The Particle World

CERN Summer School Lectures 2023

Lectures 1 and 2: The Standard Model

David Tong
Further Reading

Particle Physics
CERN Lectures

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around 240 pages! (Sorry)
What are we made of?

"If we consider protons and neutrons as elementary particles, we would have three kinds of elementary particles \([p, n, e]\). This number may seem large but, from that point of view, two is already a large number."

Paul Dirac 1933 Solvay Conference
The Standard Model

12 particles + 4 forces + Higgs boson
The Standard Model

The numbers in the table are the masses of the particles, written as multiples of the electron mass. (Hence the electron itself is assigned mass 1.) The masses of the neutrinos are known to be very small but, otherwise are only constrained within a window and not yet established individually.

Each horizontal line of this diagram is called a generation. Hence, each generation consists of an electron-like particle, two quarks, and a neutrino. The statement that each generation behaves the same means that, among other things, the electric charges of all electron-like particles in the first column are \( \frac{1}{2} \) (appropriate units); the electric charges of all quarks in the second column are \( \frac{1}{3} \) and all those in the third column \( \frac{2}{3} \) \( + \) \( \frac{1}{3} \). All neutrinos are electrically neutral.

We understand aspects of this horizontal pattern very well. In particular, various mathematical consistency conditions tell us that the particles must come in a collective of four particles, and their properties are largely fixed. In particular, we understand why the particles have the electric charges that they do: this is forced upon us by the mathematics and they simply can't be anything else. Moreover if, one day, we were to find a fourth species of electron-like particle, then we can be sure that there are also two further quarks and a neutrino to discover as well. We'll describe this more in Section 4.

We don't, however, understand the observed pattern of masses. More importantly, we don't understand the vertical direction in the pattern at all. We don't understand why there are 3 generations in the world and not, say, 17. Nonetheless, we know from both particle physics and from cosmological observations that there are no more than 3 –9–

Contents

1 Introduction

Gravity Electromagnetism Weak Strong
The Standard Model

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Our First Unification: Fields

- Particles are excitations of underlying quantum fields
- Forces are also due to fields and have associated particles
  - Electromagnetism = photon
  - Strong = gluon
  - Weak = W and Z bosons
  - Gravity = graviton
Intrinsic Angular Momentum = Spin

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These are fermions.

The Pauli exclusion principle applies to fermions.

These are all (gauge) bosons

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A Remarkable Fact

All spin $\frac{1}{2}$ particles are described by the same equation, discovered by Dirac.

$$ (i\gamma^\mu \partial_\mu - m) \psi = 0 $$

Here $m$ is the mass.

Consequence: all matter particles come with anti-particles.
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An aside: In the Standard Model, the masses of all particles are determined by the strength of interaction with the Higgs field.
Units for Mass

We measure mass in terms of energy, using $E=mc^2$. The unit of choice is the electronvolt

$$1 \text{ eV} \approx 1.6 \times 10^{-19} \text{ J}$$

Or MeV = $10^6$ eV or GeV = $10^9$ eV or TeV = $10^{12}$ eV.

More confusingly, we also measure distance in terms of inverse energy, using

$$\lambda = \frac{\hbar c}{E}$$

For a particle of mass $E$, this is the “Compton wavelength”, or the size of the particle.

Note: heavier particles are smaller!
The Masses of Particles

For each of these particles, there is an associated length scale. We get this by transforming energy \( E \) into a length using the fundamental constants of Nature \( c \) and \( \hbar \),

\[
\frac{\hbar}{E} = \frac{\hbar}{c E}.
\]

This is known as the Compton wavelength. Roughly speaking, it can be viewed as the size of the particle. For example, for the electron the Compton wavelength is \( \lambda_e \approx 2 \times 10^{-12} \text{ m} \). Perhaps somewhat surprisingly, the heavier a particle is, the smaller its Compton wavelength.

The Biggest and the Smallest

There are two further length scales that we should mention before delving into details of the subatomic world. One is associated to the strength of the gravitational force. Newton's constant is given by

\[
G_N = \frac{\mu_0}{c^3} = \frac{\pi}{6}.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}.
\]
The Masses of Particles

Note: photon and graviton both massless.

