Flavour Physics: A Taster

CERN Summer Student Lecture Programme 2023

Lecture 2 of 3: CP violation and the B factories

17-19 July 2023

Mark Williams
University of Edinburgh

Introduction

Yesterday we covered the foundations and motivations of the subject

- Quantum loops & indirect searches for new physics
- Discrete symmetries in nature
- Example: Neutral meson oscillations

Today we connect these ideas and examine them in the context of the standard model

- The CKM mechanism and quark mixing
- Complex CKM phases ⇋ CP violation
- Experimental constraints and the B factory era

Part I: Quark flavour in the SM

Quark mixing

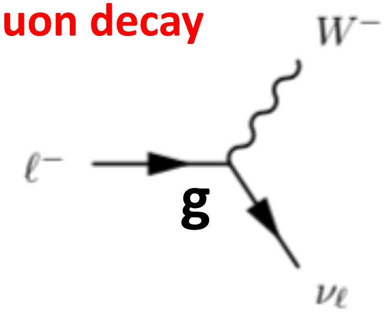
Weak interaction breaks C and P maximally, and CP a bit – **how?**

In 1960s, list of fundamental fermions was small:

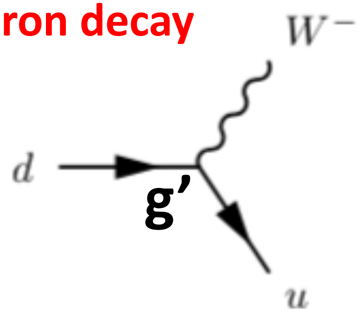
- 4 leptons (e, μ, ν_e, ν_μ)
- 3 quarks (u, d, s)

From particle lifetimes, can derive weak coupling strengths **g** for different decays…

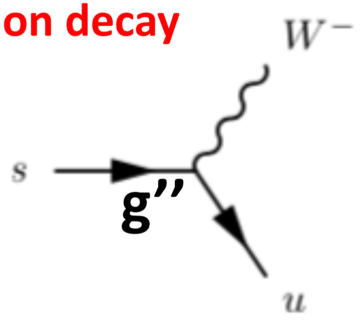
Muon decay



Neutron decay



Kaon decay



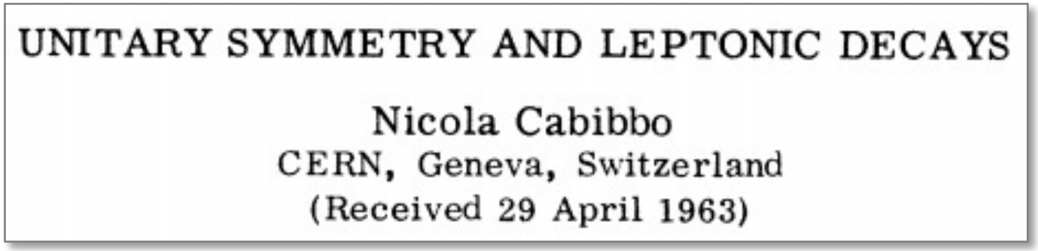
Find **g > g’ >> g’’** ⇒ why?

Quark mixing

Universal coupling can be recovered if weak interaction 'sees' rotated combination of quark flavours

θ_c = 13° from experiments

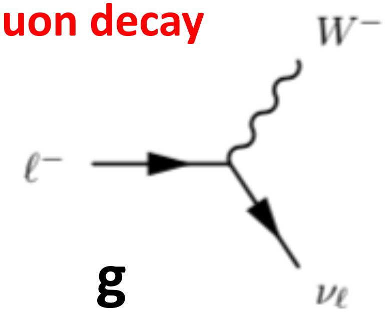
https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.10.531



UNITARY SYMMETRY AND LEPTONIC DECAYS

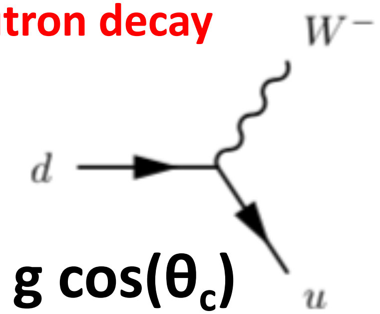
Nicola Cabibbo
CERN, Geneva, Switzerland
(Received 29 April 1963)

Muon decay



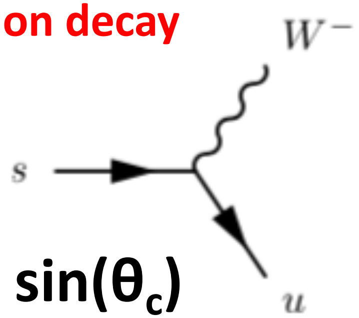
g

Neutron decay



g cos(θ_c)

Kaon decay



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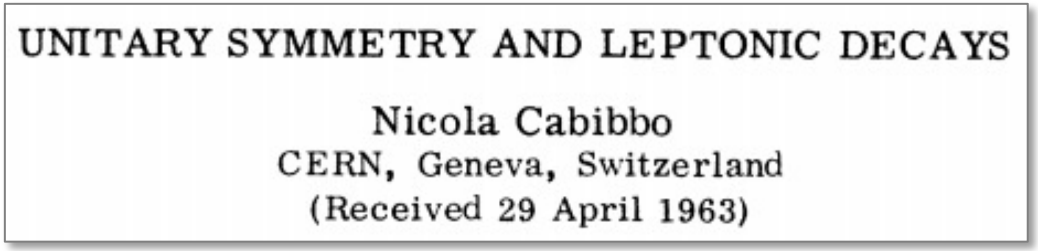

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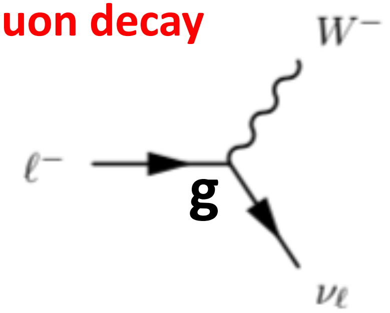
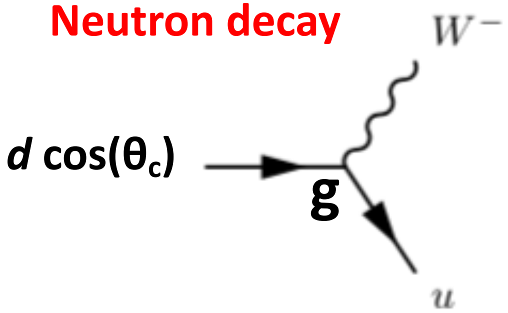
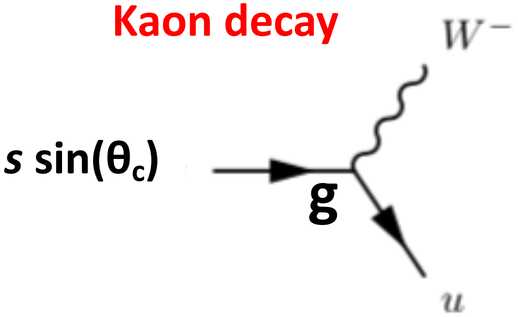


