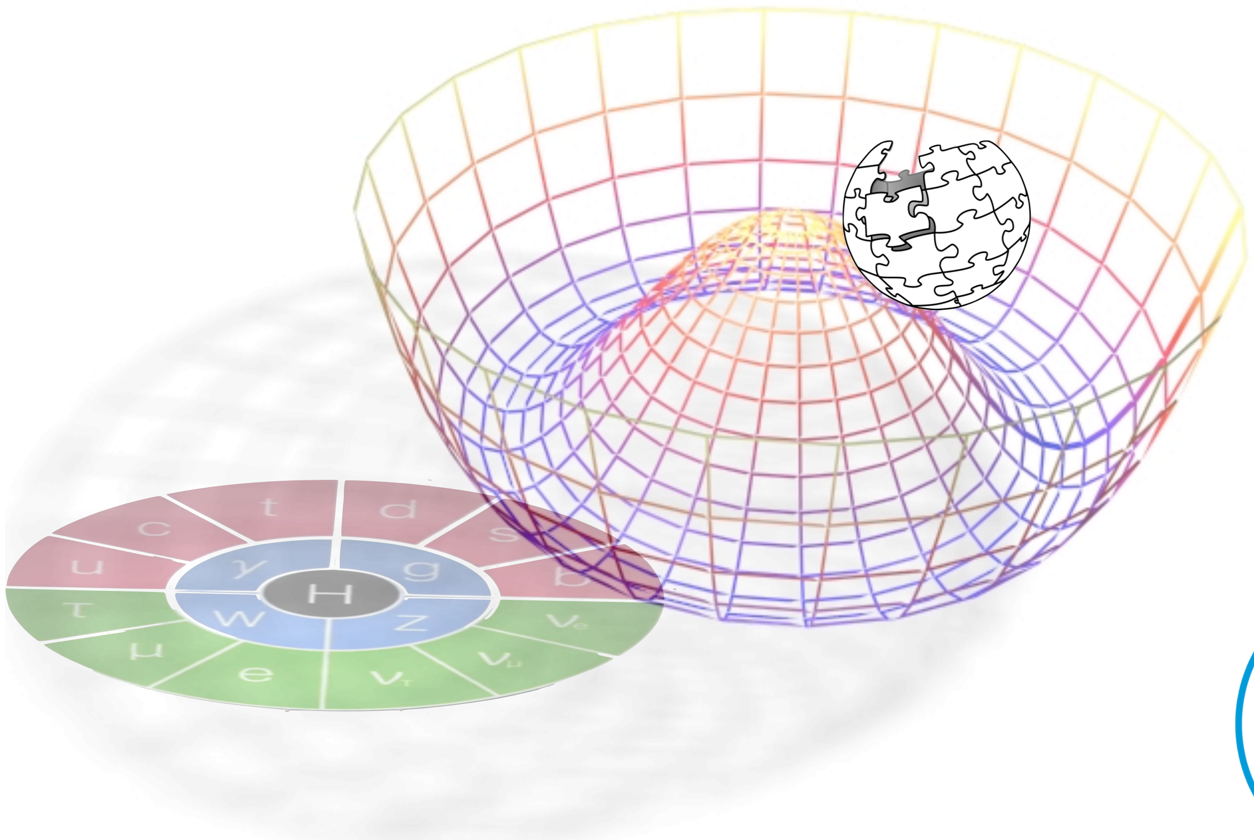
The Standard Model
of particle physics

CERN summer student lectures 2023

Lecture 4/5



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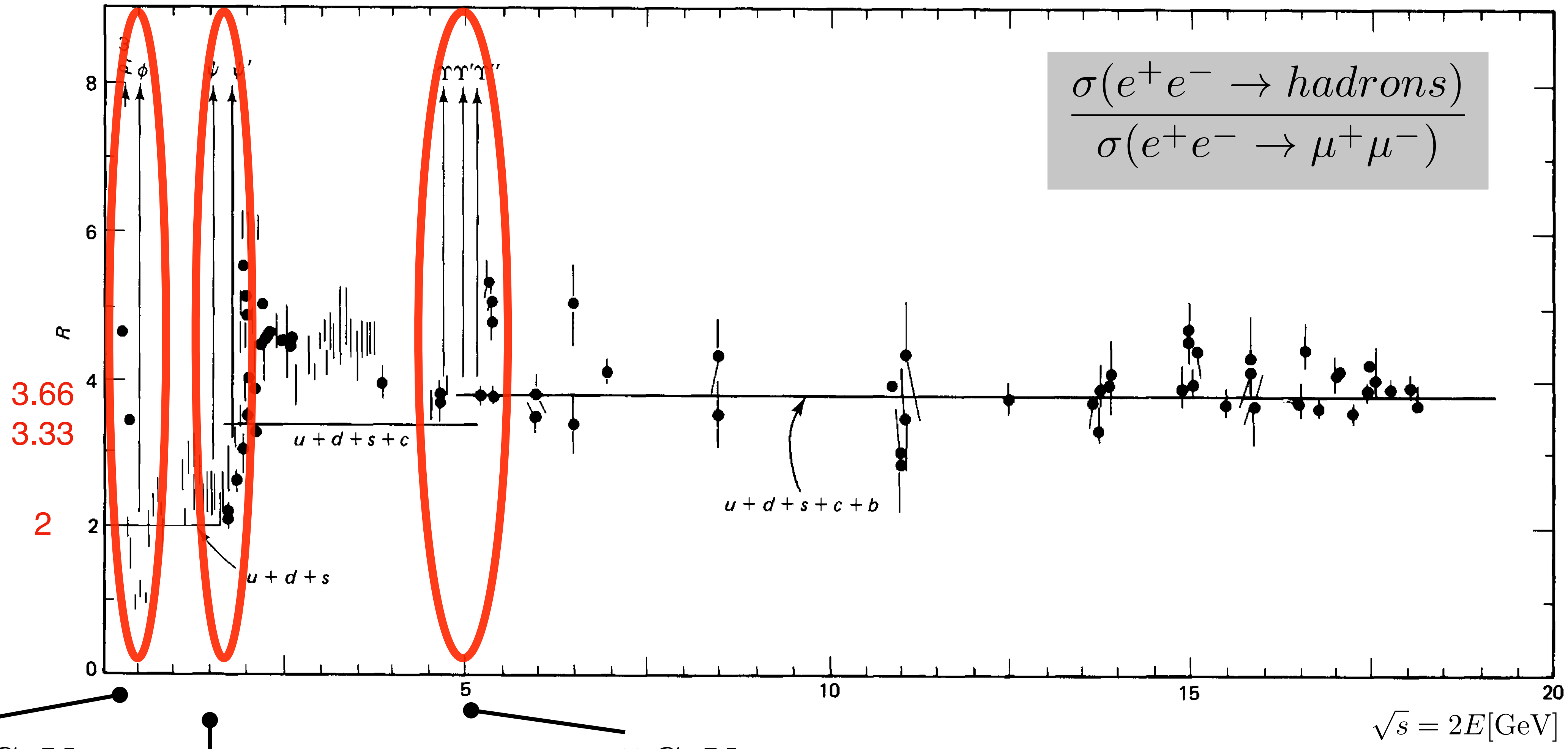
Outline

- **Monday: symmetry**
  - Lagrangians
  - Lorentz symmetry - scalars, fermions, gauge bosons
  - Gauge/local symmetry as dynamical principle - Example: U(1) electromagnetism
- **Tuesday: SM symmetries**
  - Nuclear decay, Fermi theory and weak interactions: SU(2)
  - Dimensional analysis: cross-sections and life-time computations made simple
  - Strong interactions: SU(3)
- **Wednesday: chirality of weak interactions**
  - Chirality of weak interactions
  - Pion decay
- **Thursday: Higgs mechanism**
  - More about QCD
  - Spontaneous symmetry breaking and Higgs mechanism
  - Lepton and quark masses, quark mixings
  - Neutrino masses
- **Friday: quantum effects**
  - Running couplings
  - Asymptotic freedom of QCD
  - Anomalies cancelation

SU(3) QCD



$m_s \approx 0.1\,\mathrm{GeV}$

$m_c \approx 2\,\mathrm{GeV}$

$m_b \approx 5\,\mathrm{GeV}$

$$\boxed{R(E) = \sum_{m_i<E} Q_i^2}$$

$$2 = 3 \times \left(\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2\right)$$

below 2 GeV
3 particles of charges ±2/3 and 6 particles of charge ±1/3

$$\frac{10}{3} = 3 \times \left(\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2\right)$$

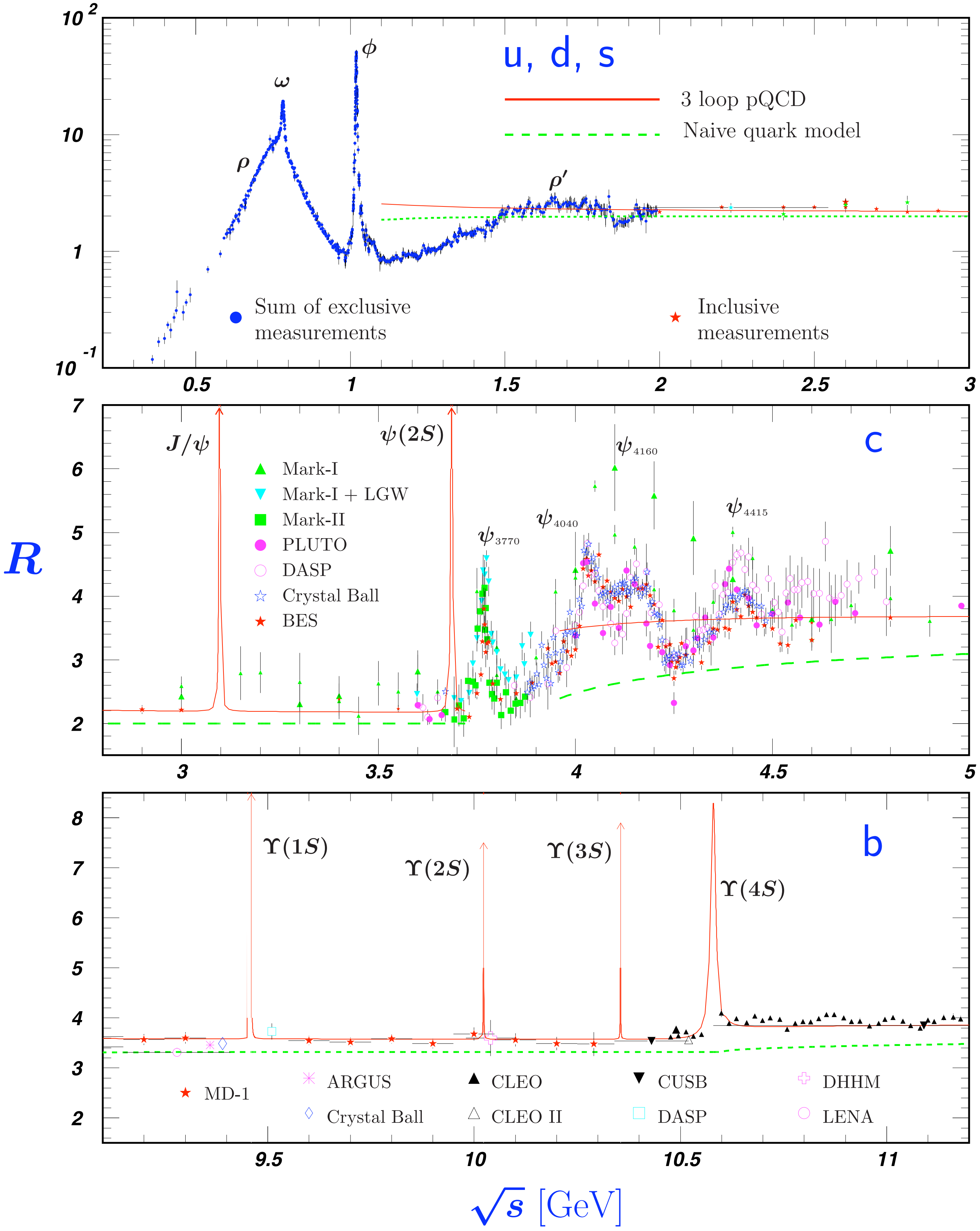
at 2 GeV, 3 more particles of charge ±2/3

$$\frac{11}{3} = 3 \times \left(\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 \left(-\frac{1}{3}\right)^2\right)$$

