The Particle World

CERN Summer School Lectures 2023

Lectures 1 and 2: The Standard Model

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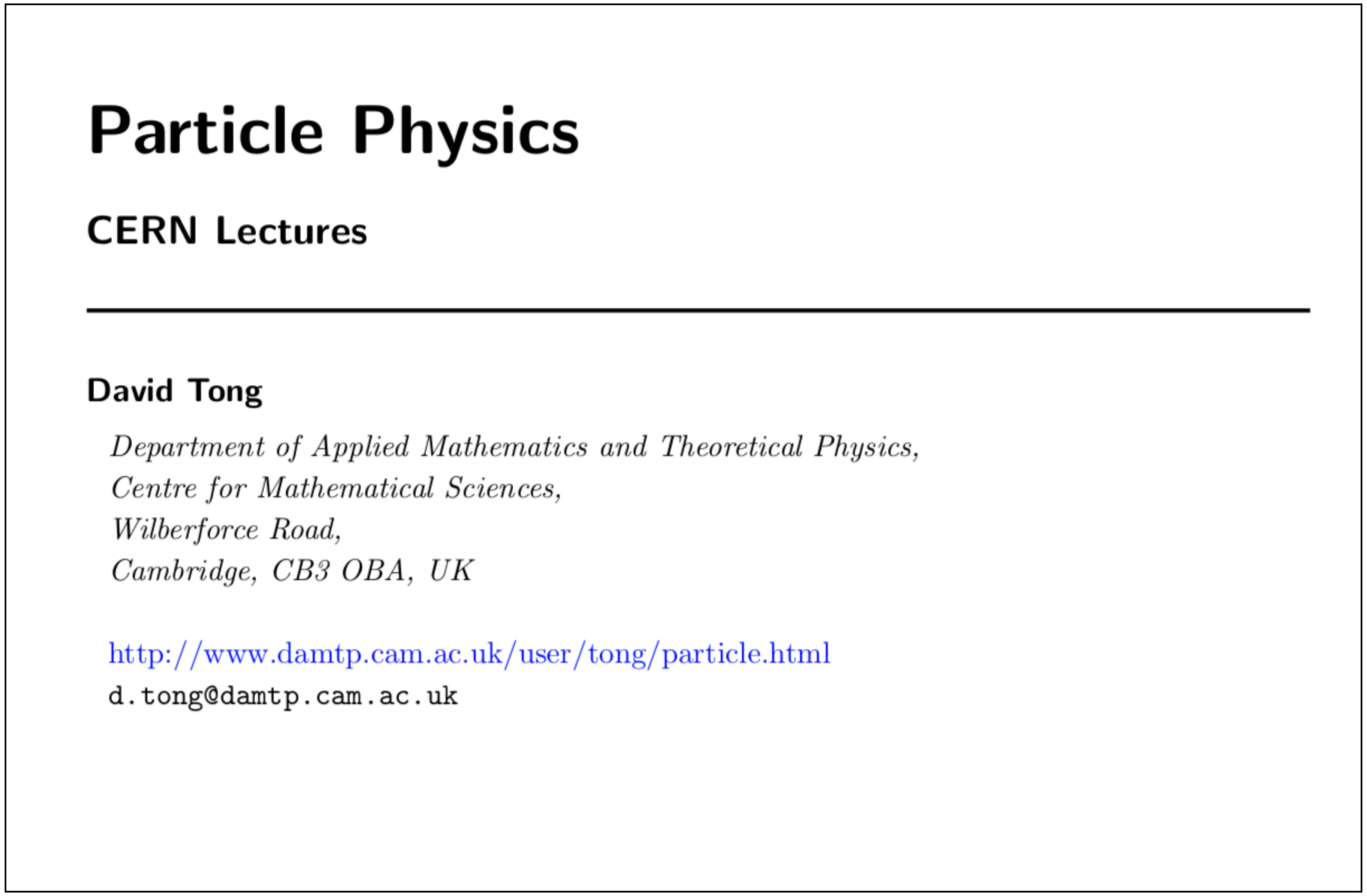
Further Reading



Particle Physics

CERN Lectures

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http://www.damtp.cam.ac.uk/user/tong/particle.html
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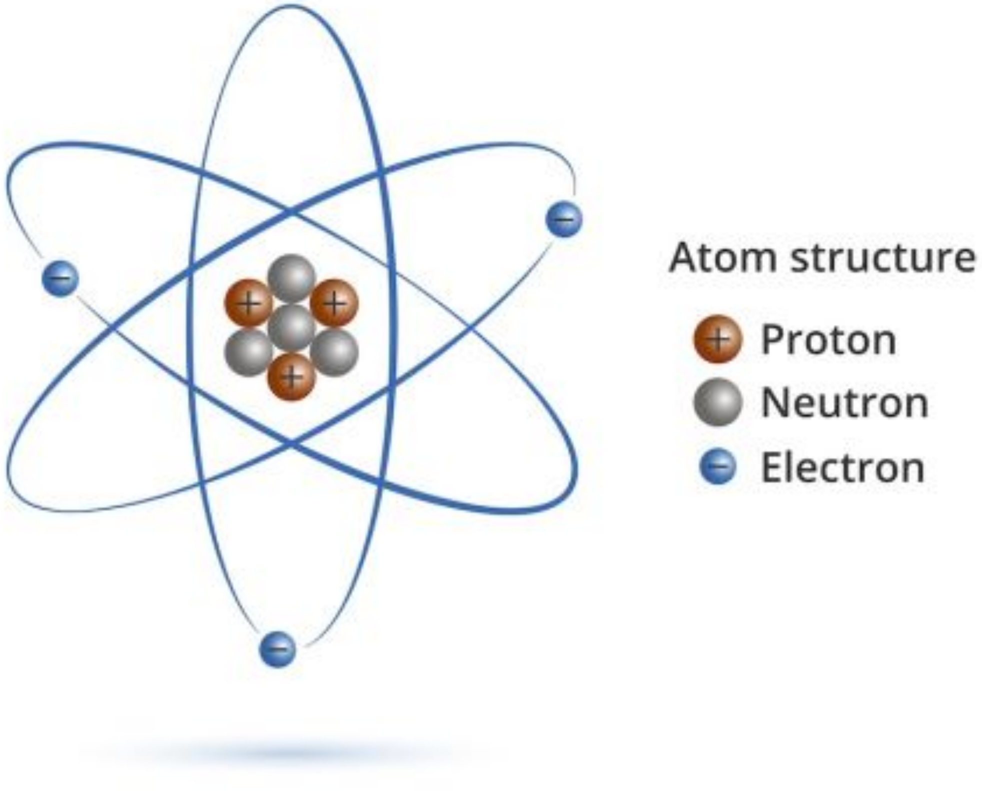
around 240 pages! (Sorry)

What are we made of?



"If we consider protons and neutrons as elementary particles, we would have three kinds of elementary particles [p,n,e].... This number may seem large but, from that point of view, two is already a large number.”

Paul Dirac 1933 Solvay Conference

The Standard Model

12 particles + 4 forces + Higgs boson

The Standard Model

Electron	Down Quark	Up Quark	Electron Neutrino
Muon	Strange Quark	Charm Quark	Muon Neutrino
Tau	Bottom Quark	Top Quark	Tau Neutrino

