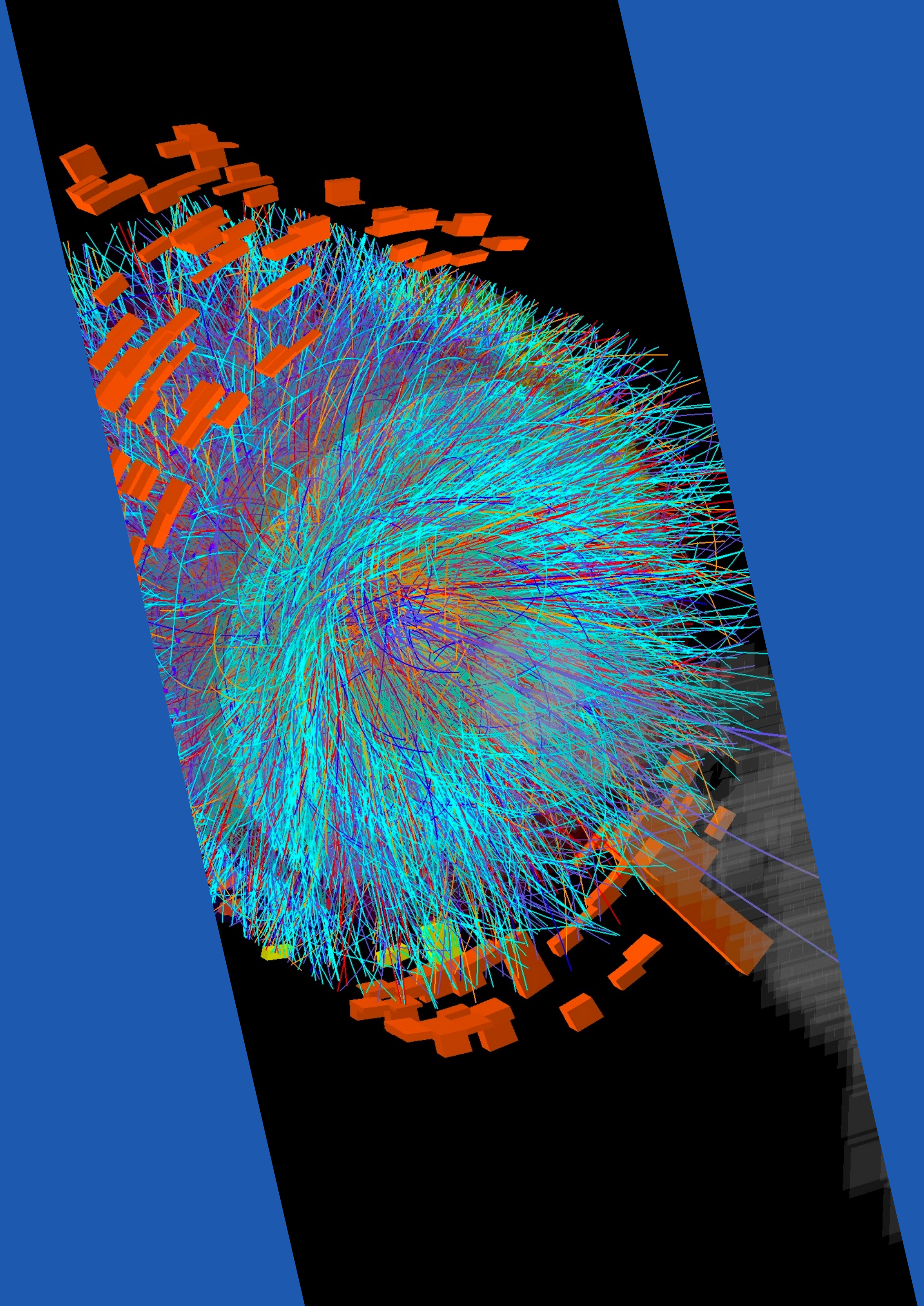
CERN Summer Student Lectures 2023



Heavy Ions 1/3

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Heavy-ion physics

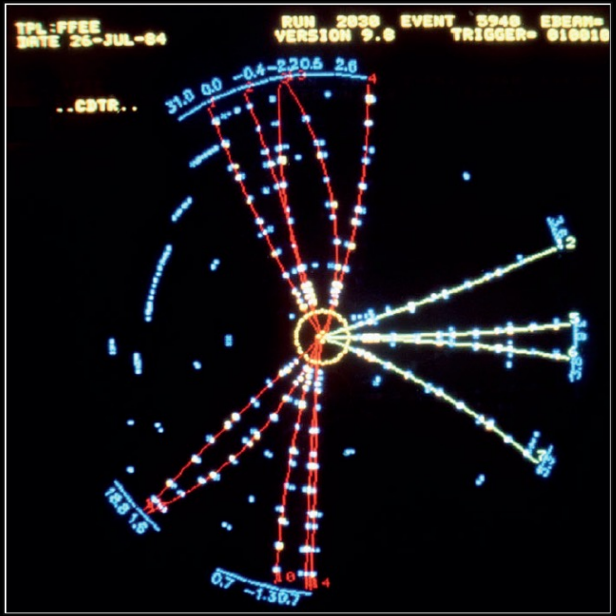
QCD physics

In these lectures:

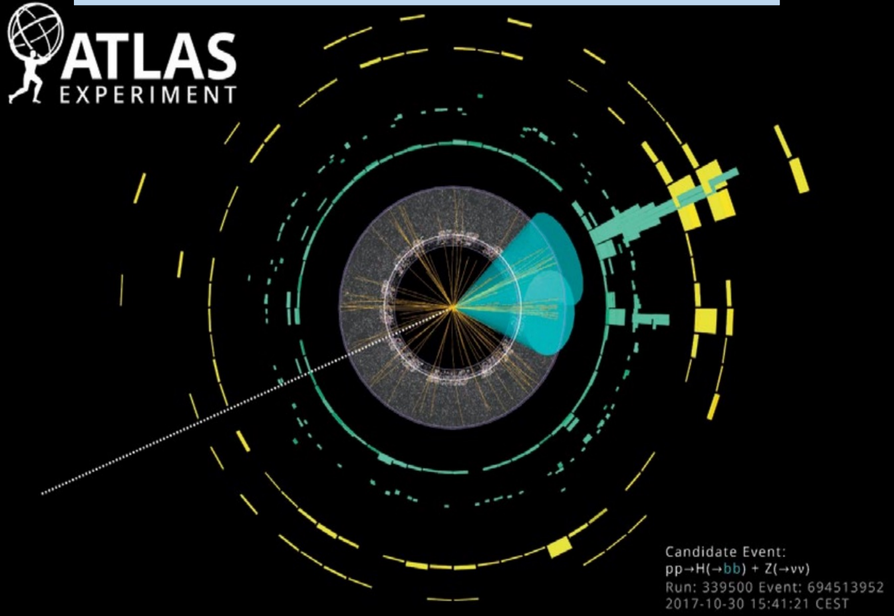
- Basic concepts of QCD and heavy-ion physics
- Experimental principles
- Production and characterization of the QGP at the LHC
- The HIP programme at the LHC: present and future

Take home message in
blue background slides

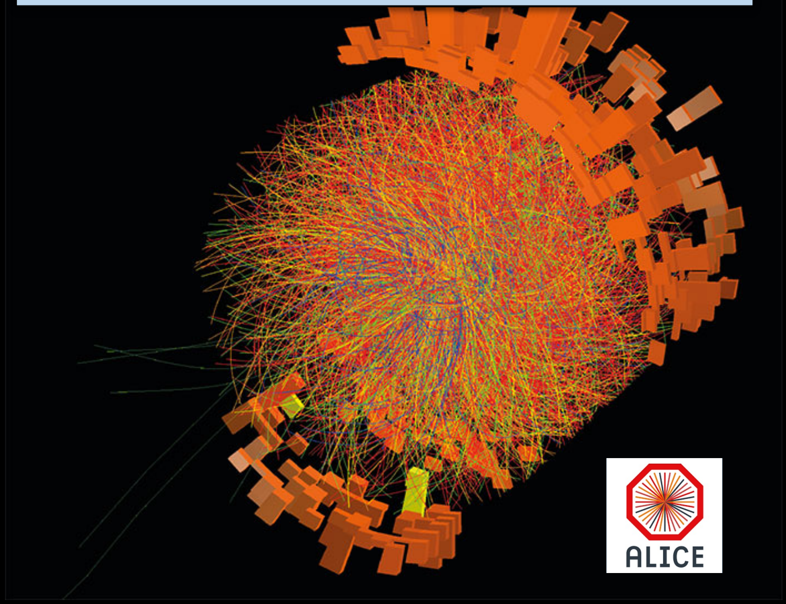
3 jets at PETRA: $e^+e^- \rightarrow q\bar{q}g$



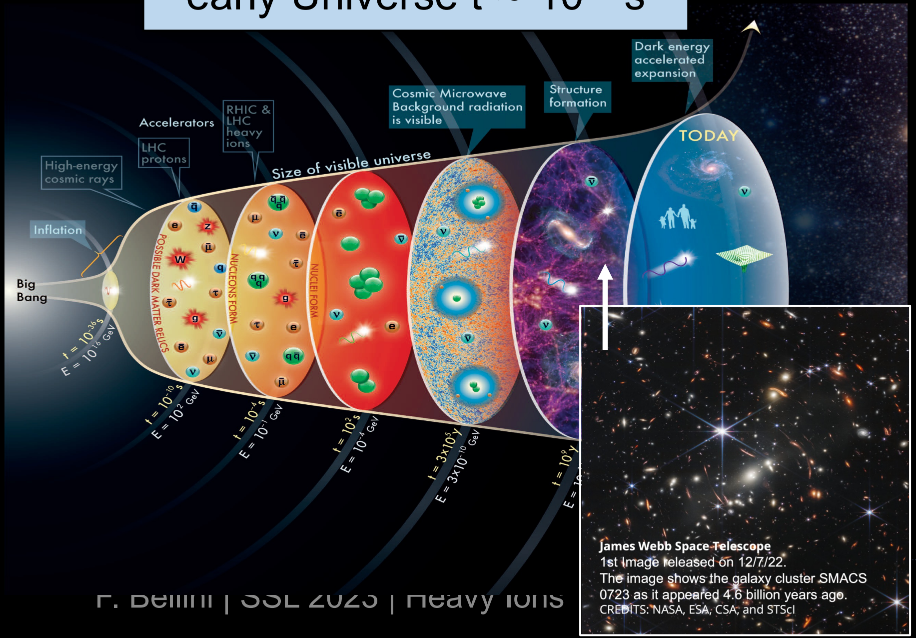
pp → H(→ bbar)+Z event at LHC



One central Pb-Pb collision at LHC



early Universe t ~ 10^{-6} s



neutron stars and NS mergers

Artist's impression of a neutron-star merger (Image: NASA)

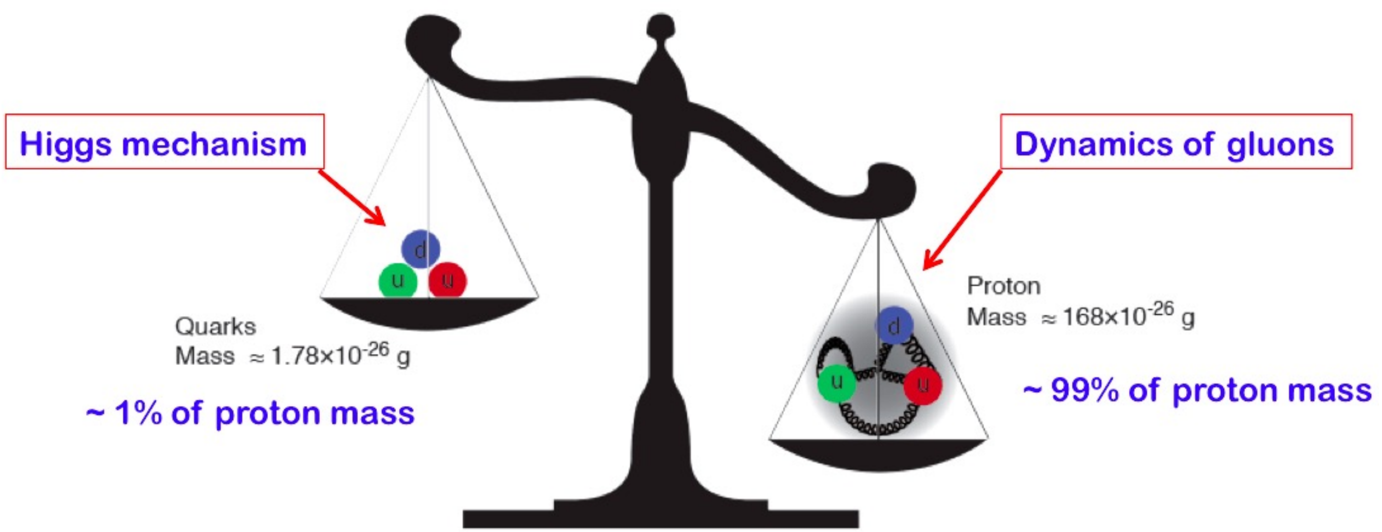
(extra)ordinary matter

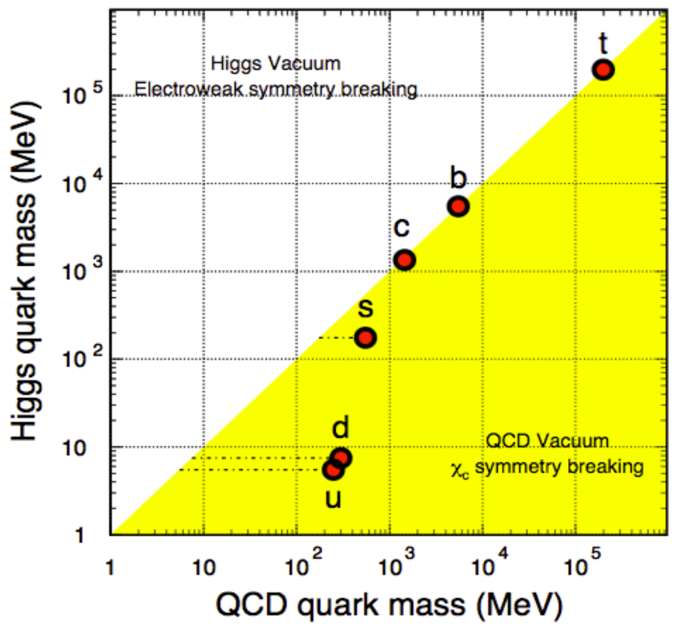
Why studying QCD 1/3

Because we have **mass**

→ Mass is concentrated in atomic nuclei, i.e. in protons and neutrons but the mass of protons and neutrons is much larger than the sum of the masses of the valence quarks that come from the Higgs mechanism

→ **Most of the hadron mass comes from the strong interaction among color charges!**





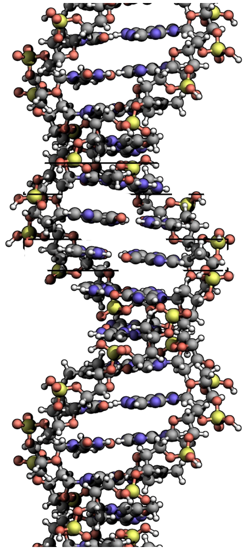
$$\mathcal{L}_{\text{Yukawa}} = -\hat{h}_{d_{ij}}\bar{q}_{L_i}\Phi\, d_{R_j} - \hat{h}_{u_{ij}}\bar{q}_{L_i}\tilde{\Phi}u_{R_j} - \hat{h}_{l_{ij}}\bar{l}_{L_i}\Phi\, e_{R_j} + h.c.$$

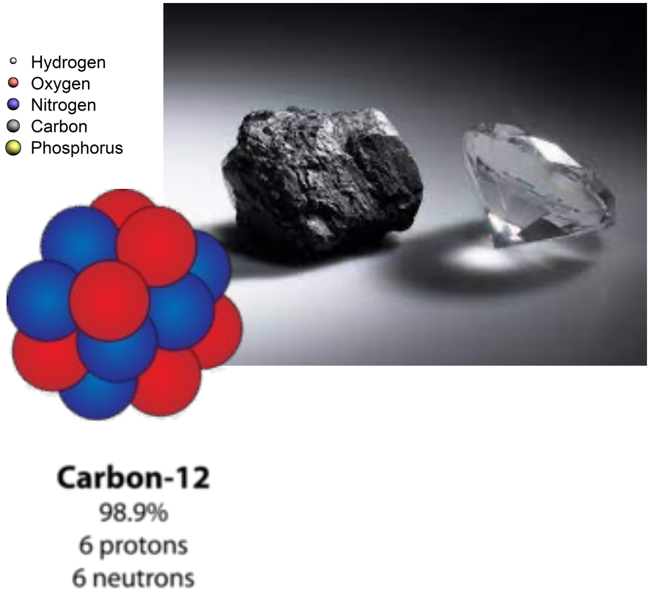
Why studying QCD 2/3

Because we are made of **bound (=strongly-interacting!) objects**

→ Nuclear many-body theories attempt to describe how the nuclear structure emerges from the basic properties of the strong interaction… but *«the description of all but the lightest nuclei at the QCD level is inefficient at best, and impossible at worst»* *[H. Hergert (2020), Front. Phys. 8:379]*

→ **We have effective field theories but not yet a complete understanding!**





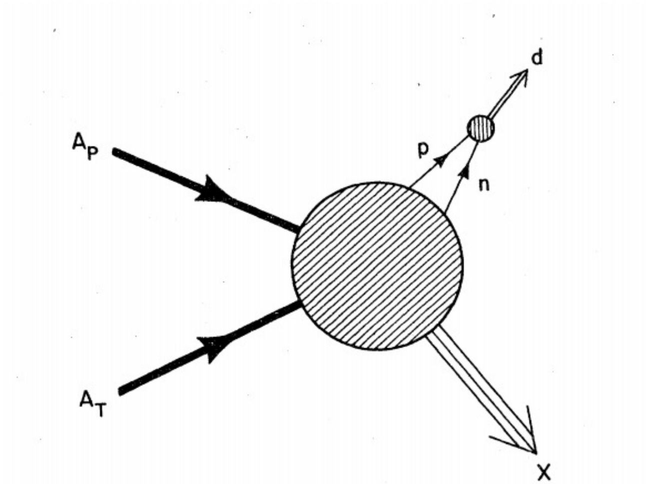
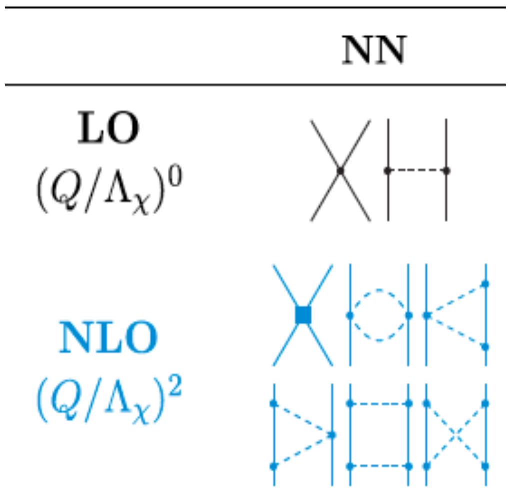


FIG. 1. Schematic for the production of a deuteron in the final state of a relativistic collision between two heavy nuclei.



