The Standard Model of particle physics

CERN summer student lectures 2023

Lecture 4/5

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Outline

- **Monday: symmetry**
  - Lagrangians
  - Lorentz symmetry - scalars, fermions, gauge bosons
  - Gauge/local symmetry as dynamical principle - Example: U(1) electromagnetism

- **Tuesday: SM symmetries**
  - Nuclear decay, Fermi theory and weak interactions: SU(2)
  - Dimensional analysis: cross-sections and life-time computations made simple
  - Strong interactions: SU(3)

- **Wednesday: chirality of weak interactions**
  - Chirality of weak interactions
  - Pion decay

- **Thursday: Higgs mechanism**
  - More about QCD
  - Spontaneous symmetry breaking and Higgs mechanism
  - Lepton and quark masses, quark mixings
  - Neutrino masses

- **Friday: quantum effects**
  - Running couplings
  - Asymptotic freedom of QCD
  - Anomalies cancelation
SU(3) QCD

\[ R(E) = \sum_{m_i < E} Q_i^2 \]

\[ 2 = 3 \times \left( \left( \frac{2}{3} \right)^2 + \left( -\frac{1}{3} \right)^2 + \left( -\frac{1}{3} \right)^2 \right) \]

below 2 GeV
3 particles of charges ±2/3 and 6 particles of charge ±1/3

\[ \frac{10}{3} = 3 \times \left( \left( \frac{2}{3} \right)^2 + \left( -\frac{1}{3} \right)^2 + \left( -\frac{1}{3} \right)^2 + \left( \frac{2}{3} \right)^2 \right) \]

at 2 GeV, 3 more particles of charge ±2/3

\[ \frac{11}{3} = 3 \times \left( \left( \frac{2}{3} \right)^2 + \left( -\frac{1}{3} \right)^2 + \left( -\frac{1}{3} \right)^2 + \left( \frac{2}{3} \right)^2 \left( -\frac{1}{3} \right)^2 \right) \]

at 5 GeV, 3 more particles of charge ±1/3
SU(3) QCD

The R-ratio:

现代化版本的图…

图8.3

马田,等（1984年，纽约：威利公司版权所有）

数据比较
SU(3) QCD

\[ R(E) = \sum_{m_i < E} Q_i^2 \times \left( 1 + \frac{\alpha_s}{\pi} + \frac{\alpha_s^2}{\pi^2} (c + \pi b \log \Lambda^2 / E^2) \right) \]

below 2 GeV

3 particles of charges ±2/3 and 6 particles of charge ±1/3

at 2 GeV, 3 more particles of charge ±2/3

at 5 GeV, 3 more particles of charge ±1/3

\[ m_s \approx 0.1 \text{ GeV} \]

\[ m_b \approx 5 \text{ GeV} \]

\[ \sigma(e^+ e^- \to \text{hadrons}) / \sigma(e^+ e^- \to \mu^+ \mu^-) \]

\[ \sqrt{s} = 2E[\text{GeV}] \]

3.66  
3.33  
2  

\[ m_s \approx 0.1 \text{ GeV} \]

\[ m_b \approx 5 \text{ GeV} \]
SU(3) QCD

Experiments in the 60’s revealed the internal structure of the neutrons and protons. Gell-Mann and others proposed that they are made of “quarks”

- Up quarks (up, charm, top): spin-1/2, Q=2/3
- Down quarks (down, strange, bottom): spin-1/2, Q=-1/3

SU(2) weak symmetry that changes neutrino into electron also changes up-quark into down-quark (to explain neutron decay)

This experiment counts the number of quarks and gives their electric charges. Another remarkable feature: at high energy, the quarks behave like muons, i.e., not sensitive to strong interactions.

Asymptotic freedom of QCD!
(consequence of non-abelian nature of strong interaction - see tomorrow lecture)
SU(3) QCD

Experiments in the 60’s revealed the internal structure of the neutrons and protons. Gell-Mann and others proposed that they are made of “quarks”

<table>
<thead>
<tr>
<th>Quark</th>
<th>Spin</th>
<th>Charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Up quarks (up, charm, top)</td>
<td>1/2</td>
<td>2/3</td>
</tr>
<tr>
<td>Down quarks (down, strange, bottom)</td>
<td>1/2</td>
<td>-1/3</td>
</tr>
</tbody>
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SU(2) weak symmetry that changes neutrino into electron also changes up-quark into down-quark (to explain neutron decay)
**SU(3) QCD**

Experiments in the 60’s revealed the internal structure of the neutrons and protons. Gell-Mann and others proposed that they are made of "quarks".