Gravity	Electromagnetism	Weak	Strong

Higgs

The Standard Model

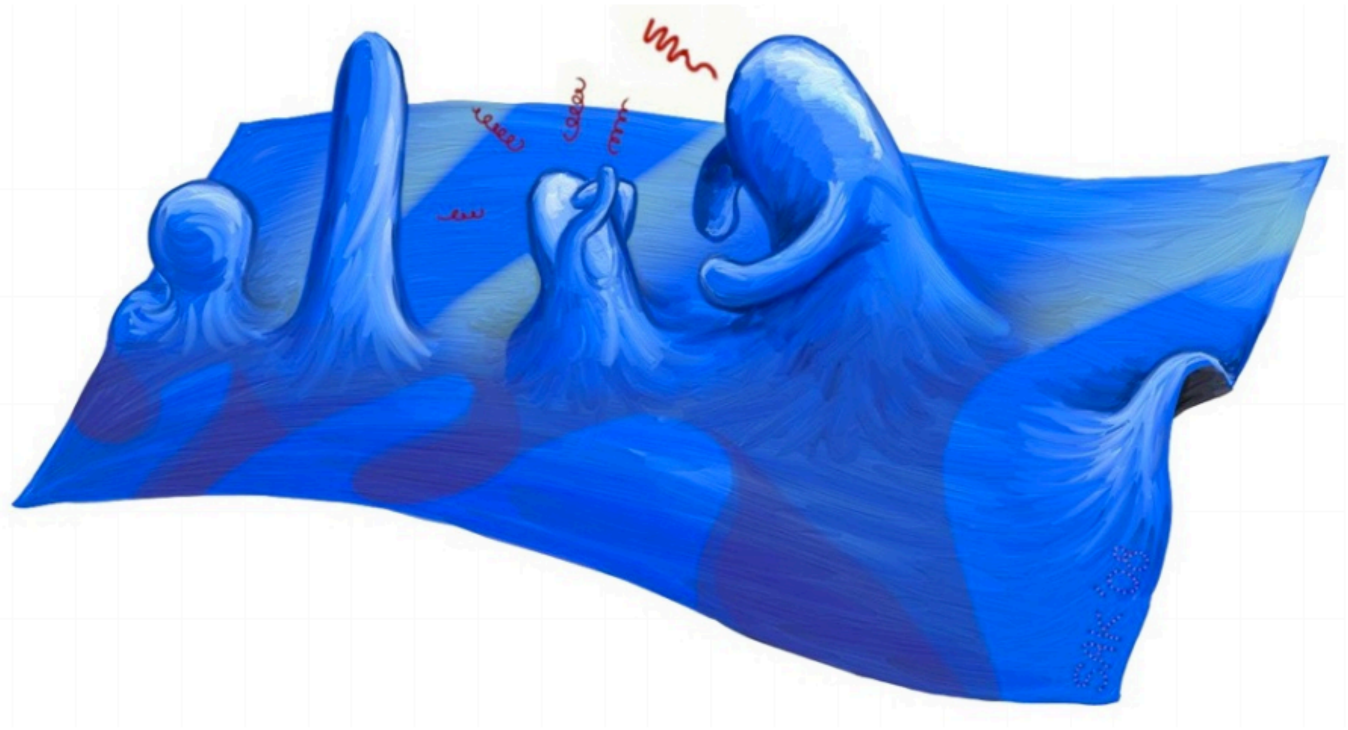
Electron	Down Quark	Up Quark	Electron Neutrino
Muon	Strange Quark	Charm Quark	Muon Neutrino
Tau	Bottom Quark	Top Quark	Tau Neutrino

Hypercharge

Gravity	~~Electromagnetism~~	Weak	Strong

Higgs

Our First Unification: Fields

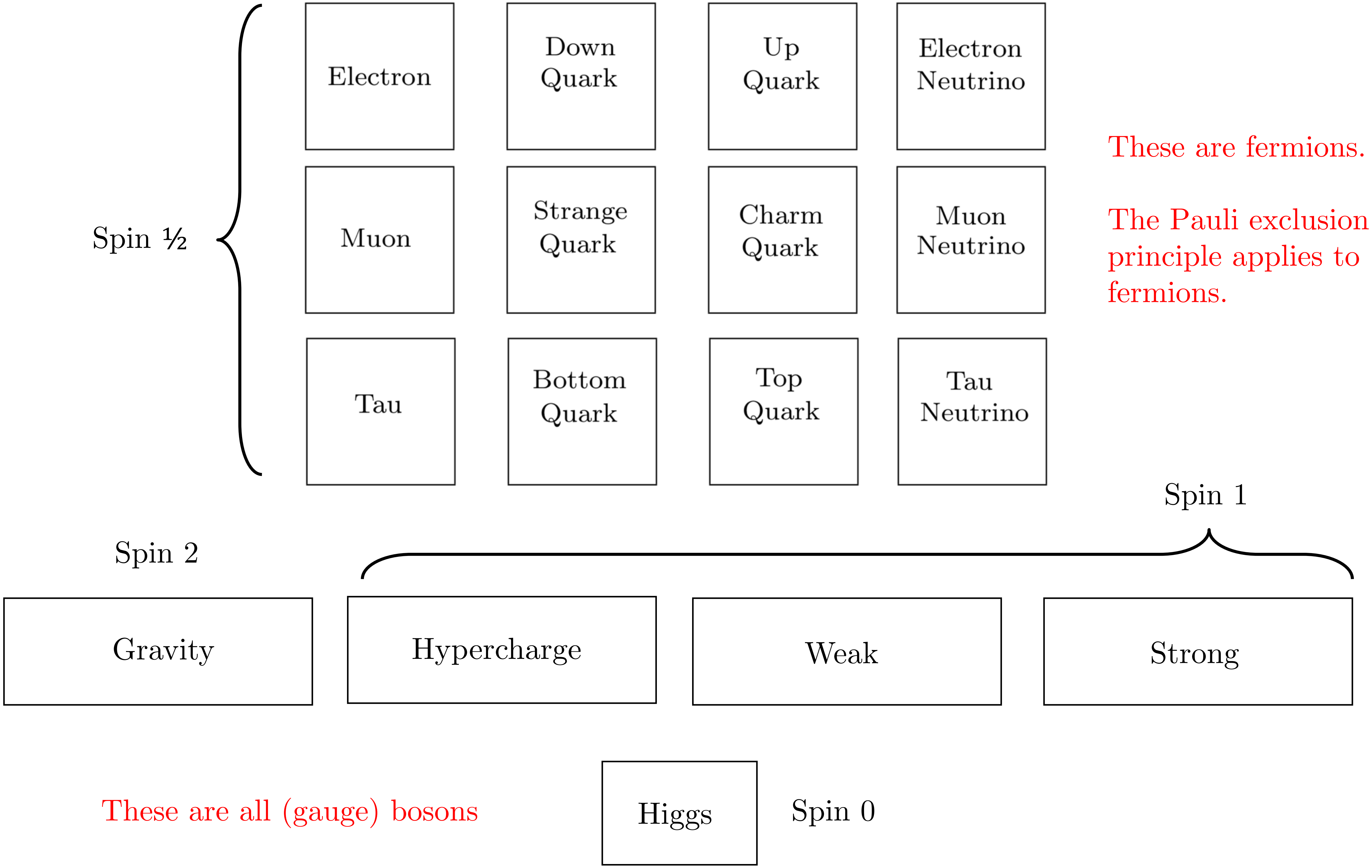


- Particles are excitations of underlying quantum fields

- Forces are also due to fields and have associated particles

    - Electromagnetism = photon
    - Strong = gluon
    - Weak = W and Z bosons
    - Gravity = graviton

Intrinsic Angular Momentum = Spin



Spin ½

Electron

Down Quark

Up Quark

Electron Neutrino

Muon

Strange Quark

Charm Quark

Muon Neutrino

Tau

Bottom Quark

Top Quark

Tau Neutrino

These are fermions.

The Pauli exclusion principle applies to fermions.

Spin 2

Gravity

Spin 1

Hypercharge

Weak

Strong

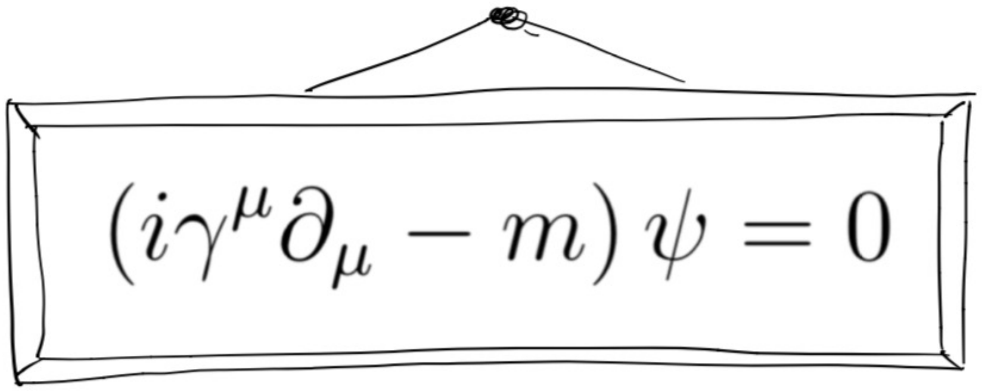
These are all (gauge) bosons

Higgs

Spin 0

A Remarkable Fact

All spin ½ particles are described by the same equation, discovered by Dirac



$$(i\gamma^{\mu}\partial_{\mu} - m)\,\psi = 0$$

Here m is the mass.

