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Standard Model | **Homework 5** | **Date: 07.07.2023**

Feel free to send me (`christophe.grojean@desy.de`) your solutions and I'll give you feedback.

Exercice 1: Multiple choice questions

Give the correct answer(s) (more than one answer can be correct and it could also be that none of the solution proposed is correct). Remember that $\hbar c \sim 200\,\mathrm{MeV}\cdot\mathrm{fm}$.

1) A certain radioactive element has a half-life of 20 days. The time it will take for 7/8 of the atoms originally present to decay is

1) 20 days 2) 40 days 3) 60 days 4) it cannot be predicted

2) A particle moves in such a way that its kinetic energy just equals its rest energy. The velocity of this particle is

1) $c/4$ 2) $c/2$ 3) $0.87c$ 4) c 5) it can never happen

3) An antiproton is a particle that has

1) the mass of a proton and the charge of an electron
2) the mass of an electron and the charge of a proton
3) the mass of a neutron and the charge of a proton
4) the mass of a proton and the charge of a neutron

4) Which of the following statements are correct (according to the Standard Model):

1) a hadron is always a baryon
2) a baryon is always a hadron
3) an unstable baryon can decay into a meson, leaving no other baryon

5) When the β-decay of ^{60}Co nuclei is observed at low temperatures in a magnetic field that aligns the spins of the nuclei, it is found that the electrons are emitted preferentially in a direction opposite to the ^{60}Co spin. Which of the following invariance is violated by this decay?

1) Gauge invariance
2) Time invariance
3) Translational invariance
4) Rotational invariance
5) Reflection invariance
6) Energy conservation

6) The wave function for identical fermions is anti-symmetric under particle interchange. Which of the following is a consequence of this property?

1) Pauli exclusion principle
2) Bohr correspondence principle
3) Heisenberg uncertainty principle
4) Fermi's golden rule
5) Fermi–Dirac statistics

6) Energy conservation

7) The primary decay mode for the negative pion is $\pi^- \to \mu^- + \bar{\nu}_\mu$. Given that $m_{\pi^-} = 139.6\,\text{MeV}$, $m_\mu = 105.7\,\text{MeV}$ and $m_{\bar{\nu}_\mu} \sim 0$, the kinetic energy of muon in the pion rest frame is:

1) $E_\mu = 4.1\,\text{MeV}$ 2) $E_\mu = 29.8\,\text{MeV}$ 3) $E_\mu = 33.9\,\text{MeV}$

4) it can never be determined but it varies between 0 and 33.9 MeV .

8) The neutral pion, π^0, of mass 135 MeV and of lifetime 0.8×10^{-16} s, decays mostly (with a branching ratio of 98.8%) to two photons. The uncertainty in the energy released in this decay is

1) 67.5 MeV 2) 1 MeV 3) 10 eV 4) 0.1 eV

9) Which of the following processes are forbidden (the valence-quark contain of the baryons and mesons is indicated in parentheses)?

1) $\mu^- \to e^+ e^- e^+$

2) $\Delta^+(uud) \to n(udd)\, \pi^+(u\bar{d})$ ($m_{\Delta^+} = 1210\,\text{MeV}$, $m_n = 940\,\text{MeV}$ and $m_{\pi^+} = 140\,\text{MeV}$)

3) $\Sigma^0(uds) \to \Lambda(uds)\, \gamma$ ($m_{\Sigma^0} = 1193\,\text{MeV}$ and $m_\Lambda = 1116\,\text{MeV}$).

4) $\gamma \to e^+ e^-$.

10) What is the correct ratio of the decay widths of the neutral pion $\pi^0 \to \gamma\gamma : \pi^0 \to \gamma e^+ e^- : \pi^0 \to e^+ e^- e^+ e^- : \pi^0 \to e^+ e^-$:

1) $1 : 10^{-2} : 10^{-4} : 10^{-7}$

2) $10^{-2} : 10^{-4} : 10^{-6} : 1$

3) $0 : 10^{-2} : 10^{-4} : 1$

4) $1 : 10^{-2} : 10^{-4} : 0$

Exercice 2: Mirror world and dark photon

We consider the following model:

- the local symmetry group is $\text{U}(1)_{\text{EM}} \times \text{U}(1)_{\text{D}}$. We denote the corresponding gauge bosons by A_μ and C_μ, and their field strengths by $F_{\mu\nu}$ and $C_{\mu\nu}$.
- There are four Weyl spin-1/2 fields:

$$e_L(-1,0), \quad e_R(-1,0), \quad d_L(0,-1), \quad d_R(0,-1),$$

  where the first number in the parenthesis is the charge under $\text{U}(1)_{\text{EM}}$ and the second is the charge under $\text{U}(1)_{\text{D}}$.
- There is a single complex scalar:

$$\phi(q_{\text{EM}}, q_{\text{D}}).$$

1) Write down the covariant derivative D_μ for the four fermion fields (use a normalisation such that the coupling constants of the two groups are the same, $g_{\text{EM}} = g_{\text{D}} = e$.