Example of chiral EFT π-exchange diagrams

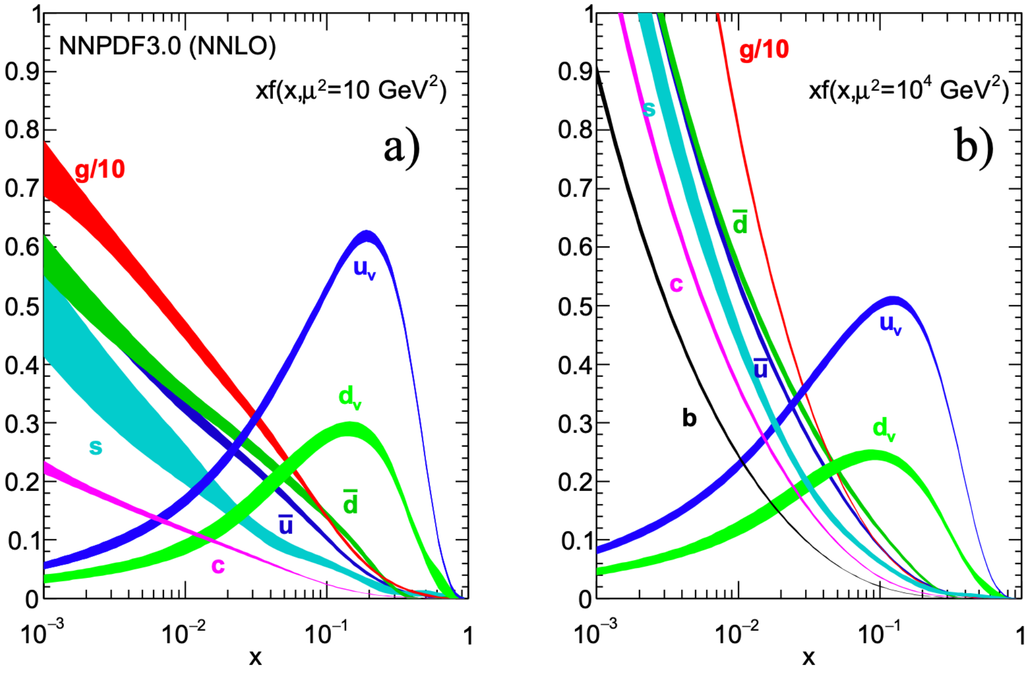
Why studying QCD 3/3

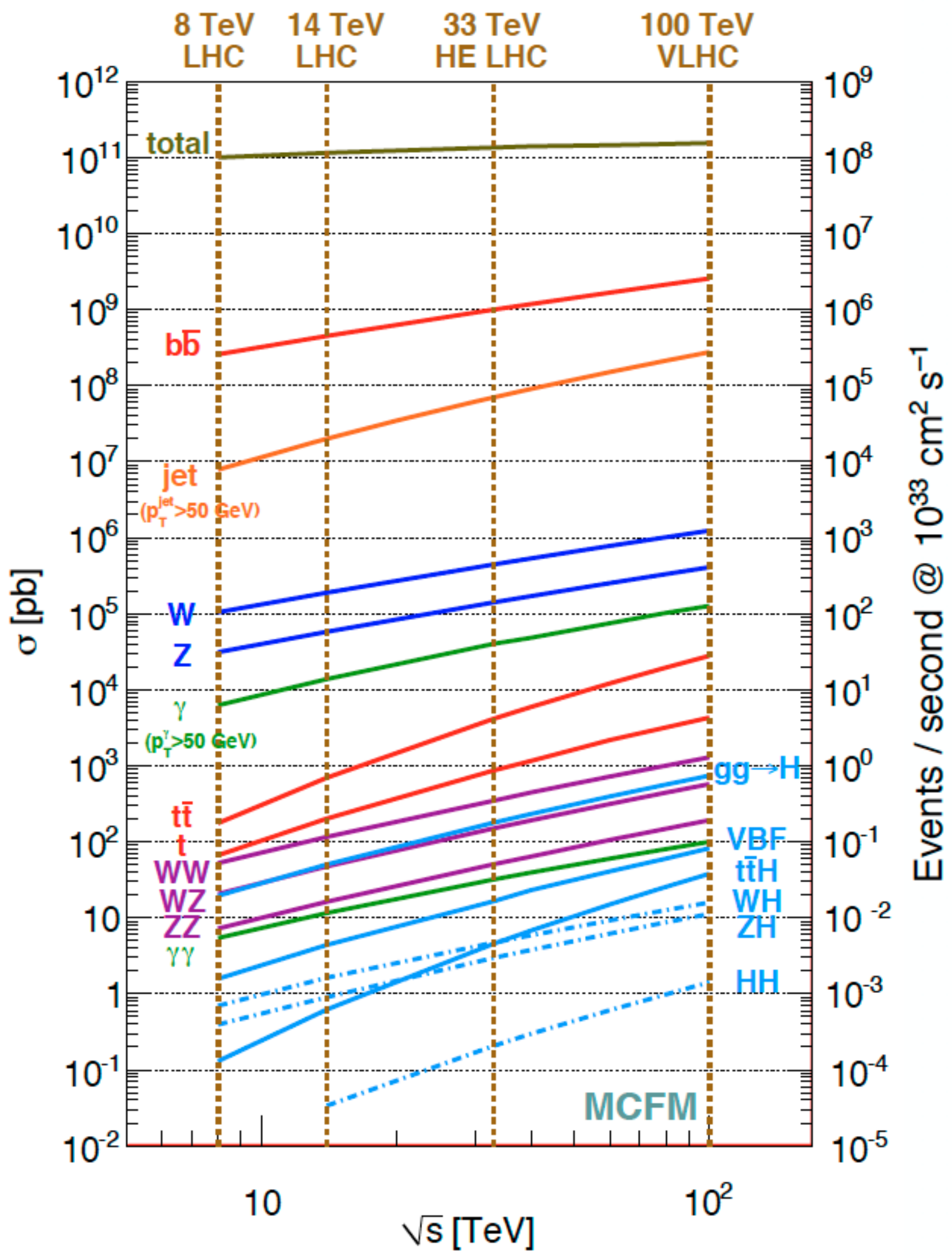
Because we **need** to understand it!

→ In the era of hadron colliders, we need to understand **the main background** for the signals of interest,

→ This is needed both for Standard Model precision measurements and searches for new physics BSM

Proton parton distribution functions (PDFs)



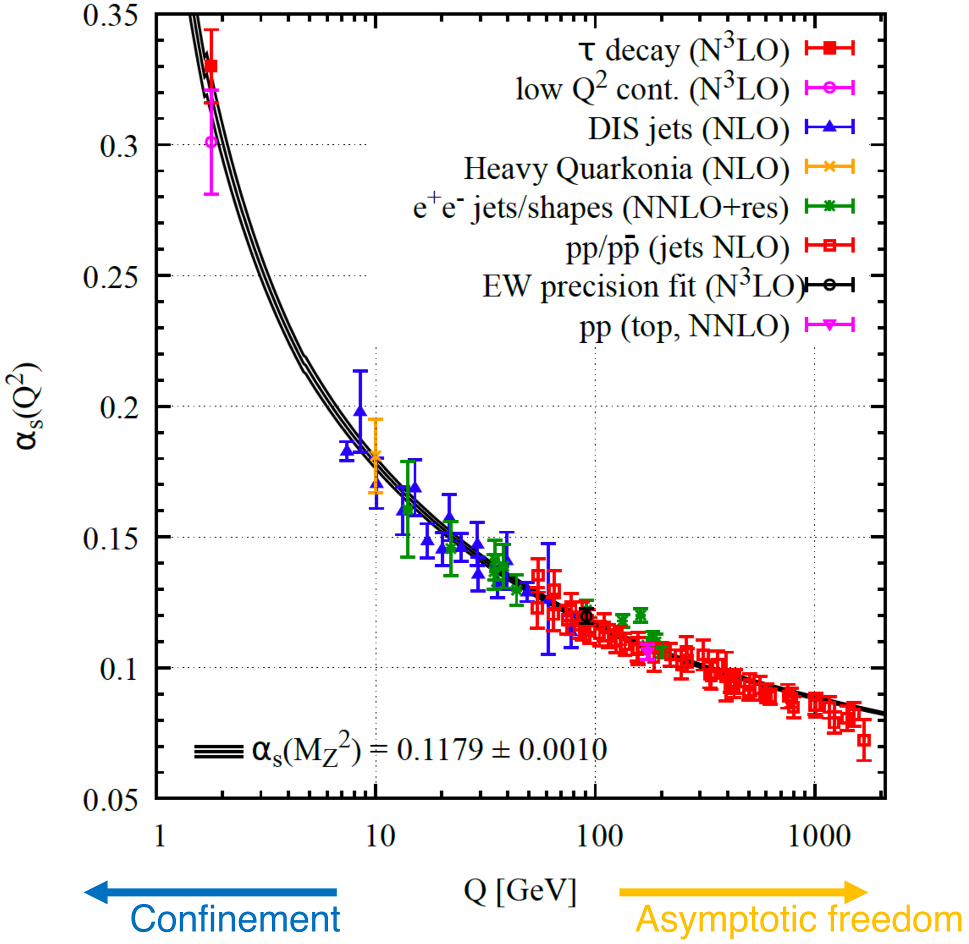


Recall: fundamental properties of QCD

Quarks and gluons exist in nature as confined in colorless hadrons: ordinary matter at room temperature
→ confining property of QCD

The strong coupling, i.e. the effective strength of the interaction, becomes weaker for processes involving larger quadrimomentum exchange
→ asymptotic freedom

Chiral symmetry is explicitly broken by non-zero quark masses: (light) quarks acquire mass dynamically
→ the mass of hadrons is a consequence of the strong interaction acting among their constituents



The QCD phase transition (a very simplified picture)

At low temperature and "normal" density, i.e. us in this room at T ~ 1/40 eV, colored partons are confined in hadrons with chiral symmetry being broken (giving 99% mass to the proton!)

Idea developed back in the 1970's:

by **heating** hadronic matter up to high T and **compressing** it at high pressures, we can observe a **phase transition** from **confined** matter to a deconfined state made of colored quarks and gluons

→ What are the critical conditions at which the QCD phase transition occurs?

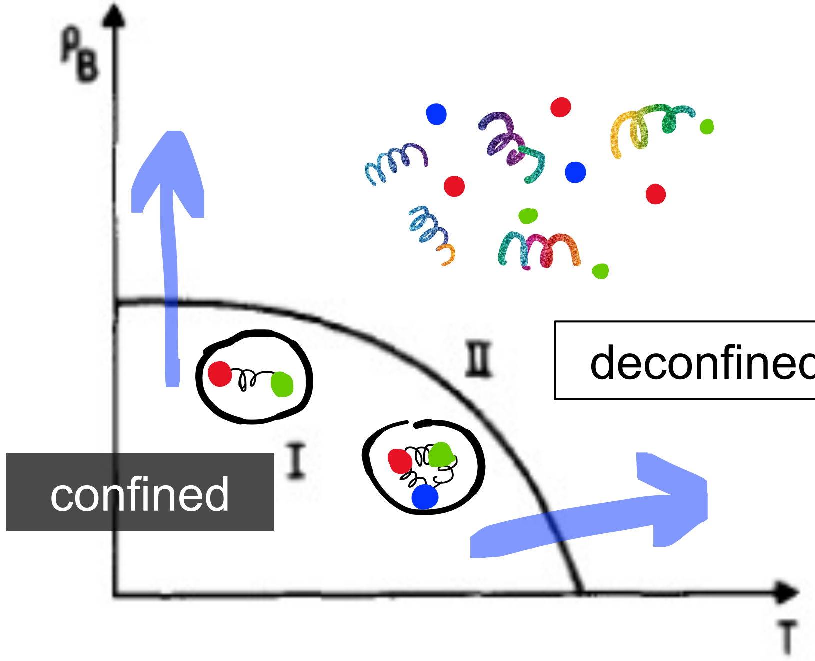


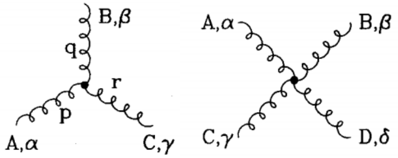
Fig. 1. Schematic phase diagram of hadronic matter. ρ_B is the density of baryonic number. Quarks are confined in phase I and unconfined in phase II.

N. Cabibbo, G. Parisi, Phys. Lett. B59 (1975) 67
J.C. Collins, M.J. Perry, Phys. Rev. Lett. 34 (1975) 1353

Understanding confinement

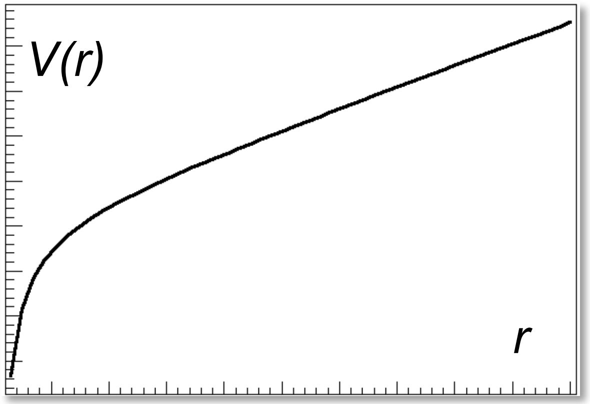
Properties of the QCD vacuum:

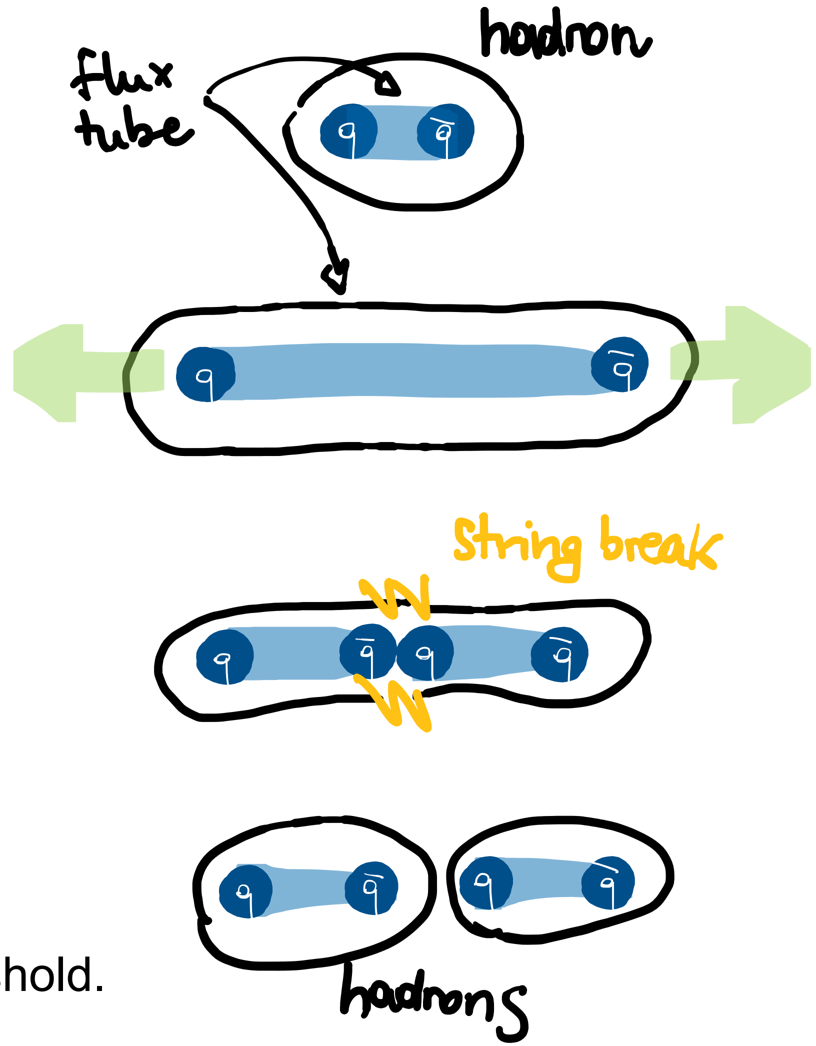
- Gluon-gluon self-interaction (non-abelian)
- QCD field lines compressed in flux tube (or "string")



The q-qbar potential is of the form (Cornell potential):

$$V(r) = -\frac{a}{r} + \sigma r$$





- The potential grows with distance.
- If pulled apart, the energy in the string increases
- A new q-qbar pair is created once the energy is above production threshold.
- No free quark can be obtained by breaking a flux tube → confinement

The MIT Bag model

A simple phenomenological model, describes confinement by assuming that hadrons are confined in bubbles of perturbative (= empty) vacuum and are surrounded by QCD vacuum (a fluid of gg pairs) exerting pressure.

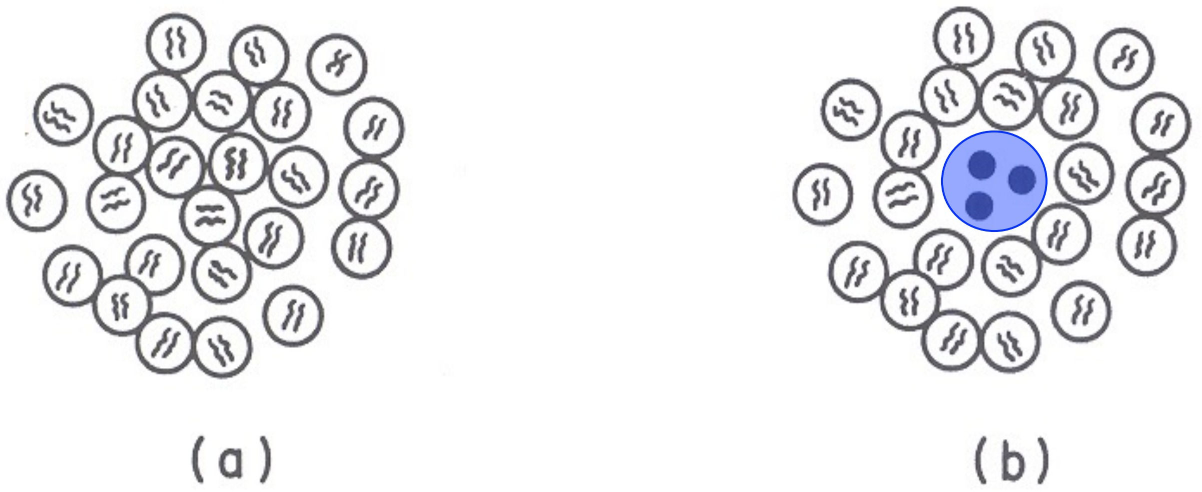


FIG. 9. The QCD vacuum state is depicted in (a). It is a random distribution of cells that contain a gluon pair in a color and spin singlet state. Quarks (in a color singlet configuration) displace these cells, creating a region (or "bag") of "empty" vacuum, as shown in (b).

*A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, and V. F. Weisskopf, Phys. Rev. D **9**, 3471;*
T. DeGrand, R.L. Jaffe, K. Johnson, J. Kiskis, Phys. Rev. D12 (1975) 2060

The MIT Bag model (2)

Inside the bag, quarks have very small masses and the interaction is weak

Outside the bag, quarks are not allowed to propagate, no colored partons, but quark and gluon condensates

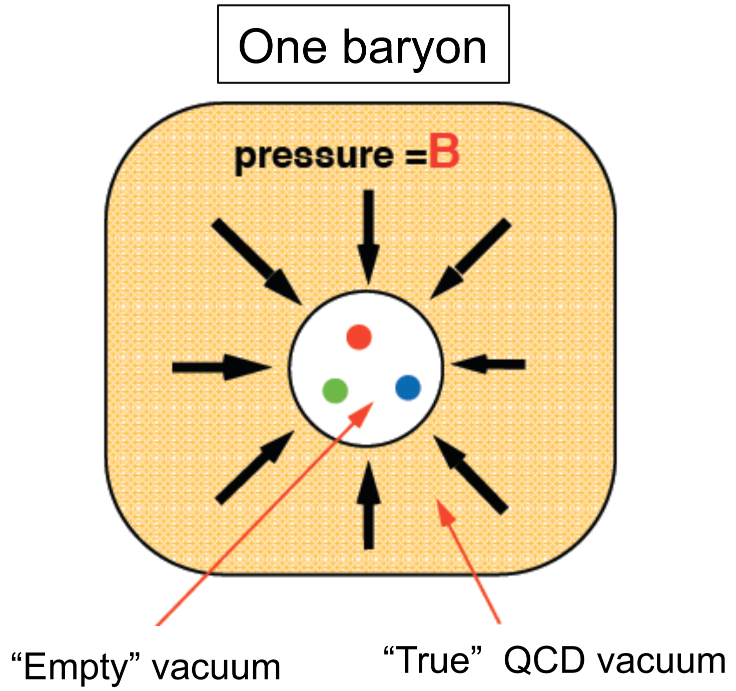
The equilibrium between the kinetic pressure of the quarks inside the hadron vs the pressure of the surrounding QCD vacuum ("bag pressure", *B*) defines the radius *R* of the hadron.