Consequence: all matter particles come with anti-particles

Mass

Electron 1	Down Quark 9	Up Quark 4	Electron Neutrino ~10^{-6}
Muon 207	Strange Quark 186	Charm Quark 2495	Muon Neutrino ~10^{-6}
Tau 3483	Bottom Quark 8180	Top Quark 340,000	Tau Neutrino ~10^{-6}

An aside: In the Standard Model, the masses of all particles are determined by the strength of interaction with the Higgs field.

Units for Mass

We measure mass in terms of energy, using E=mc^2. The unit of choice is the electronvolt

$$1 \text{ eV} \approx 1.6 \times 10^{-19} \text{ J}$$

Or MeV = 10^6 eV or GeV = 10^9 eV or TeV = 10^{12} eV.

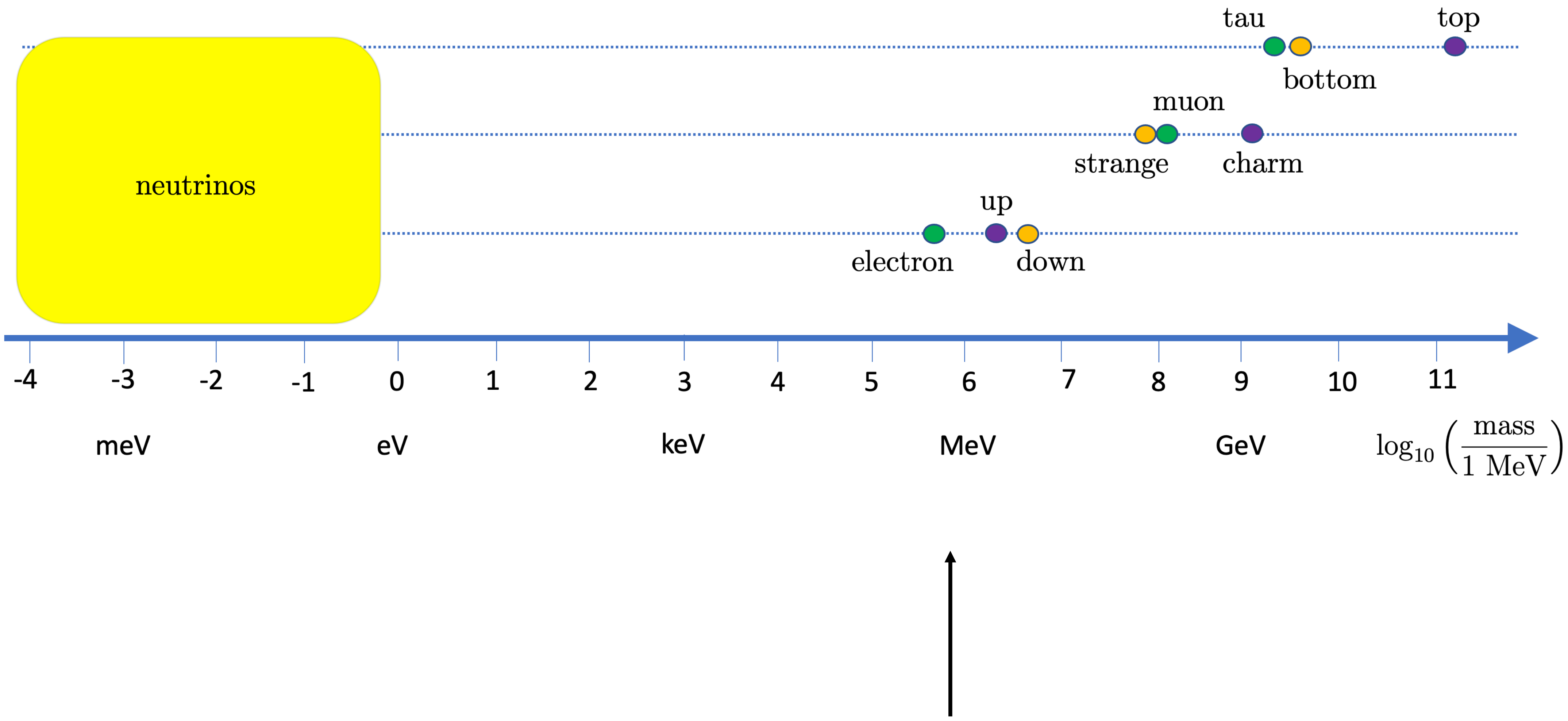
More confusingly, we also measure distance in terms of inverse energy, using

$$\lambda = \frac{\hbar c}{E}$$

For a particle of mass E, this is the “Compton wavelength”, or the size of the particle.

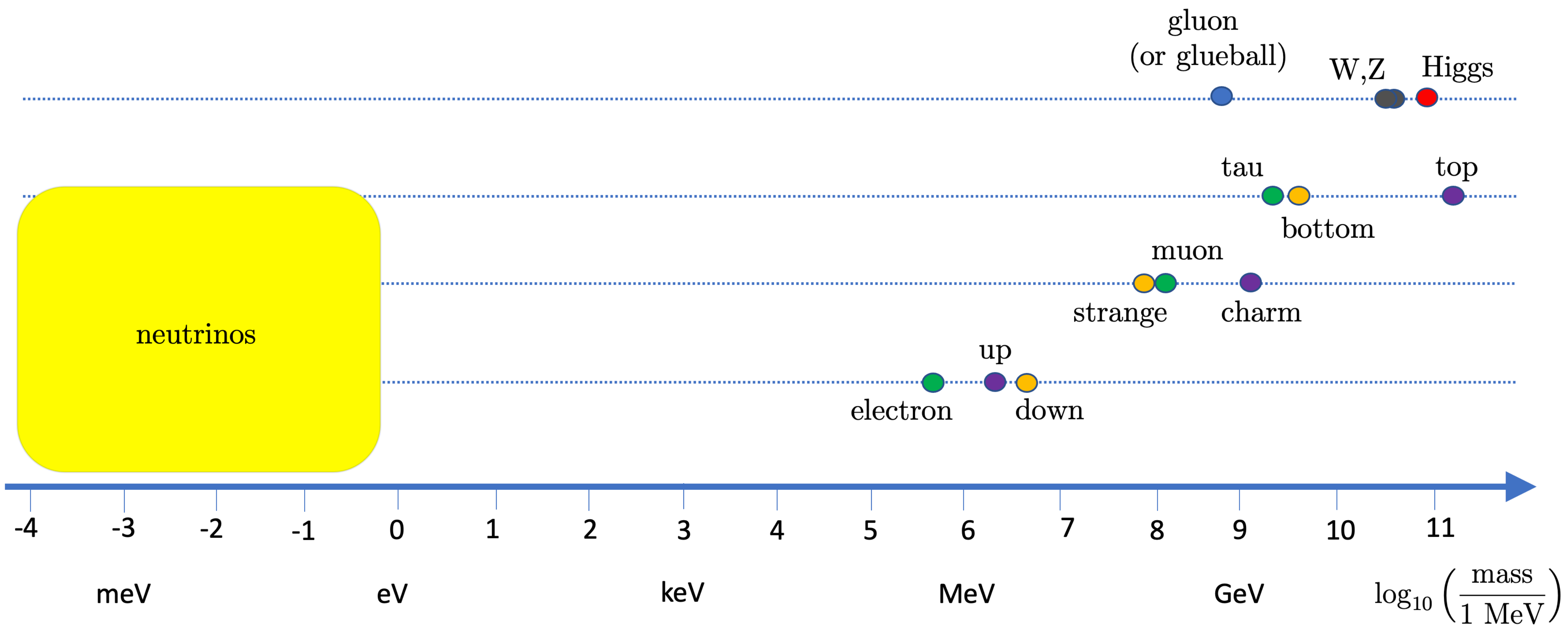
Note: heavier particles are smaller!