(The gluon is a little subtle...see later.)
The Masses of Particles

The biggest

\[ H \approx 2 \times 10^{-33} \text{ eV} \]

\[ L_{\text{universe}} \approx 9 \times 10^{26} \text{ m} \]

The smallest

\[ M_{\text{pl}} = \sqrt{\frac{\hbar c}{8\pi G}} \approx 2 \times 10^{18} \text{ GeV} \]

\[ L_{\text{pl}} = \sqrt{\frac{8\pi \hbar G}{c^3}} \approx 8 \times 10^{-35} \text{ m} \]
Electric Charge

<table>
<thead>
<tr>
<th>Charge</th>
<th>-1</th>
<th>-1/3</th>
<th>+2/3</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Down Quark</td>
<td>9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Up Quark</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Electron Neutrino</td>
<td>~10^-6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Muon</td>
<td>207</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Strange Quark</td>
<td>186</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Charm Quark</td>
<td>2495</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Muon Neutrino</td>
<td>~10^-6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tau</td>
<td>3483</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bottom Quark</td>
<td>8180</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top Quark</td>
<td>340,000</td>
<td></td>
<td></td>
<td></td>
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<td>~10^-6</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The electric charge characterizes the (relative) strength of the electromagnetic interaction.
Electromagnetism (or QED)

The Maxwell Equations

\[
\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} , \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\
\nabla \cdot \mathbf{B} = 0 , \quad \nabla \times \mathbf{B} = \mu_0 \left( \mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)
\]

This implies the Coulomb force which, in natural units, reads

\[
F = \alpha \frac{Q_1 Q_2}{r^2}
\]

with the fine structure constant

\[
\alpha = \frac{e^2}{4\pi \varepsilon_0 \hbar c} \approx \frac{1}{137}
\]
Feynman Diagrams

An important fact: quantum field theory is hard!

We are saved in QED because $\alpha \approx \frac{1}{137} \ll 1$. This allows us to write down an approximate solution

e.g. what is the probability for a photon to scatter off an electron in some direction?

$$\text{Probability} = \left| O(\alpha) + O(\alpha^2) + \ldots \right|^2$$

More complicated diagrams, like , are less and less important.
Renormalisation

Look close at the electron.

\[ E = \frac{e}{4\pi\varepsilon_0 r^2} \hat{r} \]

Large energy density near electron. This allows for the creation of particle-anti-particle pairs.

This is the physics behind the increasingly complicated diagrams like this.
As you look more closely, the charge of an electron gets bigger!

\[ \alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c} \]

Constants of nature are not constant!
The Strong and Weak Force

Both nuclear forces have associated “electric” and “magnetic” fields

\[ \mathbf{E} = (E_x, E_y, E_z) \quad \mathbf{B} = (B_x, B_y, B_z) \]

But each component is now itself a matrix.

- 1 x 1 matrix  \(\iff\)  Electromagnetism
- 2 x 2 matrix  \(\iff\)  Weak force
- 3 x 3 matrix  \(\iff\)  Strong force

These fields are governed by the Yang-Mills equations

\[ \mathcal{D}_\mu F^{\mu\nu} = J^\nu \]
The Strong Force (or QCD)

Each quark comes in three *colours*, which we take to be red, green and blue.

(Note: a better counting is that each generation contains $1 + 3 + 3 + 1 = 8$ particles.)
Why is the Strong Force Strong?

At high energy, say $E=100$ GeV, we have $\alpha_s \approx 0.1$. But the strong force gets stronger as we go to larger distances. (Asymptotic freedom.)

Taken naively, $\alpha_s \rightarrow \infty$ at the energy scale: $\Lambda_{\text{QCD}} \approx 200$ MeV

This corresponds to a distance scale $R_{\text{QCD}} = \frac{1}{\Lambda_{\text{QCD}}} \approx 5 \times 10^{-15}$ m
Confinement

At short distances, $F(r) \sim \frac{\alpha_s}{r^2}$ but at long distances $F(r)$ becomes constant.

In terms of the potential energy, $V(r) \sim -\frac{\alpha_s}{r}$ at short distances, but at long distances

$$V(r) \sim \Lambda_{\text{QCD}}^2 r$$

This is confinement. We don’t see isolated quarks.