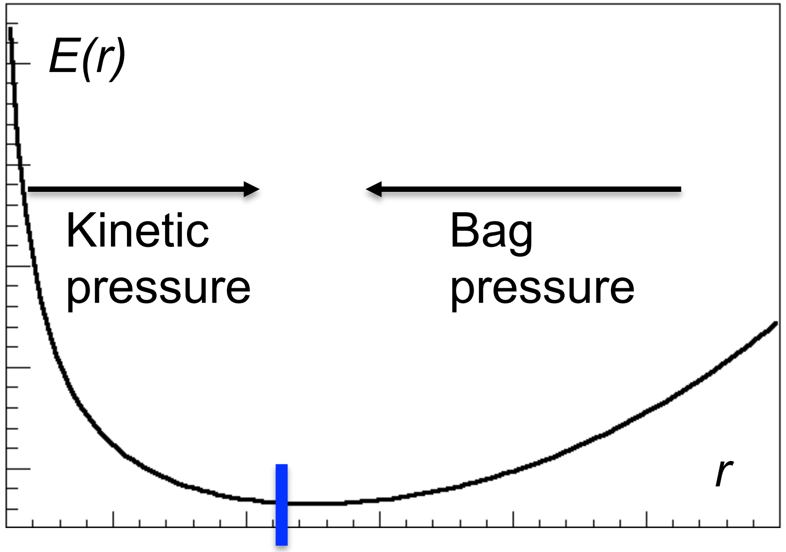
If the hadron can be modeled as *N* massless Dirac fermions in a spherical cavity,

$$E = \frac{2.04N}{R} + \frac{4\pi}{3}R^3 B$$

By asking $\partial E/\partial R = 0$ and $R(p) \sim 0.8$ fm

→ $B_{MIT} \sim (200\ \mathrm{MeV})^4$





Deconfinement

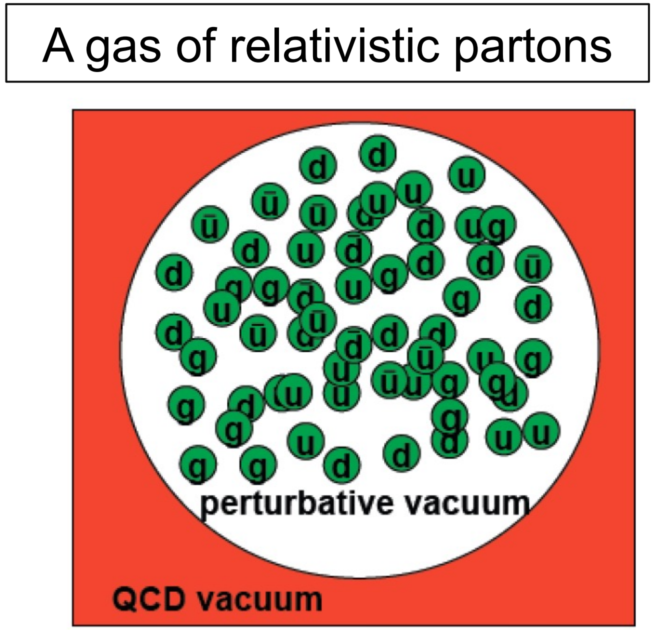
For a gas of massless, relativistic partons the pressure can be calculated from the Stefan-Boltzmann law

$$P = \left(n_g + \frac{7}{8}n_f\right)\frac{\pi^2 T^4}{90}$$

where the degrees of freedom of the system are

n_g = 8 gluons x 2 spin = 16

n_f = 2 quark flavors x 2 spin x 3 colors + anti-q = 24

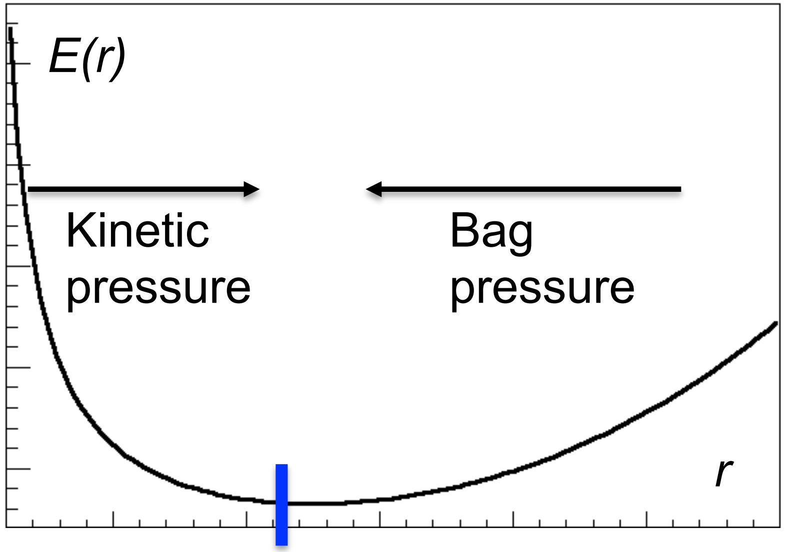


The systems gets **deconfined** if the kinetic pressure exceeds the bag pressure

$P > B \rightarrow T^4 > (200\ \text{MeV})^4 * 90 / (16+7/3) / \pi^2$

$\rightarrow$ $T_c > 141$ MeV (critical temperature)

Above T_c, the system undergoes a phase transition to a state of matter where quark and gluons are (quasi) free, the Quark-Gluon Plasma

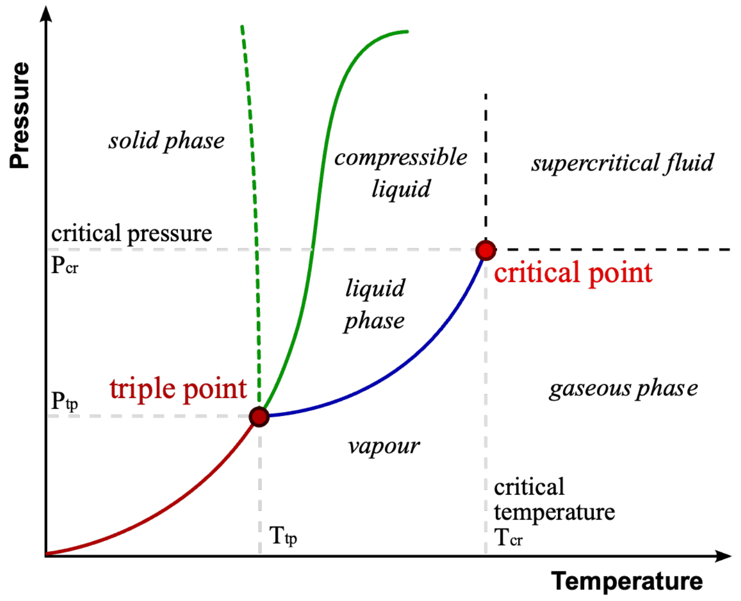


Recall: phase transitions

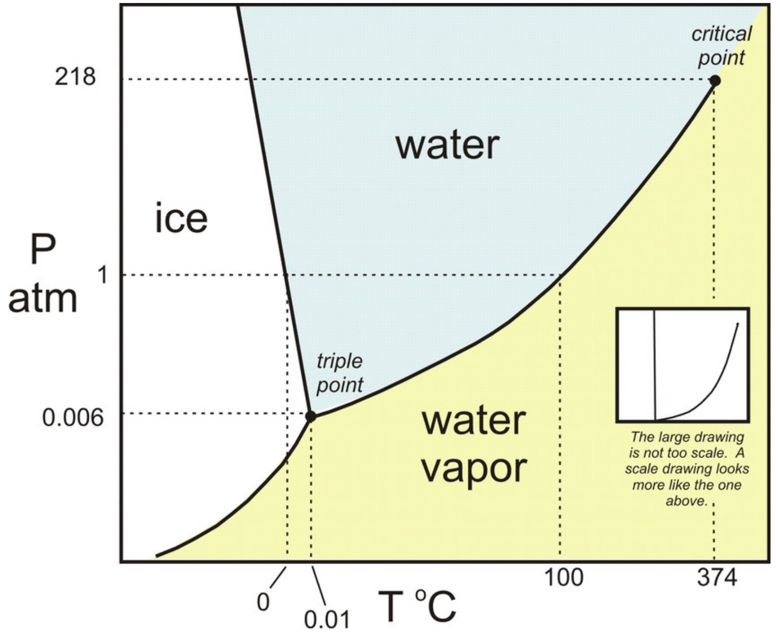
A **phase transition** is the transformation of a thermodynamic system from one phase (or state of matter) to another.
e.g. ice ⇔ water ⇔ vapour
e.g. confinement ⇔ deconfinement in QCD

During a phase transition, certain properties of the medium change, often discontinuously, as a result of external conditions
e.g. pressure, temperature, …

The measurement of these external conditions at which the transformation occurs is called the phase transition point.



Source: wikipedia



Source:http://serc.carleton.edu/research_education/equilibria/phaserule.html%5D

The QCD phase diagram

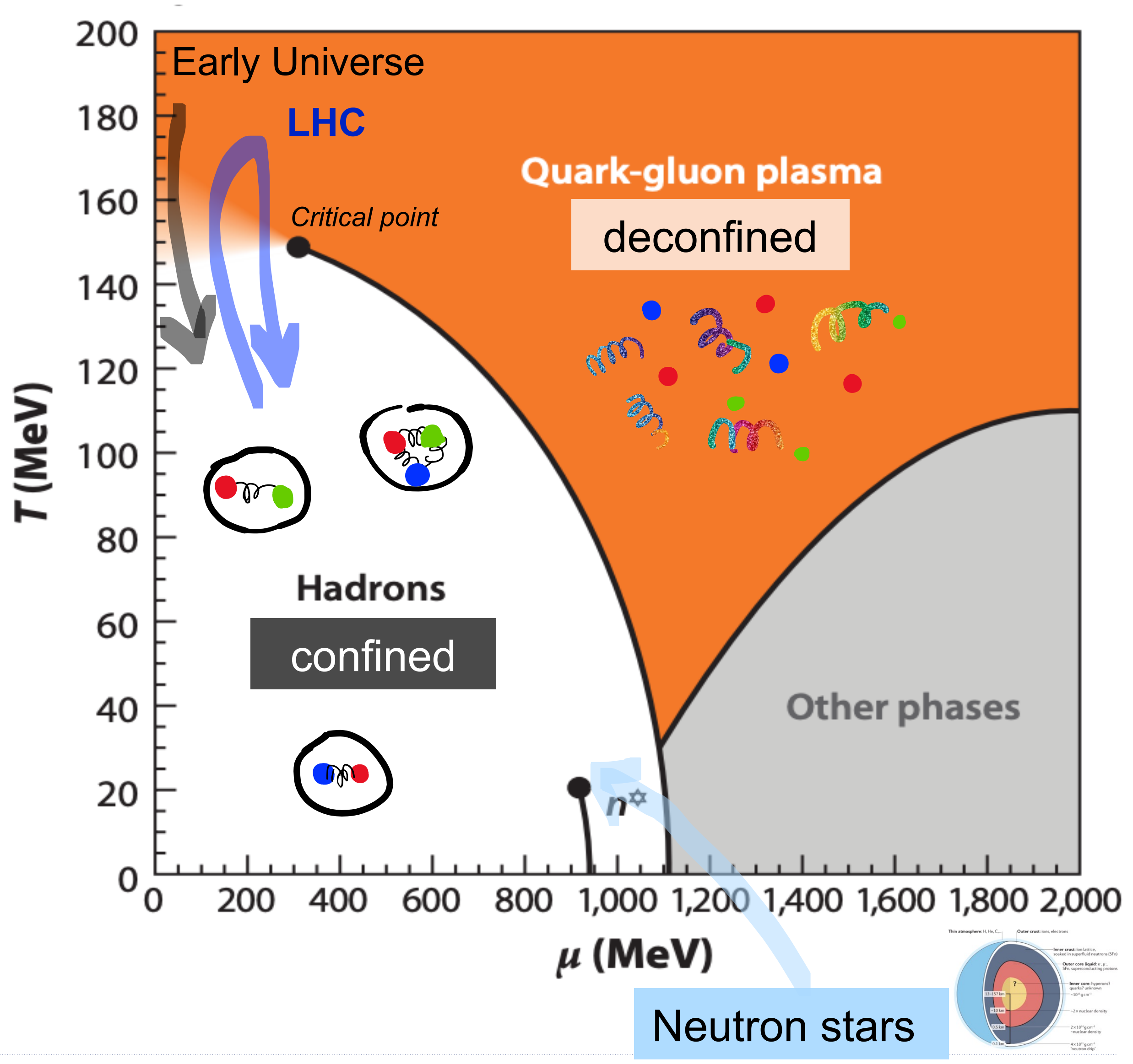
The phases of QCD matter can be summarized in a phase diagram as a function of two parameters:

temperature T and
baryochemical potential μ_B

$$\mu_B = \frac{\partial E}{\partial n_B}, \quad n_B = n(B) - n(\overline{B})$$

$\mu_B = 0$ → antimatter / matter = 1
as at the LHC and in the Early Universe!

The quark-gluon plasma is the deconfined phase of strongly-interacting matter.



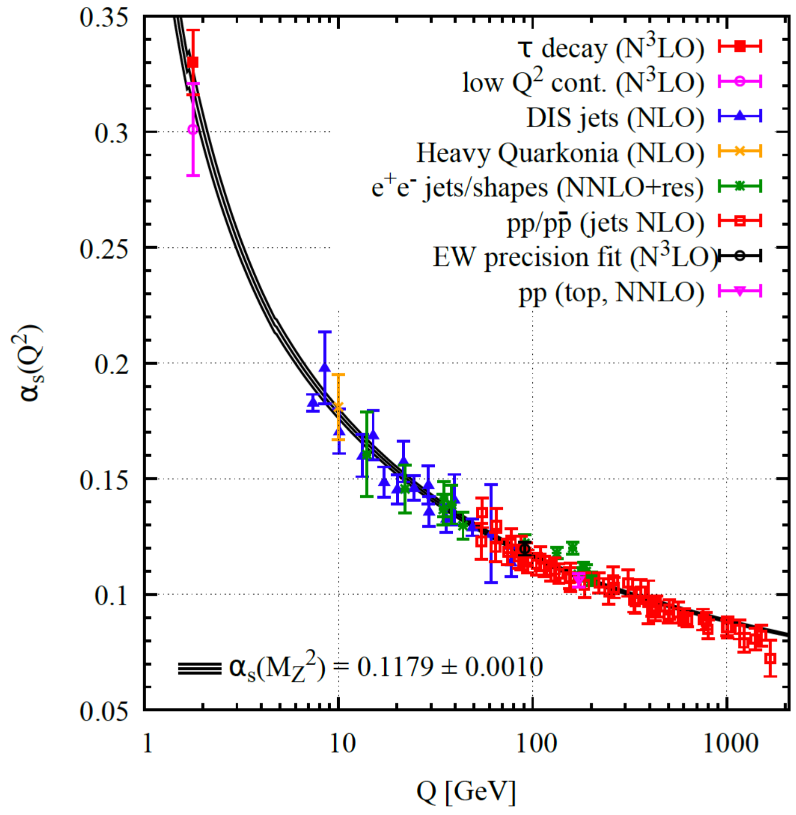
Which QCD energy regime are we dealing with?

Having in mind:
- MIT bag model estimate for the critical temperature: T_c ~ 140 MeV
- $\Lambda_{QCD}(m_{Z}, N_f = 3)$ = 244 MeV

But also that at *T = 200 MeV*, the typical kinetic energy
- for a non-relativistic particle is $E = 3/2\, k_B T$ = 300 MeV
- for a relativistic particle is $E = 3k_B T$ = 600 MeV

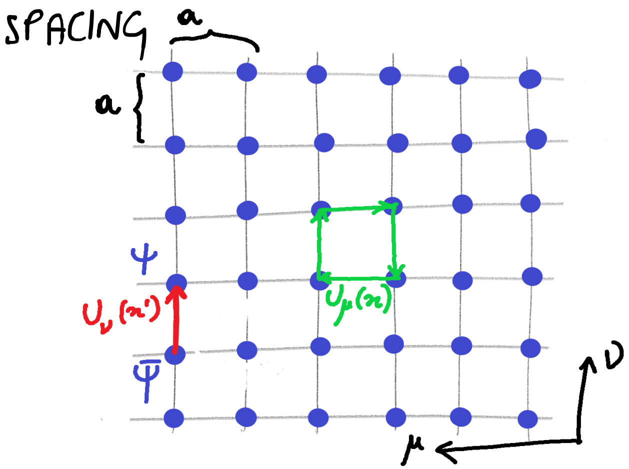
Low Q → α_s is not small!
→ The QCD transition is a non-perturbative QCD problem

- Need models to deal with (phenomenology)
- Use Lattice QCD for calculations from first principles



Lattice QCD basics

Lattice QCD (lQCD) is a well-established non-perturbative method to solve QCD starting from first principles, i.e. the QCD Lagrangian.



IBM BlueGene
Image: Argonne Nat. Lab.

The Euclidean space-time is discretized in a 4D-hypercubic lattice with 3 real spatial and 1 imaginary time.

The finite lattice **spacing *a*** acts as an ultraviolet cutoff for the theory.
The quark fields $\psi(x)$ are defined on lattice sites.
The gluon fields $U_\mu(x)$ are defined as links between lattice sites.

The QCD Lagrangian in the Minkowski space is transformed into the action in Euclidean space and then the theory is solved by numerical integration.

$$\mathcal{L} = \sum_q \bar{\psi}_{q,a}(i\gamma_\mu \mathcal{D}^\mu - m)_{ab}\psi_{q,b} - \frac{1}{4}F^A_{\mu\nu}F^{\mu\nu}_A$$

$$i\int \mathrm{d}^4x\mathcal{L}(x) \rightarrow S_E$$

$$S_E = \int \mathrm{d}^4x\left[\sum_{\mu,\nu}\frac{1}{2}Tr\{t\cdot F^{\mu\nu}(x)t\cdot F^{\mu\nu}(x)\} + \bar{\psi}(x)(\sum_\mu \hat{\gamma}^\mu\partial^\mu + m)\psi(x)\right]$$

Gluon action | Fermion action

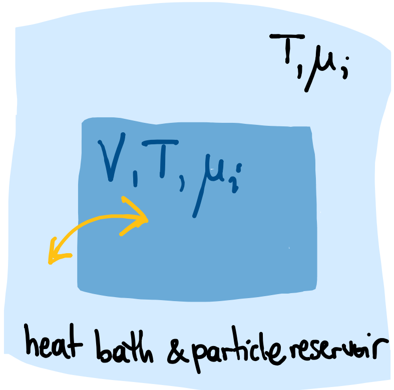
Thermodynamics of QCD matter on the lattice

Thermodynamical properties of a quantum system can be calculated according to the principles of statistical mechanics from the **partition function, Z.**

$$Z = Tr\left[e^{-\beta(\hat{H}-\mu_i \hat{N}_i)}\right]$$

$$P = T\frac{\partial \log Z}{\partial V}; \qquad N_i = T\frac{\partial \log Z}{\partial \mu_i};$$

$$S = \frac{\partial T \log Z}{\partial T} \qquad E = -PV + TS + \mu_i N_i$$



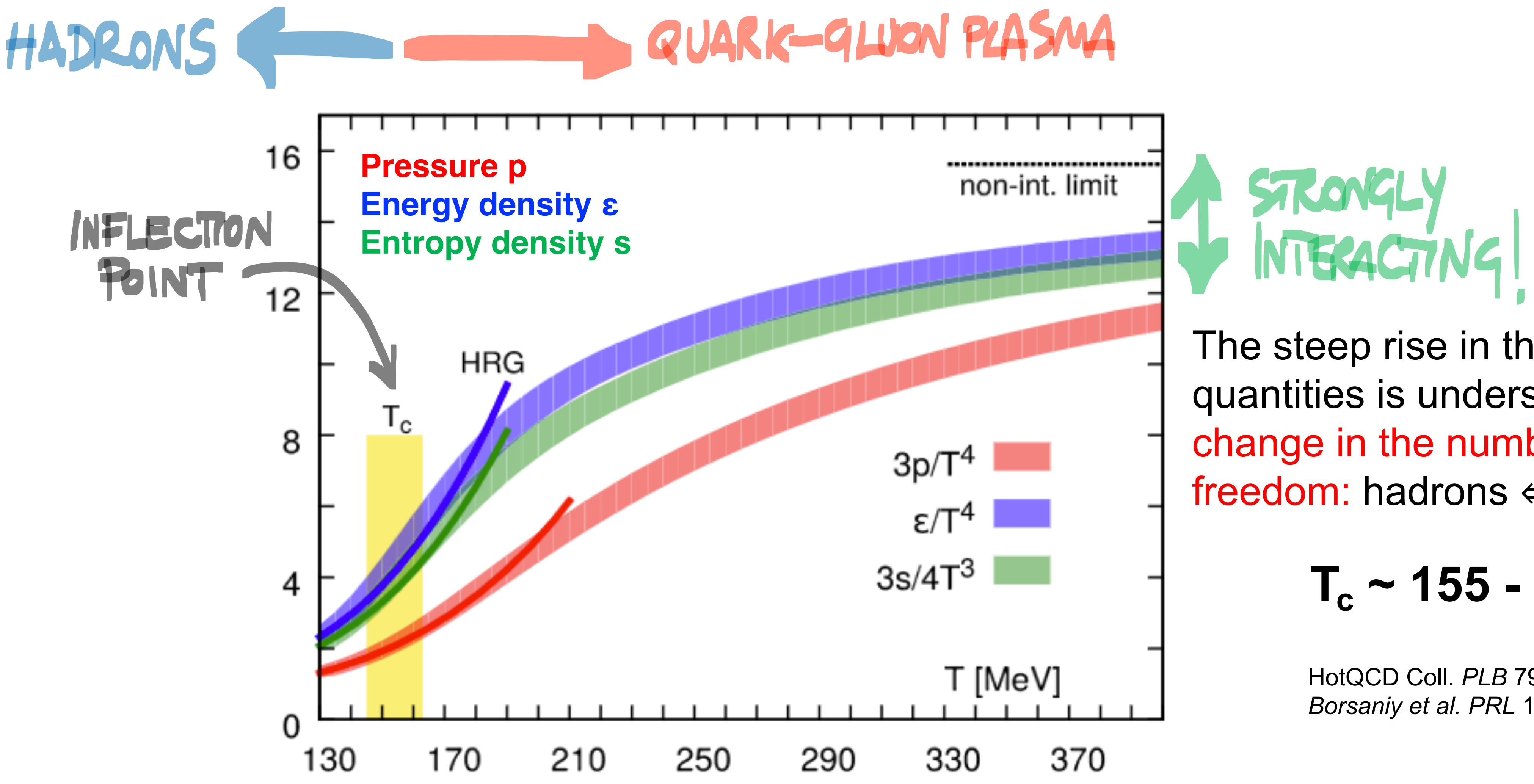
A system of **QCD degrees of freedom** can be represented by a **grand canonical** (GC) ensemble, where a given volume element can exchange particles and heat with its surrounding (heat bath and particle reservoir).

$$\mathcal{Z}(T,V,\vec{\mu}) = \int \prod_{\mu} \mathcal{D}A_{\mu} \prod_{f=u,d,s\dots} \mathcal{D}\psi_f \mathcal{D}\bar{\psi}_f \, e^{-S_E(T,V,\vec{\mu})}$$

On the lattice, the GC partition function can be used to extract the expectation value of the physical observables.

$$\langle \mathcal{O} \rangle = \frac{1}{Z(T,V,\vec{\mu})} \int \prod_{\mu} \mathcal{D}A_{\mu} \prod_{f} \mathcal{D}\psi_f \mathcal{D}\bar{\psi}_f \, \mathcal{O} \, e^{-S_E(T,V,\vec{\mu})}$$

Equation of State (EoS) from lattice QCD



A. Bazavov et al., Phys. Rev. D 90 (2014) 094503

The steep rise in the thermodynamic quantities is understood as due to a change in the number of degrees of freedom: hadrons ⇔ partons

T_c ~ 155 - 159 MeV

HotQCD Coll. *PLB* 795 (2019) 15-21,
Borsaniy et al. PRL 125 (2020) 5, 052001

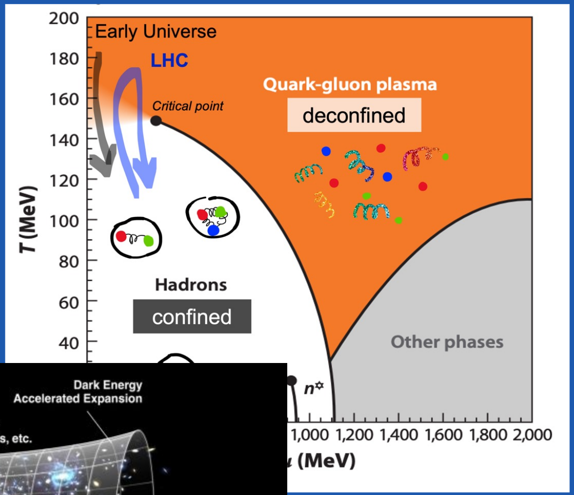
The QGP is a state of strongly-interacting matter resulting from the phase transition of nuclear/hadronic (color-neutral) matter under extreme conditions of pressure or temperature