The Masses of Particles



Compton wavelength of electron $\lambda_e \approx 2 \times 10^{-12}$ m

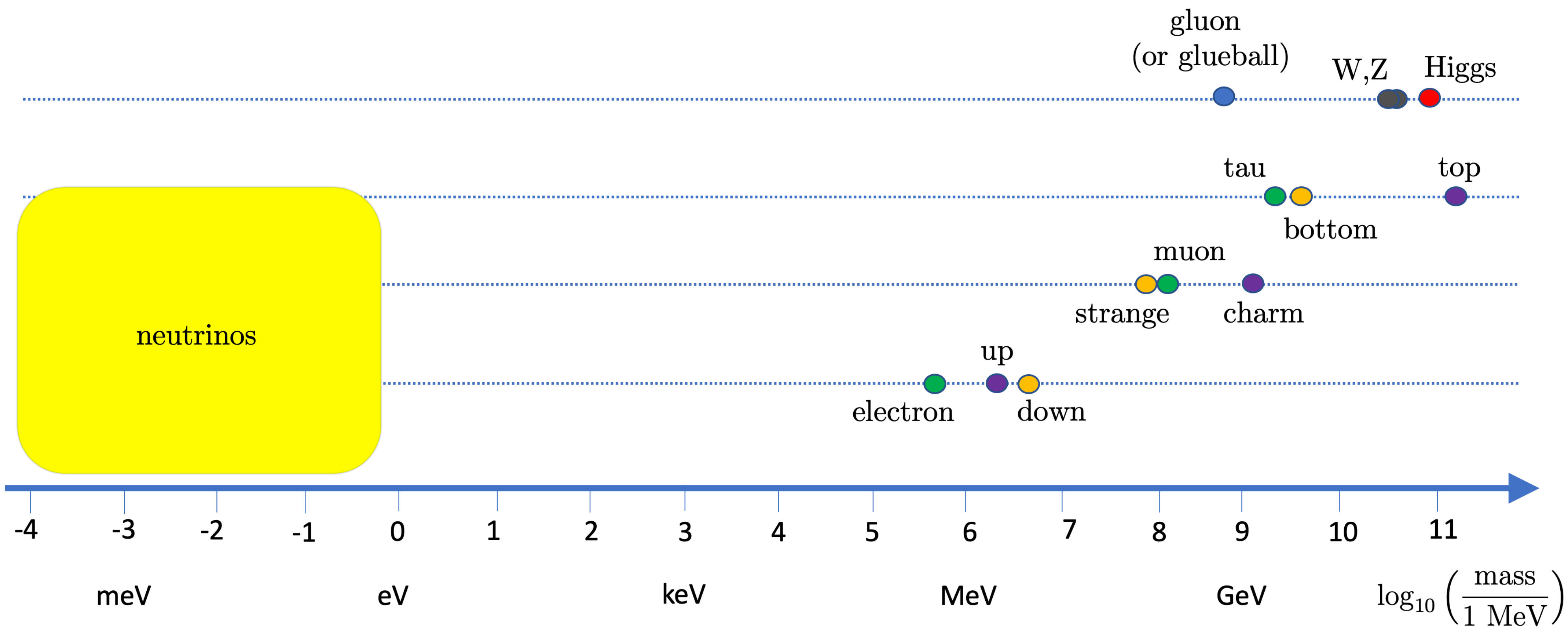
The Masses of Particles



Note: photon and graviton both massless.

(The gluon is a little subtle...see later.)

The Masses of Particles



The biggest

$$H \approx 2 \times 10^{-33}\ \text{eV}$$

$$L_{\text{universe}} \approx 9 \times 10^{26}\ \text{m}$$

The smallest

$$M_{\text{pl}} = \sqrt{\frac{\hbar c}{8\pi G}} \approx 2 \times 10^{18}\ \text{GeV}$$

$$L_{\text{pl}} = \sqrt{\frac{8\pi \hbar G}{c^3}} \approx 8 \times 10^{-35}\ \text{m}$$

Electric Charge

Charge =	-1	$-1/3$	$+2/3$	0
	Electron 1	Down Quark 9	Up Quark 4	Electron Neutrino $\sim 10^{-6}$
	Muon 207	Strange Quark 186	Charm Quark 2495	Muon Neutrino $\sim 10^{-6}$
	Tau 3483	Bottom Quark 8180	Top Quark 340,000	Tau Neutrino $\sim 10^{-6}$

The electric charge characterizes the (relative) strength of the electromagnetic interaction

Electromagnetism (or QED)

The Maxwell Equations

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \quad , \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0 \quad , \quad \nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$

or

$$\boxed{\partial_\mu F^{\mu\nu} = J^\nu}$$

This implies the Coulomb force which, in natural units, reads

$$F = \alpha \frac{Q_1 Q_2}{r^2}$$

with the fine structure constant

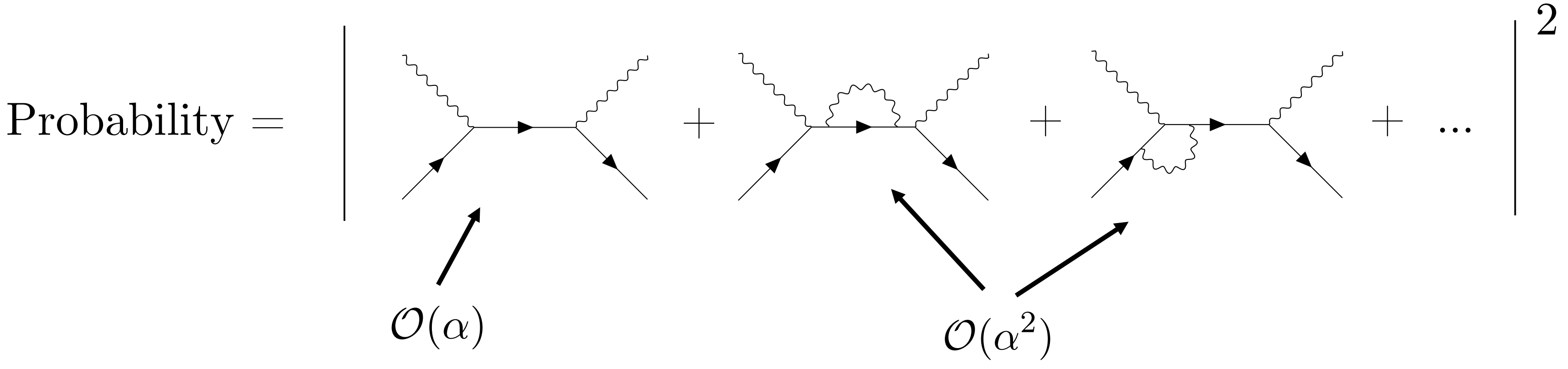
$$\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c} \approx \frac{1}{137}$$

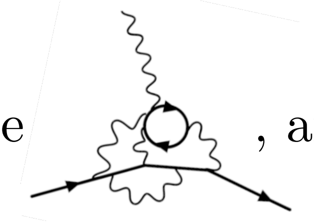
Feynman Diagrams

An important fact: quantum field theory is hard!

We are saved in QED because $\alpha \approx \dfrac{1}{137} \ll 1$. This allows us to write down an approximate solution

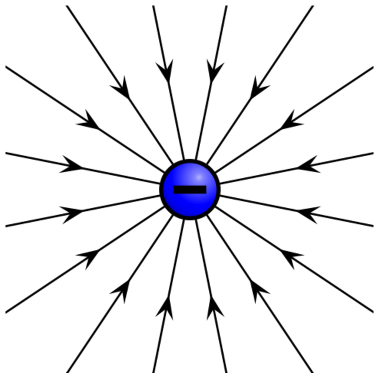
e.g. what is the probability for a photon to scatter off an electron in some direction?



More complicated diagrams, like  , are less and less important.

