CERN Summer Student Lectures 2023

– Cosmology –

Plan:

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I) the expanding Universe

II) BBN + CMB

III) inflation

Literature:

- The early Universe; Kolb, Turner (1990)
- Cosmology in gauge field theory and string theory
  Bailin, Love (2004)
- Inflation, TASI Lectures, Baumann 0907.5424

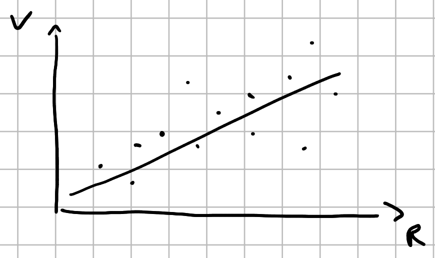
Units

$\hbar = c = k_B = 1$

$(8\pi G)^{-1} = M_p^2$ (sometimes $M_p = 1$)

I The expanding Universe

Hubble, 1929, velocity - distance relation of galaxies:



Hubble const ~ 70 $\frac{km}{s\,Mpc}$

$$V = \underline{H_0} \cdot R$$

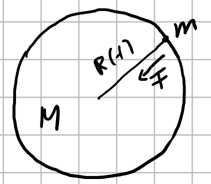
scale factor

expansion of space: $R(t) = \underline{a(t)}R_0$

$$\hookrightarrow V = \dot R = \frac{\dot a}{a} R \equiv H \cdot R$$

1) Friedmann equations: evolution of a(t)

$\rho(t)$



spherical region in homogeneous universe

$$F = -\frac{GMm}{R^2} = m\ddot R$$

$\int \ddot R\, dR = \int \frac{d\dot R}{dt} dR = \int \dot R\, d\dot R = \frac{1}{2}\dot R^2$

$ma \to \frac{1}{2}mv^2$

$\rightarrow$ energy conservation

$$\frac{1}{2}\dot R^2 - \frac{GM}{R(t)} = U = \text{const.}$$

$$M = \frac{4\pi}{3}R^3\rho(t), \quad R(t) = a(t)R_0$$

$$\rightarrow \boxed{\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{2U}{R_0^2}\frac{1}{a^2}}$$

Friedmann eq. links $a(t) \leftrightarrow \rho(t)$

> 0

$\to 0$ for $a \to \infty$

$\rho = 0$ limit

	$U > 0$	expands forever
	$U < 0$	stops and re-collapses
observations $\longrightarrow$	$U = 0$	limiting case, 'flat' universe

from now on: $U = 0$.

Homogeneous universe ~ modelled by fluid:

$$\frac{dE}{dt} = -p\frac{dV}{dt} \qquad \text{1st law of thermodynamics}$$

$$E = \rho V, \quad V \propto a^3$$

$\dot\rho V + \rho\frac{3\dot a}{a}V = -P\frac{3\dot a}{a}V$

$\hookrightarrow \frac{d\rho}{dt} = -3\frac{\dot a}{a}(\rho+P)$

$$\rightarrow \boxed{\frac{d\rho}{dt} = -3\frac{\dot a}{a}(\rho + P)}$$

continuity equation

$$\frac{d\rho}{dt} = -3\underbrace{\frac{\dot a}{a}}_{H}(\rho+P) \underset{U=0}{=} \frac{3}{8\pi G}\frac{d}{dt}\left(\frac{\dot a}{a}\right)^2 = \frac{3\cdot 2}{8\pi G}\underbrace{\frac{a\ddot a - \dot a^2}{a^2}}_{\ddot a/a - H^2}$$

$$\rightsquigarrow \text{(F2)}: (8\pi G)^{-1}\frac{\ddot a}{a} = -\frac{1}{6}H(\rho + 3p)$$

