Classifying the CP properties of the Higgs–gluon interaction

and the quest for interpretable ML

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based on 2309.03146

in collaboration with E. Fuchs, M. Hannig, and M. Menen

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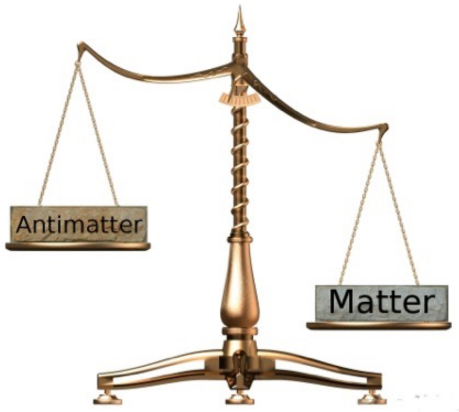
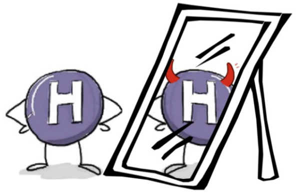
ML4Jets, DESY Hamburg, 8.11.23

CP violation in the Higgs sector



- New sources of CP violation are necessary to explain the baryon asymmetry of the Universe.
- One possibility: CP violation in the Higgs sector.

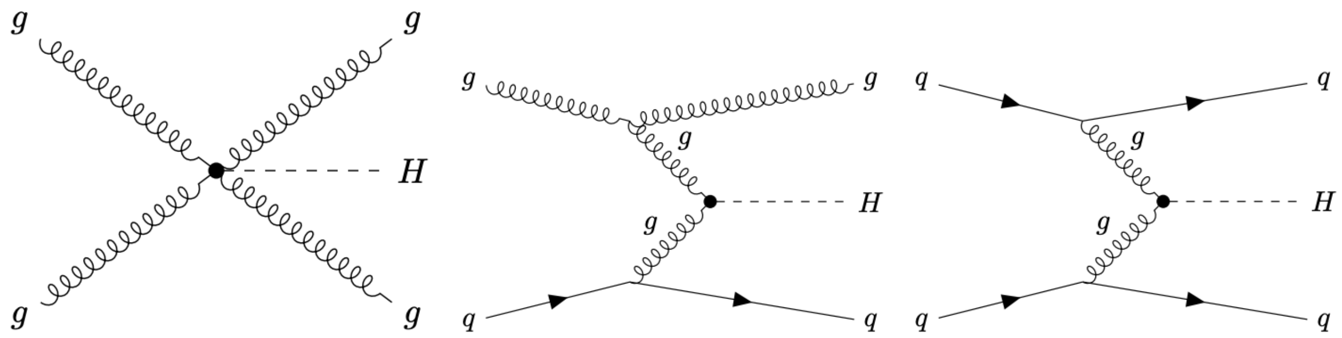


Is the Higgs boson a CP-admixed state?

- Why use ggF2j production for CP tests? [Hankele, Klamke, Zeppenfeld `06,`07, …]
  - Gluon fusion is the largest Higgs production channel → wealth of data.
  - Two additional jets in the final state allow to construct CP-odd observables.

  → CP sensitivity beyond total rate information.

ggF2j production



(a) gg-initiated (b) gq-initiated (c) qq-initiated

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$$\mathcal{L}_{Hgg} = -\frac{1}{4v} H \left(-\frac{\alpha_s}{3\pi} c_g G^a_{\mu\nu} G^{a,\mu\nu} + \frac{\alpha_s}{2\pi} \tilde{c}_g G^a_{\mu\nu} \tilde{G}^{a,\mu\nu} \right)$$

(heavy top limit enforced by p_T cut)

- Amplitude splits up into three pieces:

$$\left|\mathcal{M}_{\text{ggF2j}}\right|^2 = c_g^2 |\mathcal{M}_{\text{even}}|^2 + \underbrace{2 c_g \tilde{c}_g Re[\mathcal{M}_{\text{even}} \mathcal{M}_{\text{odd}}{}^*]}_{\text{interference}} + \tilde{c}_g^2 |\mathcal{M}_{\text{odd}}|^2$$

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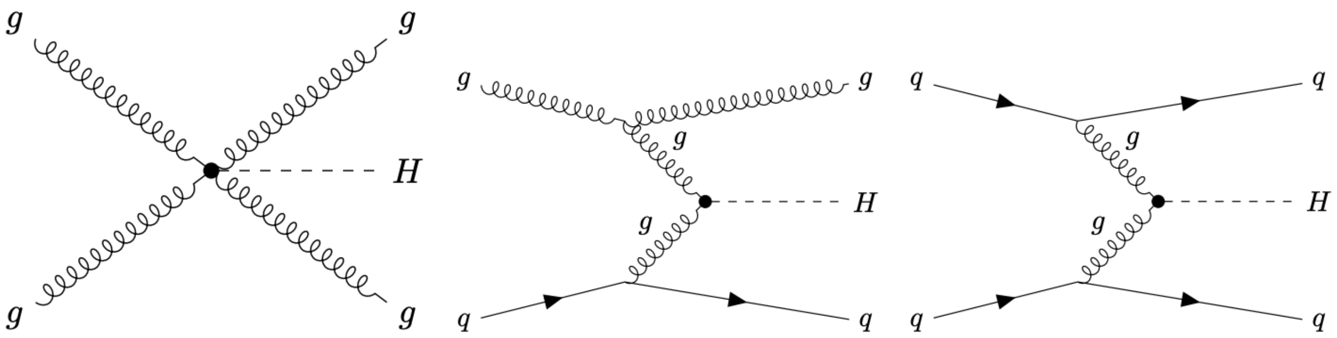

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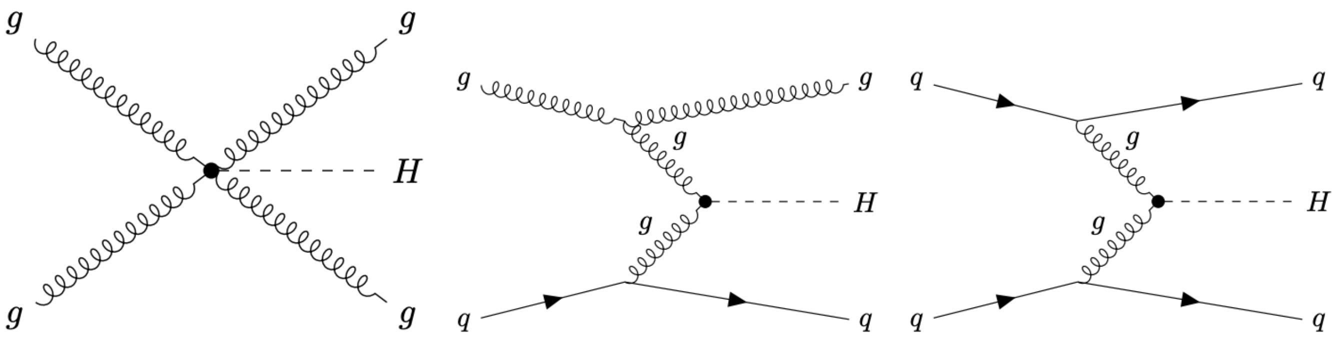

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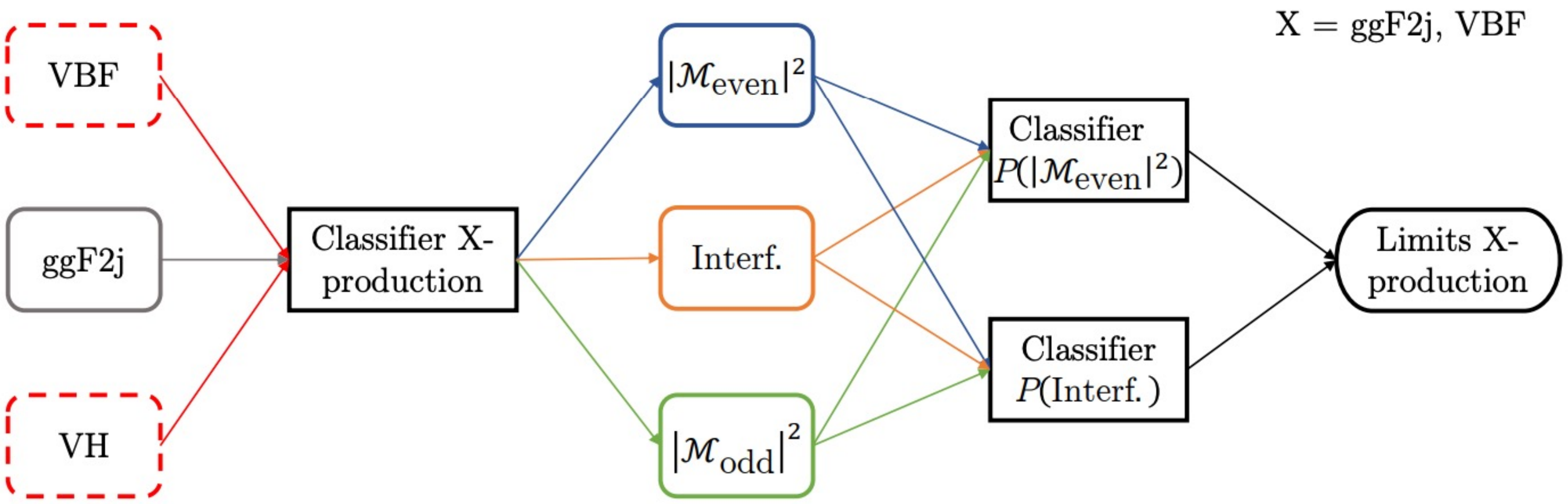
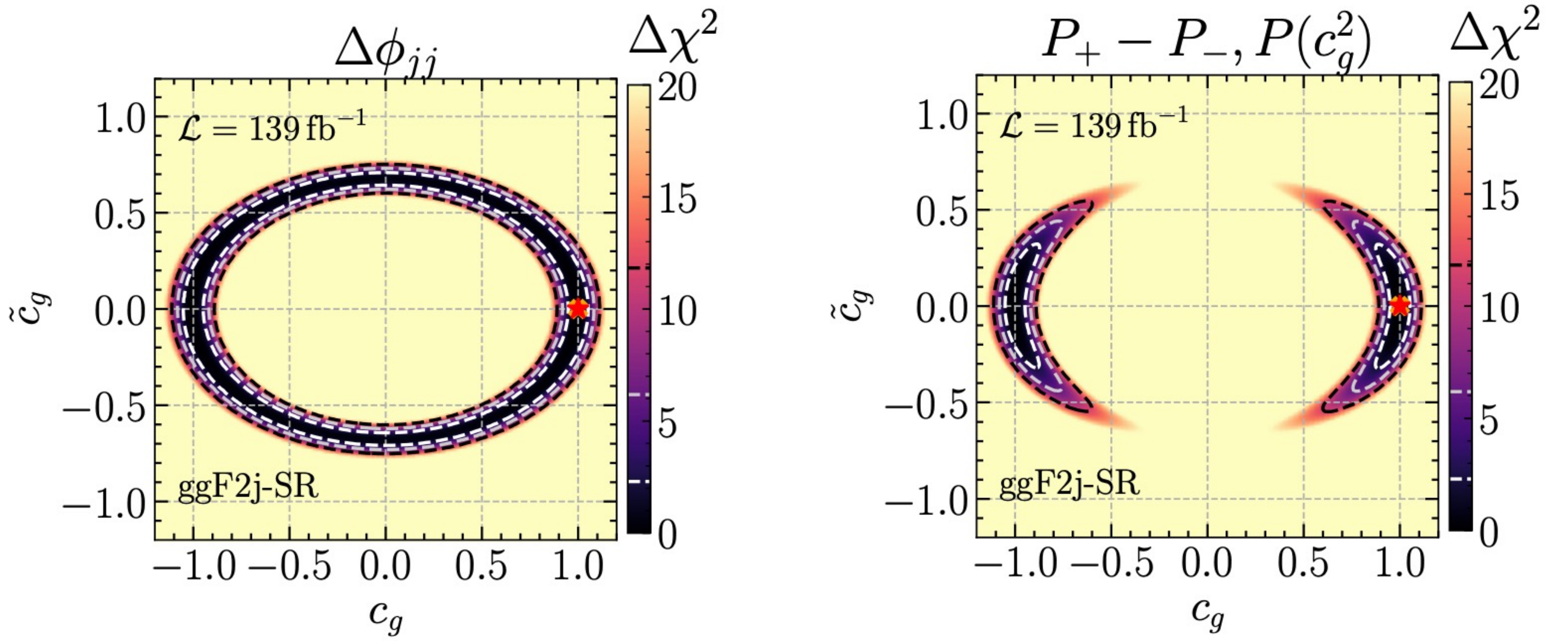
Can we do better?

