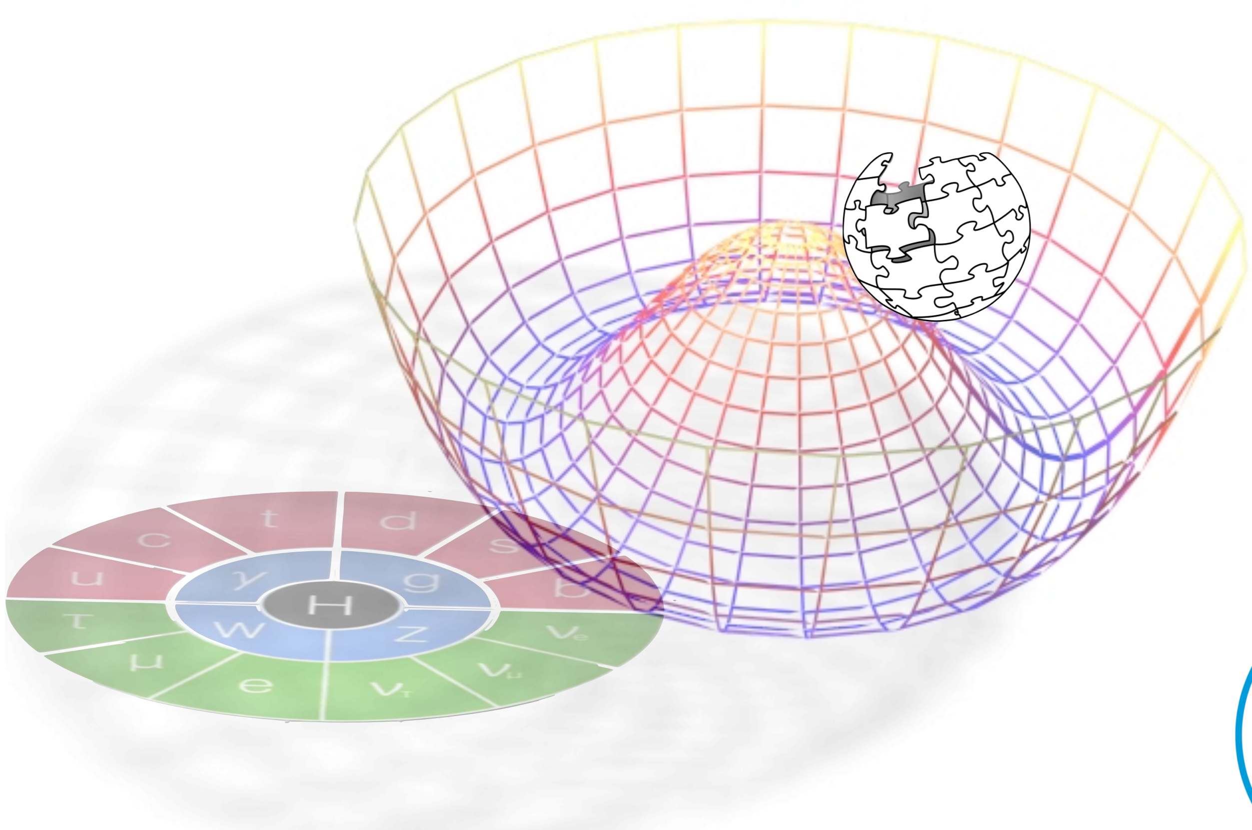
The Standard Model
of particle physics

CERN summer student lectures 2023

Lecture 1/5



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Citius, Altius, Fortius

How high can a human jump with a pole?

Physics (energy conservation) tells us that longer poles don't help!

$$\Delta h = \frac{v^2}{2g}$$

footspeed: 44.72km/h

(Usain Bolt, Berlin, August 2009, between 60m and 80m)

$$\Delta h = 7.62\,\mathrm{m}$$

Over the years, we have learnt a few other **conservation laws** that tell us what an athlete/a particle can do or cannot do.