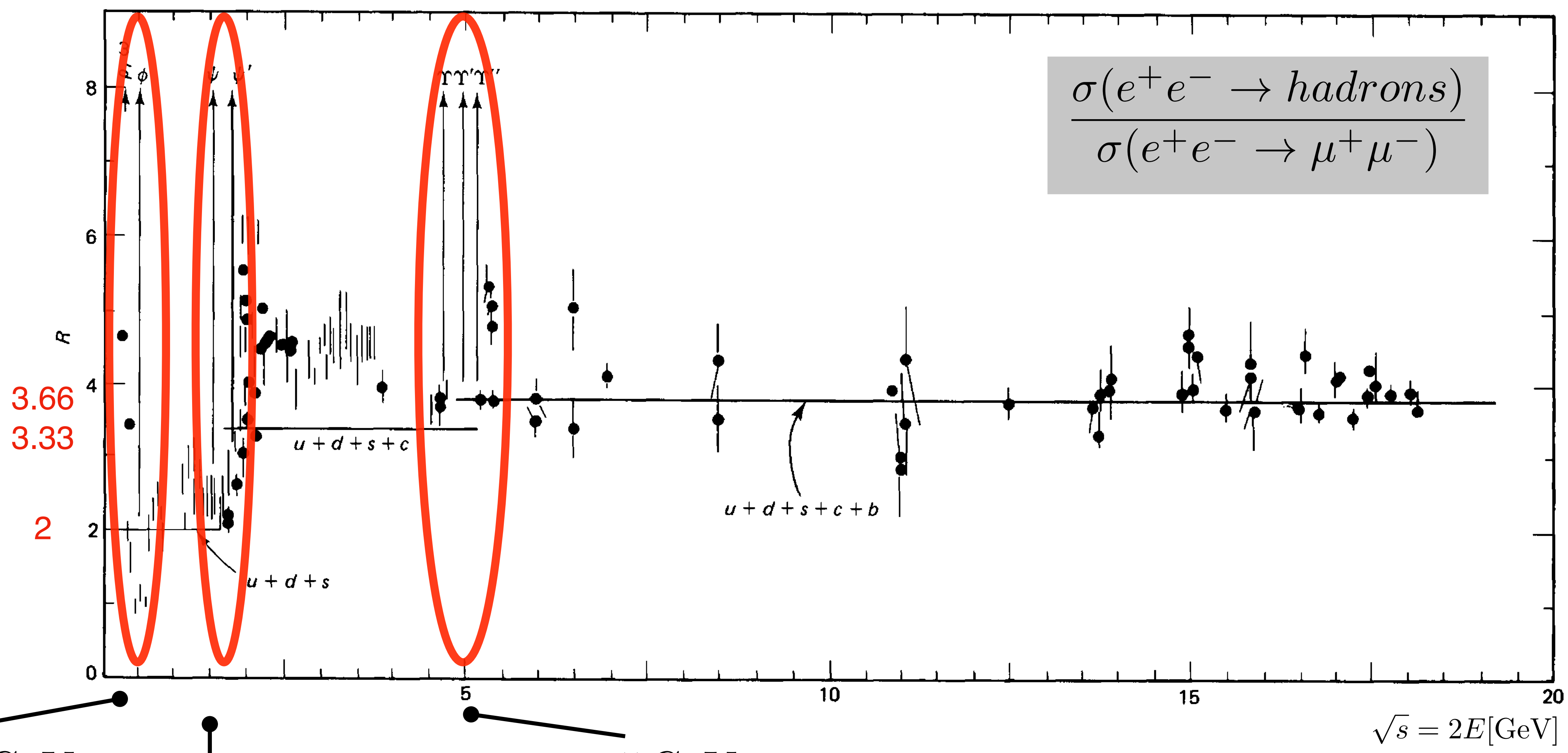
at 5 GeV, 3 more particles of charge ±1/3

SU(3) QCD

modern version of the plot…



SU(3) QCD



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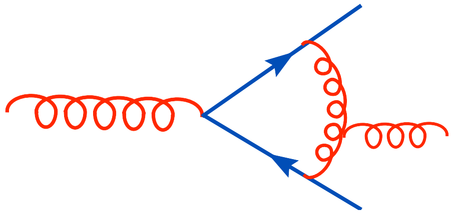

$$\boxed{R(E) = \sum_{m_i<E} Q_i^2} \times \left(1 + \frac{\alpha_s}{\pi} + \frac{\alpha_s^2}{\pi^2}(c + \pi b \log \Lambda^2/E^2)\right)$$

$$2 = 3 \times \left(\left(\frac{2}{3}\right)^{-} + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2\right)$$

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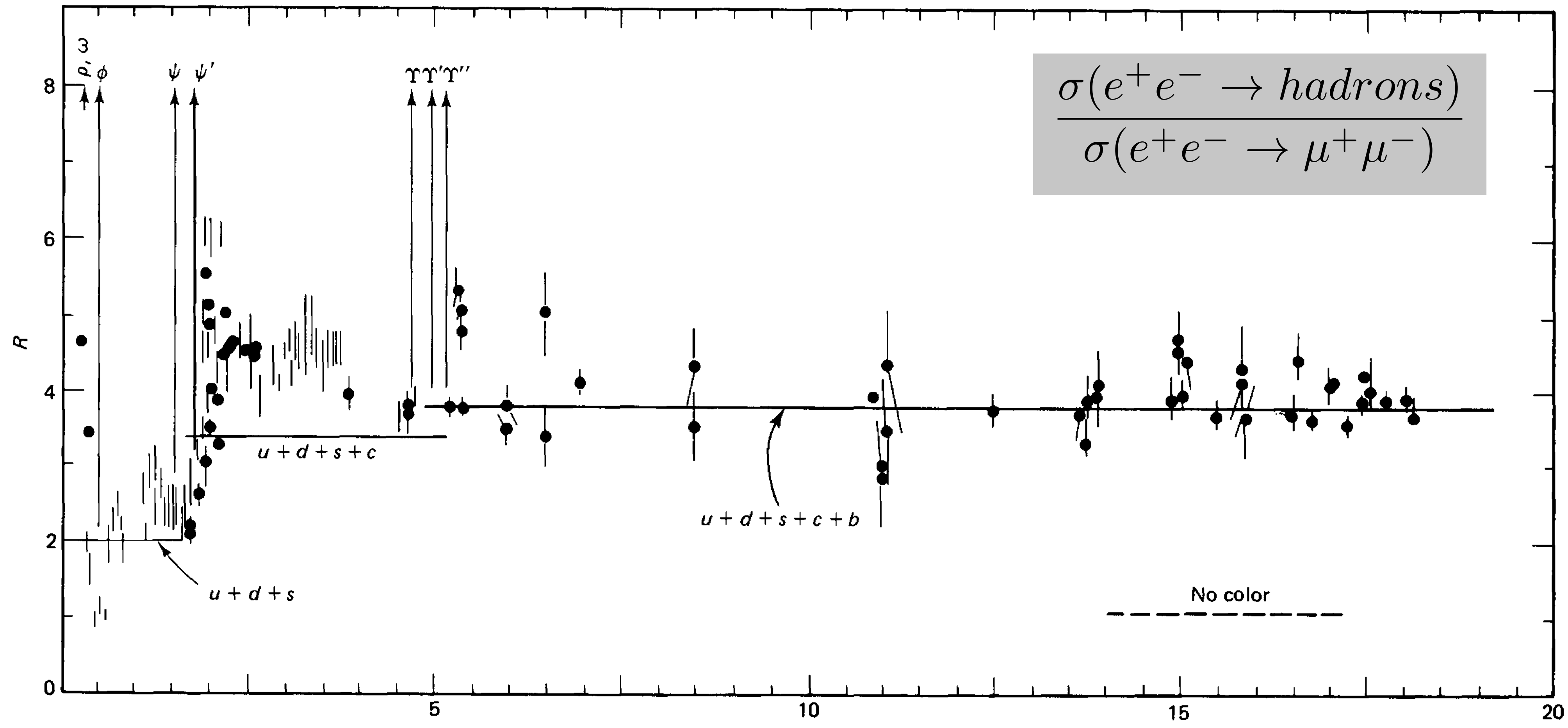
at 5 GeV, 3 more particles of charge ±1/3

SU(3) QCD

Experiments in the 60’s revealed the internal structure of the neutrons and protons
Gell-Mann and others proposed that they are made of “**quarks**”

Up quarks (up, charm, top): spin-1/2, Q=2/3
Down quarks (down, strange, bottom): spin-1/2, Q=-1/3

SU(2) weak symmetry that changes neutrino into electron also changes up-quark into down-quark
(to explain neutron decay)



$$\frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

This experiment counts the number of quarks and gives their electric charges.
Another remarkable feature: at high energy, the quarks behave like muons,
i.e., not sensitive to strong interactions.

Asymptotic freedom of QCD!

(consequence of non-abelian nature of strong interaction - see tomorrow lecture)

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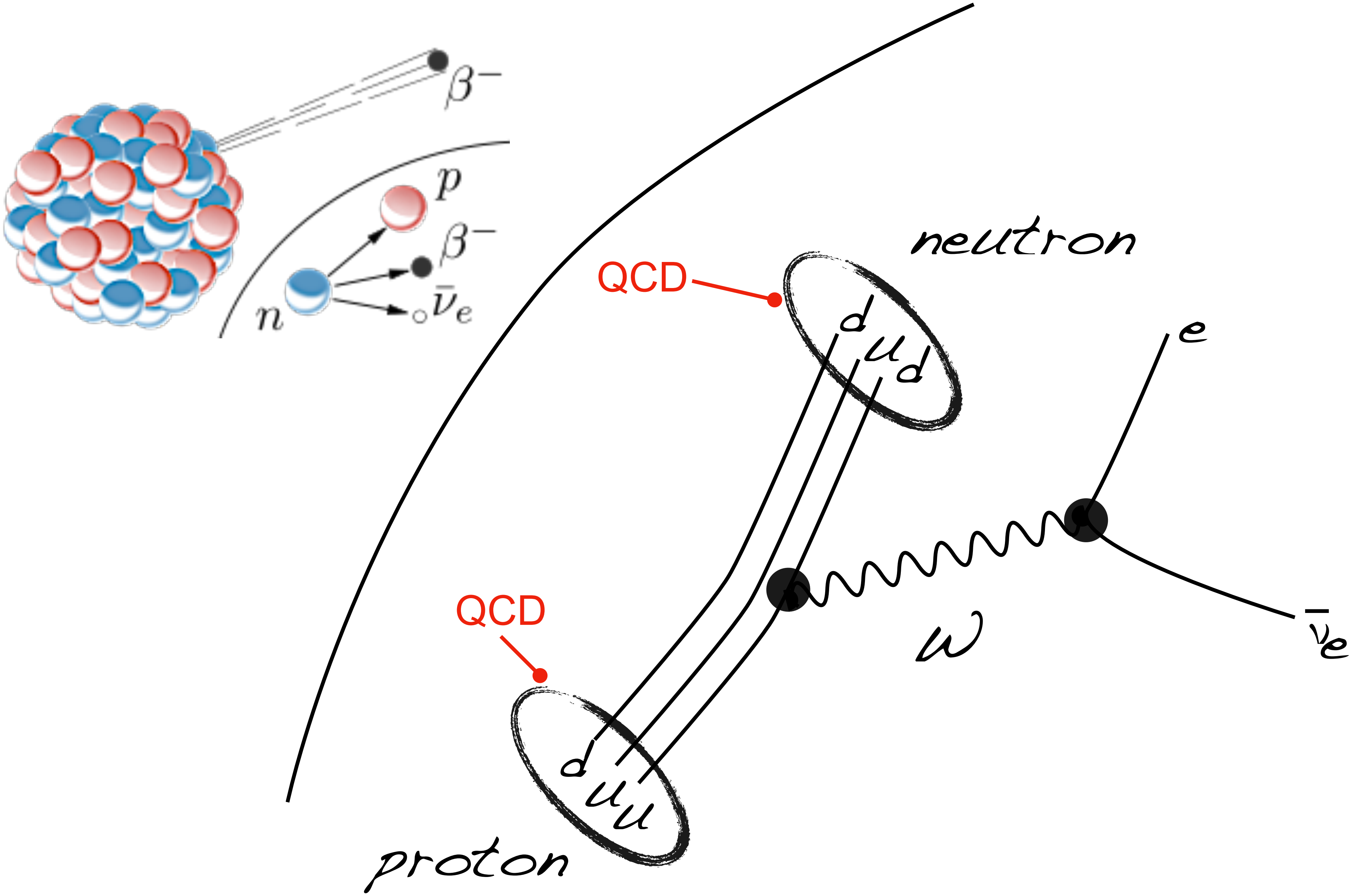