Renormalisation

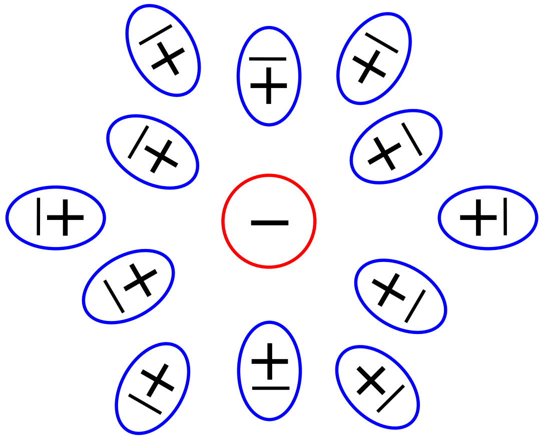
Look close at the electron.



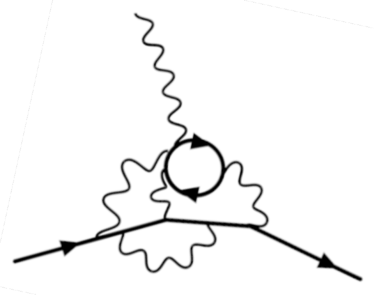
$$\mathbf{E} = \frac{e}{4\pi\epsilon_0 r^2}\hat{\mathbf{r}}$$

⇒

Large energy density near electron, This allows for the creation of particle-anti-particle pairs

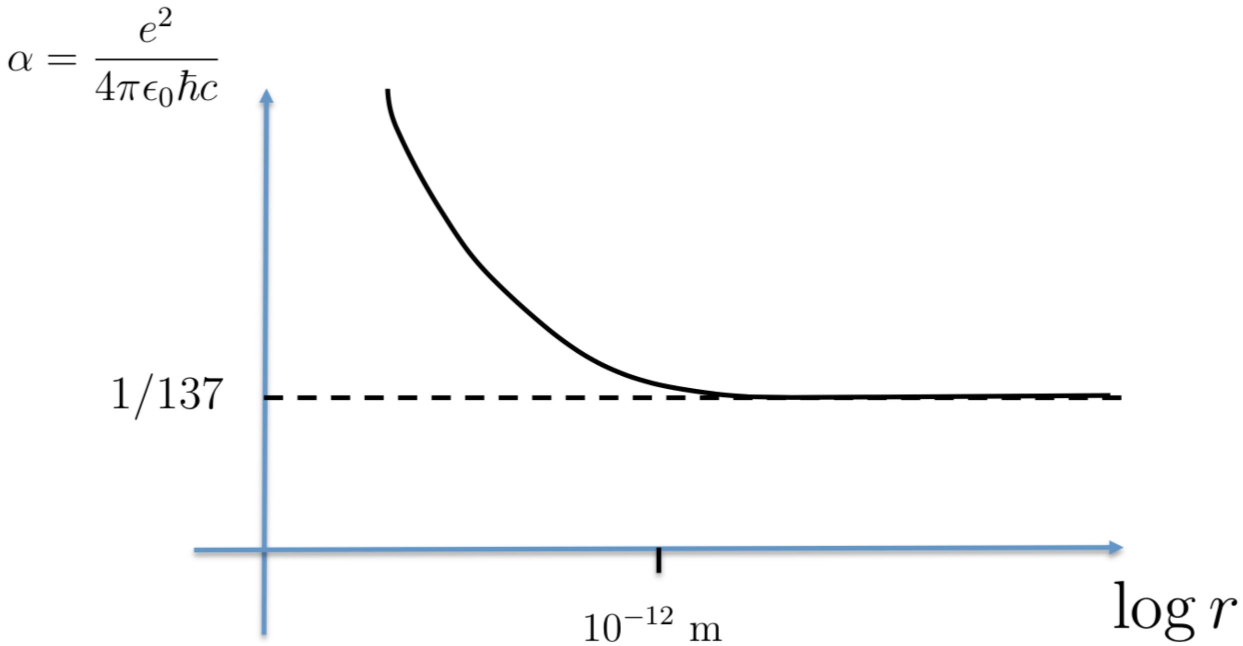


This is the physics behind the increasingly complicated diagrams like this



Renormalisation

As you look more closely, the charge of an electron gets bigger!



Constants of nature are not constant!

The Strong and Weak Force

Both nuclear forces have associated “electric” and “magnetic” fields

$$\mathbf{E} = (E_x, E_y, E_z) \qquad \mathbf{B} = (B_x, B_y, B_z)$$

But each component is now itself a matrix.

- 1 x 1 matrix ⟹ Electromagnetism
- 2 x 2 matrix ⟹ Weak force
- 3 x 3 matrix ⟹ Strong force

or U(1) x SU(2) x SU(3)

These fields are governed by the Yang-Mills equations

$$\boxed{\mathcal{D}_\mu F^{\mu\nu} = J^\nu}$$

The Strong Force (or QCD)

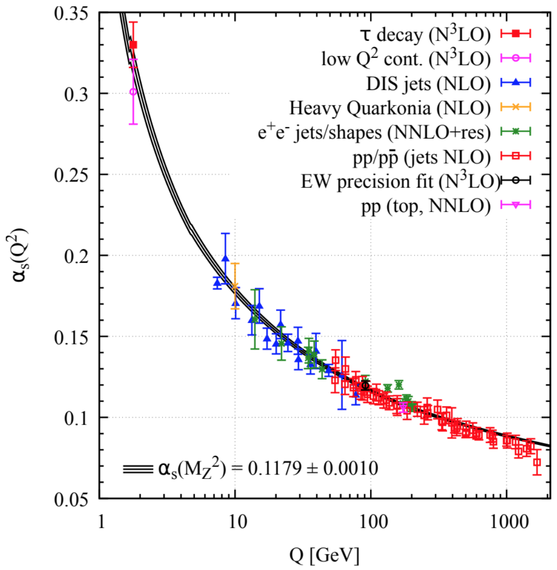
No	Yes	Yes	No
Electron 1	Down Quark 9	Up Quark 4	Electron Neutrino ~10^{-6}
Muon 207	Strange Quark 186	Charm Quark 2495	Muon Neutrino ~10^{-6}
Tau 3483	Bottom Quark 8180	Top Quark 340,000	Tau Neutrino ~10^{-6}

Each quark comes in three *colours*, which we take to be red, green and blue.

(Note: a better counting is that each generation contains 1+3+3+1=8 particles.)

Why is the Strong Force Strong?

strong coupling constant



energy = 1/distance

At high energy, say E=100 GeV, we have $\alpha_s \approx 0.1$. But the strong force gets stronger as we go to larger distances. (Asymptotic freedom.)

Taken naively, $\alpha_s \to \infty$ at the energy scale:

$$\boxed{\Lambda_{\text{QCD}} \approx 200 \text{ MeV}}$$

This corresponds to a distance scale $R_{\text{QCD}} = \dfrac{1}{\Lambda_{\text{QCD}}} \approx 5 \times 10^{-15}$ m

Confinement

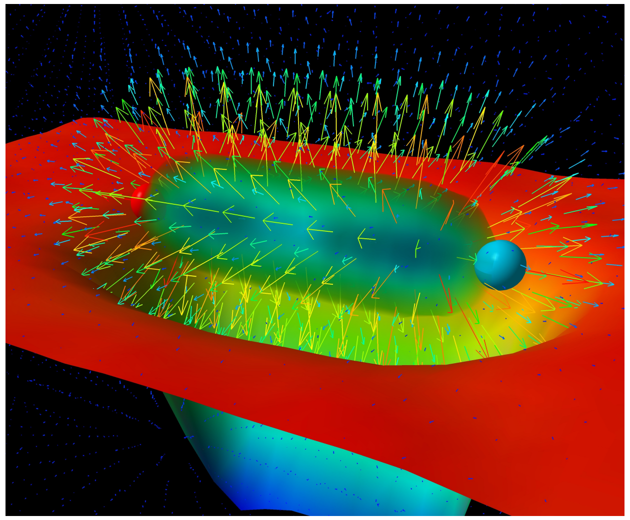
At short distances, $F(r) \sim \frac{\alpha_s}{r^2}$ but at long distances $F(r)$ becomes constant.

