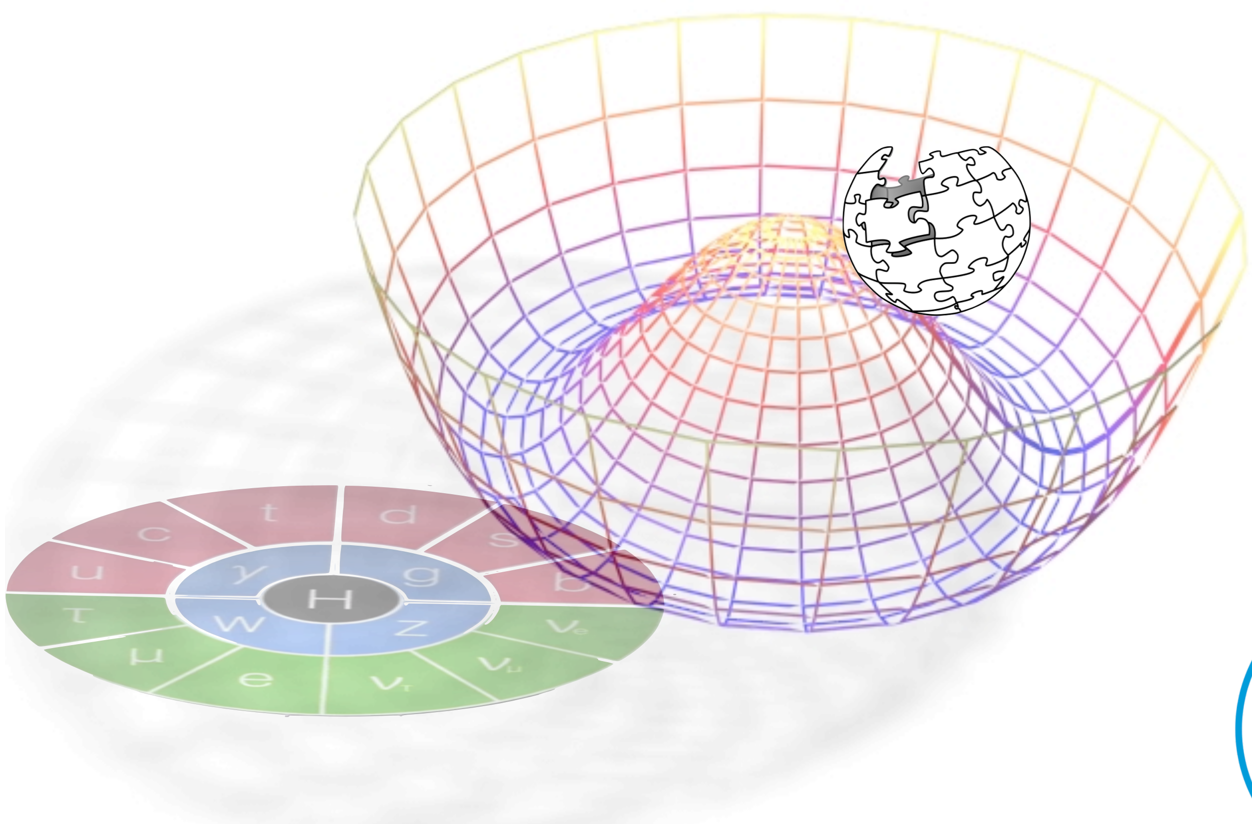
The Standard Model
of particle physics

CERN summer student lectures 2023

Lecture 3/5



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Outline

Monday: symmetry
- Lagrangians
- Lorentz symmetry - scalars, fermions, gauge bosons
- Gauge/local symmetry as dynamical principle - Example: U(1) electromagnetism

Tuesday: SM symmetries
- Nuclear decay, Fermi theory and weak interactions: SU(2)
- Dimensional analysis: cross-sections and life-time computations made simple
- Strong interactions: SU(3)

Wednesday: chirality of weak interactions
- Chirality of weak interactions
- Pion decay

Thursday: Higgs mechanism
- Spontaneous symmetry breaking and Higgs mechanism
- Lepton and quark masses, quark mixings
- Neutrino masses

Friday: quantum effects
- Running couplings
- Asymptotic freedom of QCD
- Anomalies cancelation

Universality of Weak Interactions

$$\mu \to e \nu_\mu \bar{\nu}_e \qquad\qquad n \to p\, e\, \bar{\nu}_e$$

$\tau_\mu \approx 10^{-6}$s $\qquad$ $\tau_n \approx 900$s

$$\mathcal{L} = G_F\, \psi^4$$

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} \sim 1/10^{-6''} \qquad\qquad \Gamma_n = \frac{G_F^2 \Delta m^5}{192\pi^3} \sim 1/15'$$

[factor 192 not exactly correct
because n and p are not elementary particles
form factors are involves]

$$\mathcal{L} \stackrel{?}{=} G_F\ (\bar{n} p \bar{e} \nu_e + \bar{\mu} \nu_\mu \bar{e} \nu_e)$$

By analogy with electromagnetism, one can see the Fermi force as a current-current interaction (vector-vector interaction instead of scalar-scalar interaction)

$$\mathcal{L} = G_F\, J_\mu^* J^\mu \qquad \text{with} \qquad J^\mu \stackrel{?}{=} (\bar{n}\gamma^\mu p) + (\bar{e}\gamma^\mu \nu_e) + (\bar{\mu}\gamma^\mu \nu_\mu) + \ldots$$

it can be show (thanks to the transformation law of spin-1/2 field given before) that this Lagrangian is invariant under Lorentz transformation

The cross-terms generate both neutron decay and muon decay.
The life-times of the neutron and muon tell us that the relative factor between the e and the μ in the current is of order one: the weak force has the **same strength for e and μ**.

Pion decay(s)

What about $\pi^{\pm}$ decay $\tau_{\pi} \approx 10^{-8}$s?

$$\pi^- \to \mu \bar{\nu}_\mu \qquad\qquad \pi^- \to e^- \bar{\nu}_e$$

experimentally the pions decay dominantly into muons and not electrons.

