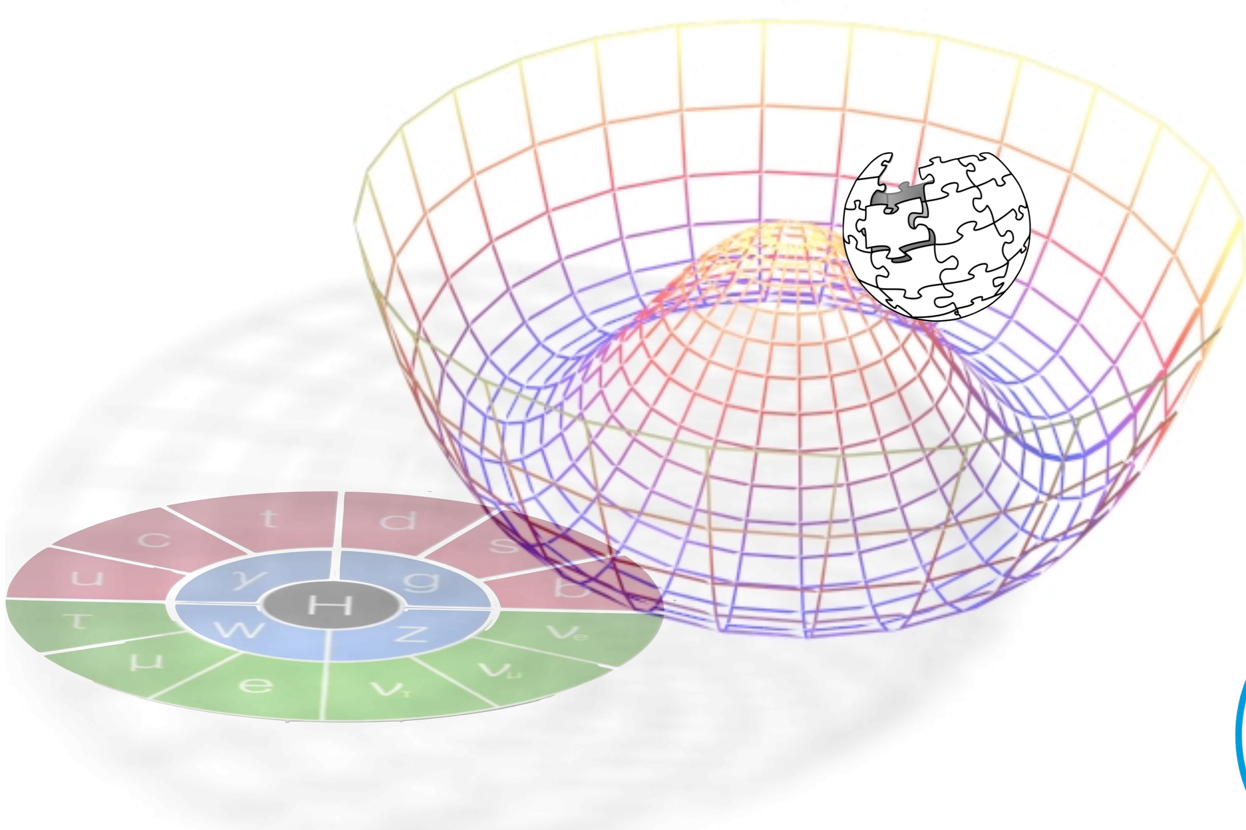
The Standard Model
of particle physics

CERN summer student lectures 2023

Lecture 2/5



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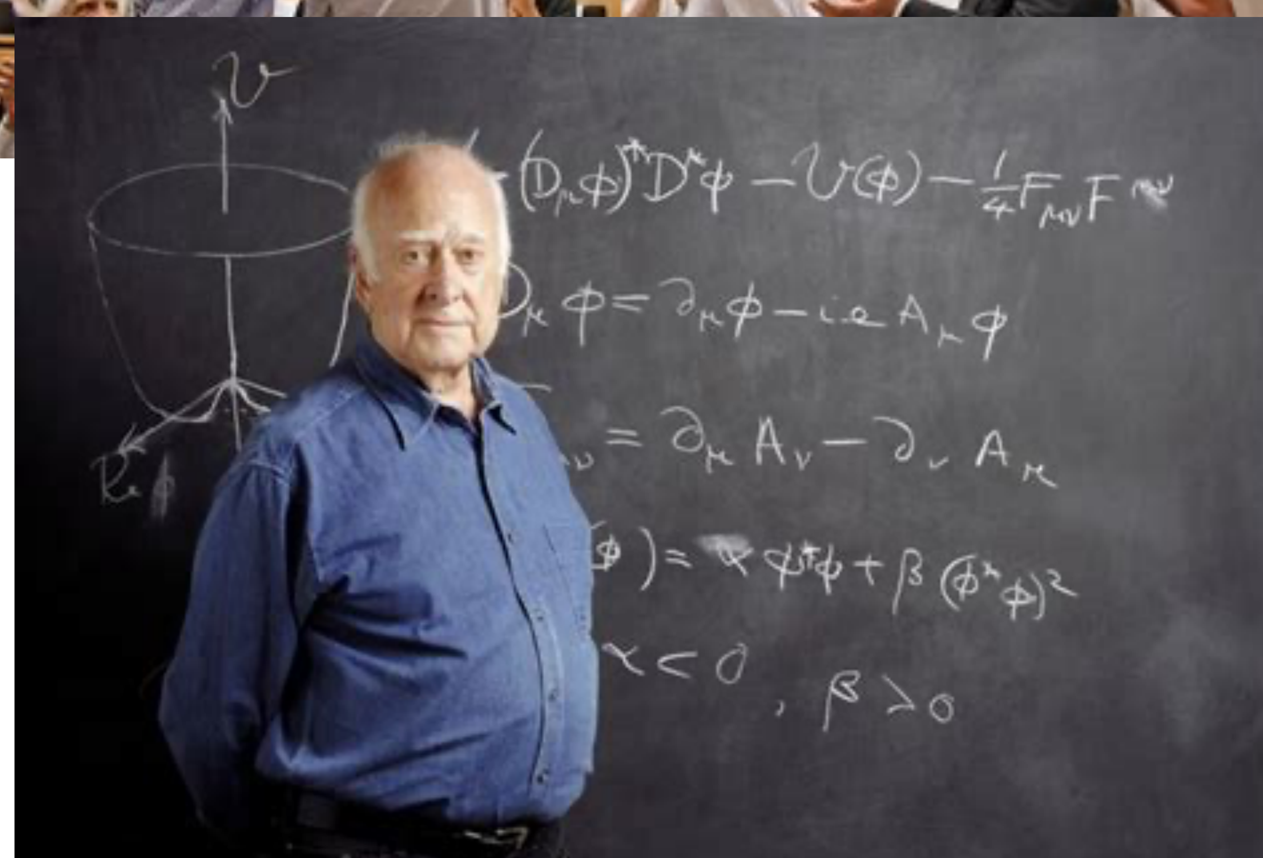
Happy birthday, Higgs boson!

