The Standard Model of particle physics

CERN summer student lectures 2023

Lecture 3/5

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## Outline

### Monday: symmetry
- Lagrangians
- Lorentz symmetry - scalars, fermions, gauge bosons
- Gauge/local symmetry as dynamical principle - Example: U(1) electromagnetism

### Tuesday: SM symmetries
- Nuclear decay, Fermi theory and weak interactions: SU(2)
- Dimensional analysis: cross-sections and life-time computations made simple
- Strong interactions: SU(3)

### Wednesday: chirality of weak interactions
- Chirality of weak interactions
- Pion decay

### Thursday: Higgs mechanism
- Spontaneous symmetry breaking and Higgs mechanism
- Lepton and quark masses, quark mixings
- Neutrino masses

### Friday: quantum effects
- Running couplings
- Asymptotic freedom of QCD
- Anomalies cancelation
Universality of Weak Interactions

\[ \mu \rightarrow e \nu_\mu \bar{\nu}_e \quad \tau_\mu \approx 10^{-6} \text{s} \]
\[ n \rightarrow p \, e \, \bar{\nu}_e \quad \tau_n \approx 900 \text{s} \]

\[ \mathcal{L} = G_F \psi^4 \]

\[ \Gamma_\mu = \frac{G_F^2 m_\mu^5}{192 \pi^3} \sim 1/10^{-6''} \]
\[ \Gamma_n = \frac{G_F^2 \Delta m^5}{192 \pi^3} \sim 1/15' \]

By analogy with electromagnetism, one can see the Fermi force as a current-current interaction (vector-vector interaction instead of scalar-scalar interaction)

\[ \mathcal{L} = G_F \, J^*_\mu \, J^{\mu} \quad \text{with} \quad J^\mu = (\bar{n} \gamma^\mu p) + (\bar{e} \gamma^\mu \nu_e) + (\bar{\mu} \gamma^\mu \nu_\mu) + \ldots \]

it can be show (thanks to the transformation law of spin-1/2 field given before) that this Lagrangian is invariant under Lorentz transformation

The cross-terms generate both neutron decay and muon decay.
The life-times of the neutron and muon tell us that the relative factor between the e and the \( \mu \) in the current is of order one: the weak force has the **same strength for e and \( \mu \)**.
Pion decay(s)

What about $\pi^{\pm}$ decay $\tau_\pi \approx 10^{-8}$s?

$$\pi^- \rightarrow \mu \bar{\nu}_\mu \quad \quad \pi^- \rightarrow e^- \bar{\nu}_e$$

experimentally the pions decay dominantly into muons and not electrons.

Why $\frac{\Gamma(\pi^- \rightarrow e^- \bar{\nu}_e)}{\Gamma(\pi^- \rightarrow \mu \bar{\nu}_\mu)} \sim 10^{-4}$? And not $\frac{\Gamma(\pi^- \rightarrow e^- \bar{\nu}_e)}{\Gamma(\pi^- \rightarrow \mu \bar{\nu}_\mu)} \sim \frac{(m_\pi - m_e)^5}{(m_\pi - m_\mu)^5} \sim 500$? EXP

Does it mean that our way to compute decay rate is wrong?
Is pion decay mediated by another interaction?
The pion is a composite particle: does is mean that the form factors drastically change our estimates?
Is the weak interaction non universal, i.e. is the value of $G_F$ processus dependent?
Pathology at High Energy

What about weak scattering process, e.g. $e\nu_e \rightarrow e\nu_e$?

\[
\mathcal{L} = G_F J^*_\mu J^\mu \quad \text{with} \quad J^\mu = (\bar{\nu}\gamma^\mu p) + (\bar{e}\gamma^\mu \nu_e) + (\bar{\mu}\gamma^\mu \nu_\mu) + \ldots
\]

The same Fermi Lagrangian will thus also contain a term

\[
G_F (\bar{e}\gamma^\mu \nu_e)(\bar{\nu}_e \gamma^\mu e)
\]

that will generate $e\nu_e$ scattering whose cross-section can be guessed by dimensional arguments

\[
\sigma \propto G_F^2 E^2
\]

It means that, at high-energy, the quantum corrections to the classical contribution can be sizeable:

\[
\sigma \propto G_F^2 E^2 + \frac{1}{16\pi^2} G_F^4 E^6 + \ldots
\]

The theory becomes non-perturbative at an energy $E_{\text{max}} = \frac{2\sqrt{\pi}}{\sqrt{G_F}} \sim 100 \text{ GeV–1 TeV}$

unless new degrees of freedom appear before to change the behaviour of the scattering
The Fermi interaction is not a fundamental interaction of Nature. It is a low energy effective interaction.
Electroweak Interactions