2) Consider the term $C_{\mu\nu}F^{\mu\nu}$, called kinetic mixing term. Argue that this term is gauge invariant, Lorentz invariant and has mass dimension 4. Would such a term exist if the two gauge symmetries were non-Abelian? For the rest of the exercise, we'll assume that this kinetic mixing term is absent.

3) There are five specific charge assignments that allow Yukawa interactions, i.e. couplings

of mass dimension 4 between ϕ and the fermions. What are these assignments? *Hint: think of using the charge conjugate fermions.*
4) For this question, we assume that ϕ has some charges $(-1, 1)$. Show that the process $e\bar{e} \to d\bar{d}$ is now allowed. Draw a tree-level Feynman diagram for this process. Assuming that m_ϕ, the mass of the scalar is much larger that E, the centre of mass energy of the incoming electron-positron, how does $\sigma(e\bar{e} \to d\bar{d})$ scale with m_ϕ?
5) Consider for this question a new Dirac fermion b of charges $(0, -2)$ in addition to $\phi(-1, 1)$. Argue that $e\bar{e} \to b\bar{b}$ does not occur at tree-level. Draw instead a one-loop diagram.

We now assume different charges for the scalar field ϕ such that all Yukawa interactions are forbidden.
6) Write the scalar potential (limiting yourself to interactions of mass dimension at most 4). What is the condition for ϕ to acquire a vacuum expectation value $\langle\phi\rangle$?
7) One way to make the model possibly consistent with Nature is to have partial spontaneous symmetry breaking, such that the photon A_μ is massless but the dark photon C_μ is massive. What are the conditions on q_{EM}, and q_{D} (the U(1)'s charges of the scalar ϕ) for this to happen? Compute the mass of the dark photon in terms of the model parameters.
8) Consider a case where both $q_{\text{EM}} \neq 0$, and $q_{\text{D}} \neq 0$. What is the symmetry breaking pattern when $\langle\phi\rangle \neq 0$? We denote the massless gauge boson A'_μ and the massive one, C'_μ. Find these mass-eigenstate vectors in terms of the original gauge bosons A_μ and C_μ.
9) Derive the couplings of the fermions to A'_μ and C'_μ.
10) We now assume that $q_{\text{EM}} \ll q_{\text{D}}$. In this case, we can think of A'_μ as a small deviation from A_μ, and still call it the photon. We further assume that $m_d \sim m_e$. Experimentally, a particle with a mass of order the electron mass and with EM charge larger than about 10^{-3} that of the electron, is ruled out. Obtain the resulting constraint on $q_{\text{EM}}/q_{\text{D}}$.

Exercice 3: Colour Higgs, B, L breakings, $B-L$ conservation

In the Standard Model (SM), all the interactions at the classical level respect both a baryon and lepton number global symmetry. It is an *accident* due to the particular gauge symmetry transformations of the different particles. This feature usually does not survive in presence of new physics. In this exercise, we can consider a model where, in addition to the SM quarks and leptons, we introduce a set of scalars transforming as a doublet under the weak $SU(2)_L$ gauge symmetry and as a triplet under the colour $SU(3)_C$ symmetry: h^i_a ($i = 1, 2, a = 1, 2, 3$).
1) Give a table with the different transformation properties of all the quarks and leptons under the SM gauge symmetries.
2) Find the four Yukawa interactions (i.e. the interactions of the scalar h with two fermions) that are invariant under $SU(2)_L$ and $SU(3)_C$ and of course also invariant under Lorentz symmetry (in some cases, you'll need to use the charge conjugated quarks). You'll write explicitly the structure of the colour and weak charges.
3) In each case, determine the hypercharge of the scalar h to ensure that the Yukawa interactions are also invariant under the hypercharge $U(1)_Y$ gauge symmetry. For the rest of the exercise, you'll consider the hypercharge charge assignment such that at least two Yukawa interactions can be present simultaneously.
4) With the two Yukawa interactions turned on, show that it is *not* possible to define a ba-

ryon and a lepton numbers that would be both conserved (remember that by definition, the lepton number of all SM leptons is one, -1 for the antileptons, while the baryon number of the SM quarks is 1/3 and -1/3 for the antiquarks). However, prove that one can define a $B-L$ quantum number that is a good global symmetry of the model. This is a welcome feature linked to charge neutrality of the hydrogen atom.

5) Phenomenologically, is it viable for h to acquire a vacuum expectation value?