Why $\left.\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)}\right._{\text{EXP}} \sim 10^{-4}$? And not $\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} \sim \left.\frac{(m_\pi - m_e)^5}{(m_\pi - m_\mu)^5}\right._{\text{TH}} \sim 500$?

Does it mean that our way to compute decay rate is wrong?
Is pion decay mediated by another interaction?
The pion is a composite particle: does is mean that the form factors drastically change our estimates?
Is the weak interaction non universal, i.e. is the value of G_F processus dependent?

Pathology at High Energy

What about weak scattering process, e.g . $e\nu_e \to e\nu_e$?

$$\mathcal{L} = G_F\, J^*_\mu J^\mu \qquad \text{with} \qquad J^\mu = (\bar{n}\gamma^\mu p) + (\bar{e}\gamma^\mu \nu_e) + (\bar{\mu}\gamma^\mu \nu_\mu) + \ldots$$

The same Fermi Lagrangian will thus also contain a term

$$G_F\,(\bar{e}\gamma^\mu \nu_e)(\bar{\nu}_e\gamma^\mu e)$$

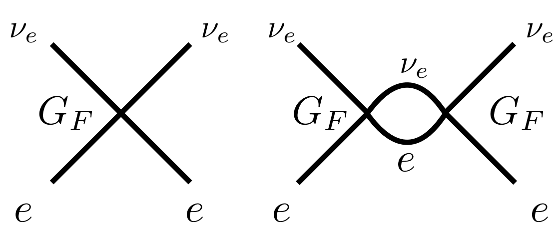
that will generate e-ν_e scattering whose cross-section can be guessed by dimensional arguments

$$\sigma \propto G_F^2 E^2$$

$[\text{mass}]^{-2}$ $\quad [\text{mass}]^{-2\times 2}$ $\quad [\text{mass}]^2$

non conservation of probability
(non-unitary theory)
inconsistent at high energy

It means that, at high-energy, the quantum corrections to the classical contribution can be sizeable:



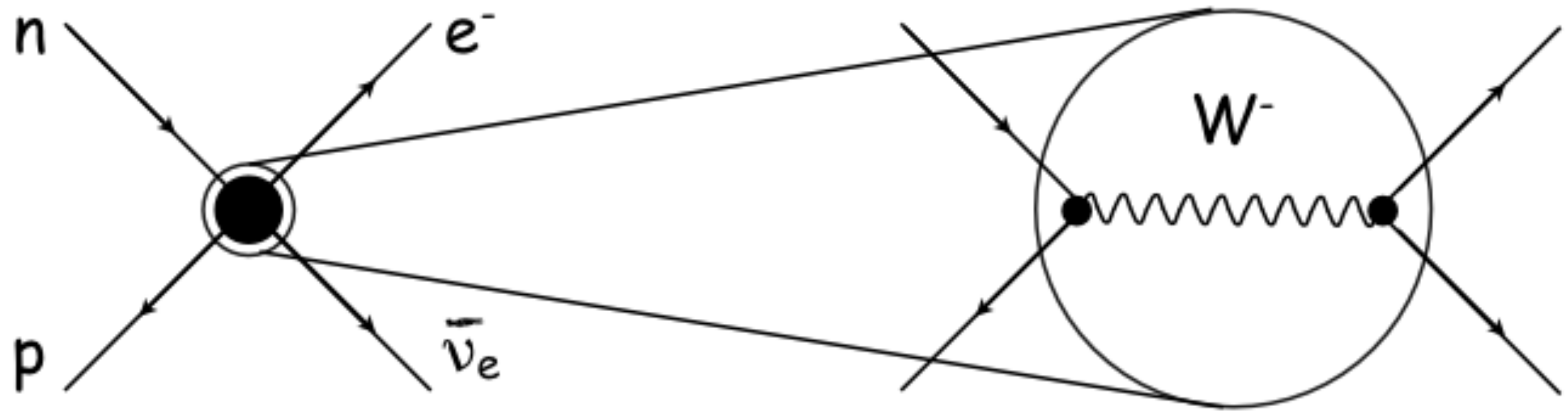
$$\sigma \propto G_F^2 E^2 + \frac{1}{16\pi^2} G_F^4 E^6 + \ldots$$

The theory becomes non-perturbative at an energy $E_{\max} = \dfrac{2\sqrt{\pi}}{\sqrt{G_F}} \sim 100\,\mathrm{GeV}\text{–}1\,\mathrm{TeV}$

unless new degrees of freedom appear before to change the behaviour of the scattering

Electroweak Interactions

Low energy | High energy



$$\sigma \propto G_F^2 E^2$$

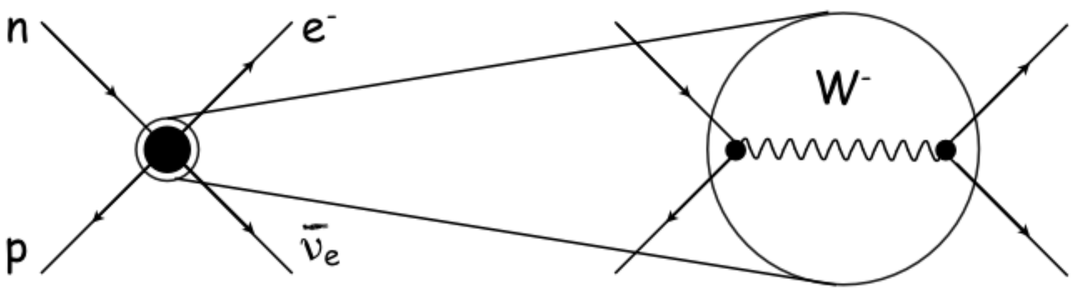
$$\sigma \propto g^4 \frac{E^2}{m_W^2\,(E^2 + m_W^2)}$$

— matching —

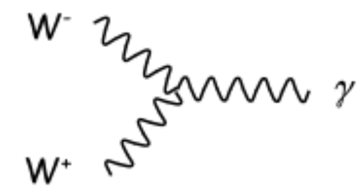
$$G_F \propto \frac{g^2}{m_W^2}$$

The Fermi interaction is not a fundamental interaction of Nature.
It is a low energy effective interaction.