Analysis flow



- Focus on $H \to \gamma\gamma$ decay channel.
- Two signal regions: **ggF2j-SR**, VBF-SR
- For each signal region: train signal–background classifier.
- Then, train two classifiers to distinguish $|\mathcal{M}_{\text{even}}|^2$ vs. $|\mathcal{M}_{\text{odd}}|^2$ and (positive intf.) vs (negative intf).
- Build two observables: CP-even $P(c_g^2)$ and CP-odd $P_+ - P_-$.

ggF2j signal region



- ggF2j signal region outperforms VBF signal region (not shown),
- $\Delta\phi_{jj}$ limit is significantly worse.

Which observables drive these constraints? → interpretable ML?!

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[shapleyvalue.com]

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Leibniz has no value: $\phi_{\text{Leibniz}} = 0$, $\phi_{\text{Newton}} = 100$

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⇒ Can we formalize this for more complex situations?

Shapley values

[Shapley, `51]

$$\phi_j(val) = \sum_{S \subseteq \{1,\ldots,p\} \setminus \{j\}} \frac{|S|!\,(p - |S| - 1)!}{p!} \left(val(S \cup \{j\}) - val(S)\right)$$

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- $\sum_{S \subseteq \{1,\ldots,p\} \setminus \{j\}}$: Sum over all sets without player j
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- Dummy player: $val(S \cup \{j\}) = val(S)$ for all $S \subseteq \{1, \ldots, p\} \setminus \{j\} \Rightarrow \phi_j = 0$

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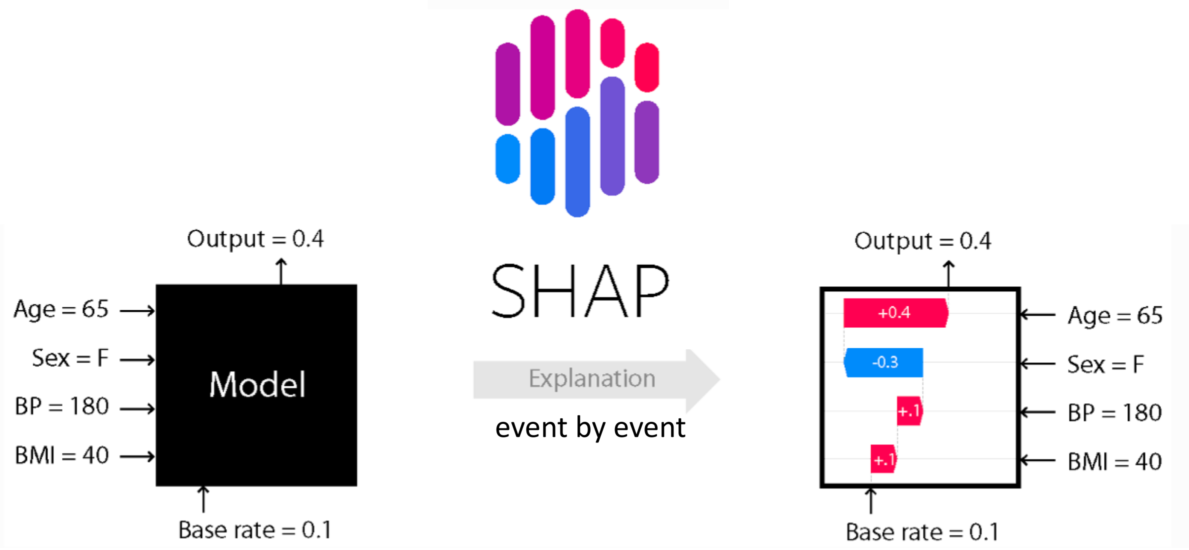

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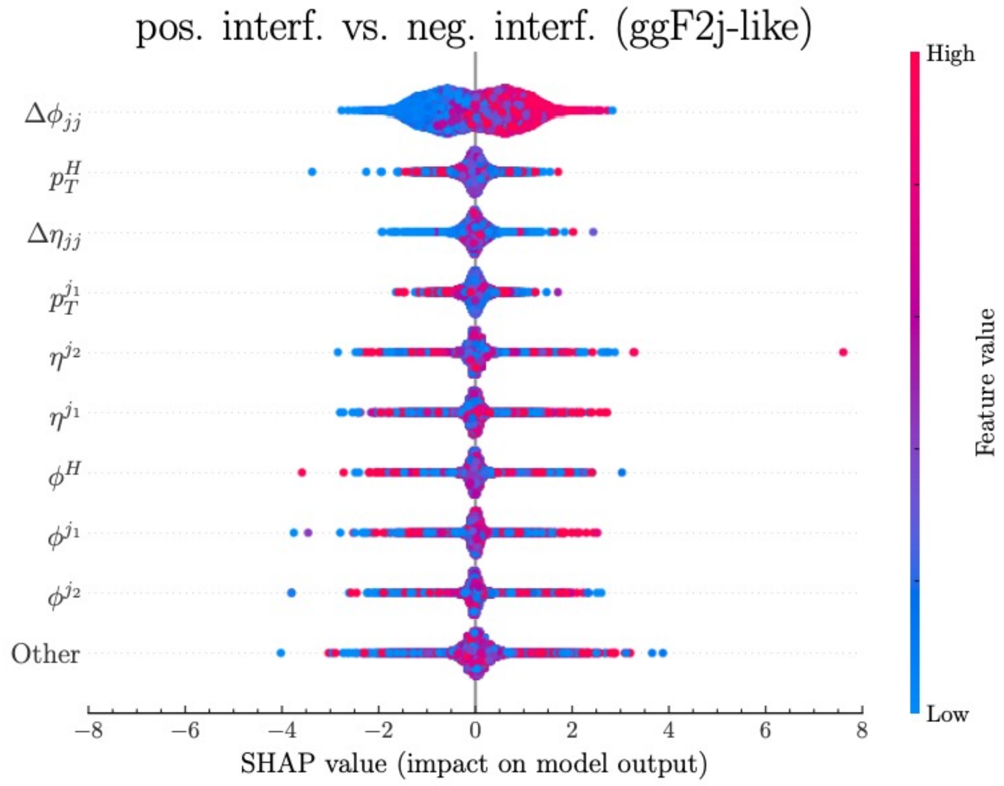