In terms of the potential energy, $V(r) \sim -\frac{\alpha_s}{r}$ at short distances, but at long distances

$$\boxed{V(r) \sim \Lambda_{\text{QCD}}^2\, r}$$

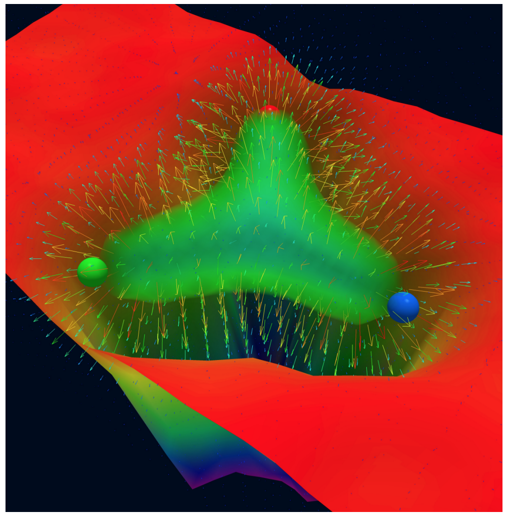
This is *confinement*. We don't see isolated quarks.

Also, the force carrying field is not massless. The gluons stick together to form glueballs, with mass around $m_{\text{gluon}} \approx \Lambda_{\text{QCD}}$. This is the "mass gap" problem.



Hadrons (Stuff Made of Quarks)

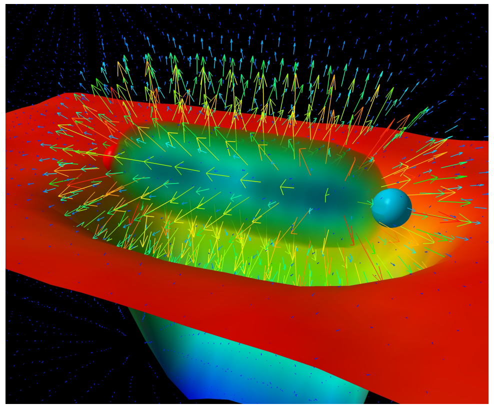
- Baryons: three quarks. For example



$$\begin{array}{ll} n\ (ddu) & m_n \approx 939.57\ \text{MeV} \\ p\ (uud) & m_p \approx 938.28\ \text{MeV} \end{array}$$

A puzzle: $\mathrm{m}_{\mathrm{down}}$= 5 meV and m_{up}=2 MeV. Where does the mass come from?

- Mesons: quark-anti-quark pair. For example, pions



$$\begin{array}{lll} \pi^+ & (\bar{d}u) & m \approx 139\ \text{MeV} \\ \pi^0 & \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d) & m \approx 135\ \text{MeV} \\ \pi^- & (\bar{u}d) & m \approx 139\ \text{MeV} \end{array}$$

Note: Pions have spin 0 and so should be thought of as “force carrying” particles! So ...

Decay

All hadrons other than the proton are unstable. They decay.

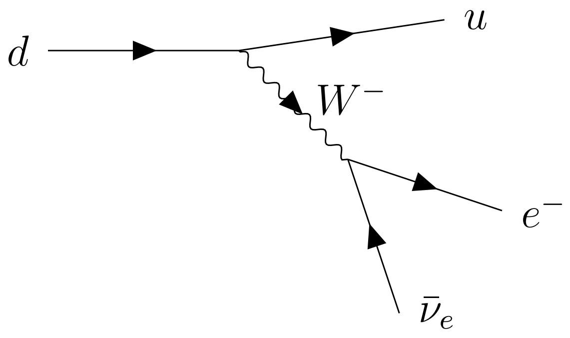
- Strong decay: $\sim 10^{-22}$ to 10^{-24} seconds. $\Delta^{++}\,(uuu) \rightarrow p + \pi^+$
- Electromagnetic decay: $\sim 10^{-16}$ to 10^{-21} seconds. $\Sigma^0\,(dus) \rightarrow \Lambda^0 + \gamma$ · with $\Lambda^0\,(dus)$
- Weak decay: $\sim 10^{-7}$ to 10^{-13} seconds. $\pi^+\,(ud) \;\rightarrow\; \mu^+ + \nu_\mu$

The most famous weak decay is how we first discovered the weak force

$$n \;\rightarrow\; p + e^- + \bar{\nu}_e$$

Or, if you look more closely,

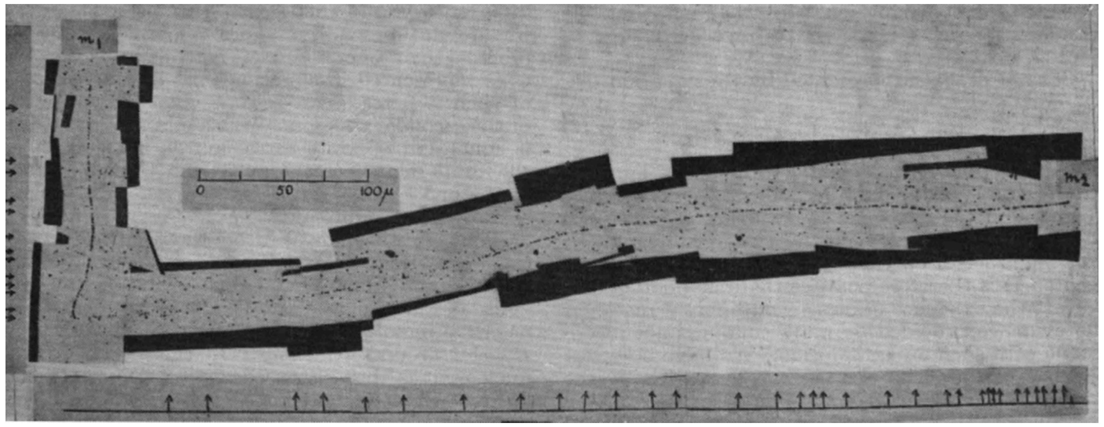
$$d \;\rightarrow\; u + e^- + \bar{\nu}_e$$



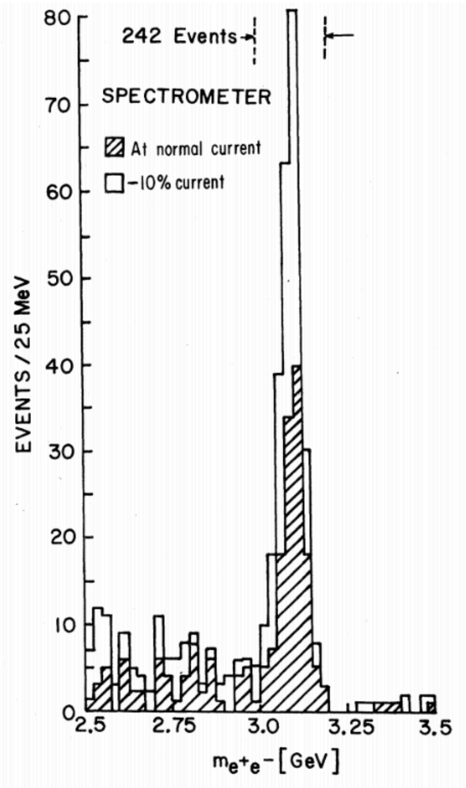
Particles vs Resonances

- Strong decay: $\sim 10^{-22}$ to 10^{-24} seconds.
- Electromagnetic decay: $\sim 10^{-16}$ to 10^{-21} seconds.
- Weak decay: $\sim 10^{-7}$ to 10^{-13} seconds.

If a particle decays through the weak force, we can take a photograph of it!



If it decays through the strong force, or EM, then we see it more indirectly



The Weak Force

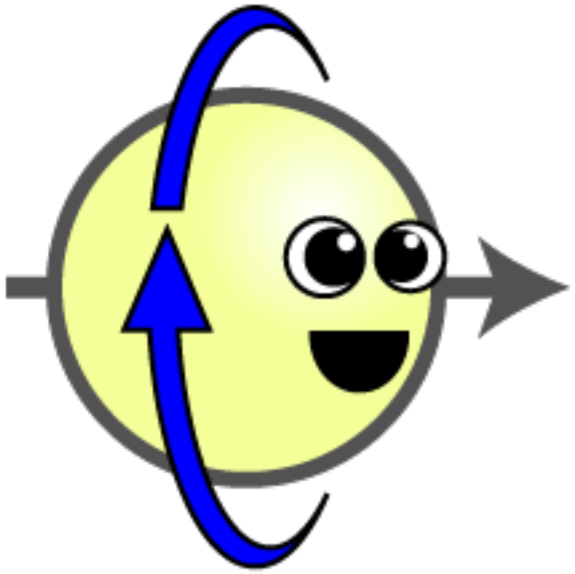
half of each particle!