Electroweak Interactions



charged W ⇛ must couple to photon:



⇛ non-abelian gauge symmetry [Q,T±]=±T±

1. No additional "force" (Georgi, Glashow '72) mathematical consistency ⇛ extra matter

$SU(2)$

$$[T^a, T^b] = i\epsilon^{abc}T^c$$

$$[T^+, T^-] = Q \qquad [Q, T^\pm] = +\pm T^\pm$$

$$T^\pm = \frac{1}{\sqrt{2}}(T^1 \pm iT^2)$$

$\mathrm{Tr}_{\mathrm{irrep}} T^3 = 0$ ⇛ extra matter $\begin{pmatrix} X_L \\ \nu_L \\ e_L \end{pmatrix} \begin{pmatrix} X_R \\ \nu_R \\ e_R \end{pmatrix}$

$SU(1,1)$

$$[T^+, T^-] = -Q$$

$$[Q, T^\pm] = +\pm T^\pm$$

non-compact

unitary rep. has dim ∞

E_2

2D Euclidean group

$$[T^+, T^-] = 0$$

$$[Q, T^\pm] = +\pm T^\pm$$

only one unitary rep.

of finite dim. = trivial rep.

2. No additional "matter" (Glashow '61, Weinberg '67, Salam '68): SU(2)xU(1)

⇛ **extra force**

$Q = T^3?$

as Georgi-Glashow

⇛ extra matter

$Q = Y?$

$Q(e_L) = Q(\nu_L)$

$Q = T^3 + Y!$

Gell-Mann '56, Nishijima-Nakano '53

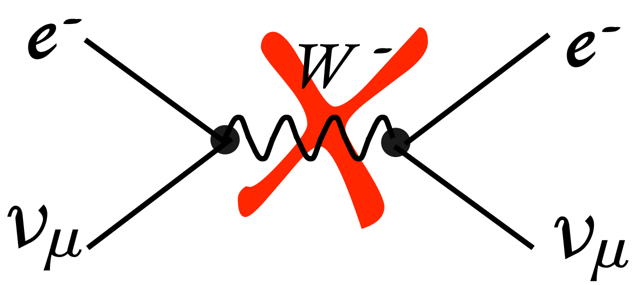
Electroweak Interactions

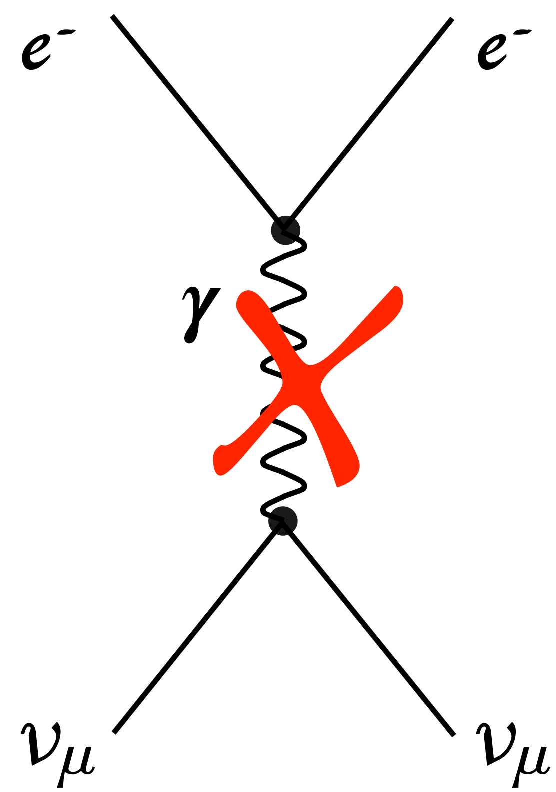
Gargamelle experiment ’73 first established the SU(2)xU(1) structure

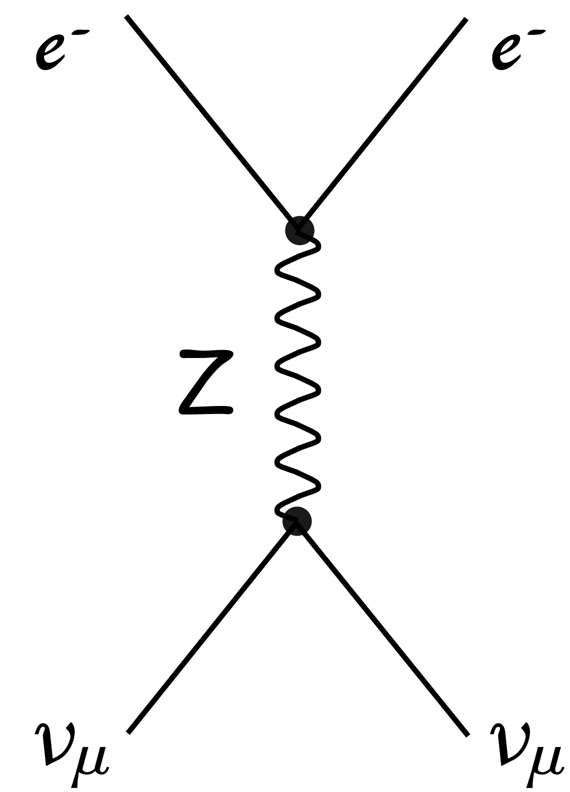
How?

rely on a particle that doesn’t interact with photon to prove the existence a new neutral current process!