<table>
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<th>Up quarks (up, charm, top): spin-1/2, Q=2/3</th>
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<tbody>
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<td>Down quarks (down, strange, bottom): spin-1/2, Q=-1/3</td>
</tr>
</tbody>
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**SU(2) weak symmetry** that changes neutrino into electron also changes up-quark into down-quark (to explain neutron decay).

Quarks carry yet another quantum number: "colour"

There are 3 possible colours and Nature is colour-blind, i.e., Lagrangian should remain the same when the colours of the quarks are changed, i.e., when we perform a rotation in the colour-space of quarks.

\[
Q^a \rightarrow U^{a \,b} Q^b
\]

\[U: \text{3x3 matrix satisfying } \quad U^\dagger U = 1_3 \quad \text{SU(3)}\]

such that the quark kinetic term is invariant

**hadrons** (spin-1/2, #hadronic=1):

\[ p = uud \quad n = udd \]

**mesons** (spin-0, #hadronic=0):

\[
\pi^0 = \frac{u\bar{u} + d\bar{d}}{\sqrt{2}} \quad \pi^+ = u\bar{d} \quad \pi^- = d\bar{u}
\]

(Each quark carries a baryon number =1/3)

There are (heavier) quarks and hence other baryons and mesons.

All the interactions of the SM preserve baryon and lepton numbers:

\[
\begin{align*}
\mu &\rightarrow e\nu_{\mu}\bar{\nu}_e \\
n &\rightarrow p e \bar{\nu}_e \\
\pi^- &\rightarrow \mu^- \nu_{\mu} \\
\pi^0 &\rightarrow \gamma\gamma \\
p &\rightarrow \pi^0\bar{e}
\end{align*}
\]
Inside Hadrons

One can break matter into pieces to learn what it is made of. But this is not always possible (not sharp enough knife, not enough energy…). Fortunately, remember the boiled egg experiment:

https://youtu.be/r1ygKQbcqh4

The way the egg is spinning can tell if it is boiled (one piece) or raw (internal structure with different components moving independently from each others)
Inside Hadrons

One can break matter into pieces to learn what it is made of. But this is not always possible (not sharp enough knife, not enough energy…). Fortunately, remember the boiled egg experiment:

\( \lambda \gg r_p \)  
\( \lambda \sim r_p \)  
\( \lambda < r_p \)  
\( \lambda \ll r_p \)

electron-proton scattering (1960’s) reveals the proton intimate structure (3 elementary spin-1/2 quarks that exist in 3 colours bounded by strong interactions that become feeble at large energies — asymptotic freedom).
The Standard Model: Interactions

- **U(1)_Y**  
  - **electromagnetic interactions**
  - Photon \( \gamma \)

- **SU(2)_L**  
  - **weak interactions**
  - Bosons \( W^\pm, Z^0 \)

- **SU(3)_c**  
  - **strong interactions**
  - Gluons \( g^a \)

**Light**
- Atoms
- Molecules

\[ \begin{align*}
\beta \text{ decay} \\
& n \xrightarrow{W^\pm} p + e^- + \bar{\nu}_e \\
e^+ + e^- & \xrightarrow{Z^0} D^{+}_{(cs)} + D^{-}_{(\bar{cs})}
\end{align*} \]

**Atomic nuclei**
- Alpha decay

\[ \begin{align*}
\alpha \text{ decay} \\
& \frac{238}{92}U \rightarrow \frac{234}{90}Th + \frac{4}{2}He
\end{align*} \]