today is the

11th anniversary of its discovery

in the very same room you are seating now



formula you are encountering in the lectures today

Outline

Monday: symmetry
- Lagrangians
- Lorentz symmetry - scalars, fermions, gauge bosons
- Gauge/local symmetry as dynamical principle - Example: U(1) electromagnetism

Tuesday: SM symmetries
- Nuclear decay, Fermi theory and weak interactions: SU(2)
- Strong interactions: SU(3)
- Dimensional analysis: cross-sections and life-time computations made simple

Wednesday: chirality of weak interactions
- Chirality of weak interactions
- Pion decay

Thursday: Higgs mechanism
- Spontaneous symmetry breaking and Higgs mechanism
- Lepton and quark masses, quark mixings
- Neutrino masses

Friday: quantum effects
- Running couplings
- Asymptotic freedom of QCD
- Anomalies cancelation

Recap from Lecture #1

- **Lorentz transformation:**

$$x^\mu \to x'^\mu = \Lambda^\mu{}_\nu x^\nu \quad \text{with} \quad \eta_{\mu\nu} = \eta_{\mu'\nu'}\Lambda^{\mu'}{}_\mu \Lambda^{\nu'}{}_\nu$$

At linear order, $\Lambda^\mu{}_\nu \approx \delta^\mu{}_\nu + \omega^\mu{}_\nu$, it simply writes $\omega_{\mu\nu} + \omega_{\nu\mu} = 0$ where $\omega_{\mu\nu} \equiv \eta_{\mu\mu'}\omega^{\mu'}{}_\nu$

- **Scalar (aka spin-0) field:** $\phi(x) \to \phi'(x') = \phi(x)$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$$

Eq. of motion: $\delta\mathcal{L} = 0$ ➡ $\Box\phi = -V'(\phi)$ Klein-Gordon equation

- **Spin-1/2 field:** $\psi(x) \to \psi'(x') = \left(1_4 + \frac{1}{8}\omega_{\mu\nu}[\gamma^\mu, \gamma^\nu]\right)\psi(x)$

$$\mathcal{L} = \psi^\dagger\gamma^0\left(i\gamma^\mu\partial_\mu - m\right)\psi$$

Eq. of motion: $\delta\mathcal{L} = 0$ ➡ $\left(i\gamma^\mu\partial_\mu - m\right)\psi = 0$ Dirac equation

- **U(1) gauge symmetry** $\psi \to e^{i\theta}\psi$ θ const. = global symm., θ(x) = local symm.

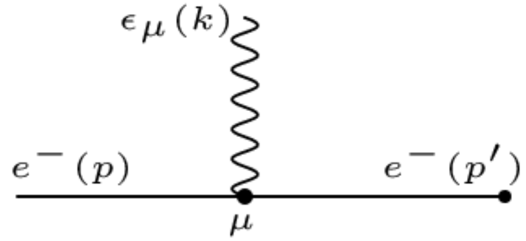
Need to promote space-time derivative to covariant derivative: $D_\mu\psi = \partial_\mu\psi + ieA_\mu\psi$

Gauge field, Aμ, transforms non-trivially under gauge transformation: $A_\mu \to A_\mu - \frac{1}{e}\partial_\mu\theta$

$$\mathcal{L}_{\text{kin}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

⬇ Maxwell equations

Dictate EM interactions photon-electron:



SU(N) non-Abelian Gauge Symmetry

We generalise the QED construction by considering general transformation of a N-vector

$$\phi \to U\phi$$

We build a **covariant derivative** that again has nice homogeneous transformations

$$D_\mu \phi = \partial_\mu \phi + igA_\mu \phi \to UD_\mu \phi \qquad \text{iff} \qquad A_\mu \to UA_\mu U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1}$$

g is the gauge coupling and defines the strength of the interactions

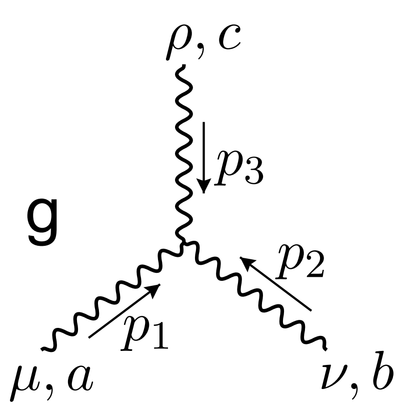
For the field strength to transform homogeneously, one needs to add a non-Abelian piece

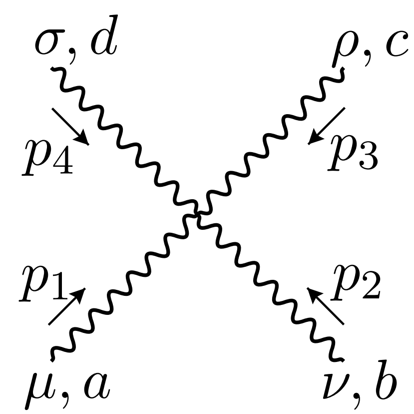
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu] \to UF_{\mu\nu}U^{-1}$$

Contrary to the Abelian case, the gauge fields are now charged and interact with themselves

$$\mathcal{L}_{\text{kin}} = \text{Tr}F_{\mu\nu}F^{\mu\nu} \supset g\partial AAA + g^2 AAAA$$

g





g^2

gauge boson self-interactions for non-abelian symmetries

Natural & Planck Units

- $[G_N]$=mass^{-1} L^3 T^{-2}
- $[\hbar]$=mass L^2 T^{-1}
- [c]=L T^{-1}

- Planck mass: $M_{\rm Pl} = \sqrt{\frac{\hbar c}{G_{\rm N}}} \sim 10^{19}\,{\rm GeV/c^2} \sim 2 \times 10^{-5}\,{\rm g}$
- Planck length: $l_{\rm Pl} = \sqrt{\frac{\hbar G_{\rm N}}{c^3}} \sim 10^{-33}\,{\rm cm}$
- Planck time: $\tau_{\rm Pl} = \sqrt{\frac{\hbar G_{\rm N}}{c^5}} \sim 10^{-44}\,{\rm s}$

In High Energy Physics, it is a current practise to use a system of units for which ħ=1 and c=1

energy~ mass ~ distance^{-1} ~ time^{-1}

Unit conversion: SI ↔ HEP

E	T	L
1eV	10^{-16}s	10^{-7}m
10^{-16}eV	1s	10^{9}m
10^{-7}eV	10^{-9}s	1m

- The string theorists will remember:

$$M_{\rm Pl} \sim 10^{19}\,{\rm GeV} \quad \leftrightarrow \quad \tau_{\rm Pl} \sim 10^{-44}\,{\rm s} \quad \leftrightarrow \quad l_{\rm Pl} \sim 10^{-33}\,{\rm cm}$$

- The nuclear physicists will remember:

$$\hbar c \sim 200\,{\rm MeV} \cdot {\rm fm}$$
$$10^8\,{\rm eV} \quad \leftrightarrow \quad 10^{-15}\,{\rm m} \quad \leftrightarrow \quad 10^{-24}\,{\rm s}$$

- The others will remember:

average mosquito m~10^{-3}g=100M_{Pl}
Compton wavelength 0.01L_{Pl}=10^{-35}cm, Schwarzschild radius 100L_{Pl}=10^{-31}cm
(much smaller than its physical size, so a mosquito is not a Black Hole)

Dimensional Analysis

$$[S]_m = 0 \quad \Rightarrow \quad [\mathcal{L}]_m = 4$$

$$S = \int d^4x\, \mathcal{L}$$

Scalar field $\mathcal{L} = \partial_\mu \phi \partial^\mu \phi + \ldots$ $\Rightarrow$ $[\phi]_m = 1$

Spin-1/2 field $\mathcal{L} = \psi^\dagger \gamma^0 \gamma^\mu \partial_\mu \psi$ $\Rightarrow$ $[\psi]_m = 3/2$

Spin-1 field $\mathcal{L} = F_{\mu\nu} F^{\mu\nu} + \ldots \text{ with } F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \ldots$ $\Rightarrow$ $[A_\mu]_m = 1$

Particle lifetime of a (decaying) particle: $[\tau]_m = -1$ Width: $[\Gamma = 1/\tau]_m = 1$

Cross-section (“area” of the target): $[\sigma]_m = -2$

Lifetime "Computations"

muon and neutron are unstable particles

$$\mu \to e\nu_\mu\bar{\nu}_e$$

$$n \to p\, e\, \bar{\nu}_e$$

We'll see that the interactions responsible for the decay of muon and neutron are of the form

$$\mathcal{L} = G_F\, \psi^4 \quad \Rightarrow \quad \Gamma \propto G_F^2 m^5$$

$\mathcal{L}$: $[\mathrm{mass}]^4$; G_F: $[\mathrm{mass}]^{-2}$; ψ^4: $[\mathrm{mass}]^{3/2\times 4}$; m: $[\mathrm{mass}]$