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Quarks carry yet another quantum number: "**colour**"
There are 3 possible colours and Nature is colour-blind, i.e, Lagrangian should remain the same when the colours of the quarks are changed, i.e., when we perform a rotation in the colour-space of quarks.

$$Q^a \to U^a{}_b Q^b$$

U: 3x3 matrix satisfying $U^\dagger U = 1_3$ SU(3)
such that the quark kinetic term is invariant

hadrons (spin-1/2, #hadronic=1): $p = uud$ $n = udd$

mesons (spin-0, #hadronic=0): $\pi^0 = \dfrac{u\bar{u} + d\bar{d}}{\sqrt{2}}$ $\pi^+ = u\bar{d}$ $\pi^- = d\bar{u}$

(Each quark carries a baryon number =1/3)

There are (heavier) quarks and hence other baryons and mesons

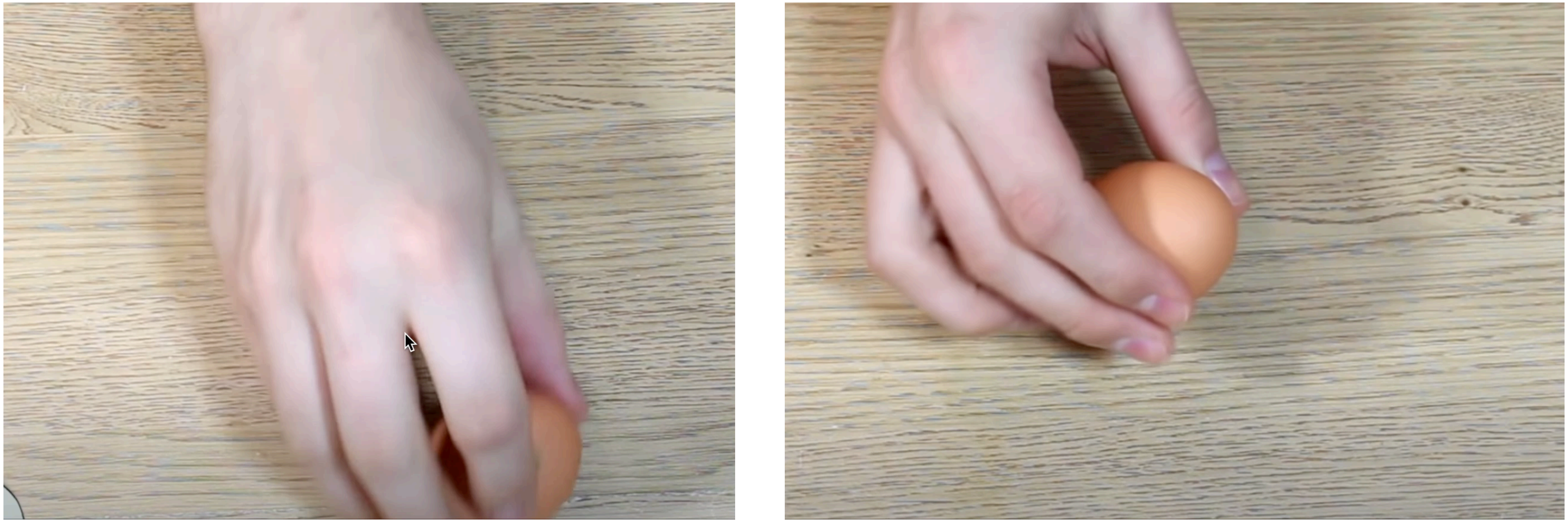
All the interactions of the SM preserve baryon and lepton numbers

$$\mu \to e \nu_\mu \bar{\nu}_e \qquad n \to p\, e\, \bar{\nu}_e \qquad \pi^- \to \mu^- \bar{\nu}_\mu \qquad \pi^0 \to \gamma\gamma \qquad p \cancel{\to} \pi^0 \bar{e}$$

Inside Hadrons

One can break matter into pieces to learn what it is made of.
But this is not always possible (not sharp enough knife, not enough energy…).
Fortunately, remember the boiled egg experiment:

https://youtu.be/r1ygKQbcqh4



The way the egg is spinning can tell if it is boiled (one piece) or raw (internal structure with different components moving independently from each others)

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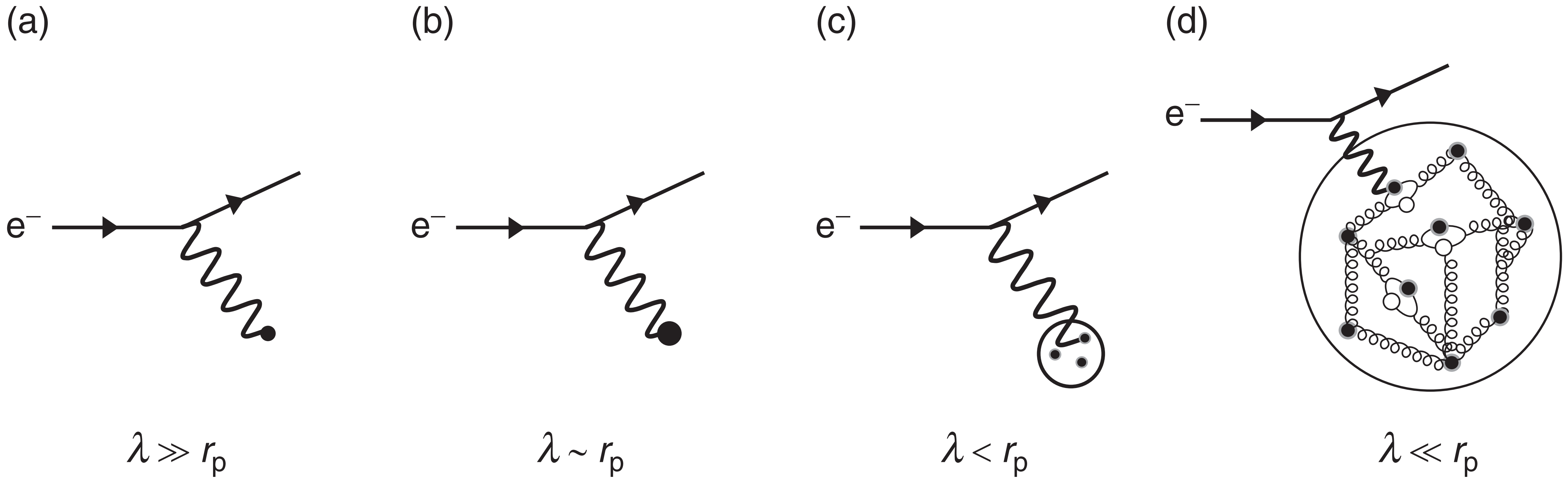


electron-proton scattering (1960's) reveals the proton intimate structure
(3 elementary spin-1/2 quarks that exist in 3 colours bounded by strong interactions that become feeble at large energies — asymptotic freedom).

The Standard Model: Interactions

- $U(1)_Y$ ***electromagnetic interactions***

  Photon γ

  light

  atoms

  molecules

- $SU(2)_L$ ***weak interactions***

  bosons $W^{\pm}, Z^0$

  β decay

  $$n \xrightarrow{W^{\pm}} p + e^- + \bar{\nu}_e$$

  $$e^+ + e^- \xrightarrow{Z^0} D^+_{(c\bar{s})} + D^-_{(\bar{c}s)}$$

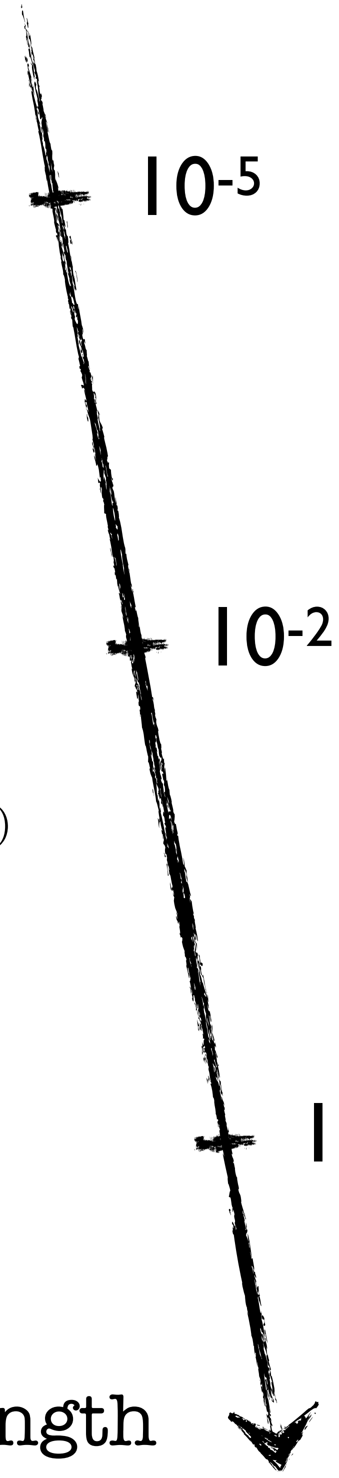
- $SU(3)_c$ ***strong interactions***

  gluons g^a

  atomic nuclei

  α decay

  $${}^{238}_{92}\mathrm{U} \longrightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He}$$



SM Summary

			color	chirality	hypercharge	weak isospin	electric charge	effective coupling to Z boson	
	SPIN	PARTICLES	$SU(3)_C$ ×	$SU(2)_L$ ×	$U(1)_Y$	T_{3_L}	$Q = T_{3_L} + Y$	g_{eff}	MEANING
LEPTONS	1/2	$L=\begin{pmatrix}\nu\\e\end{pmatrix}_L$	1	2	$\begin{pmatrix}-1/2\\-1/2\end{pmatrix}$	$\begin{pmatrix}1/2\\-1/2\end{pmatrix}$	$\begin{pmatrix}0\\-1\end{pmatrix}$	$\begin{pmatrix}1/2\\-1/2+\sin^2\theta_W\end{pmatrix}$	doublet under SU(2), singlet under SU(3)
		e_R	1	1	-1	0	-1	$\sin^2\theta_W$	singlet under SU(2) and SU(3)
QUARKS		$Q=\begin{pmatrix}u\\d\end{pmatrix}_L$	3	2	$\begin{pmatrix}1/6\\1/6\end{pmatrix}$	$\begin{pmatrix}1/2\\-1/2\end{pmatrix}$	$\begin{pmatrix}2/3\\-1/3\end{pmatrix}$	$\begin{pmatrix}1/2-\frac{2}{3}\sin^2\theta_W\\-1/2+\frac{1}{3}\sin^2\theta_W\end{pmatrix}$	doublet under SU(2), triplet under SU(3)
		u_R	3	1	2/3	0	2/3	$-\frac{1}{3}\sin^2\theta_W$	singlet under SU(2), triplet under SU(3)
		d_R	3	1	-1/3	0	-1/3	$\frac{1}{3}\sin^2\theta_W$	singlet under SU(2), triplet under SU(3)
HIGGS	0	$H=\begin{pmatrix}h^+\\h^0\end{pmatrix}$	1	2	$\begin{pmatrix}1/2\\1/2\end{pmatrix}$	$\begin{pmatrix}1/2\\-1/2\end{pmatrix}$	$\begin{pmatrix}1\\0\end{pmatrix}$	×	doublet under SU(2), singlet under SU(3)

