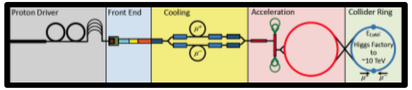
Report from WG on RF: Progress on the design and simulation of RF systems for RCS

Fabian Batsch, Heiko Damerau, Ivan Karpov

Acknowledgements*: David Amorim, Scott Berg, Fulvio Boattini, Luca Bottura, Christian Carli, Antoine Chancé, Alexej Grudiev, Elias Metral, Ursula Van Rienen, Daniel Schulte, Sosoho-Abasi Udongwo

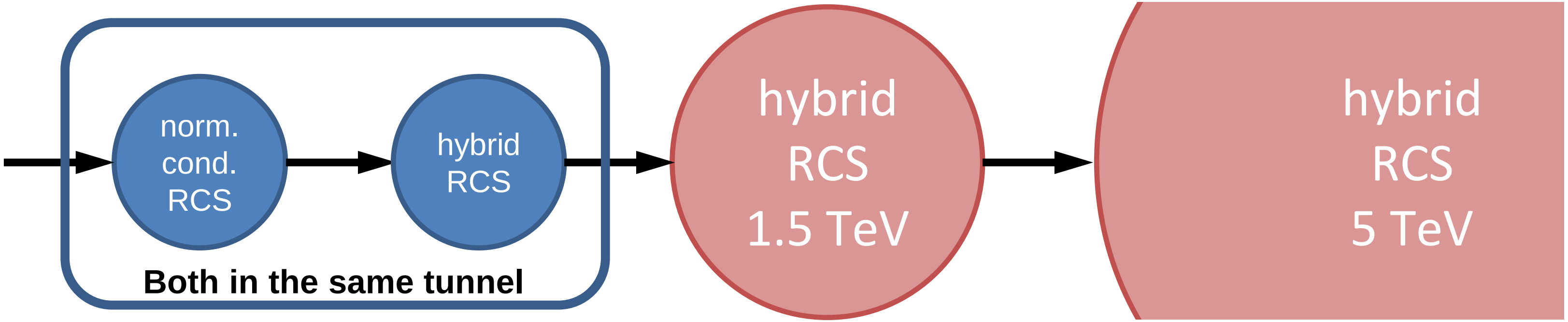
Presented on the Accelerator design meeting, 27/2/2023

Reminder: the muon RCSs



Details on RCS:
See talk by H. Damerau

- Chain of rapid cycling synchrotrons, counter-rotating μ^+/μ^- beams → 63 GeV → 314 GeV → 750 GeV → 1.5 TeV (→ 5 TeV)



- Hybrid RCSs have intersecting normal conducting (NC) and superconducting (SC) magnets
- Studies presented aim to determine the RF (cavity) and lattice parameters (number of RF stations, momentum compaction factor,…)

H. Damerau

Outline

- Activity summary
- Beam induced power estimates for muon RCS RF systems
- Studies on synchronous phase and consequences on the acceleration
- Summary and Outlook

Activity summary / reminder

- **From MC Collaboration Meeting: [reminder, see presentation for details]**
  1. **In total, short-range wakefields and beam loading cause induced voltage of ~2.2 MV/m per cavity, or 10% of V_{acc}, but do not harm beam transport**
  2. **On the order n_{RF} = 32 RF stations needed to ensure a sufficiently low synchrotron tune between stations, less but n_{RF} > 16 for RCS3**

Activity summary / reminder

- From MC Collaboration Meeting: [reminder, see presentation for details]
  1. In total, short-range wakefields and beam loading cause induced voltage of ~2.2 MV/m per cavity, or 10% of V_{acc}, but do not harm beam transport
  2. On the order n_{RF} = 32 RF stations needed to ensure a sufficiently low synchrotron tune per station, less but n_{RF} > 16 for RCS3
- Muon RCS in LHC tunnel?→ limited to around 4.2 TeV due to required amount of magnets [preliminary studies by D. Amorim and me, see here]
- Question of possibly high HOM power for the TESLA cavity raised during collaboration meeting → 1st topic of today

Beam-induced power for the TESLA cavity

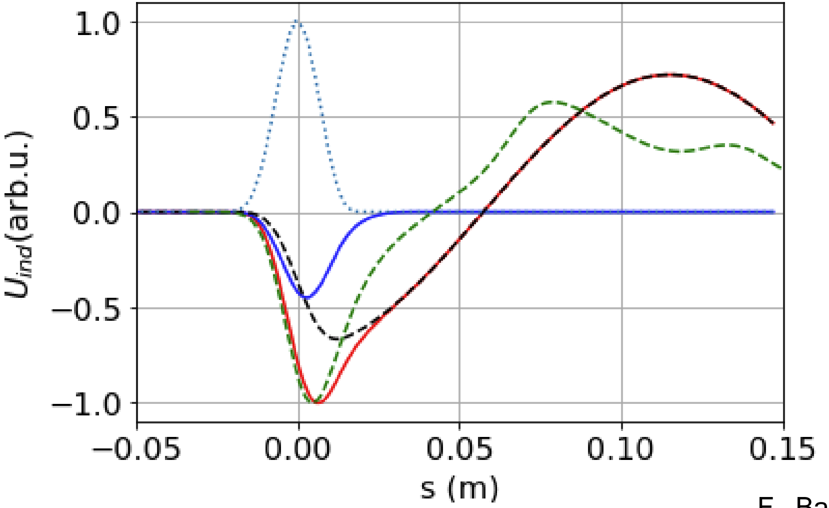
- First estimate of HOM power assume a constant current in ring:
  - Bunch population 2.54×10^{12} μ/b, T_{rev} = 20 µs → I = 20.4 mA
  - Induced voltage from short-range wakefields $U_{\mathrm{ind,SR}}$ = 1.1 MV/m
  - → Rough limit estimate per cavity: P = 20.4 mA x 1.1 MV = 22.4 kW

- Calculation of HOM power in TESLA / ILC 1.3 GHz cavity in two ways for a single bunch from loss factors using:
  - Approximated wake potentials in macro-particle tracking simulations (BLonD)
  - The output of ABCI code for detailed RF structure

Beam-induced power with BLonD

- The geometry of the cavity defines all HOM, i.e. for single-bunch cases, the short-range wakefield from K. Bane [ref, see appendix for details] includes these, but not the long-range fundamental mode

→ Use short-range wake potential $W_{||,SR}$ to compute power



Plot shows:

bunch charge density, $\sigma = 6.6$mm

U_{ind}, short-range

U_{ind}, fundamental mode

Total U_{ind}

Wake potential from ABCI

(for RCS1, $n_{\text{RF}} = 32$, parameter in appendix)

F. Batsch

Beam-induced power with BLonD

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  → Use short-range wake potential $W_{||,SR}$ to compute power

- Calculate power loss through loss factor $k_{||}$ for each simulation step / RF station:

$k_{||} = \int \lambda(t) W_{||,SR}(t) dt$, with bunch charge density $\lambda(t)$

$$\Big\downarrow$$

$P_{HOM} = k_{||} * \frac{Q^2}{T_\mathrm{B}}$ with bunch charge Q and bunch spacing $T_\mathrm{B} = T_\mathrm{rev}$

Beam-induced power from mode analysis

- Second possibility uses an approximation for short Gaussian bunches to compute loss factor

$$k_{||} = \left|\frac{R}{Q}\right| \frac{\omega_r}{2} * e^{-(\omega_r \sigma)^2} \quad (\frac{\omega_r}{4} \text{ for Linac norm})$$

- This gives the loss factor per mode, for longitudinal modes, see here (TESLA) & paper (ILC LL)

→ Total HOM loss factor is sum over all HOMs:

$$k_{||} = \sum k_{||,\mathrm{i}}\,, \qquad P_{HOM} = k_{||} * \frac{Q^2}{T_\mathrm{B}}$$

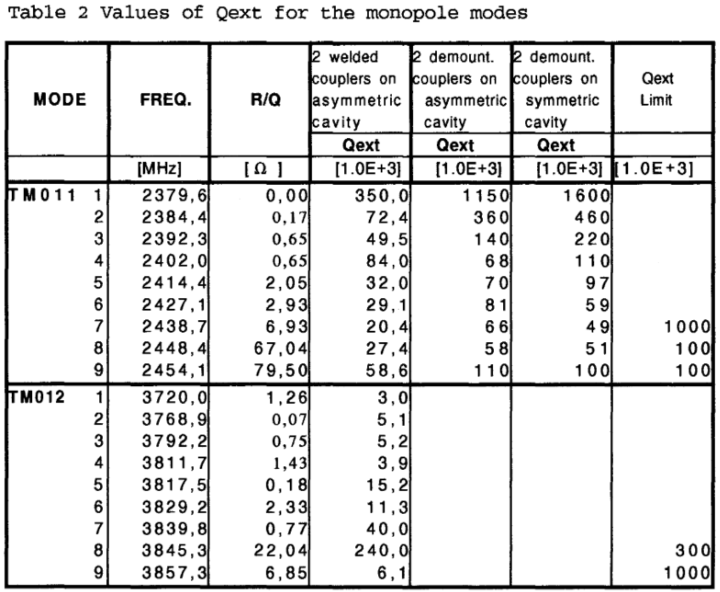


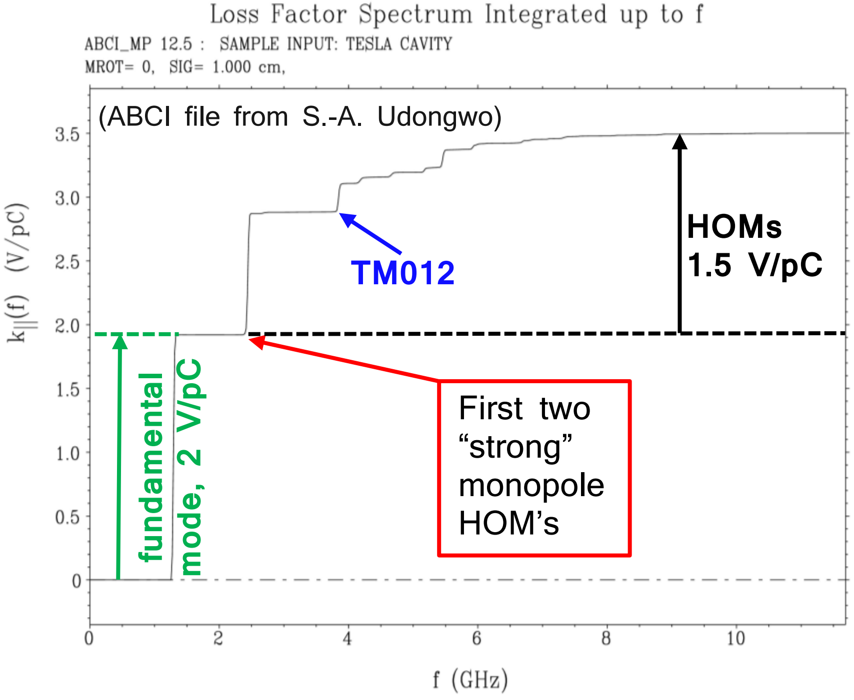
Table 2 Values of Qext for the monopole modes

MODE		FREQ.	R/Q	2 welded couplers on asymmetric cavity	2 demount. couplers on asymmetric cavity	2 demount. couplers on symmetric cavity	Qext Limit
				Qext	Qext	Qext	
		[MHz]	[Ω]	[1.0E+3]	[1.0E+3]	[1.0E+3]	[1.0E+3]
TM011	1	2379,6	0,00	350,0	1150	1600	
	2	2384,4	0,17	72,4	360	460	
	3	2392,3	0,65	49,5	140	220	
	4	2402,0	0,65	84,0	68	110	
	5	2414,4	2,05	32,0	70	97	
	6	2427,1	2,93	29,1	81	59	
	7	2438,7	6,93	20,4	66	49	1000
	8	2448,4	67,04	27,4	58	51	100
	9	2454,1	79,50	58,6	110	100	100
TM012	1	3720,0	1,26	3,0			
	2	3768,9	0,07	5,1			
	3	3792,2	0,75	5,2			
	4	3811,7	1,43	3,9			
	5	3817,5	0,18	15,2			
	6	3829,2	2,33	11,3			
	7	3839,8	0,77	40,0			
	8	3845,3	22,04	240,0			300
	9	3857,3	6,85	6,1			1000

From
“Higher order mode coupler for TESLA”, J. Sekutowisz

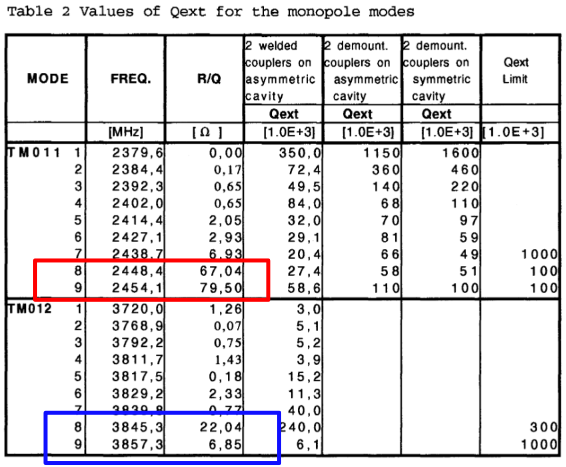
Beam-induced power from mode analysis

- **We obtain the modes through ABCI:**



(ABCI file from S.-A. Udongwo)

First two "strong" monopole HOM's



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(ABCI file from Sosoho U.)