Neutron decay



Kaon decay



Weak eigenstates are a **mixture** (superposition) of flavour states:

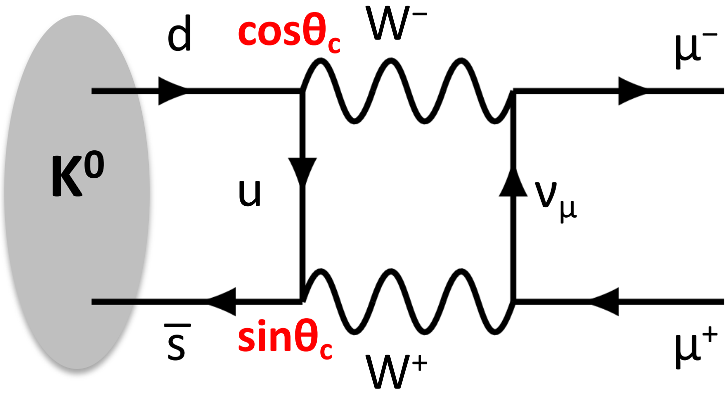
$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

✓ Saves universality of weak interaction, introduces concept of quark mixing

✗ Predicts additional kaon decays well above observed experimental limits…

"GIM" and the charm quark

Following Cabibbo, questions remain – some apparently allowed decays are never observed



Process $K^0 \rightarrow \mu^+\mu^-$ apparently highly suppressed (based on exp.) – but **why**?

“GIM” and the charm quark

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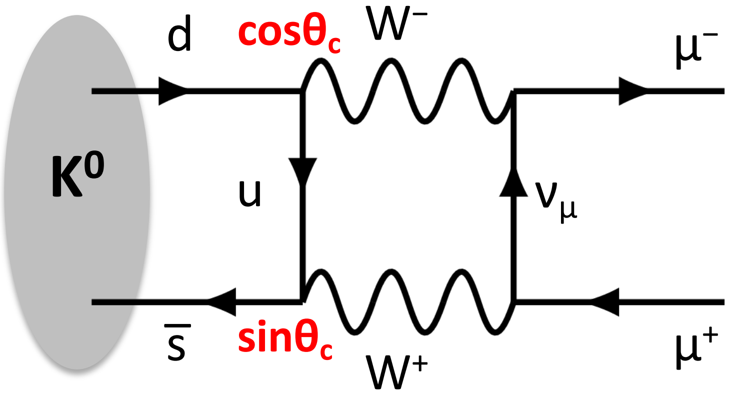

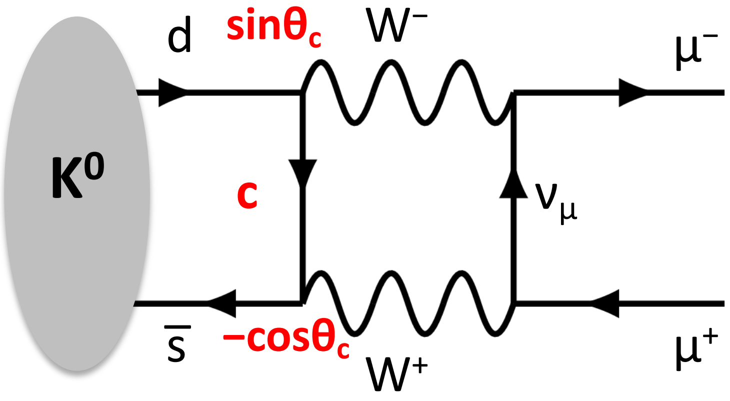

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Add charm quark ⇒ add second diagram (= amplitude)

“GIM” and the charm quark

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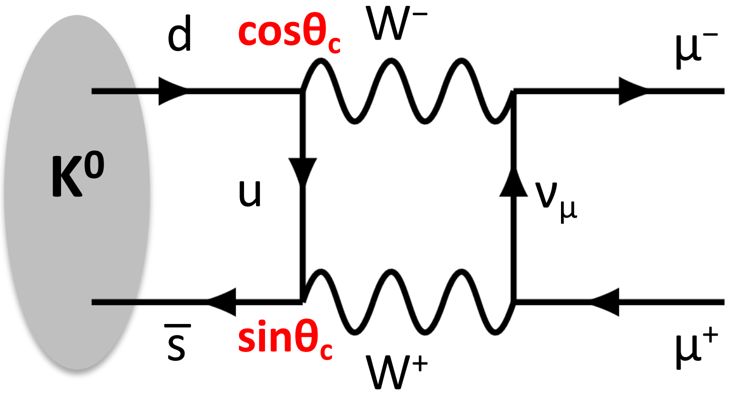
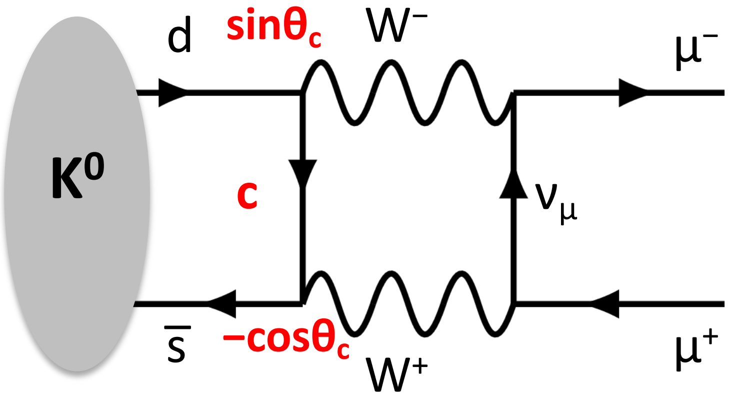




Process $K^0 \rightarrow \mu^+\mu^-$ apparently highly suppressed (based on exp.) – but **why**?

Add charm quark ⇒ add second diagram (= amplitude)

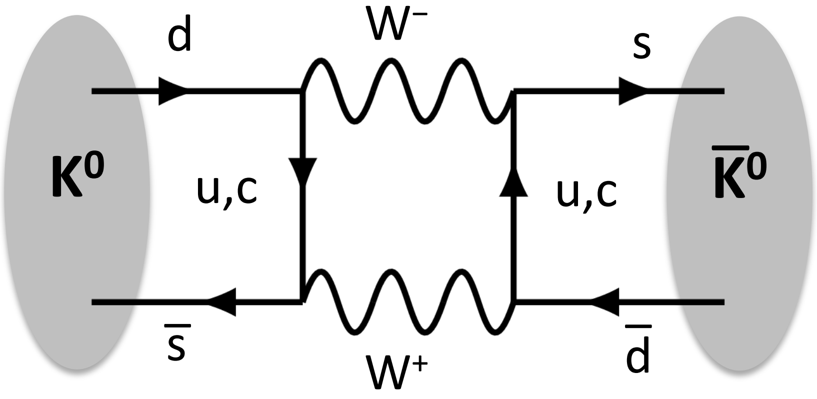
Two amplitudes ~equal and have opposite sign
⇒ total amplitude **highly suppressed!**

Cancellation not perfect because u and c quarks have different mass.