Are we done?

$$m_e \bar{e}_L e_R + h.c.$$

is not gauge invariant

Y=1/2 Y=-1

$$m_W^2 W^+{}_\mu W^-_\nu \eta^{\mu\nu}$$

is not gauge invariant

$$A_\mu \to U A_\mu U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1}$$

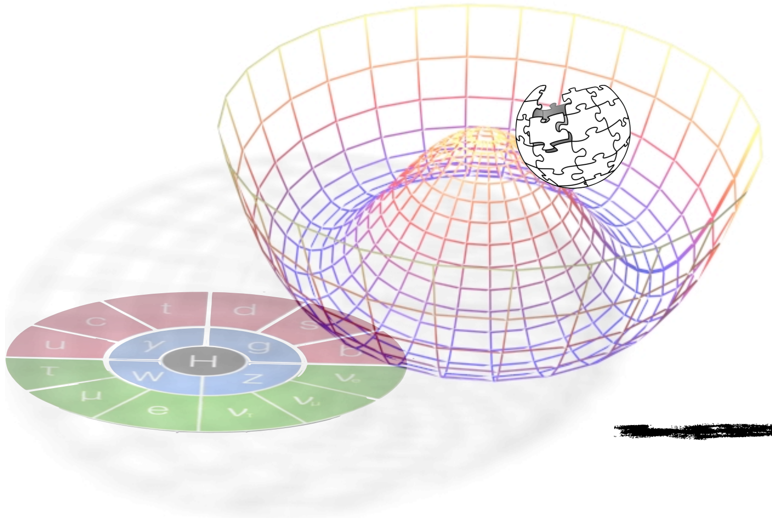
Remember May 1, 2003:

“Mission accomplished” speech by G.W. Bush.

That was certainly not the end of the story and there were (are) still a lot things to do!

Spontaneous Symmetry Breaking

Short-distance interactions ≠ Long-distance interactions
The masses are emergent due to a non-trivial structure of the vacuum

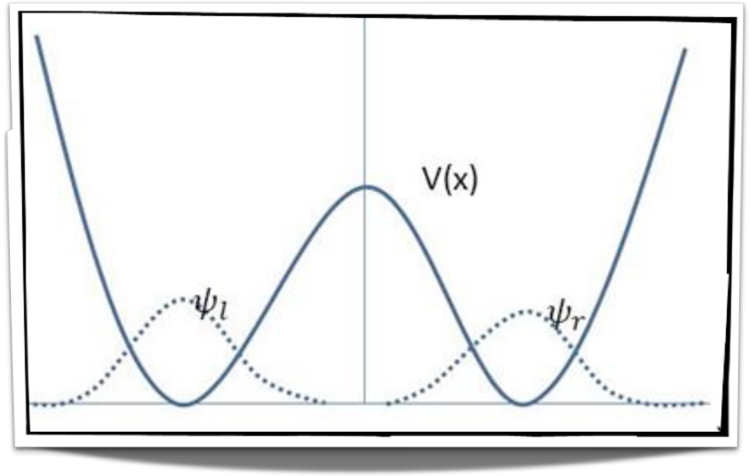


vacuum = ~~a space entirely devoid of matter~~
Oxford English

vacuum = a space filled with BEH substance
Physics English

QM vs QFT

(courtesy of J. Lykken@Aspen2014)



Ground state of QM double well potential
is a superposition of two states each localised on one minimum,
and this superposition preserves the Z_2 symmetry of the potential

In QFT, it is more difficult to transition between degenerate vacua
and spontaneous symmetry breaking can occur
(or more correctly, the symmetry is non-linearly realised in Hilbert space)

The vacuum of the SM breaks SU(2)xU(1) to $U(1)_{em}$
via the dynamics of an elementary scalar field

The Brout-Englert-Higgs Boson

(postulated in 1964 — discovered in 2012)

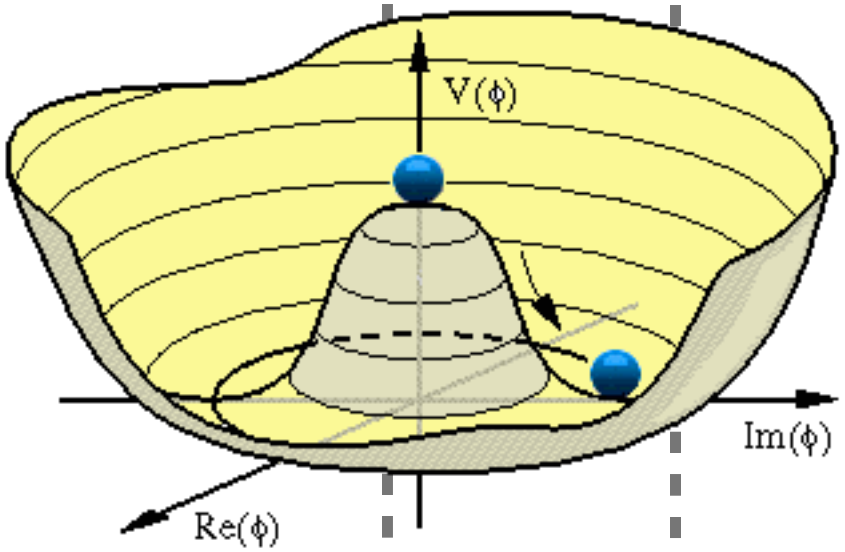
Spontaneous Symmetry

Symmetry of the Lagrangian

$$SU(2)_L \times U(1)_Y$$

Higgs Doublet

$$H_{1/2} = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}$$



Symmetry of the Vacuum

$$U(1)_{e.m.}$$

Vacuum Expectation Value

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \quad \text{with} \quad v \approx 246 \text{ GeV}$$

Most general Higgs (renormalisable) potential

$$V(H) = \lambda \left(|H|^2 - v^2/2 \right)^2$$

$v^2>0$ EW symmetry breaking, $v^2<0$ no breaking
Why Nature has decided that $v^2>0$? No dynamics explains it.

vacuum invariant under $U(1)_{EM}$

$$\delta_{SU(2)} \langle H \rangle = \frac{i}{2} \left(\theta^1 \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} + \theta^2 \begin{pmatrix} & -I \\ I & \end{pmatrix} + \theta^3 \begin{pmatrix} 1 & \\ & -1 \end{pmatrix} \right) \langle H \rangle \neq 0$$

$$\delta_Y \langle H \rangle = i\theta_Y \begin{pmatrix} 1/2 & \\ & 1/2 \end{pmatrix} \langle H \rangle \neq 0$$

$$\delta_Q \langle H \rangle = i\theta_{QED} \begin{pmatrix} 1 & \\ & 0 \end{pmatrix} \langle H \rangle = 0 \qquad \theta_{QED} = \theta_Y = \theta_3 \qquad Q = Y + T_{3L}$$

Higgs Boson

Before EW symmetry breaking

- 4 massless gauge bosons for SU(2)x(1): 4 x 2 = 8 dofs
- Complex scalar doublet: 4 dofs

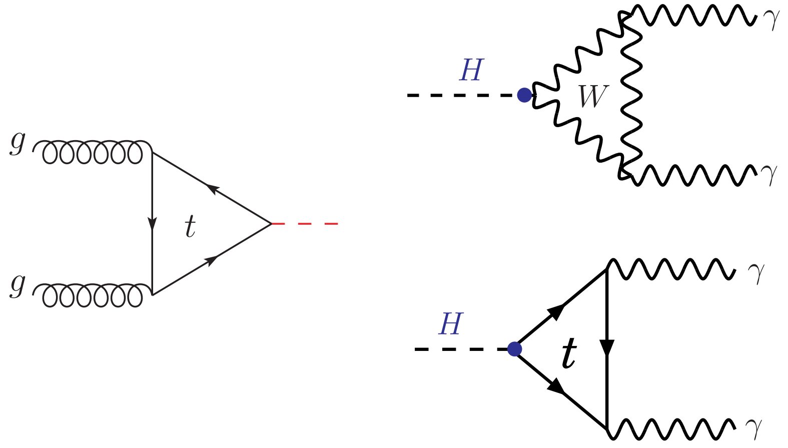
After EW symmetry breaking

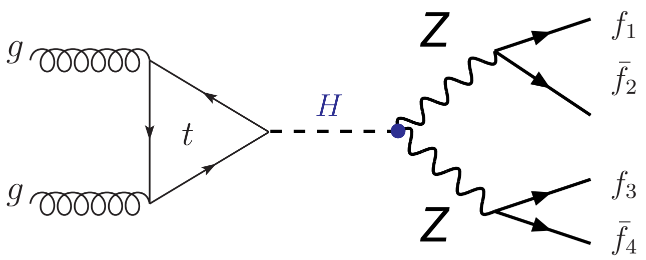
- 1 massless gauge boson, photon: 2 dofs
- 3 massive gauge bosons, $W^{\pm}$ and Z: 3 x 3 = 9 dofs
- 1 real scalar: 1 dof

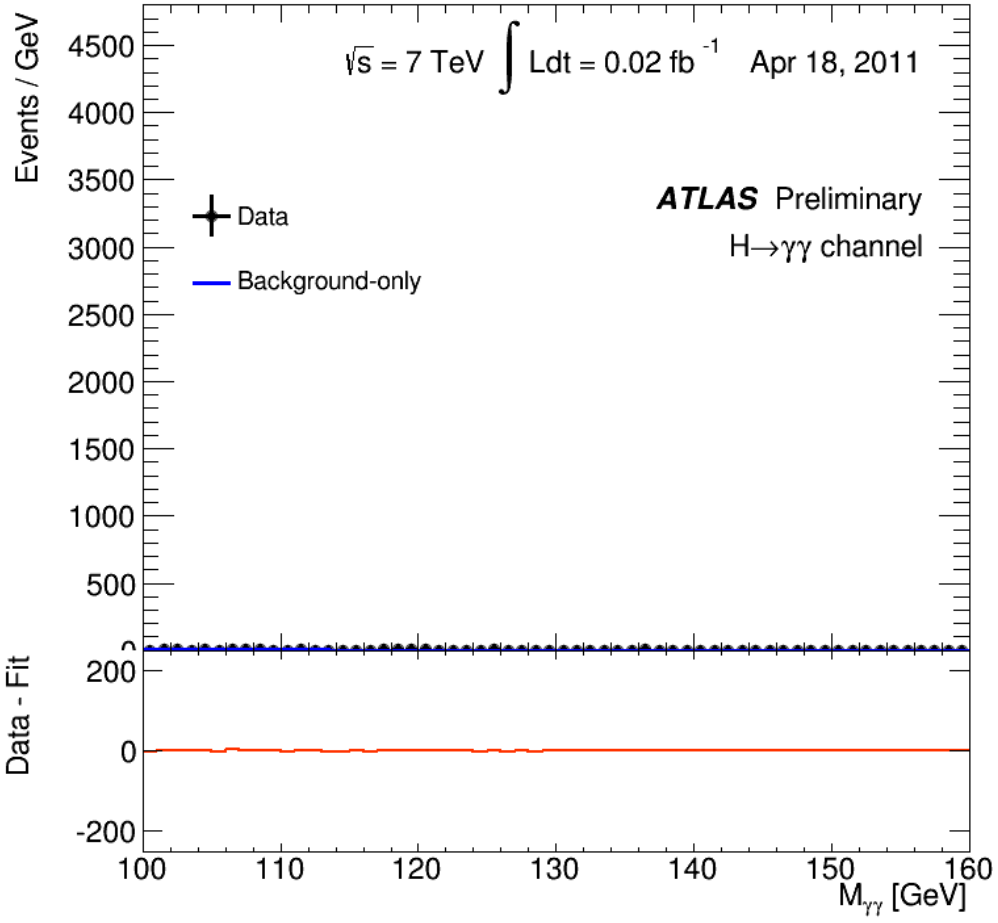
$$H = \left(\begin{array}{c} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{array} \right)$$