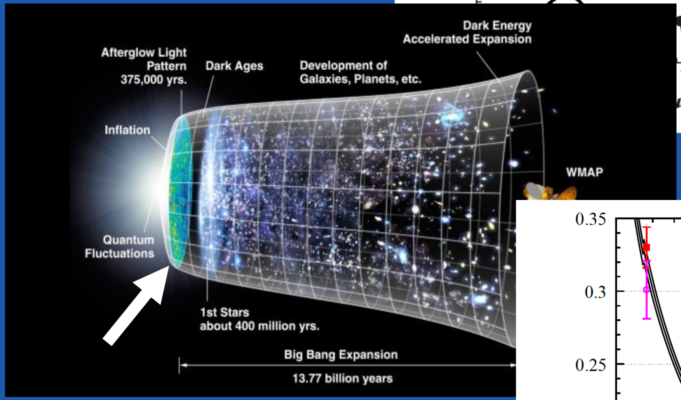
→ the **Universe** up to O(1-10μs) after the Big Bang

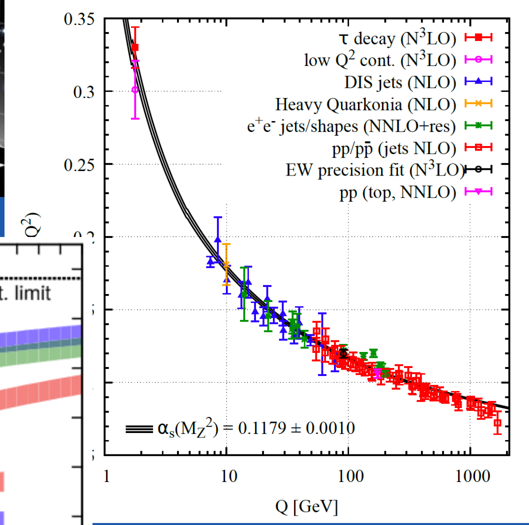
→ the properties of the QGP **emerge** from the fundamental properties of the strong interaction

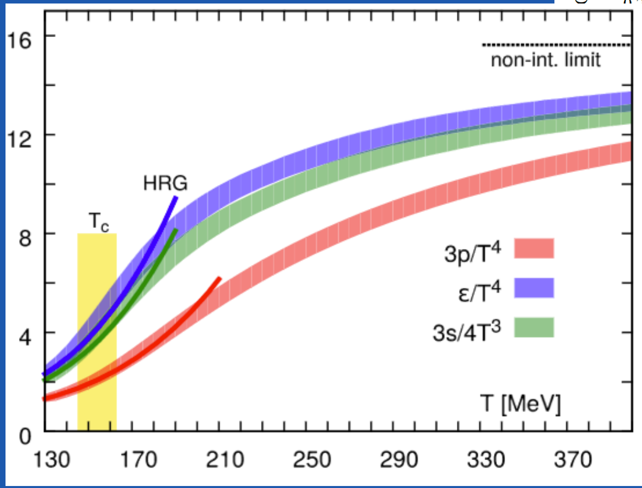
→ physics of **condensed** QCD matter

Next: the experimental quest towards a **quantitative characterization** of the QGP









Experimental principles

QCD in extreme conditions in the laboratory

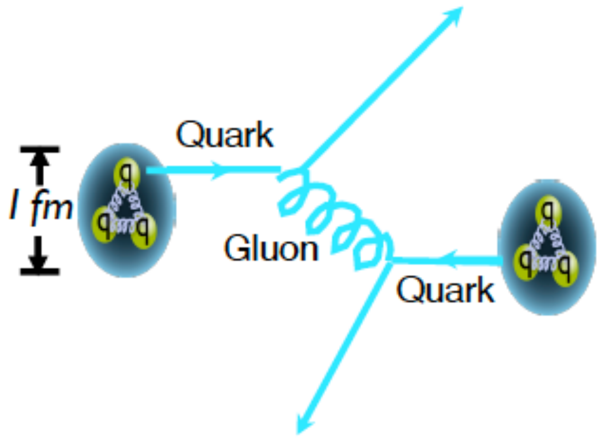
A QGP can be formed by compressing large amount of energy in a small volume

→ collide **heavy nuclei** (multiple, ~simultaneous nucleon-nucleon collisions)

→ **control the energy deposited** in the collision region by varying the collision system

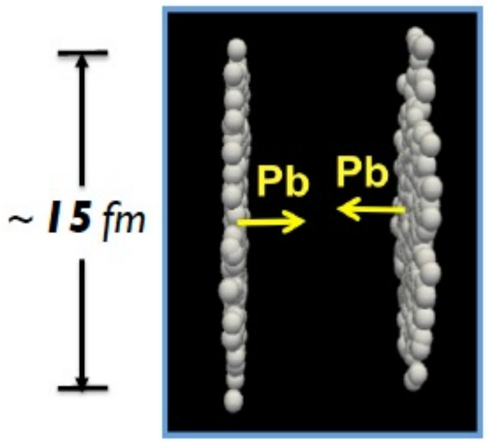
- nuclear species, p-Pb, pp
- vary impact parameter (centrality)

Colliding protons

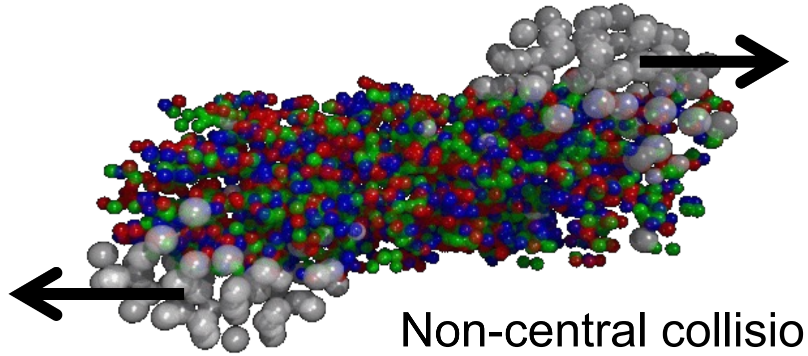


Colliding large nuclei

T.D. Lee, RMP 47, 267(1975)



Head-on collision, b = 0



Non-central collision
b > 0

Hadron and ion colliders

With symmetric proton beams with energy E, the centre-of-mass energy is $\sqrt{s} = 2E$.

With heavy-nuclei, only protons can be accelerated, but neutrons are there too:

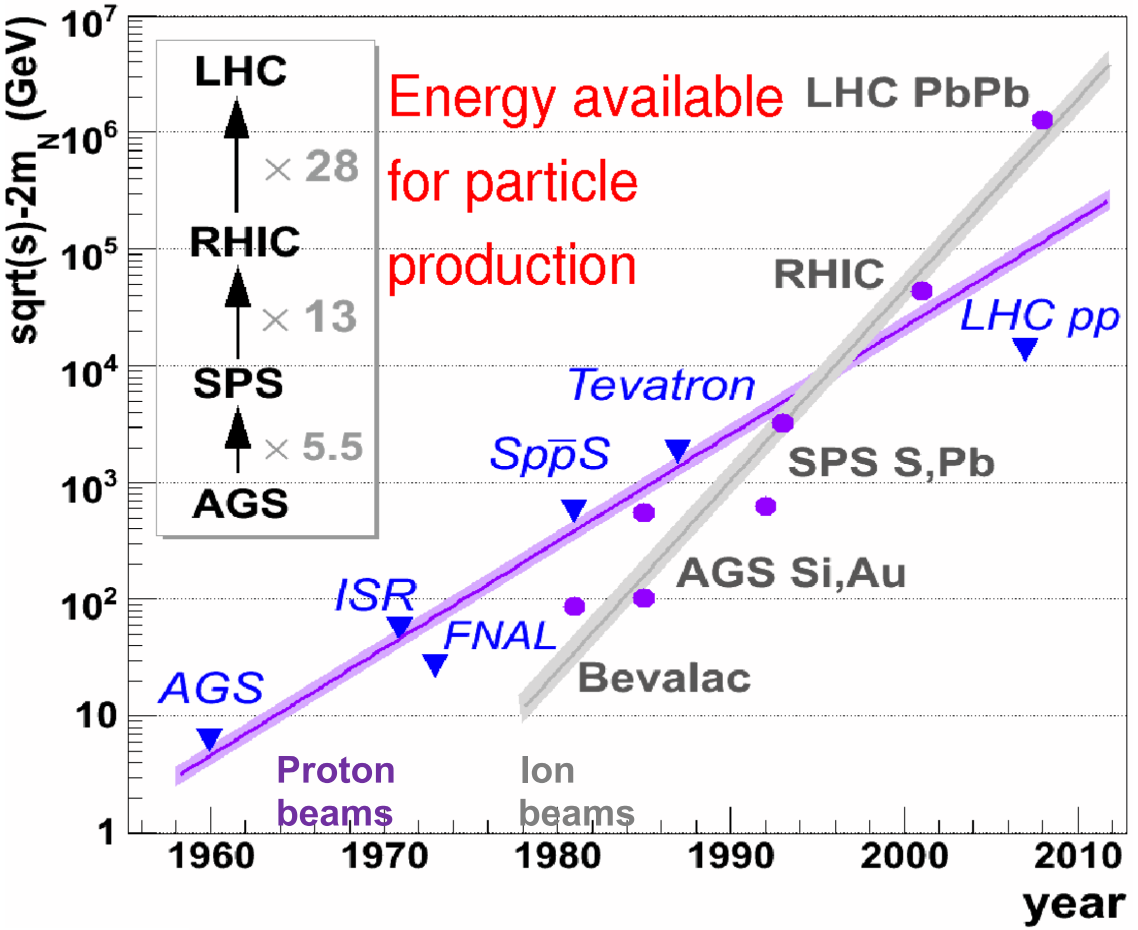
$$p_A = Z/A \; p_{proton}$$

At the LHC, the rigidity of accelerated particles is fixed by the magnet field configuration (B_{max} = 8.3 T).

For the **$^{208}Pb^{82+}$** ions used at the LHC:

$p_{Pb} = 82 / 208 \; p_{proton}$

p_{proton} = 6.5 TeV (Run 2) ⟶ p_{Pb} = 2.56 TeV
⟶ $\sqrt{s_{NN}}$ = 5.02 TeV ⟶ **$\sqrt{s}$ ~ 1.04 PeV**



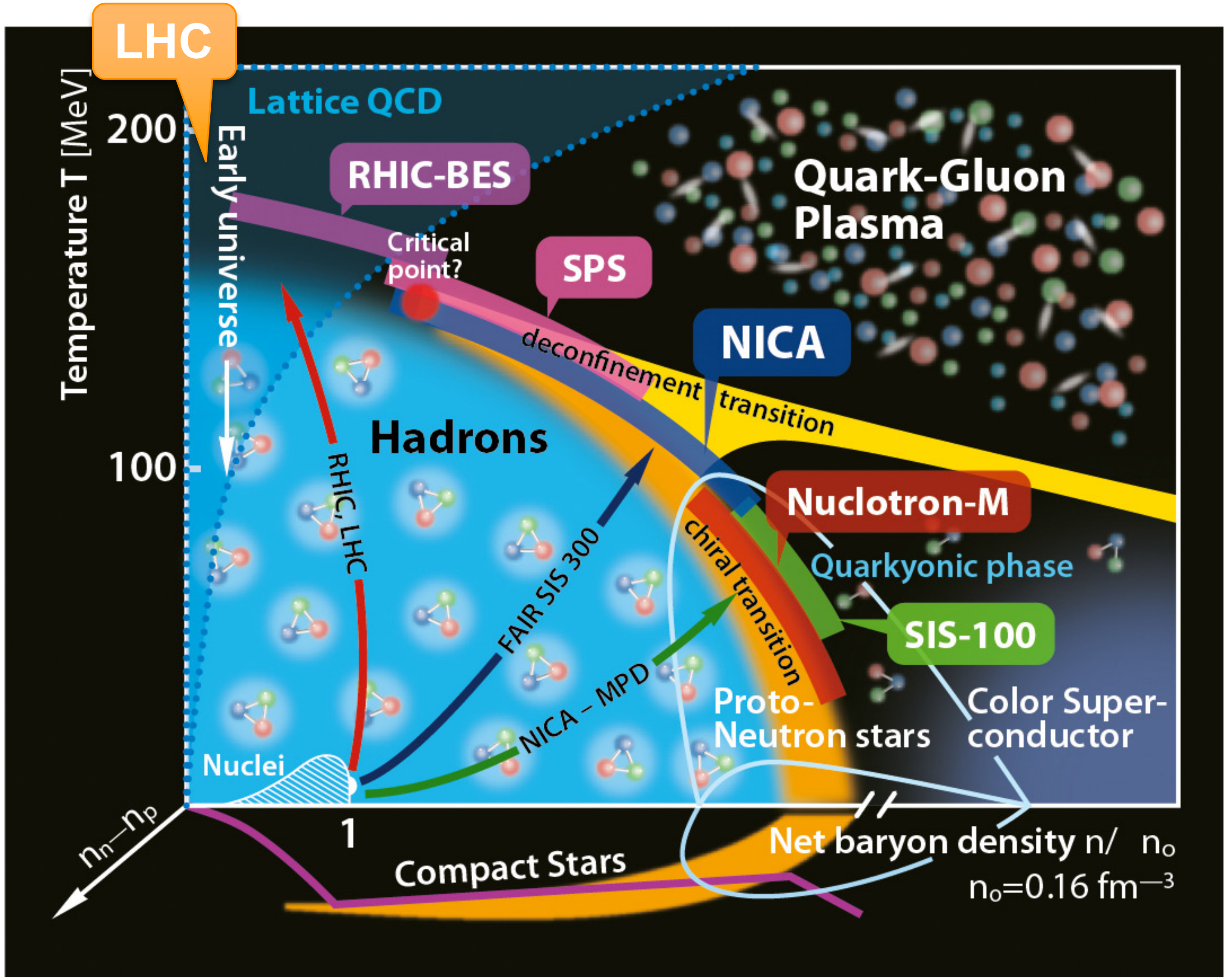
Some numbers (colliders):
RHIC @ BNL (2000-) $\sqrt{s_{NN}}$ <200 GeV
[beam energy scan $\sqrt{s_{NN}}$ = 7.7, 11.5, 19.6, 27, 39, and 62.4 GeV]
LHC @ CERN (Run I, 2009-2013) $\sqrt{s_{NN}}$ = 2.76 TeV
LHC @ CERN (Run II, 2015-2018) $\sqrt{s_{NN}}$ = 5.02 TeV
HL-LHC @ CERN (Run III+IV, 2022-2030) $\sqrt{s_{NN}}$ = 5.5 TeV
NICA @ JINR (2021) $3 < \sqrt{s_{NN}} < 11$ GeV

Experimental exploration of the QCD phase diagram

RHIC and the LHC explore the region of the phase diagram for $\mu_B \sim 0$

which is also the region of the phase diagram where lattice QCD calculations can be performed

Low energy (& high μ_B) are the conditions to study the 1st order transition and the search for the critical point, the key regime being $2.5 < \sqrt{s_{NN}} < 8$ GeV



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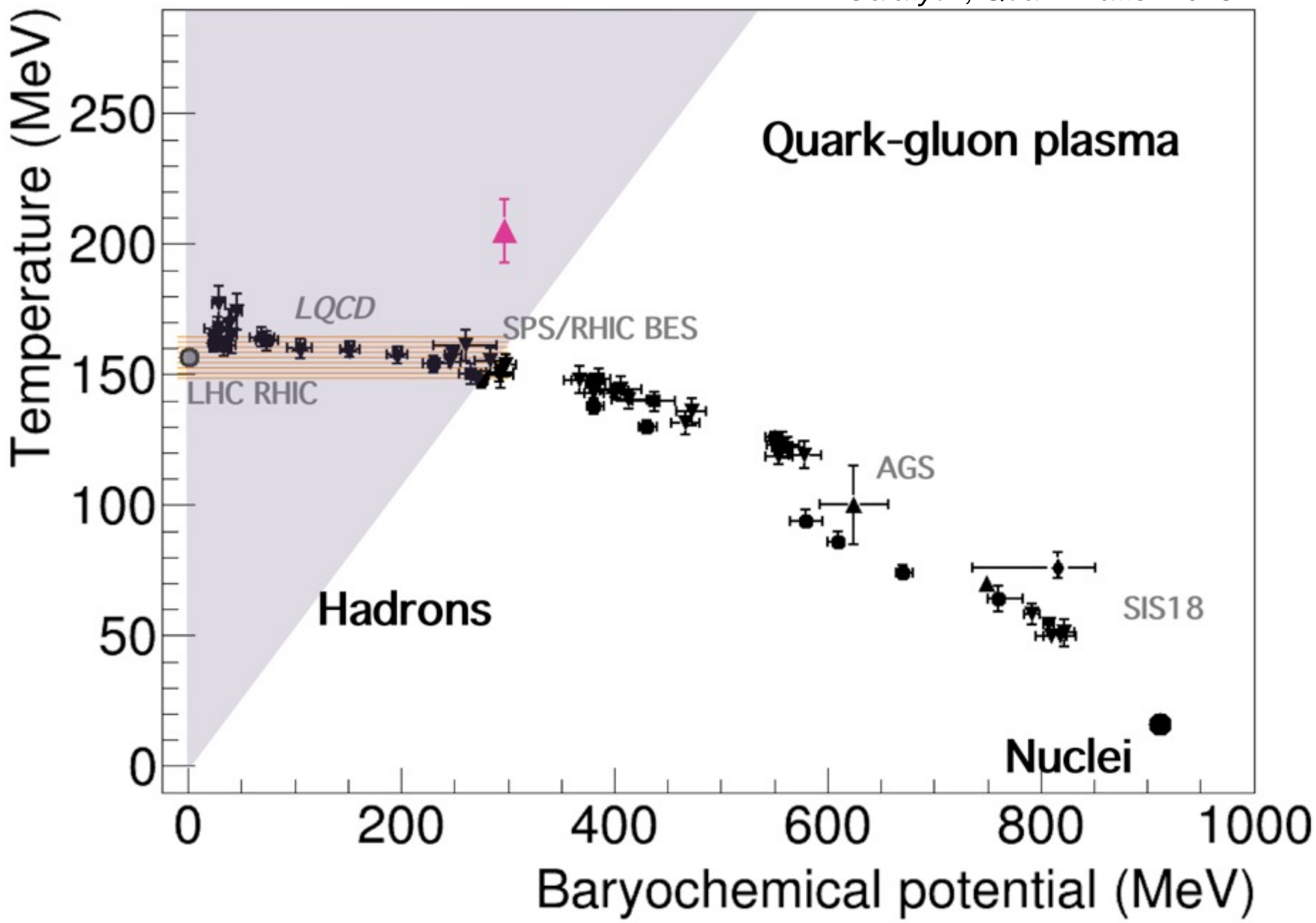
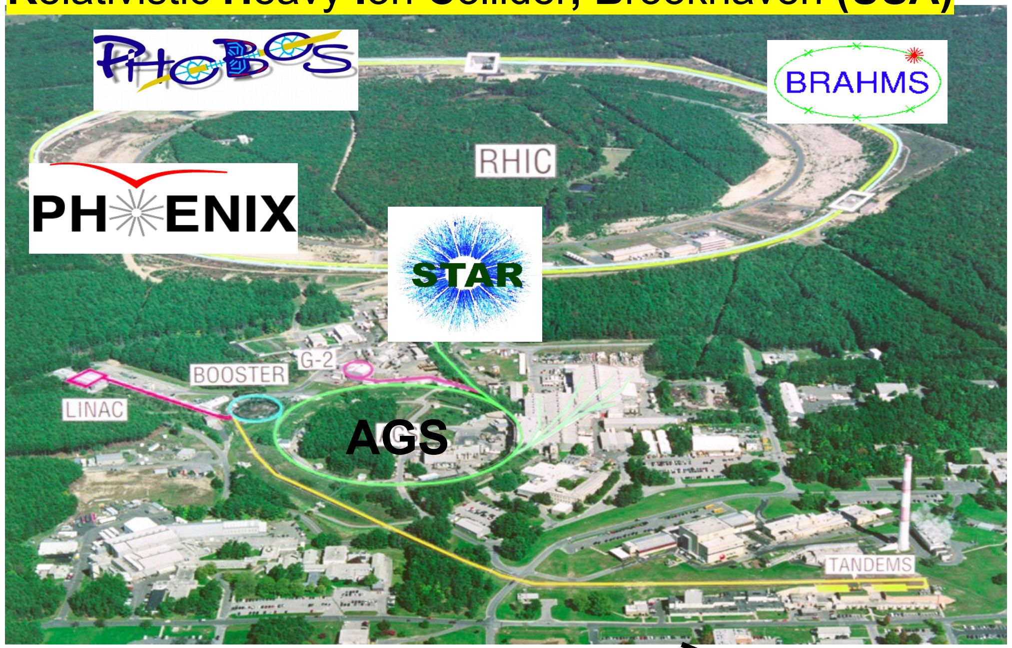
T. Galatyuk, Quark Matter 2018

