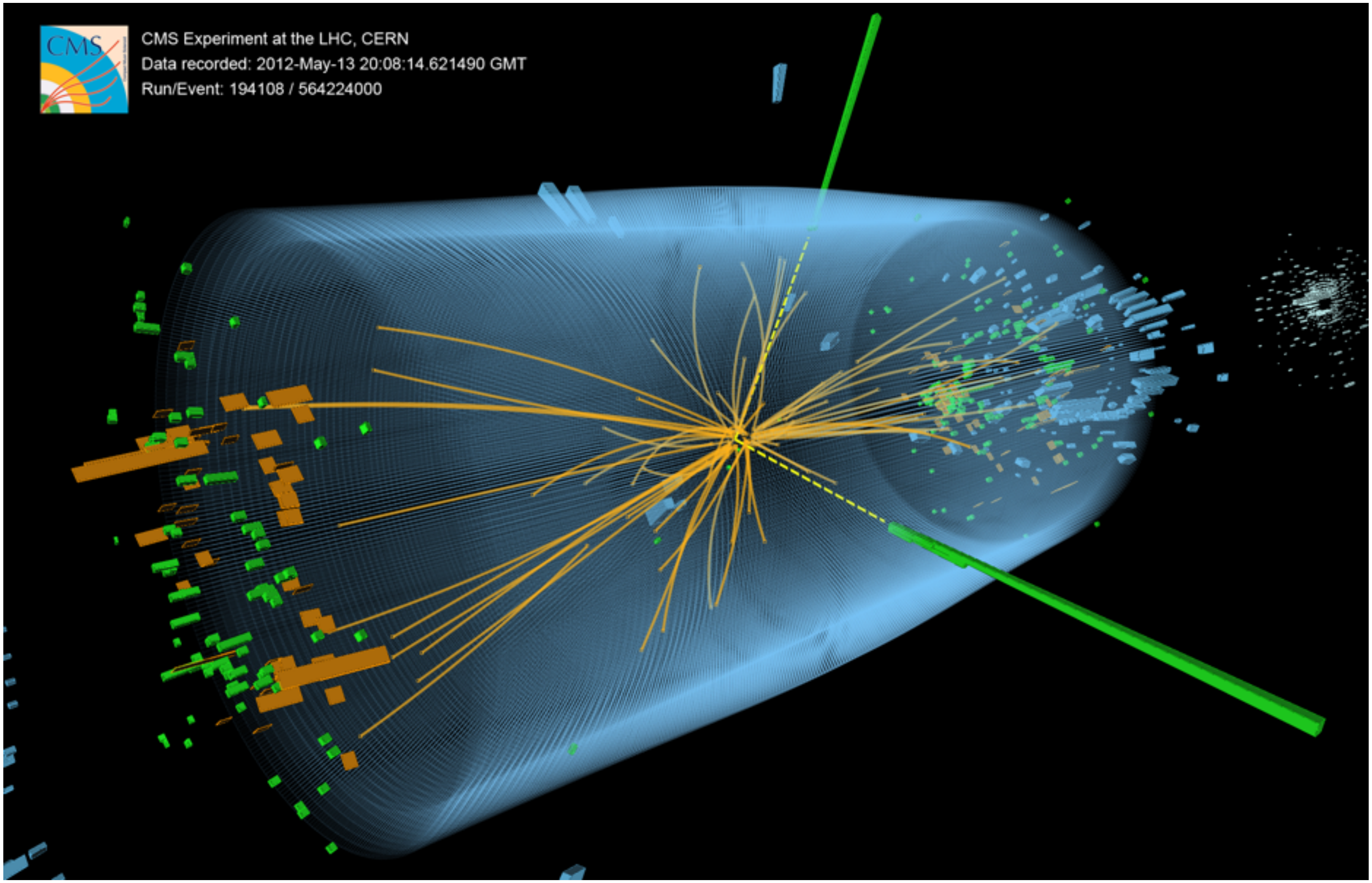
Fundamental Problem in Particle Physics: Binary Discrimination



CMS-PHO-EVENTS-2013-003

Is this event from a Higgs decay (signal) or continuum production (background)?