G_F = Fermi constant: $G_F \sim \dfrac{10^{-5}}{m_{\mathrm{proton}}} \sim 10^{-5}\,\mathrm{GeV}^{-2}$

For the **muon**, the relevant mass scale is the muon mass m_μ=105MeV:

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} \sim 10^{-19}\,\mathrm{GeV} \quad \text{i.e.} \quad \tau_\mu \sim 10^{-6}\,\mathrm{s}$$

For the **neutron**, the relevant mass scale is (m_n-m_p)≈1.29MeV:

$$\Gamma_n = \mathcal{O}(1)\frac{G_F^2 \Delta m^5}{\pi^3} \sim 10^{-28}\,\mathrm{GeV} \quad \text{i.e.} \quad \tau_{\mathrm{n}} \sim 10^3\,\mathrm{s}$$

$1 = \hbar c \sim 200\,\mathrm{MeV}\cdot\mathrm{fm}$

E	T	L
1eV	10^{-16}s	10^{-7}m

What if particles were spin-0?

$$\mathcal{L} = G_F \phi^4$$

$[\text{mass}]^4$ $[\text{mass}]^0$ $[\text{mass}]^{1\times 4}$

$$\Gamma \propto G_F^2 m$$

$[\text{mass}]$

$$\Gamma_\mu = \left(10^{-6\prime\prime}\right)^{-1} = \frac{G_F^2 m_\mu}{192\pi^2}$$

$$\Gamma_n = \left(15^\prime\right)^{-1} = \frac{G_F^2 (m_n - m_p)}{192\pi^2}$$

$$10^{11} = \frac{\Gamma_\mu}{\Gamma_n} \stackrel{?}{=} \frac{m_\mu}{m_n - m_p} = 10^2$$

TH prediction

It could still have been true but we would need to give up universality of the Fermi interactions.
Remember theorists like to connect phenomena are are seemingly different.
Even more true when they follow from simple assumptions.

Beta decay

$$ {}^{40}_{19}\mathrm{K} \to {}^{40}_{20}\mathrm{Ca}^+ + e^- \qquad {}^{64}_{29}\mathrm{Cu} \to {}^{64}_{30}\mathrm{Zn}^+ + e^- \qquad {}^{3}_{1}\mathrm{H} \to {}^{3}_{2}\mathrm{He}^+ + e^- $$

- Two body decays: A→B+C

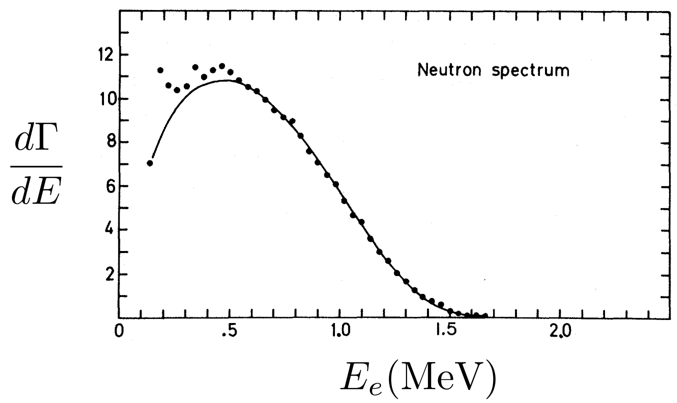
$$ E_B = \frac{m_A^2 + m_B^2 - m_C^2}{2m_A} c^2 \qquad p = \frac{\sqrt{\lambda(m_A, m_B, m_C)}}{2m_A} c $$

$$ \lambda(m_A, m_B, m_C) = (m_A + m_B + m_C)(m_A + m_B - m_C)(m_A - m_B + m_C)(m_A - m_B - m_C) $$

fixed energy of daughter particles

(pure SR kinematics, independent of the dynamics)

TH prediction



EXP measurements

⇛ non-conservation of energy?

Pauli '30: ∃ **neutrino**, very light since end-point of spectrum is close to 2-body decay limit

ν first observed in '53 by Cowan and Reines

- N-body decays: A→B₁+B₂+...+B$_N$

$$ E_{B_1}^{\min} = m_{B_1} c^2 \qquad E_{B_1}^{\max} = \frac{m_A^2 + m_{B_1}^2 - (m_{B_2} + \ldots + m_{B_N})^2}{2m_A} c^2 $$

— How are neutrinos produced? —

$\pi \to \mu\bar{\nu}$ (more about pion decay later later)

$\mu \to e\bar{\nu}_e\nu_\mu$ need 2 neutrino flavours and flavour conservation since $\mu \not\to e\gamma$

Lederman, Schwartz, Steinberger '62: $p\,\bar{\nu}_\mu \to n\,\mu^+$ but $p\,\bar{\nu}_\mu \not\to n\,e^+$

Fermi theory '33

(paper rejected by Nature: declared too speculative !)

$$ \mathcal{L} = G_{\mathcal{F}}(\bar{n}p)(\bar{\nu}_e e) $$

exp: $G_F = 1.166\times10^{-5}\ \mathrm{GeV}^{-2}$

We'll see later that the structure is a bit more complicated

Universality of Weak Interactions

How can we be sure that muon and neutron decays proceed via the same interactions?