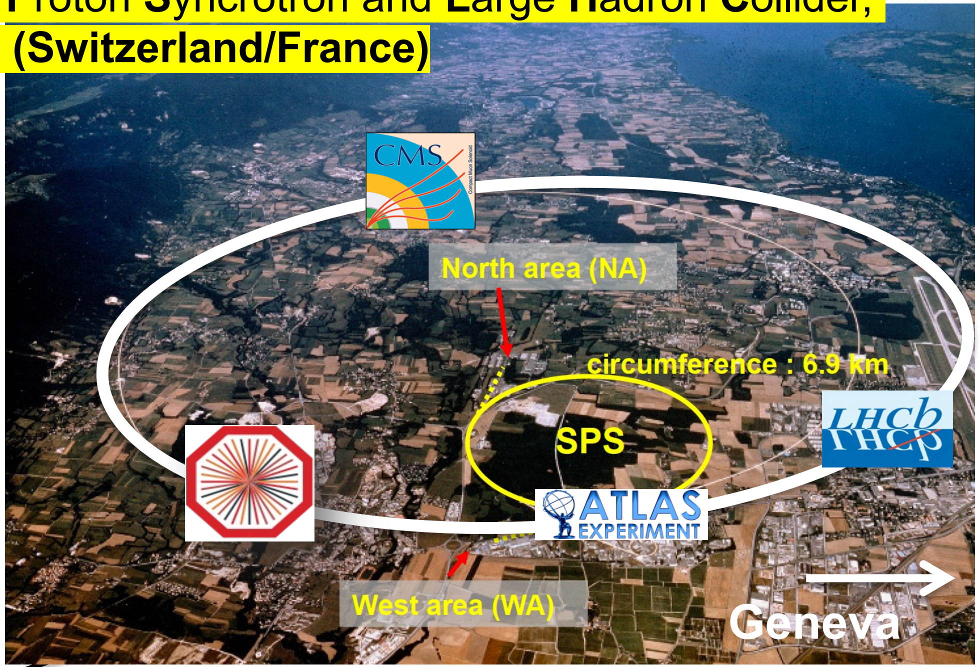


Heavy-ion physics worldwide: present / high energy

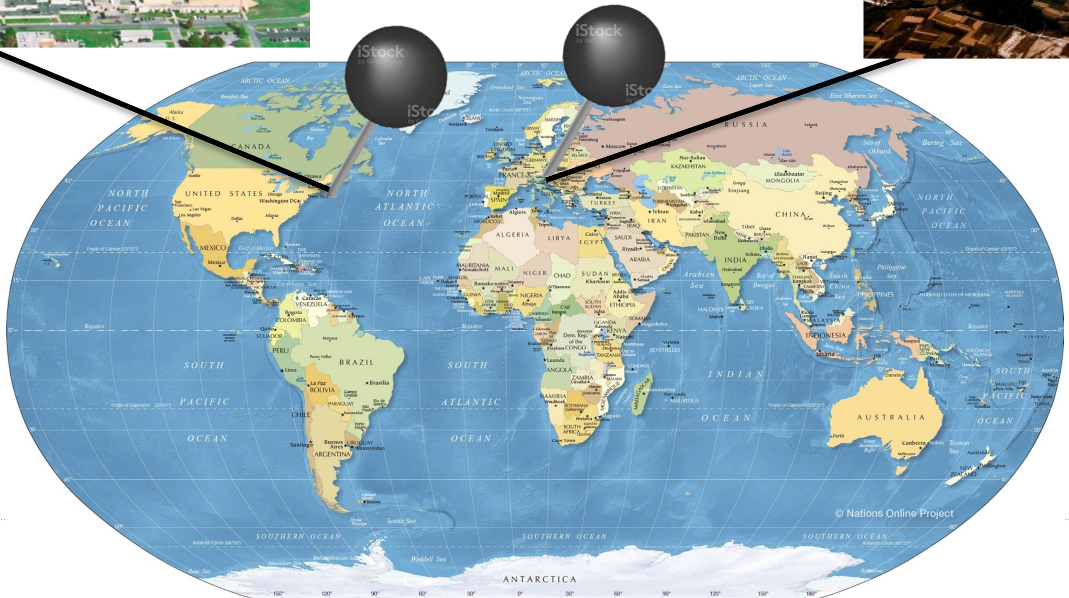
Relativistic Heavy Ion Collider, Brookhaven (USA)



Super Proton Syncrotron and Large Hadron Collider, CERN (Switzerland/France)



CERN SPS
- Operating since 1986
- Circumference 6.9 Km
- max p = 450 A/Z GeV
- $\sqrt{s_{NN}}$ < 20 GeV
- Ongoing: NA61/Shine

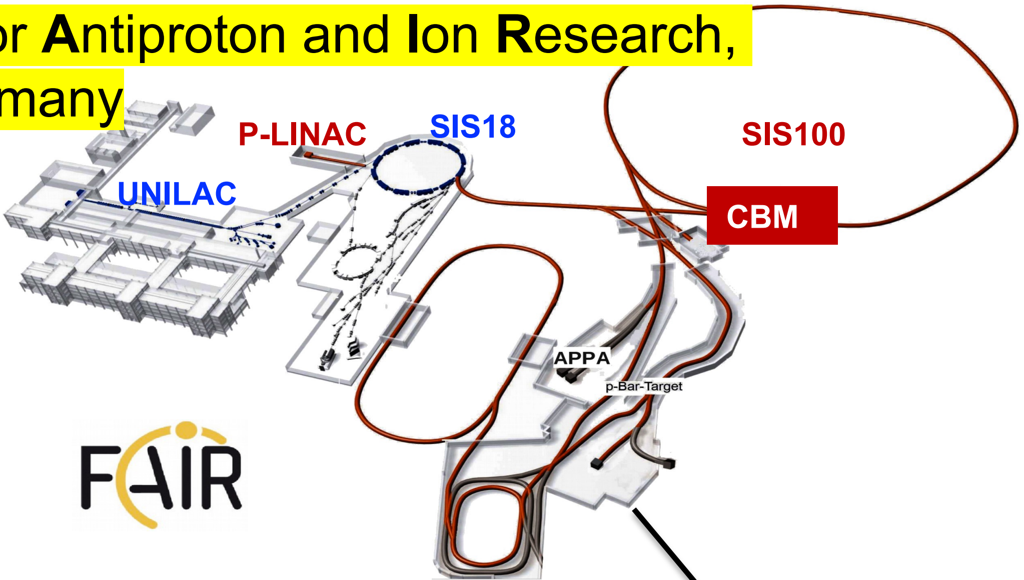


Brookhaven RHIC
- Operating since 2000
- Circumference 3.83 km, 2 rings
- Superconducting magnets
- $\sqrt{s_{NN}}$ = 3 - 200 GeV in Au-Au
- Beam energy scan I: 2010-11
- Beam energy scan II: 2019-22
- Ongoing exp: STAR

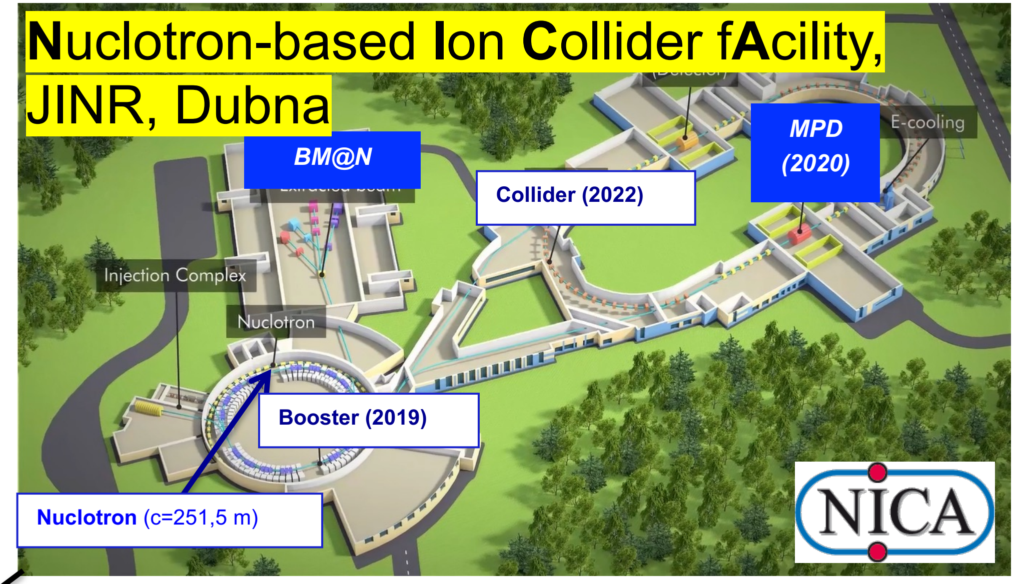
CERN LHC
- Operating since 2009
- Run III: started in 2022
- Circumference: 27 km
- B-field: 8 T, superconducting
- pp $\sqrt{s}$ = 0.9 – 13.6 TeV
- Pb-Pb $\sqrt{s_{NN}}$ = 2.76-5.5 TeV
- Main ongoing: ALICE, ATLAS, CMS, LHCb

Heavy-ion physics worldwide: future / low energy

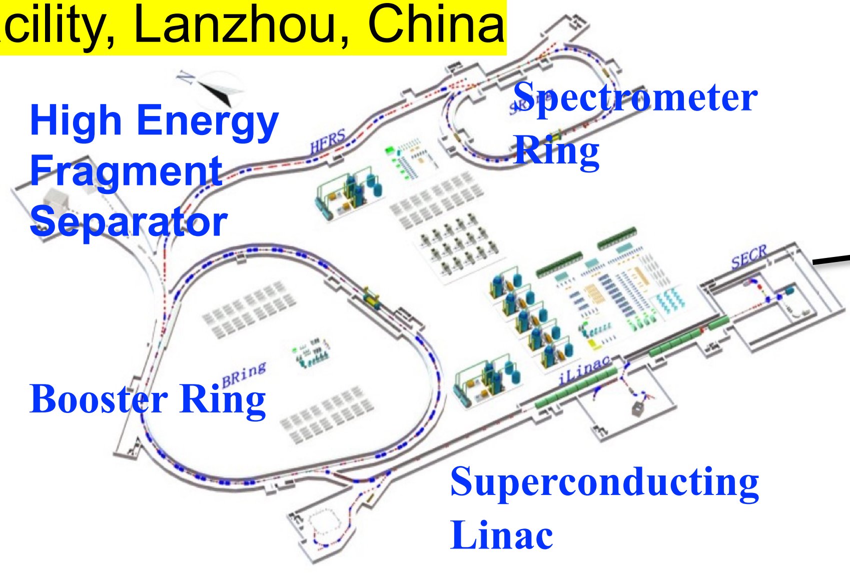
Facility for **A**ntiproton and **I**on **R**esearch, GSI, Germany

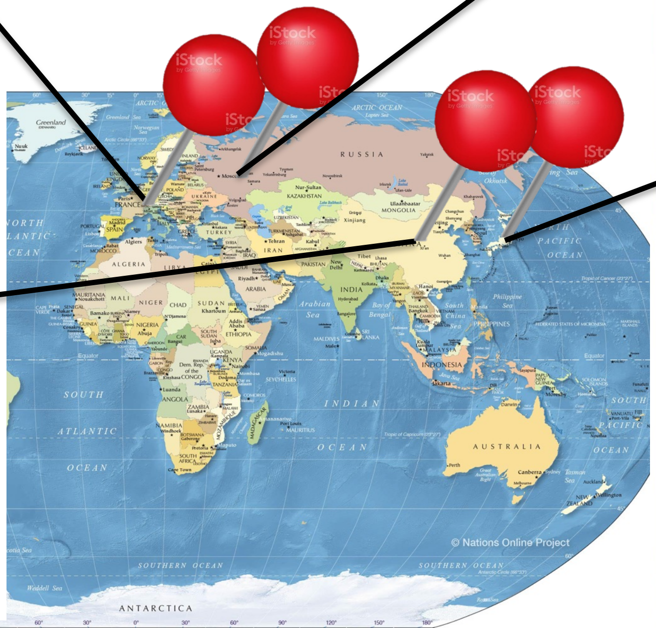


Nuclotron-based **I**on **C**ollider f**A**cility, JINR, Dubna

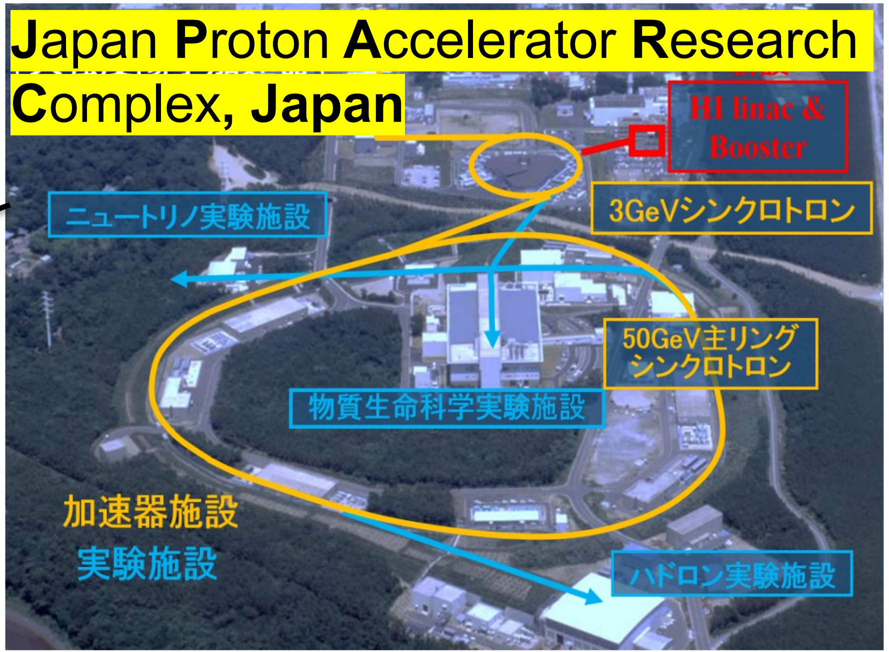


High-**I**ntensity Heavy Ion **A**ccelerator **F**acility, Lanzhou, China





Japan **P**roton **A**ccelerator **R**esearch **C**omplex, Japan



Heavy-ion physics at the LHC

2010-2012		2015-2018		2022-2024		2028-2030		2032-2034		2036-2039
Run I	LS1	Run 2	LS2	Run 3	LS3	Run 4	LS4	Run 5	LS5	Run 6

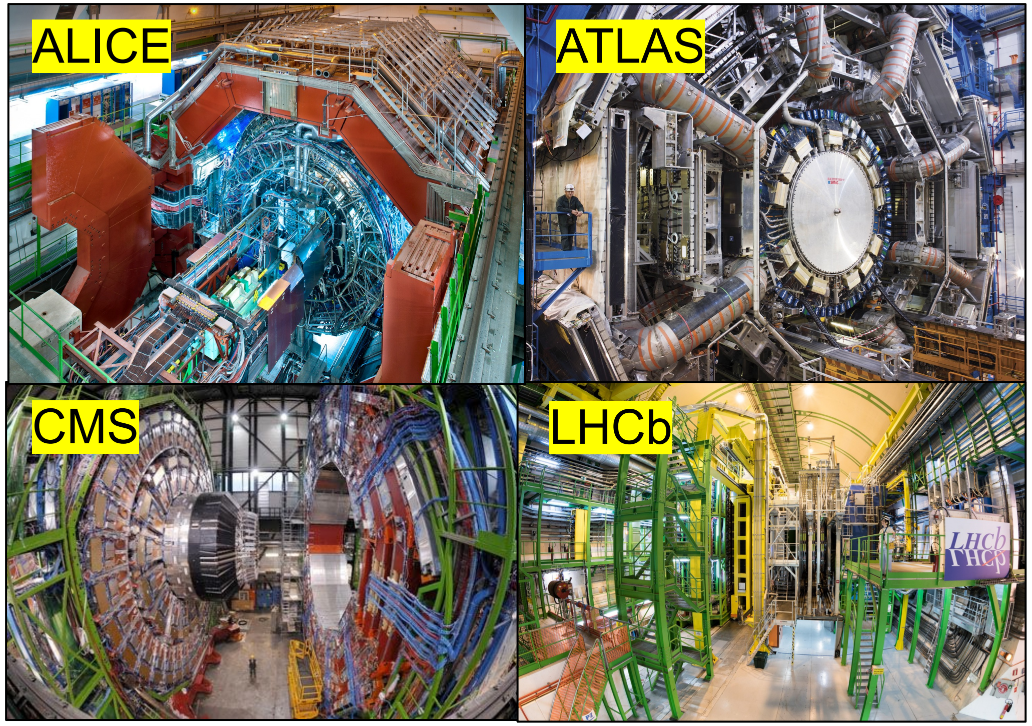
High luminosity LHC

Major upgrades for **ALICE and LHCb**

We are here

ATLAS and CMS phase II
ALICE ITS3 and FoCal

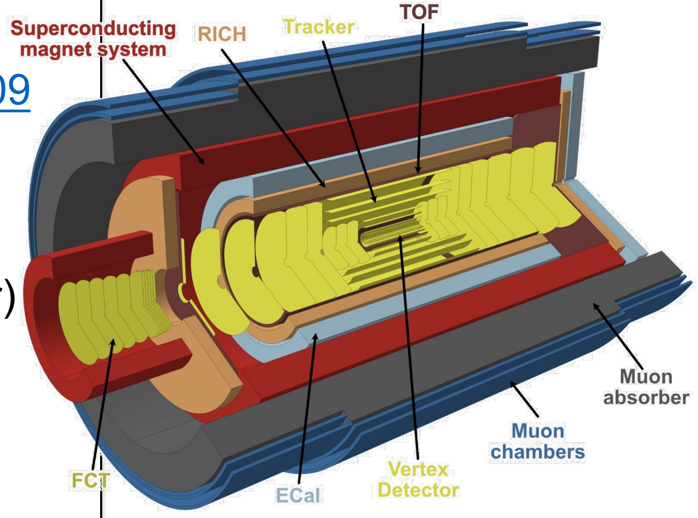
ALICE 3: a whole new dedicated HI detector!
LHCb upgrade II



ALICE 3

- Letter of Intent: CERN-LHCC-2022-009
- next-generation HI experiment
- all-Si MAPS tracker
- ultimate vertex detector
- minimal mass (essentially only sensor)
- 5 mm from beam (LHC aperture)

Physics focus:
- low-p_T heavy-flavour
- electromagnetic radiation from QGP



Intermezzo: kinematic variables

Momentum and transverse momentum: $p = \sqrt{p_L^2 + p_T^2}$

Transverse mass: $m_T := \sqrt{m^2 + p_T^2}$

Rapidity (generalizes longitudinal velocity $\beta_L = p_L / E$): $y := \operatorname{arctanh} \beta_L = \frac{1}{2} \ln \frac{1+\beta_L}{1-\beta_L} = \frac{1}{2} \ln \frac{E+p_L}{E-p_L}$

- In a collider where 2 beams of different ions: $y_{CM} = \frac{1}{2} \ln \frac{Z_1 A_2}{A_1 Z_2}$
- In fixed-target mode: $y_{CM} = (y_{\text{target}} + y_{\text{beam}})/2 = y_{\text{beam}}/2$

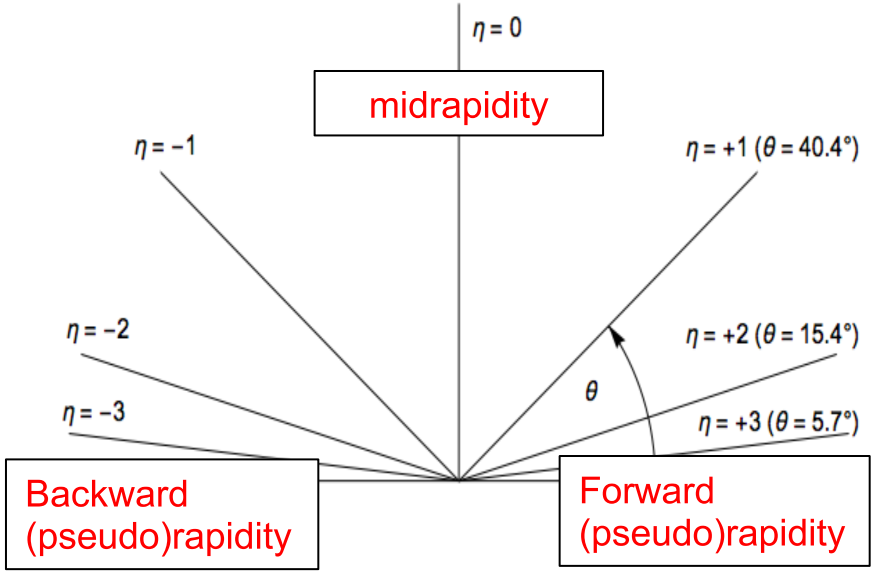
The rapidity can be approximated by **pseudorapidity** in the ultra-relativistic limit (*p>>m*):

$$y = \frac{1}{2} \ln \frac{E + p\cos\vartheta}{E - p\cos\vartheta} \overset{p \gg m}{\approx} \frac{1}{2} \ln \frac{1+\cos\vartheta}{1-\cos\vartheta} = \frac{1}{2} \ln \frac{2\cos^2\frac{\vartheta}{2}}{2\sin^2\frac{\vartheta}{2}} = -\ln\left[\tan\frac{\vartheta}{2}\right] =: \eta$$

$$\cos(2\alpha) = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha$$

where ϑ is the angle between the direction of the beam and the particle.