Optimal Discriminant Observable Solution: Likelihood Ratio

$$\hat{\mathcal{L}}(\Phi) = \frac{p_B(\Phi)}{p_S(\Phi)}$$

Neyman, Pearson 1933

Optimal Discriminant Observable Solution: Likelihood Ratio

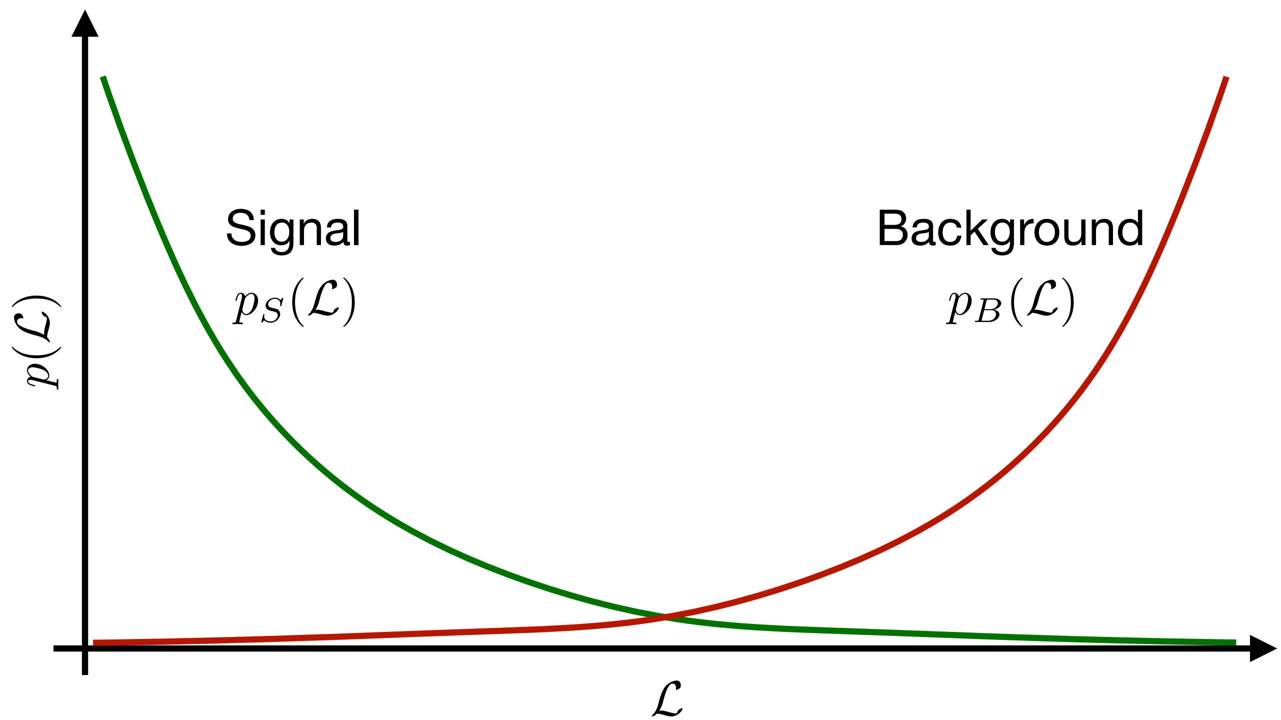
$$\hat{\mathcal{L}}(\Phi) = \frac{p_B(\Phi)}{p_S(\Phi)}$$

Background Distribution ($p_B(\Phi)$)

Signal Distribution ($p_S(\Phi)$)

Phase Space (Φ)

Neyman, Pearson 1933



Problem reduced to calculating signal/background distributions on phase space

Can be systematically predicted if the likelihood ratio is **Infrared and Collinear Safe**

Example: $H \to b\bar{b}$ versus $g \to b\bar{b}$

This problem jump-started modern jet substructure!