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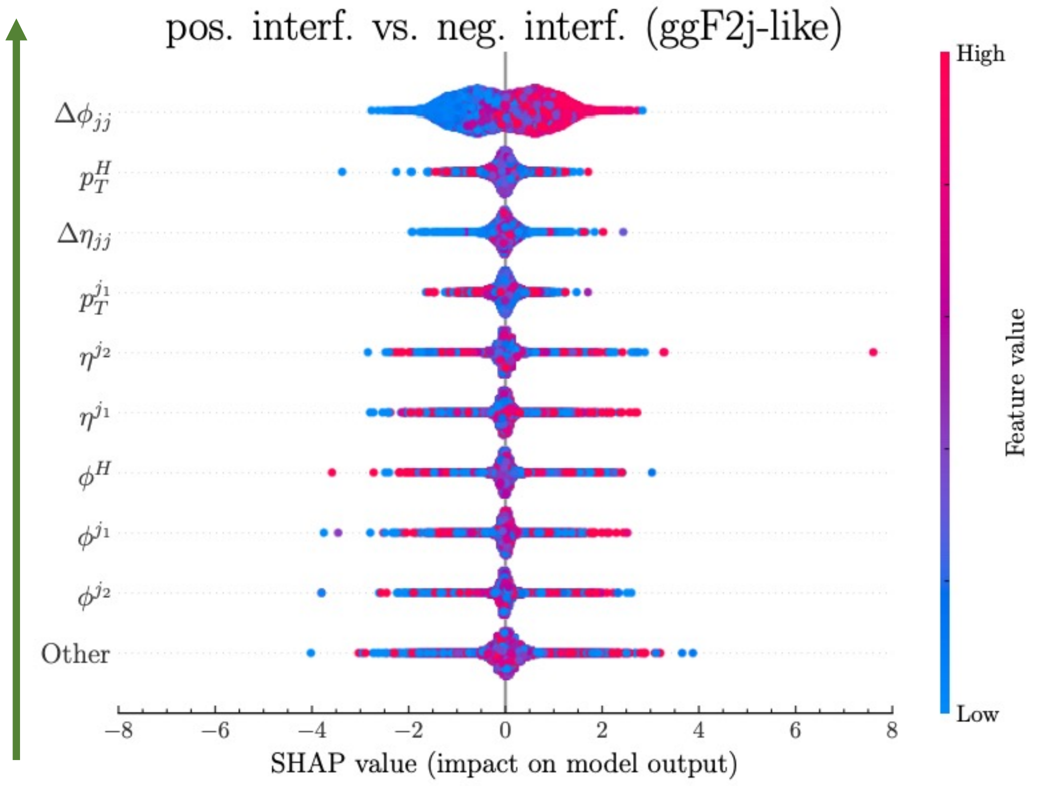
($\sim \sum_{\text{events}} |\phi_j|$)



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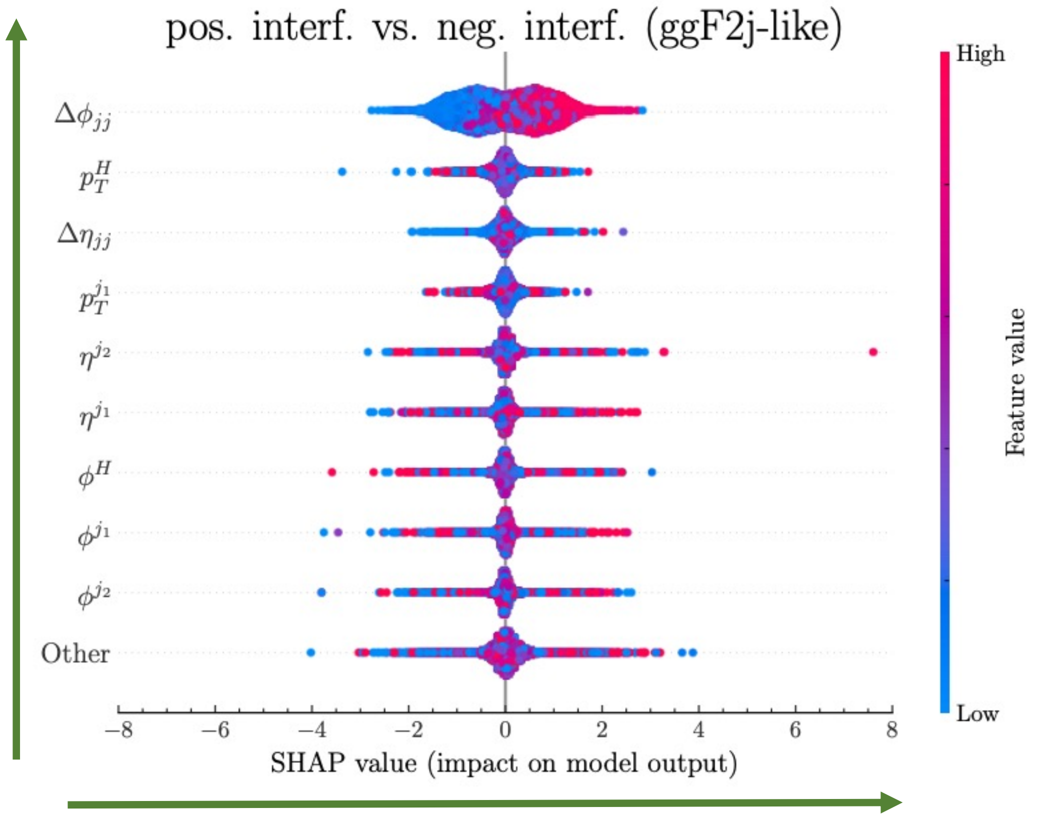

Shapley value for each event (shown as dot)

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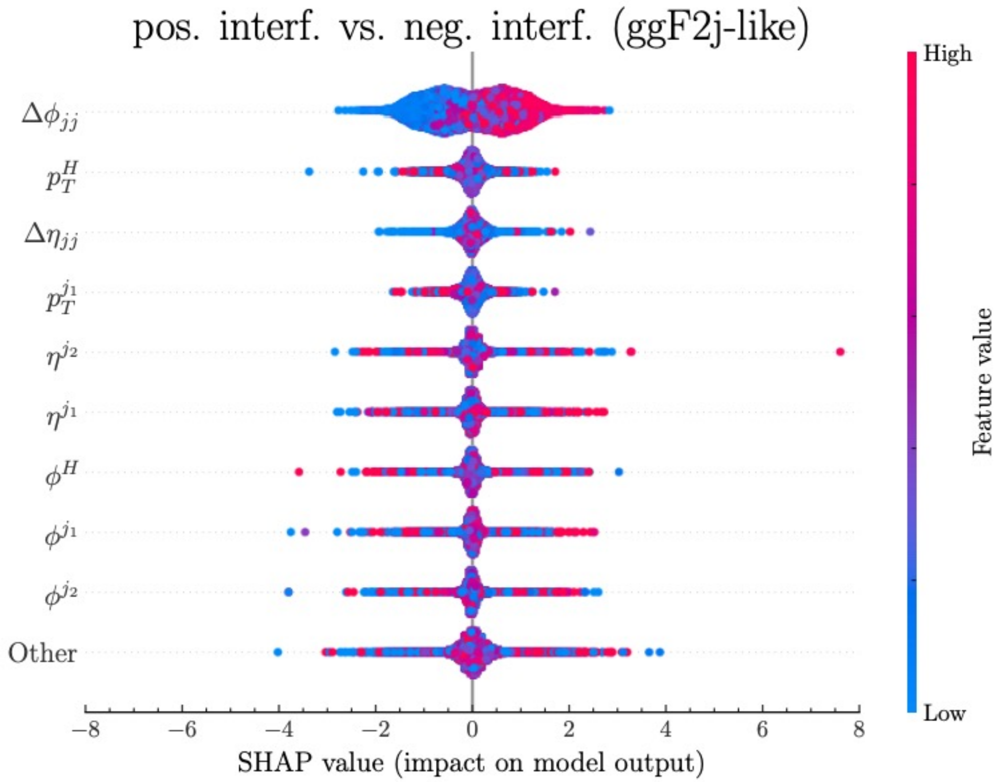

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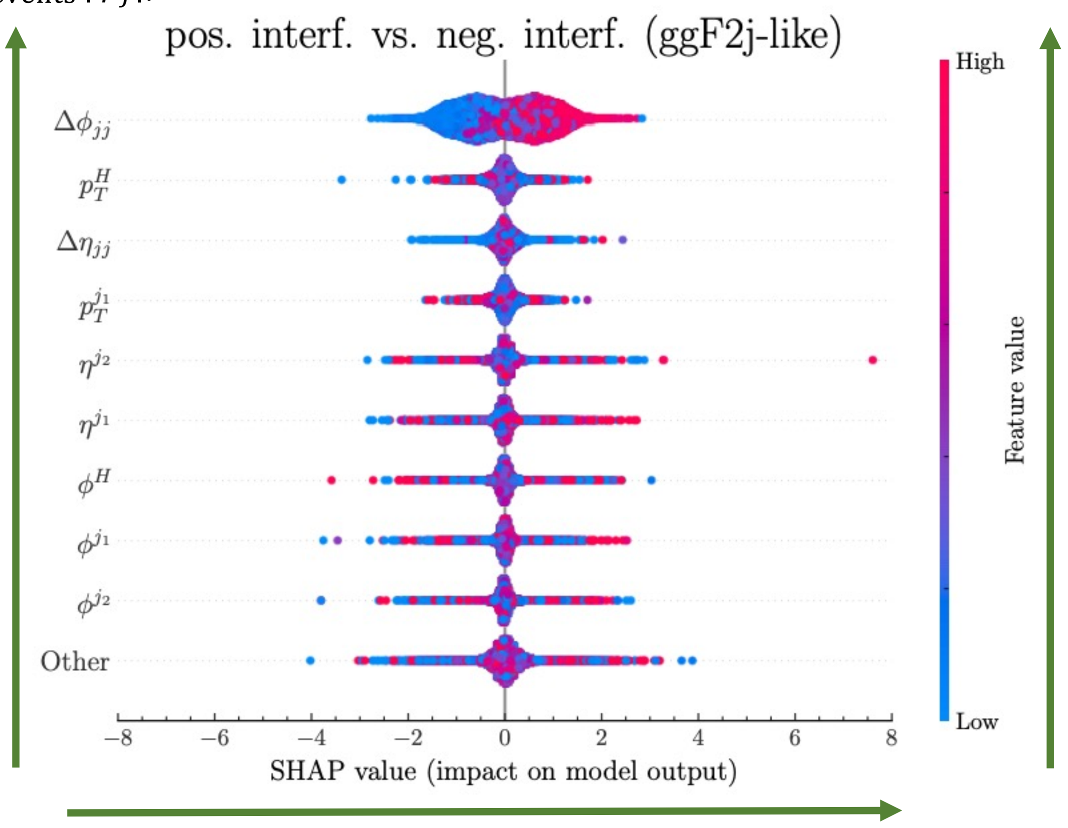