$\tau_\mu \approx 10^{-6}$s vs. $\tau_{neutron} \approx 900$s

By analogy with electromagnetism, one can see the Fermi force as a current-current interaction

$$\mathcal{L} = G_F\, J^*_\mu J^\mu \qquad \text{with} \qquad J^\mu = (\bar{n}\gamma^\mu p) + (\bar{e}\gamma^\mu \nu_e) + (\bar{\mu}\gamma^\mu \nu_\mu) + \dots$$

The cross-terms generate both neutron decay and muon decay.
The life-times of the neutron and muon tell us that the relative factor between the electron and the muon in the current is of order one, i.e., the weak force has the same strength for electron and muon.

What about $\pi^\pm$ decay $\tau_\pi \approx 10^{-8}$s?

Why $\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} \underset{\text{Exp}}{\sim} 10^{-4}$? And not $\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} \sim \frac{(m_\pi - m_e)^5}{(m_\pi - m_\mu)^5} \underset{\text{Th}}{\sim} 500$?

Does it mean that our way to compute decay rate is wrong?
Is pion decay mediated by another interaction?
Is the weak interaction non universal, i.e. is the value of G_F processus dependent?

Pathology at High Energy

What about weak scattering process, e.g . $e\nu_e \to e\nu_e$?

$$\mathcal{L} = G_F \, J^*_\mu J^\mu \qquad \text{with} \qquad J^\mu = (\bar{n}\gamma^\mu p) + (\bar{e}\gamma^\mu \nu_e) + (\bar{\mu}\gamma^\mu \nu_\mu) + \ldots$$

The same Fermi Lagrangian will thus also contain a term

$$G_F \, (\bar{e}\gamma^\mu \nu_e)(\bar{\nu}_e \gamma^\mu e)$$

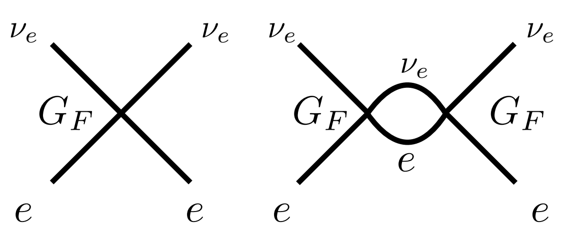
that will generate e-ν_e scattering whose cross-section can be guessed by dimensional arguments

$$\sigma \propto G_F^2 E^2$$

$[\text{mass}]^{-2}$ $[\text{mass}]^{-2\times 2}$ $[\text{mass}]^2$

non conservation of probability
(non-unitary theory)
inconsistent at high energy

It means that at high-energy the quantum corrections to the classical contribution can be sizeable:



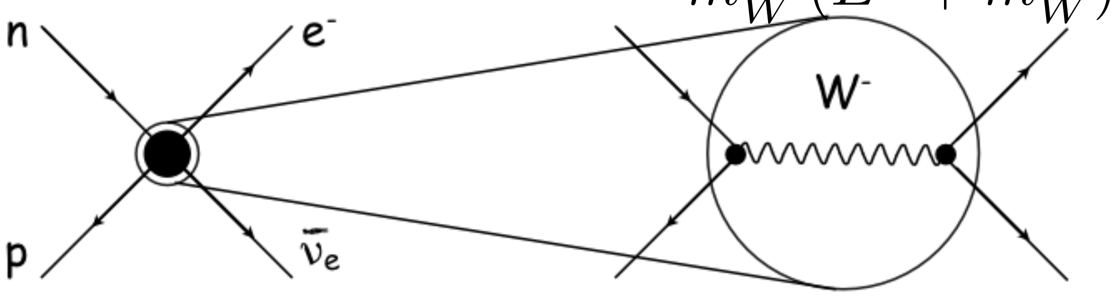
$$\sigma \propto G_F^2 E^2 + \frac{1}{16\pi^2} G_F^4 E^6 + \ldots$$

The theory becomes non-perturbative at an energy $E_{\max} = \frac{2\sqrt{\pi}}{\sqrt{G_F}} \sim 100\,\text{GeV–}1\,\text{TeV}$

unless new degrees of freedom appear before to change the behaviour of the scattering

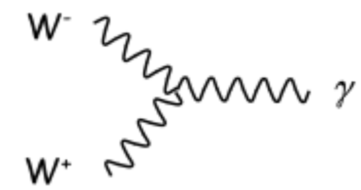
Electroweak Interactions

$\sigma \propto G_F^2 E^2$ $\qquad$ $\sigma \propto g^4 \frac{E^2}{m_W^2 (E^2 + m_W^2)}$



$G_F \propto \frac{g^2}{m_W^2}$

charged W ⇛ must couple to photon:



⇛ non-abelian gauge symmetry $[Q,T^\pm]=\pm T^\pm$

1. No additional "force" (Georgi, Glashow '72) mathematical consistency ⇛ extra matter

$SU(2)$

$$[T^a, T^b] = i\epsilon^{abc}T^c$$

$$[T^+, T^-] = Q \qquad [Q, T^\pm] = + \pm T^\pm$$

$$T^\pm = \frac{1}{\sqrt{2}}(T^1 \pm iT^2)$$

$\mathrm{Tr}_{\mathrm{irrep}} T^3 = 0$ ⇛ extra matter $\begin{pmatrix} X_L \\ \nu_L \\ e_L \end{pmatrix} \begin{pmatrix} X_R \\ \nu_R \\ e_R \end{pmatrix}$

$SU(1,1)$

$$[T^+, T^-] = -Q$$

$$[Q, T^\pm] = + \pm T^\pm$$

non-compact

unitary rep. has dim ∞

E_2

2D Euclidean group

$$[T^+, T^-] = 0$$

$$[Q, T^\pm] = + \pm T^\pm$$

only one unitary rep.

of finite dim. = trivial rep.