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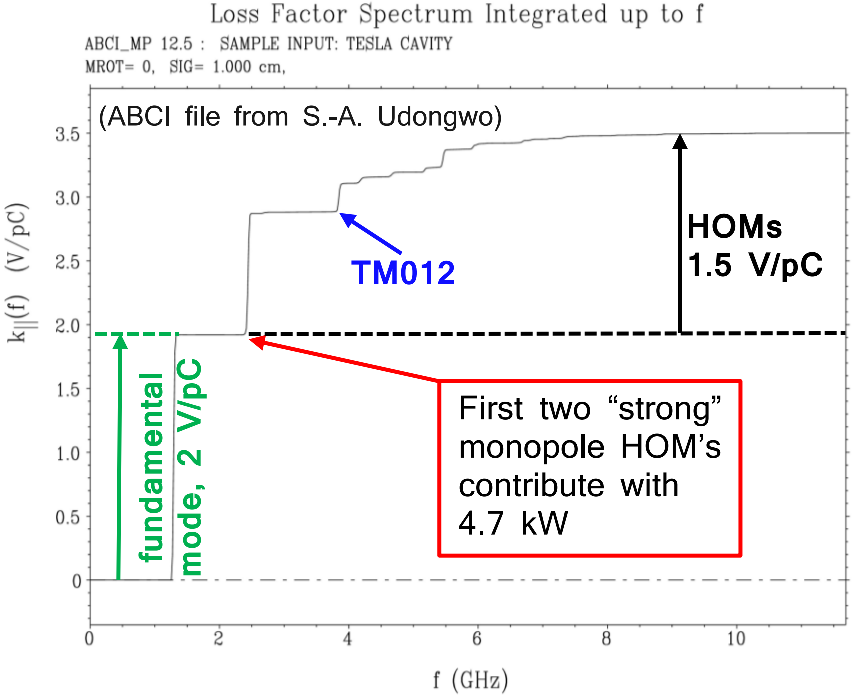


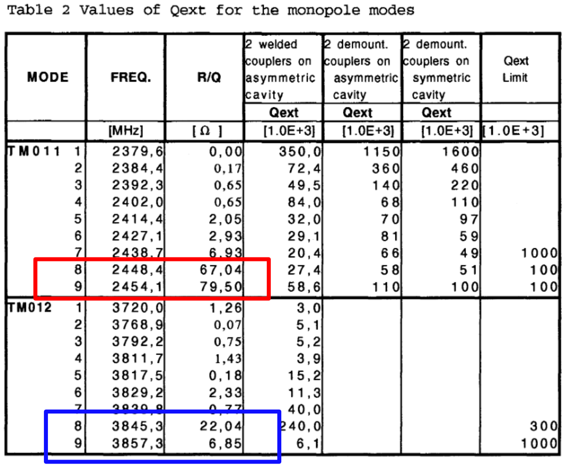
(ABCI file from S.-A. Udongwo)

TM012

HOMs
1.5 V/pC

fundamental mode, 2 V/pC

First two "strong" monopole HOM's contribute with 4.7 kW



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From "Higher order mode coupler for TESLA", J. Sekutowisz

- **1.5 V/pC results in 7.9 kW!**

→ Consistent with upper limit of 22.4 kW

- Power for fundamental mode is 10 kW

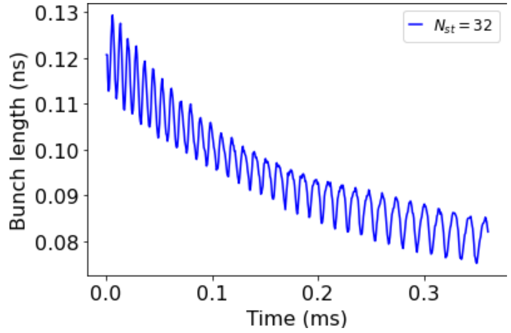
Beam-induced power using BLonD

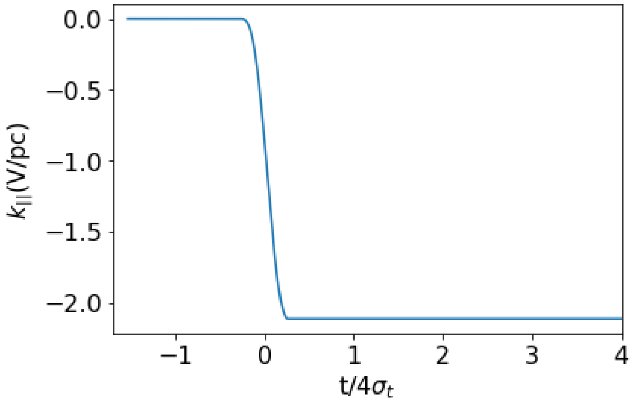
- **Parameters in BLonD: RCS1, n_{RF} = 32 RF stations, 696 cavities, 90% survival, bunch length $4\sigma_z$ = 0.1 ns = 30 mm, 1 bunch**

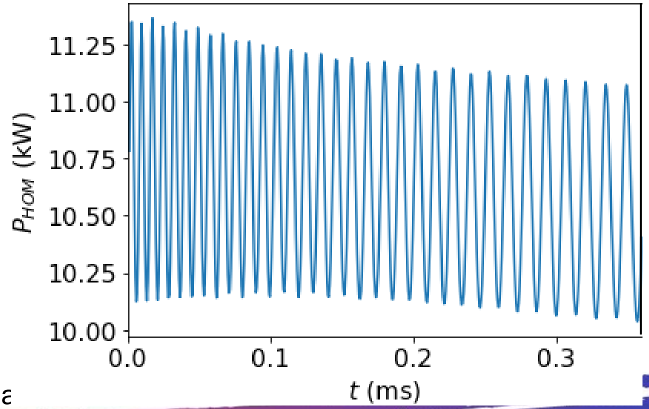
→ $k_{||,SR} = \int \lambda(t) W_{||,SR}(t)dt$ **= -2.11 V/pC**

→ **The HOM power loss per cavity reaches 10.4 kW**

→ **Consistent with ABCI**

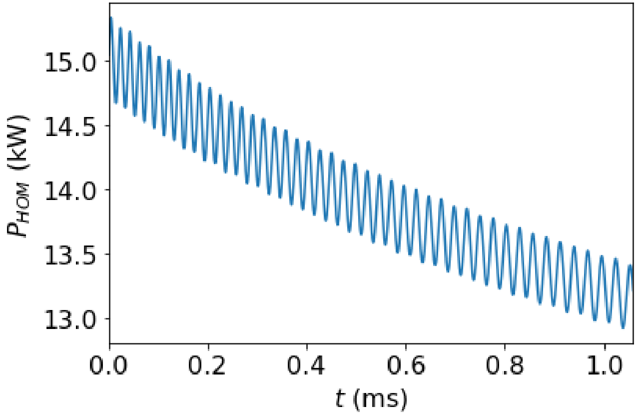




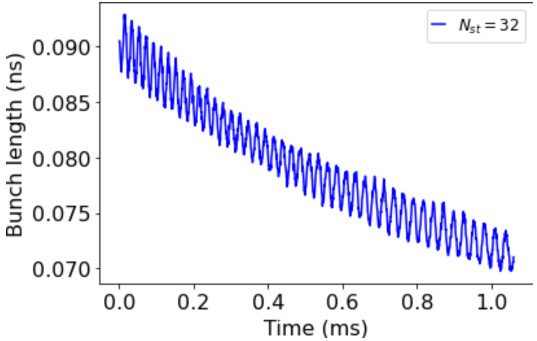


P_{HOM} for other RCS

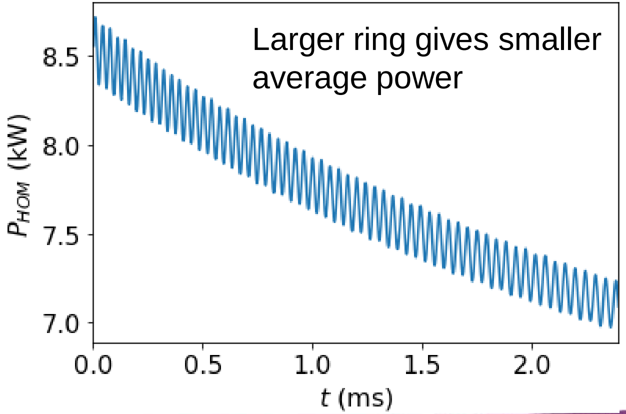
- **RCS2, n_{RF} = 32**



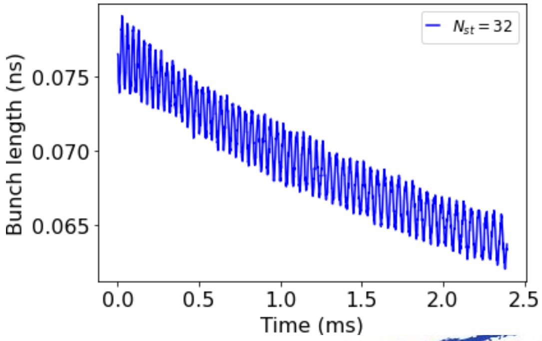
Larger power loss due to shorter bunches in RCS2:



- **RCS3, n_{RF} = 32**



Larger ring gives smaller average power



Summary (1)

- The induced power is very large, up to 13 kW for RCS1&2 per bunch and cavity

→ Bunch crossings inside the cavity increases power up to 4 times, to be avoided

- HOM power capability limit is 1 kW, 3-4 kW under development → up to 20 kW per cavity estimate

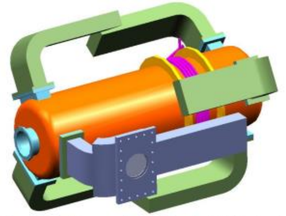


Figure 1: JLab Ampere class cavity with HOM loads and waveguide fundamental power coupler.

from [R.Rimmer]

See also PhD thesis of S. Zadeh

→ Design of high-capacity power absorbers or lower RF frequency with larger iris needed (wakefields scale with $1/a^2$, a the iris radius)

→ The present parameter tables are based on the ILC cavity (1.3 GHz), but a lower frequency, e.g. 800 MHz, might be required if the HOM power cannot be handled

Studies for 801.58 MHz cavities

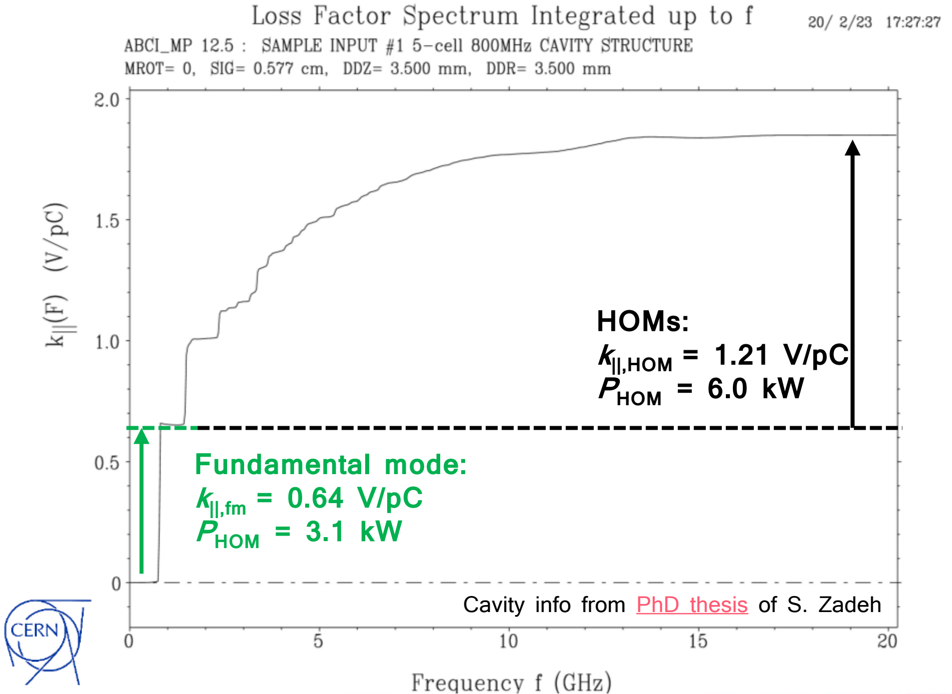
- **Some RCS parameter that change with the FCC-ee 5-cell cavity:**

	TESLA/ILC	FCC-ee
Frequency f_{RF} [MHz]	1300	801.58
Cells	9	5
Active length L_{active} [mm]	1038	935
Cavity length L_{cav} [mm]	1276	1291
Gradient [MV/m]	30 (conservative)	25
Number of cavities RCS1	696	835
Straight length RCS1	2334	2334
Straight length with RF	**38 %**	**46 %**

46% instead of 38% use of straight section, feasible!