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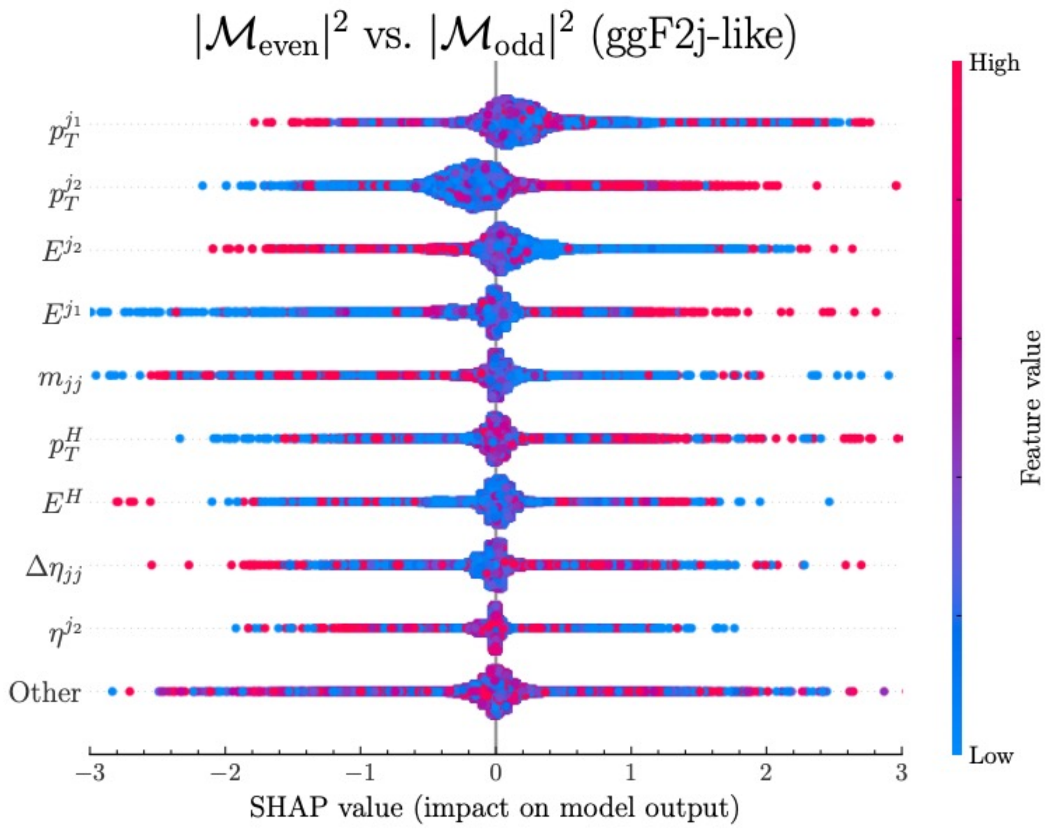
For the interference classifiers, as expected, the CP-odd $\Delta\phi_{jj}$ is most important.

Results for squared term classifiers



- p_T of jets/Higgs most important, $\Delta\phi_{jj}$ plays only subleading role.
- disadvantage: interplay between observables hard to judge.

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[work in progress, HB, Menen, Fuchs, Plehn]

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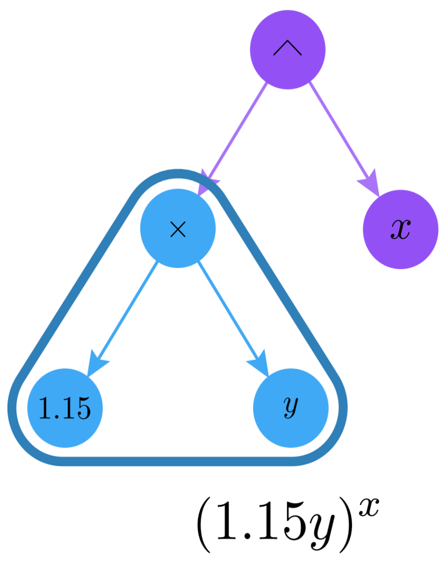

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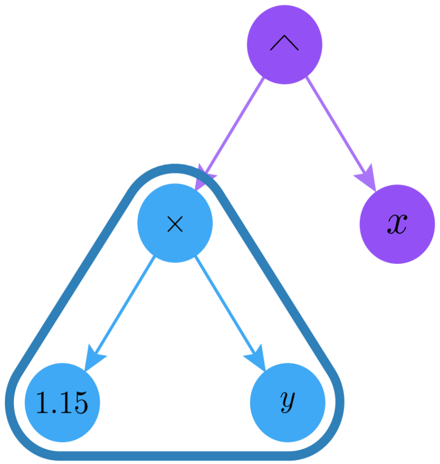

$(1.15y)^x$

Preliminary example: background discrimination VBF vs. ggF2j

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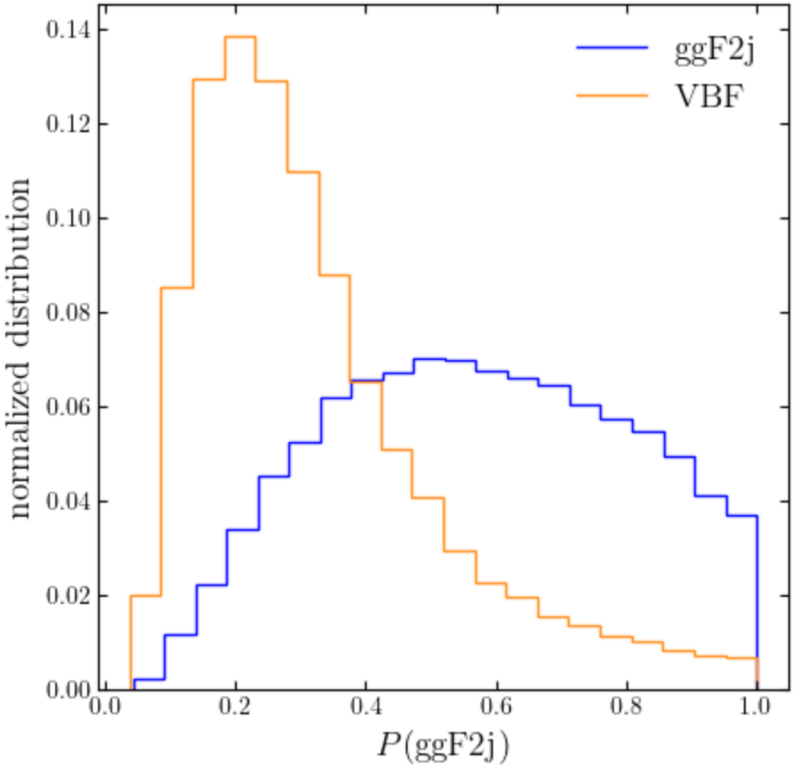
⇒ symbolic regression → $P(\mathrm{ggF2j}) \sim \mathrm{Sigmoid}(p_{\mathrm{T},j_1} \log(|\Delta\eta_{jj}|))$



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- ggF2j production is a key process to probe the Higgs CP character nature.
- Existing analysis focus on VBF-like phase-space region and/or $\Delta\phi_{jj}$.
- Including full phase space information → significantly improved limits.
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Thanks for your attention!

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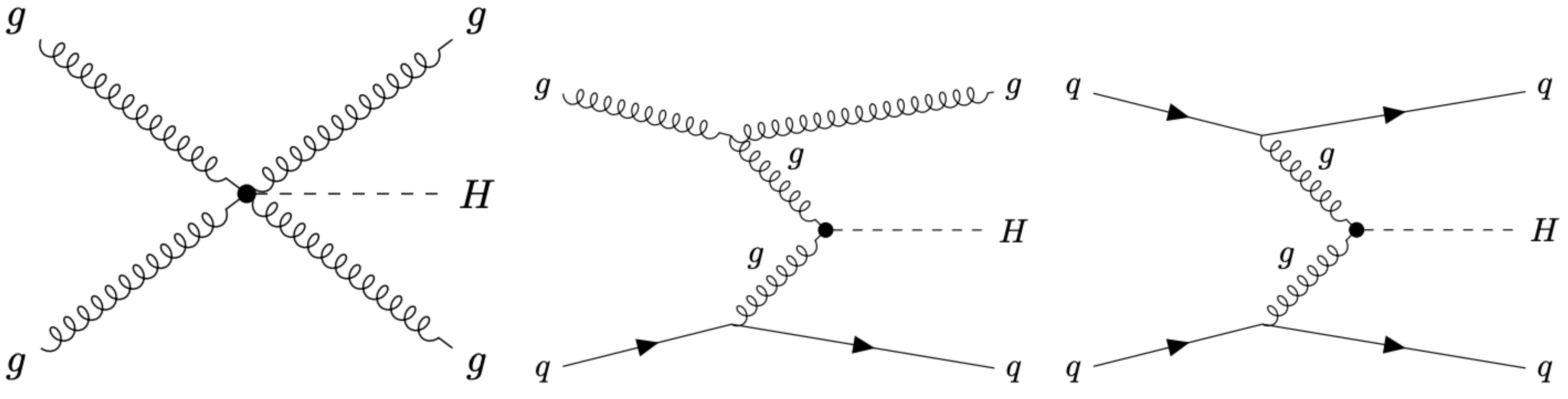

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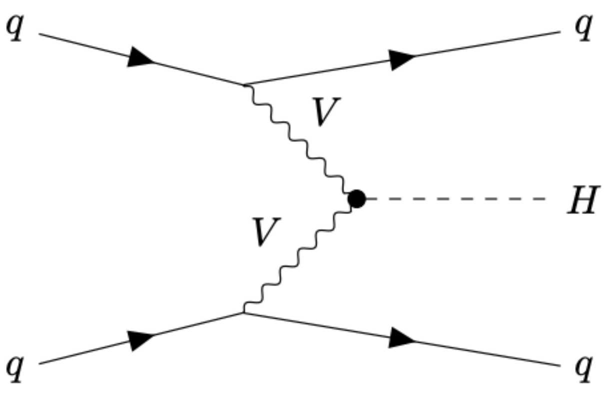
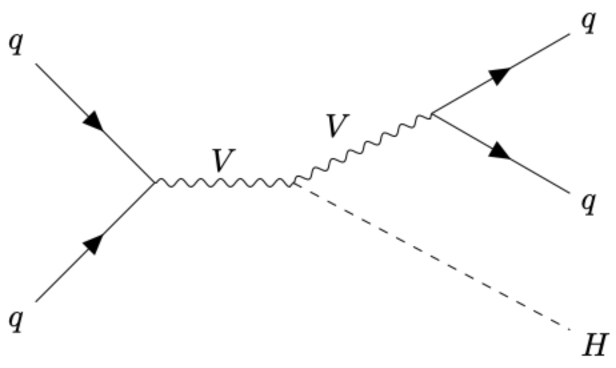
Appendix