— Remarkable breakthrough in the understanding of Nature: —

forces among particles are associated to symmetries

- conservation of E → invariance by (time)-translation
- electro-magnetic forces → (local) invariance by phase rotation of particle wavefunctions

The Standard Model of Particle Physics

Lorentz symmetry + internal SU(3)xSU(2)xU(1) symmetry

Role(s) of Symmetry

— Selection Rules —

- hydrogen atom: energy levels depends on n, but not on l, nor m
  (invariance under rotations as well as another symmetry that leaves the Runge-Lenz vector invariant)
- electric charge conservation: $e^+e^- \xrightarrow{✓} \gamma$ but $e^+\gamma \xrightarrow{✗}$ e-

— Dynamical Principle —

Requiring that theory describing SM particles is invariant under some (local) symmetries require the existence of interactions among these particles. And these interactions have a particular structure.

Outline

Monday: symmetry
- Lagrangians
- Lorentz symmetry - scalars, fermions, gauge bosons
- Gauge/local symmetry as dynamical principle - Example: U(1) electromagnetism

Tuesday: SM symmetries
- Nuclear decay, Fermi theory and weak interactions: SU(2)
- Strong interactions: SU(3)
- Dimensional analysis: cross-sections and life-time computations made simple

Wednesday: chirality of weak interactions
- Chirality of weak interactions
- Pion decay

Thursday: Higgs mechanism
- Spontaneous symmetry breaking and Higgs mechanism
- Lepton and quark masses, quark mixings
- Neutrino masses

Friday: quantum effects
- Running couplings
- Asymptotic freedom of QCD
- Anomalies cancelation

SM= S(R+Q)M

The fundamental constituents of matter obey the laws of **Quantum Mechanics** and **Special Relativity**
They are described in the framework of **Quantum Field Theory (QFT)**

QFT offers a way
1) To organise our knowledge
2) To parametrise our ignorance

Describe collider data

Play a crucial role in the evolution of the Universe

Likely to fail to describe gravity @ quantum level

"Before breaking the rules, you first need to master them”

Goals of the lectures

1. Explain QFT to describe the SM particles and their interactions
2. Introduce the principles to build a model of Nature
3. Explain how to compute cross-section and decay rate
4. Unveil clues where the SM might fail

Lagrangians

The Newton law of classical mechanics

$$\vec{F} = m\vec{a} \quad \text{or} \quad V'(x) = -m\ddot{x}$$

can be obtained by requiring the least action principle

$$\delta S = 0$$

where

the action: $S = \int_{t_1}^{t_2} dt \mathcal{L}(x, \dot{x})$ with the (classical) Lagrangian: $\mathcal{L}(x, \dot{x}) = \frac{1}{2} m \dot{x}^2 - V(x)$

(Hamiltonian/energy: $\mathcal{H} = \dot{x}\frac{\delta \mathcal{L}}{\delta \dot{x}} - \mathcal{L} = \frac{1}{2} m \dot{x}^2 + V(x)$)

Euler-Lagrange equations

$$\delta S = \int_{t_1}^{t_2} dt \left(\frac{\delta \mathcal{L}}{\delta x} - \frac{d}{dt}\frac{\delta \mathcal{L}}{\delta \dot{x}} \right) \delta x + \text{boundary terms} = 0 \quad \Rightarrow \quad \frac{\delta \mathcal{L}}{\delta x} = \frac{d}{dt}\frac{\delta \mathcal{L}}{\delta \dot{x}}$$

For the classical Lagrangian: $-V'(x) = m\ddot{x}$

Lagrangians for Particle Physics

Equations of motion, like $\vec{F} = m\vec{a}$, are **covariant** under the action of a symmetry.

Lagrangians are **invariant**.

That makes identifying the symmetries of Nature much easier.