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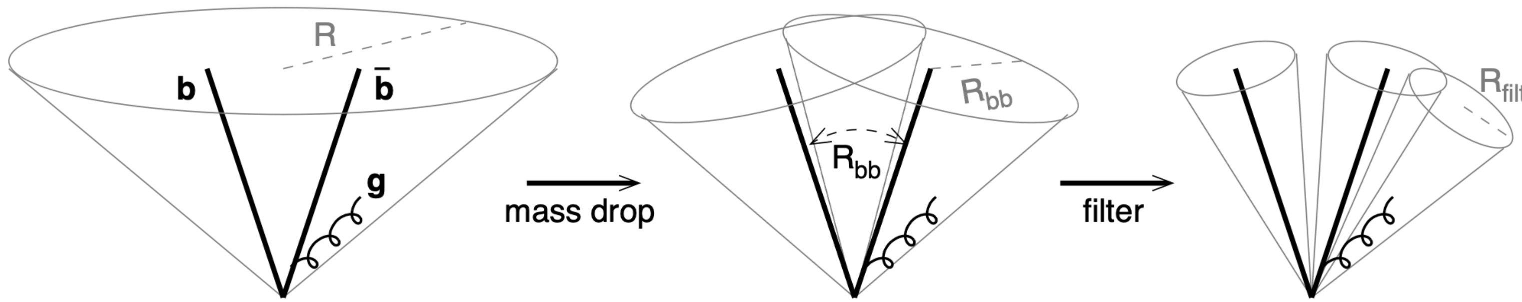
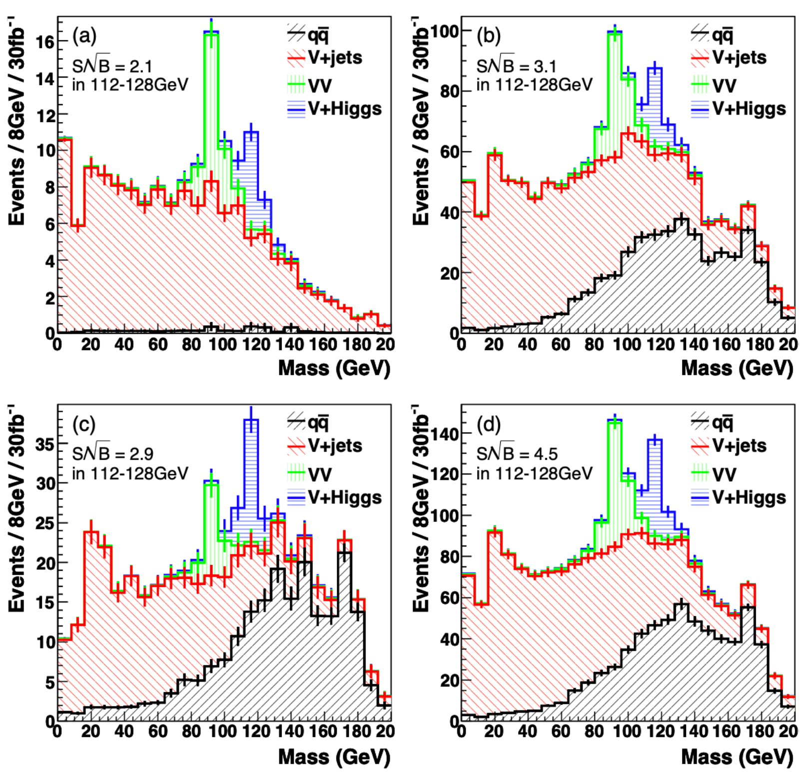




Instead of bespoke designing an observable/technique, let matrix elements on phase space do the work for us

Establish theoretically optimal discrimination performance for a DNN implementation of this problem

Likelihood at Leading Order

$$\hat{\mathcal{L}}(\Phi) = \frac{p_g^{(0)}(\Phi^{(0)})}{p_H^{(0)}(\Phi^{(0)})} + \mathcal{O}(\alpha_s)$$

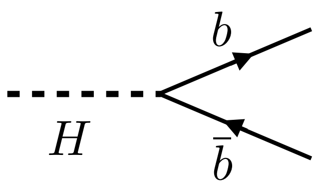
$$d\Phi^{(0)} = dz\, dm^2\, \delta(m^2 - m_H^2)$$