Studies for 801.58 MHz cavities

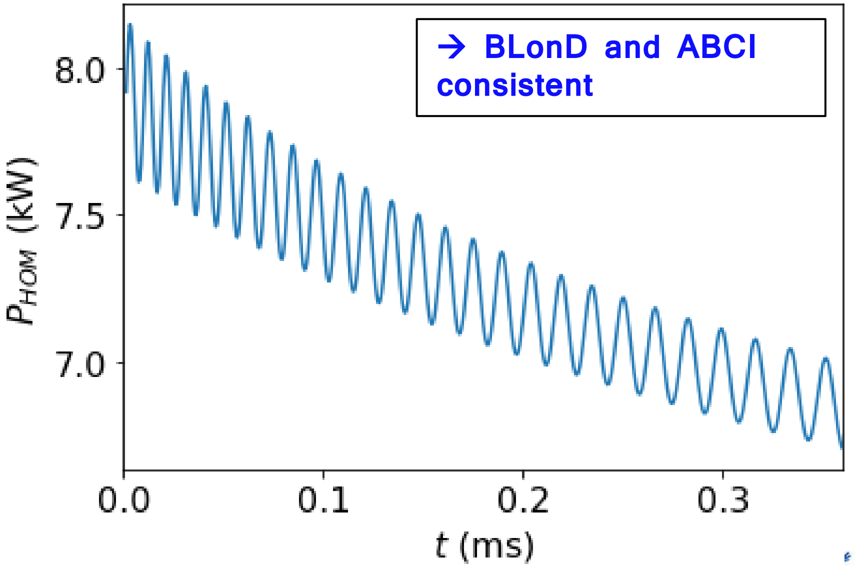
- **The loss factors and beam-induced powers approximately half their values:**



HOMs:
$k_{||,HOM}$ = 1.21 V/pC
P_{HOM} = 6.0 kW

Fundamental mode:
$k_{||,fm}$ = 0.64 V/pC
P_{HOM} = 3.1 kW

Cavity info from PhD thesis of S. Zadeh



→ BLonD and ABCI consistent

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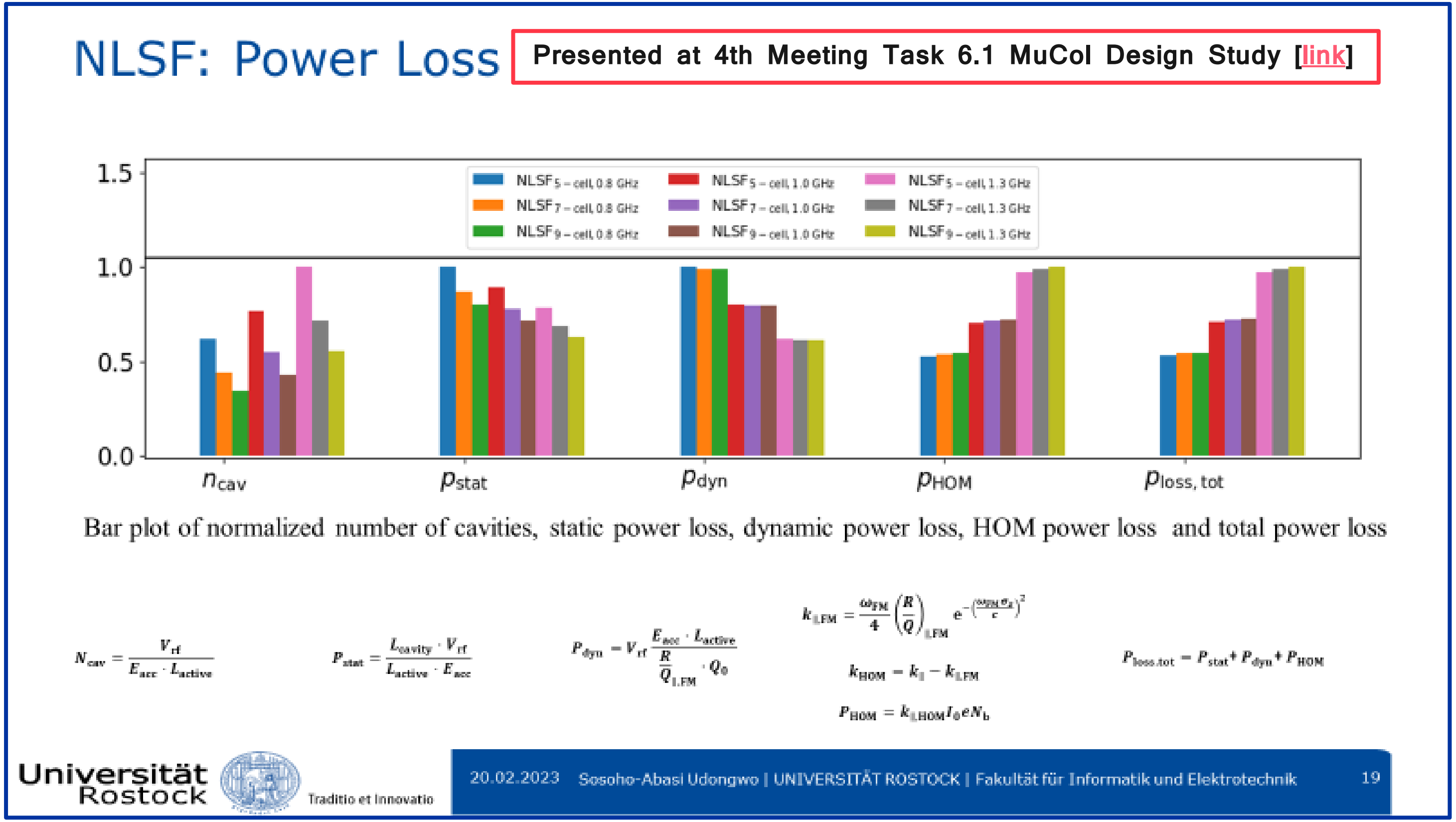
Complementary studies by S.-A. Udongwo



NLSF: Power Loss

Presented at 4th Meeting Task 6.1 MuCol Design Study [link]

Bar plot of normalized number of cavities, static power loss, dynamic power loss, HOM power loss and total power loss

$$N_{cav} = \frac{V_{rf}}{E_{acc} \cdot L_{active}}$$

$$P_{stat} = \frac{L_{cavity} \cdot V_{rf}}{L_{active} \cdot E_{acc}}$$

$$P_{dyn} = V_{rf} \frac{E_{acc} \cdot L_{active}}{\frac{R}{Q}_{\parallel,FM} \cdot Q_0}$$

$$k_{\parallel,FM} = \frac{\omega_{FM}}{4} \left(\frac{R}{Q}\right)_{\parallel,FM} e^{-\left(\frac{\omega_{FM} \sigma_z}{c}\right)^2}$$

$$k_{HOM} = k_{\parallel} - k_{\parallel,FM}$$

$$P_{HOM} = k_{\parallel,HOM} I_0 e N_b$$

$$P_{loss,tot} = P_{stat} + P_{dyn} + P_{HOM}$$

Universität Rostock

Traditio et Innovatio

20.02.2023 Sosoho-Abasi Udongwo | UNIVERSITÄT ROSTOCK | Fakultät für Informatik und Elektrotechnik 19

800 MHz vs 1.3 GHz: power decreases by factor ½ for scaled cavities

Outline

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- Beam induced power estimates for muon RCS RF systems
- Studies on synchronous phase and consequences on the acceleration
- Summary and Outlook

The synchronous phase and its influence

- The synchronous phase ϕ_s as it becomes more important with HOM discussion:
- The bucket area changes with ϕ_s, which affects the HOM power to a small extend
- The synchronous phase ϕ_s strongly influences the main RF requirements:

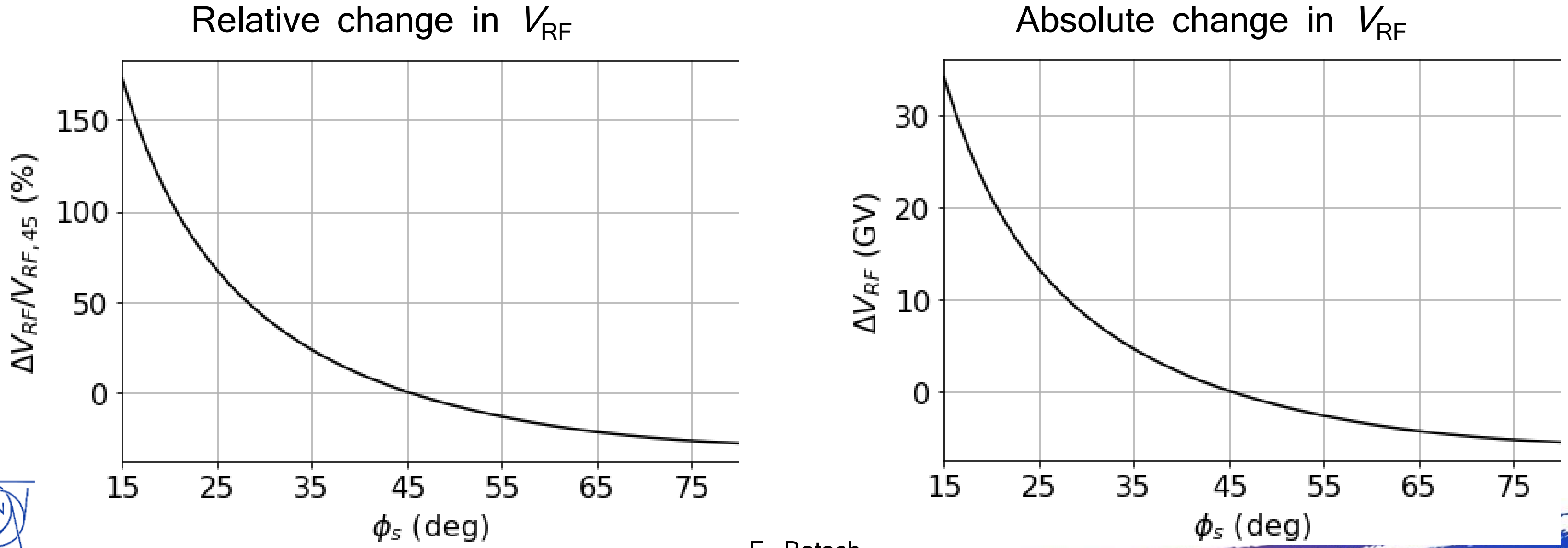
Energy gain of the synchronous particle $\Delta E_s = V_{RF} * \sin \phi_s =$ 14.75 GeV per turn.

For $\phi_s = 45°$ → $V_{RF} = 21$ GV, i.e., large overvoltage

→ Increase the synchronous phase and consequently reduce bucket area to possibly decrease V_{RF}

Over-voltages due to ϕ_s

- **Independent of the cavity frequency or RCS, the overvoltage in the RF voltage $V_{RF} = \Delta E_s/\sin(\phi_s)$, compared to $\phi_s = 45°$ is:**



F. Batsch

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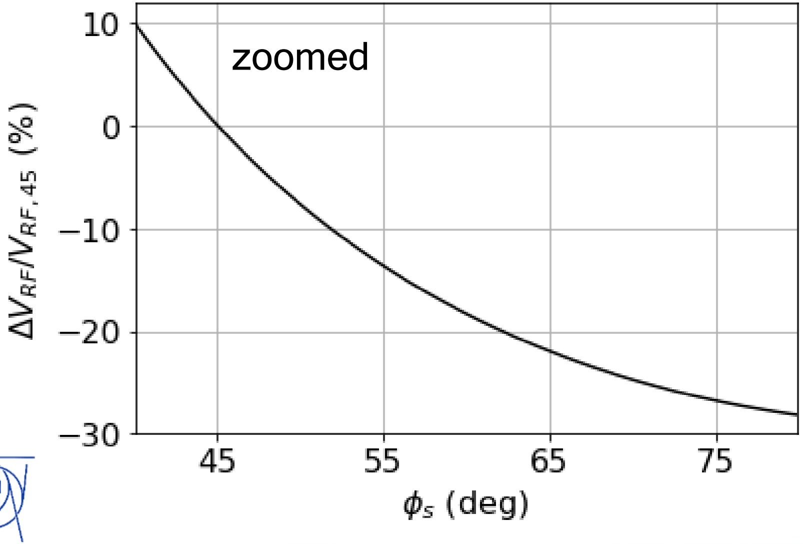
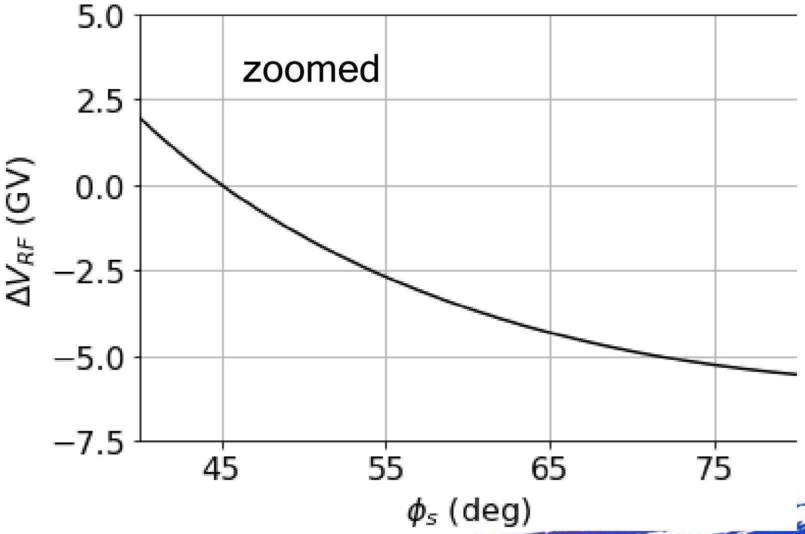
Relative change in V_{RF}