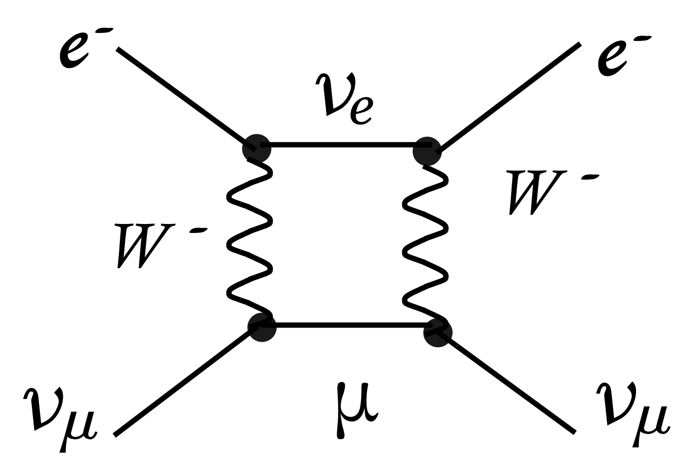
$$\nu_\mu \, e^- \rightarrow \nu_\mu \, e^-$$







loop-suppressed contribution from W:



From Gauge Theory to Fermi Theory

We can derive the Fermi current-current contact interactions by "integrating out" the gauge bosons, i.e., by replacing in the Lagrangian the W's by their equation of motion. Here is a simple derivation:
(a better one should take taking into account the gauge kinetic term and the proper form of the fermionic current that we'll figure out tomorrow, for the moment, take it as a heuristic derivation)

$$\mathcal{L} = -m_W^2 W_\mu^+ W_\nu^- \eta^{\mu\nu} + g W_\mu^+ J_\nu^- \eta^{\mu\nu} + g W_\nu^- J_\nu^+ \eta^{\mu\nu}$$

$$J^{+\mu} = \bar{n}\gamma^\mu p + \bar{e}\gamma^\mu \nu_e + \bar{\mu}\gamma^\mu \nu_\mu + \dots \qquad \text{and} \qquad J^{-\mu} = \left(J^{+\mu}\right)^*$$

The equation of motion for the gauge fields: $\dfrac{\partial \mathcal{L}}{\partial W_\mu^+} = 0 \quad \Rightarrow \quad W_\mu^- = \dfrac{g}{m_W^2} J_\mu^-$

Plugging back in the original Lagrangian, we obtain an *effective Lagrangian* (valid below the mass of the gauge bosons):

$$\boxed{\mathcal{L} = \frac{g^2}{m_W^2} J_\mu^+ J_\nu^- \eta^{\mu\nu}}$$

which is the Fermi current-current interaction. The Fermi constant is given by
(the correct expression involves a different normalisation factor)

$$\boxed{G_F = \frac{g^2}{m_W^2}}$$

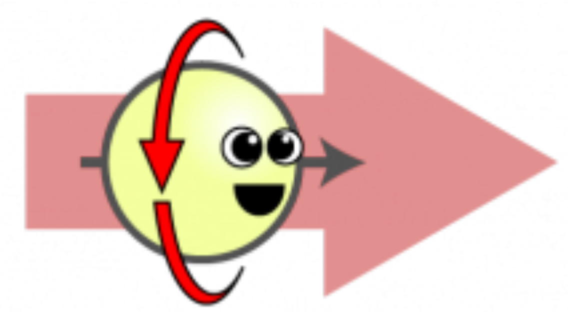
But what is the origin of the W mass?
By the way, it is not invariant under SU(2) gauge transformation…
That's what the Higgs mechanism will take care of!

Chirality & Masslessness

Quantum Mechanics 1.0.1

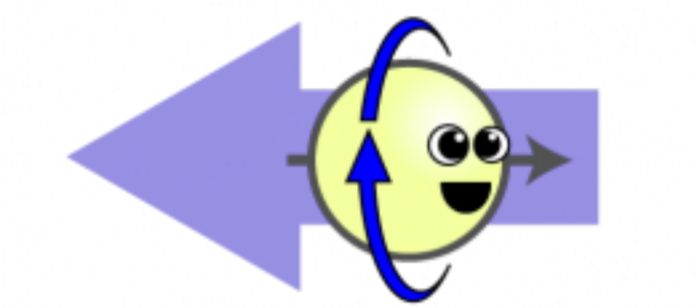
Particle of spin s has 2s+1 polarisation states