**Strength**
## SM Summary

<table>
<thead>
<tr>
<th>SPIN</th>
<th>PARTICLES</th>
<th>SU(3)_C × SU(2)_L × U(1) _Y</th>
<th>$T_{3L}$</th>
<th>$Q = T_{3L} + Y$</th>
<th>$g_{\text{eff}}$</th>
<th>MEANING</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEPTONS</td>
<td>$L = (\nu_e)_L$</td>
<td>1</td>
<td>2</td>
<td>$(-1/2)$</td>
<td>$(1/2)$</td>
<td>$(0)$</td>
</tr>
<tr>
<td>1/2</td>
<td>$e_R$</td>
<td>1</td>
<td>1</td>
<td>$-1$</td>
<td>0</td>
<td>$-1$</td>
</tr>
<tr>
<td>QUARKS</td>
<td>$Q = (u_d)_L$</td>
<td>3</td>
<td>2</td>
<td>$(1/6)$</td>
<td>$(1/2)$</td>
<td>$2/3$</td>
</tr>
<tr>
<td></td>
<td>$u_R$</td>
<td>3</td>
<td>1</td>
<td>$2/3$</td>
<td>0</td>
<td>$2/3$</td>
</tr>
<tr>
<td></td>
<td>$d_R$</td>
<td>3</td>
<td>1</td>
<td>$-1/3$</td>
<td>0</td>
<td>$-1/3$</td>
</tr>
<tr>
<td>HIGGS</td>
<td>$H = (h^+_0) _L$</td>
<td>1</td>
<td>2</td>
<td>$(1/2)$</td>
<td>$(1/2)$</td>
<td>$(0)$</td>
</tr>
</tbody>
</table>
Are we done?

\[ m_e \bar{e} L e_R + h.c. \]

is not gauge invariant

\[ Y = 1/2 \quad \text{and} \quad Y = -1 \]

\[ m_W^2 W^+ \mu W^- \eta^{\mu\nu} \]

is not gauge invariant

\[ A_\mu \rightarrow U A_\mu U^{-1} + \frac{i}{g} (\partial_\mu U) U^{-1} \]

Remember May 1, 2003:

“Mission accomplished” speech by G.W. Bush.

That was certainly not the end of the story and there were (are) still a lot things to do!
Spontaneous Symmetry Breaking

Short-distance interactions ≠ Long-distance interactions
The masses are emergent due to a non-trivial structure of the vacuum

vacuum = a space entirely devoid of matter
Oxford English

vacuum = a space filled with BEH substance
Physics English

QM vs QFT

Ground state of QM double well potential
is a superposition of two states each localised on one minimum,
and this superposition preserves the $Z_2$ symmetry of the potential

In QFT, it is more difficult to transition between degenerate vacua
and spontaneous symmetry breaking can occur
(or more correctly, the symmetry is non-linearly realised in Hilbert space)

The vacuum of the SM breaks $SU(2) \times U(1)$ to $U(1)_{em}$
via the dynamics of an elementary scalar field

The Brout-Englert-Higgs Boson
(postulated in 1964 — discovered in 2012) 🌟
Spontaneous Symmetry

Symmetry of the Lagrangian

\[ SU(2)_L \times U(1)_Y \]

Higgs Doublet

\[ H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}_{1/2} \]

Symmetry of the Vacuum

\[ U(1)_{e.m.} \]

Vacuum Expectation Value

\[ \langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \quad \text{with} \quad v \approx 246 \text{ GeV} \]

Most general Higgs (renormalisable) potential

\[ V(H) = \lambda \left( |H|^2 - \frac{v^2}{2} \right)^2 \]

\( v^2 > 0 \) EW symmetry breaking, \( v^2 < 0 \) no breaking

Why Nature has decided that \( v^2 > 0 \)? No dynamics explains it.

\[ \delta_{SU(2)} \langle H \rangle = \frac{i}{2} \left( \theta^1 \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} + \theta^2 \begin{pmatrix} I & -I \\ I & I \end{pmatrix} + \theta^3 \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} \right) \langle H \rangle \neq 0 \]

\[ \delta_Y \langle H \rangle = i \theta_Y \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix} \langle H \rangle \neq 0 \]

\[ \delta_Q \langle H \rangle = i \theta_{QED} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \langle H \rangle = 0 \quad \theta_{QED} = \theta_Y = \theta_3 \quad Q = Y + T_{3L} \]
Higgs Boson

Before EW symmetry breaking

- 4 massless gauge bosons for SU(2)x(1): 4 x 2 = 8 dofs
- Complex scalar doublet: 4 dofs

After EW symmetry breaking

- 1 massless gauge boson, photon: 2 dofs
- 3 massive gauge bosons, $W^\pm$ and $Z$: 3 x 3 = 9 dofs
- 1 real scalar: 1 dof

\[ H = \begin{pmatrix} 0 \\ \frac{v + h(x)}{\sqrt{2}} \end{pmatrix} \]

$h(x)$ describes the Higgs boson (the fluctuation above the VEV).