Electron 1	Down Quark 9	Up Quark 4	Electron Neutrino $\sim 10^{-6}$
Muon 207	Strange Quark 186	Charm Quark 2495	Muon Neutrino $\sim 10^{-6}$
Tau 3483	Bottom Quark 8180	Top Quark 340,000	Tau Neutrino $\sim 10^{-6}$

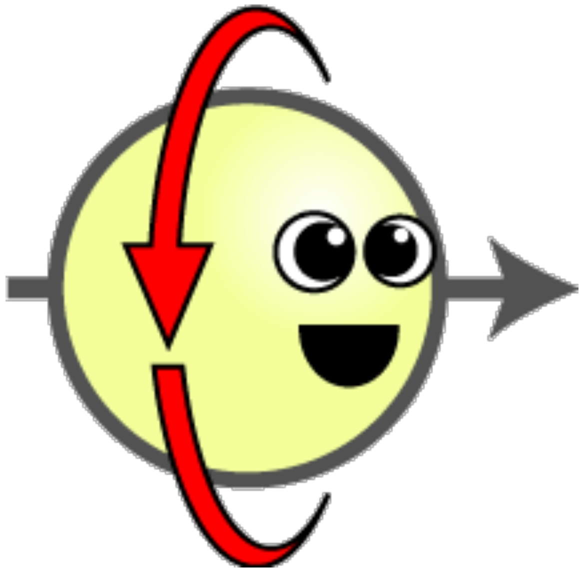
Parity Violation

$$^{60}\mathrm{Co} \ \rightarrow \ ^{60}\mathrm{Ni} + e^- + \bar{\nu}_e + 2\gamma$$

Chiral Fermions



left-handed fermion



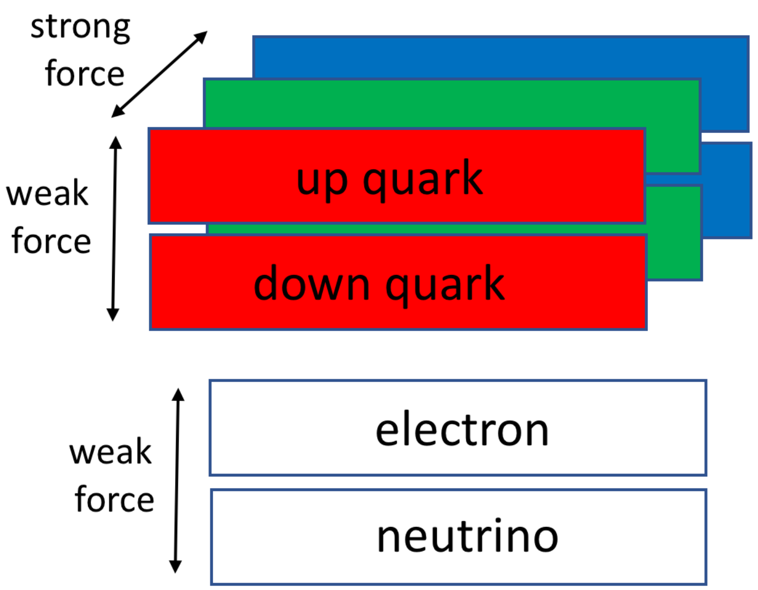
right-handed fermion

Left-handed particles experience the weak force, right-handed do not.

The Forces of the Standard Model

Each generation splits into 8x2 sets of particles

Left-Handed Particles



strong
force

weak
force

up quark

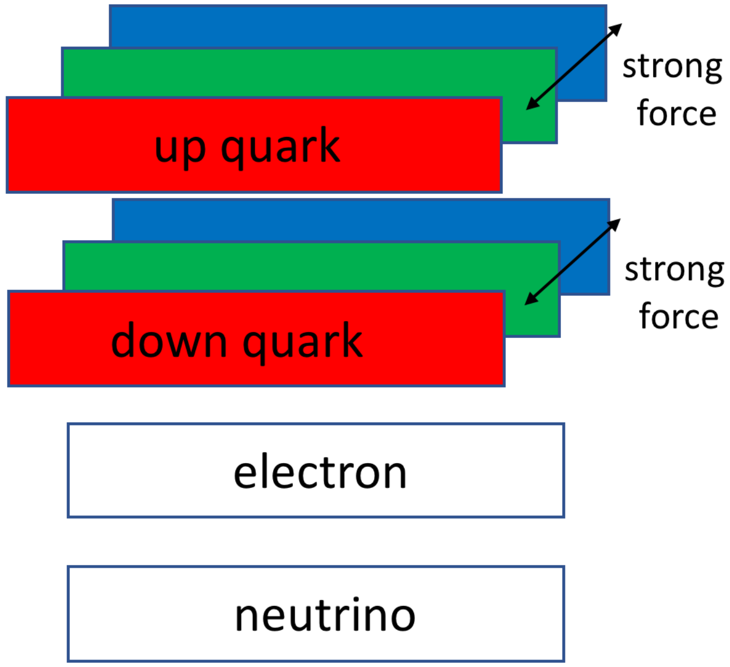
down quark

weak
force

electron

neutrino

Right-Handed Particles



up quark

strong
force

down quark

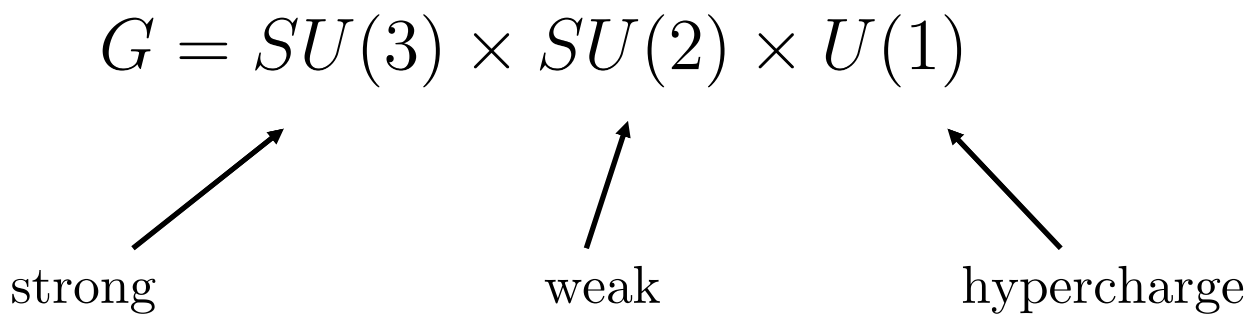
strong
force

electron

neutrino

Note: We don't yet know if the right-handed neutrino exists.

The Structure of the Standard Model



$$G = SU(3) \times SU(2) \times U(1)$$

strong

weak

hypercharge

Particles		Strong	Weak	Hypercharge
Left-handed	quarks	yes	yes	+1/6
	leptons	no	yes	-1/2
Right-handed	up quark	yes	no	+2/3
	down quark	yes	no	-1/3
	electron	no	no	-1
	neutrino	no	no	0

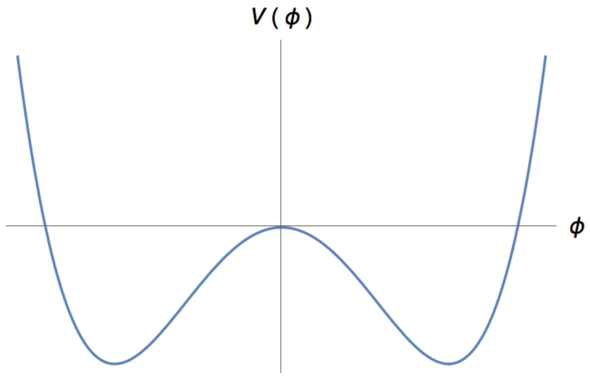
A perfect jigsaw: Anomaly cancellation means that it could hardly be any other way!