2. No additional "matter" (Glashow '61, Weinberg '67, Salam '68): SU(2)xU(1)

⇛ **extra force**

$Q = T^3?$

as Georgi-Glashow

⇛ extra matter

$Q = Y?$

$Q(e_L) = Q(\nu_L)$

$Q = T^3 + Y!$

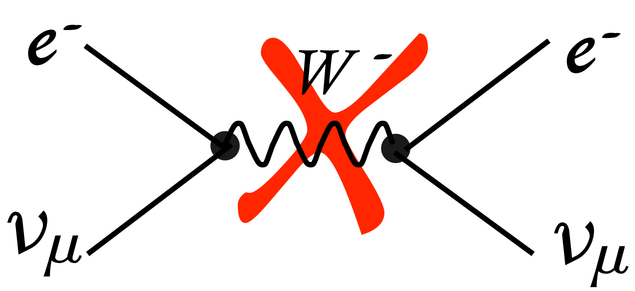
Gell-Mann '56, Nishijima-Nakano '53

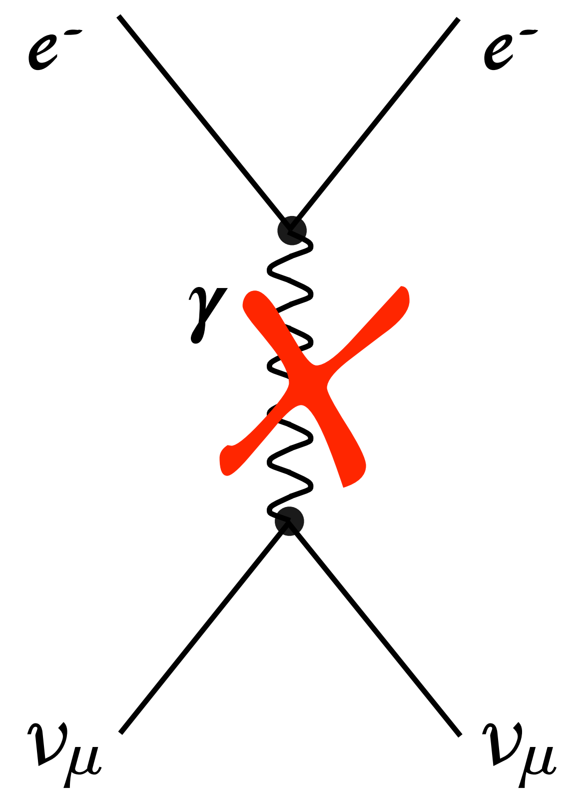
Electroweak Interactions

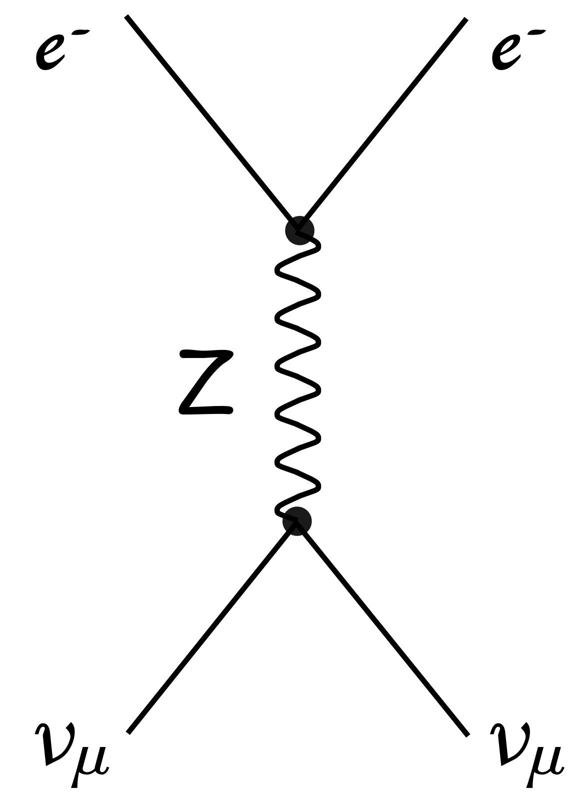
Gargamelle experiment '73 first established the SU(2)xU(1) structure

How?
rely on a particle that doesn't interact with photon to prove the existence a new neutral current process!