Absolute change in V_{RF}



Over-voltages due to ϕ_s

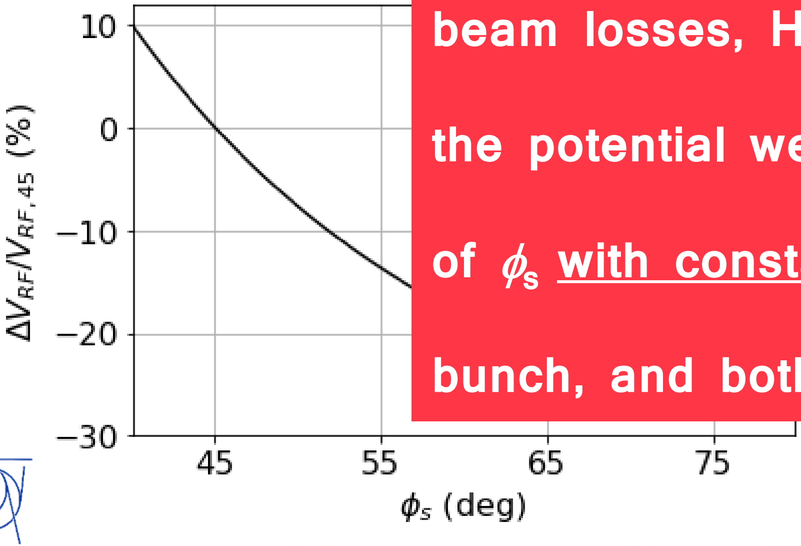
- **Independent of the cavity frequency or RCS, the overvoltage in the RF voltage**

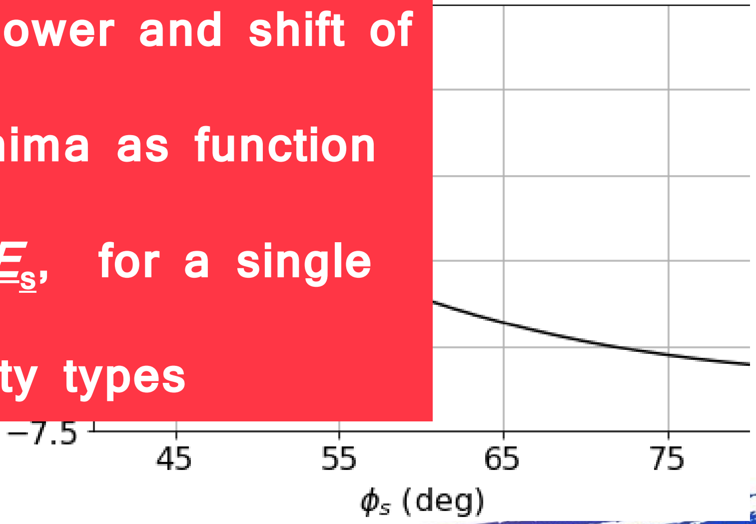
$V_{RF} = \Delta E_s / \sin(\phi_s)$

→ Studied bunch length, bucket area, beam losses, HOM power and shift of the potential well minima as function of ϕ_s <u>with constant ΔE_s</u>, for a single bunch, and both cavity types

Relative cha

e in V_{RF}

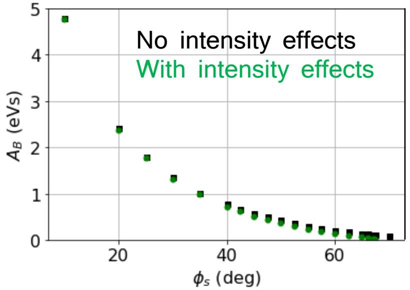


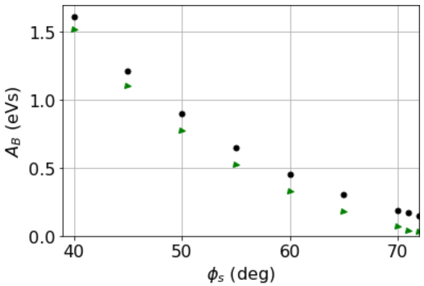


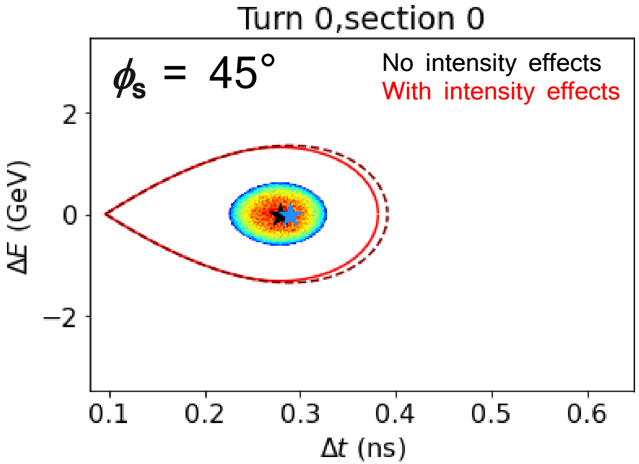
Studies of synchronous phase: RCS1

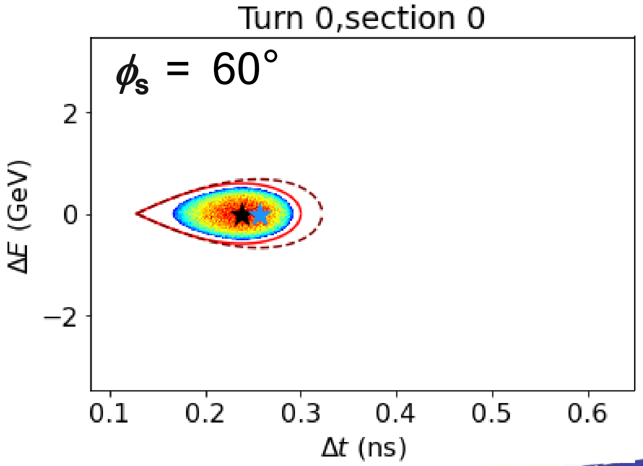
- With increasing ϕ_s … (for RCS1, 1.3 GHz, emittance 0.1 eVs n_{RF} = 32)

1. The bucket area A_b shrinks, the potential well shift due to U_{ind} becomes relevant









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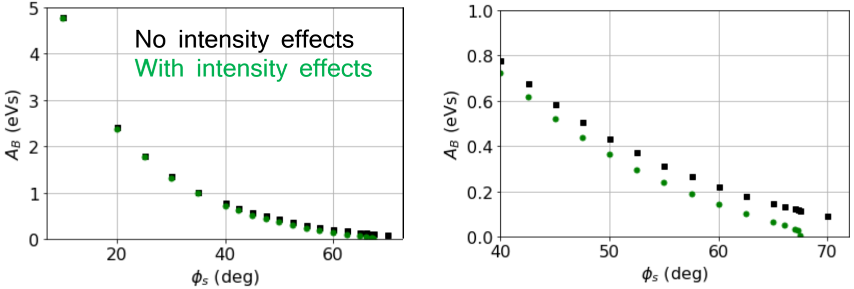
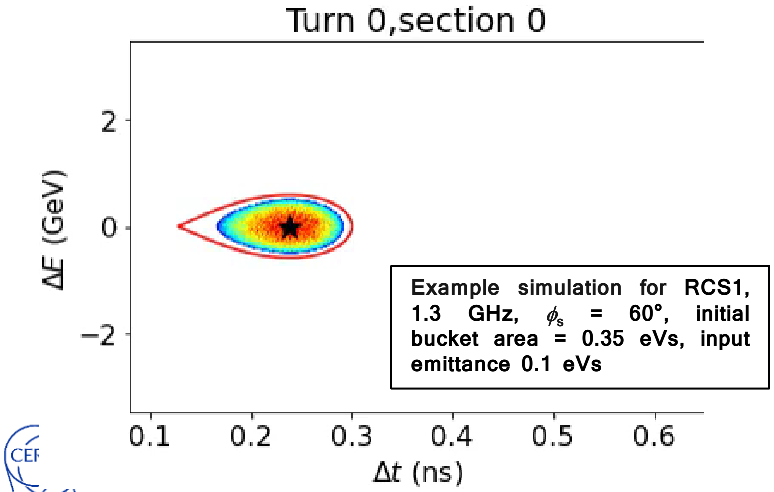
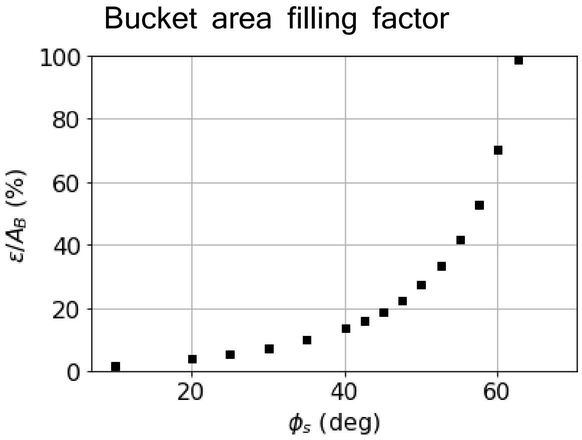




Example simulation for RCS1, 1.3 GHz, ϕ_s = 60°, initial bucket area = 0.35 eVs, input emittance 0.1 eVs



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- With increasing ϕ_s … (for RCS1, 1.3 GHz, emittance 0.1 eVs n_{RF} = 32)

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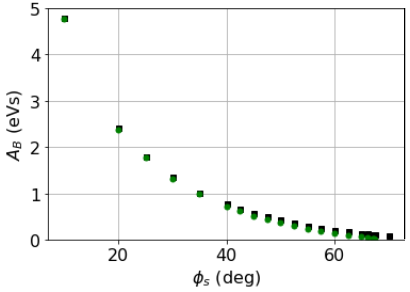
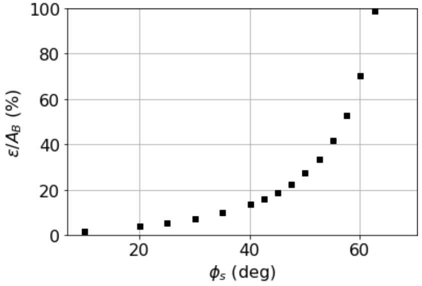
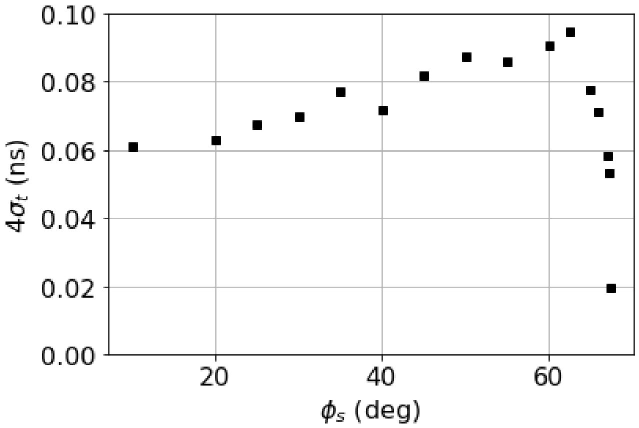




2. The bunch length increases:

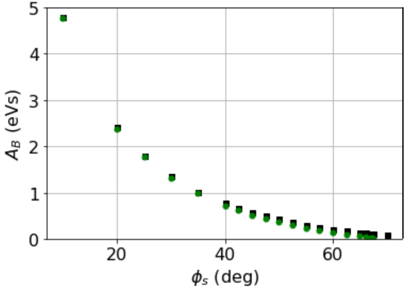


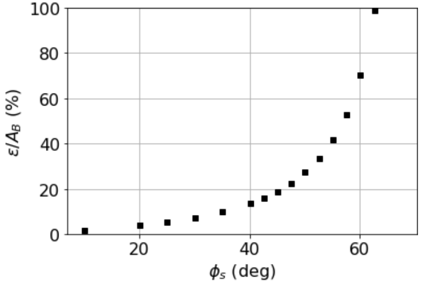
F. Batsch

Studies of synchronous phase: RCS1

- With increasing ϕ_s … (for RCS1, 1.3 GHz, emittance 0.1 eVs n_{RF} = 32)

1. The bucket area A_b shrinks, the potential well shift due to U_{ind} becomes relevant

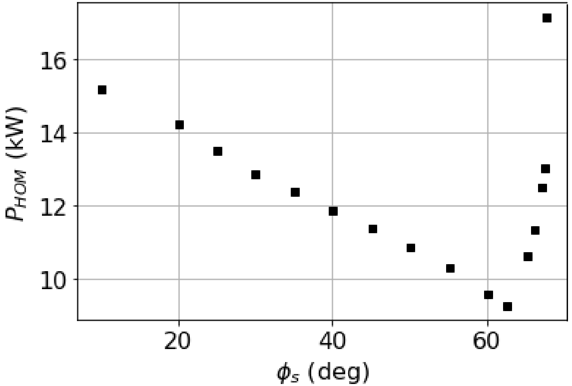


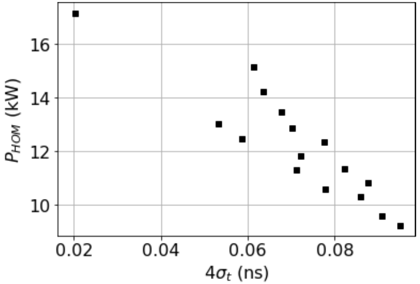


2. The bunch length increases:

3. The HOM power decreases

… until the bucket becomes too small and the bunch is lost (> 60°)





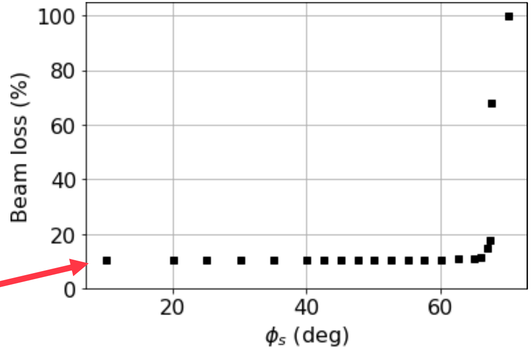
Power loss is correlated with bunch length!