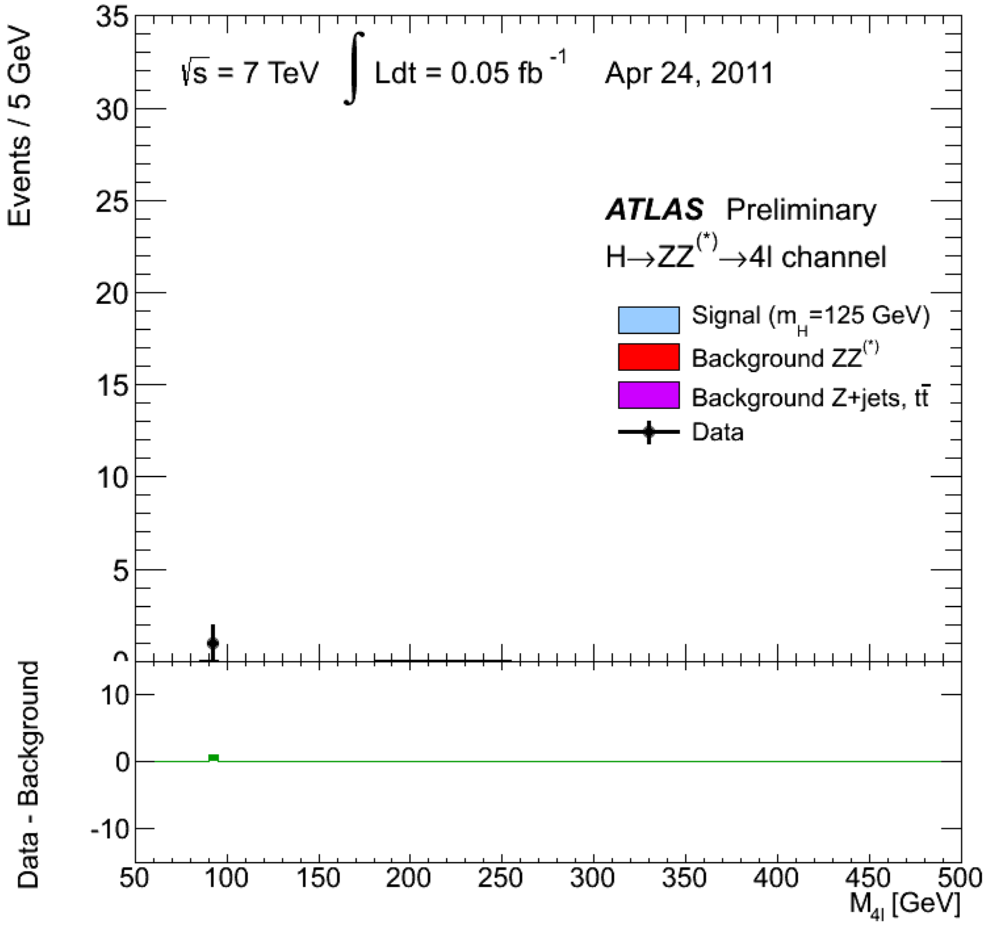
h(x) describes the Higgs boson
(the fluctuation above the VEV).
The other components of the Higgs doublet H become
the longitudinal polarisations of the $W^{\pm}$ and Z

The 2012 Scalar Discovery







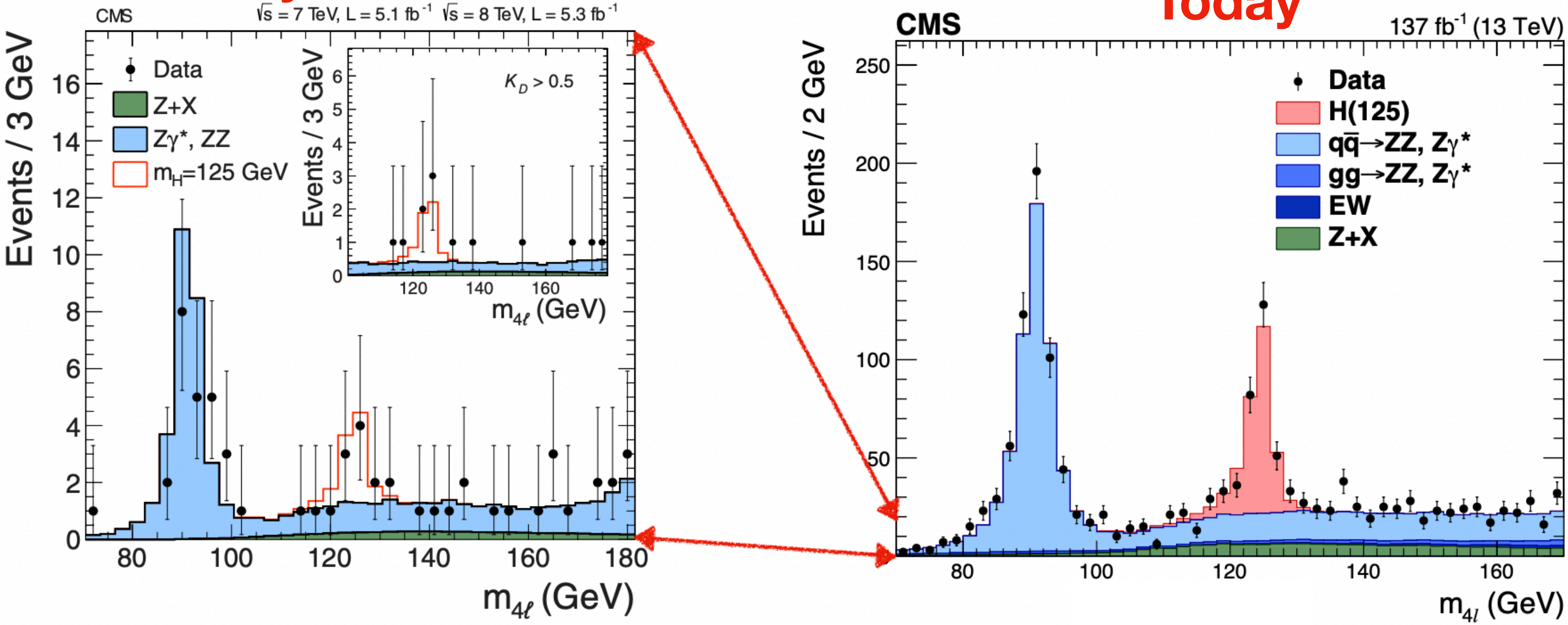


Not the most abundant BEH modes, but the “cleanest” ones

The 2012 Scalar Discovery

July 4, 2012

Today



See Paris Sphicas' talk for more details

The LHC Scalar Harvest

(8M Brout-Englert-Higgs bosons produced so far)

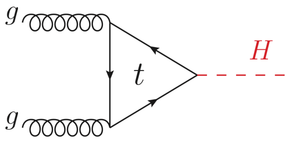
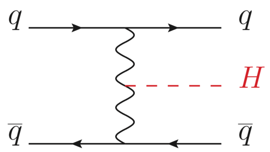
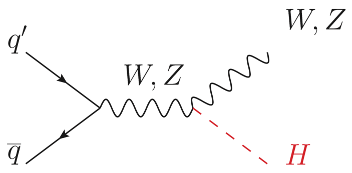
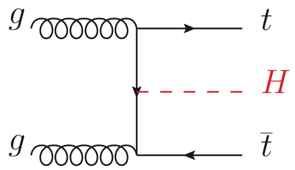
	Channel categories	Br	ggF	VBF	VH	ttH
			 ~8 M evts produced	 ~600 k evts produced	 ~400 k evts produced	 ~80 k evts produced
	Cross Section 13 TeV (8 TeV)		48.6 (21.4) pb*	3.8 (1.6) pb	2.3 (1.1) pb	0.5 (0.1) pb
Observed modes	γγ	0.2 %	✓	✓	✓	✓
	ZZ	3%	✓	✓	✓	✓
	WW	22%	✓	✓	✓	✓
	ττ	6.3 %	✓	✓	✓	✓
	bb	55%	✓	✓	✓	✓
Remaining to be observed	Zγ and γγ*	0.2 %	✓	✓	✓	✓
	μμ	0.02 %	✓	✓	✓	✓
Limits	Invisible	0.1 %	✓ (monojet)	✓	✓	✓

Table courtesy to M. Kado

Fermion Masses

SM is a **chiral** theory (≠ QED that is vector-like)

$$m_e \underset{\overset{\nearrow}{\texttt{Y=1/2}}}{\bar{e}_L} \underset{\overset{\nwarrow}{\texttt{Y=-1}}}{e_R} + h.c.$$

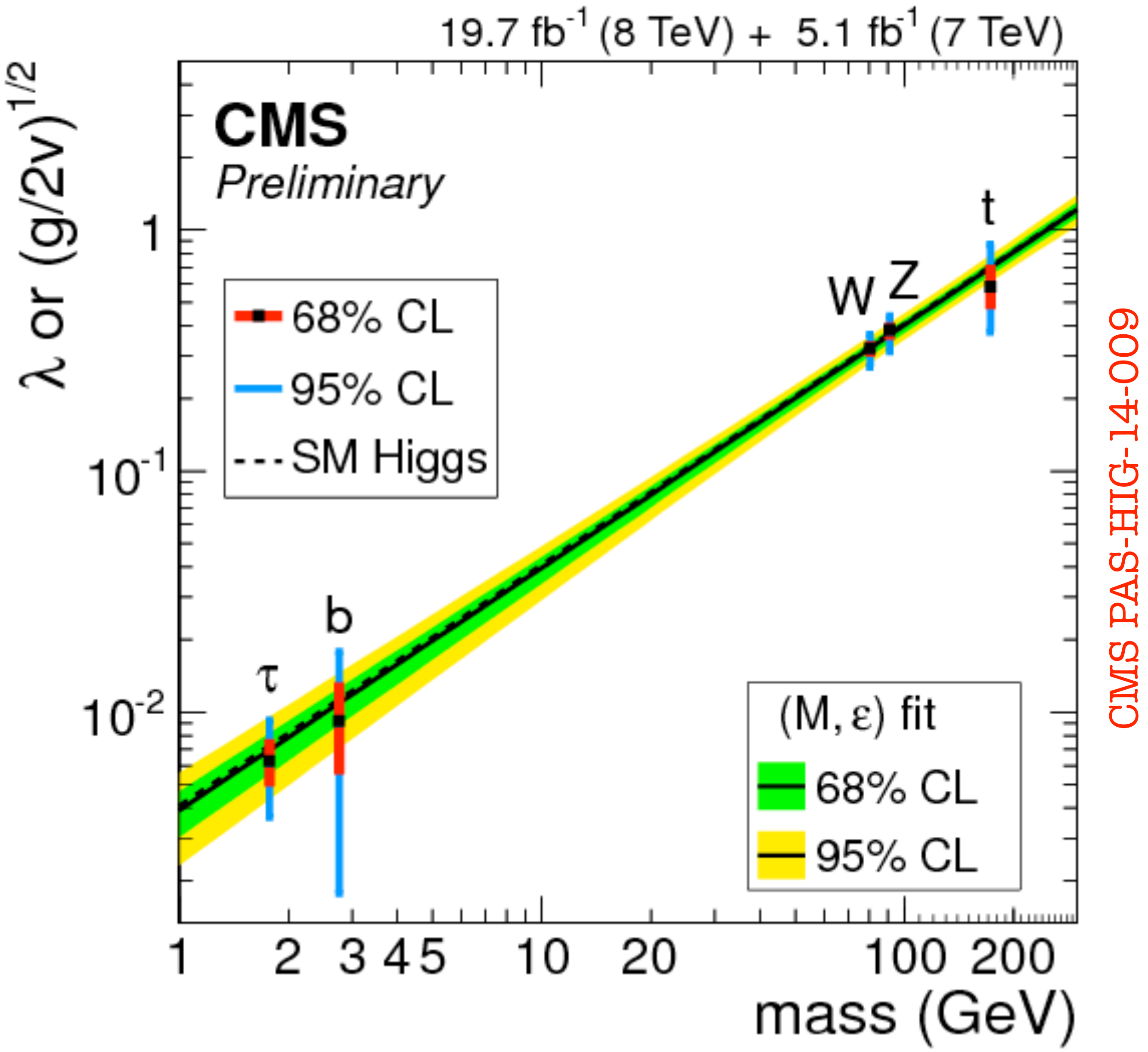
is not gauge invariant

The SM Lagrangian cannot contain fermion mass term.
Fermion masses are **emergent** quantities
that originate from **interactions with Higgs VEV**