⇒ GIM mechanism

[Neutral kaon mixing]

Same diagrams cause kaon mixing



Mixing rate strongly depends on charm quark mass – if we can observe kaon mixing we can **predict** this mass

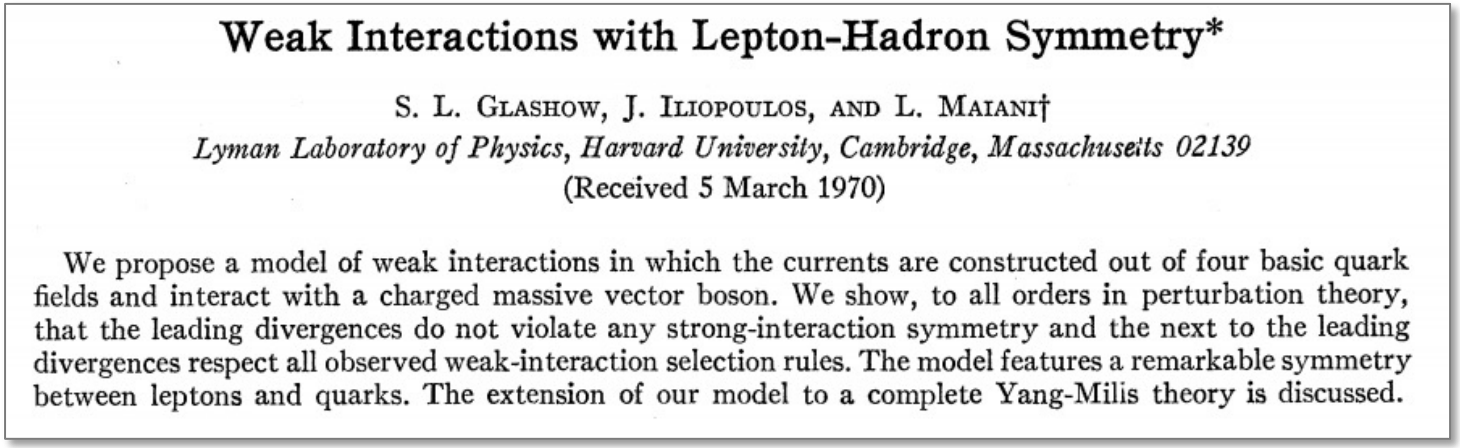
Kaon mixing experimentally confirmed since 1960s

Measurement of Δm_k (=oscillation frequency) gave prediction **m_c = 1.5 GeV**

$$\Delta m_k = \frac{G_F^2}{4\pi} m_K f_K^2 \boxed{m_c^2} \left| V_{cs} V_{cd} \right|^2$$

“GIM” and the charm quark

https://journals.aps.org/prd/abstract/10.1103/PhysRevD.2.1285



Weak Interactions with Lepton-Hadron Symmetry*

S. L. Glashow, J. Iliopoulos, and L. Maiani†
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02139
(Received 5 March 1970)

We propose a model of weak interactions in which the currents are constructed out of four basic quark fields and interact with a charged massive vector boson. We show, to all orders in perturbation theory, that the leading divergences do not violate any strong-interaction symmetry and the next to the leading divergences respect all observed weak-interaction selection rules. The model features a remarkable symmetry between leptons and quarks. The extension of our model to a complete Yang-Milis theory is discussed.

Leads to remarkable symmetry
between quark and lepton sector

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L , \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$$

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L , \begin{pmatrix} c \\ s' \end{pmatrix}_L$$

Makes testable prediction of existence and mass of charm quark…

“GIM” and the charm quark

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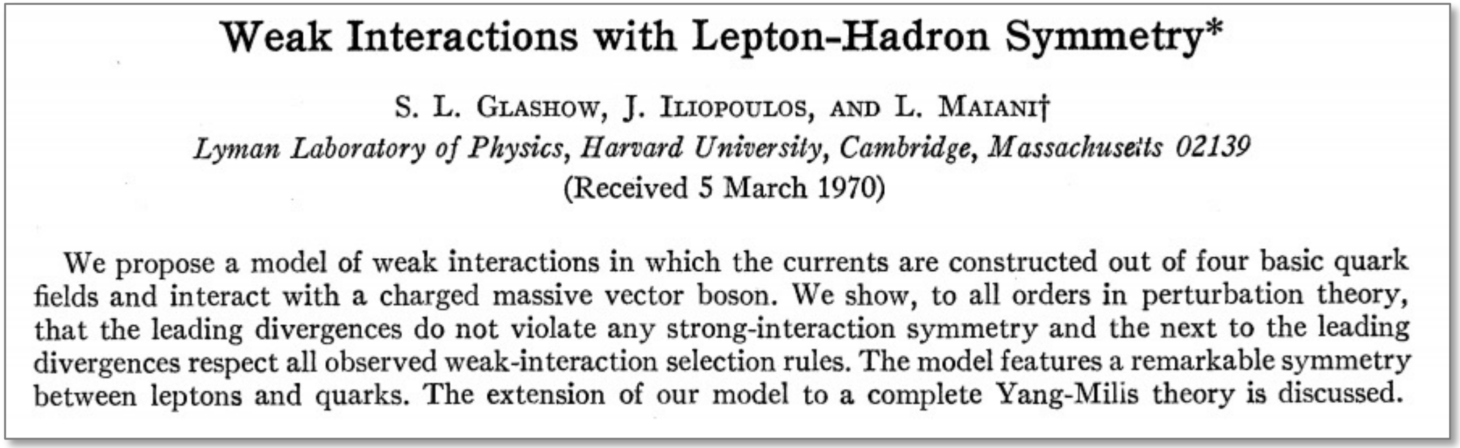

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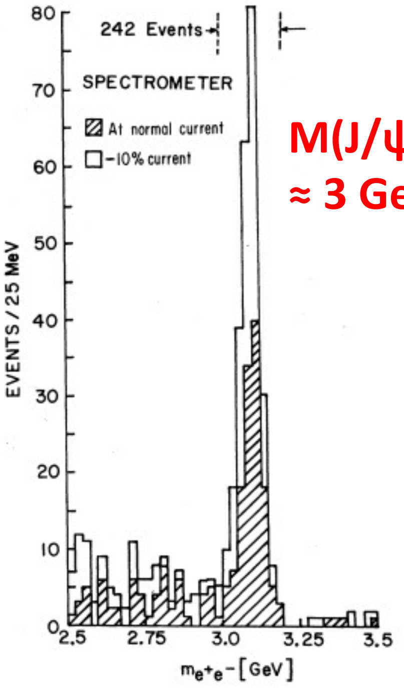
$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$$

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} c \\ s' \end{pmatrix}_L$$

J/ψ meson
($c\bar{c}$ bound state)
discovered simultaneously at BNL and SLAC in 1974