Also, the force carrying field is not massless. The gluons stick together to form glueballs, with mass around $m_{\text{gluon}} \approx \Lambda_{\text{QCD}}$. This is the “mass gap” problem.
Hadrons (Stuff Made of Quarks)

• Baryons: three quarks. For example

\[ n \ (d\bar{d}u) \quad m_n \approx 939.57 \text{ MeV} \]
\[ p \ (uud) \quad m_p \approx 938.28 \text{ MeV} \]

A puzzle: \( m_{\text{down}} = 5 \text{ meV} \) and \( m_{\text{up}} = 2 \text{ MeV} \). Where does the mass come from?

• Mesons: quark-anti-quark pair. For example, pions

\[ \pi^+ \ (\bar{d}u) \quad m \approx 139 \text{ MeV} \]
\[ \pi^0 \ \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d) \quad m \approx 135 \text{ MeV} \]
\[ \pi^- \ (\bar{u}d) \quad m \approx 139 \text{ MeV} \]

Note: Pions have spin 0 and so should be thought of as “force carrying” particles! So ...
Decay

All hadrons other than the proton are unstable. They decay.

- Strong decay: \(\sim 10^{-22}\) to \(10^{-24}\) seconds.
- Electromagnetic decay: \(\sim 10^{-16}\) to \(10^{-21}\) seconds.
- Weak decay: \(\sim 10^{-7}\) to \(10^{-13}\) seconds.

The most famous weak decay is how we first discovered the weak force

\[
n \rightarrow p + e^- + \bar{\nu}_e
\]

Or, if you look more closely,

\[
d \rightarrow u + e^- + \bar{\nu}_e
\]

\[
\begin{align*}
\Delta^{++} (uuu) & \rightarrow p + \pi^+ \\
\Sigma^0 (dus) & \rightarrow \Lambda^0 + \gamma \text{ with } \Lambda^0 (dus) \\
\pi^+ (ud) & \rightarrow \mu^+ + \nu_\mu
\end{align*}
\]
Here "almost conserved" means conserved by the strong interaction. The strong interactions cannot change the number of strange quarks and so particles like $\bar{\psi}^\pm$, $\psi^0$ and $\bar{\psi}^0$, do not decay straight away. The decay only proceeds through the weak interaction, and this takes significantly longer. We'll describe how these decays occur in Section 4. In contrast, particles like the baryons decay directly through the strong interaction, and this happens much faster.

(As an aside: there's always one complication. It turns out that, among the collection of strange baryons, there is one which is unstable: the $\bar{\psi}^0 = \psi^0 + \pi^+$ with a lifetime of around $10^{-20}$ seconds. But this is allowed by the strong force because the number of strange quarks is unchanged. The $\psi^0$, as we've seen, then waits another $10^{10}$ seconds before it too decays.)

As a general rule of thumb, hadrons can decay through one of the three forces: strong (like the $\psi^+$'s), electromagnetic (like $\psi^0$) or weak (like $\bar{\psi}^\pm$, $\psi^0$ and $\bar{\psi}^0$). The lifetimes of these particles reflect the decay process:

- Strong decay: $\sim 10^{-22}$ to $10^{-24}$ seconds.
- Electromagnetic decay: $\sim 10^{-16}$ to $10^{-21}$ seconds.
- Weak decay: $\sim 10^{-7}$ to $10^{-13}$ seconds.

If a particle decays through the weak force, we can take a photograph of it!

If it decays through the strong force, or EM, then we see it more indirectly

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**Particles vs Resonances**

- Strong decay: $\sim 10^{-22}$ to $10^{-24}$ seconds.
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The Weak Force

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<tr>
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<th>Mass (electron mass)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron</td>
<td>1</td>
</tr>
<tr>
<td>Muon</td>
<td>207</td>
</tr>
<tr>
<td>Tau</td>
<td>3483</td>
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</tbody>
</table>

Half of each particle!
4.1 The Structure of the Standard Model

When describing the strong force, we saw that it affects some particles (we call these quarks) while leaving other untouched (we call these leptons). Our first task now should be to describe which particles are affected by the weak force.

You might think that we could simply list those particles that feel the weak force. But, as we will see, things are not quite so straightforward. It turns out that the weak force acts on exactly half of the particles in the universe. But it does so by acting on exactly half of each and every particle!