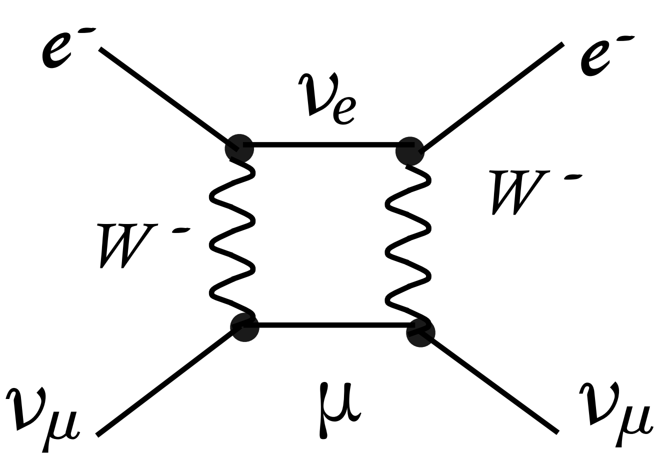
$\nu_\mu e^- \to \nu_\mu e^-$







loop-suppressed contribution from W:



From Gauge Theory to Fermi Theory

We can derive the Fermi current-current contact interactions by "integrating out" the gauge bosons, i.e., by replacing in the Lagrangian the W's by their equation of motion. Here is a simple derivation:
(a better one should take taking into account the gauge kinetic term and the proper form of the fermionic current that we'll figure out tomorrow, for the moment, take it as a heuristic derivation)

$$\mathcal{L} = -m_W^2 W_\mu^+ W_\nu^- \eta^{\mu\nu} + g W_\mu^+ J_\nu^- \eta^{\mu\nu} + g W_\nu^- J_\nu^+ \eta^{\mu\nu}$$

$$J^{+\mu} = \bar{n}\gamma^\mu p + \bar{e}\gamma^\mu \nu_e + \bar{\mu}\gamma^\mu \nu_\mu + \dots \qquad \text{and} \qquad J^{-\mu} = \left(J^{+\mu}\right)^*$$

The equation of motion for the gauge fields: $\dfrac{\partial \mathcal{L}}{\partial W_\mu^+} = 0 \quad \Rightarrow \quad W_\mu^- = \dfrac{g}{m_W^2} J_\mu^-$

Plugging back in the original Lagrangian, we obtain an *effective Lagrangian* (valid below the mass of the gauge bosons):

$$\boxed{\mathcal{L} = \frac{g^2}{m_W^2} J_\mu^+ J_\nu^- \eta^{\mu\nu}}$$

which is the Fermi current-current interaction. The Fermi constant is given by
(the correct expression involves a different normalisation factor)

$$\boxed{G_F = \frac{g^2}{m_W^2}}$$

But what is the origin of the W mass?
By the way, it is not invariant under SU(2) gauge transformation…
That's what the Higgs mechanism will take care of!

SU(3) QCD

Deep inelastic experiments in the 60's revealed the internal structure of the neutrons and protons
Gell-Mann and others proposed that they are made of "**quarks**"

Up quark: spin-1/2, Q=2/3
Down quark: spin-1/2, Q=-1/3

SU(2) weak symmetry that changes neutrino into electron also changes up-quark into down-quark

But **quarks** carry yet another quantum number: "**colour**"
There 3 possible colours and Nature is colour-blind, i.e, Lagrangian should remain the same when the colours of the quarks are changed, i.e., when we perform a rotation in the colour-space of quarks

$$Q^a \to U^a{}_b Q^b$$

U: 3x3 matrix satisfying $U^\dagger U = 1_3$ SU(3)
such that the quark kinetic term is invariant

hadrons (spin-1/2, #hadronic=1):	$p = uud \qquad n = udd$
mesons (spin-0, #hadronic=0):	$\pi^0 = \frac{u\bar{u} + d\bar{d}}{\sqrt{2}} \qquad \pi^+ = u\bar{d} \quad \pi^- = d\bar{u}$

(Each quark carries a baryon number =1/3)

There are other (heavier) quarks and hence other baryons and mesons

All the interactions of the SM preserve baryon and lepton numbers

$\mu \to e\nu_\mu\bar{\nu}_e \qquad n \to p\, e\, \bar{\nu}_e \qquad \pi^- \to \mu^-\bar{\nu}_\mu \qquad \pi^0 \to \gamma\gamma \qquad p \not\to \pi^0 \bar{e}$

The Standard Model: Interactions

- $U(1)_Y$ ***electromagnetic interactions***
  - Photon γ
  - light, atoms, molecules
- $SU(2)_L$ ***weak interactions***
  - bosons $W^\pm, Z^0$
  - β decay
  - $n \xrightarrow{W^\pm} p + e^- + \bar{\nu}_e$
  - $e^+ + e^- \xrightarrow{Z^0} D^+_{(c\bar{s})} + D^-_{(\bar{c}s)}$
- $SU(3)_c$ ***strong interactions***
  - gluons g^a
  - atomic nuclei
  - α decay
  - ${}^{238}_{92}\mathrm{U} \longrightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He}$

strength: 10^{-5}, 10^{-2}, 1

Technical Details
for Advanced Students

Compton vs Schwarzschild Scales

Compton radius: for an object of mass m, one can define a length scale that will measure its quantum size