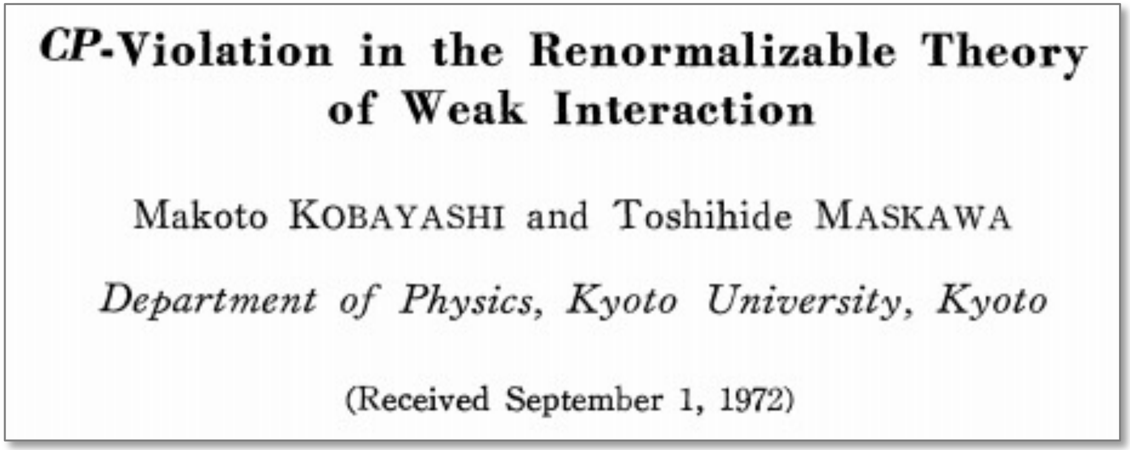
M(J/ψ) ≈ 3 GeV !

Makes testable prediction of existence and mass of charm quark…

Where’s the CP violation?

CP violation experimentally verified in weak interaction, but couldn’t fit into existing theory…

https://doi.org/10.1143/PTP.49.652



CP-Violation in the Renormalizable Theory of Weak Interaction

Makoto KOBAYASHI and Toshihide MASKAWA

Department of Physics, Kyoto University, Kyoto

(Received September 1, 1972)

KM realised that **we need 3 generations** to allow CP violation…

Cabibbo

$$\begin{bmatrix} d' \\ s' \end{bmatrix} = \begin{bmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{bmatrix} \begin{bmatrix} d \\ s \end{bmatrix}$$

1 (real) parameter: mixing angle θ_c

Cabibbo Kobayashi Maskawa (CKM)

$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix}$$

4 parameters: 3 real mixing angles
1 complex phase!

Where’s the CP violation?

CP violation experimentally verified in weak interaction, but couldn’t fit into existing theory…

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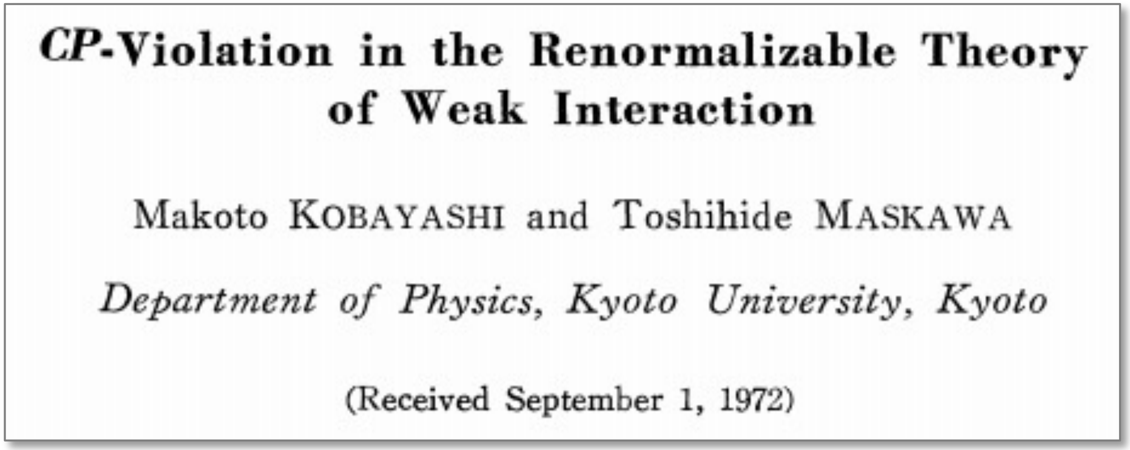

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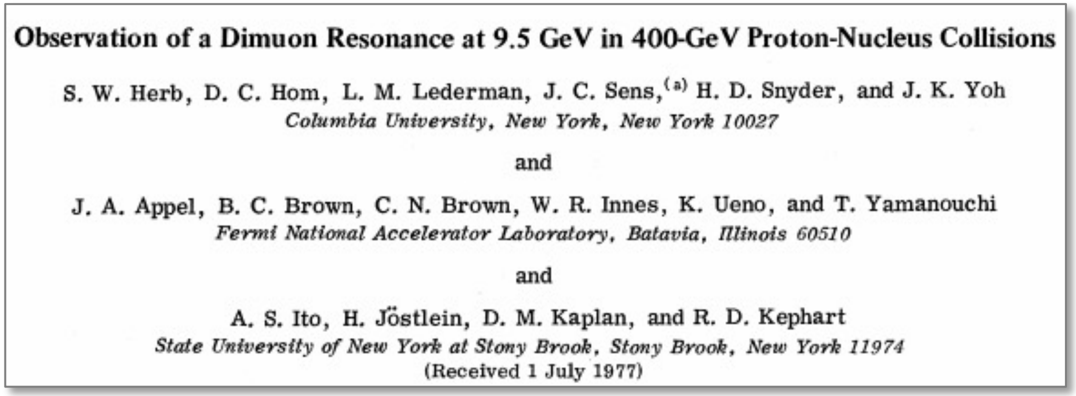
Prediction of another 2 new quarks even before charm was discovered!

⇒ b (t) quark not discovered until 1977 (1994)!

[Discovering beauty/bottom]



Observation of a Dimuon Resonance at 9.5 GeV in 400-GeV Proton-Nucleus Collisions

S. W. Herb, D. C. Hom, L. M. Lederman, J. C. Sens,(a) H. D. Snyder, and J. K. Yoh
Columbia University, New York, New York 10027

and

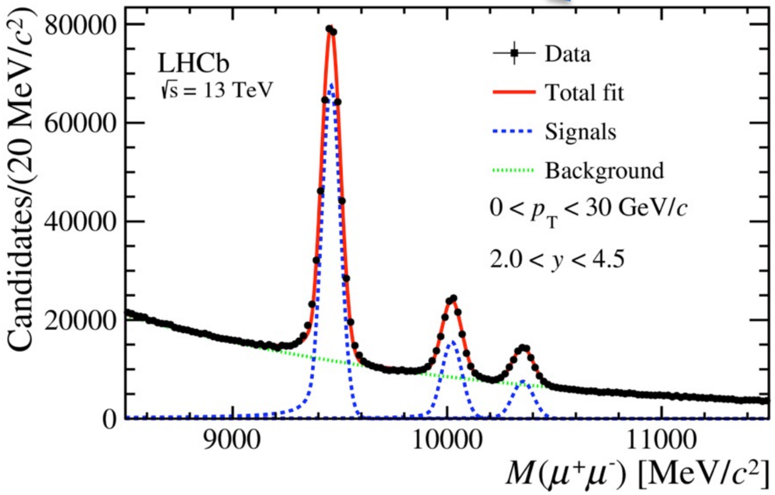
J. A. Appel, B. C. Brown, C. N. Brown, W. R. Innes, K. Ueno, and T. Yamanouchi
Fermi National Accelerator Laboratory, Batavia, Illinois 60510

and

A. S. Ito, H. Jöstlein, D. M. Kaplan, and R. D. Kephart
State University of New York at Stony Brook, Stony Brook, New York 11974
(Received 1 July 1977)