$$\mathcal{L} = y_e \underset{\overset{\uparrow}{\texttt{Y=1/2}}}{\begin{pmatrix} \bar{\nu}_L \\ \bar{e}_L \end{pmatrix}} \cdot \underset{\overset{\uparrow}{\texttt{Y=1/2}}}{\begin{pmatrix} H^+ \\ H^0 \end{pmatrix}} \underset{\overset{\uparrow}{\texttt{Y=-1}}}{e_R} \overset{\overset{H = \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}}{\downarrow}}{=} \frac{y_e\, v}{\sqrt{2}} \left(\bar{e}_L e_R + \frac{1}{v}\, \bar{e}_L e_R \underset{\overset{|}{\text{Higgs Boson}}}{\boxed{h}} \right)$$

Higgs couplings proportional to the mass of particles

The Higgs PR plot



CMS PAS-HIG-14-009

Higgs couplings
are proportional
to the masses of the particles

$$\lambda_\psi \propto \frac{m_\psi}{v}\,, \qquad \lambda_V^2 \equiv \frac{g_{VVh}}{2v} \propto \frac{m_V^2}{v^2}$$

Fermion Masses

In SM, the Yukawa interactions are the only source of the fermion masses

$$y_{ij}\bar{f}_{L_i} H f_{R_j} = \frac{y_{ij} v}{\sqrt{2}} \bar{f}_{L_i} f_{R_j} + \frac{y_{ij}}{\sqrt{2}} h \bar{f}_{L_i} f_{R_j}$$

mass

Higgs-fermion interactions

both matrices are simultaneously diagonalisable

⇩ ⇩ ⇩

no tree-level Flavor Changing Current induced by the Higgs

Once the mass terms are diagonal, the Higgs interactions are diagonal too

Not true anymore if the SM fermions mix with vector-like partners or for non-SM Yukawa

$$y_{ij}\left(1 + c_{ij}\frac{|H|^2}{f^2}\right)\bar{f}_{L_i} H f_{R_j} = \frac{y_{ij} v}{\sqrt{2}}\left(1 + c_{ij}\frac{v^2}{2f^2}\right)\bar{f}_{L_i} f_{R_j} + \left(1 + 3c_{ij}\frac{v^2}{2f^2}\right)\frac{y_{ij}}{\sqrt{2}} h \bar{f}_{L_i} f_{R_j}$$

Look for SM forbidden Flavour Violating decays h → μτ and h → eτ

(look also at t→hc)

- weak indirect constrained by flavour data (μ→ eγ): BR<10%
- ATLAS and CMS have the sensitivity to set bounds O(1%)
- ILC/CLIC/FCC-ee can certainly do much better

Fermion Masses: Quark Mixings

In SM, the Yukawa interactions are the only source of the fermion masses

$$\mathcal{L}_{\rm Yuk} = y^U_{ij}\bar{Q}^i_L H^\star u^i_R + y^D_{ij}\bar{Q}^i_L H d^i_R$$

$$\mathcal{U}_L^\dagger \left(\frac{v}{\sqrt{2}} y^U_{ij}\right)\mathcal{U}_R = \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} \qquad \mathcal{D}_L^\dagger \left(\frac{v}{\sqrt{2}} y^D_{ij}\right)\mathcal{D}_R = \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix}$$

$$\mathcal{L}_{\rm Yuk} = (\bar{u}_L \bar{c}_L \bar{t}_L)\begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix}\begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix} + (\bar{d}_L \bar{s}_L \bar{b}_L)\begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix}\begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix}$$

$$\mathcal{L}_{\rm gauge} = \frac{e}{\sqrt{2}\sin\theta_w}\left[W^+_\mu \bar{u} V \gamma^\mu \left(\frac{1-\gamma_5}{2}\right) d + W^-_\mu \bar{d} V^\dagger \gamma^\mu \left(\frac{1-\gamma_5}{2}\right) u\right]$$

$$V = \mathcal{D}_L^\dagger \mathcal{U}_L$$

$$V_{\rm CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Note: one complex phase → CP violation

Neutrino Masses

The same construction doesn’t work for neutrinos
since in the SM there are only Left Handed neutrinos

For an uncharged particle, it is possible to write a Majorana mass
another Lorentz-invariant quadratic term in the Lagrangian
(it involves the charge-conjugate spinor, see lecture #3-technical slides)

$$\mathcal{L}_{\text{Majorana}} = m\bar{\psi}_C\,\psi = m\left(\bar{\psi}_{L_C}\psi_L + \bar{\psi}_{R_C}\psi_R\right)$$

can build such a term with LH field only!

In SM, such neutrino Majorana mass can be obtained from dim-5 operator:

$$\mathcal{L} = \frac{y_\nu}{\Lambda} \underset{\substack{\uparrow \\ \text{mass}^{3/2}}}{\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}_C} \cdot \underset{\substack{\uparrow \\ \text{mass}}}{\begin{pmatrix} H^+ \\ H^0 \end{pmatrix}} \underset{\substack{\uparrow \\ \text{mass}^{3/2}}}{\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}} \cdot \underset{\substack{\uparrow \\ \text{mass}}}{\begin{pmatrix} H^+ \\ H^0 \end{pmatrix}} = \frac{y_\nu\, v^2}{\Lambda}\nu_{LC}\nu_L$$

Seesaw: $m_\nu = \dfrac{y_\nu v^2}{\Lambda}$

Order eV
for y_ν~1 and Λ~10^{14}GeV

Note that such an operator breaks Lepton Number by 2 units

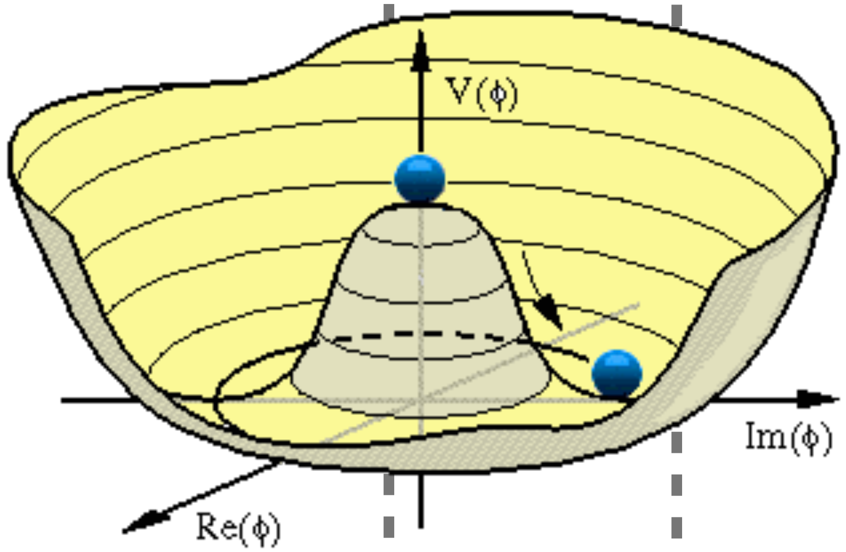
Higgs Mechanism

Symmetry of the Lagrangian

$$SU(2)_L \times U(1)_Y$$

Higgs Doublet

$$H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}$$



Symmetry of the Vacuum

$$U(1)_{e.m.}$$

Vacuum Expectation Value

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \quad \text{with} \quad v \approx 246 \text{ GeV}$$

$$D_\mu H = \partial_\mu H - \frac{i}{2} \begin{pmatrix} g W_\mu^3 + g' B_\mu & \sqrt{2} g W_\mu^+ \\ \sqrt{2} g W_\mu^- & -g W_\mu^3 + g' B_\mu \end{pmatrix} H \quad \text{with} \quad \mathrm{W}_\mu^\pm = \tfrac{1}{\sqrt{2}} \left(\mathrm{W}_\mu^1 \mp \mathrm{W}_\mu^2 \right)$$

$$|D_\mu H|^2 = \tfrac{1}{4}\, g^2 v^2\, W_\mu^+ W^{-\,\mu} + \tfrac{1}{8} \left(W_\mu^3\; B_\mu \right) \begin{pmatrix} g^2 v^2 & -g g' v^2 \\ -g g' v^2 & g'^2 v^2 \end{pmatrix} \begin{pmatrix} W^{3\,\mu} \\ B^\mu \end{pmatrix}$$

- Gauge boson spectrum
  - electrically charged bosons $\boxed{M_W^2 = \frac{1}{4} g^2 v^2}$
  - electrically neutral bosons $\boxed{\begin{aligned} Z_\mu &= c W_\mu^3 - s B_\mu \\ \gamma_\mu &= s W_\mu^3 + c B_\mu \end{aligned}}$

Weak mixing angle

$$c = \frac{g}{\sqrt{g^2 + g'^2}} \qquad s = \frac{g'}{\sqrt{g^2 + g'^2}}$$

$$\boxed{M_Z^2 = \tfrac{1}{4}(g^2 + g'^2) v^2}$$

$$\boxed{M_\gamma = 0}$$

The Brout-Englert-Higgs Boson is Special

The scalar discovery in 2012 has been an important milestone for HEP. Many of us are still excited about it. Others should be too.

BEH = **new forces** of different nature than the interactions known so far

- No underlying local symmetry
- No quantised charges
- Deeply connected to the space-time vacuum structure

The knowledge of the values of the **BEH couplings** is essential to understand the deep structure of matter/Universe

m_W, m_Z ↔ BEH couplings

↑ ✓

lifetime of stars

(why $t_{Sun} \sim t_{life\ evolution}$?)

m_e, m_u, m_d ↔ BEH couplings

?

size of atoms ↗ ↖ nuclei stability

EWSB @ $t \sim 10^{-10}$s ↔ BEH self-coupling

?

matter/anti-matter ↔ CPV in BEH sector

?

The Brout-Englert-Higgs Boson is Special

LHC will make remarkable progress
but it won’t be enough
A new collider will be needed!

The knowledge of the values of the **BEH couplings** is essential to understand the deep structure of matter/Universe

m_W, m_Z ↔ BEH couplings
✓
lifetime of stars
(why $t_{\text{Sun}} \sim t_{\text{life evolution}}$?)

m_e, m_u, m_d ↔ BEH couplings
?
size of atoms
nuclei stability

EWSB @ $t \sim 10^{-10}$s ↔ BEH self-coupling
?

matter/anti-matter ↔ CPV in BEH sector
?