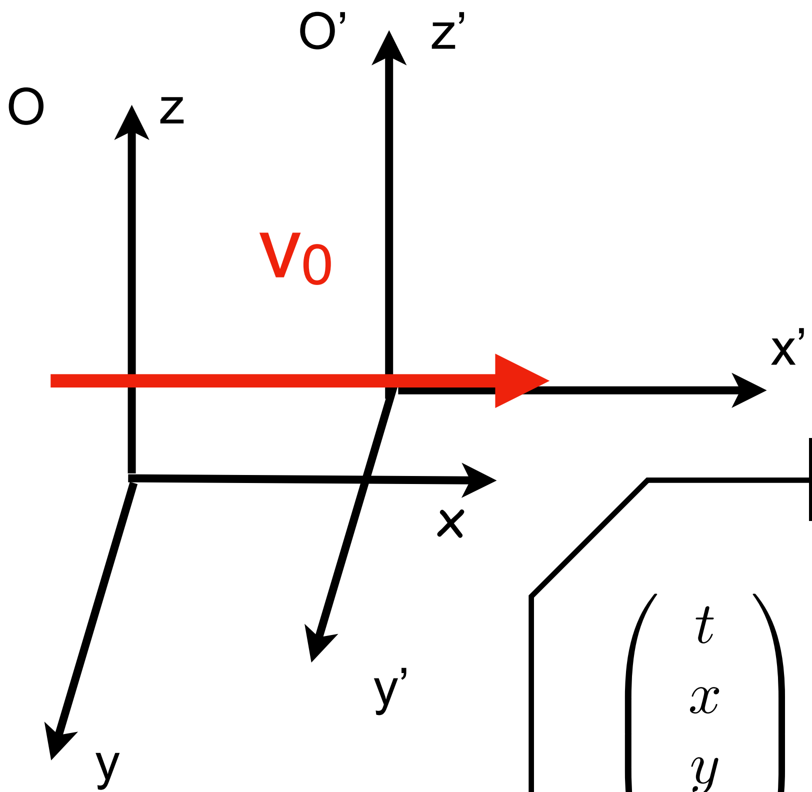
—Particle Physics—

particles $\leftrightarrow$ fields with specific transformation properties under some fundamental symmetries

build a Lagrangian (i.e. a function of the these fields and their space-time derivatives) that remains invariant under the action of the symmetry transformations.

Which symmetries?

Lorentz Transformations



Consider two observers

in relative motion with a constant speed v_0 along the x-axis

they use their own systems of coordinates (t,x,y,z) and (t',x',y',z')

Galilean transformations

$$\begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} t' = t \\ x' = -\beta_0 ct + x \\ y' = y \\ z' = z \end{pmatrix} \text{ with } \beta_0 = \frac{v_0}{c}$$

in particular

$$\boxed{v' = v - v_0}$$

the speed can be arbitrarily large.

Lorentz transformations

$$\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} ct' = \gamma_0 (ct - \beta_0 x) \\ x' = \gamma_0 (-\beta_0 ct + x) \\ y' = y \\ z' = z \end{pmatrix} \text{ with } \beta_0 = \frac{v_0}{c} \quad \gamma_0 = \frac{1}{\sqrt{1-\beta_0^2}}$$

Note: $\Delta^2 \equiv (ct)^2 - x^2 - y^2 - z^2 = (ct')^2 - x'^2 - y'^2 - z'^2 \equiv \Delta'^2$

in particular

$$\boxed{v' = \frac{v - v_0}{1 - v \cdot v_0 / c^2}}$$

The speed of light is

the same for all observers:

if v=c than v'=c too

Einstein Algebra

$$x^\mu \equiv \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \qquad \mu = 0, 1, 2, 3$$

$$\Delta^2 \equiv (ct)^2 - x^2 - y^2 - z^2 = \eta_{\mu\nu} x^\mu x^\nu$$ with $$\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$$ Minkowski metric

$x^\mu \to x'^\mu = \Lambda^\mu{}_\nu x^\nu$ leaves Δ^2 invariant iff $\eta_{\mu\nu} = \eta_{\mu'\nu'} \Lambda^{\mu'}{}_\mu \Lambda^{\nu'}{}_\nu$

At linear order, $\Lambda^\mu{}_\nu \approx \delta^\mu{}_\nu + \omega^\mu{}_\nu$, the invariance of Δ^2 simply writes $\omega_{\mu\nu} + \omega_{\nu\mu} = 0$

where we have defined $\omega_{\mu\nu} \equiv \eta_{\mu\mu'} \omega^{\mu'}{}_\nu$

boost along x-direction

$$\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \to \begin{pmatrix} ct' = \gamma_0 \left(ct - \beta_0 x\right) \\ x' = \gamma_0 \left(-\beta_0 ct + x\right) \\ y' = y \\ z' = z \end{pmatrix}$$ corresponds to $$\Lambda^\mu{}_\nu = \begin{pmatrix} \gamma & -\gamma\beta & & \\ -\gamma\beta & \gamma & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

indeed satisfies $\eta_{\mu\nu} = \eta_{\mu'\nu'} \Lambda^{\mu'}{}_\mu \Lambda^{\nu'}{}_\nu$ since $\gamma^2(1-\beta^2) = 1$

Exercise: find the expression of $\Lambda^\mu{}_\nu$ for a boost along a general space direction

Scalar Lagrangian

A (real) **scalar** field ϕ
is a real function of space-time coordinates that doesn't change under Lorentz transformations

$$\boxed{\begin{gathered} x^\mu \to x'^\mu = \Lambda^\mu{}_\nu x^\nu \\ \phi(x) \to \phi'(x') = \phi(x) \end{gathered}}$$

Lorentz invariant Lagrangian for scalar field?