1977, Lederman et al (proton beam on fixed target)

2018, LHCb (pp collisions)

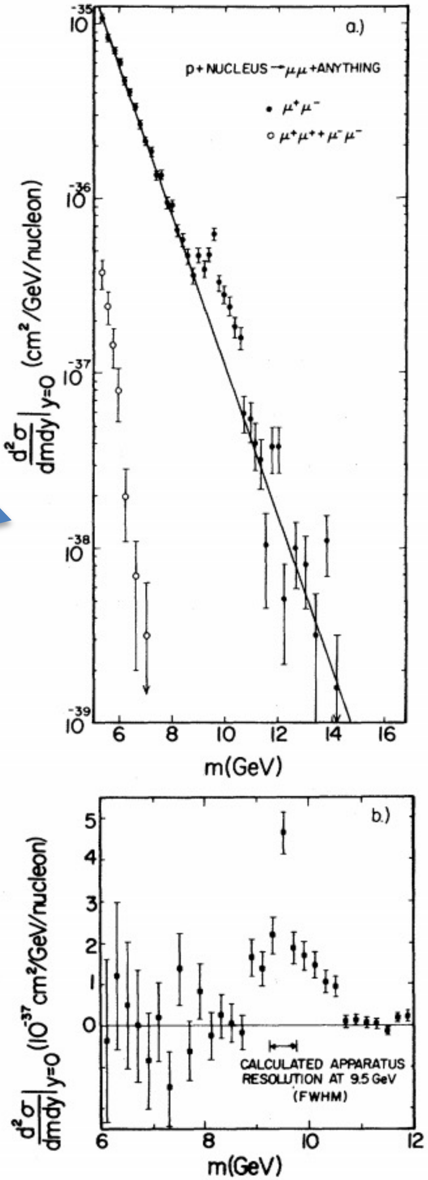


'bottomonium'

($b\bar{b}$ bound state)

$\Upsilon \rightarrow \mu^+\mu^-$

M ≈ 9.5 GeV



CKM structure

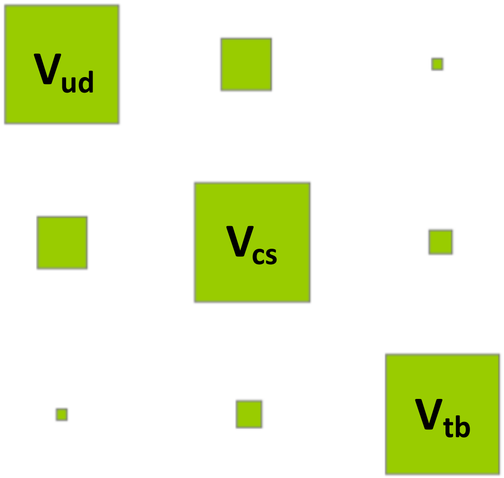
Current experimental status:

https://pdg.lbl.gov/2023/reviews/rpp2022-rev-cp-violation.pdf

$$\begin{bmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{bmatrix} = \begin{bmatrix} 0.97435 \pm 0.00016 & 0.22500 \pm 0.00067 & 0.00369 \pm 0.00011 \\ 0.22486 \pm 0.00067 & 0.97349 \pm 0.00016 & 0.04182^{+0.00085}_{-0.00074} \\ 0.00857^{+0.00020}_{-0.00018} & 0.04110^{+0.00083}_{-0.00072} & 0.999118^{+0.000031}_{-0.000036} \end{bmatrix}$$

Magnitudes $|V_{ij}|^2$ appear in probabilities (=rates) of decays.

Magnitudes have suggestive pattern
No known reason!

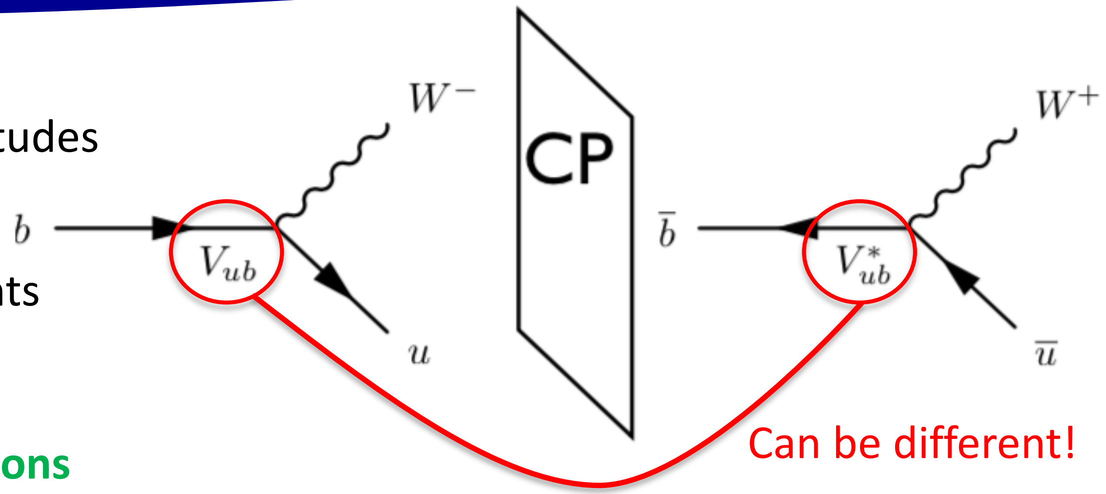


Transitions within same generation : "**Cabibbo Favoured**" (CF)

Processes with 1 (2) off-diagonal elements :
"**Singly (doubly) Cabibbo Suppressed**" (SCS / DCS)

CKM and CP violation

CP operator
⇒ complex conjugation of amplitudes



Can be different!

With 3 generations, CKM elements V_{ij} can be complex

A universe with 2 (or 1) generations could not have CP violation this way!

Highly predictive (= good theory!)