Particle spinning anticlockwise wrt its direction of motion



electron
has 2 polarisation

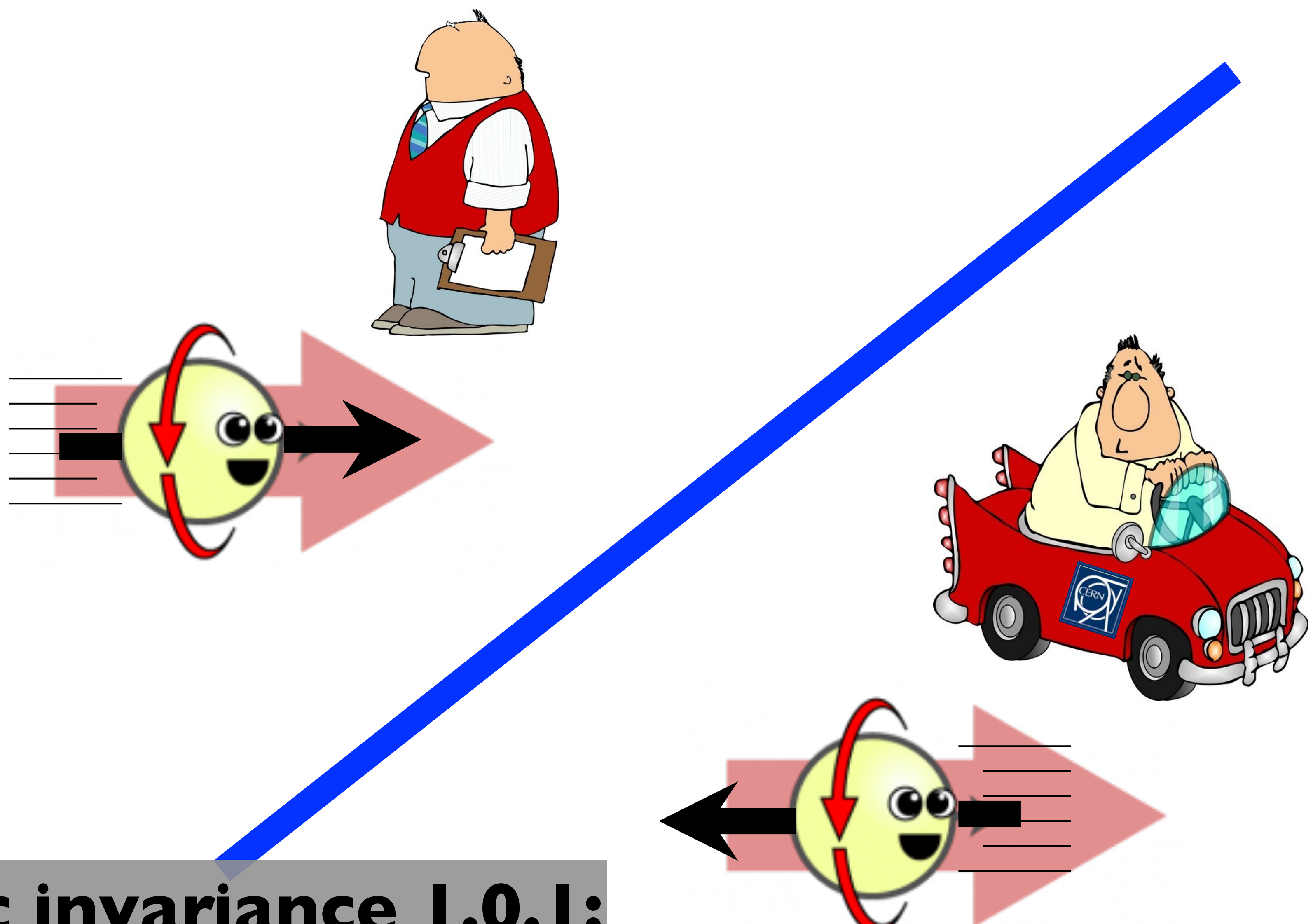
Particle spinning clockwise wrt its direction of motion



Picture courtesy to G. Giudice

Chirality & Masslessness

Picture courtesy to G. Giudice



Relativistic invariance 1.0.1:

there must be no distinction for massive particles between particles spinning clockwise or anti-clockwise

[chirality operator doesn't commute with the Hamiltonian]

Theorem

If your theory sees a difference between e_L and e_R, either your theory is wrong or m_e=0

Chirality of SM & Mass problem

Weak interaction
(force responsible for
neutron decay)
is chiral!

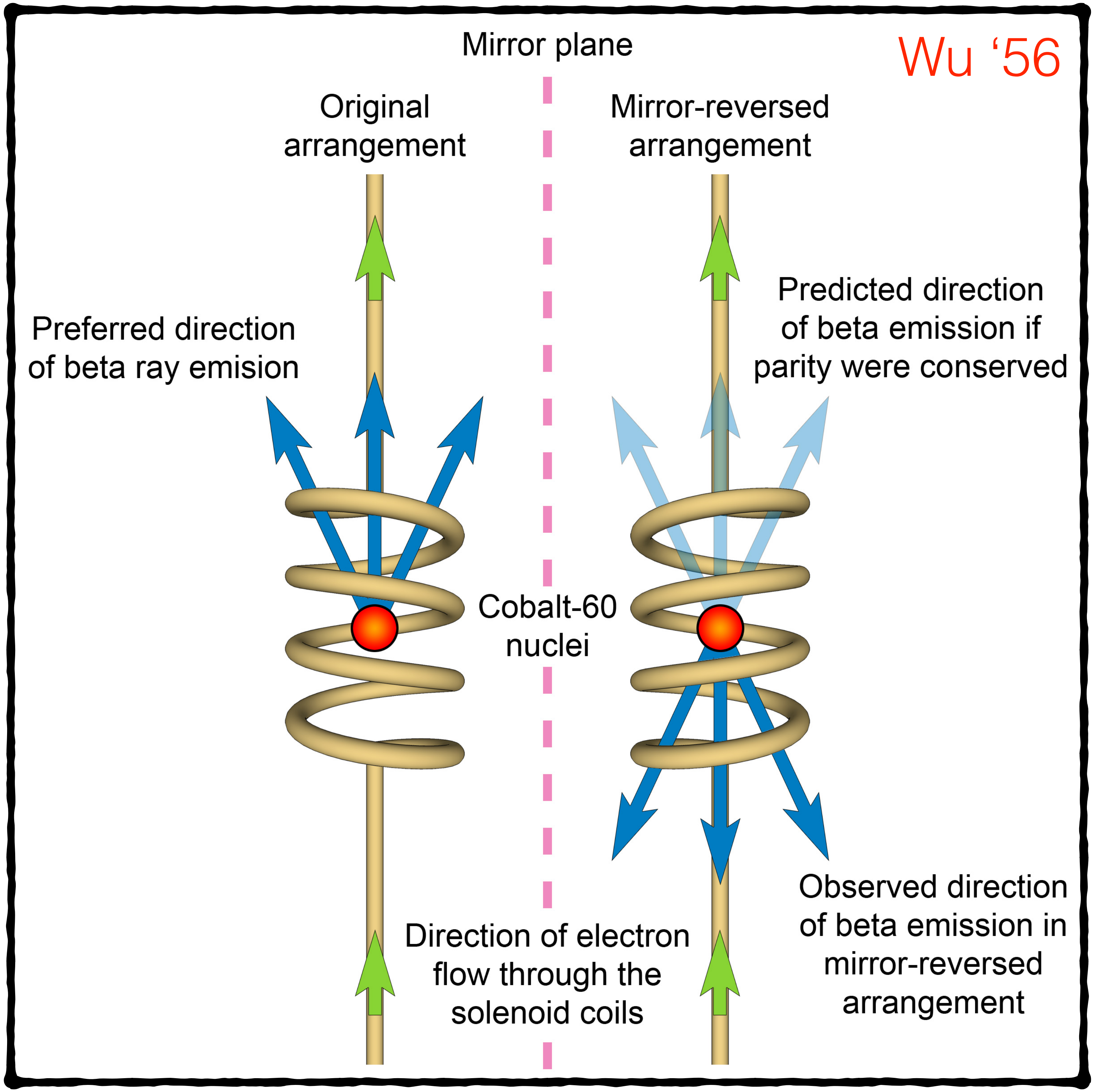
[e_L and e_R are fundamentally two different particles
Only an accident of the history of physics that they are both called electron]