┌ from GR: $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$

metric: $ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega\right)$

matter / perfect fluid
$T_{\mu\nu} = \mathrm{diag}(\rho, -p, -p, -p)$

→ Friedmann eq, continuity eq. ┘

2) epochs of cosmological history

cosmological fluids are described by constant eq. of state

$$w \equiv p/\rho \quad \xrightarrow[\text{c.e.}]{} \quad \rho = a^{-3(1+w)}$$

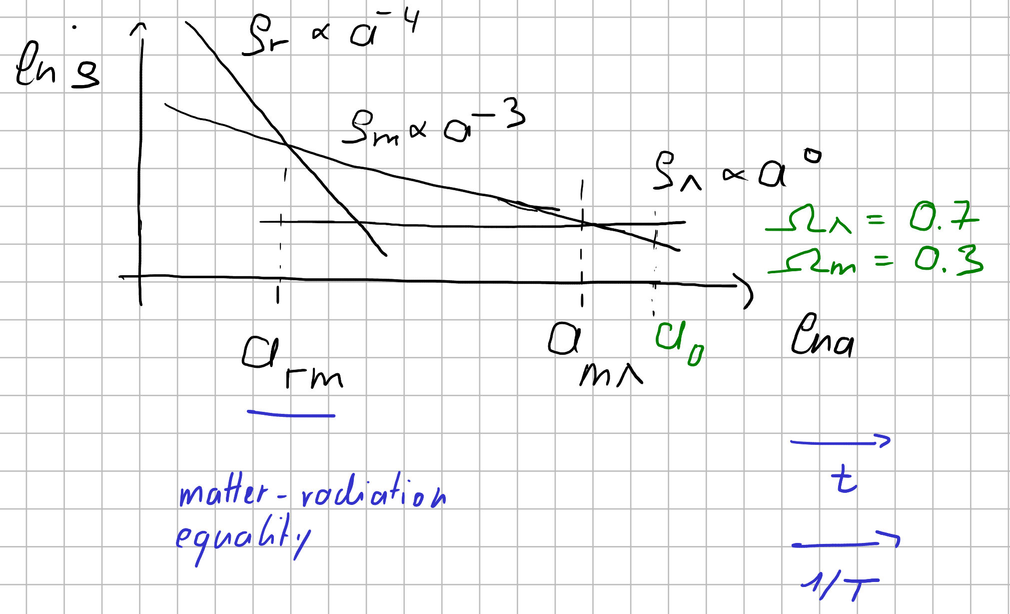
e.g. * gas of relativistic particles:

$$\rho = \underbrace{(N_B + \tfrac{7}{8}N_F)}_{g_*} \frac{\pi^2 T^4}{30}, \quad p = g_* \frac{\pi^2 T^4}{90}, \quad s = g_* \frac{2\pi^2 T^3}{45}$$

$$\to w = 1/3 \quad \to \quad \rho = a^{-4} \quad \text{(redshift + volume)}$$

* non-relativstic particles: $p = 0 \to w = 0 \to \rho = a^{-3}$
* cosmological constant: $w = -1$, $p = -\rho = \text{const.}$

⇒



matter-radiation equality

matter-radiation equality:

$$\rho_m(T) = \overbrace{3M_p^2H_0^2}^{\rho_0}\;\Omega_m\left(\frac{T}{T_0}\right)^3$$

$T_0 = 2.7\,K$
CMB-Temp.
$= 2\cdot 10^{-4}\,eV$

$$\rho_r(T) = \frac{\pi^2}{30}\, g_{*,eff}\, T^4$$

$$2 + \frac{7}{8}\cdot 6\cdot\left(\frac{4}{11}\right)^{4/3} \simeq 3.36$$

photons   neutrinos

T_γ-reheating after ν-decoupling due to $e^+e^- \to \gamma\gamma$

$g_*(\gamma, e^\pm) = 2 + \frac{7}{2} = \frac{11}{2}$

$g_*(\gamma) = 2$

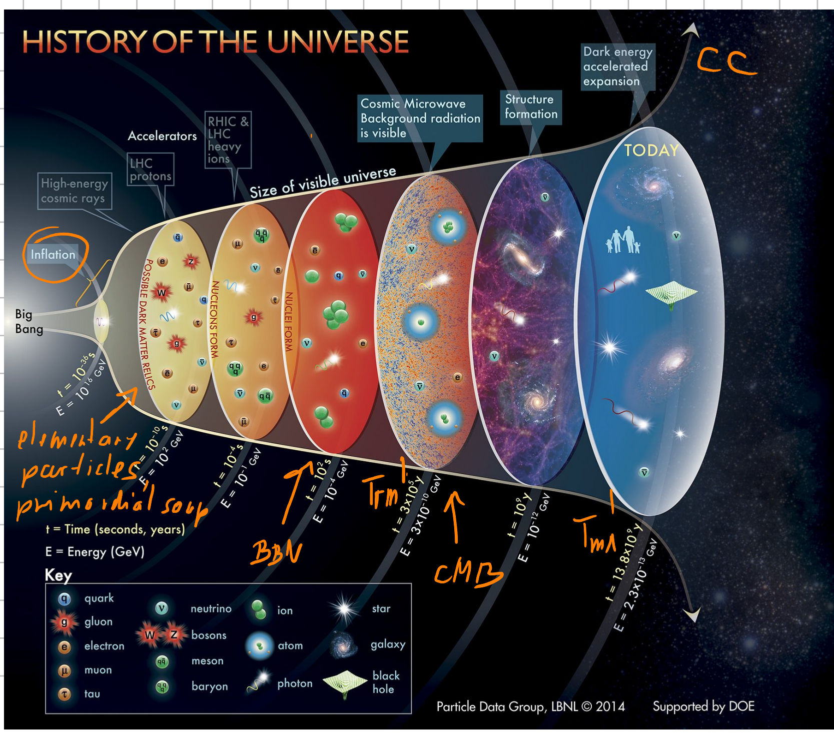
$\rho_m = \rho_r$

$\hookrightarrow T_{rm} = 5.68\,\Omega_m h^2\,eV \simeq eV$ $h \simeq 0.7$

$\rho_m = \rho_\Lambda$

$\hookrightarrow T_{m\Lambda} = 3.5\,K \simeq 3\cdot 10^{-4}\,eV$

⇒ Hot big bang standard model of cosmology



→ Cosmology as a laboratory for high-energy particle physics.