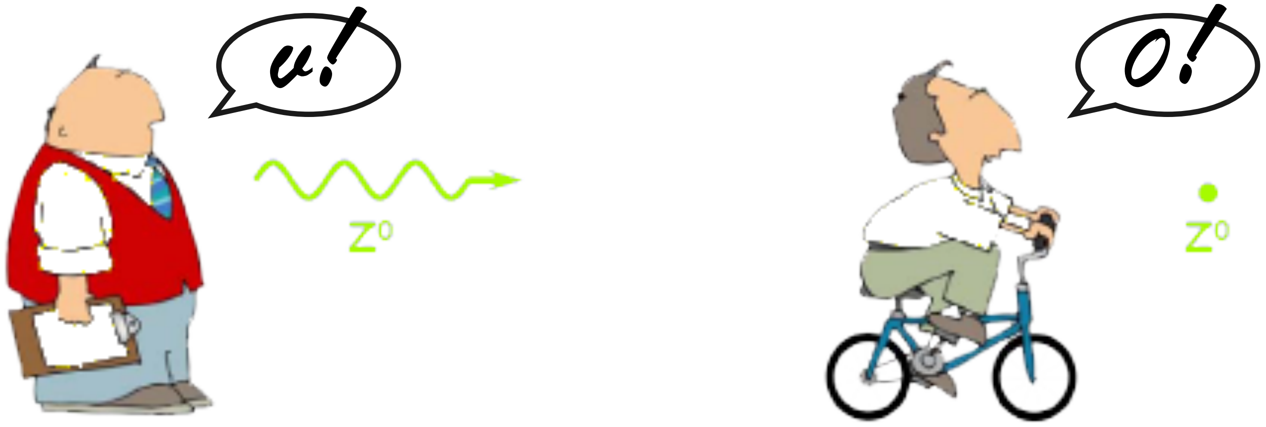
Technical Details for Advanced Students

The longitudinal polarisation of massive W, Z

a massless particle is never at rest: always possible to distinguish (and eliminate!) the longitudinal polarisation

3=2+1

Guralnik et al '64



the longitudinal polarisation is physical for a massive spin-1 particle

(pictures: courtesy of G. Giudice)

symmetry breaking: new phase with more degrees of freedom

$$\epsilon_{\parallel} = \left(\frac{|\vec{p}|}{M}, \frac{E}{M}\frac{\vec{p}}{|\vec{p}|}\right)$$ polarization vector grows with the energy

The longitudinal polarisation of massive W, Z

Indeed a massive spin 1 particle has 3 physical polarizations:

$$A_\mu = \epsilon_\mu \; e^{ik_\mu x^\mu}$$

$$\epsilon^\mu \epsilon_\mu = -1 \qquad k^\mu \epsilon_\mu = 0$$

$$k^\mu = (E, 0, 0, k)$$

with $k_\mu k^\mu = E^2 - k^2 = M^2$

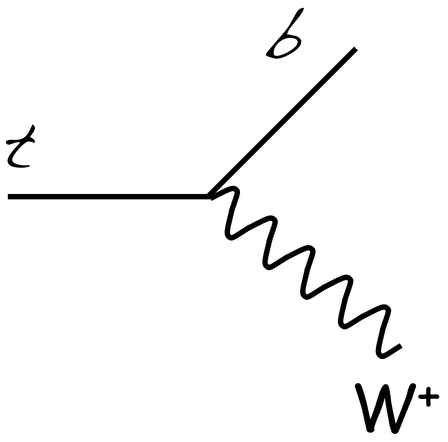
- 2 transverse: $\begin{cases} \epsilon_1^\mu = (0,1,0,0) \\ \epsilon_2^\mu = (0,0,1,0) \end{cases}$
- 1 longitudinal: $\epsilon_{\|}^\mu = (\frac{k}{M}, 0, 0, \frac{E}{M}) \approx \frac{k^\mu}{M} + \mathcal{O}(\frac{E}{M})$

(in the R-ξ gauge, the time-like polarization ($\epsilon^\mu \epsilon_\mu = 1 \quad k^\mu \epsilon_\mu = M$) is arbitrarily massive and decouple)

in the particle rest-frame, no distinction between L and T polarisations
in a frame where the particle carries a lot of kinetic energy,
the L polarisation "dominates"

The BEH mechanism: “V_L=Goldstone bosons”

At high energy, the physics of the gauge bosons becomes simple



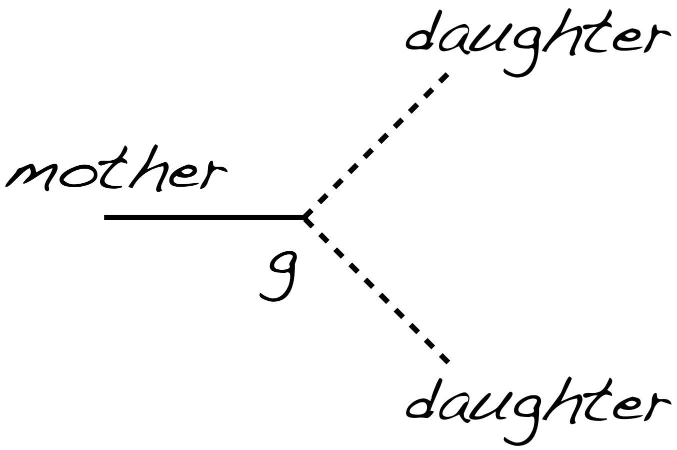
$$\Gamma(t \to bW_L) = \frac{g^2}{64\pi}\boxed{\frac{m_t^2}{m_W^2}}\frac{(m_t^2 - m_W^2)^2}{m_t^3}$$

$$\Gamma(t \to bW_T) = \frac{g^2}{64\pi}\frac{2(m_t^2 - m_W^2)^2}{m_t^3}$$

- at threshold ($m_t \sim m_W$) democratic decay
- at high energy ($m_t >> m_W$) W_L dominates the decay

At high energy, the dominant degrees of freedom are W_L

~~ why you should be stunned by this result: ~~



we expect:
(dimensional analysis)

$$\Gamma \sim g^2\, m_{\mathrm{mother}}$$

instead $\Gamma \propto m^3_{\mathrm{mother}}$ means $g \propto m$ like the Higgs couplings!

very efficient way to get energy from the mother particle $\tau \ll \tau_{\mathrm{naive}}$

Goldstone equivalence theorem

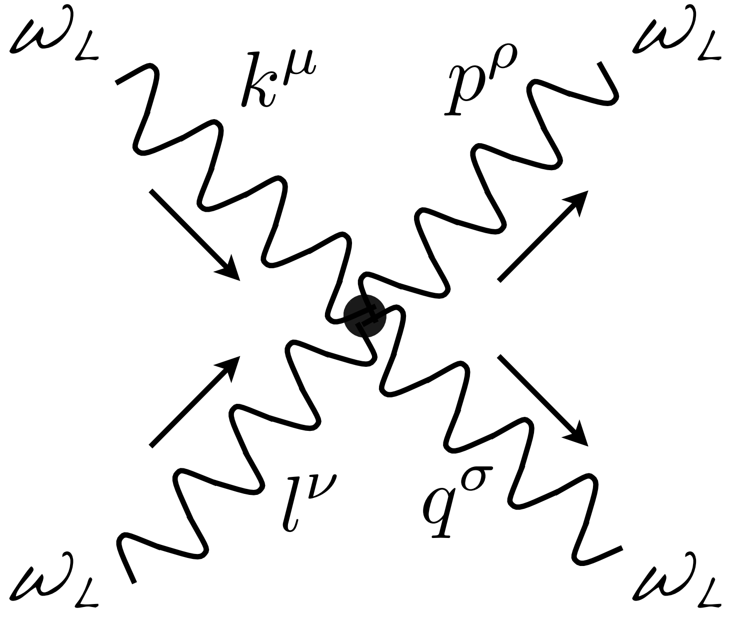
$W^{\pm}_L$, Z_L ≈ SO(4)/SO(3)

This is the physics that was understood at LEP
The pending question was then: is there something else?
That was the job of the LHC

Call for extra degrees of freedom

NO LOSE THEOREM

Bad high-energy behaviour for the scattering of the longitudinal polarisations



$$\mathcal{A} = \epsilon^\mu_\parallel(k)\epsilon^\nu_\parallel(l)g^2\left(2\eta_{\mu\rho}\eta_{\nu\sigma} - \eta_{\mu\nu}\eta_{\rho\sigma} - \eta_{\mu\sigma}\eta_{\nu\rho}\right)\epsilon^\rho_\parallel(p)\epsilon^\sigma_\parallel(q)$$

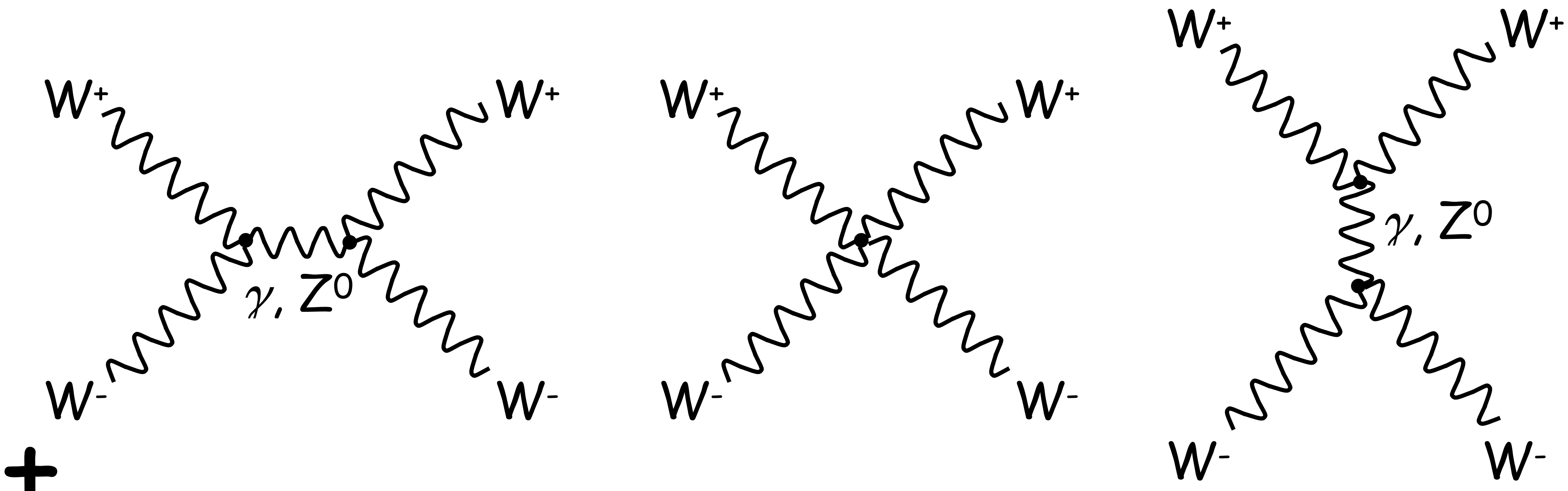
$$\boxed{\mathcal{A} = g^2\,\frac{E^4}{4M_W^4}}$$

violations of perturbative unitarity around E ~ M/√g (actually M/g)

Extra degrees of freedom are needed to have a good description of the W and Z masses at higher energies

numerically: E ~ 3 TeV ⬅ the LHC was sure to discover something!