m_e=0

but since we know it is not true, we

need a new phenomena to generate mass: Higgs mechanism

TH: Yang&Lee '56. EXP: Wu '57



Chirality of SM & Mass problem

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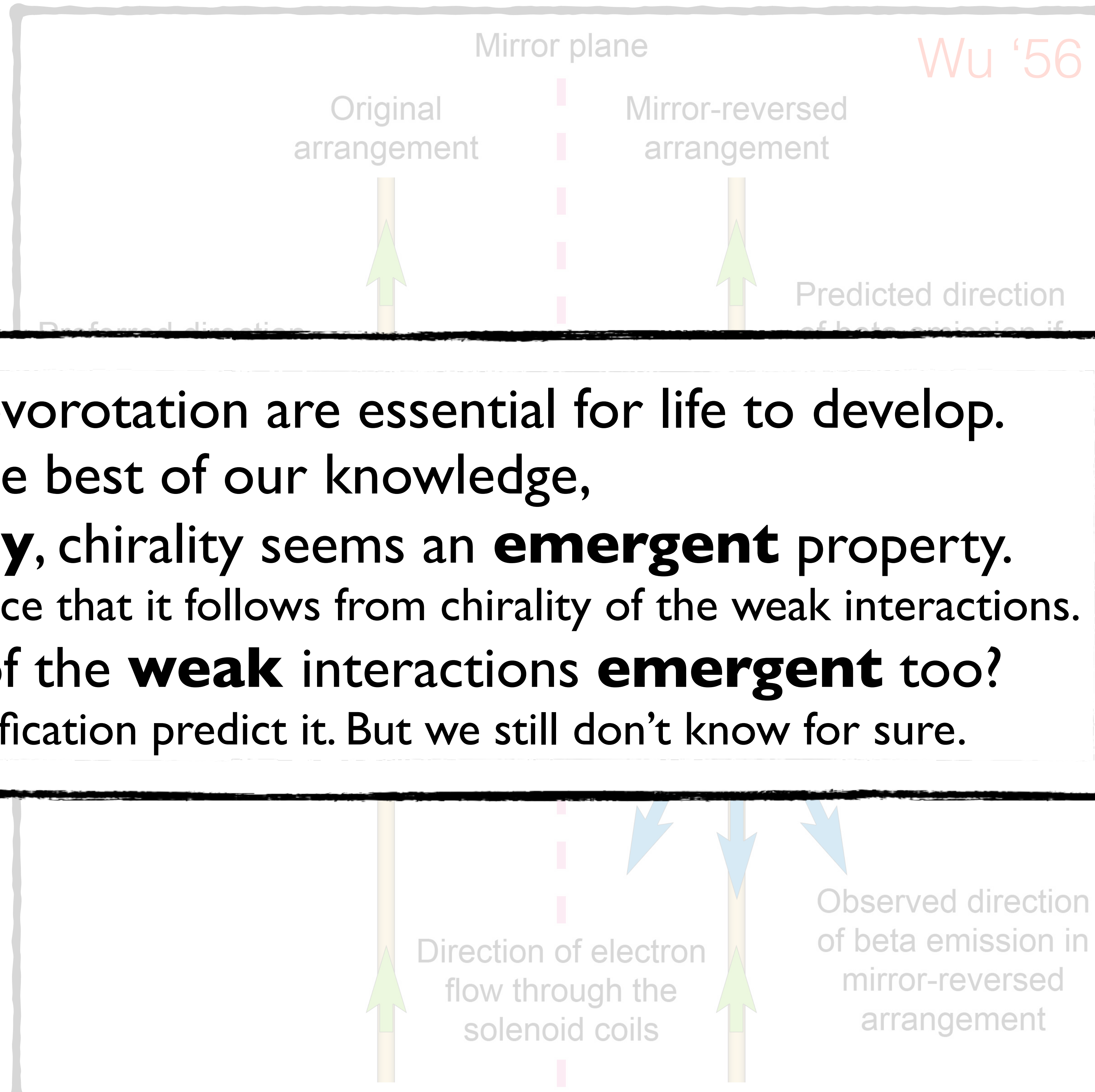


Dextrorotation and Levorotation are essential for life to develop.
To the best of our knowledge,
in **molecular biology**, chirality seems an **emergent** property.
At least, there is no clear evidence that it follows from chirality of the weak interactions.
Are the chiral nature of the **weak** interactions **emergent** too?
Some models of grand unification predict it. But we still don’t know for sure.