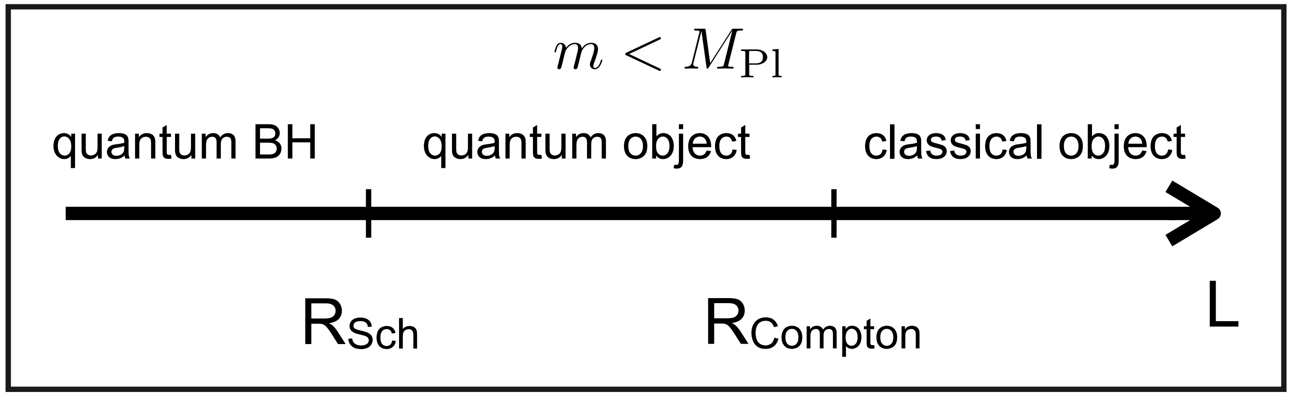
$$R_{\text{Compton}} = \frac{\hbar}{mc}$$

Schwarzschild radius: for an object of mass m, one can define a length scale that will measure its gravitational strength

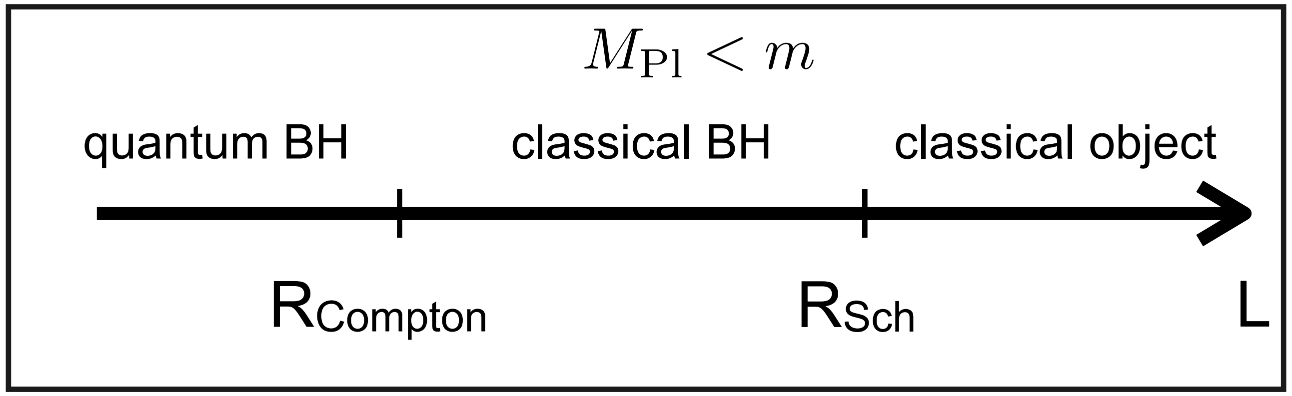
$$R_{\text{Sch}} = \frac{G_{\text{N}} m}{c^2} = \frac{m}{M_{\text{Pl}}} l_{\text{Pl}}$$

$$R_{\text{Compton}} < R_{\text{Sch}} \quad \text{iff} \quad M_{\text{Pl}} < m$$

— elementary particles —



— macroscopic objects —

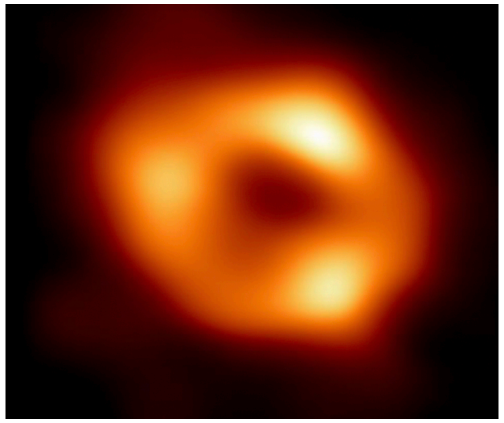


Black Holes

Neutron stars: $m \sim 10^{30}$kg, $R \sim 10^{4}$m (density of human population concentrated in a sugar cube): $R_{Sch} \sim 10^{3}$m: ~~BH~~

Stellar BHs: $m \sim 10^{31}$kg, $R \sim 10^{4}$m: $R_{Sch} \sim 10^{4}$m: BH

Supermassive BHs: $m \sim 10^{37}$kg, $R \sim 10^{10}$m: $R_{Sch} \sim 10^{10}$m: BH



Event Horizon Telescope
Sagittarius A*
$m = 4.3 \times 10^{6}\ M_{sun}$
$R = 23.5 \times 10^{6}$ km

LHC Black Holes: $m \sim 1$TeV, $R \sim 10^{-19}$m: $R_{Compton} \sim 10^{-19}$m, $R_{Sch} \sim 10^{-51}$m (ordinary gravity) but

$R_{Sch} \sim 10^{-19}$m if M_{Pl} is lowered to 1TeV as in models with large extra dimensions