Background processes

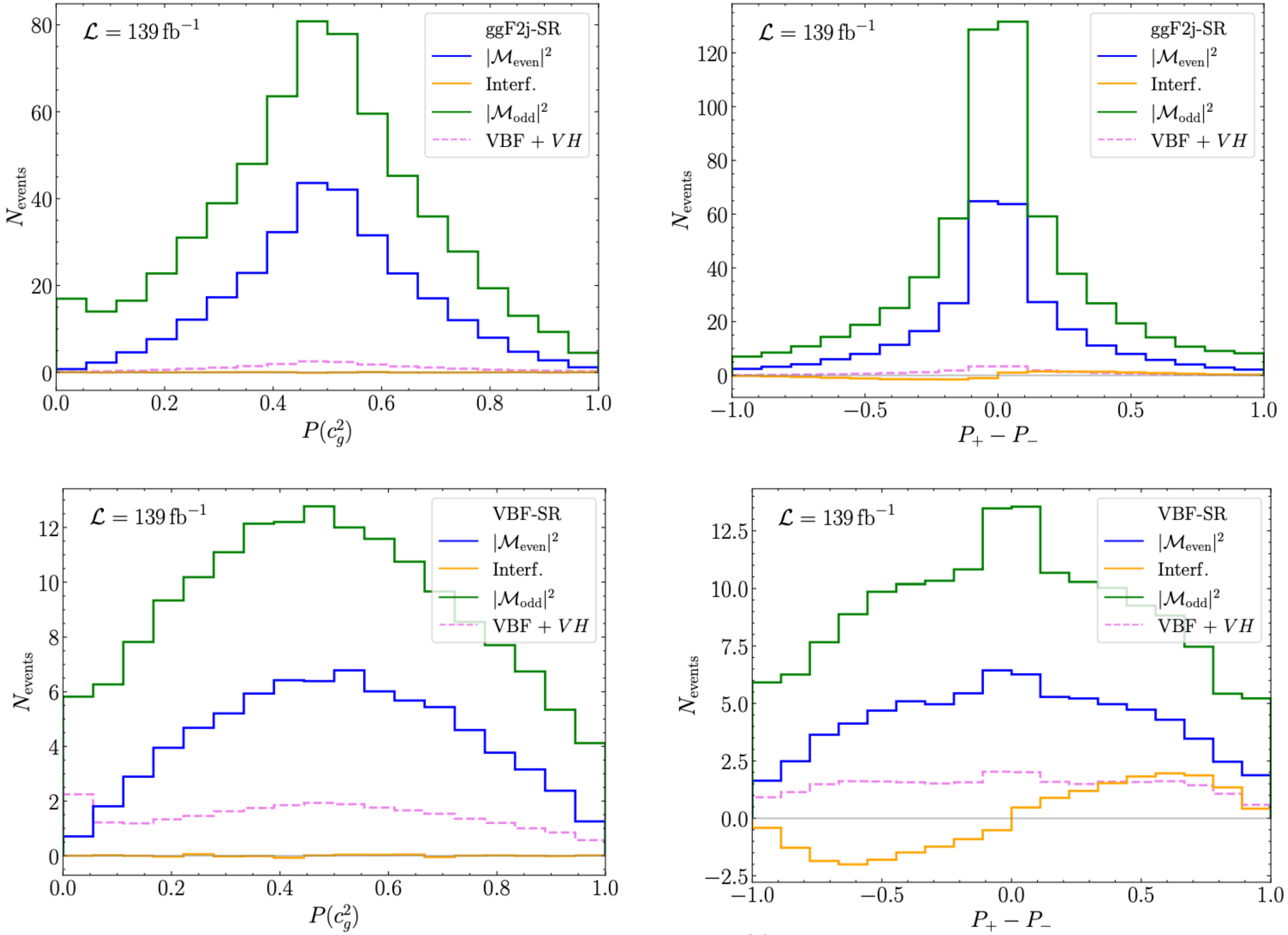


(a) VBF

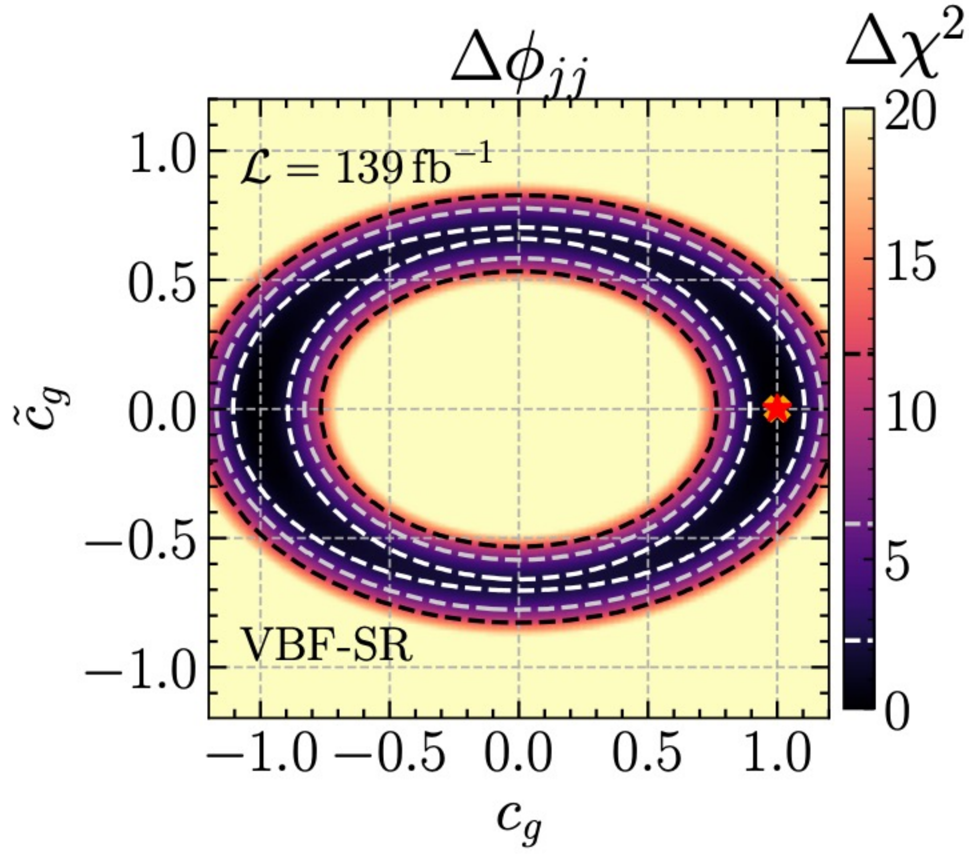


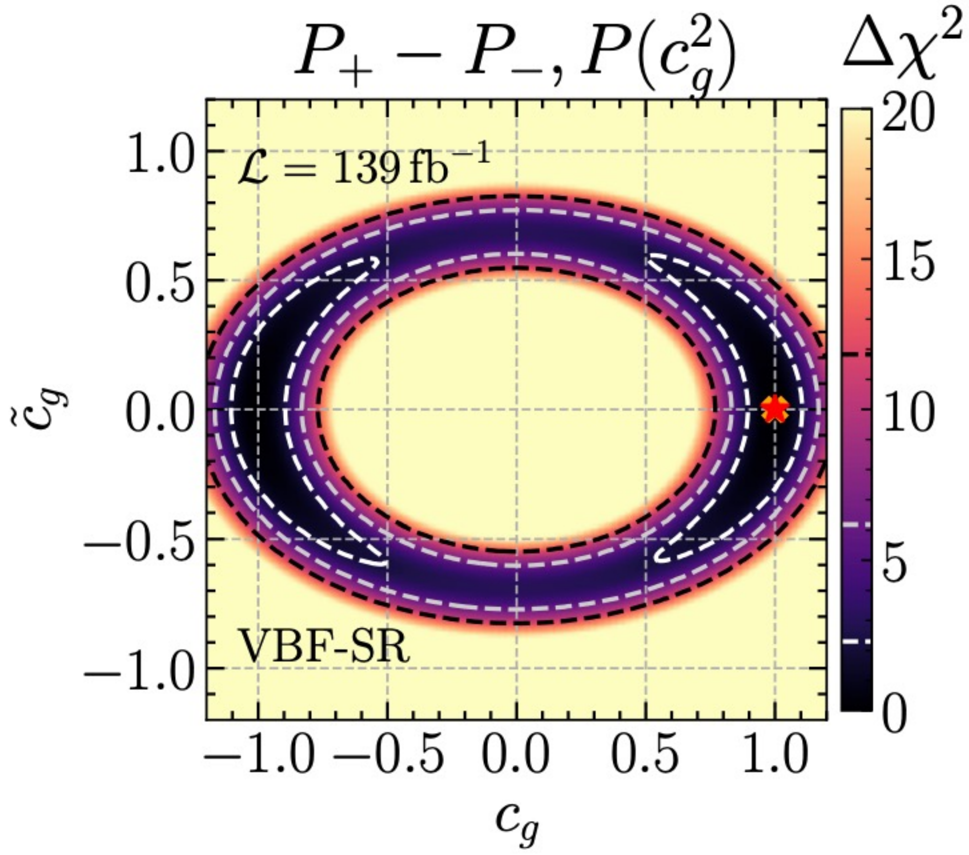
(b) VH

Classifier scores

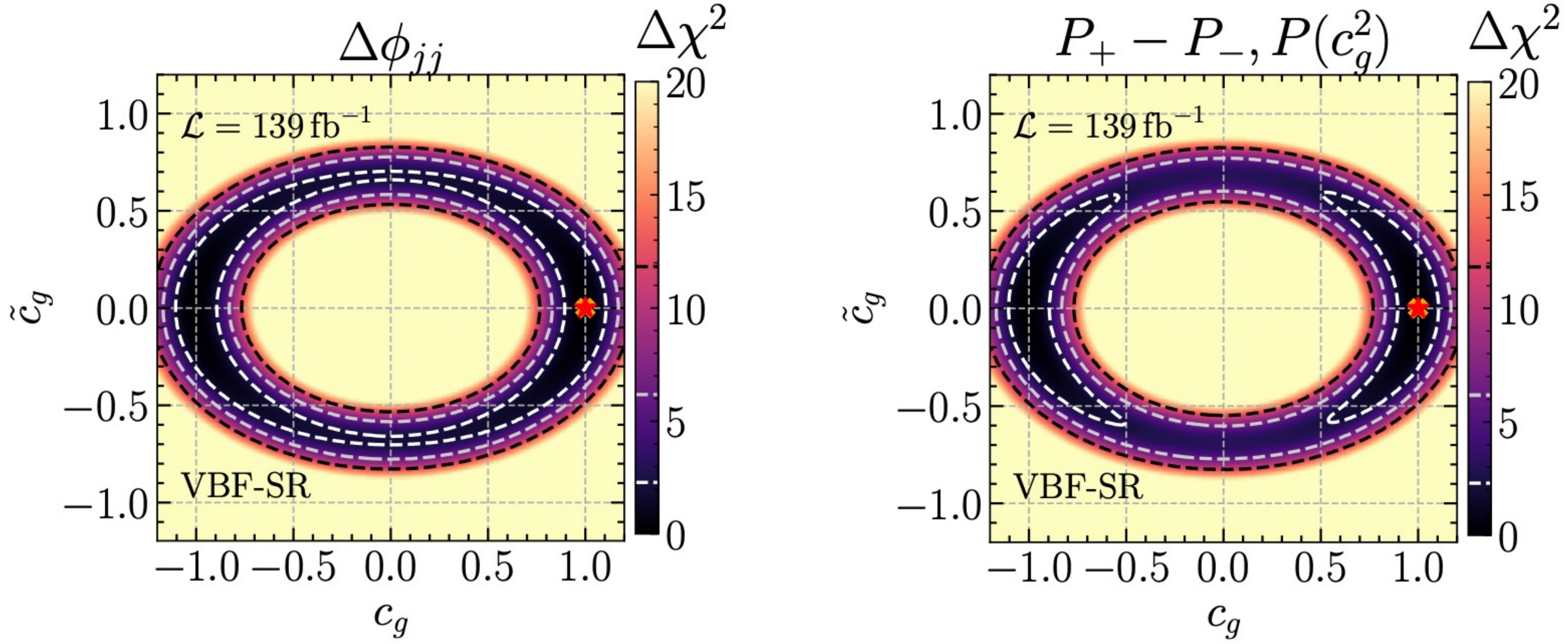


VBF signal region



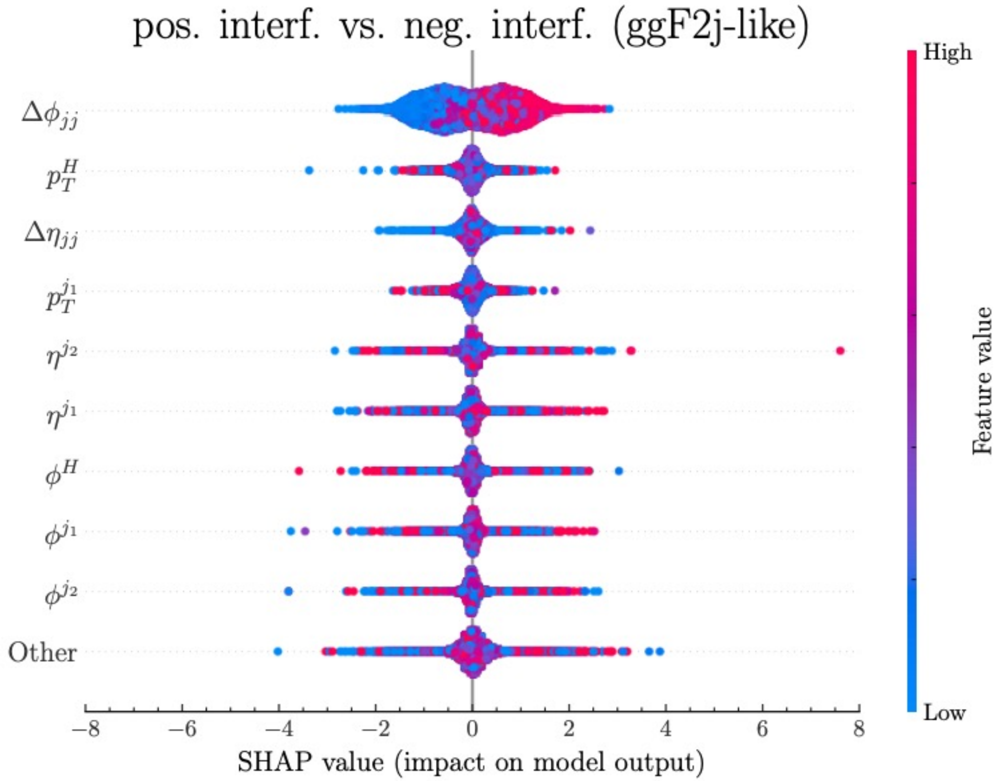


VBF signal region