1. No additional “force” (Georgi, Glashow ’72) mathematical consistency ⇒ extra matter

- **SU(2)**
  - \([T^a, T^b] = i\epsilon^{abc}T^c\)
  - \([T^+, T^-] = Q\)
  - \([Q, T^\pm] = \pm T^\pm\)
  - \(T^\pm = \frac{1}{\sqrt{2}}(T^1 \pm iT^2)\)
  - \(\text{Tr}_{\text{irrep}} T^3 = 0\) ⇒ extra matter

- **SU(1, 1)**
  - \([T^+, T^-] = -Q\)
  - \([Q, T^\pm] = \pm T^\pm\)
  - non-compact
  - unitary rep. has dim \(\infty\)

- **E_2**
  - 2D Euclidean group
  - \([T^+, T^-] = 0\)
  - \([Q, T^\pm] = \pm T^\pm\)
  - only one unitary rep. of finite dim. = trivial rep.

2. No additional “matter” (Glashow ’61, Weinberg ’67, Salam ’68): \(\text{SU}(2)\times\text{U}(1)\) ⇒ extra force

- \(Q = T^3\)?
  - as Georgi-Glashow
  - ⇒ extra matter

- \(Q = Y\)?
  - \(Q(e_L) = Q(\nu_L)\)
  - Gell-Mann ’56, Nishijima-Nakano ’53

charged W ⇒ must couple to photon:

\[ \nonumber \]

⇒ non-abelian gauge symmetry \([Q, T^\pm] = \pm T^\pm\)
**Electroweak Interactions**

**Gargamelle** experiment ’73 first established the SU(2)xU(1) structure

How? rely on a particle that doesn’t interact with photon to prove the existence a new neutral current process!

$\nu_\mu e^- \rightarrow \nu_\mu e^-$

- loop-suppressed contribution from W:
From Gauge Theory to Fermi Theory

We can derive the Fermi current-current contact interactions by “integrating out” the gauge bosons, i.e., by replacing in the Lagrangian the W's by their equation of motion. Here is a simple derivation:
(a better one should take taking into account the gauge kinetic term and the proper form of the fermionic current that we’ll figure out tomorrow, for the moment, take it as a heuristic derivation)

\[
\mathcal{L} = -m_W^2 W_\mu^+ W_\nu^- \eta^{\mu\nu} + g W_\mu^+ J_\nu^- \eta^{\mu\nu} + g W_\nu^- J_\nu^+ \eta^{\mu\nu}
\]

\[
J^{+\mu} = \bar{n} \gamma^\mu p + \bar{e} \gamma^\mu \nu_e + \bar{\mu} \gamma^\mu \nu_\mu + \ldots \quad \text{and} \quad J^{-\mu} = (J^{+\mu})^*
\]

The equation of motion for the gauge fields:

\[
\frac{\partial \mathcal{L}}{\partial W_\mu^+} = 0 \quad \Rightarrow \quad W_\mu^- = \frac{g}{m_W^2} J_\mu^-
\]

Plugging back in the original Lagrangian, we obtain an effective Lagrangian (valid below the mass of the gauge bosons):

\[
\mathcal{L} = \frac{g^2}{m_W^2} J_\mu^+ J_\nu^- \eta^{\mu\nu}
\]

which is the Fermi current-current interaction. The Fermi constant is given by
(the correct expression involves a different normalisation factor)

\[
G_F = \frac{g^2}{m_W^2}
\]

But what is the origin of the W mass?
By the way, it is not invariant under SU(2) gauge transformation…
That’s what the Higgs mechanism will take care of!
Chirality & Masslessness

**Quantum Mechanics 1.0.1**

Particle of spin \( s \) has \( 2s+1 \) polarisation states

Particle spinning **anticlockwise** wrt its direction of motion

**electron** has 2 polarisation

Particle spinning **clockwise** wrt its direction of motion
Chirality & Masslessness

Relativistic invariance 1.0.1:
there must be no distinction for massive particles between particles spinning clockwise or anti-clockwise
[chirality operator doesn’t commute with the Hamiltonian]

If your theory sees a difference between $e_L$ and $e_R$, either your theory is wrong or $m_e = 0$
**Weak** interaction (force responsible for neutron decay) is chiral!