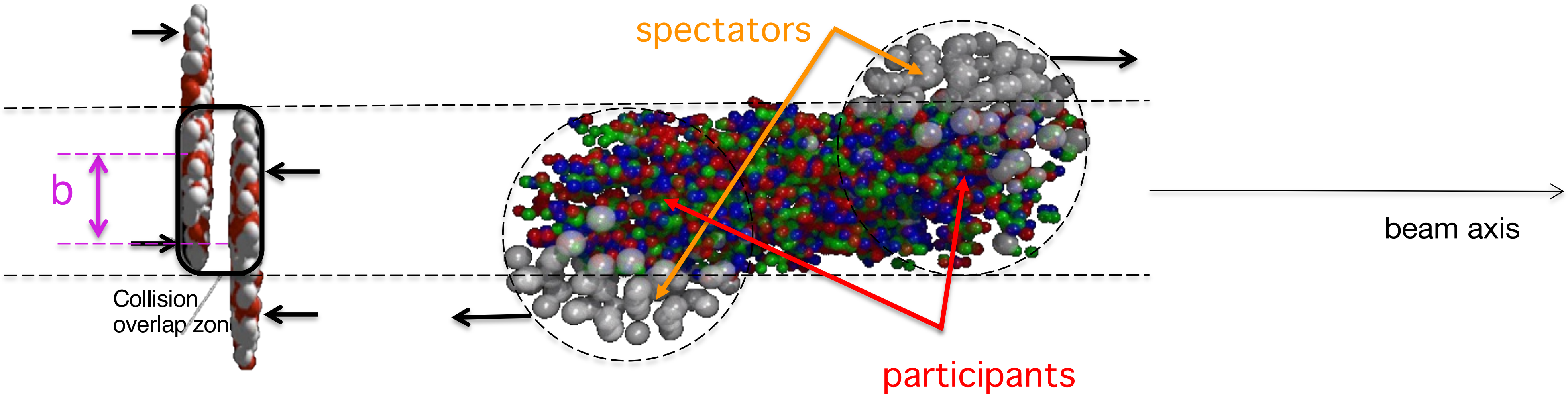
In general y ≠ η, especially at low momenta.



Geometry of heavy-ion collisions 1/2

We can control **a posteriori** the geometry of the collision by selecting in **centrality**.

Centrality = fraction of the total hadronic cross section of a nucleus-nucleus collision, typically expressed in percentile, and related to the impact parameter (**b**)

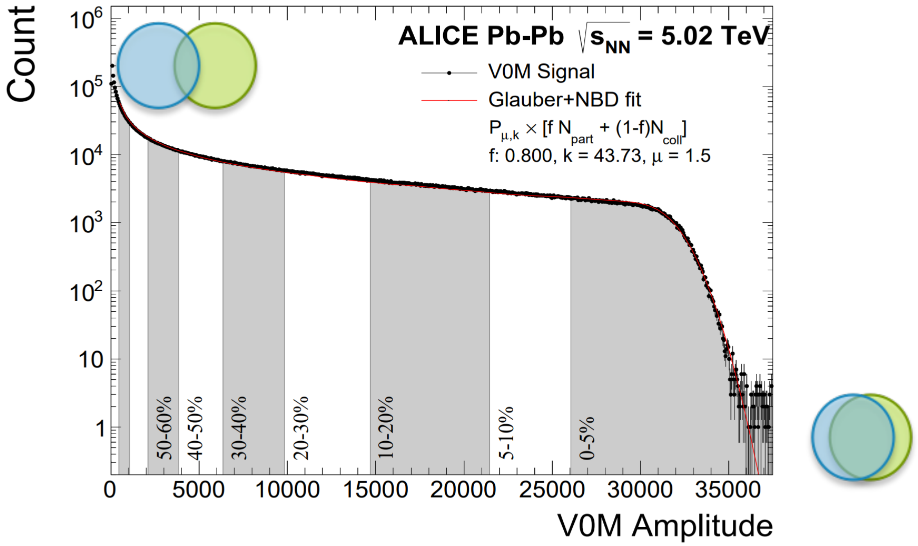


Other variables related to centrality:

- N_{coll}, number of binary nucleon-nucleon collisions
- N_{part} number of participating nucleons

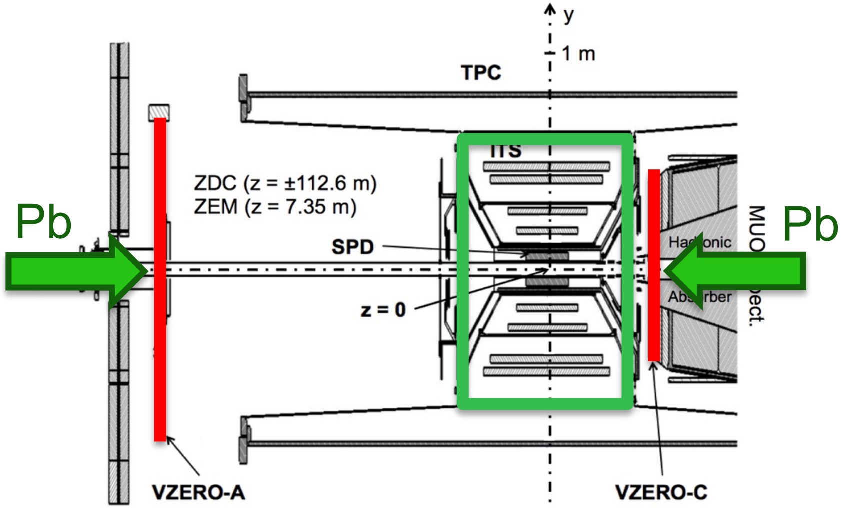
Geometry of heavy-ion collisions 2/2

More **central**, ie. “head-on” collisions
→ smaller impact parameter
→ larger overlap region
→ more participants
→ more particles produced



ALICE, PRL 106 (2011) 032301, PRC 91 (2015) 064905

More **peripheral** collision
→ larger impact parameter
→ smaller overlap region
→ less participants
→ fewer particles produced



Centrality is determined by counting the number of particles (multiplicity) or measuring the energy deposition in a region of phase space *independent* from the measurement, to avoid biases/autocorrelations in the results.

Production and characterization of the QGP at the LHC

Rapidity distributions in HI collisions

Before the collision: beams with given rapidity

E.g. at RHIC:

- p_{BEAM} = 100 GeV/c per nucleon
- $E_{BEAM} = \sqrt{(m_p^2+p_{BEAM}^2)}$ = 100.0044 per nucleon
- β = 0.999956, $\gamma_{BEAM} \approx 100$
- $y_{BEAM1} = -y_{BEAM2} = 5.36 \rightarrow \Delta y = 10.8$

After the collision, 2 possible scenarios

1. **Nuclei stopping**
   - For $\sqrt{s_{NN}}$ ~ 5 -10 GeV (AGS,…)
2. **Transparency**
   - For $\sqrt{s_{NN}}$ > 100 GeV (RHIC, **LHC**)
   - nuclei slow down to lower γ and y
   - particles are produced with a ''plateau'' at midrapidity

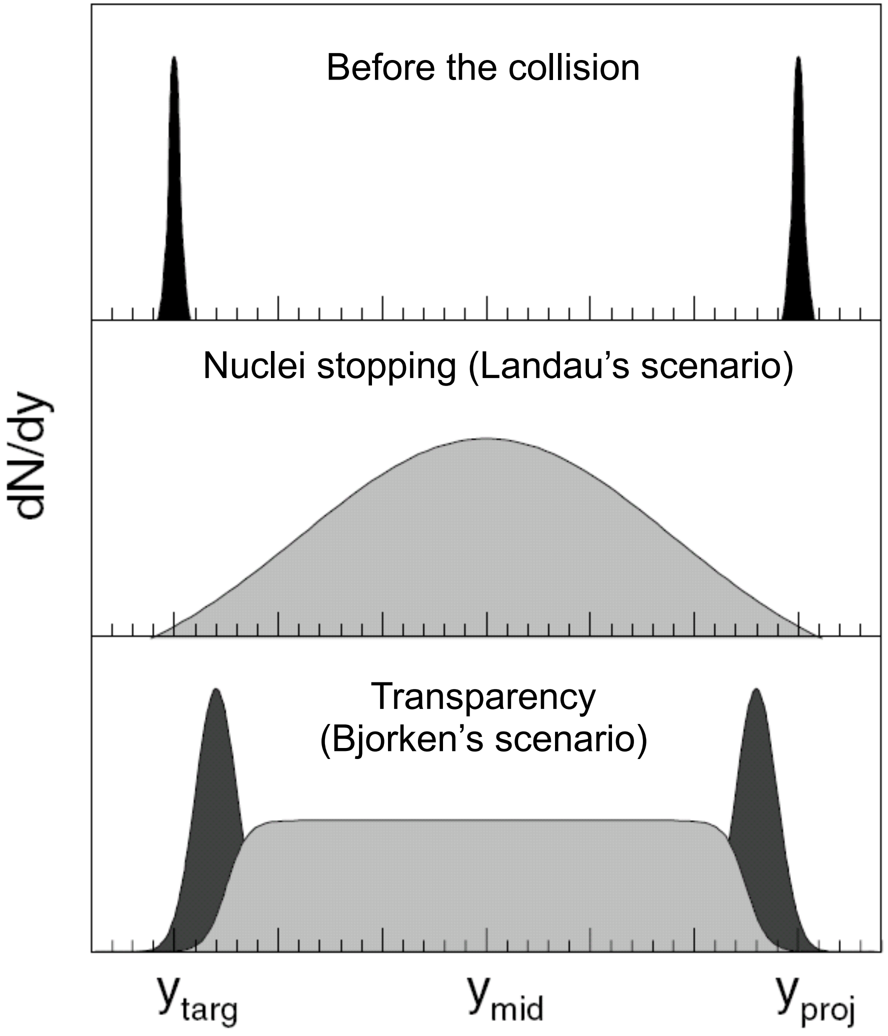
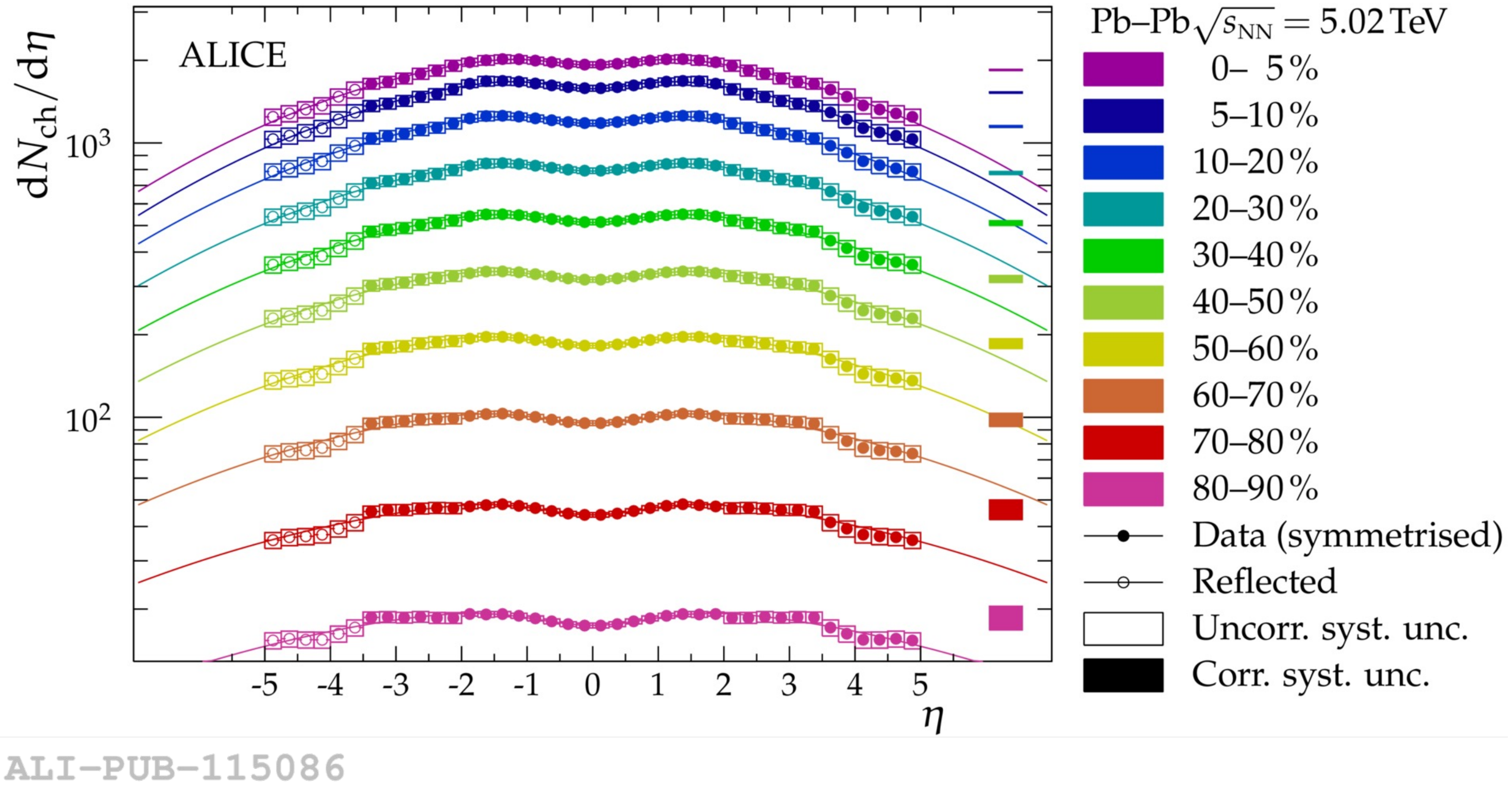


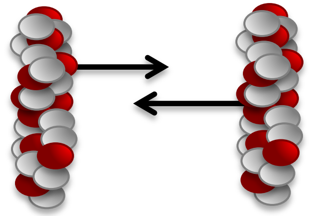
Figure from K. Reygers

Charged particle multiplicity vs centrality

Charged particle pseudorapidity density

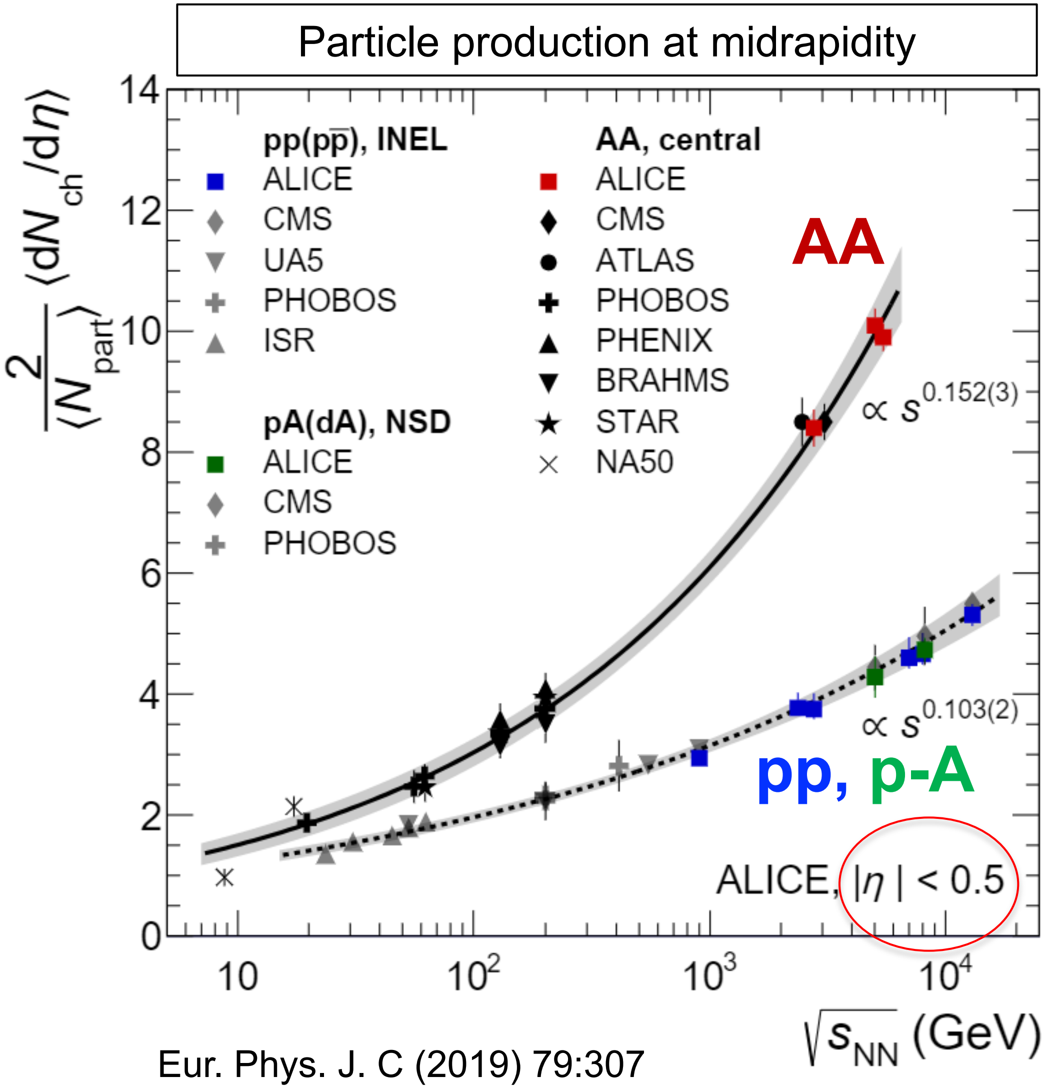


ALICE, Phys.Lett. B 772 (2017) 567-577



Pb-Pb

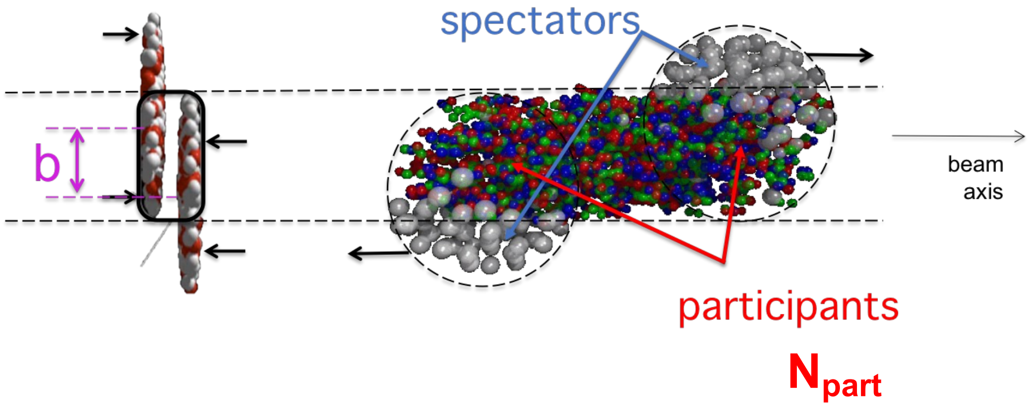
Charged particle production in central HI collisions



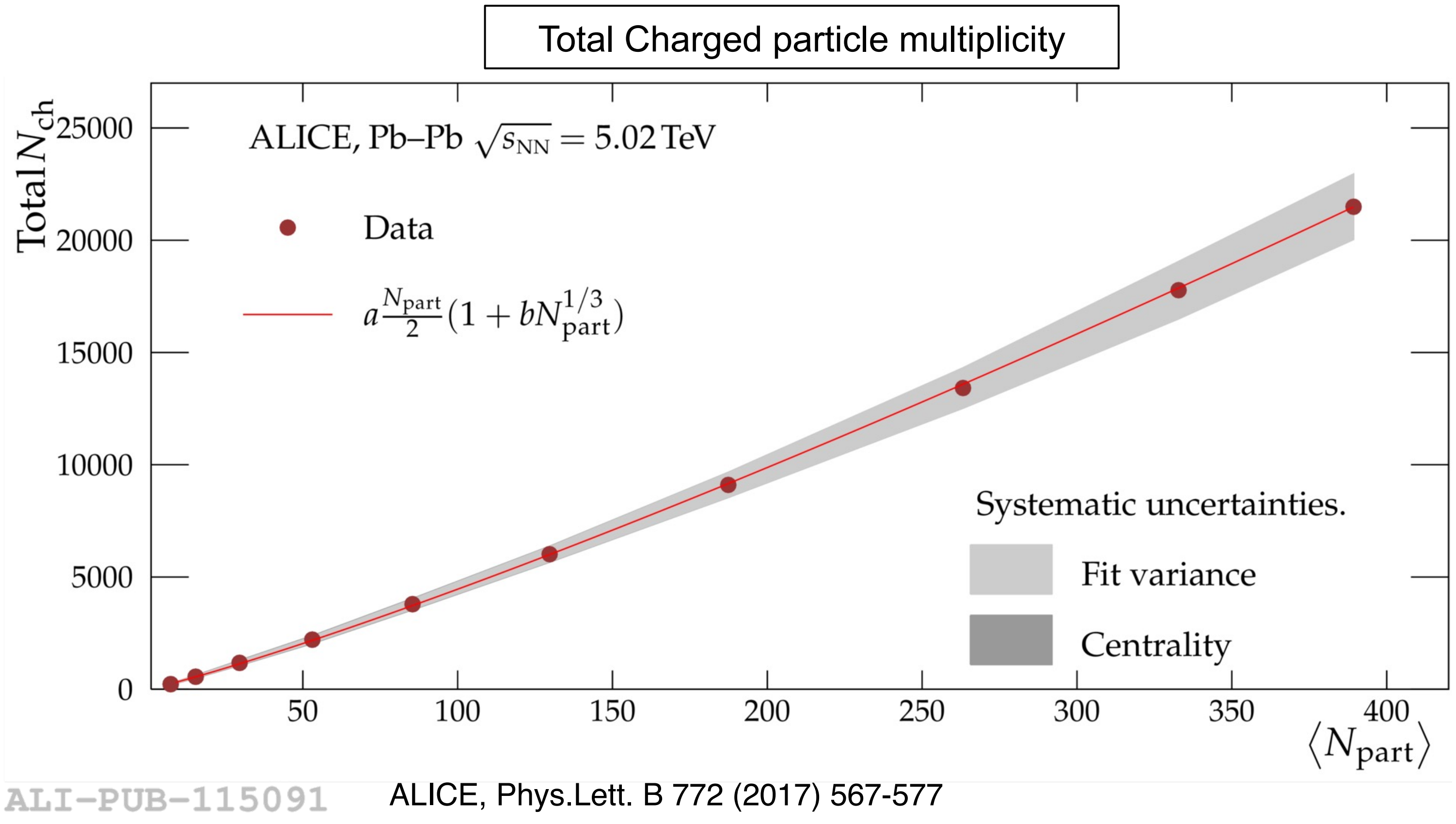
Eur. Phys. J. C (2019) 79:307

Particle production per participant in HI collisions follows a steeper power law than in pp, pA and increases by 2-3x from RHIC to the LHC

Heavy-ion collisions are more efficient in transferring energy from beam- to mid- rapidity than pp



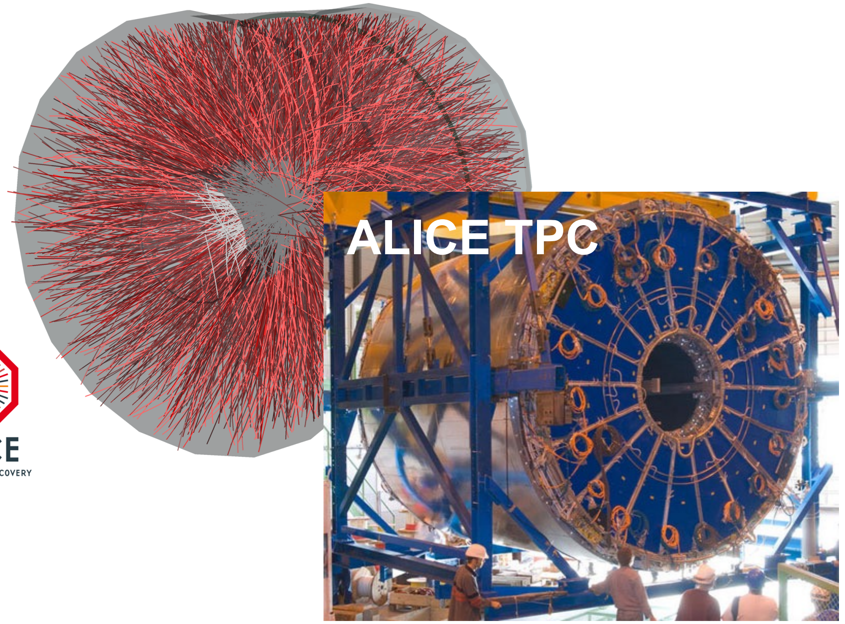
How many particles are created in a collision?