The other components of the Higgs doublet $H$ become the longitudinal polarisations of the $W^\pm$ and $Z$
At high-energy hadron colliders, the Higgs boson production mechanism with the largest cross section is gluon fusion production, as well as uncertainties due to hadronisation and parton-shower events. The main production mechanisms at the Tevatron collider and the LHC are gluon fusion (ggF), weak-boson fusion (VBF), associated production with a gauge boson (VH), di-boson production (WW, WZ, ZZ), and (f-g) production in association with a single top quark. Very important when we will discuss the Higgs production at high energy, for too heavy to be produced directly, and the higher–order QCD corrections will be postponed to the leading order.

Since gluons and photons are massless particles, they do not substantially for Higgs boson masses. These decays are thus extremely interesting since their couplings to the Higgs boson grow with the masses. For fermions, only the heavy top quark and to a lesser extent bottom quarks would be obese since its total decay width becomes competitive to the Higgs width.

The 2012 Scalar Discovery

Not the most abundant BEH modes, but the “cleanest” ones
At high-energy hadron colliders, the Higgs boson production mechanism with the largest cross section is the gluon-fusion process, which is used to search for the Higgs boson mass of 125 GeV. For parameters involved in the calculations, these loop decays are important only for Higgs bosons of mass 100 GeV or less. For fermions, only the heavy top quark and to a lesser extent charged fermions loops, while the coupling is mediated only by quark loops; Fig. 2.14. For masses much larger than the Higgs boson mass, these virtual corrections are too heavy to be produced directly. These partial widths will be suppressed by the third power of the Higgs mass, since their strength is sensitive to—leading order (NLO) QCD corrections. The discussion of these partial widths will be challenging conditions too heavy to be produced directly. For masses much larger than the Higgs boson mass, these virtual corrections are too heavy to be produced directly. These partial widths will be suppressed by the third power of the Higgs mass, since their strength is sensitive to—leading order (NLO) QCD corrections. The discussion of these partial widths will be challenging conditions too heavy to be produced directly. These partial widths will be suppressed by the third power of the Higgs mass, since their strength is sensitive to—leading order (NLO) QCD corrections. The discussion of these partial widths will be challenging conditions too heavy to be produced directly. These partial widths will be suppressed by the third power of the Higgs mass, since their strength is sensitive to—leading order (NLO) QCD corrections. The discussion of these partial widths will be challenging conditions too heavy to be produced directly.
# The LHC Scalar Harvest

(8M Brout-Englert-Higgs bosons produced so far)

<table>
<thead>
<tr>
<th>Channel categories</th>
<th>Br</th>
<th>ggF</th>
<th>VBF</th>
<th>VH</th>
<th>ttH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross Section 13 TeV (8 TeV)</td>
<td>48.6 (21.4) pb*</td>
<td>3.8 (1.6) pb</td>
<td>2.3 (1.1) pb</td>
<td>0.5 (0.1) pb</td>
<td></td>
</tr>
<tr>
<td>γγ</td>
<td>0.2 %</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>ZZ</td>
<td>3%</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>WW</td>
<td>22%</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>ττ</td>
<td>6.3 %</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>bb</td>
<td>55%</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Zγ and γγ</td>
<td>0.2 %</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>μμ</td>
<td>0.02 %</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Invisible</td>
<td>0.1 %</td>
<td>✓ (monojet)</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

- ~8 M evts produced
- ~600 k evts produced
- ~400 k evts produced
- ~80 k evts produced

Table courtesy to M. Kado
Fermion Masses

SM is a **chiral** theory (≠ QED that is vector-like)

\[ m_e \bar{e}_L e_R + h.c. \]

is not gauge invariant

\[ \begin{align*}
Y=1/2 & \quad \text{Y=1/2} \\
Y=-1 & \\
\end{align*} \]

The SM Lagrangian cannot contain fermion mass term.