$[e_L$ and $e_R$ are fundamentally two different particles
Only an accident of the history of physics that they are both called electron]

$m_e=0$

but since we know it is not true, we need a new phenomena to generate mass: Higgs mechanism

TH: Yang&Lee ‘56. EXP: Wu ‘57
Dextrorotation and Levorotation are essential for life to develop. To the best of our knowledge, in molecular biology, chirality seems an emergent property. At least, there is no clear evidence that it follows from chirality of the weak interactions. Are the chiral nature of the weak interactions emergent too? Some models of grand unification predict it. But we still don’t know for sure.
SM is a Chiral Theory

Weak interactions maximally violates P

\[ \frac{^{60}_{27}\text{Co}}{} \rightarrow \frac{^{60}_{28}\text{Ni}}{} + e^- + \bar{\nu}_e \] only left-handed (LH) e^- produced

Weak interactions act only on LH particles (and RH anti-particles)

this property has an important consequence (aka selection rule) for pion decay

Conservation of momentum and spin imposes to have a RH e^-

Weak decays proceed only w/ LH e^-
So the amplitude is prop. to \( m_e \)

\[ \mathcal{L}_{\text{Dirac}} = \bar{\psi}_L \gamma^\mu \partial_\mu \psi_L + \bar{\psi}_R \gamma^\mu \partial_\mu \psi_R + m \left( \bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L \right) \]
Lorentz structure of fermion mass

\[ \frac{\Gamma(\pi^- \rightarrow e^- \bar{\nu}_e)}{\Gamma(\pi^- \rightarrow \mu^- \bar{\nu}_\mu)} \propto \frac{m_e^2}{m_\mu^2} \sim 2 \times 10^{-5} \sim 10^{-4} \]
Extra phase-space factor

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Extra phase-space factor
SU(3) QCD

Experiments in the 60’s revealed the internal structure of the neutrons and protons. Gell-Mann and others proposed that they are made of “quarks”

<table>
<thead>
<tr>
<th>Quark</th>
<th>Type</th>
<th>spin</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Up</td>
<td>(up, charm, top)</td>
<td>1/2</td>
<td>2/3</td>
</tr>
<tr>
<td>Down</td>
<td>(down, strange, bottom)</td>
<td>1/2</td>
<td>-1/3</td>
</tr>
</tbody>
</table>

SU(2) weak symmetry that changes neutrino into electron also changes up-quark into down-quark

Counts the number of quarks and gives their electric charges

Another remarkable feature: at high energy, the quarks behave like muons, i.e., not sensitive to strong interactions

Asymptotic freedom of QCD!
SU(3) QCD

Deep inelastic experiments in the 60’s revealed the internal structure of the neutrons and protons
Gell-Mann and others proposed that they are made of “quarks”

<table>
<thead>
<tr>
<th>Up quark: spin-1/2, Q=2/3</th>
<th>Down quark: spin-1/2, Q=-1/3</th>
</tr>
</thead>
</table>

SU(2) weak symmetry that changes neutrino into electron also changes up-quark into down-quark

Quarks carry yet another quantum number: “colour”
There are 3 possible colours and Nature is colour-blind, i.e, Lagrangian should remain the same when
the colours of the quarks are changed, i.e., when we perform a rotation in the colour-space of quarks.

\[ Q^a \rightarrow U_{ab} Q^b \]

\[ U: \text{3x3 matrix satisfying } U^\dagger U = 1_3 \]

such that the quark kinetic term is invariant

SU(3)

\[ \text{hadrons (spin-1/2, #hadronic=1): } p = uud \quad n = udd \]

\[ \text{mesons (spin-0, #hadronic=0): } \pi^0 = \frac{u\bar{u} + d\bar{d}}{\sqrt{2}} \quad \pi^+ = u\bar{d} \quad \pi^- = d\bar{u} \]

(Each quark carries a baryon number =1/3)

There are (heavier) quarks and hence other baryons and mesons

All the interactions of the SM preserve baryon and lepton numbers

\[ \mu \rightarrow e\nu_\mu \bar{\nu}_e \quad n \rightarrow p e \bar{\nu}_e \quad \pi^- \rightarrow \mu^- \bar{\nu}_\mu \quad \pi^0 \rightarrow \gamma \gamma \quad p \rightarrow \pi^0 \bar{\nu}_e \]
The Standard Model: Interactions