But since we know it is not true, we

need a new phenomena to generate mass: Higgs mechanism

SM is a Chiral Theory

Weak interactions maximally violates P

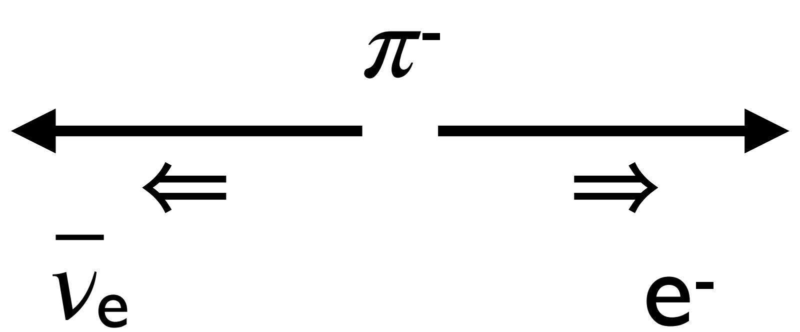
$^{60}_{27}\mathrm{Co} \to ^{60}_{28}\mathrm{Ni} + e^- + \bar{\nu}_e$ only left-handed (LH) e^- produced

Weak interactions act only on LH particles (and RH anti-particles)

this property has an important consequence (aka selection rule) for pion decay

$\rightarrow$ direction of momentum

$\Rightarrow$ direction of spin



Conservation of momentum and spin imposes to have a RH e^-

Weak decays proceed only w/ LH e^-
So the amplitude is prop. to m_e

$$\mathcal{L}_{\mathrm{Dirac}} = \bar{\psi}_L \gamma^\mu \partial_\mu \psi_L + \bar{\psi}_R \gamma^\mu \partial_\mu \psi_R + m\left(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L\right)$$

Lorentz structure of fermion mass

$$\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} \propto \frac{m_e^2}{m_\mu^2} \sim 2 \times 10^{-5} \sim 10^{-4}_{\mathrm{obs}}$$

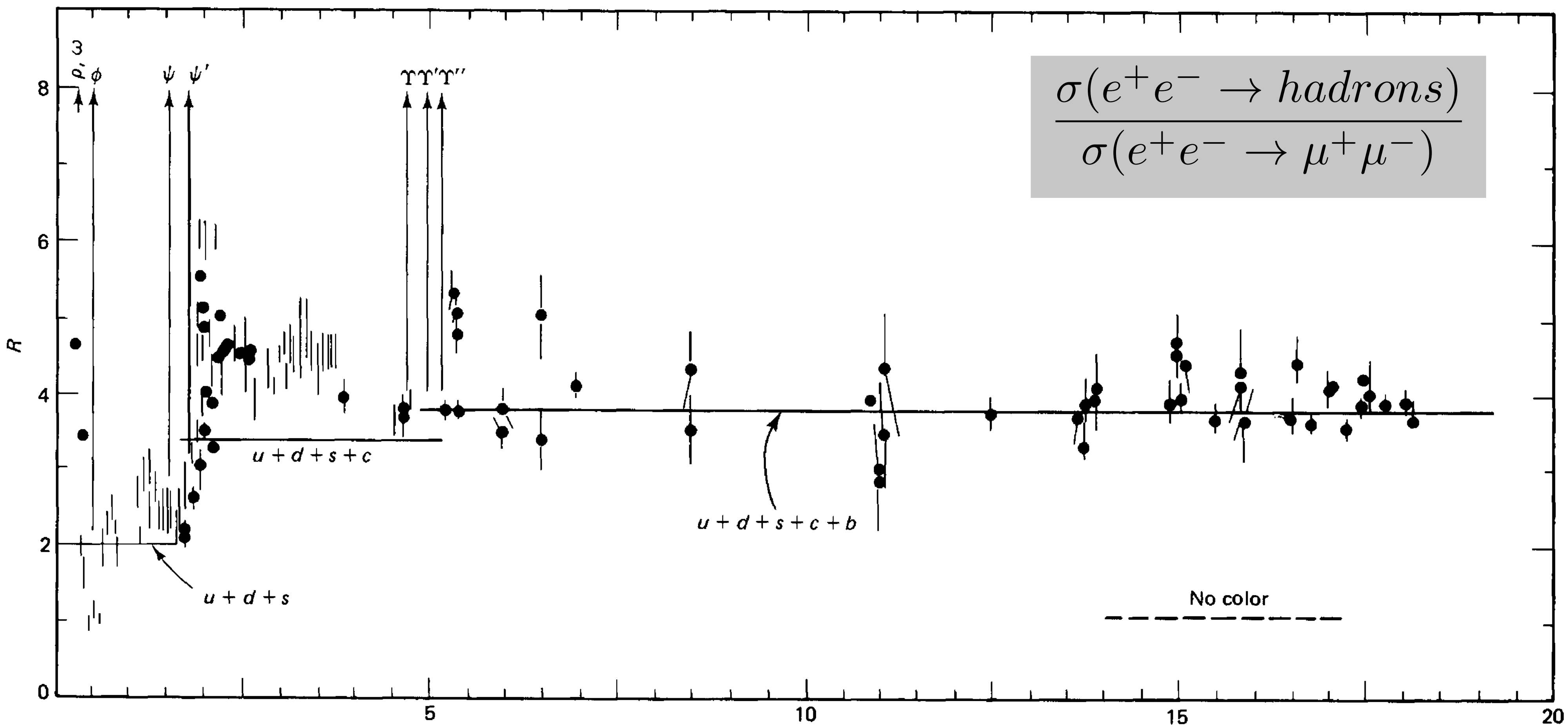
Extra phase-space factor

SU(3) QCD

Experiments in the 60's revealed the internal structure of the neutrons and protons
Gell-Mann and others proposed that they are made of "**quarks**"

Up quark (up, charm, top): spin-1/2, Q=2/3
Down quark (down, strange, bottom): spin-1/2, Q=-1/3

SU(2) weak symmetry that changes neutrino into electron also changes up-quark into down-quark



counts the number of quarks and gives their electric charges
another remarkable feature: at high energy, the quarks behaves like muons,
i.e., not sensitive to strong interactions
Asymptotic freedom of QCD!