The Higgs Field

This is both the simplest and most complicated field in the Standard Model!

Particle	Strong	Weak	Hypercharge
Higgs	no	yes	+1/2

$$\boxed{\mathcal{D}_\mu \mathcal{D}^\mu \phi - V(\phi) = \lambda \psi \psi}$$



Two relevant scales:

- Mass $m_H \approx 125$ GeV
- Condensate $\langle \phi \rangle \approx 246$ GeV

It is the condensate that gives the Higgs its Midas touch: everything that it touches gets a mass

How Particles Get a Mass

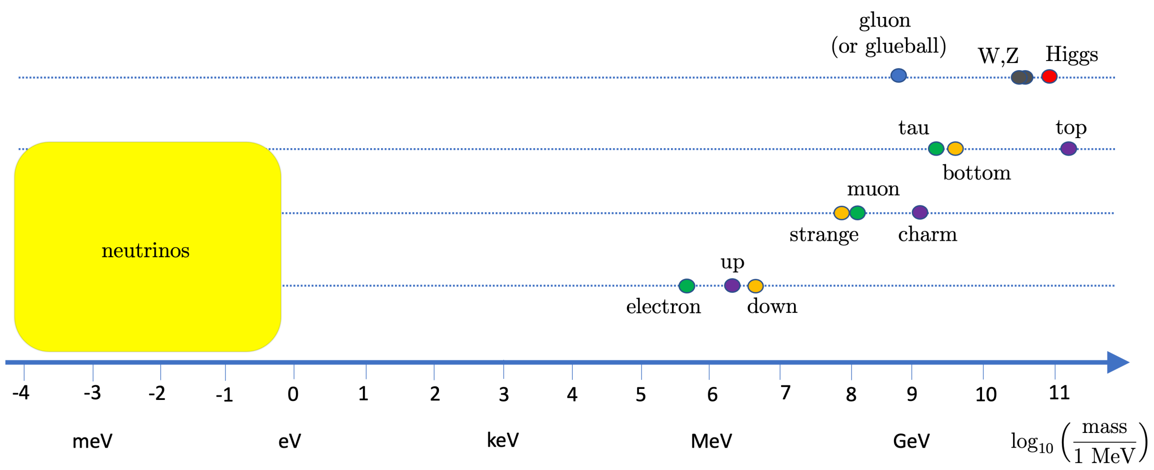
In the Standard Model, all fermions and gauge bosons are obliged to be fundamentally massless

They get a mass by interaction with the Higgs.

$$\text{Mass} = g \times \langle\phi\rangle$$

some dimensionless coupling

$\langle\phi\rangle \approx 246$ GeV



- The Higgs gives mass to the W-boson and Z-boson and all fermions.
- The photon remains massless: it is the one that got away!
- Recall: the mass of the proton and neutron do not come from the Higgs!

One Last Thing: Quark Mixing

There is a misalignment between the interactions with the Higgs and the interaction with the weak force.

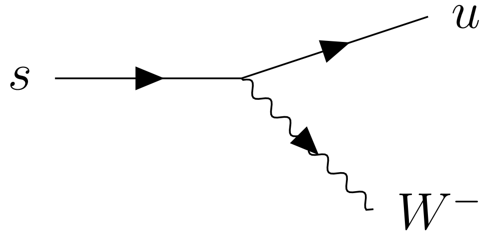
It turns out that you can choose to have the up-sector aligned. But then the down sector is not. The result is a superposition of particles.

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

These particles interact with weak force

These particles interact with Higgs, and so definite mass.

This is how, for example, mesons with strange quarks decay



One Last Thing: and Lepton Mixing

There is a similar statement for neutrinos

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

These particles interact with weak force and are produced in, say, beta decay

These particles have definite mass. These are energy eigenstates that travel unchanged through space.

This gives rise to neutrino oscillations

The Mixing Matrices

For quarks, we have the CKM matrix

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 0.97 & 0.22 & 0.004 \\ 0.22 & 0.97 & 0.04 \\ 0.009 & 0.04 & 0.999 \end{pmatrix}$$

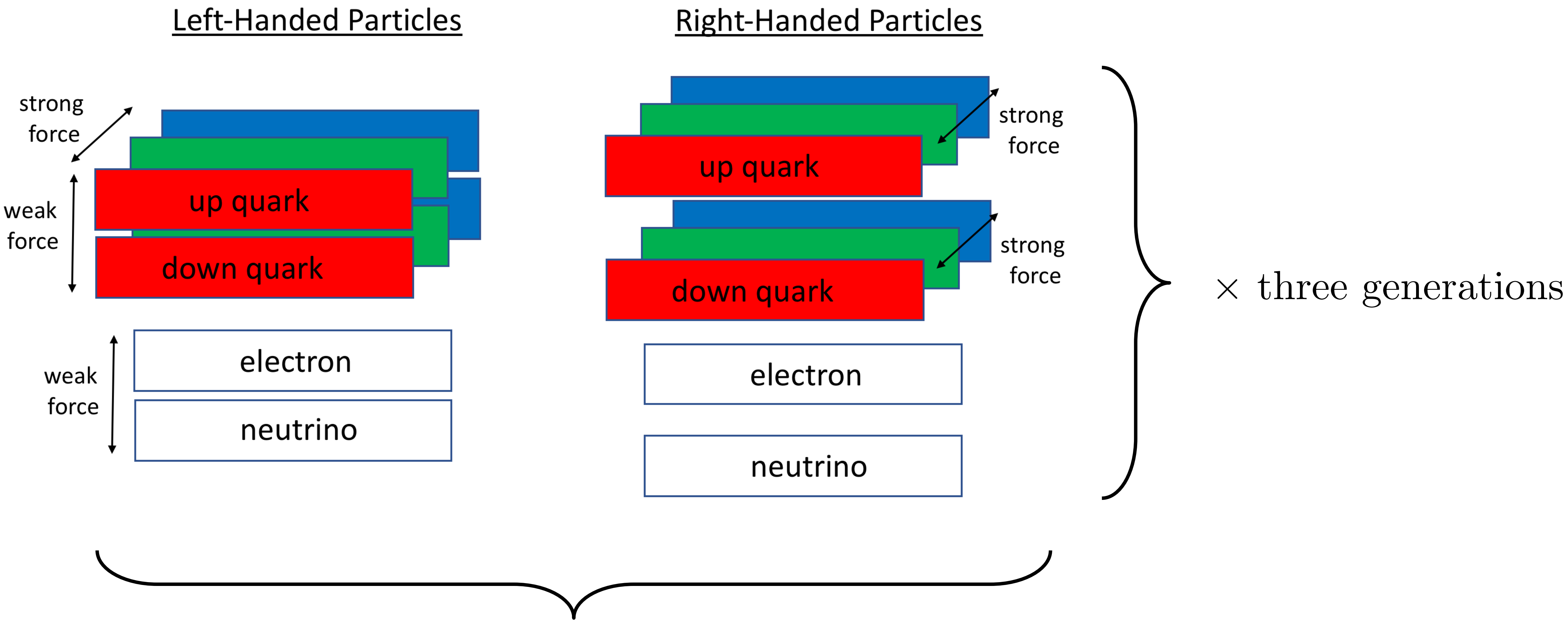
For neutrinos, we have the PMNS matrix

$$\begin{pmatrix} |U_{e1}| & |U_{e2}| & |U_{e3}| \\ |U_{\mu 1}| & |U_{\mu 2}| & |U_{\mu 3}| \\ |U_{\tau 1}| & |U_{\tau 2}| & |U_{\tau 3}| \end{pmatrix} \approx \begin{pmatrix} 0.8 & 0.5 & 0.1 \\ 0.3 & 0.5 & 0.7 \\ 0.4 & 0.6 & 0.6 \end{pmatrix}$$

We only know these parameters by experimental measurement. Why do they take these values? Why are the matrices so different?

Summary: The Greatest Theory of All Time

$$G = SU(3) \times SU(2) \times U(1)$$



Left-Handed Particles

strong force

weak force

up quark

down quark

weak force

electron

neutrino

Right-Handed Particles

up quark

strong force

down quark

strong force

electron

neutrino

× three generations

with all complications coming from interactions with Higgs!