ALICE, Phys.Lett. B 772 (2017) 567-577

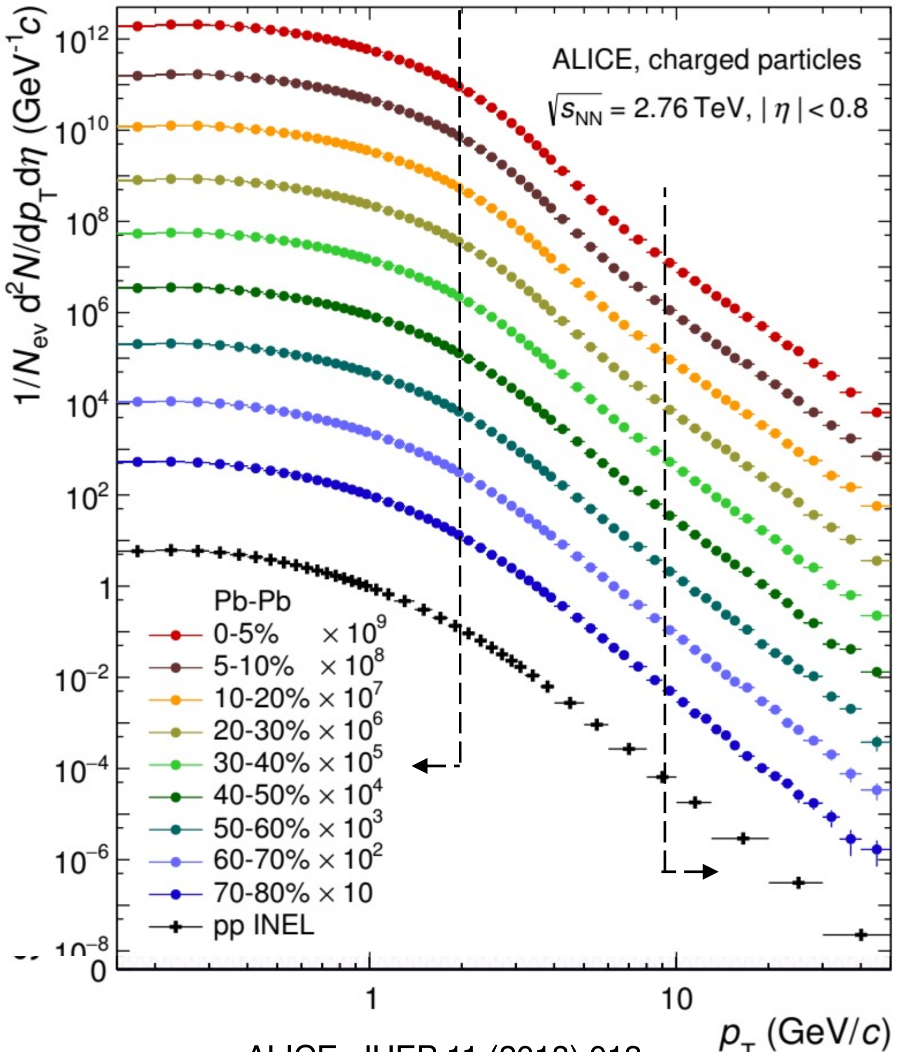
In a central Pb-Pb collision at the LHC, more than 20000 charged tracks must be reconstructed.

→ High granularity tracking systems, primary importance of tracking, vertexing calibration



Particle "spectra"

Charged particle transverse momentum distribution



ALICE, JHEP 11 (2018) 013

Low p_T *(< 2 GeV/c)*

- Particle spectra are described by a Boltzmann distribution → "thermal", ~ $\exp(-1/k_B T)$
- "Bulk" dominated by light flavor particles
- Non-perturbative QCD regime

High p_T *(> 8-10 GeV/c)*

- Particle spectra described by a power law
- Dominated by parton fragmentation (jets)
- Perturbative QCD regime

Mid p_T *(2 to 8 GeV/c)*

- Interplay of parton fragmentation and recombination of partons from QGP

Heavy-ion and high-energy physics have different goals and thus different detector requirements.

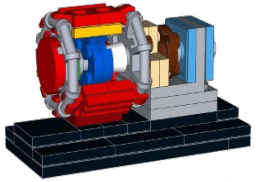
<u>Observables</u>:

- soft (low p_T) and hard (high p_T) probes
- hadron production rates (needs PID)
- flow (needs acceptance coverage)
- photon/W/Z (calorimetry)
- jets (coverage, high p_T)

In HI physics also emphasis on:

- **midrapidity** measurements
- **identification** of hadron species
- soft (non-perturbative) regime, i.e. **low p_T**
- **minimum bias** events

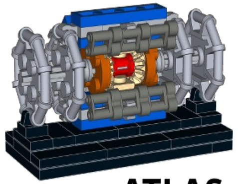
Complementarity of the LHC experiments



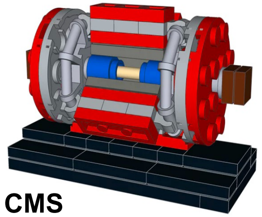
ALICE

ALICE

- Low p_T
- PID
- Low material budget next to IP



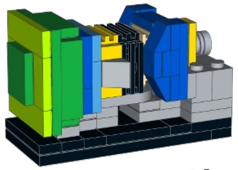
ATLAS



CMS

ATLAS/CMS

- Wide pseudorapidity coverage
- High $p_{T,}$ jets



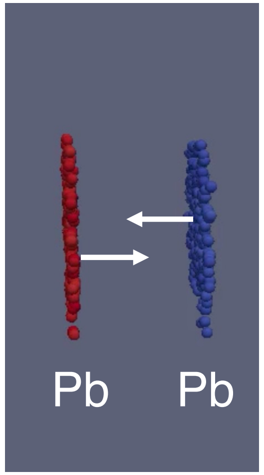
LHCb

LHCb

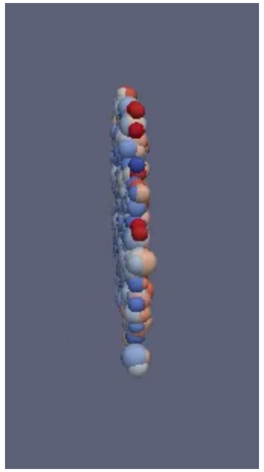
- Forward pseudorapidity
- PID
- Fixed target

The standard model of heavy-ion collisions

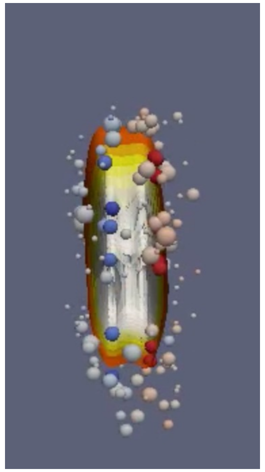
Initial state



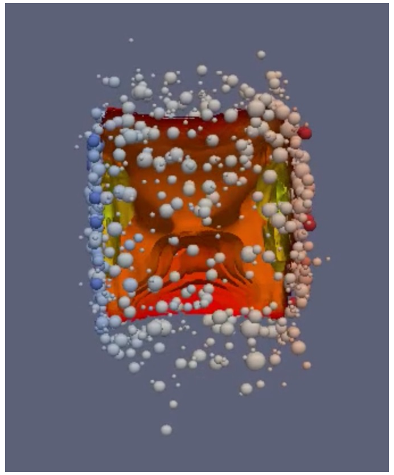
Hard scatterings



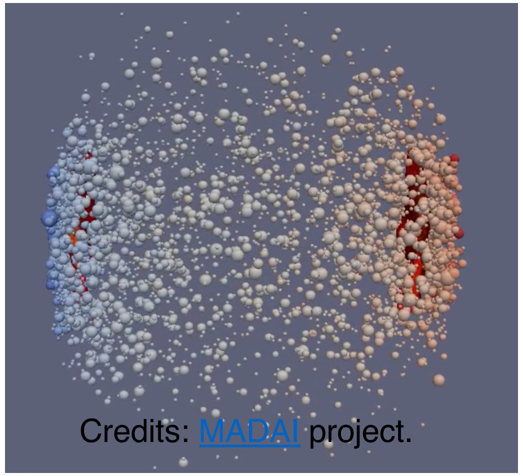
QGP formation



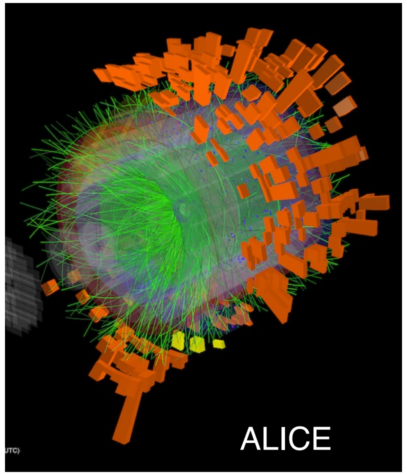
Hydrodynamic expansion



Hadronization and freeze-out



Detection



Time: $\tau = 0$ — $\tau \sim 1$ fm/c — $\tau \sim 10$ fm/c — $\tau > 10$ pm/c

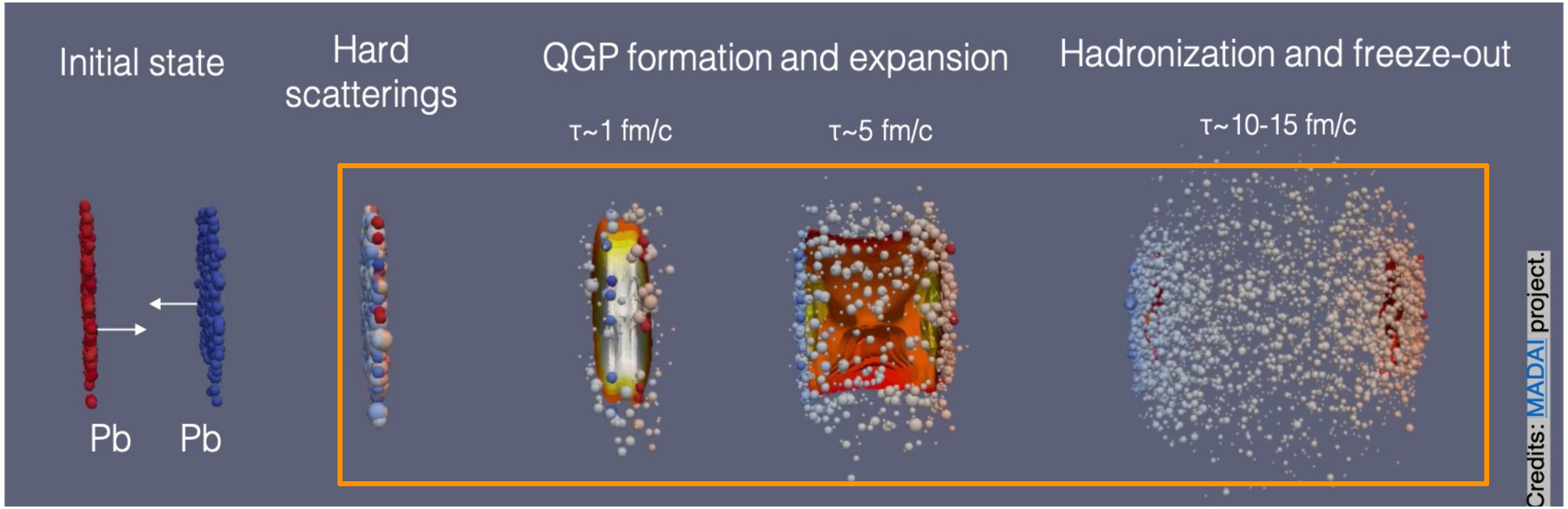
Temperature: T ~ 200-800 MeV [1] — T ~ 155-100 MeV [4]

$T_{c,\, lQCD} \sim$ 155-159 MeV [2,3]

No direct observation of the QGP is possible
→ rely on emerging particles as "probes"

[1] F. Gardim et al. Nature Phys. 16 (2020) 6, 615-619
[2] A. Bazavov et al., Phys. Lett. B 795 (2019)
[3] Borsaniy et al. PRL 125 (2020) 5, 052001
[4] A. Andronic et al., Nature 561 (2018) 7723, 321-330

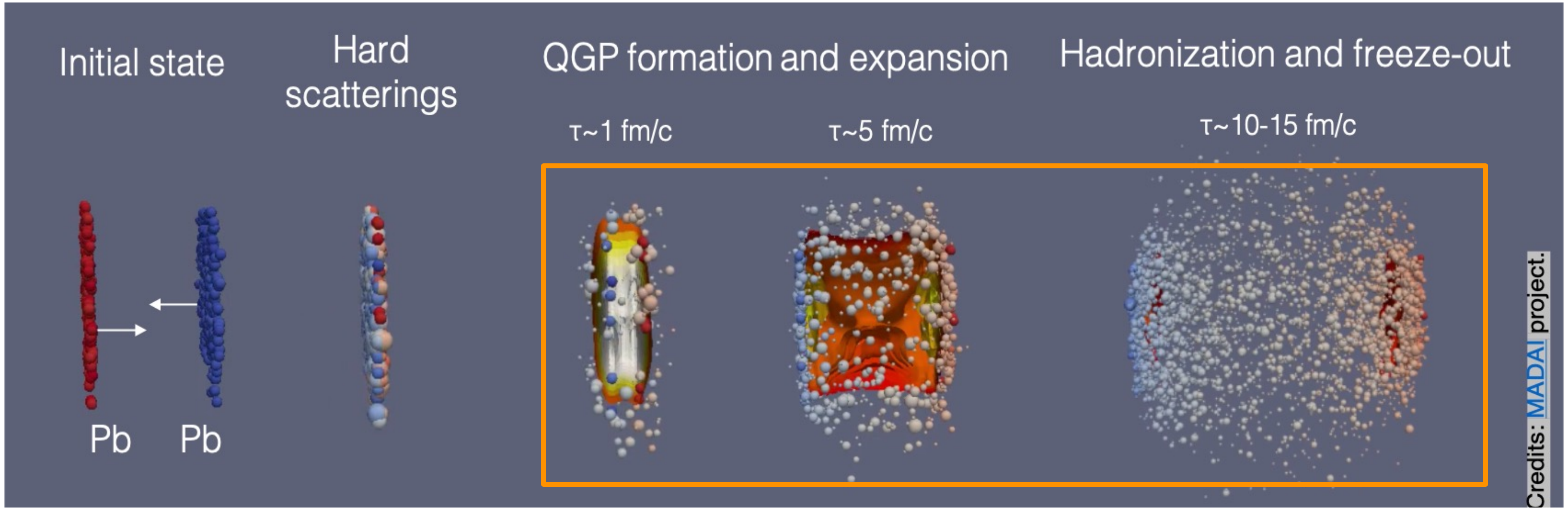
Probes 1/2



1 fm/c = 3x10^{-24} s, 1 MeV ~ 10^{10} K

High-p_T partons (→ jets), charm and beauty quarks (→ open HF, quarkonia)
produced in the early stages in hard processes,
traverse the QGP interacting with its constituents = colored probes in a colored medium
→ rare, calibrated probes, perturbative QCD
→ in-medium interaction (energy loss) and transport properties
→ in-medium modification of the strong force and of fragmentation

Probes 2/2



1 fm/c = 3x10^{-24} s, 1 MeV ~ 10^{10} K

Low-p_T particles, light flavour hadrons (u,d,s, +nuclei)
produced from hadronization of the strongly-interacting, thermalized QGP constitute the bulk of the system
→ **non-perturbative QCD regime**
→ **thermodynamical, hydrodynamical and transport properties**

Bonus material

Characteristics of a heavy-ion detector: ALICE

ALICE is the dedicated heavy-ion detector at the LHC, designed and built specifically for this purpose.

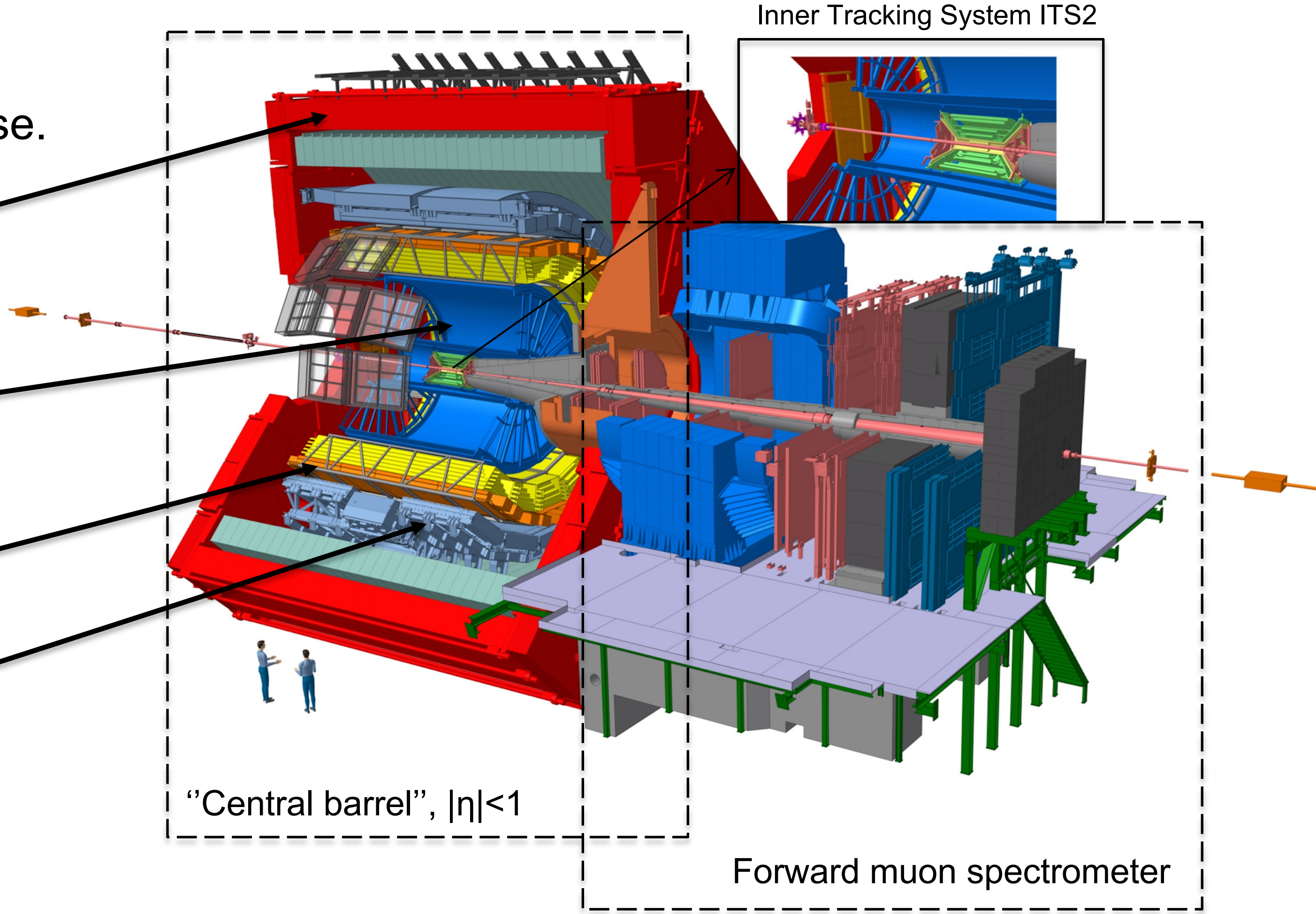
Solenoid: magnetic field B = 0.5 T

Inner Tracking System + Time Projection Chamber: vertexing and tracking + identification (TPC) down to very low p_T ~ 0.1 GeV/c