4.1.1 Parity Violation

There is one defining characteristic of the weak force and hypercharge that differentiates them from the strong force (and from electromagnetism). They do not respect the symmetry of parity.

This fact was discovered by Chien-Shiung Wu, on a cold winters day, in New York City, in December 1956.

Wu’s experiment was technically challenging, but conceptually very simple. She placed a bunch of Cobalt atoms in a magnetic field and watched them die. Cobalt undergoes beta decay

\[ ^{60}\text{Co} \rightarrow ^{60}\text{Ni} + e^- + \bar{\nu}_e + 2\gamma \]
Chiral Fermions

Left-handed particles experience the weak force, right-handed do not.
The Forces of the Standard Model

Each generation splits into 8x2 sets of particles

Note: We don’t yet know if the right-handed neutrino exists.
The Structure of the Standard Model

To write down a theory which violates parity is then straightforward: we simply need to ensure that the left-handed particles experience a different force from the right-handed particles.

The weak force accomplishes this in the most extreme way possible: only left-handed particles experience the weak force. Right-handed particles do not feel it at all. For reasons that we now explain, this is the key property of the weak force and one of the key properties of the Standard Model.

There are quite a few things that we will need to unpick regarding the weak force. Not least is the fact that, as stressed above, the distinction between left-handed and right-handed particles is only valid when the particles are massless. A remarkable and shocking consequence of parity violation is that, at the fundamental level, all elementary spin-\(\frac{1}{2}\) particles are indeed massless! The statement that elementary particles—like electrons, quarks and neutrinos—are fundamentally massless seems to be in sharp contradiction with what we know about these particles! We learn in school that electrons and quarks have mass. Indeed, in the introduction to these lecture notes we included a table with the masses of all elementary particles. How can this possibly be reconciled with the statement that they are, at heart, massless? Clearly we have a little work ahead of us to explain this! We'll do so in Section 4.2 where we introduce the Higgs boson.

4.1.2 A Weak Left-Hander

We're now in a position to explain how the three forces of the Standard Model act on the matter particles. The short-hand mathematical notation for the forces is

\[ G = SU(3) \times SU(2) \times U(1) \]

Let's first recall some facts from the previous chapter. The strong force is associated to the "\(SU(3)\)" term in the equation above. As we explained in Chapter 3, the analog of the electric and magnetic fields for the strong force are called gluons, and are described by \(3 \times 3\) matrices. (This is what the "3" in \(SU(3)\) means.) Correspondingly, each quark carries an additional label, that we call colour that comes in one of three variants which we take to be red, green or blue.

While quarks come in three, colour-coded varieties, the leptons—i.e. the electron and neutrino—do not experience the strong force and hence they come in just a single, colourless variety. In the introduction, we said that each generation contains four particles: two quarks and two leptons. However, a better counting, including colour, is:

<table>
<thead>
<tr>
<th>Particles</th>
<th>Strong</th>
<th>Weak</th>
<th>Hypercharge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left-handed</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>quarks</td>
<td>yes</td>
<td>yes</td>
<td>+1/6</td>
</tr>
<tr>
<td>leptons</td>
<td>no</td>
<td>yes</td>
<td>-1/2</td>
</tr>
<tr>
<td>Right-handed</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>up quark</td>
<td>yes</td>
<td>no</td>
<td>+2/3</td>
</tr>
<tr>
<td>down quark</td>
<td>yes</td>
<td>no</td>
<td>-1/3</td>
</tr>
<tr>
<td>electron</td>
<td>no</td>
<td>no</td>
<td>-1</td>
</tr>
<tr>
<td>neutrino</td>
<td>no</td>
<td>no</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2. The Standard Model forces acting on each of the fermions in a single generation.

For the right-handed particles coincide with their electromagnetic charge. This, as we shall see, is no coincidence. However, the hypercharges for the left-handed particles are rather different.

Before we move, I should mention that there is one caveat. (Isn't there always!) We don't yet have direct evidence for the existence of the right-handed neutrino and there is a possibility that it doesn't exist! Indeed, many people would say that the right-handed neutrino should not be included in the list of particles in the Standard Model. From the table you can see that the right-handed neutrino is neutral under all three of the forces in the Standard Model and this makes it very challenging to detect. We'll see the indirect evidence for its existence in Section 4.4 where we describe more about neutrinos in general.