- any potential V(Φ) is automatically invariant
- kinetic term?

$$\begin{gathered} x^\mu \to x'^\mu = \Lambda^\mu{}_\nu x^\nu \\ \phi(x) \to \phi'(x') = \phi(x) \end{gathered} \Rightarrow \partial_\mu \phi = \Lambda^\nu{}_\mu \, \partial'_\nu \phi' \Rightarrow \partial_\mu \phi \partial^\mu \phi = \underbrace{\eta^{\mu\nu} \Lambda^{\mu'}{}_\mu \, \Lambda^{\nu'}{}_\nu}_{\eta^{\mu'\nu'}} \partial'_{\mu'} \phi' \partial'_{\nu'} \phi' = \eta^{\mu'\nu'} \partial'_{\mu'} \phi' \partial'_{\nu'} \phi'$$

(Lorentz transformation)

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi)$$

Eq. of motion: $0 = \delta \mathcal{L} = \left(-\partial_\mu \partial^\mu \phi - \frac{\partial V}{\partial \phi} \right) \delta \phi$ i.e. $\Box \phi = -V'(\phi)$ Klein-Gordon equation

Equations of Motion of Elementary Particles

Schrödinger Equation (1926):

$$\left(i\hbar\frac{\partial}{\partial t}+\frac{\hbar^2}{2m}\Delta - V\right)\Phi = 0$$

$$E=\frac{p^2}{2m}+V$$

classical $\leftrightarrow$ quantum correspondance

$E \to i\hbar\frac{\partial}{\partial t}$ & $p \to i\hbar\frac{\partial}{\partial x}$

Klein-Gordon Equation (1927):

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\Delta+\frac{m^2c^2}{\hbar^2}\right)\Phi = 0$$

$$\frac{E^2}{c^2}=p^2+m^2c^2$$

Dirac Equation (1928):

$$\left(i\gamma^\mu\partial_\mu - \frac{mc}{\hbar}\right)\Psi = 0$$

$$E=\begin{cases} +\sqrt{p^2c^2+m^2c^4} & \text{matter} \\ -\sqrt{p^2c^2+m^2c^4} & \text{antimatter}\end{cases}$$

$$E=\vec{\alpha}\vec{p}\,c+\beta mc^2$$

$$\gamma^0=\beta,\ \gamma^i=\beta\alpha^i,\ \{\gamma^\mu,\gamma^\nu\}=2\eta^{\mu\nu}$$

positron (e^+) discovered by C. Anderson in 1932

Fermion Lagrangian

$$\boxed{\mathcal{L} = \psi^\dagger \gamma^0 \left(i\gamma^\mu \partial_\mu - m\right) \psi}$$

ψ 4-component Dirac spinor describes a spin-1/2 particle when quantised

$\gamma^\mu \ (\mu = 0, 1, 2, 3)$ are four 4x4 matrices

- **Equation of motion:**

$$0 = \delta\mathcal{L} = \psi^\dagger \gamma^0 \left(i\gamma^\mu \partial_\mu - m\right) \delta\psi$$

Dirac equation $\left(i\gamma^\mu \partial_\mu - m\right) \psi = 0$

- **Lorentz invariance:** (see technical slides at the end of the lecture)

$$\boxed{\begin{array}{c} x^\mu \to x'^\mu = (\delta^\mu{}_\nu + \omega^\mu{}_\nu) x^\nu \quad \text{with} \quad \omega_{\mu\nu} + \omega_{\nu\mu} = 0 \\ \psi(x) \to \psi'(x') = \left(1_4 + \frac{1}{8}\omega_{\mu\nu}[\gamma^\mu, \gamma^\nu]\right) \psi(x) \end{array}}$$