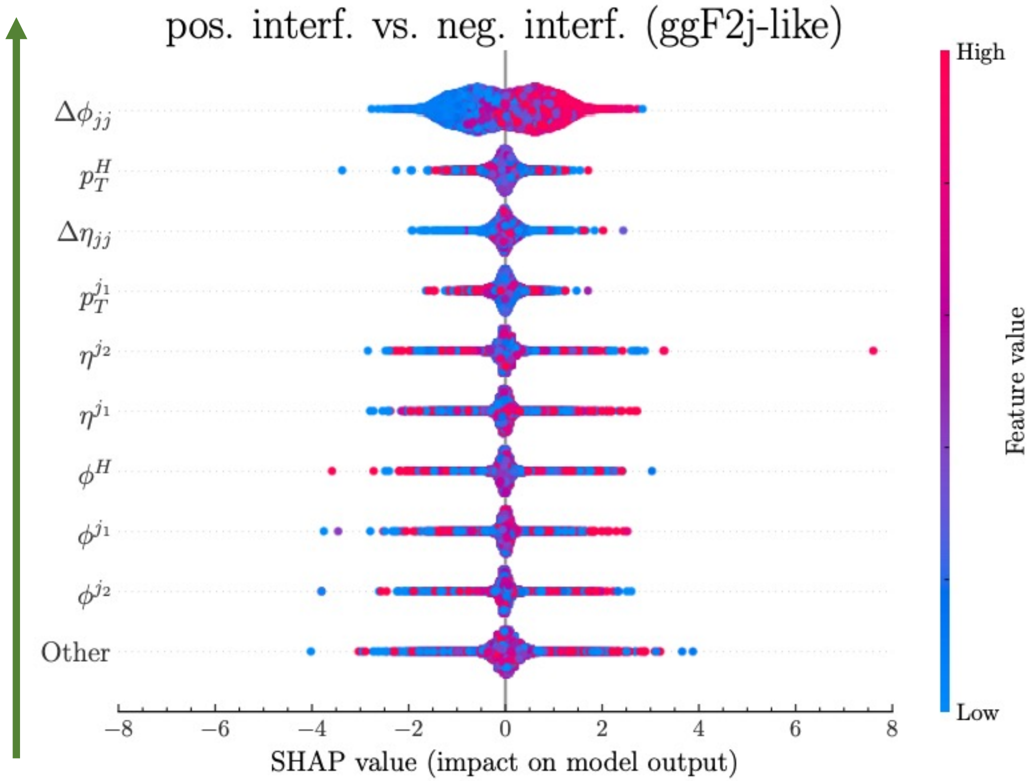
→ $\Delta\phi_{jj}$ limit only slightly worse than limit based on classifiers.

Results for interference classifiers



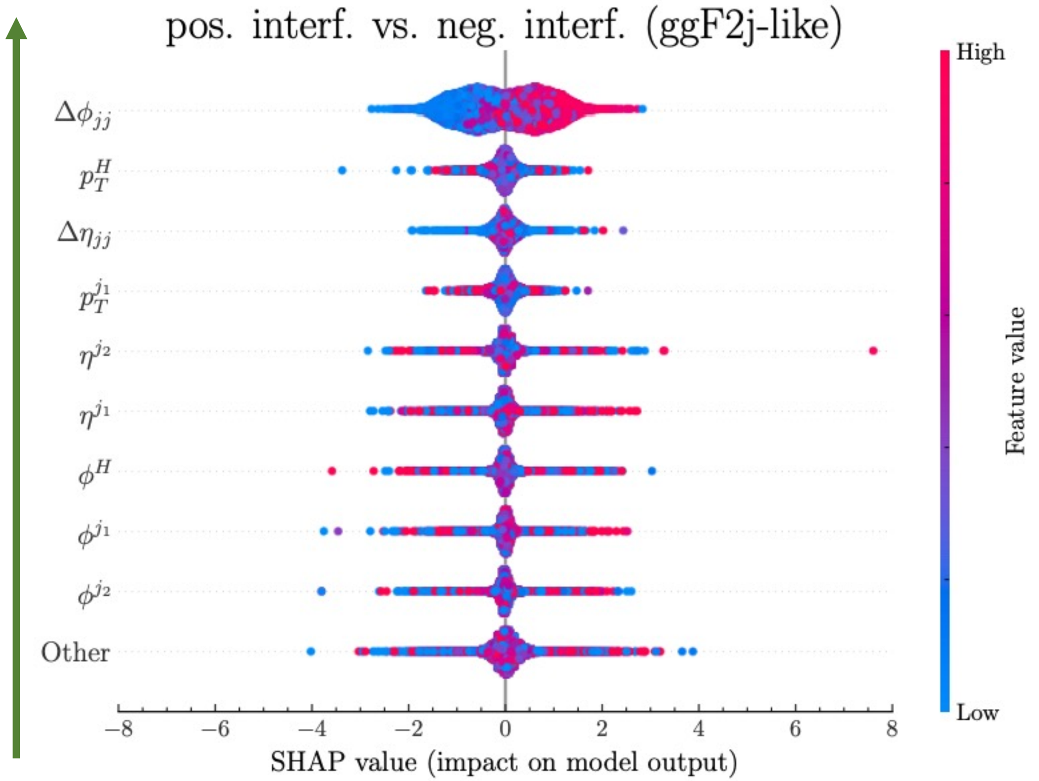
Results for interference classifiers

Importance of observable
($\sim \sum_{\text{events}} |\phi_j|$)