- Can make many independent measurements of V_{ij} from different systems
- Test if these are self-consistent

Next job: measure the magnitudes and phases of these complex parameters V_{ij}

CKM parameterization: `PDG'

$s_{ij} = \sin\theta_{ij}$

$c_{ij} = \cos\theta_{ij}$

Decompose into three rotation matrices:

$$V_{\rm CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

Parameters:

- 3 rotation angles θ_{12},θ_{13},θ_{23}
- CP-violating phase δ

Observed hierarchy motivates an alternative parameterisation…

CKM parameterization: Wolfenstein

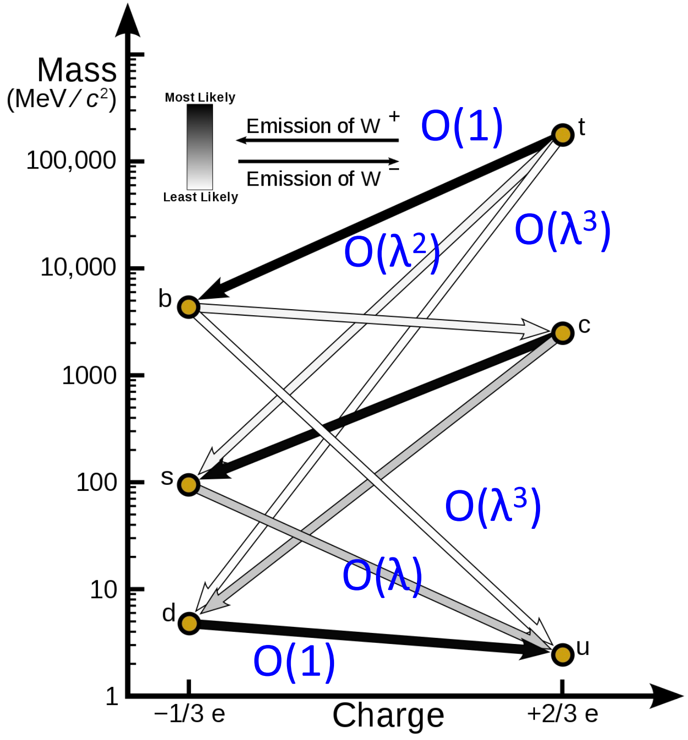
$$\begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$

Expand CKM matrix elements in powers of λ ≈ 0.22 (i.e. $\sin\theta_c$)

Here shown to order λ^3

Parameters: A, λ, ρ, η

Quantify CP violation



Part II: Testing the CKM mechanism

a. Magnitudes

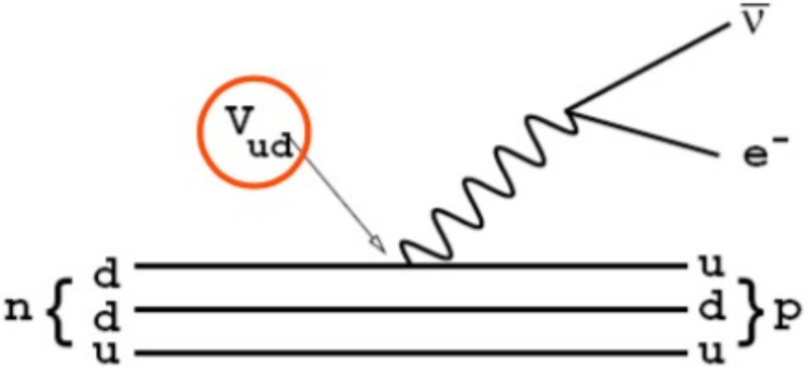
Testing the CKM mechanism

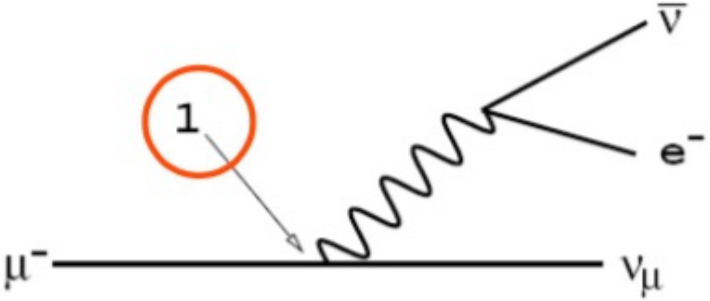
How to measure CKM matrix elements?
⇒ magnitudes control rates of particle decays

⇒ Ratio of **decay rates** proportional to ratio of **|amplitude|^2**

For **V_{ud}**, compare neutron (β decay) and muon decay rates

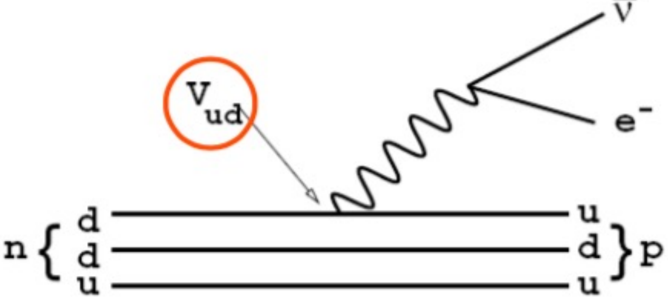
$$\begin{pmatrix} |V_{ud}| & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$





Testing the CKM mechanism

β-decays



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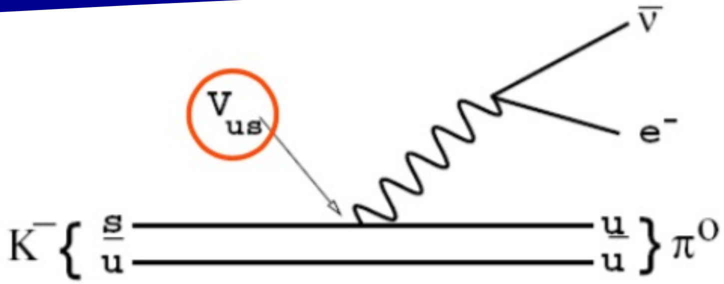
Testing the CKM mechanism



β-decays

Semileptonic (or leptonic) Kaon decays

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix}$$

Testing the CKM mechanism

β-decays

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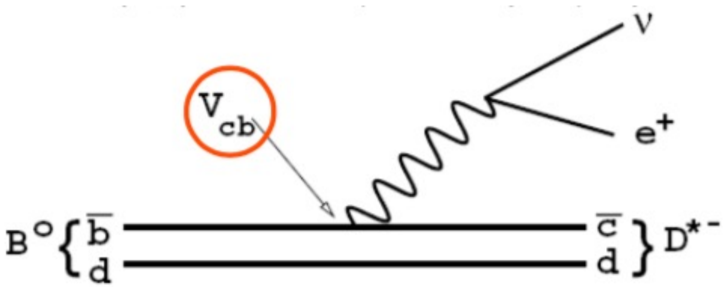

