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Standard Model | **Homework 4** | **Date: 06.07.2023**

Feel free to send me (`christophe.grojean@desy.de`) your solutions and I'll give you feedback.

Exercice 1: Standard Model: interactions and conservation laws

Are the following decays permitted in the Standard Model? If not, why?

1. $n \to p\mu^- \bar{\nu}_\mu$
2. $\mu^- \to e^- e^- e^+$
3. $n \to p\nu_e \bar{\nu}_e$
4. $p \to e^+ \pi^0$
5. $\pi^0 \to \gamma\gamma$
6. $\tau^- \to \mu^- \gamma$
7. $K^0 \to \mu^+ e^-$ (the kaon, K^0, is a $d\bar{s}$ meson)
8. $\mu^- \to \pi^- \nu_\mu$
9. $\mu \to e\gamma$
10. $\mu \to e\nu_e \bar{\nu}_\mu$

Exercice 2: BSM proton decay

With the particle content of the SM, baryon number is an accidental symmetry when restricting to renormalisable (dimension-4) interactions. One can however build dimension-6 four-fermion interactions among quarks and leptons that do break the baryon and lepton numbers. On dimension ground, derive an estimate of the proton life-time as a function of the Fermiconstant of these interactions. Given that the current experimental lower bound on the life-time of the proton in 10^{34} years, find the lower bound on the scale governing these contact interactions.

Exercice 3: Bound of hypothetical scalar leptoquark

A leptoquark is a hypothetical bosonic field which transforms as a triplet of $SU(3)_C$, and consequently (with appropriate $SU(2)_L \times U(1)_Y$ quantum numbers) can couple to quark-lepton pairs. In this exercise, we add to the SM a single scalar field, F, that is a doublet of $SU(2)_L$ and has a hypercharge Y_F.

1) We require that the following Yukawa couplings are allowed: $\lambda^{QeF} \bar{Q}_L e_R F$. Determine Y_F and find the baryon and lepton numbers of F.

2) We denote the components of the F-doublet as (F_u, F_d). What are the electric charges of F_u and F_d?

3) The model is ruled out if $\langle F \rangle \neq 0$. Explain why this is the case. In what follows, we then assume that $\langle F \rangle = 0$.

4) Write explicitly, in the quark mass basis, the Yukawa interactions of F_u and F_d introduced in **1)**. Denote the Yukawa matrices by $\lambda_{ij}^{Fu\ell}$ and $\lambda_{ij}^{Fd\ell}$ respectively (i, j are the flavour indices).

5) The Higgs vacuum expectation value introduces a splitting between the masses of F_u and F_d. Write the mass-squared terms for F and the couplings to the Higgs field. Calculate the masses-squared of F_u and F_d, and explicitly write the mass-squared splitting. Note that there are two independent ways to contract the $SU(2)_L$ indices in the terms that involve the Higgs and the F fields. Makre sure you include both of them.

From this point on, assume that the splitting between F_u and F_d is negligible.
6) In the Standard Model, the decay $b \to s\mu^+e^-$ is forbidden. Explain why.
7) F mediates the decay $b \to s\mu^+e^-$. Draw the tree-level Feynman diagram for this decay and estimate the amplitude.
8) Estimate the F-mediated amplitude $b \to s\mu^-e^+$.

Next, we derive a lower bound on m_F. To do so, we compare the rate of the F-mediated $b \to s\mu^+e^-$ decay rate to that of the W-mediated $b \to ce^-\bar{\nu}_e$.
9) Draw the tree-level diagram for $b \to ce^-\bar{\nu}_e$.
10) Estimate the ratio $\Gamma(b \to s\mu^+e^-)/\Gamma(b \to ce^-\bar{\nu}_e)$ in terms of $\lambda^{Fd\ell}, m_F, g, m_W$ and the CKM matrix elements. Assume that $m_F \gg m_b$ and neglect phase space effects. Make sure you write explicitly the flavour structure of the couplings.
11) Assuming that $\lambda^{QeF}_{ij} \sim g$ for all i and j, and using the experimental data: $\text{Br}(b \to ce^-\bar{\nu}_e) \sim 10^{-1}$, $|V^{\text{CKM}}_{cb}| \sim 0.04$ and $\text{Br}(b \to s\mu^+e^-) < 10^{-5}$, estimate the lower bound on m_F.