Results for interference classifiers

Importance of observable
($\sim \sum_{\text{events}} |\phi_j|$)

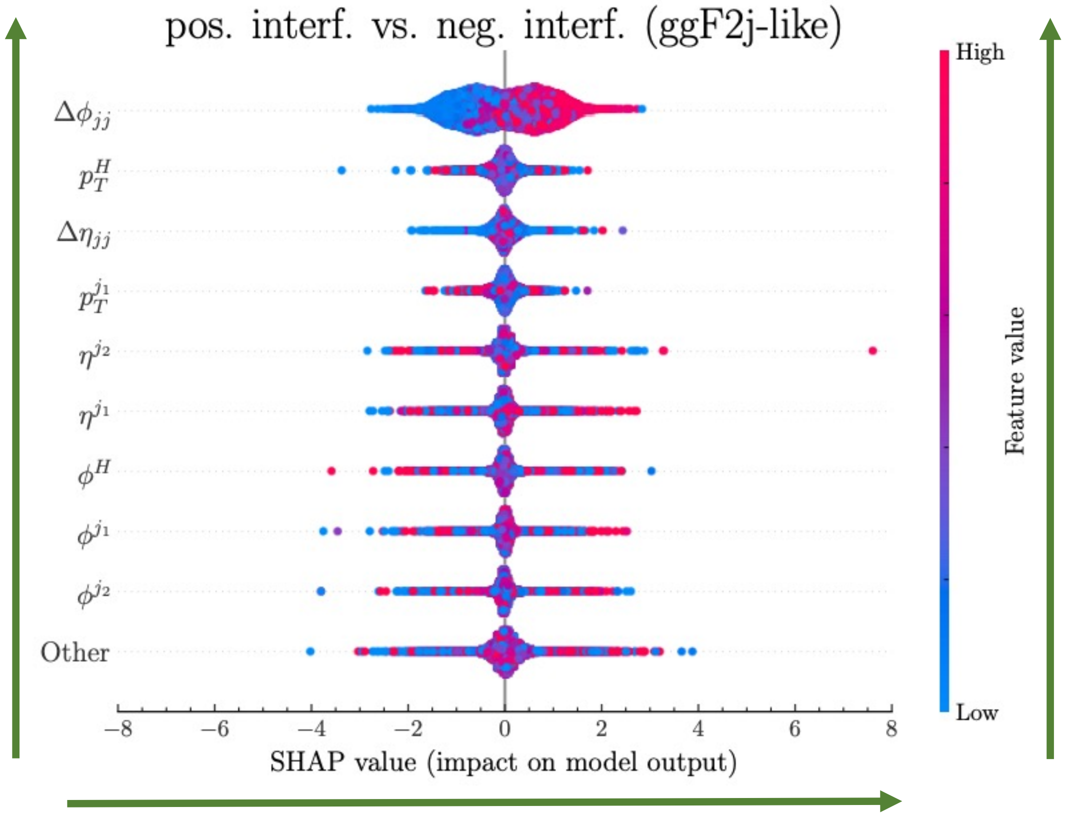


Shapley value for each event (shown as dot)

Results for interference classifiers

Importance of observable
($\sim \sum_{\text{events}} |\phi_j|$)

Value of observable

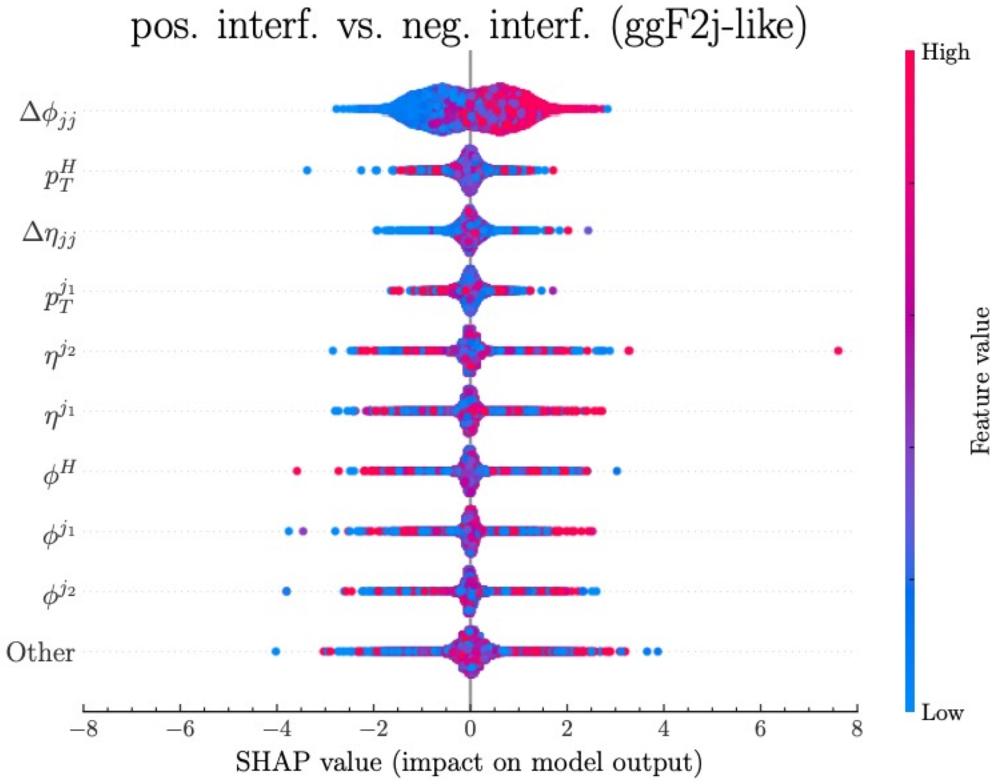


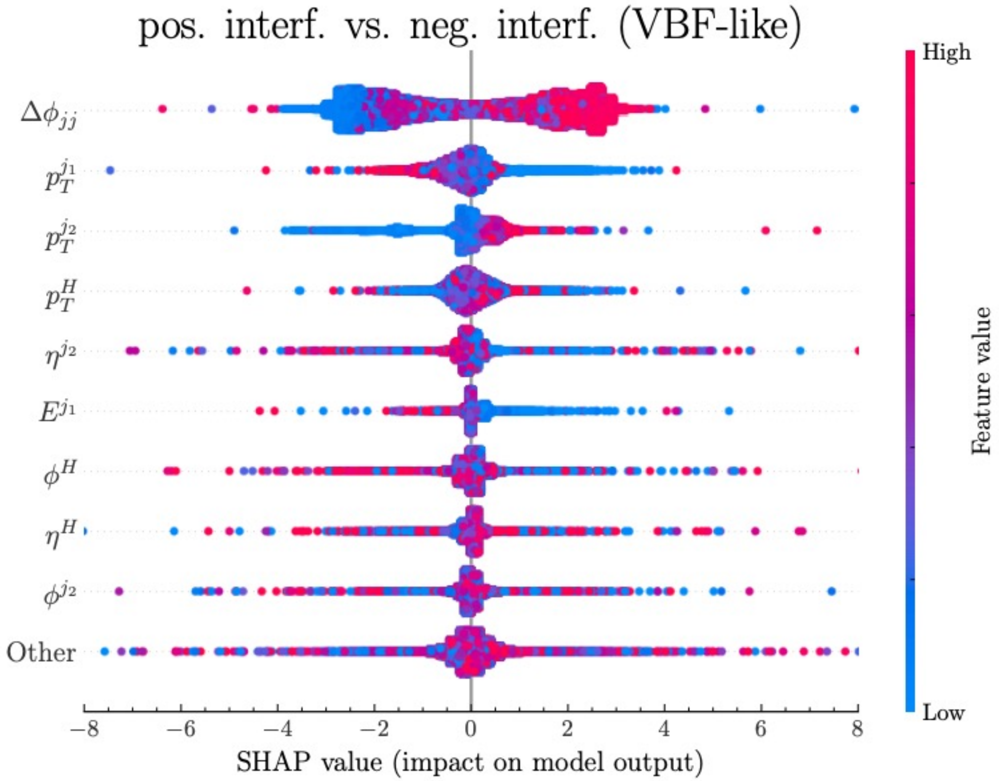
Shapley value for each event (shown as dot)

Results for interference classifiers

Importance of observable
($\sim \sum_{\text{events}} |\phi_j|$)