F. Batsch

Studies of synchronous phase: RCS1

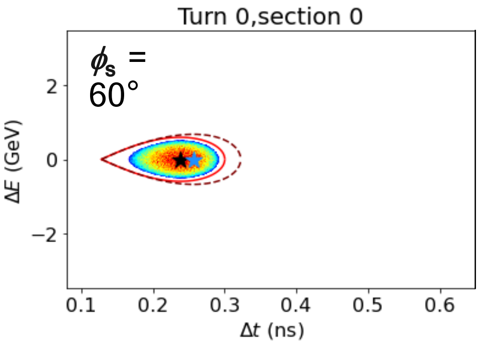
- With increasing ϕ_s … (for RCS1, 1.3 GHz, emittance 0.1 eVs n_{RF} = 32)

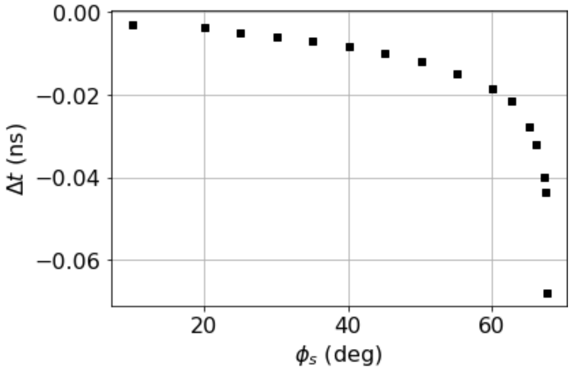
4. The losses increase

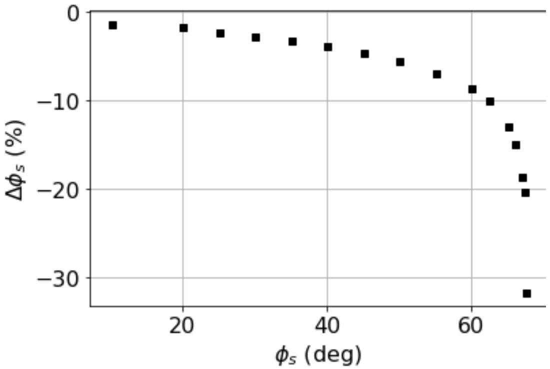


10% loss due to muon decay!

5. The potential well minimum shifts







Studies of synchronous phase: RCS2

- **With increasing ϕ_s … (for RCS1, 1.3 GHz, emittance 0.1 eVs n_{RF} = 32)**

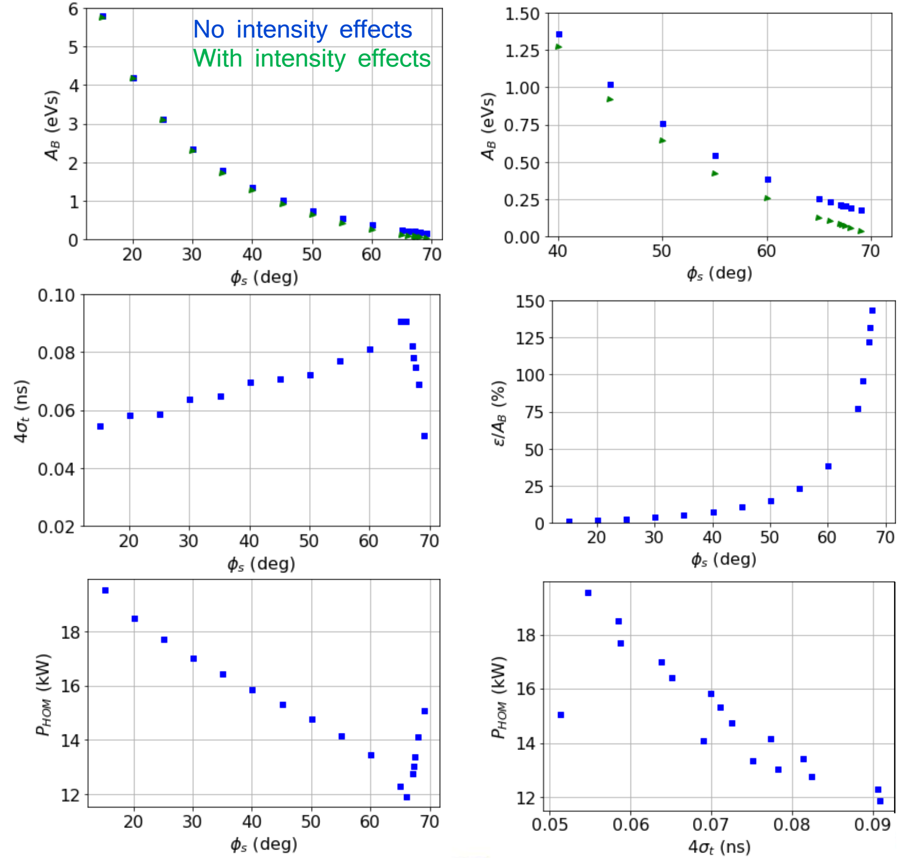
1. **The bucket area A_b shrinks, the potential well shift due to U_{ind} becomes relevant**

2. **The bunch length increases:**

3. **The HOM power decreases**

… until the bucket becomes too small

and the bunch is lost (> 65°)



Studies of synchronous phase: RCS2

- **With increasing ϕ_s … (for RCS1, 1.3 GHz, emittance 0.1 eVs n_{RF} = 32)**

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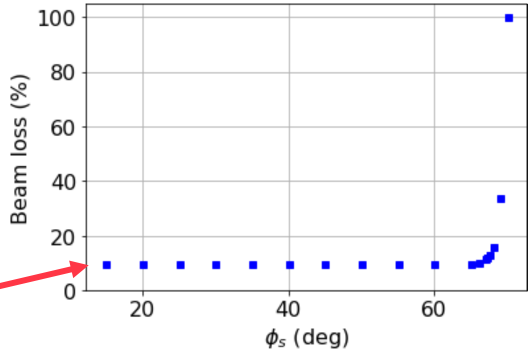
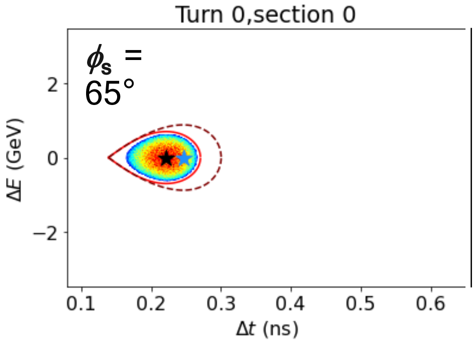
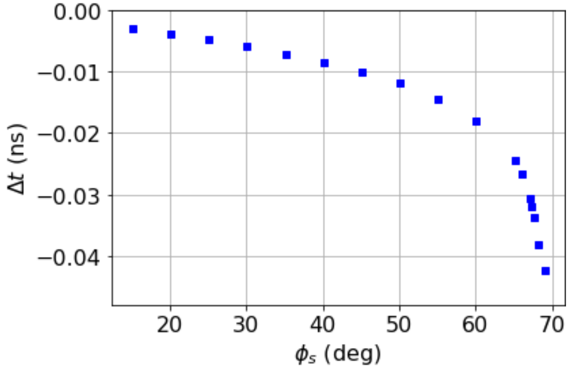
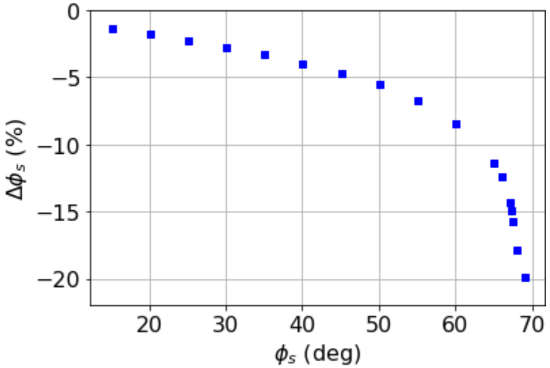


10% loss due to muon decay!

5. **The potential well minimum shifts**







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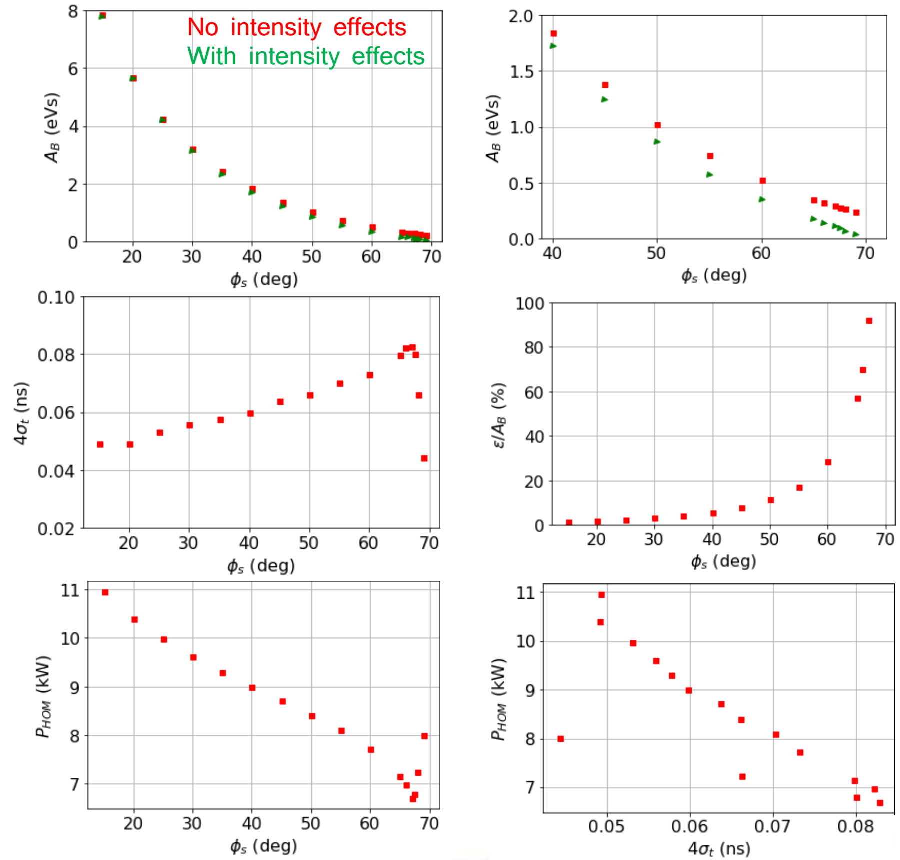
Studies of synchronous phase:

RCS3

- With increasing ϕ_s … (for RCS1, 1.3 GHz, emittance 0.1 eVs n_{RF} = 32)

1. The bucket area A_b shrinks, the potential well shift due to U_{ind} becomes relevant
2. The bunch length increases:
3. The HOM power decreases

… until the bucket becomes too small and the bunch is lost (> 66°)



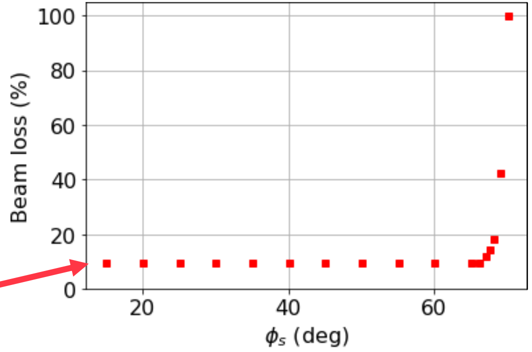
F. Batsch

Studies of synchronous phase:

RCS3

- **With increasing ϕ_s … (for RCS1, 1.3 GHz, emittance 0.1 eVs n_{RF} = 32)**

4. The losses increase



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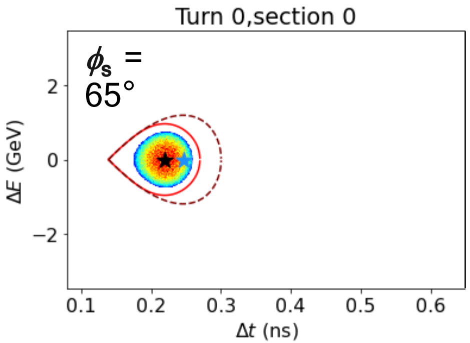

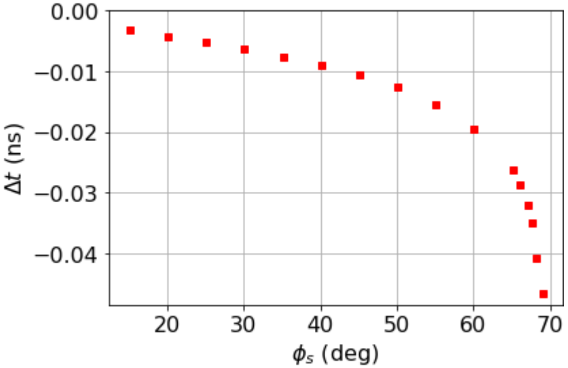

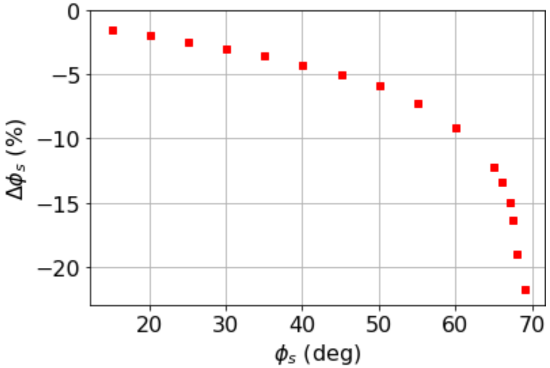

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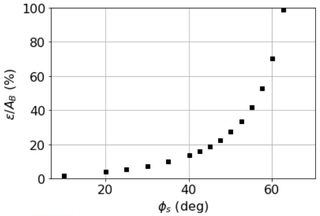
Summary (2)