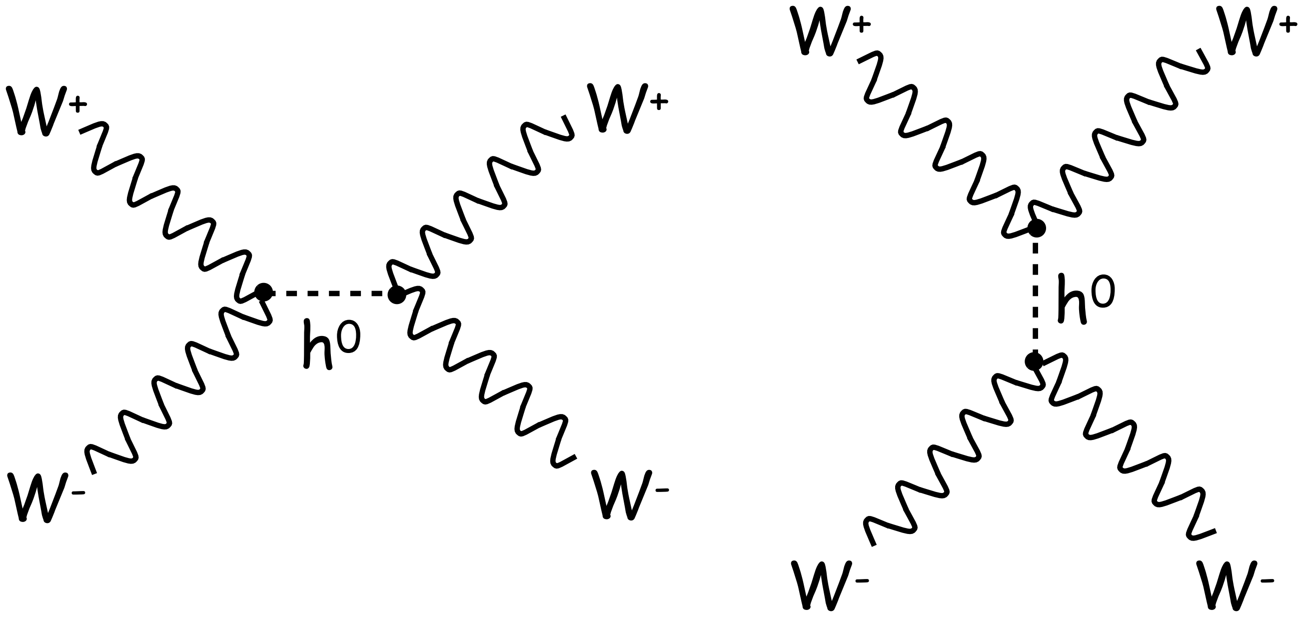
Call for extra degrees of freedom



$$\mathcal{A} = g^2 \left(\frac{E}{M_W} \right)^2$$

+

+



$$\mathcal{A} = -g^2 \left(\frac{E}{M_W} \right)^2$$

$$\mathcal{A} = g^2 \left(\frac{M_H}{2M_W} \right)^2$$

The Higgs boson unitarizes the W scattering
(if its mass is below ~ 1 TeV)

W_L scattering = pion scattering
Goldstone equivalence theorem

Lewellyn Smith '73
Dicus, Mathur '73
Cornwall, Levin, Tiktopoulos '73
Lee, Quigg, Thacker '77

What is the SM Higgs?

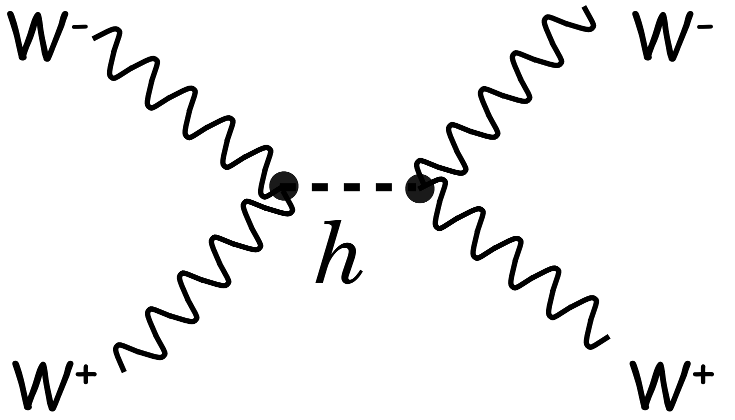
A single scalar degree of freedom that couples to the mass of the particles

$\Sigma = e^{i\pi^a\sigma^2/v}$ parametrises the coset SO(4)/SO(3)

$$\begin{tikzcd}
 & m_W^2 W_\mu^+ W_\mu^+ + \frac{1}{2} m_Z Z_\mu Z^\mu \\
\mathcal{L} = \frac{v^2}{4} \mathrm{Tr}\, D_\mu \Sigma^\dagger D^\mu \Sigma \arrow[ur, "\Sigma = \mathbb{1}"] \arrow[dr, "g = g' = 0"'] & \\
 & \frac{1}{2}(\partial\pi)^2 + \frac{1}{v^2}\partial^2\pi^4 + \dots
\end{tikzcd}$$

$$\mathcal{L}_{\mathrm{EWSB}} = m_W^2 W_\mu^+ W_\mu^+ \left(1 + 2a\frac{h}{v} + b\frac{h^2}{v^2}\right) - m_\psi \bar{\psi}_L \psi_R \left(1 + c\frac{h}{v}\right)$$

‘a’, ‘b’ and ‘c’ are arbitrary free couplings



$$\mathcal{A} = \frac{1}{v^2}\left(s - \frac{a^2 s^2}{s - m_h^2}\right)$$

growth cancelled for
a = 1
restoration of perturbative unitarity

What is the Higgs the name of?

A single scalar degree of freedom that couples to the mass of the particles

$$\mathcal{L}_{\text{EWSB}} = m_W^2 W_\mu^+ W_\mu^+ \left(1 + 2a\frac{h}{v} + b\frac{h^2}{v^2}\right) - m_\psi \bar{\psi}_L \psi_R \left(1 + c\frac{h}{v}\right)$$

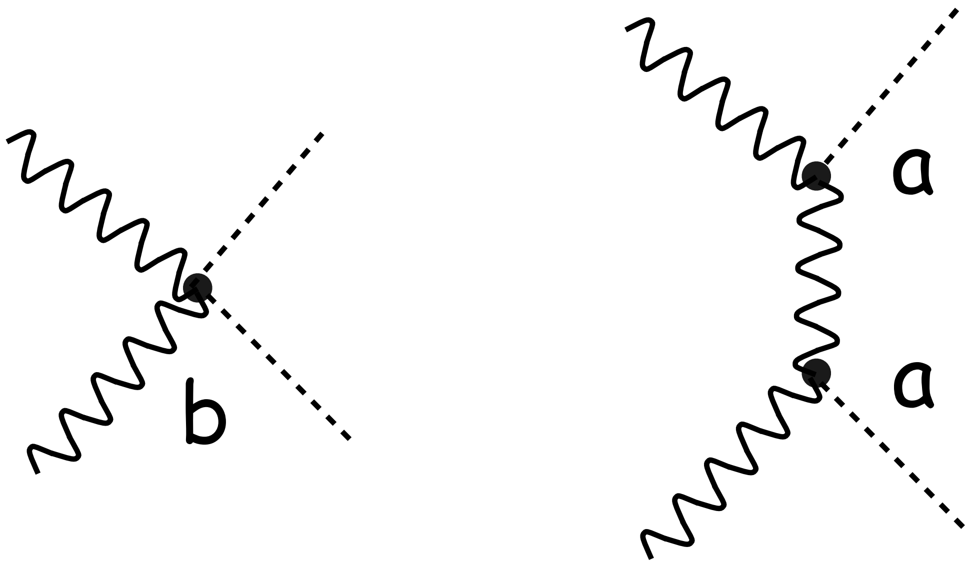
'a', 'b' and 'c' are arbitrary free couplings

For a=1: perturbative unitarity in elastic channels WW → WW

For b = a^2: perturbative unitarity in inelastic channels WW → hh

Cornwall, Levin, Tiktopoulos '73

Contino, Grojean, Moretti, Piccinini, Rattazzi '10



What is the Higgs the name of?

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For ac=1: perturbative unitarity in inelastic WW → ψ ψ

Cornwall, Levin, Tiktopoulos ’73

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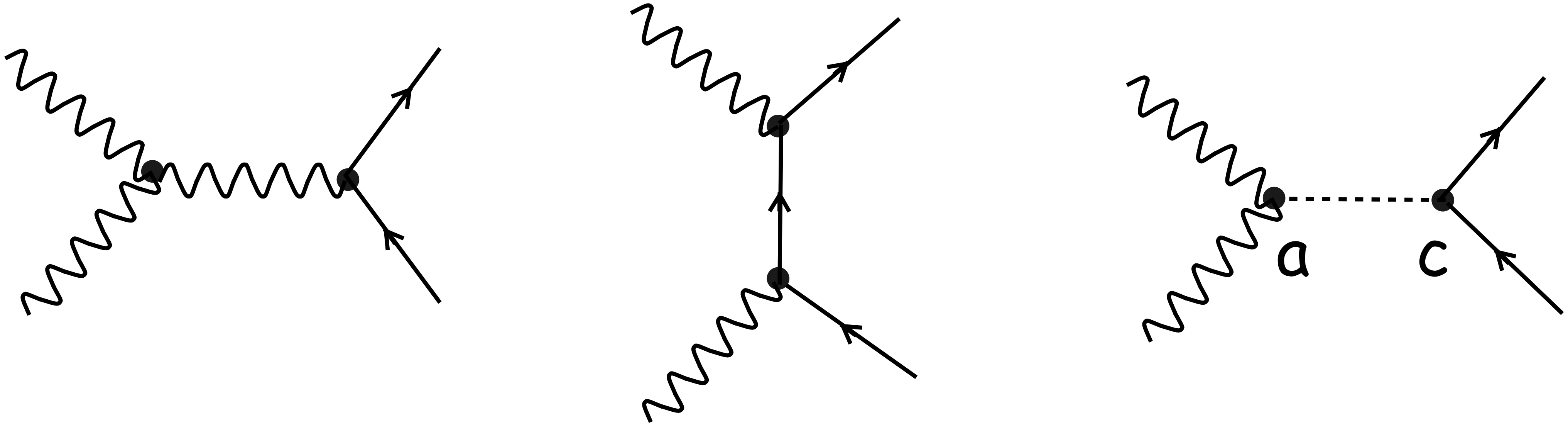

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For ac=1: perturbative unitarity in inelastic WW → *ψ ψ*

Cornwall, Levin, Tiktopoulos ’73

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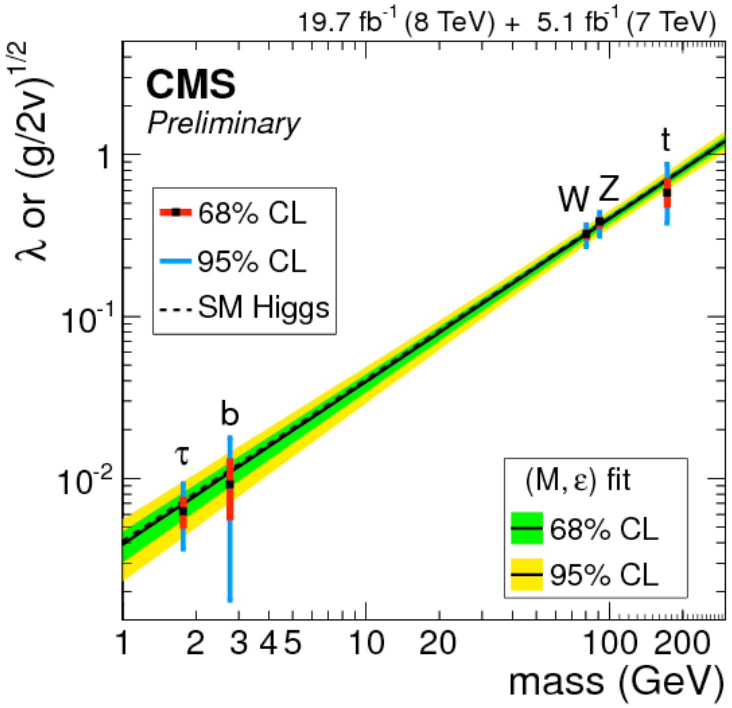

CMS PAS-HIG-14-009

Higgs couplings
are proportional
to the masses of the particles

$$\lambda_\psi \propto \frac{m_\psi}{v}, \qquad \lambda_V^2 \equiv \frac{g_{VVh}}{2v} \propto \frac{m_V^2}{v^2}$$