4.1.3 A Perfect Jigsaw

The particles and forces listed in Table 2 summarise 150 years of work (dated from Röntgen's discovery of X-rays), dedicated to understanding the structure of matter at the most fundamental level. The first thing that comes to mind when you see it is: what a mess! The individual elements comprise some of the most gorgeous objects in theoretical physics— the Dirac, Maxwell and Yang-Mills equations. And yet any semblance of elegance would seem to have been jettisoned at the last, with the different components thrown together in this strange higgledy-piggledy fashion. Why this collection of forces and particles? In particular, why this strange collection of hypercharges?

Happily, there is an wonderful and astonishing answer to these questions. The beautiful truth is simply: it could barely have been any other way.

A perfect jigsaw: Anomaly cancellation means that it could hardly be any other way!
The Higgs Field

This is both the simplest and most complicated field in the Standard Model!

<table>
<thead>
<tr>
<th>Particle</th>
<th>Strong</th>
<th>Weak</th>
<th>Hypercharge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Higgs</td>
<td>no</td>
<td>yes</td>
<td>+1/2</td>
</tr>
</tbody>
</table>

\[ \mathcal{D}_\mu \mathcal{D}^\mu \phi - V(\phi) = \lambda \psi \psi \]

Two relevant scales:

- Mass \( m_H \approx 125 \text{ GeV} \)
- Condensate \( \langle \phi \rangle \approx 246 \text{ GeV} \)

It is the condensate that gives the Higgs its Midas touch: everything that it touches gets a mass
How Particles Get a Mass

In the Standard Model, all fermions and gauge bosons are obliged to be fundamentally massless. They get a mass by interaction with the Higgs.

\[ \text{Mass} = g \times \langle \phi \rangle \]

some dimensionless coupling \( \langle \phi \rangle \approx 246 \text{ GeV} \)

- The Higgs gives mass to the W-boson and Z-boson and all fermions.
- The photon remains massless: it is the one that got away!
- Recall: the mass of the proton and neutron do not come from the Higgs!
One Last Thing: Quark Mixing

There is a misalignment between the interactions with the Higgs and the interaction with the weak force.

It turns out that you can choose to have the up-sector aligned. But then the down sector is not. The result is a superposition of particles.

\[
\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}
\]

These particles interact with weak force.

This is how, for example, mesons with strange quarks decay.
One Last Thing: and Lepton Mixing

There is a similar statement for neutrinos

$$\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix}
= 
\begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\
U_{\tau 1} & U_{\tau 2} & U_{\tau 3}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}$$

These particles interact with weak force and are produced in, say, beta decay

These particles have definite mass. These are energy eigenstates that travel unchanged through space.

This gives rise to neutrino oscillations.
The Mixing Matrices

For quarks, we have the CKM matrix

\[
\begin{pmatrix}
|V_{ud}| & |V_{us}| & |V_{ub}| \\
|V_{cd}| & |V_{cs}| & |V_{cb}| \\
|V_{td}| & |V_{ts}| & |V_{tb}|
\end{pmatrix}
\approx
\begin{pmatrix}
0.97 & 0.22 & 0.004 \\
0.22 & 0.97 & 0.04 \\
0.009 & 0.04 & 0.999
\end{pmatrix}
\]

For neutrinos, we have the PMNS matrix

\[
\begin{pmatrix}
|U_{e1}| & |U_{e2}| & |U_{e3}| \\
|U_{\mu 1}| & |U_{\mu 2}| & |U_{\mu 3}| \\
|U_{\tau 1}| & |U_{\tau 2}| & |U_{\tau 3}|
\end{pmatrix}
\approx
\begin{pmatrix}
0.8 & 0.5 & 0.1 \\
0.3 & 0.5 & 0.7 \\
0.4 & 0.6 & 0.6
\end{pmatrix}
\]

We only know these parameters by experimental measurement. Why do they take these values? Why are the matrices so different?
Summary: The Greatest Theory of All Time

\[ G = SU(3) \times SU(2) \times U(1) \]

with all complications coming from interactions with Higgs!