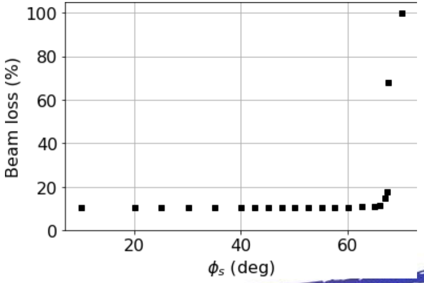
- The synchronous phase ϕ_s can be increased with 1.3 GHz up to approx. 60° (RCS1) or 65° (RCS2 and 3), around 5° more for a 800 MHz cavity

$\phi_{s,max}$	TESLA/ILC	FCC-ee
RCS1	60°	65°
RCS2	65°	70°
RCS3	66°	71°

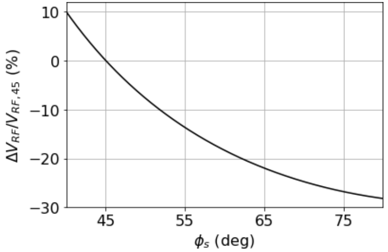
$A_{B,min}$	TESLA/ILC	FCC-ee
RCS1	**70%**	**54%**
RCS2	**77%**	**70%**
RCS3	**70%**	**71%**

- Minimum 70% of the bucket can be filled, deeper look for range up to 90% required
- Instabilities seem no issue, only bucket filling factor





Summary (2)



- The synchronous phase could be a mean to reduce P_{HOM} due to increasing bunch lengths
- Larger ϕ_s , can reduce required V_{RF} by >20%, even more for a 800 MHz system

<u>Outlook:</u>

- Define range of ϕ_s for harmonic ramping and full simulation through all 3 RCS
- Beam-crossings must be avoided in cavities → assumed to be possible as bunch positions are precisely controlled for collider

→ Inclusion of multi-turn effects in simulation (also for counter-rotating bunches)

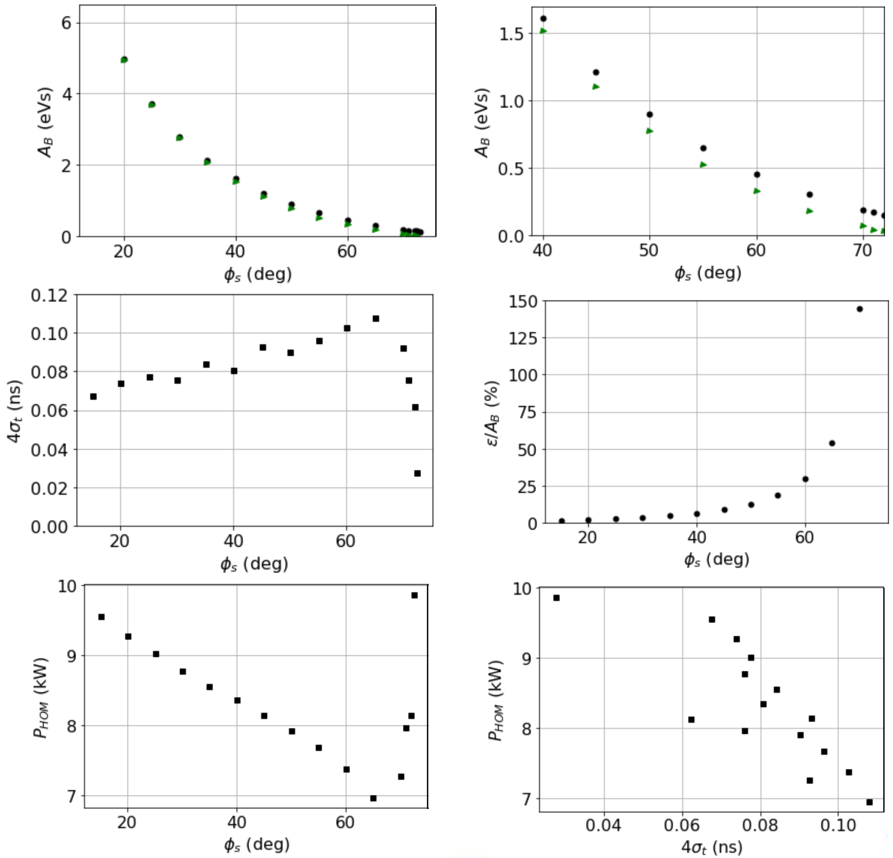
CERN

Studies of synchronous phase: RCS1

- With increasing ϕ_s … (for RCS1, 800MHz, emittance 0.1 eVs $n_{RF} = 32$)

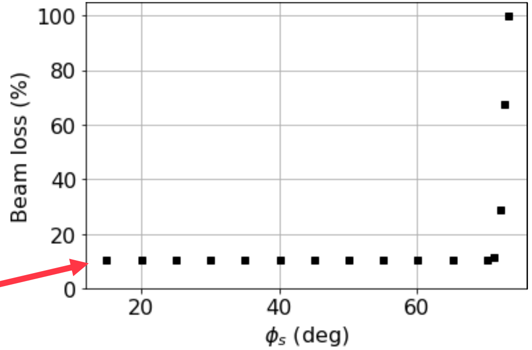
1. The bucket area A_b shrinks, the potential well shift due to U_{ind} becomes relevant
2. The bunch length increases:
3. The HOM power decreases

… until the bucket becomes too small and the bunch is lost (> 65°)



Studies of synchronous phase: RCS1

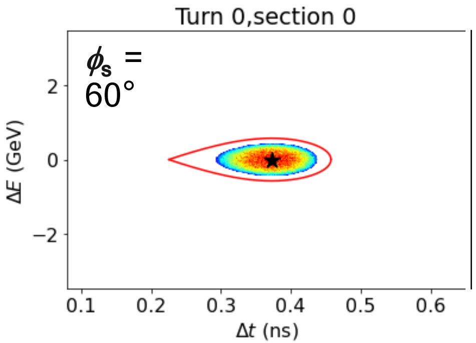
- With increasing ϕ_s … (for RCS1, 800MHz, emittance 0.1 eVs n_{RF} = 32)

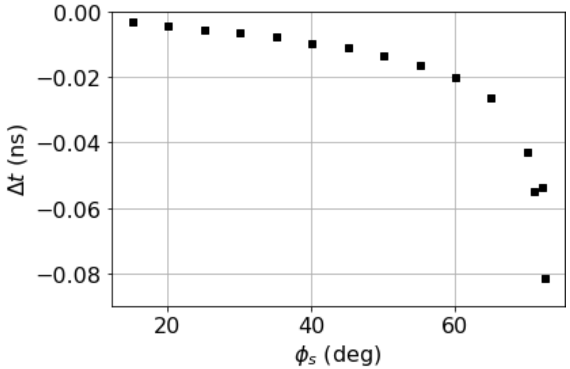


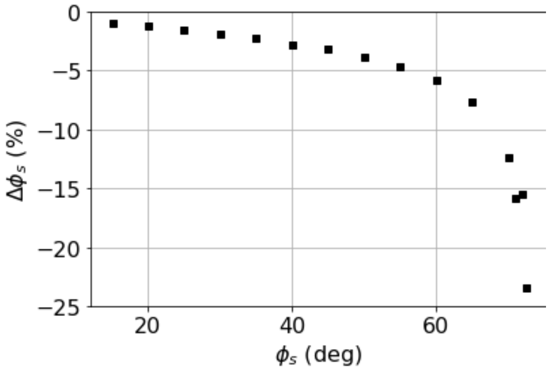
4. The losses increase

10% loss due to muon decay!

5. The potential well minimum shifts







F. Batsch

Studies of synchronous phase:

RCS2

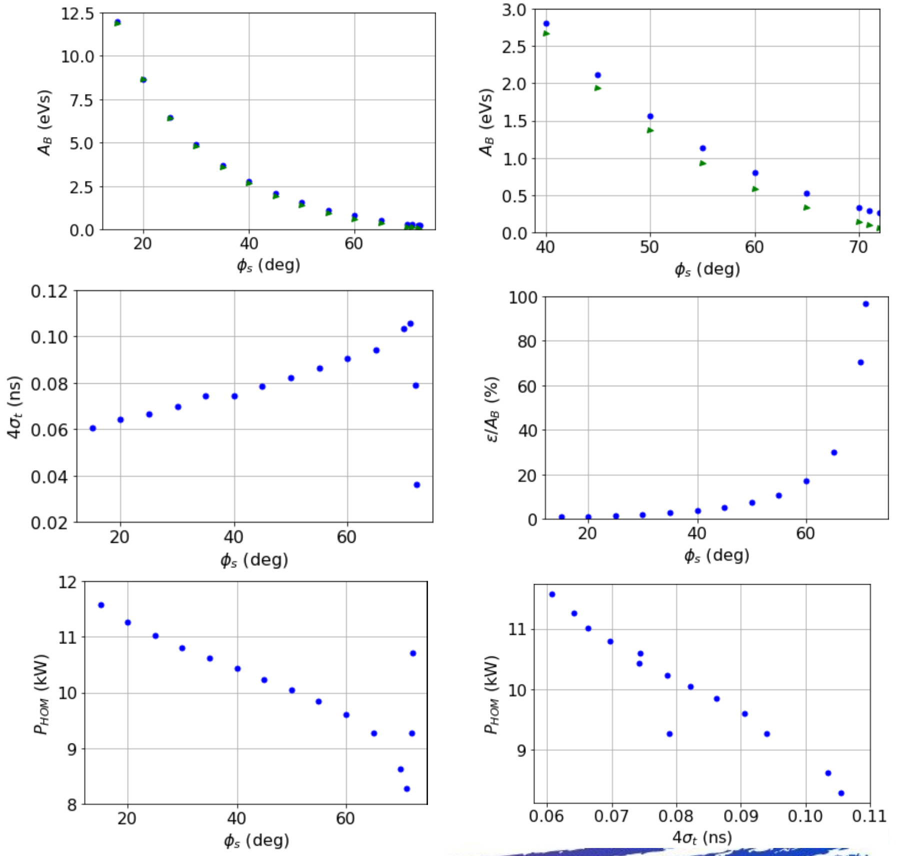
- With increasing ϕ_s … (for RCS1, 800MHz, emittance 0.1 eVs n_{RF} = 32)

1. The bucket area A_b shrinks, the potential well shift due to U_{ind} becomes relevant

2. The bunch length increases:

3. The HOM power decreases

… until the bucket becomes too small and the bunch is lost (> 70°)



Studies of synchronous phase:

RCS2

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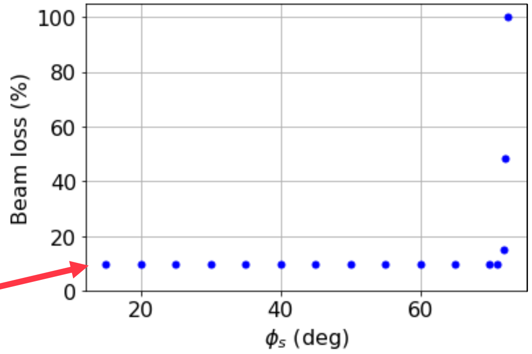

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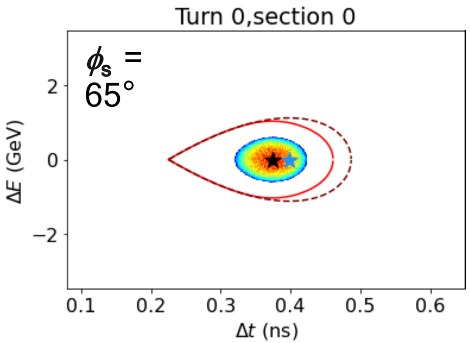
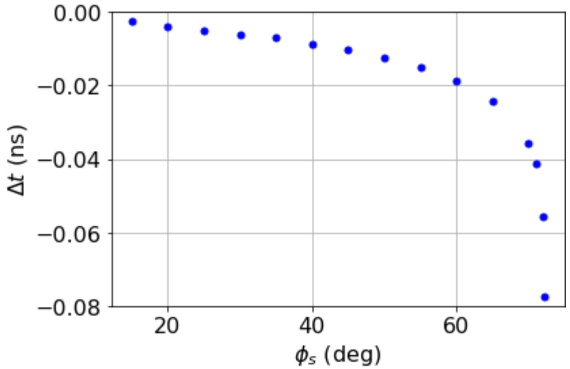
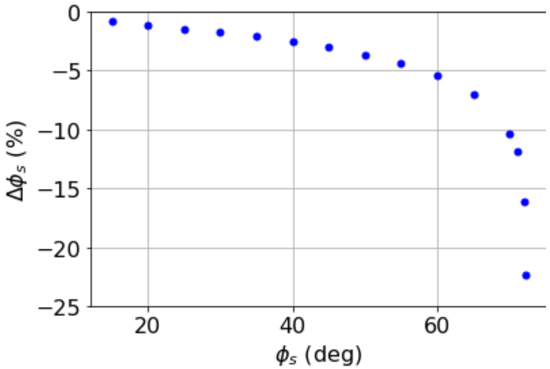