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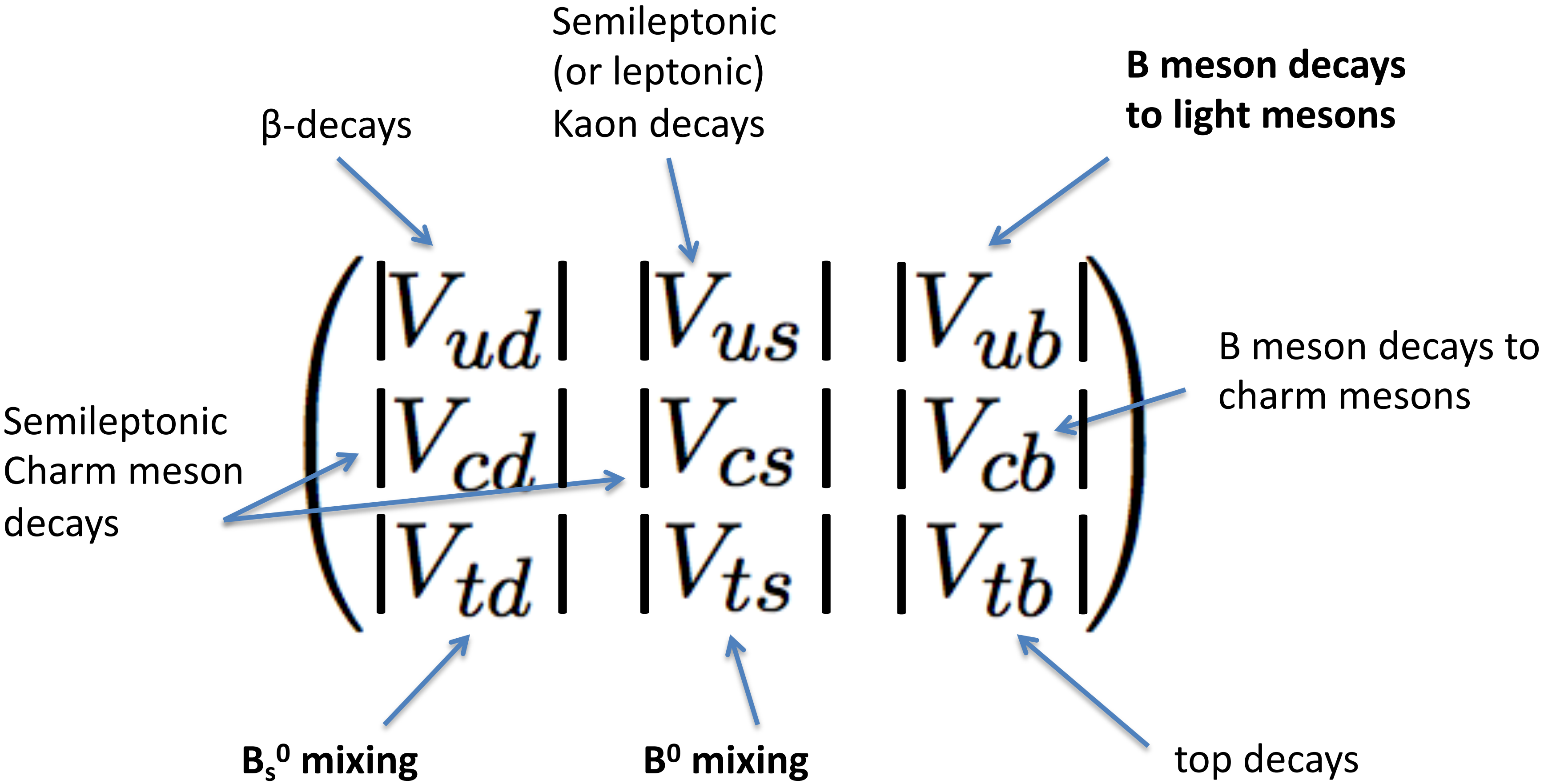
B meson decays to charm mesons

Testing the CKM mechanism



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β-decays

Semileptonic
(or leptonic)
Kaon decays

**B meson decays
to light mesons**

B meson decays to
charm mesons

Semileptonic
Charm meson
decays

B_s^0 mixing

B^0 mixing

top decays

**Often require theory inputs to relate
hadron measurements to quark-level CKM**

Unitarity triangle(s)

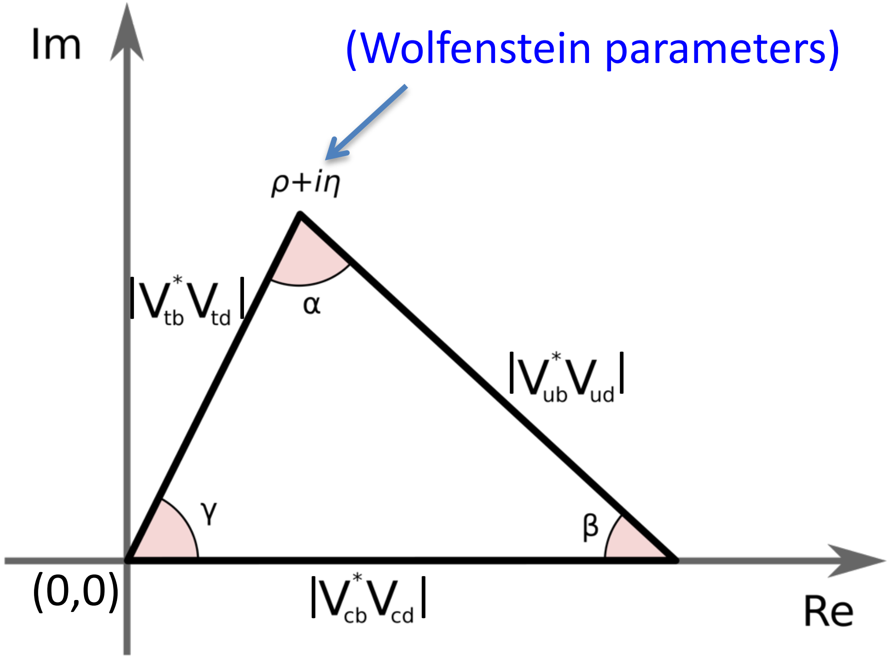
CKM matrix is unitary: $\mathbf{V_{CKM}V^{\dagger}_{CKM} = I}$

Provides 9 constraints relating elements, e.g.

$$\boxed{V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0}$$

Sum of three complex numbers = 0
⇒ triangle on Argand plane

There are in fact 6 triangles
(one per quark pair)
– this one (‘bd’) is most insightful


(Wolfenstein parameters)

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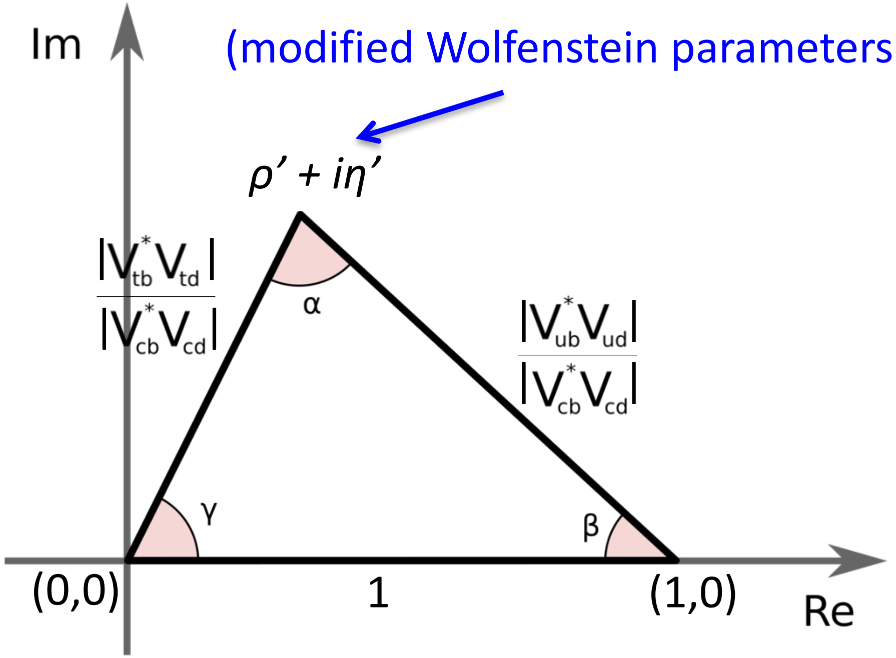
Sum of three complex numbers = 0
⇒ triangle on Argand plane

Rescale by dividing all sides by $|V_{cd}V_{cb}^*|$

$$\begin{aligned}\beta &= \phi_1 = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)\\ \alpha &= \phi_2 = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)\\ \gamma &= \phi_3 = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)\end{aligned}$$



(modified Wolfenstein parameters)

Unitarity triangle(s)

CKM matrix is unitary: $\mathbf{V_{CKM}V^{\dagger}_{CKM} = I}$

Provides 9 constraints relating elements, e.g.