Energy Fraction → dz

Invariant Mass → dm^2

Fixed to Higgs Mass → $\delta(m^2 - m_H^2)$

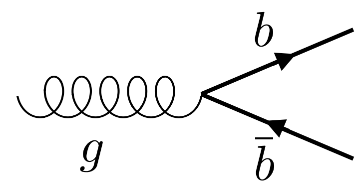
$p_H^{(0)}(\Phi^{(0)})$ =



On-shell, unpolarized decay

Constant

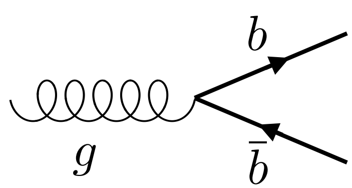
$p_g^{(0)}(\Phi^{(0)})$ =

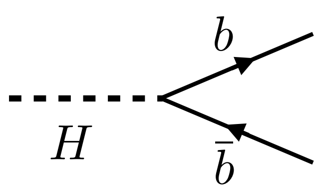


$g \to b\bar{b}$ splitting function at fixed mass

$$p_g^{(0)}(z) \propto z^2 + (1-z)^2$$

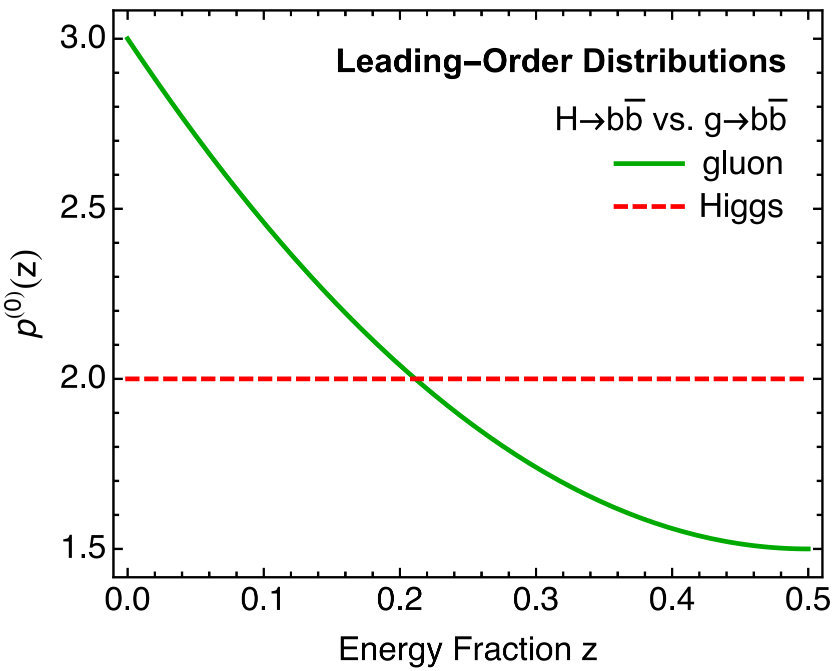
$\hat{\mathcal{L}}(\Phi) \approx$

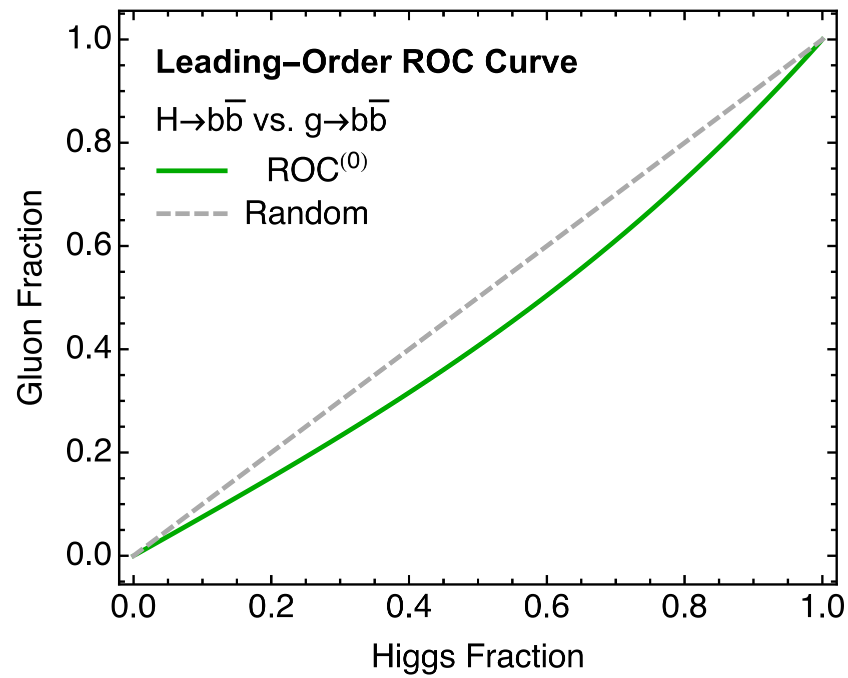




$$\left(1 + \mathcal{O}(\alpha_s) \right) \approx z^2 + (1-z)^2$$

Likelihood at Leading Order





Only sensitive to relative energy fractions of decay products

Very poor discrimination power

Missing key physics information at this order:
Higgs is color singlet, gluon is color octet

Likelihood at Next-to-Leading Order

$$\hat{\mathcal{L}}(\Phi)^{*} = \frac{p_g^{(0)}(\Phi^{(0)}) + \frac{\alpha_s}{2\pi}\, p_g^{(1)}(\Phi^{(1)}) + \mathcal{O}(\alpha_s^2)}{p_H^{(0)}(\Phi^{(0)}) + \frac{\alpha_s}{2\pi}\, p_H^{(1)}(\Phi^{(1)}) + \mathcal{O}(\alpha_s^2)}$$

$$= \frac{p_g^{(0)}(\Phi^{(0)})}{p_H^{(0)}(\Phi^{(0)})} + \frac{\alpha_s}{2\pi}\, \frac{p_g^{(0)}(\Phi^{(0)})}{p_H^{(0)}(\Phi^{(0)})} \left(\frac{p_g^{(1)}(\Phi^{(1)})}{p_g^{(0)}(\Phi^{(0)})} - \frac{p_H^{(1)}(\Phi^{(1)})}{p_H^{(0)}(\Phi^{(0)})} \right) + \mathcal{O}(\alpha_s^2)$$