F. Batsch

Studies of synchronous phase:
RCS3

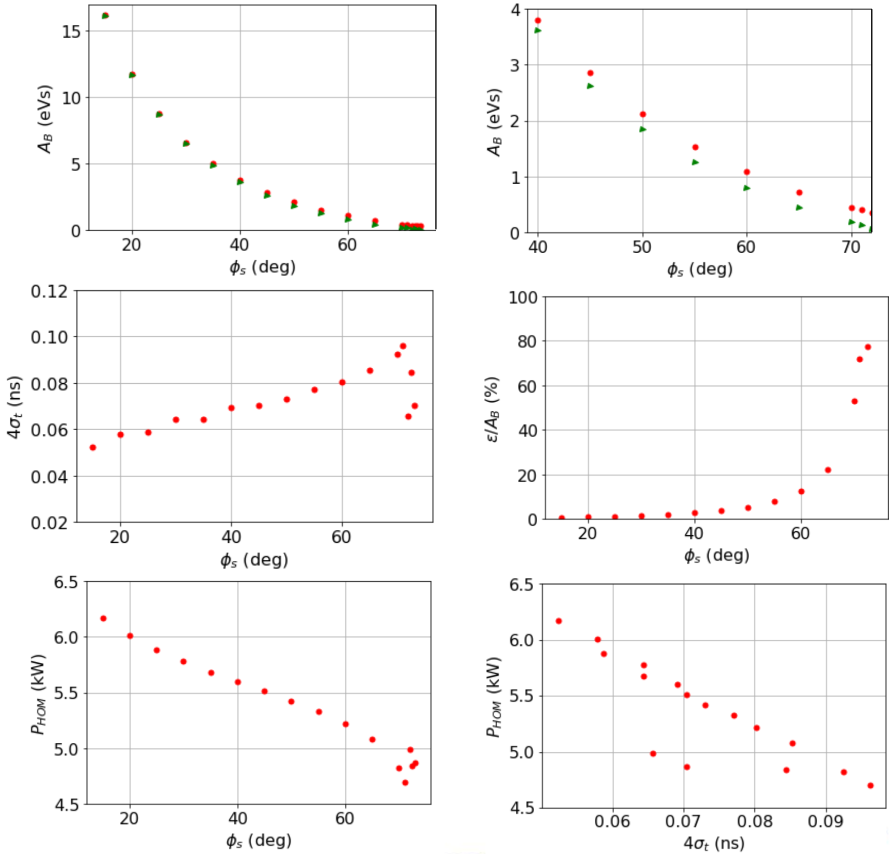
- With increasing ϕ_s … (for RCS1, 800MHz, emittance 0.1 eVs n_{RF} = 32)

1. The bucket area A_b shrinks, the potential well shift due to U_{ind} becomes relevant

2. The bunch length increases:

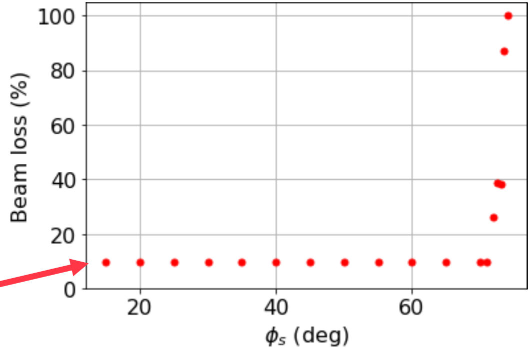
3. The HOM power decreases

… until the bucket becomes too small and the bunch is lost (> 71°)



Studies of synchronous phase:
RCS3

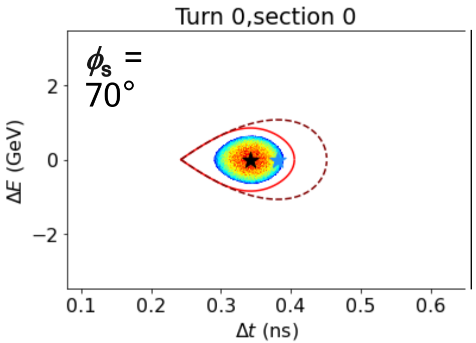
- **With increasing ϕ_s … (for RCS1, 800MHz, emittance 0.1 eVs n_{RF} = 32)**

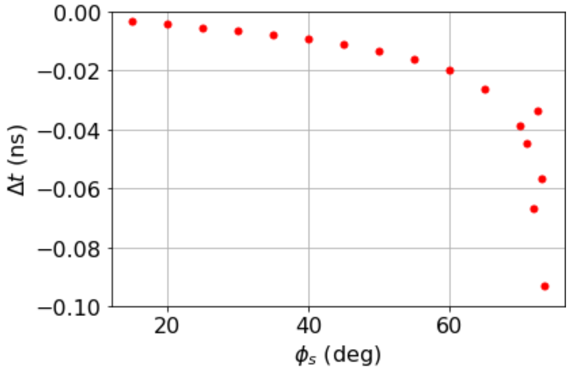


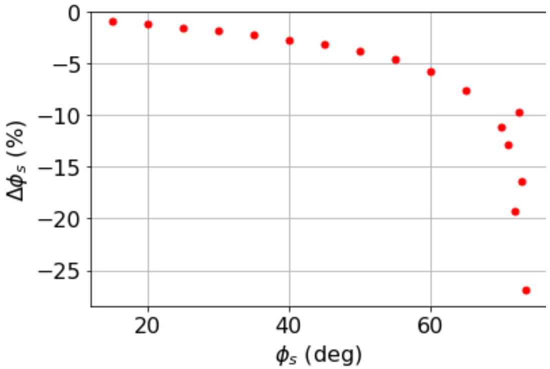
4. **The losses increase**

10% loss due to muon decay!

5. **The potential well minimum shifts**





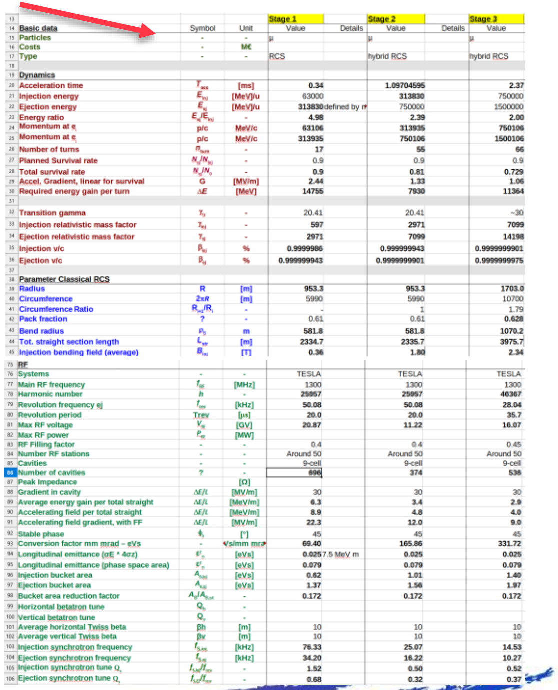


F. Batsch

Parameters and tools: General parameter

- **Detailed parameter table:** https://cernbox.cern.ch/index.php/s/l0VnlTpeUeGBti=

	RCS1→314 GeV	RCS2→750 GeV	RCS3→1.5 TeV
Circumference, $2\pi R$ [m]	5990	5590	10700
Energy factor, E_{ej}/E_{inj}	5.0	2.4	2.0
Repetition rate, f_{rep} [Hz]	5 (asym.)	5 (asym.)	5 (asym.)
Number of bunches	$1\mu^+$, $1\mu^-$	$1\mu^+$, $1\mu^-$	$1\mu^+$, $1\mu^-$
Bunch population	$2.5\text{x}10^{12}$	$2.3\text{x}10^{12}$	$2.2\text{x}10^{12}$
Survival rate per ring	90%	90%	90%
Acceleration time [ms]	0.34	1.04	2.37
Number of turns	17	55	66
Energy gain per turn, ΔE [GeV]	14.8	7.9	11.4
Acc. gradient for survival [MV/m]	2.4	1.3	1.1
Acc. field in RF cavity [MV/m]	30	30	30



			Stage 1		Stage 2		Stage 3
Basic data	Symbol	Unit	Value	Details	Value	Details	Value
Particles	-	-	μ		μ		μ
Costs	-	M€					
Type	-	-	RCS		hybrid RCS		hybrid RCS
Dynamics							
Acceleration time	T_{acc}	[ms]	0.34		1.09704595		2.37
Injection energy	E_{inj}	[MeV]/u	63000		313830		750000
Ejection energy	E_{ej}	[MeV]/u	313830	defined by m	750000		1500000
Energy ratio	E_{ej}/E_{inj}	-	4.98		2.39		2.00
Momentum at e_i	p/c	MeV/c	63106		313935		750106
Momentum at e_j	p/c	MeV/c	313935		750106		1500106
Number of turns	n_{turn}	-	17		55		66
Planned Survival rate	N_{ej}/N_{inj}	-	0.9		0.9		0.9
Total survival rate	N_{ej}/N_0	-	0.9		0.81		0.729
Accel. Gradient, linear for survival	G	[MV/m]	2.44		1.33		1.06
Required energy gain per turn	ΔE	[MeV]	14755		7930		11364
Transition gamma	γ_{tr}	-	20.41		20.41		~30
Injection relativistic mass factor	γ_{inj}	-	597		2971		7099
Ejection relativistic mass factor	γ_{ej}	-	2971		7099		14198
Injection v/c	β_{inj}	%	0.9999986		0.999999943		0.9999999901
Ejection v/c	β_{ej}	%	0.999999943		0.9999999901		0.9999999975
Parameter Classical RCS							
Radius	R	[m]	953.3		953.3		1703.0
Circumference	$2\pi R$	[m]	5990		5990		10700
Circumference Ratio	R_{i+1}/R_i	-	-		1		1.79
Pack fraction	?	-	0.61		0.61		0.628
Bend radius	ρ_b	m	581.8		581.8		1070.2
Tot. straight section length	L_{str}	[m]	2334.7		2335.7		3975.7
Injection bending field (average)	B_{inj}	[T]	0.36		1.80		2.34
RF							
Systems	-	-	TESLA		TESLA		TESLA
Main RF frequency	f_{RF}	[MHz]	1300		1300		1300
Harmonic number	h	-	25957		25957		46367
Revolution frequency ej	f_{rev}	[kHz]	50.08		50.08		28.04
Revolution period	Trev	[μs]	20.0		20.0		35.7
Max RF voltage	V_{rf}	[GV]	20.87		11.22		16.07
Max RF power	P_{rf}	[MW]					
RF Filling factor	-	-	0.4		0.4		0.45
Number RF stations	-	-	Around 50		Around 50		Around 50
Cavities	-	-	9-cell		9-cell		9-cell
Number of cavities	?	-	696		374		536
Peak Impedance		[Ω]					
Gradient in cavity	$\Delta E/L$	[MV/m]	30		30		30
Average energy gain per total straight	$\Delta E/L$	[MeV/m]	6.3		3.4		2.9
Accelerating field per total straight	$\Delta E/L$	[MeV/m]	8.9		4.8		4.0
Accelerating field gradient, with FF	$\Delta E/L$	[MV/m]	22.3		12.0		9.0
Stable phase	ϕ_s	[°]	45		45		45
Conversion factor mm mrad – eVs	-	Vs/mm mra	69.40		165.86		331.72
Longitudinal emittance (σE * 4σz)	ε^r_n	[eVs]	0.025	7.5 MeV m	0.025		0.025
Longitudinal emittance (phase space area)	ε^r_n	[eVs]	0.079		0.079		0.079
Injection bucket area	$A_{b,inj}$	[eVs]	0.62		1.01		1.40
Ejection bucket area	$A_{b,ej}$	[eVs]	1.37		1.56		1.97
Bucket area reduction factor	$A_b/A_{b,st}$	-	0.172		0.172		0.172
Horizontal betatron tune	Q_h	-					
Vertical betatron tune	Q_v	-					
Average horizontal Twiss beta	βh	[m]	10		10		10
Average vertical Twiss beta	βv	[m]	10		10		10
Injection synchrotron frequency	$f_{S,inj}$	[kHz]	76.33		25.07		14.53
Ejection synchrotron frequency	$f_{S,ej}$	[kHz]	34.20		16.22		10.27
Injection synchrotron tune Q_s	$f_{s,inj}/f_{rev}$	-	1.52		0.50		0.52
Ejection synchrotron tune Q_s	$f_{s,ej}/f_{rev}$	-	0.68		0.32		0.37

Induced voltages: Short-range wakefields

Based on K. Bane et al., ‘Calculation of the short-range longitudinal wakefields in the NLC linac’, ICAP98, 1998

$$\boxed{W_L \approx \frac{Z_0 c}{\pi a^2} \exp\left(-\frac{2\pi\alpha^2 L^2 s}{a^2 g}\right) \operatorname{erfc}\left(\frac{\alpha L}{a}\sqrt{\frac{2\pi s}{g}}\right)} \quad [s \text{ small}] \qquad (3)$$

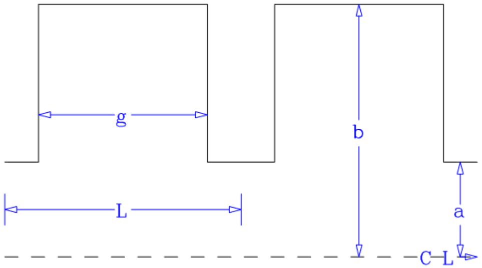
One can approximate this by a semi-analytically expression, valid for small *s* and *s/L* < 0.15:

$$W_L = \frac{Z_0 c}{\pi a^2} \exp\left(-\sqrt{s/s_0}\right)$$

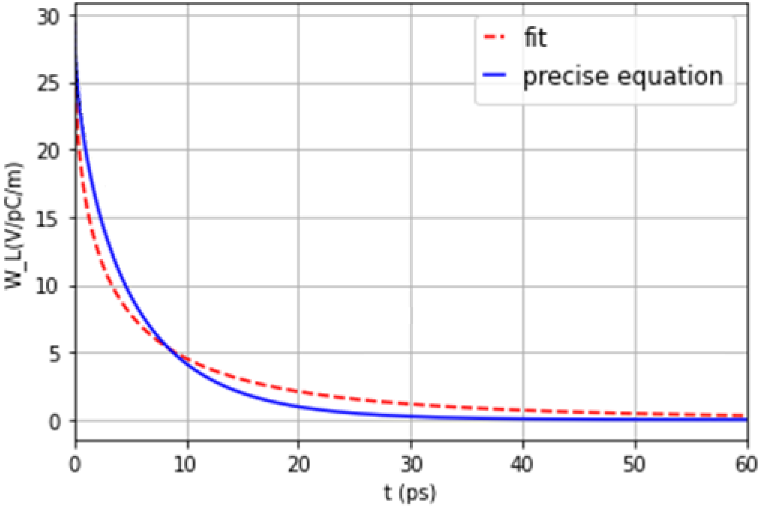
$$s_0 = 0.41 \frac{a^{1.8} g^{1.6}}{L^{2.4}}$$

The parameters for the Tesla cavity[1] gives long. wake functions on the order of 30 V/pC/m:

Adjusting ‚*a*‘ can be a powerful tool to mitigate wakefields!