- **U(1)$_Y$** electromagnetic interactions
  - Photon $\gamma$

- **SU(2)$_L$** weak interactions
  - Bosons $W^\pm, Z^0$

- **SU(3)$_c$** strong interactions
  - Gluons $g^a$

\[
\begin{align*}
\text{light} & \quad \text{atoms} \\
\text{molecules} & \quad \text{β decay} \\
& \quad n \xrightarrow{W^\pm} p + e^- + \bar{\nu}_e \\
& \quad e^+ + e^- \xrightarrow{Z^0} D_{(cs)}^+ + D_{(\bar{cs})}^- \\
\text{atomic nuclei} & \quad \text{α decay} \\
& \quad ^{238}_{92}\text{U} \rightarrow ^{234}_{90}\text{Th} + ^4\text{He}
\end{align*}
\]
Technical Details for Advanced Students
Chirality

Chirality matrix

\[ \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 \]

A few remarkable properties

\[ (\gamma^5)^2 = 1_4 \]

\[ \{\gamma^5, \gamma^\mu\} = 0 \]

\[ \gamma^{5\dagger} = \gamma^5 = -\gamma^0\gamma^5\gamma^0 \]

Chiral/Weyl spinor

A **chiral/Weyl** spinor is an eigenvector of the chirality matrix \[ \psi_{L,R} = \pm \gamma^5 \psi_{L,R} \]

From the Lorentz-transformation law of a spinor, it is obvious that the chirality condition is frame-independent

A Dirac spinor can also be written as a sum of two chiral spinors

\[ \psi = \frac{1}{2} \left( 1_4 + \gamma^5 \right) \psi + \frac{1}{2} \left( 1_4 - \gamma^5 \right) \psi \equiv \psi_L + \psi_R \]
Charge conjugation

In general, $\psi$ and $\psi^*$ do not transform in the same way under Lorentz transformations and the naive reality condition $\psi = \psi^*$ is frame dependent.

But it is possible to find a matrix $C$, called charge conjugation matrix, such that

$$\psi \quad \text{and} \quad \psi_C = C\psi^*$$

transform in the same way under Lorentz transformations.

The matrix $C$ needs to satisfy $C\gamma^* = -\gamma^\mu C$

In the Dirac and Weyl representations, $C = i\gamma^2$

In the Majorana representation, $C = 1_4$

Basic properties of the charge conjugation matrix: $C^2 = 1_4$, $C^\dagger = C$, $C^* = C$

The charge conjugated spinor, $\psi_C$, satisfies the same Dirac equation as $\psi$, with the same mass but opposite electric charge (when the spinor is minimally coupled to a U(1) gauge field).

A **Majorana** spinor satisfies the (Lorentz invariant!) condition $\psi = \psi_C$

Note that in 4D, a spinor cannot be simultaneously chiral and Majorana.
Dirac and Majorana Masses

By construction, the following two mass terms in the Lagrangian are Lorentz-invariant

**Dirac mass:** \[ \mathcal{L}_{\text{Dirac}} = m \bar{\psi} \psi \] (conserves fermion number)

**Majorana mass:** \[ \mathcal{L}_{\text{Majorana}} = m \bar{\psi} C \psi \] (changes fermion number by 2)

These two mass terms have different a chirality structure

\[ \mathcal{L}_{\text{Dirac}} = m \left( \bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L \right) \]
\[ \mathcal{L}_{\text{Majorana}} = m \left( \bar{\psi}_{L C} \psi_L + \bar{\psi}_{R C} \psi_R \right) \]

A chiral fermion can have a Majorana mass
A Dirac mass requires spinors of opposite chirality

Whether or not a Dirac or a Majorana mass can be included in the Lagrangian depends on transformation laws of the spinors under the gauge transformations

Within the SM (with the Higgs field), a Dirac mass can written for the charged leptons and the quarks while a Majorana mass can be written for the neutrinos.
Higgs Lifetime “Computation”

Using dimensional analysis arguments, compute the Higgs boson lifetime (or its inverse aka as the Higgs decay width)

— Hints —

Higgs couplings proportional are proportional to the mass of the particles it couples to. It will therefore decay predominantly decay into the heaviest particle that is lighter than $m_H/2$

$$ \Gamma \sim \frac{1}{8\pi} \left( \frac{m_b}{\nu} \right)^2 m_h \sim \frac{1}{10} \left( \frac{4 \text{ GeV}}{246 \text{ GeV}} \right)^2 125 \text{ GeV} \sim 1 \text{ MeV} $$

Putting all factors and considering the other decay modes, Higgs width = 4MeV in the SM

(for Z gauge boson: $\Gamma_Z = \frac{7}{48\pi} g^2 m_Z \sim 2 \text{ GeV} \quad \text{i.e.} \quad \tau_Z \sim 10^{-25} \text{ s}$)