*Ignoring virtual terms

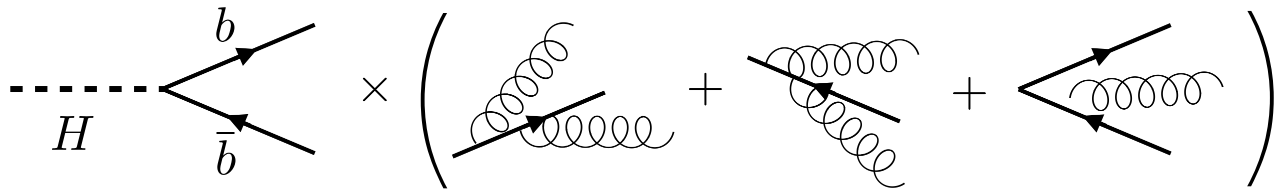
Likelihood at Next-to-Leading Order

$$\hat{\mathcal{L}}(\Phi)^{*} = \frac{p_g^{(0)}(\Phi^{(0)}) + \frac{\alpha_s}{2\pi}\, p_g^{(1)}(\Phi^{(1)}) + \mathcal{O}(\alpha_s^2)}{p_H^{(0)}(\Phi^{(0)}) + \frac{\alpha_s}{2\pi}\, p_H^{(1)}(\Phi^{(1)}) + \mathcal{O}(\alpha_s^2)}$$

$$= \frac{p_g^{(0)}(\Phi^{(0)})}{p_H^{(0)}(\Phi^{(0)})} + \frac{\alpha_s}{2\pi}\,\frac{p_g^{(0)}(\Phi^{(0)})}{p_H^{(0)}(\Phi^{(0)})}\left(\frac{p_g^{(1)}(\Phi^{(1)})}{p_g^{(0)}(\Phi^{(0)})} - \frac{p_H^{(1)}(\Phi^{(1)})}{p_H^{(0)}(\Phi^{(0)})}\right) + \mathcal{O}(\alpha_s^2)$$

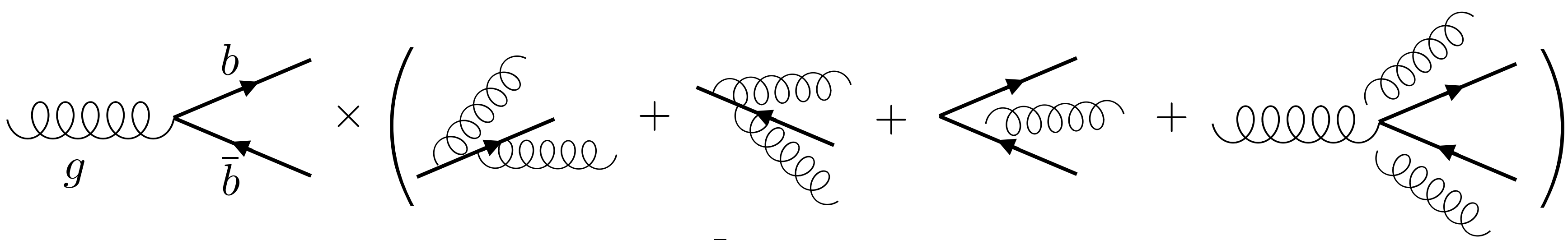
Working in soft and/or collinear limit

$p_H^{(1)}(\Phi^{(1)}) \approx$



b collinear emissions — $\bar{b}$ collinear emissions — soft Abelian dipole emissions

$p_g^{(1)}(\Phi^{(1)}) \approx$



b collinear emissions — $\bar{b}$ collinear emissions — soft Abelian dipole emissions — soft non-Abelian dipole emissions