L= 115.4 mm
g= 82 mm
a= 35 mm
b= 103.3 mm



The BLonD code convolutes the wake function with the beam profile to obtain the induced voltage

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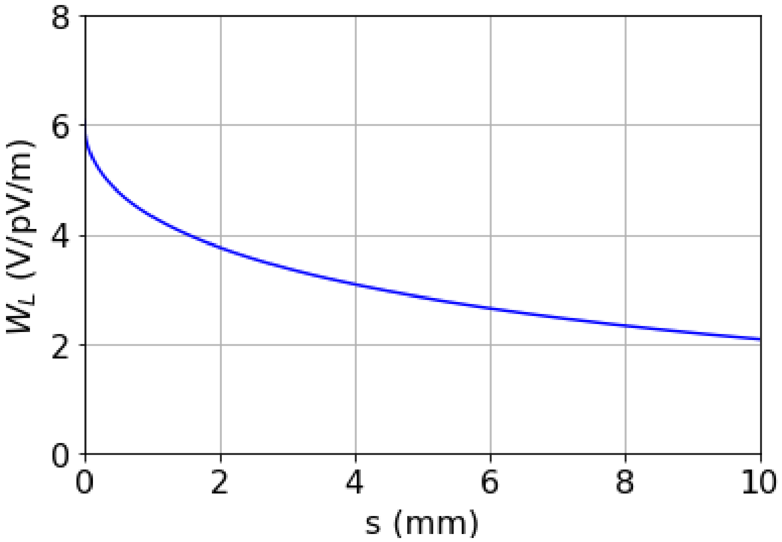
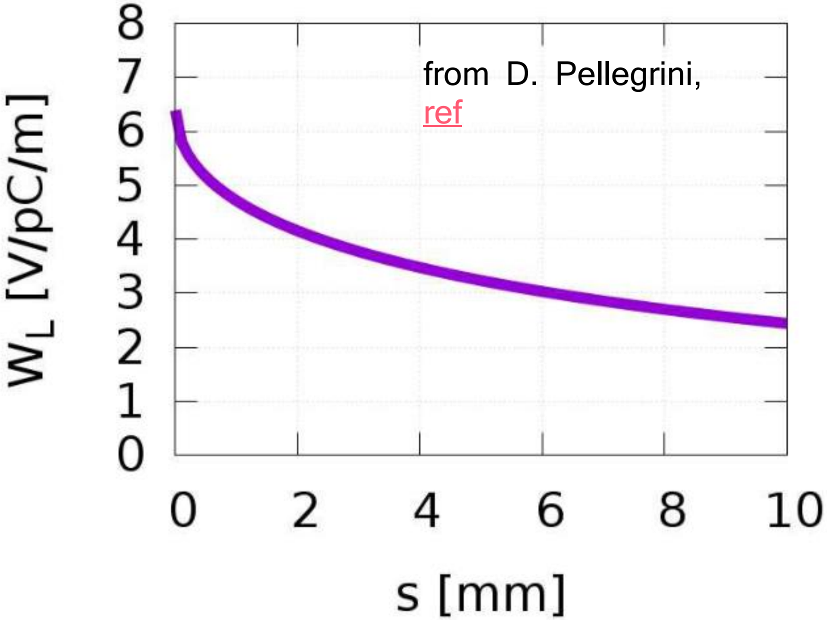
[1] Wakefield studies for the Tesla cavity shown in TESLA Report 2003-19

Induced voltages: Short-range wakefields for 800 MHZ

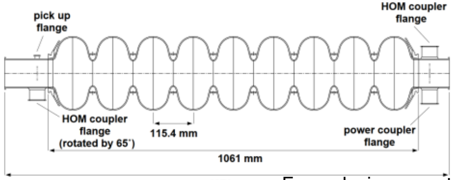
Short-range wakefields using the Bane formalism in BLonD also valid for 5-cell, 800 MHz cavity (cell length L = 187 mm):





from D. Pellegrini, ref

Parameters and tools:
RF – The TESLA cavity



From design report

- **Studies are based on the 1.3 GHz Tesla cavity (design report: Phys. Rev. ST Accel. Beams 3, 092001, 2000)**
  - → see talk by A. Yamamoto
- **Relevant beam parameter**
  - **Bunch population 2.54×10^{12}, ε_L=0.01 eVs** → large intensity effects
  - **Bunch current 20.4 / 18.8 / 10.0 mA → 2x430 kW per cavity** (with 30 MV/m accelerating gradient)
  - **700 / 374 / 532 cavities** in ring, distributed over n_{RF} RF stations
  - **Synchronous phase 45°** (above transition: γ_{tr} = 20.41, 600 < γ < 14200)
- **TESLA Cavity parameter (9 cells, *L*=1.06 m):**
  - **f_{RF} = 1.3 GHz → harmonic number *h* = 25957 to 46367**
  - ***R/Q* = 518 Ω**, total R_s = 306 GΩ
  - **Gradient 30 MV/m**

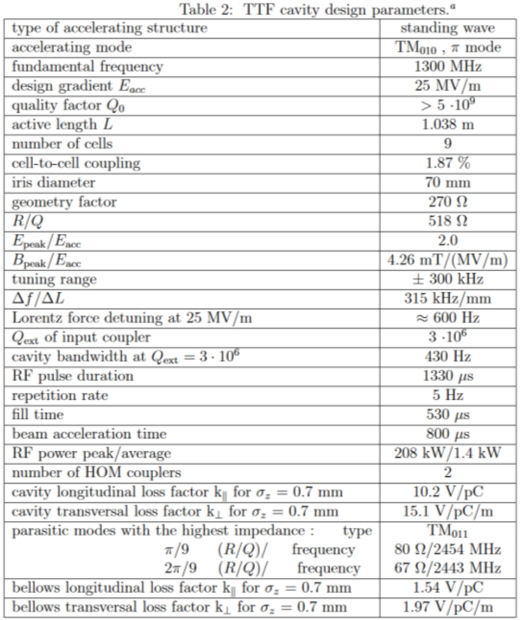


Table 2: TTF cavity design parameters.[a]

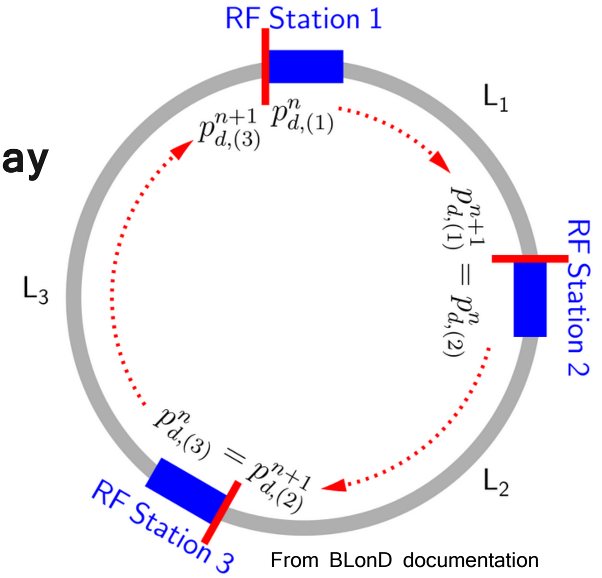
type of accelerating structure	standing wave
accelerating mode	TM_{010} , π mode
fundamental frequency	1300 MHz
design gradient E_{acc}	25 MV/m
quality factor Q_0	$> 5 \cdot 10^9$
active length L	1.038 m
number of cells	9
cell-to-cell coupling	1.87 %
iris diameter	70 mm
geometry factor	270 Ω
R/Q	518 Ω
E_{peak}/E_{acc}	2.0
B_{peak}/E_{acc}	4.26 mT/(MV/m)
tuning range	± 300 kHz
$\Delta f/\Delta L$	315 kHz/mm
Lorentz force detuning at 25 MV/m	≈ 600 Hz
Q_{ext} of input coupler	$3 \cdot 10^6$
cavity bandwidth at $Q_{ext} = 3 \cdot 10^6$	430 Hz
RF pulse duration	1330 μs
repetition rate	5 Hz
fill time	530 μs
beam acceleration time	800 μs
RF power peak/average	208 kW/1.4 kW
number of HOM couplers	2
cavity longitudinal loss factor $k_\parallel$ for $\sigma_z = 0.7$ mm	10.2 V/pC
cavity transversal loss factor $k_\perp$ for $\sigma_z = 0.7$ mm	15.1 V/pC/m
parasitic modes with the highest impedance : type $\pi/9$ $(R/Q)/$ frequency $2\pi/9$ $(R/Q)/$ frequency	TM_{011} 80 Ω/2454 MHz 67 Ω/2443 MHz
bellows longitudinal loss factor $k_\parallel$ for $\sigma_z = 0.7$ mm	1.54 V/pC
bellows transversal loss factor $k_\perp$ for $\sigma_z = 0.7$ mm	1.97 V/pC/m

From design report

Studies & BLonD code

(Beam Longitudinal Dynamics code)

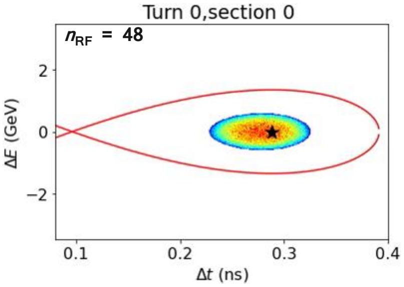
- **BLonD: macro-particle tracking code, developed at CERN since 2014**
- **Links:** documentation and github
- **MuC-specific to multiple RF stations & muon decay**
- **Using the BLonD code to observe effects of**
  - **Short-range wakefields**
  - **Fundamental beam loading**
  - **Synchrotron tune Q_s between RF stations**
- **First studies with only one bunch, 2nd to follow**

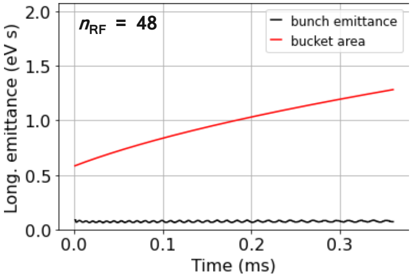


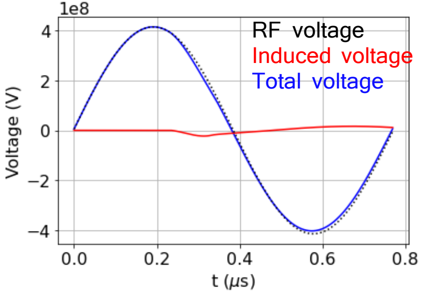
From BLonD documentation

Induced voltages: both contributions

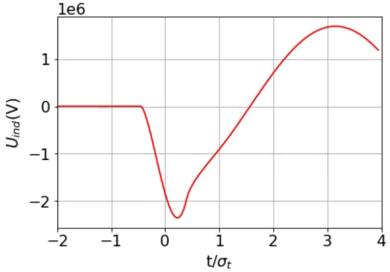
- **Both effects combined: total induced voltage in a cavity is around 2.2 MV per cavity / per meter, i.e. 10-11% of the RF voltage**

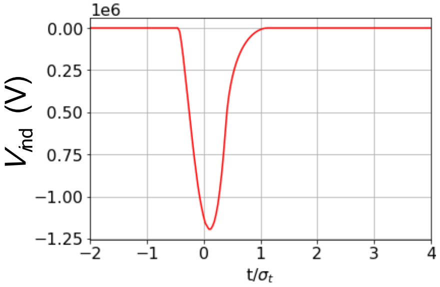


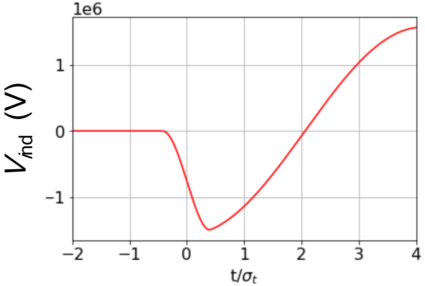




Total induced voltage per cavity = short-range wakefield + beam loading, fundamental







- **Intensity effects on bunch are mitigated by high RF voltage**