- **Dirac algebra:**

For this equation to be consistent with Einstein equation ($m^2=E^2-p^2$) or Klein-Gordon eq., the γ^μ matrices have to obey the Clifford algebra

$$\boxed{\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}}$$

- **Dirac matrices:** One particular realisation of the Dirac algebra (not unique)

$$\gamma^0 = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} & & & 1 \\ & & 1 & \\ & -1 & & \\ -1 & & & \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} & & & -i \\ & & i & \\ & i & & \\ -i & & & \end{pmatrix}, \quad \gamma^3 = \begin{pmatrix} & & 1 & \\ & & & -1 \\ -1 & & & \\ & 1 & & \end{pmatrix}$$

U(1) Gauge Symmetry — QED

Quantum ElectroDynamics : the phase of an electron is not physical and can be rotated away
(internal symmetry, same transformation in all Dirac components)

$$\boxed{\psi \to e^{i\theta}\psi}$$

If the phrase transformation is **local**, i.e., depends on space-time coordinate, then

$$\partial_\mu \psi \to e^{i\theta}\left(\partial_\mu \psi + i(\partial_\mu \theta)\psi\right)$$

and the kinetic term is no-longer invariant due to the presence of the non-homogenous piece

To make the theory invariant under **local** transformation, one needs to introduce a **gauge field** that keeps track/memory of how the phase of the electron changes from one point to another. For that, we build a **covariant derivative** that has nice homogeneous transformations

$$D_\mu \psi = \partial_\mu \psi + ieA_\mu \psi \to e^{i\theta} D_\mu \psi \quad \text{iff} \quad A_\mu \to A_\mu - \frac{1}{e}\partial_\mu \theta$$

$$\boxed{\mathcal{L} = \psi^\dagger \gamma^0 \left(i\gamma^\mu D_\mu - m\right)\psi}$$

invariant under

- Lorentz transformation
- local phase rotation

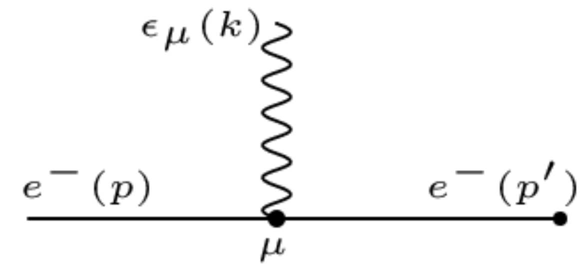
Dynamical Principle

$$\boxed{\mathcal{L} = \psi^\dagger \gamma^0 \left(i\gamma^\mu D_\mu - m \right) \psi}$$

interaction between

gauge field (aka photon) and electron

$$eA_\mu \psi^\dagger \gamma^0 \gamma^\mu \psi$$

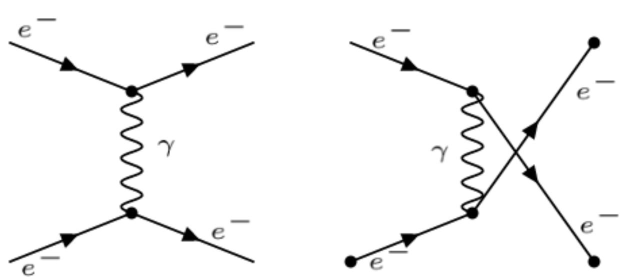


Gauge invariance is a dynamical principle: it predicts some interactions among particles.

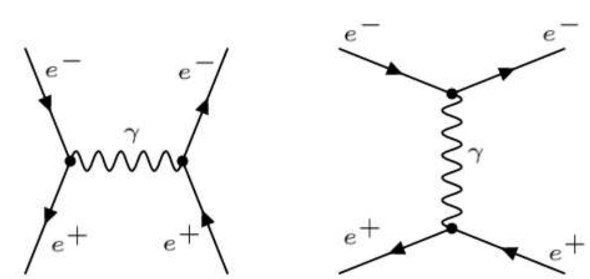
It also explains why the QED interactions are universal
(an electron interacts with a photon in the same way on Earth, on the Moon and at the outskirts of the Universe)