Time-Of-Flight, TRD, HMPID, etc.: Particle identification detectors

Electromagnetic calorimeters

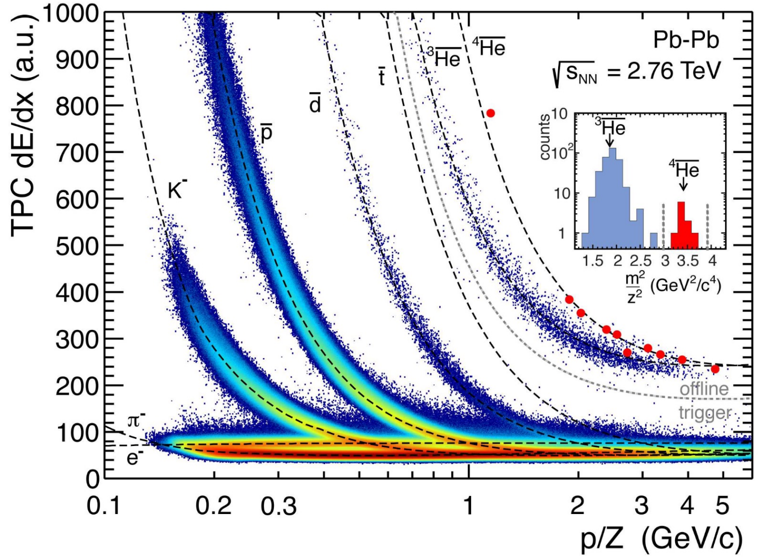
+ Forward rapidity detectors and ZDC: trigger, centrality, event time determination, …



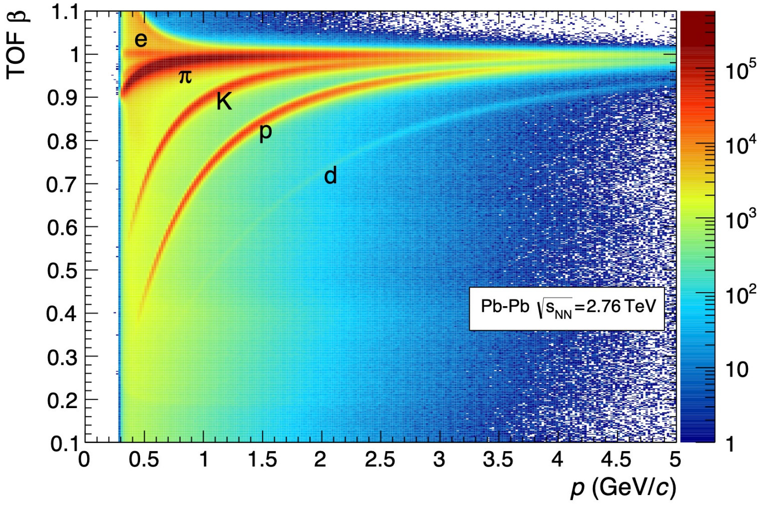
Particle identification

- Direct identification: π, K, p, light (anti)nuclei
- Electron identification using calorimeters and transition radiation detectors
- Strange and heavy-flavour hadrons:
    - reconstruction of secondary vertex and weak decay topology + PID + invariant mass reconstruction
- Photons detected in calorimeters and through pair production
- Quarkonia through leptonic decays

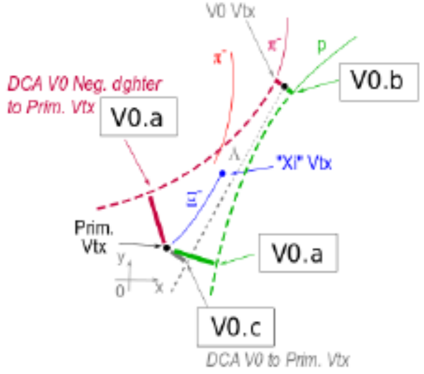
Energy loss of long lived particles in TPC



Particle velocity from TOF measurement and momentum



$\Xi \rightarrow \Lambda\pi^-$
(similarly for $\Omega \rightarrow \Lambda K^-$)



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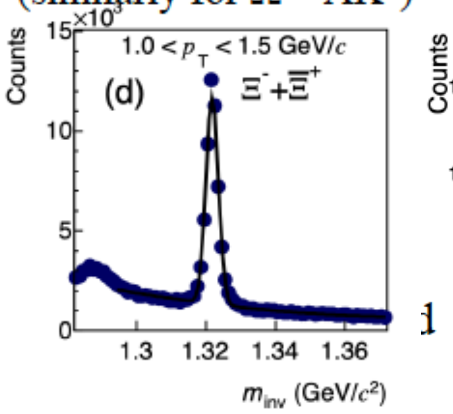
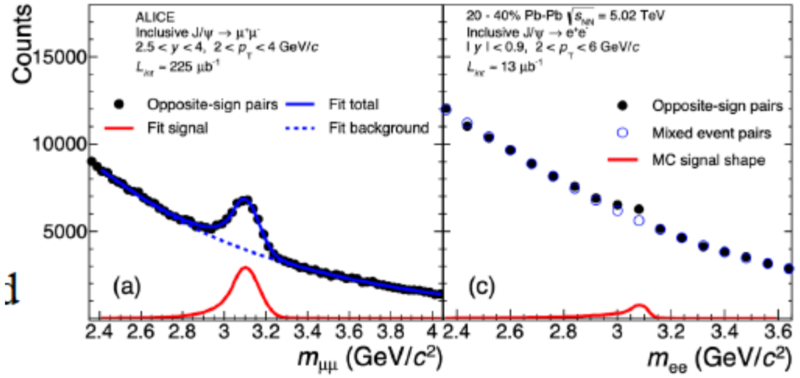
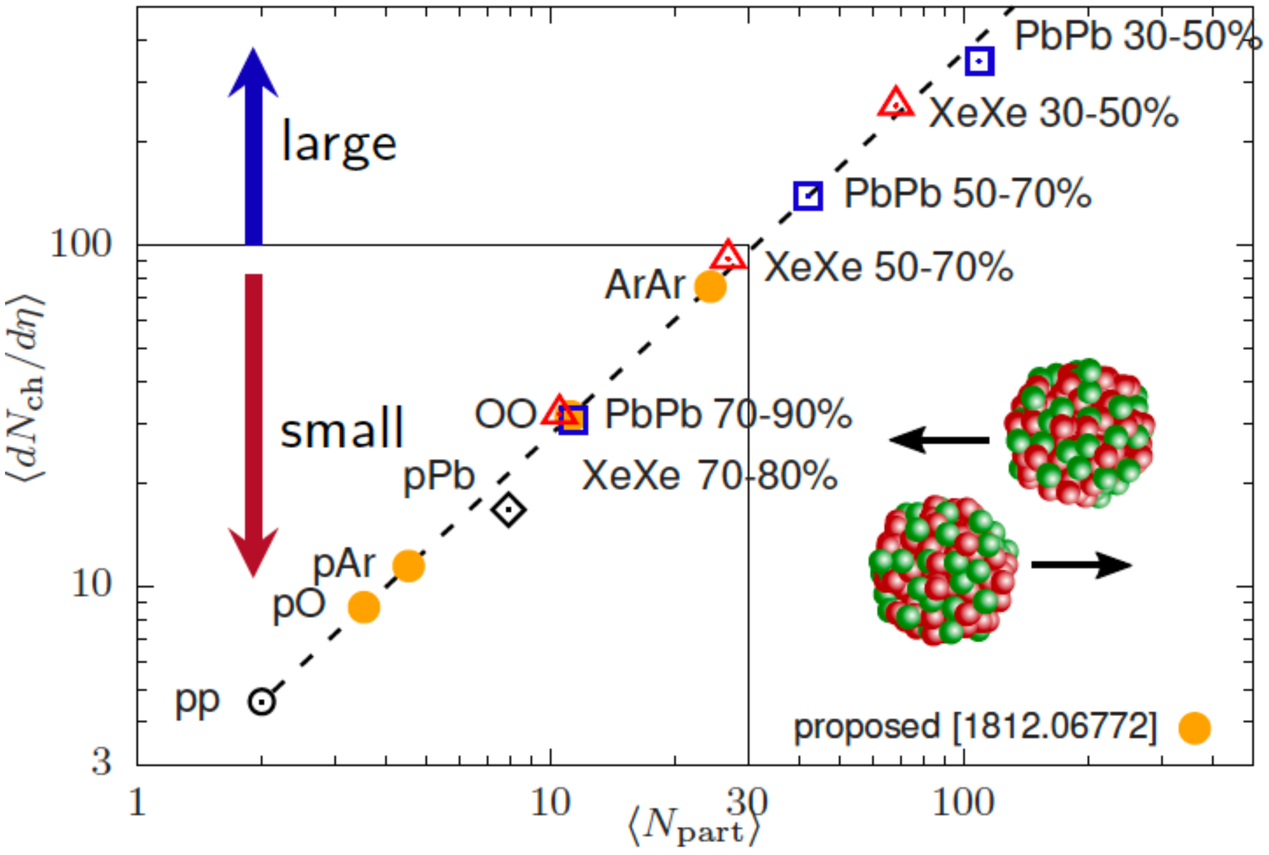
(similarly for $\Omega \rightarrow \Lambda K^-$)



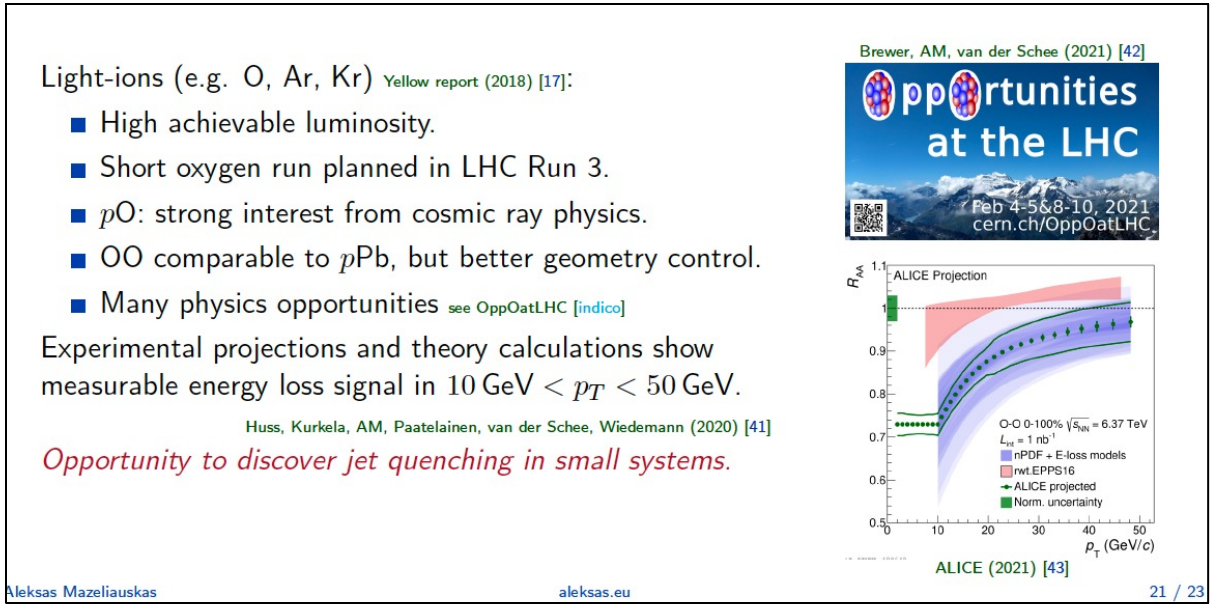
$J/\psi \rightarrow \mu^+\mu^-$ $J/\psi \rightarrow e^+e^-$



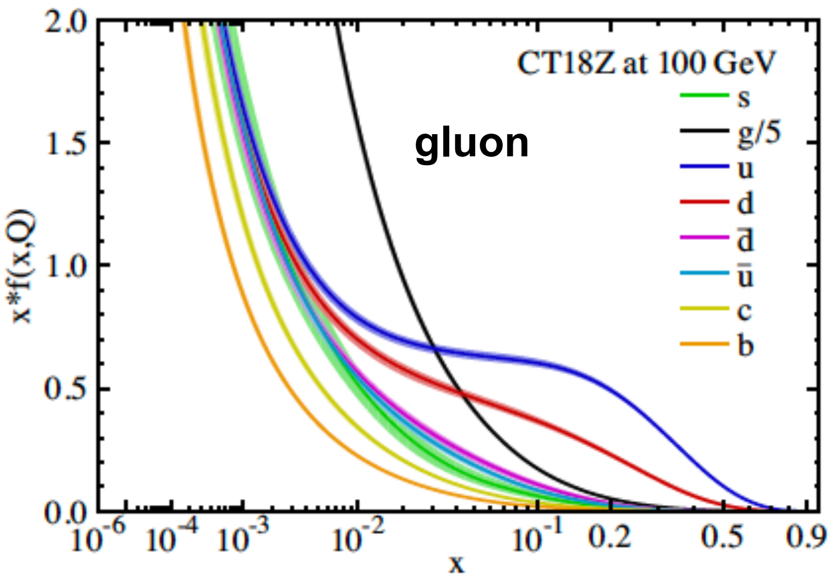
Light ions at the LHC



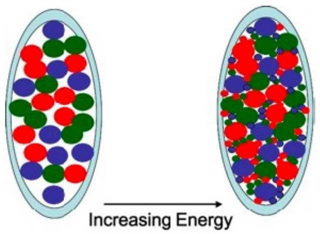
From A. Mazeliauskas, EPS-HEP 2021:



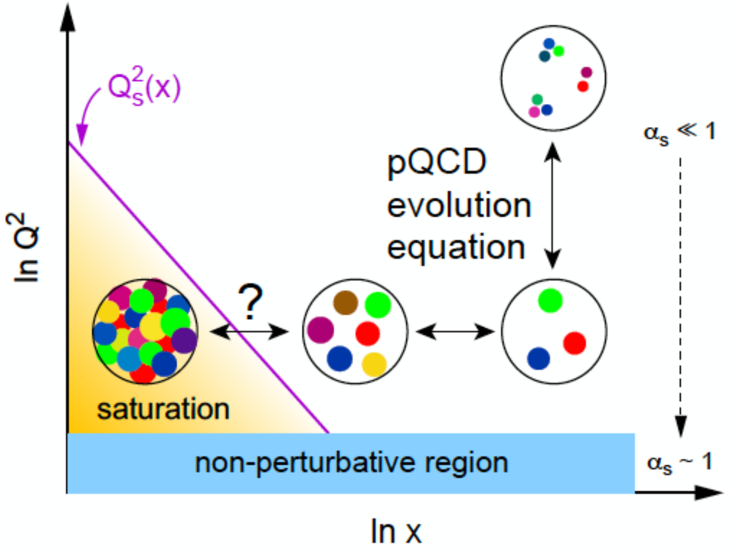
Initial stage of heavy ion collisions

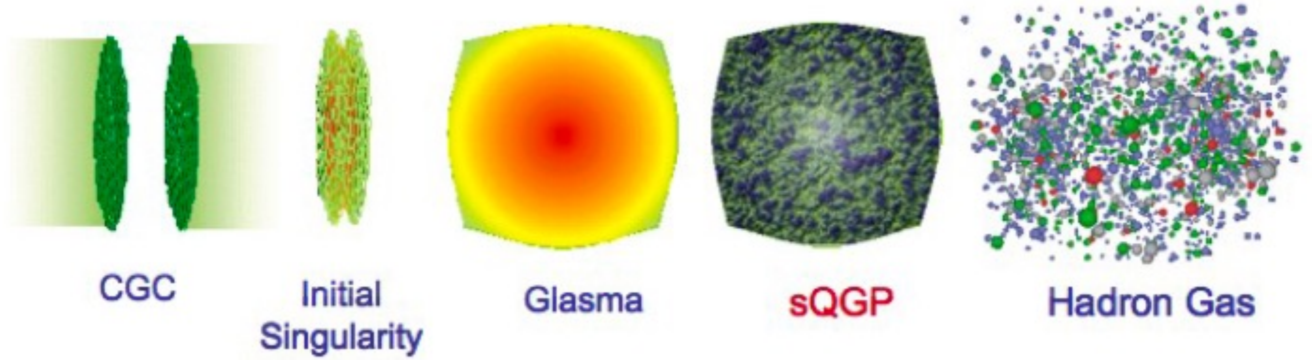


Color Glass Condensate: at high energy and small x, the hadron content is dominated by gluonic matter "packed" into high density



Saturation (momentum) scale
Q_{sat} = inverse size scale of smallest gluons which are closely packed
→ gluons of size larger than $1/Q_{sat}$ no longer fit





L. McLerran, https://bib-pubdb1.desy.de/record/296833/files/ismd08_mcl_intro-corr.pdf
+ more reviews in literature,

Glauber model

Nucleus-nucleus interaction as **incoherent superposition of nucleon-nucleon collisions** calculated in a probabilistic approach

[M. L. Miller et al., An. Rev. Nucl. Part. Sci. 57 (2007) 205-243]

- nucleons in nuclei are considered as point-like and non-interacting
- nuclei (and nucleons) have straight-line trajectories (no deflection)

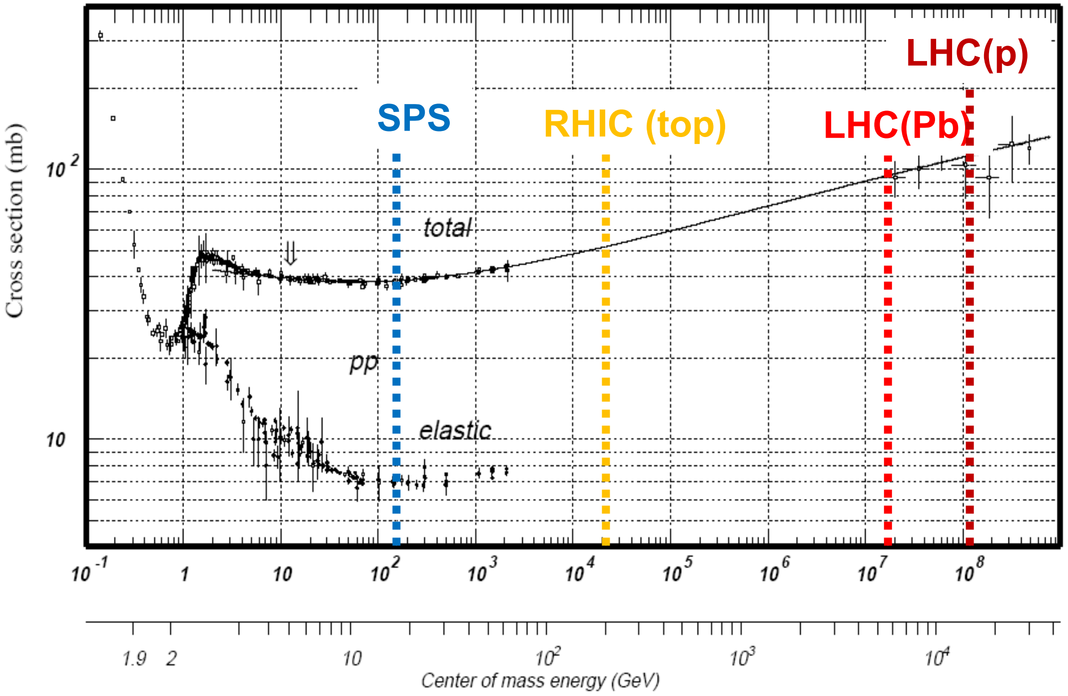
Input:

- Nucleon-nucleon inelastic cross section
- Nuclear density distribution, e.g. Fermi

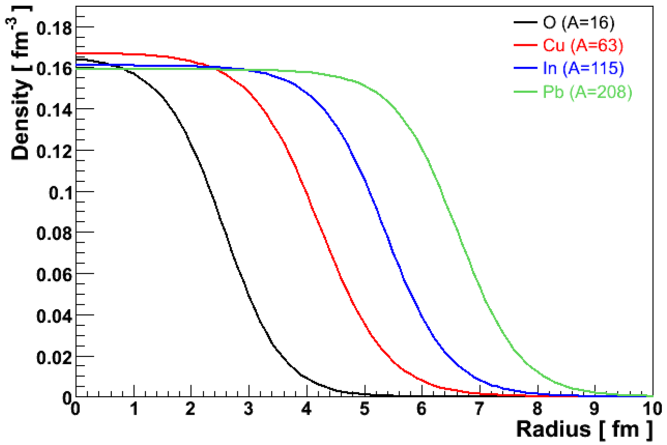
$$\rho(r) = \rho_0 \frac{1 + w(r/R)^2}{1 + \exp\left(\frac{r-R}{a}\right)}$$

ρ^0 = density in the nucleus center
R = nucleus radius
a = skin depth
w = deviations from spherical shape

Proton-proton cross section (from PDG)



Examples of density distributions of nuclei



Glauber model (2)

Output:

- Interaction probability
- **Number of elementary nucleon-nucleon collisions (N_{coll})**
- **Number of participant nucleons (N_{part})**
- **Number of spectator nucleons**
- Size of the nuclei overlap region

These variables are fundamental to study the scaling properties of observables in HIC – **Rule of thumb**:

- ***N_{part}*** scaling of soft particle production
  → bulk of the system

- ***N_{coll}*** scaling of high p_T particle production
  → hard partons produced early in the collision