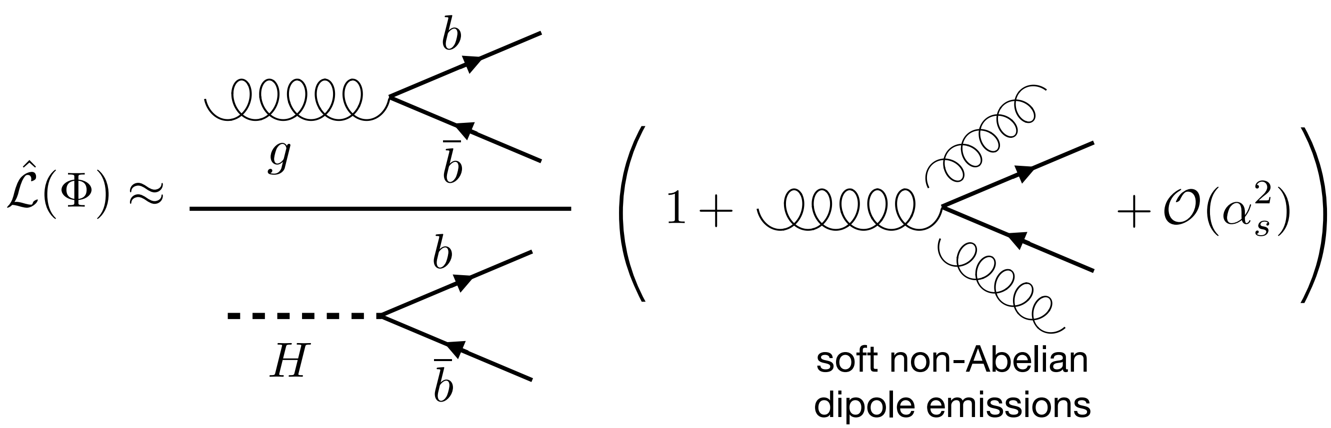
*Ignoring virtual terms

Likelihood at Next-to-Leading Order

$$\hat{\mathcal{L}}(\Phi)^{*} = \frac{p_g^{(0)}(\Phi^{(0)}) + \frac{\alpha_s}{2\pi}\, p_g^{(1)}(\Phi^{(1)}) + \mathcal{O}(\alpha_s^2)}{p_H^{(0)}(\Phi^{(0)}) + \frac{\alpha_s}{2\pi}\, p_H^{(1)}(\Phi^{(1)}) + \mathcal{O}(\alpha_s^2)}$$

$$= \frac{p_g^{(0)}(\Phi^{(0)})}{p_H^{(0)}(\Phi^{(0)})} + \frac{\alpha_s}{2\pi}\, \frac{p_g^{(0)}(\Phi^{(0)})}{p_H^{(0)}(\Phi^{(0)})} \left(\frac{p_g^{(1)}(\Phi^{(1)})}{p_g^{(0)}(\Phi^{(0)})} - \frac{p_H^{(1)}(\Phi^{(1)})}{p_H^{(0)}(\Phi^{(0)})} \right) + \mathcal{O}(\alpha_s^2)$$