— Some examples of QED processes —

- **Moeller scattering** : $e^- + e^- \to e^- + e^-$

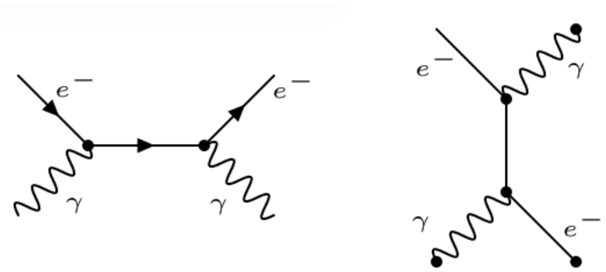


- **Bhabha scattering** : $e^- + e^+ \to e^- + e^+$

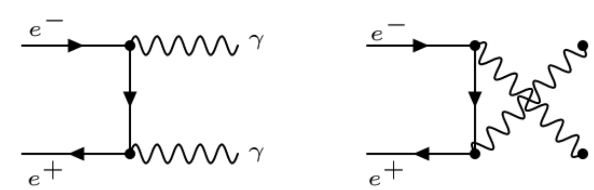


$$F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}$$

- **Compton scattering** : $e^- + \gamma \to e^- + \gamma$



- **Pair annihilation** : $e^- + e^+ \to \gamma + \gamma$



Gauge Field Kinetic Term

To build the QED Lagrangian, we had to introduce a new field A_μ

it is propagating degree of freedom we need to add a kinetic term in the Lagrangian.

Tensor field strength: $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

- **Lorentz transformations:** $x^\mu \to x'^\mu = \Lambda^\mu{}_\nu x^\nu$, $A^\mu \to A'^\mu = \Lambda^\mu{}_\nu A^\nu$ ➡ $F^{\mu\nu} \to F'^{\mu\nu} = \Lambda^\mu{}_\rho \Lambda^\mu{}_\sigma F^{\rho\sigma}$
- **U(1) gauge transformations:** $A_\mu \to A_\mu - \frac{1}{e}\partial_\mu \theta$ ➡ $F_{\mu\nu} \to F_{\mu\nu}$

$$\boxed{\mathcal{L}_{\text{kin}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}}$$

invariant under
- Lorentz transformation
- local phase rotation

equations of motion ↔ **Maxwell equations** of electromagnetism

A^0=EM scalar potential, $A^{i=1,2,3}$ = EM vector potential

$$\vec{E} = -\vec{\nabla}\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}, \quad \vec{B} = \vec{\nabla} \wedge \vec{A}$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & -\vec{E}_x & -\vec{E}_y & -\vec{E}_z \\ \vec{E}_x & 0 & -\vec{B}_z & \vec{B}_y \\ \vec{E}_y & \vec{B}_z & 0 & -\vec{B}_x \\ \vec{E}_z & -\vec{B}_y & \vec{B}_x & 0 \end{pmatrix}$$

➡ $\partial_\mu F^{\mu\nu} = J^\nu$

Remark: no interaction among photons (photons only interact with electrically charged fields)

SU(N) non-Abelian Gauge Symmetry

We generalise the QED construction by considering general transformation of a N-vector

$$\phi \to U\phi$$

We build a **covariant derivative** that again has nice homogeneous transformations

$$D_\mu \phi = \partial_\mu \phi + igA_\mu \phi \to UD_\mu \phi \qquad \text{iff} \qquad A_\mu \to UA_\mu U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1}$$

g is the gauge coupling and defines the strength of the interactions

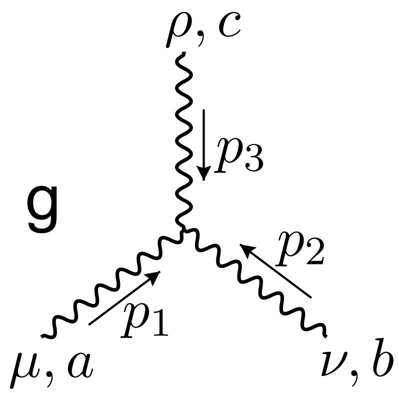
For the field strength to transform homogeneously, one needs to add a non-Abelian piece

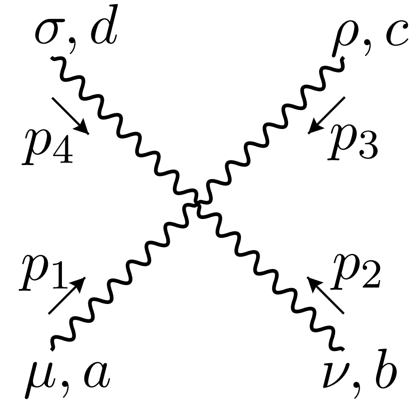
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu] \to UF_{\mu\nu}U^{-1}$$