Fermion masses are **emergent** quantities that originate from **interactions with Higgs VEV**

\[ H = \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix} \]

\[ \mathcal{L} = y_e \begin{pmatrix} \bar{\nu}_L \\ \bar{e}_L \end{pmatrix} \cdot \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} e_R = \frac{y_e v}{\sqrt{2}} \left( \bar{e}_L e_R + \frac{1}{v} \bar{e}_L e_R h \right) \]

\[ \begin{align*}
Y=1/2 & \quad \text{Y=1/2} \\
Y=1/2 & \\
Y=-1 & \\
\end{align*} \]

Higgs couplings proportional to the mass of particles
The Higgs PR plot

Higgs couplings are proportional to the masses of the particles

$$\lambda_\psi \propto \frac{m_\psi}{v}, \quad \lambda_V^2 \equiv \frac{gVVh}{2v} \propto \frac{m_V^2}{v^2}$$
Fermion Masses

In SM, the Yukawa interactions are the only source of the fermion masses

\[ y_{ij} \bar{f}_{Li} H f_{Rj} = \frac{y_{ij} v}{\sqrt{2}} \bar{f}_{Li} f_{Rj} + \frac{y_{ij} v}{\sqrt{2}} h \bar{f}_{Li} f_{Rj} \]

both matrices are simultaneously diagonalisable

no tree-level Flavor Changing Current induced by the Higgs

Once the mass terms are diagonal, the Higgs interactions are diagonal too

Not true anymore if the SM fermions mix with vector-like partners or for non-SM Yukawa

\[ y_{ij} \left( 1 + c_{ij} \frac{|H|^2}{f^2} \right) \bar{f}_{Li} H f_{Rj} = \frac{y_{ij} v}{\sqrt{2}} \left( 1 + c_{ij} \frac{v^2}{2f^2} \right) \bar{f}_{Li} f_{Rj} + \left( 1 + 3c_{ij} \frac{v^2}{2f^2} \right) \frac{y_{ij} v}{\sqrt{2}} h \bar{f}_{Li} f_{Rj} \]

Look for SM forbidden Flavour Violating decays \( h \rightarrow \mu \tau \) and \( h \rightarrow e \tau \)

(look also at \( t \rightarrow hc \))

- weak indirect constrained by flavour data (\( \mu \rightarrow e\gamma \)): BR<10\%
- ATLAS and CMS have the sensitivity to set bounds O(1\%)
- ILC/CLIC/FCC-ee can certainly do much better
Fermion Masses: Quark Mixings

In SM, the Yukawa interactions are the only source of the fermion masses

\[ \mathcal{L}_{\text{Yuk}} = y^U_{ij} \bar{Q}^i_L H^* u_R^i + y^D_{ij} \bar{Q}^i_L H d_R^i \]

\[ \mathcal{U}^\dagger_L \left( \frac{v}{\sqrt{2}} y^U_{ij} \right) \mathcal{U}_R = \left( \begin{array}{ccc} m_u & m_c & m_t \\ m_c & m_t & m_t \end{array} \right) \]

\[ \mathcal{D}^\dagger_L \left( \frac{v}{\sqrt{2}} y^D_{ij} \right) \mathcal{D}_R = \left( \begin{array}{ccc} m_d & m_s & m_b \\ m_s & m_b & m_b \end{array} \right) \]

\[ \mathcal{L}_{\text{Yuk}} = (\bar{u}_L \bar{c}_L t_L) \left( \begin{array}{ccc} m_u & m_c & m_t \\ m_c & m_t & m_t \end{array} \right) \left( \begin{array}{c} u_R \\ c_R \\ t_R \end{array} \right) + (\bar{d}_L \bar{s}_L \bar{b}_L) \left( \begin{array}{ccc} m_d & m_s & m_b \\ m_s & m_b & m_b \end{array} \right) \left( \begin{array}{c} d_R \\ s_R \\ b_R \end{array} \right) \]

\[ \mathcal{L}_{\text{gauge}} = \frac{e}{\sqrt{2} \sin \theta_w} \left[ W^+_{\mu} \bar{u} V \gamma^\mu \left( \frac{1 - \gamma_5}{2} \right) d + W^-_{\mu} \bar{d} V^\dagger \gamma^\mu \left( \frac{1 - \gamma_5}{2} \right) u \right] \]

\[ V = D^\dagger_L U_L \]

\[ V_{\text{CKM}} = \left( \begin{array}{ccc} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{array} \right) \left( \begin{array}{ccc} c_{13} & 0 & s_{13} e^{-i \delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i \delta} & 0 & c_{13} \end{array} \right) \left( \begin{array}{ccc} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{array} \right) \]

Note: one complex phase \( \rightarrow \) CP violation
Neutrino Masses

The same construction doesn’t work for neutrinos since in the SM there are only Left Handed neutrinos

For an uncharged particle, it is possible to write a Majorana mass another Lorentz-invariant quadratic term in the Lagrangian (it involves the charge-conjugate spinor, see lecture #3-technical slides)

\[ \mathcal{L}_{\text{Majorana}} = m \bar{\psi}_C \psi = m (\bar{\psi}_{L_C} \psi_L + \bar{\psi}_{R_C} \psi_R) \]

can build such a term with LH field only!