Almost Everything at NLO Cancels; Only non-Abelian Emissions Remain

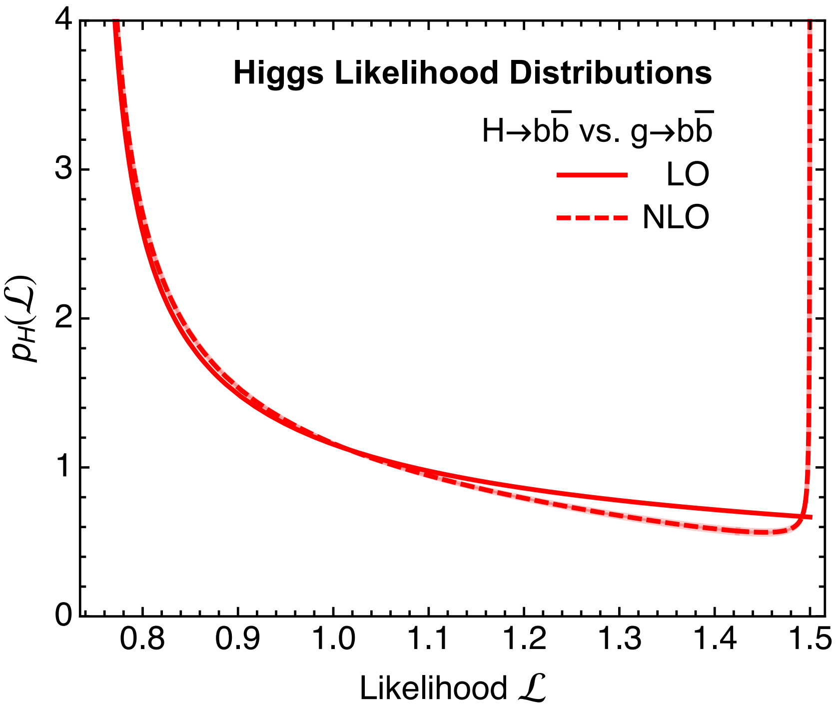


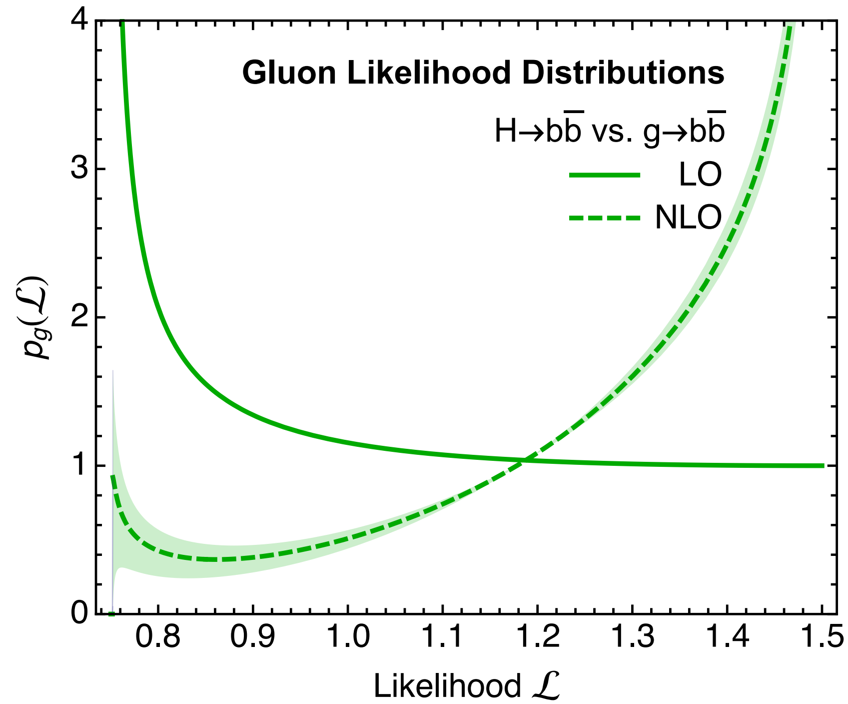
Likelihood ratio explicitly sensitive to difference in color representations!

Don’t anticipate “new physics” effects at even higher orders

*Ignoring virtual terms

Likelihood at Next-to-Leading Order





Very small modification to LO distribution for Higgs:
color singlets do not emit at wide angles

Qualitative change at NLO for gluon distribution, way beyond naive scale variations

Cf. “giant K-factors” that arise when new processes open at NLO

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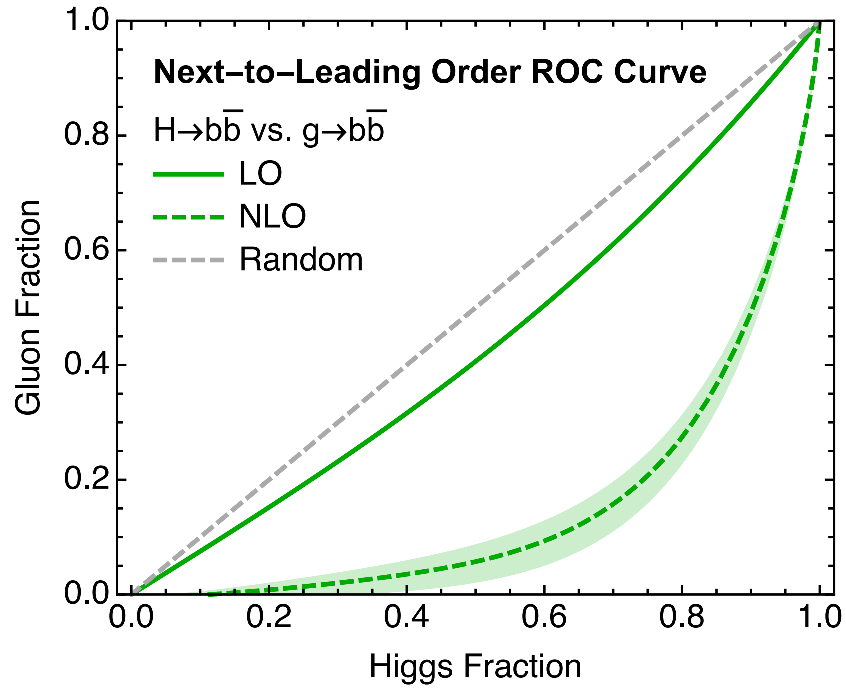
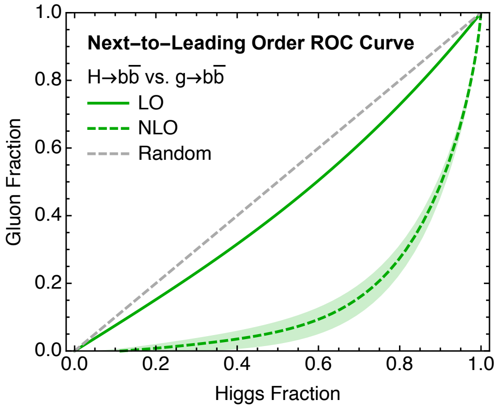
Likelihood at Next-to-Leading Order



Excellent discrimination performance almost exclusively due to sensitivity to color