Value of observable



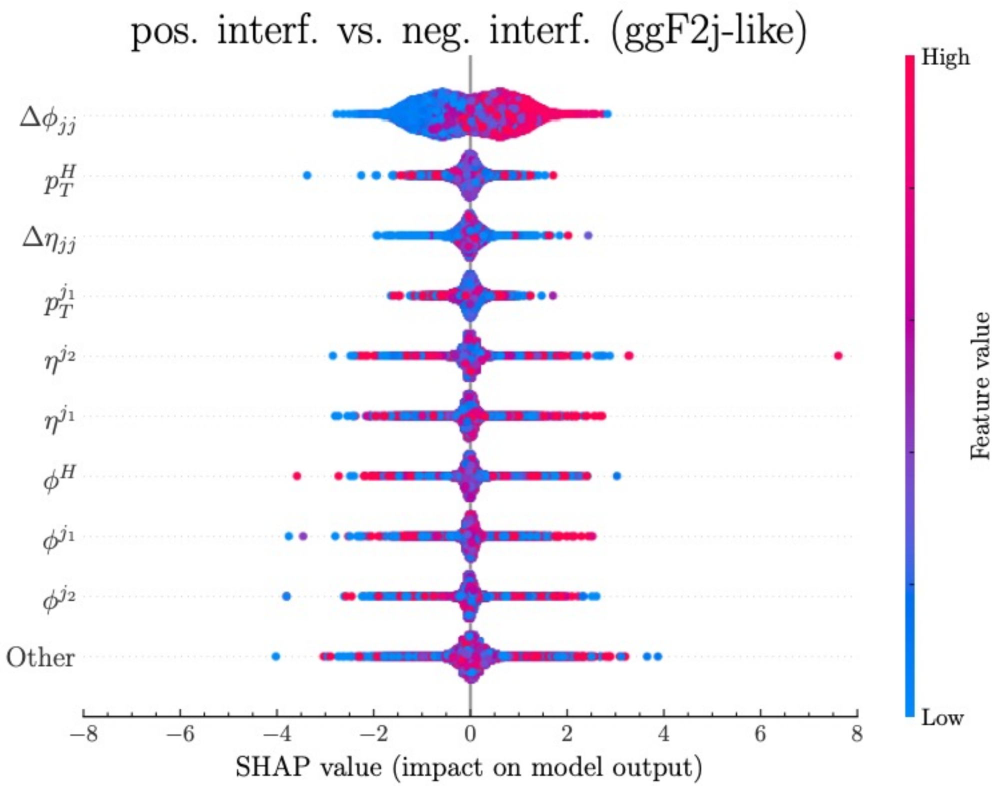


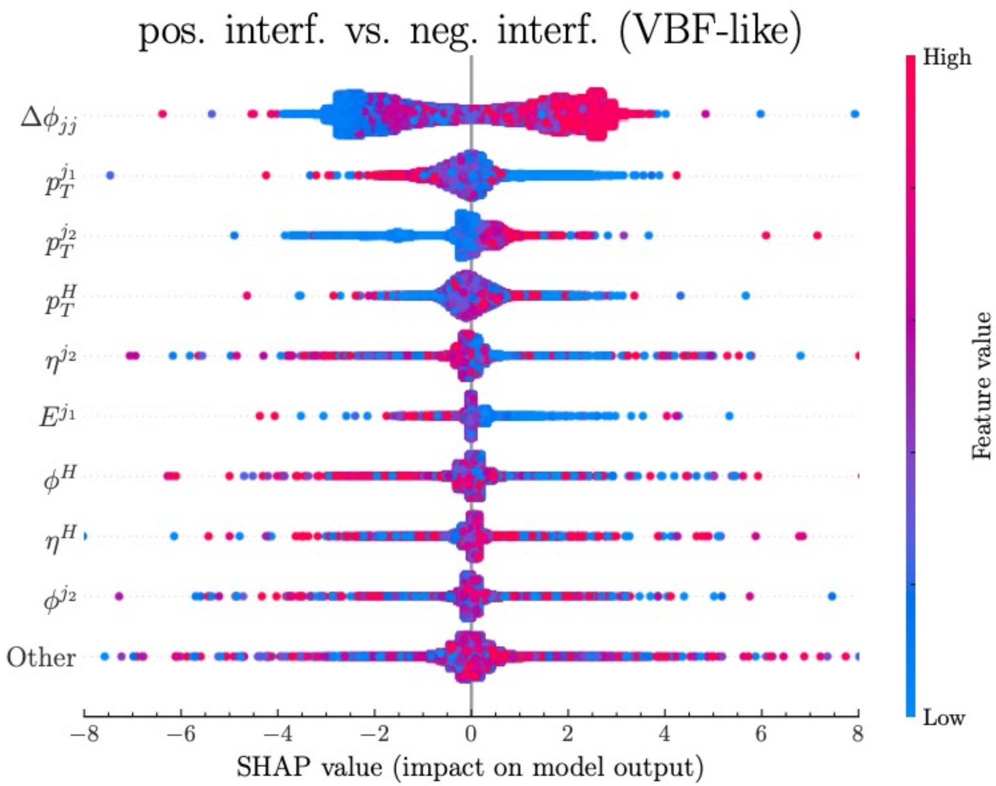
Shapley value for each event (shown as dot)

Results for interference classifiers

Importance of observable
($\sim \sum_{\text{events}} |\phi_j|$)

Value of observable

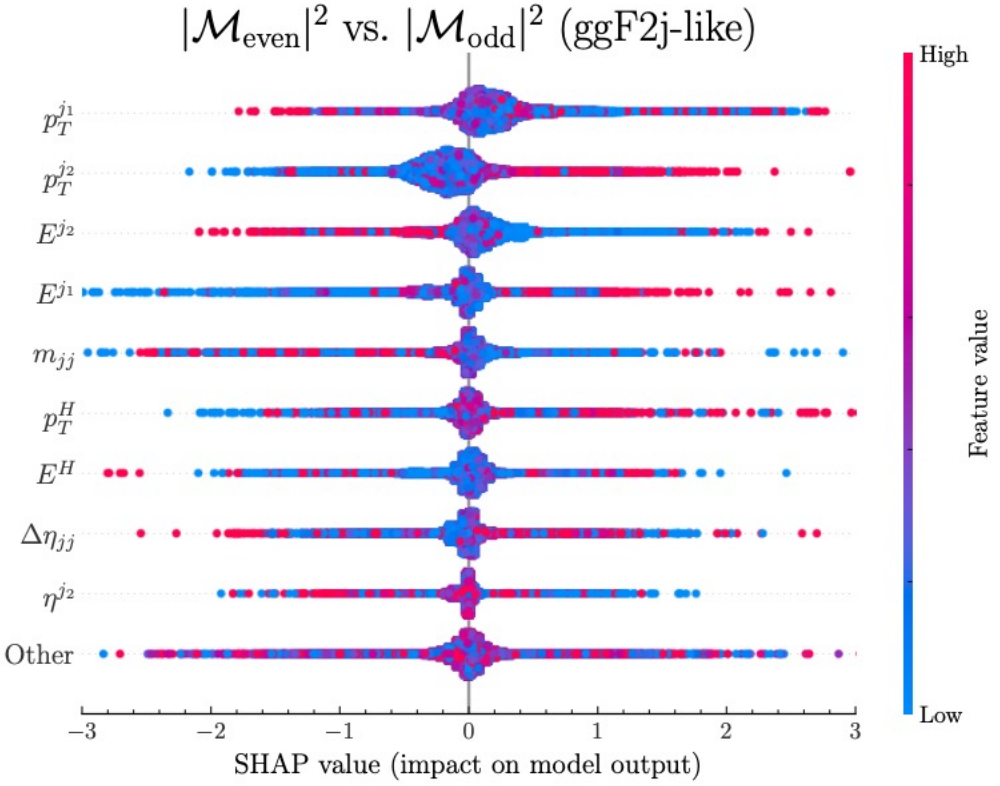


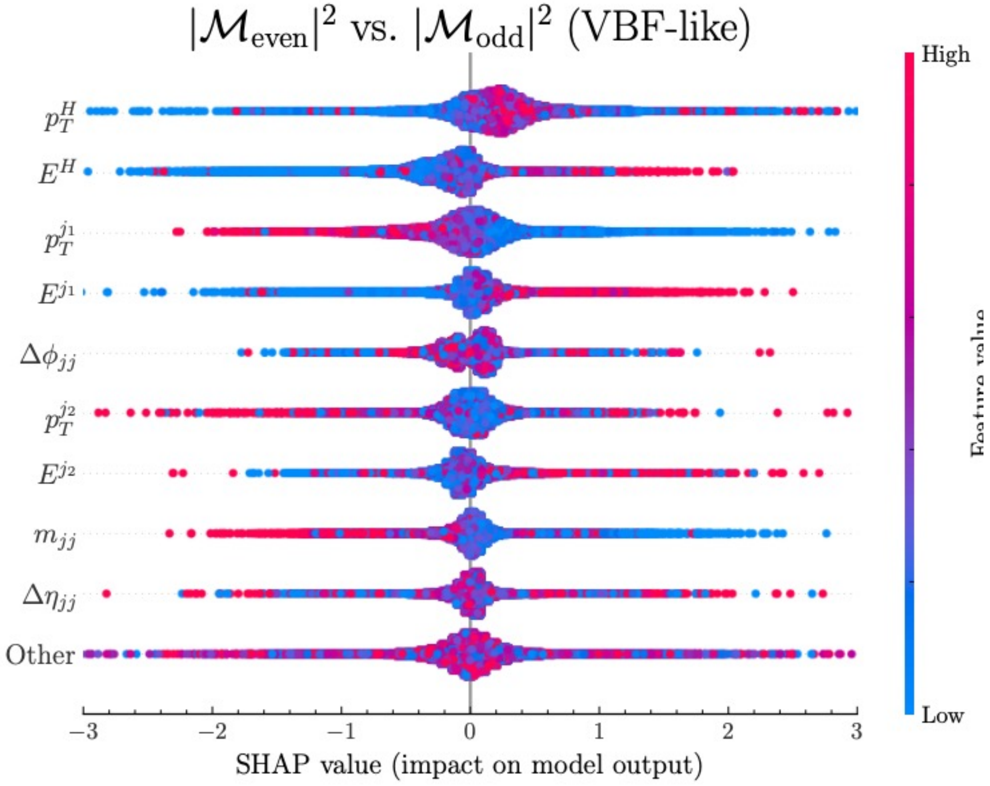


Shapley value for each event (shown as dot)

⇒ For the interference classifiers, as expected, the CP-odd $\Delta\phi_{jj}$ is most important.

Results for squared term classifiers





⇒ p_T of jets/Higgs most important, $\Delta\phi_{jj}$ plays only subleading role.
Disadvantage: interplay between observables hard to judge.