In SM, such neutrino Majorana mass can be obtained from dim-5 operator:

\[ \mathcal{L} = \frac{y_\nu}{\Lambda} \left( \begin{array}{c} \nu_L \\ e_L \end{array} \right) \cdot \left( \begin{array}{c} H^+ \\ H^0 \end{array} \right) \left( \begin{array}{c} \nu_L \\ e_L \end{array} \right) \cdot \left( \begin{array}{c} H^+ \\ H^0 \end{array} \right) = \frac{y_\nu}{\Lambda} \frac{v^2}{\Lambda} \nu_{LC} \nu_L \]

mass\(3/2\) mass mass\(3/2\) mass

Seesaw: \[ m_\nu = \frac{y_\nu v^2}{\Lambda} \]
Order eV for \(y_\nu \sim 1\) and \(\Lambda \sim 10^{14}\) GeV

Note that such an operator breaks Lepton Number by 2 units
Higgs Mechanism

**Symmetry of the Lagrangian**

$SU(2)_L \times U(1)_Y$

Higgs Doublet

$$H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}$$

**Symmetry of the Vacuum**

$U(1)_{e.m.}$

Vacuum Expectation Value

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \text{ with } v \approx 246 \text{ GeV}$$

$$D_\mu H = \partial_\mu H - \frac{i}{2} \left( \begin{array}{cc} gW_\mu^3 + g'B_\mu & \sqrt{2}gW_\mu^+ \\ \sqrt{2}gW_\mu^- & -gW_\mu^3 + g'B_\mu \end{array} \right) H \text{ with } W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \mp W_\mu^2)$$

$$|D_\mu H|^2 = \frac{1}{4} g^2 v^2 W_\mu^+ W^- \mu + \frac{1}{8} (W_\mu^3 B_\mu) \left( \begin{array}{cc} g^2 v^2 & -gg'v^2 \\ -gg'v^2 & g'^2 v^2 \end{array} \right) \left( \begin{array}{c} W_\mu^3 \\ B_\mu \end{array} \right)$$

**Gauge boson spectrum**

- electrically charged bosons
  $$M_{W}^2 = \frac{1}{4} g^2 v^2$$

- weak mixing angle
  $$c = \frac{g}{\sqrt{g^2 + g'^2}}$$
  $$s = \frac{g'}{\sqrt{g^2 + g'^2}}$$

- electrically neutral bosons
  $$M_{Z}^2 = \frac{1}{4} (g^2 + g'^2) v^2$$
  $$M_\gamma = 0$$
The Brout-Englert-Higgs Boson is Special

The scalar discovery in 2012 has been an important milestone for HEP. Many of us are still excited about it. Others should be too.

BEH = **new forces** of different nature than the interactions known so far
- No underlying local symmetry
- No quantised charges
- Deeply connected to the space-time vacuum structure

The knowledge of the values of the **BEH couplings** is essential to understand the deep structure of matter/Universe

- $m_W, m_Z \leftrightarrow$ BEH couplings
- lifetime of stars
  (why $t_{Sun} \sim t_{life\ evolution}$?)

- $m_e, m_u, m_d \leftrightarrow$ BEH couplings
- size of atoms
  nuclei stability

- EWSB @ $t \sim 10^{-10}$s $\leftrightarrow$ BEH self-coupling

- matter/anti-matter $\leftrightarrow$ CPV in BEH sector

- ?

- ?
The Brout-Englert-Higgs Boson is Special

LHC will make remarkable progress but it won’t be enough. A new collider will be needed!

The knowledge of the values of the **BEH couplings** is essential to understand the deep structure of matter/Universe.

- $m_W, m_Z \leftrightarrow$ BEH couplings
- lifetime of stars
  (why $t_{\text{Sun}} \sim t_{\text{life evolution}}$?)