Likelihood at Next-to-Leading Order

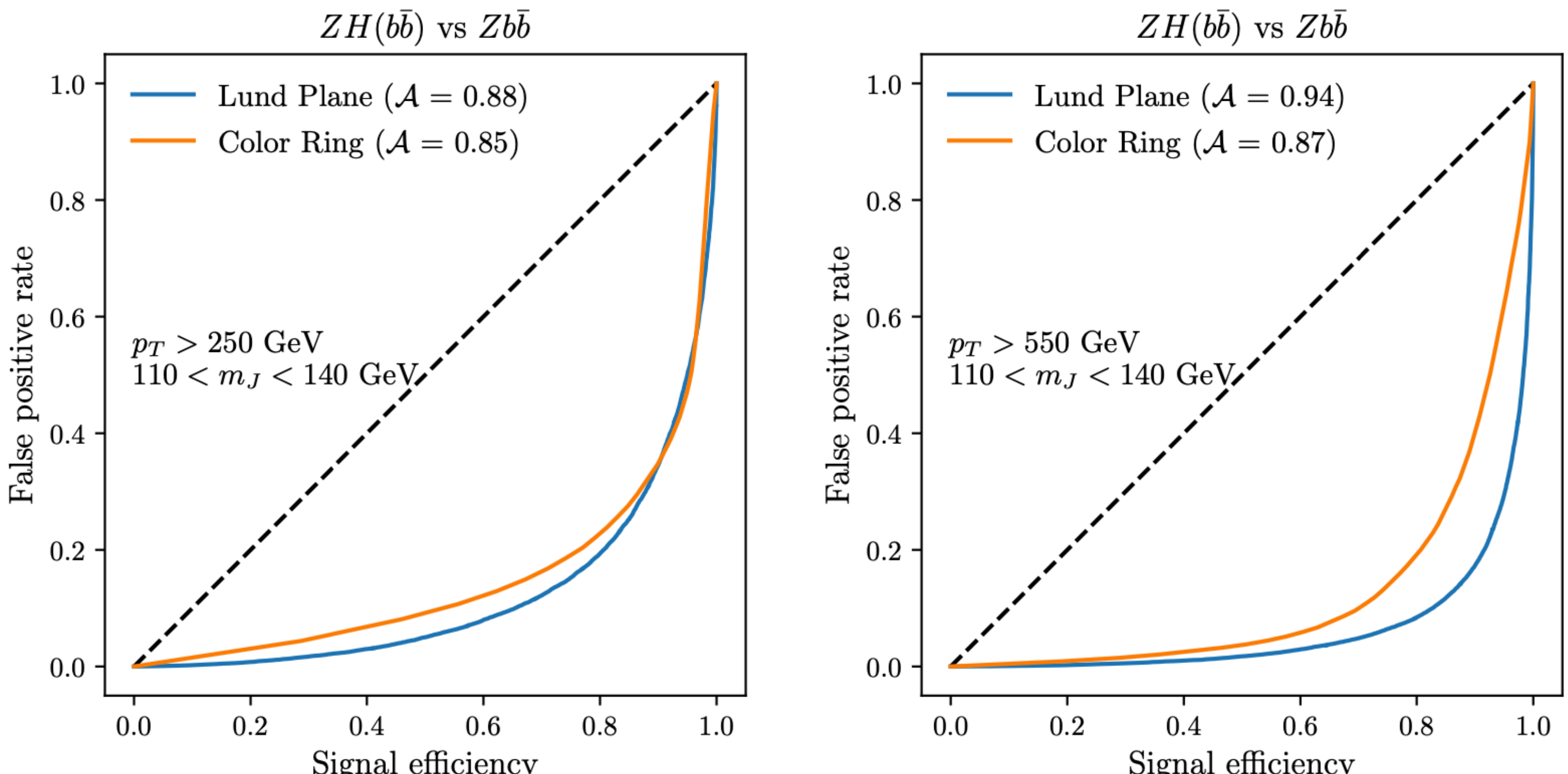


Cf. to DNN Results

NLO required for qualitative understanding

Improved discrimination as p_T increases

“Perfect” discrimination for $p_T \to \infty$



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Conclusions

Theoretical predictions for binary discrimination
establish robust bounds for DNN implementation

Likelihood for $H \to b\bar{b}$ versus $g \to b\bar{b}$ discrimination is maximally sensitive to color reps

What about application to other problems? *q* versus *g*? QCD versus top?

This work suggests that likelihood IRC safety requires indistinguishable final states

For *q* vs. *g*, must "wash out" flavor information for IRC safety

1906.01639

For QCD vs. top, only $b \to bq\bar{q}$ is indistinguishable background