SU(3) QCD

Deep inelastic experiments in the 60's revealed the internal structure of the neutrons and protons
Gell-Mann and others proposed that they are made of "**quarks**"

Up quark: spin-1/2, Q=2/3
Down quark: spin-1/2, Q=-1/3

SU(2) weak symmetry that changes neutrino into electron also changes up-quark into down-quark

Q**uarks** carry yet another quantum number: "**colour**"
There are 3 possible colours and Nature is colour-blind, i.e, Lagrangian should remain the same when the colours of the quarks are changed, i.e., when we perform a rotation in the colour-space of quarks.

$$Q^a \to U^a{}_b Q^b$$

U: 3x3 matrix satisfying $U^\dagger U = 1_3$ such that the quark kinetic term is invariant — SU(3)

hadrons (spin-1/2, #hadronic=1): $p = uud \qquad n = udd$

mesons (spin-0, #hadronic=0): $\pi^0 = \frac{u\bar{u} + d\bar{d}}{\sqrt{2}} \qquad \pi^+ = u\bar{d} \quad \pi^- = d\bar{u}$

(Each quark carries a baryon number =1/3)

There are (heavier) quarks and hence other baryons and mesons

All the interactions of the SM preserve baryon and lepton numbers

$$\mu \to e\nu_\mu\bar{\nu}_e \qquad n \to p\,e\,\bar{\nu}_e \qquad \pi^- \to \mu^-\bar{\nu}_\mu \qquad \pi^0 \to \gamma\gamma \qquad p \not\to \pi^0\bar{e}$$

The Standard Model: Interactions

- $U(1)_Y$ ***electromagnetic interactions***
  → Photon γ
  { light, atoms, molecules } — 10^{-5}

- $SU(2)_L$ ***weak interactions***
  → bosons $W^{\pm}, Z^0$
  { β decay
  $$n \xrightarrow{W^{\pm}} p + e^- + \bar{\nu}_e$$
  $$e^+ + e^- \xrightarrow{Z^0} D^+_{(c\bar{s})} + D^-_{(\bar{c}s)}$$
  } — 10^{-2}

- $SU(3)_c$ ***strong interactions***
  → gluons g^a
  { atomic nuclei
  α decay
  $$^{238}_{92}\mathrm{U} \longrightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He}$$
  } — 1

strength ↓

Technical Details for Advanced Students

Chirality

Chirality matrix

$$\boxed{\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3}$$

A few remarkable properties

$$\left(\gamma^5\right)^2 = 1_4$$

$$\{\gamma^5, \gamma^\mu\} = 0$$

$$\gamma^{5\dagger} = \gamma^5 = -\gamma^0\gamma^5\gamma^0$$

Chiral/Weyl spinor

A **chiral/Weyl** spinor is an eigenvector of the chirality matrix $\psi_{L,R} = \pm\gamma^5\psi_{L,R}$

From the Lorentz-transformation law of a spinor, it is obvious that the chirality condition is frame-independent

A Dirac spinor can also be written as a sum of two chiral spinors

$$\psi = \frac{1}{2}\left(1_4 + \gamma^5\right)\psi + \frac{1}{2}\left(1_4 - \gamma^5\right)\psi \equiv \psi_L + \psi_R$$

Charge conjugation

In general, ψ and ψ^* do not transform in the same way under Lorentz transformations and the naive reality condition $\psi = \psi^*$ is frame dependent

But it is possible to find a matrix C, called charge conjugation matrix, such that

$$\psi \quad \text{and} \quad \psi_C = C\,\psi^*$$

transform in the same way under Lorentz transformations

The matrix C needs to satisfy $C\gamma^* = -\gamma^\mu C$

In the Dirac and Weyl representations, $C = i\gamma^2$

In the Majorana representation, $C = 1_4$

Basic properties of the charge conjugation matrix: $C^2 = 1_4,\ C^\dagger = C,\ C^* = C$

The charge conjugated spinor, ψ_C, satisfies the same Dirac equation as ψ, with the same mass but opposite electric charge (when the spinor is minimally coupled to a U(1) gauge field)

A **Majorana** spinor satisfies the (Lorentz invariant!) condition $\psi = \psi_C$

Note that in 4D, a spinor cannot be simultaneously chiral and Majorana

Dirac and Majorana Masses

By construction, the following two mass terms in the Lagrangian are Lorentz-invariant

Dirac mass: $\mathcal{L}_{\mathrm{Dirac}} = m\bar{\psi}\psi$ (conserves fermion number)

Majorana mass: $\mathcal{L}_{\mathrm{Majorana}} = m\bar{\psi}_C\,\psi$ (changes fermion number by 2)

These two mass terms have different a chirality structure

$$\mathcal{L}_{\mathrm{Dirac}} = m\left(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\right)$$

$$\mathcal{L}_{\mathrm{Majorana}} = m\left(\bar{\psi}_{L_C}\psi_L + \bar{\psi}_{R_C}\psi_R\right)$$

A chiral fermion can have a Majorana mass
A Dirac mass requires spinors of opposite chirality

Whether or not a Dirac or a Majorana mass can be included in the Lagrangian depends on transformation laws of the spinors under the gauge transformations

Within the SM (with the Higgs field), a Dirac mass can written for the charged leptons and the quarks while a Majorana mass can be written for the neutrinos.

Higgs Lifetime “Computation”

Using dimensional analysis arguments,
compute the Higgs boson lifetime (or its inverse aka as the Higgs decay width)

— Hints —

Higgs couplings proportional are proportional to the mass of the particles it couples to.
It will therefore decay predominantly decay into the heaviest particle that is lighter than $m_H/2$

$$\Gamma \sim \frac{1}{8\pi}\left(\frac{m_b}{v}\right)^2 m_h \sim \frac{1}{10}\left(\frac{4\,\mathrm{GeV}}{246\,\mathrm{GeV}}\right)^2 125\,\mathrm{GeV} \sim 1\,\mathrm{MeV}$$

phase space

couplings to b-quark

dimensionally $[\Gamma]_m=1$

$$\tau \sim 10^{-22}\,\mathrm{s}$$

E	T	L
1eV	10^{-16}s	10^{-7}m

Putting all factors and considering the other decay modes, Higgs width = 4MeV in the SM

(for Z gauge boson: $\Gamma_Z = \frac{7}{48\pi} g^2 m_Z \sim 2\,\mathrm{GeV}$ i.e. $\tau_Z \sim 10^{-25}\,\mathrm{s}$)