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- EWSB @ $t \sim 10^{-10} s \leftrightarrow$ BEH self-coupling

- matter/anti-matter $\leftrightarrow$ CPV in BEH sector
Technical Details for Advanced Students
The longitudinal polarisation of massive W, Z

symmetry breaking: new phase with more degrees of freedom

\[ \epsilon_{||} = \left( \frac{|\mathbf{p}|}{M}, \frac{E}{M} \frac{\mathbf{p}}{|\mathbf{p}|} \right) \]

polarization vector grows with the energy

a massless particle is never at rest: always possible to distinguish (and eliminate!) the longitudinal polarisation

the longitudinal polarisation is physical for a massive spin-1 particle

\[ 3 = 2 + 1 \]

(pictures: courtesy of G. Giudice)
Indeed a massive spin 1 particle has 3 physical polarizations: $A_\mu = \epsilon_\mu \ e^{i k_\mu x^\mu}$

$\epsilon^\mu \epsilon_\mu = -1 \quad k^\mu \epsilon_\mu = 0$

$k^\mu = (E, 0, 0, k)$

with $k_\mu k^\mu = E^2 - k^2 = M^2$

**2 transverse:**

\[
\begin{align*}
\epsilon_1^\mu &= (0, 1, 0, 0) \\
\epsilon_2^\mu &= (0, 0, 1, 0)
\end{align*}
\]

**1 longitudinal:**

$\epsilon_{\parallel}^\mu = (\frac{k^\mu}{M}, 0, 0, \frac{E}{M}) \approx \frac{k^\mu}{M} + O(\frac{E}{M})$

( in the R-\(\xi\) gauge, the time-like polarization ($\epsilon^\mu \epsilon_\mu = 1 \quad k^\mu \epsilon_\mu = M$) is arbitrarily massive and decouple )

in the particle rest-frame, no distinction between L and T polarisations

in a frame where the particle carries a lot of kinetic energy,

the L polarisation “dominates”
The BEH mechanism: \( \mathcal{V}_L \equiv \text{Goldstone bosons} \)

At high energy, the physics of the gauge bosons becomes simple

\[
\Gamma(t \to bW_L) = \frac{g^2}{64\pi} \frac{m_t^2}{m_W^2} \frac{(m_t^2 - m_W^2)^2}{m_t^3}
\]

\[
\Gamma(t \to bW_T) = \frac{g^2}{64\pi} \frac{2(m_t^2 - m_W^2)^2}{m_t^3}
\]

At high energy, the dominant degrees of freedom are \( W_L \)

\~\~ why you should be stunned by this result: \~\~

we expect: (dimensional analysis) \( \Gamma \sim g^2 m_{\text{mother}} \)

instead \( \Gamma \propto m_{\text{mother}}^3 \) means \( g \propto m \) like the Higgs couplings!

very efficient way to get energy from the mother particle \( \tau \ll \tau_{\text{naive}} \)

Goldstone equivalence theorem
\( W_L^\pm, Z_L \approx \text{SO}(4)/\text{SO}(3) \)

This is the physics that was understood at LEP
The pending question was then: is there something else?
That was the job of the LHC
Call for extra degrees of freedom

**NO LOSE THEOREM**

Bad high-energy behaviour for the scattering of the longitudinal polarisations

\[
A = \epsilon^\mu_\parallel(k)\epsilon^\nu_\parallel(l)g^2(2\eta_{\mu\rho}\eta_{\nu\sigma} - \eta_{\mu\nu}\eta_{\rho\sigma} - \eta_{\mu\sigma}\eta_{\nu\rho})\epsilon^\rho_\parallel(p)\epsilon^\sigma_\parallel(q)
\]

\[
A = g^2 \frac{E^4}{4M_W^4}
\]

violations of perturbative unitarity around \( E \sim M/\sqrt{g} \) (actually \( M/g \))

Extra degrees of freedom are needed to have a good description of the W and Z masses at higher energies

numerically: \( E \sim 3 \text{ TeV} \) the LHC was sure to discover something!
Call for extra degrees of freedom

The Higgs boson unitarizes the W scattering (if its mass is below ~ 1 TeV)

\[ W_L \text{ scattering} = \text{pion scattering} \]

Goldstone equivalence theorem

\[ A = g^2 \left( \frac{E}{M_W} \right)^2 \]

\[ A = -g^2 \left( \frac{E}{M_W} \right)^2 \]

\[ A = g^2 \left( \frac{M_H}{2M_W} \right)^2 \]

Lewellyn Smith '73
Dicus, Mathur '73
Cornwall, Levin, Tiktopoulos '73
Lee, Quigg, Thacker '77
What is the SM Higgs?