Contrary to the Abelian case, the gauge fields are now charged and interact with themselves

$$\mathcal{L}_{\rm kin} = {\rm Tr} F_{\mu\nu}F^{\mu\nu} \supset g\partial AAA + g^2 AAAA$$

g



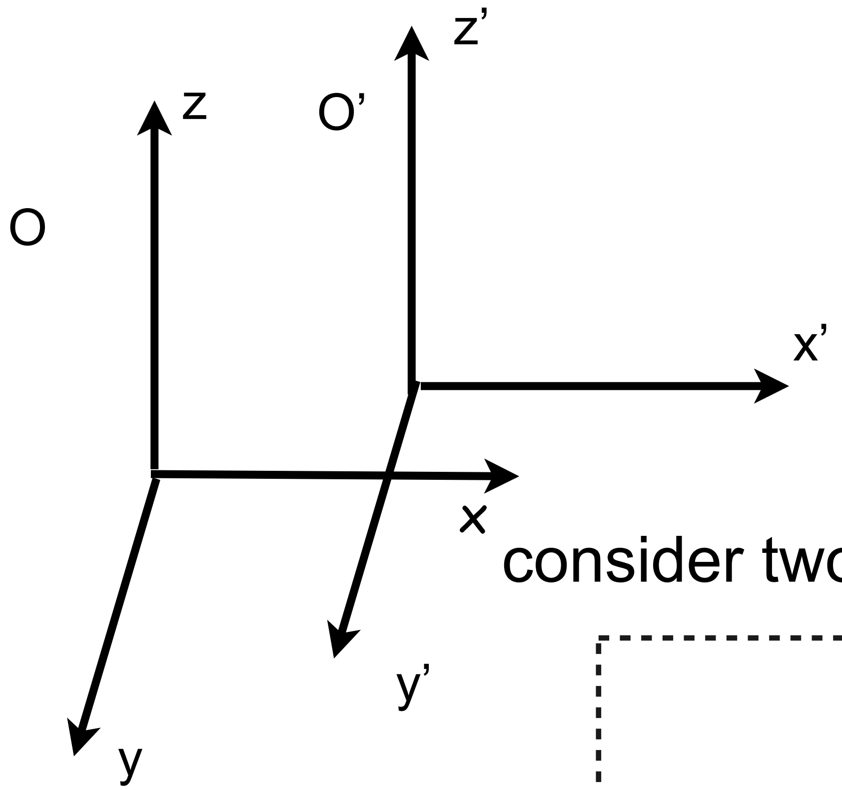


g^2

∃ gauge boson self-interactions

Technical Details
for Advanced Students

Time-ordering ≠ Causality



$$\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} ct' = \gamma_0\left(ct - \beta_0 x\right) \\ x' = \gamma_0\left(-\beta_0 ct + x\right) \\ y' = y \\ z' = z \end{pmatrix}$$

"time dilation + space contraction"

consider two events E_1 and E_2 characterised by their space-time coordinates

E_1	
$t_1 = 0$	$t'_1 = 0$
$x_1 = 0$	$x'_1 = 0$

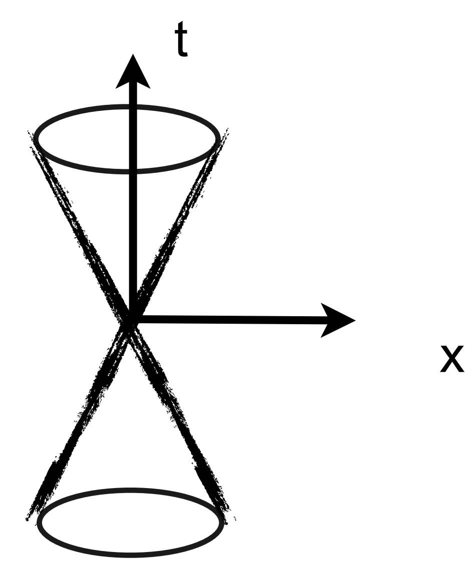
E_2	
$t_2 > 0$	$ct'_2 = \gamma\left(ct_2 - \beta x_2\right)$
$x_2 > 0$	$x'_2 = \gamma\left(-\beta ct_2 + x_2\right)$

t'_2 can be positive or negative
causality ≠ time ordering

Proper space-time distance Δ is independent of the observer:

$$\Delta'^2 = \left(ct'_2\right)^2 - \left(x'_2\right)^2 = \left(ct_2\right)^2 - x_2^2 = \Delta^2$$