A single scalar degree of freedom that couples to the mass of the particles

\[ \Sigma = e^{i\pi^a \sigma^2 / \nu} \]

parametrises the coset SO(4)/SO(3)

\[ \mathcal{L} = \frac{v^2}{4} \text{Tr} \, D_\mu \Sigma^\dagger D^\mu \Sigma \]

\[ \Sigma = 1 \quad \Rightarrow \quad m_W^2 W^+_\mu W^+_{\mu} + \frac{1}{2} m_Z Z_\mu Z^\mu \]

\[ g = g' = 0 \quad \Rightarrow \quad \frac{1}{2} (\partial \pi)^2 + \frac{1}{v^2} \partial^2 \pi^4 + \ldots \]

\[ \mathcal{L}_{\text{EWSB}} = m_W^2 W^+_\mu W^+_{\mu} \left( 1 + 2a \frac{h}{v} + b \frac{h^2}{v^2} \right) - m_\psi \bar{\psi}_L \psi_R \left( 1 + c \frac{h}{v} \right) \]

‘a’, ‘b’ and ‘c’ are arbitrary free couplings

\[ \mathcal{A} = \frac{1}{v^2} \left( s - \frac{a^2 s^2}{s - m_h^2} \right) \]

growth cancelled for \( a = 1 \)

restoration of perturbative unitarity
What is the Higgs the name of?

A single scalar degree of freedom that couples to the mass of the particles

\[ \mathcal{L}_{\text{EWSB}} = m_W^2 W^+_{\mu} W^+_{\mu} \left( 1 + 2a \frac{h}{v} + b \frac{h^2}{v^2} \right) - m_\psi \bar{\psi}_L \psi_R \left( 1 + c \frac{h}{v} \right) \]

‘a’, ‘b’ and ‘c’ are arbitrary free couplings

For a=1: perturbative unitarity in elastic channels WW → WW

For b = a^2: perturbative unitarity in inelastic channels WW → hh

Cornwall, Levin, Tiktopoulos ’73

Contino, Grojean, Moretti, Piccinini, Rattazzi ’10
What is the Higgs the name of?

A single scalar degree of freedom that couples to the mass of the particles

\[ \mathcal{L}_{\text{EWSB}} = m_W^2 W^+_\mu W^+_{\mu} \left( 1 + 2a \frac{h}{v} + b \frac{h^2}{v^2} \right) - m_\psi \bar{\psi}_L \psi_R \left( 1 + c \frac{h}{v} \right) \]

‘a’, ‘b’ and ‘c’ are arbitrary free couplings

For \( a=1 \): perturbative unitarity in elastic channels \( WW \rightarrow WW \)

For \( b = a^2 \): perturbative unitarity in inelastic channels \( WW \rightarrow hh \)

For \( ac=1 \): perturbative unitarity in inelastic \( WW \rightarrow \psi \psi \)

Cornwall, Levin, Tiktopoulos ’73

Contino, Grojean, Moretti, Piccinini, Rattazzi ’10
What is the Higgs the name of?
A single scalar degree of freedom that couples to the mass of the particles

\[ \mathcal{L}_{\text{EWSB}} = m_W^2 W_\mu^+ W_\mu^+ \left( 1 + 2a \frac{h}{v} + b \frac{h^2}{v^2} \right) - m_\psi \bar{\psi}_L \psi_R \left( 1 + c \frac{h}{v} \right) \]

‘a’, ‘b’ and ‘c’ are arbitrary free couplings

For \( a=1 \): perturbative unitarity in elastic channels \( WW \rightarrow WW \)

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For \( ac=1 \): perturbative unitarity in inelastic \( WW \rightarrow \psi \psi \)

Cornwall, Levin, Tiktopoulos ’73
Contino, Grojean, Moretti, Piccinini, Rattazzi ’10

Higgs couplings are proportional to the masses of the particles

\[ \lambda_\psi \propto \frac{m_\psi}{v}, \quad \lambda_V^2 \equiv \frac{gVVh}{2v} \propto \frac{m_V^2}{v^2} \]