Only events inside the past/future light cones are causally connected
The light cones are invariant under Lorentz transformations



Spinor Transformation

Transformation law: $\psi(x) \to \psi'(x') = S(\Lambda)\psi(x)$

We want the Dirac equation to take the same form in the two systems of coordinates x and x'

$$(i\gamma^\mu \partial_\mu - m)\psi = 0 \qquad\qquad (i\gamma^\mu \partial'_\mu - m)\psi' = 0$$

This implies the condition: $S\gamma^\mu \Lambda^\nu{}_\mu S^{-1} = \gamma^\nu$

We consider small Lorentz transformations: $\Lambda_\mu{}^\nu = \delta^\mu_\nu + \omega^\mu{}_\nu \qquad S = 1 - \frac{i}{4}\sigma^{\mu\nu}\omega_{\mu\nu}$

The covariance of the Dirac equation then implies that the matrices $\sigma_{\mu\nu}$ have to satisfy the relation

$$[\gamma^\nu, \sigma^{\rho\sigma}] = 2i(\eta^{\nu\rho}\gamma^\sigma - \eta^{\nu\sigma}\gamma^\rho)$$

It is easy to check that the following matrices fit the bill: $\sigma^{\rho\sigma} = \frac{i}{2}[\gamma^\rho, \gamma^\sigma]$

$$\boxed{\begin{gathered} x^\mu \to x'^\mu = (\delta^\mu{}_\nu + \omega^\mu{}_\nu)x^\nu \quad \text{with} \quad \omega_{\mu\nu} + \omega_{\nu\mu} = 0 \\ \psi(x) \to \psi'(x') = \left(1_4 + \frac{1}{8}\omega_{\mu\nu}[\gamma^\mu, \gamma^\nu]\right)\psi(x) \end{gathered}}$$

Lorentz-invariant Lagrangian

$\mathcal{L} = \psi^\dagger M\,(i\gamma^\mu \partial_\mu - m)\,\psi$ is Lorentz-invariant iff $\gamma^0[\gamma^\nu, \gamma^\mu]\gamma^0 M + M[\gamma^\mu, \gamma^\nu] = 0$

$M = \gamma^0$ is a solution and it defines the Dirac Lagrangian. $\boxed{\bar\psi \equiv \psi^\dagger \gamma^0}$

Symmetries and invariants

SU(N)

the transformations among the components of a complex N-vector that leaves its norm invariant

$$|\phi|^2 = \phi_1^* \phi_1 + \ldots \phi_N^* \phi_N \rightarrow |\phi'|^2 = |\phi|^2$$

SU(N,M)

the transformations among the components of a complex (N+M)-vector that leaves its (N,M) norm invariant

$$|\phi|^2 = \phi_1^* \phi_1 + \ldots \phi_N^* \phi_N + \phi_{N+1}^* \phi_{N+1} - \ldots - \phi_{N+M}^* \phi_{N+M} \rightarrow |\phi'|^2 = |\phi|^2$$

SO(N)

the transformations among the components of a real N-vector that leaves its norm invariant

$$|\phi|^2 = \phi_1^2 + \ldots \phi_N^2 \rightarrow |\phi'|^2 = |\phi|^2$$

SO(N,M)

the transformations among the components of a real (N+M)-vector that leaves its (N,M) norm invariant

$$|\phi|^2 = \phi_1^2 + \ldots \phi_N^2 + \phi_{N+1}^2 - \ldots - \phi_{N+M}^2 \rightarrow |\phi'|^2 = |\phi|^2$$

The Lorentz group is thus SO(1,3)

Lorentz transformation

SO(1,3)

The elements of SO(1,3) satisfy $U^t \eta U = \eta$ where =diag(1,-1,-,1,-1)

The infinitesimal transformations are $U = e^{\theta^a T^a} \approx 1 + \theta^a T^a + \dots$

The generators satisfy the constraints: $T^{a\,t}\eta + \eta T^a = 0$

One particular generator is $T = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

We obtain $e^{\theta T} = \begin{pmatrix} \cosh\theta & \sinh\theta & 0 & 0 \\ \sinh\theta & \cosh\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

We indeed recover the usual Lorentz transformation with the identification

$$\gamma = \cosh\theta \quad \text{and} \quad \beta\gamma = \sinh\theta$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \quad \Leftrightarrow \quad \cosh^2\theta - \sinh^2\theta = 1$$