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

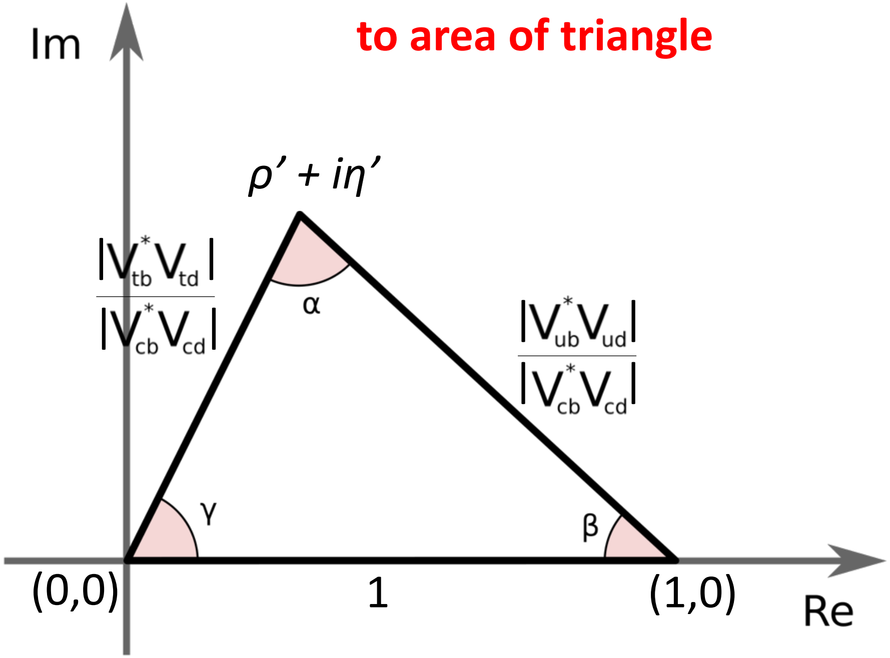
Sum of three complex numbers = 0
⇒ triangle on Argand plane

Rescale by dividing all sides by $|V_{cd}V_{cb}^*|$

Now experimental measurements form constraints of various shape on the position of the apex

- **Length of sides (x2)**
- **Angles (x3)**

Amount of CPV related to area of triangle



SM CP violation and the universe

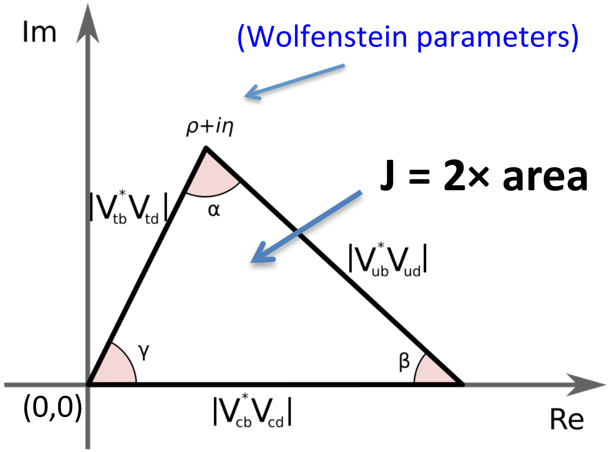
https://doi.org/10.1103/PhysRevLett.55.1039 (1985)

Jarlskog parameter J: Convention-invariant measure of CPV in quark sector

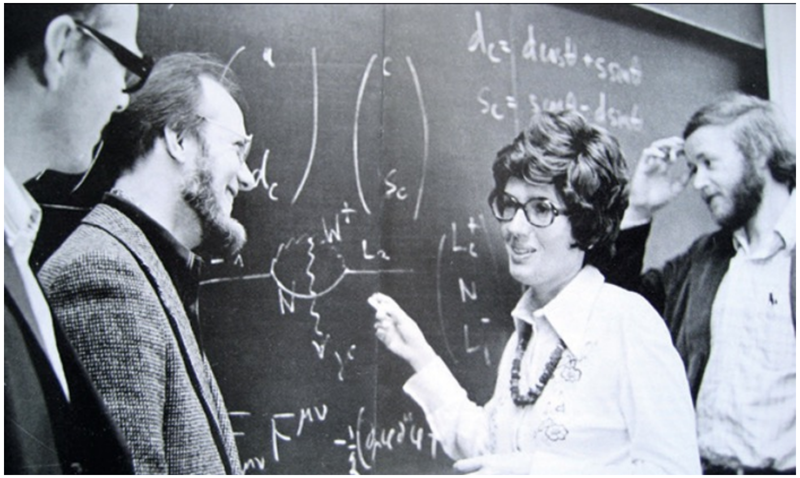
$$J = \pm\, \mathrm{Im}(V_{us}V_{cb}V^{*}_{ub}V^{*}_{cs})$$

Expressed as Wolfenstein parameters:

$J = A^2\lambda^6\eta(1 - \lambda^2/2) + O(\lambda^{10})$ **≈ 3 × 10^{-5}**


(Wolfenstein parameters)

J = 2× area

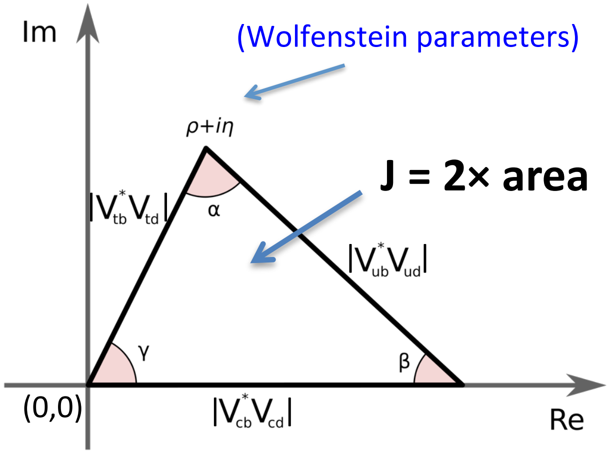


Cecilia Jarlskog with colleagues at the Nordic Institute of Theoretical Physics (NORDITA) in Copenhagen, in the early 1980s.

SM CP violation and the universe

Jarlskog parameter J: Convention-invariant measure of CPV in quark sector

$J = \pm\, \mathrm{Im}(V_{us}V_{cb}V^{*}_{ub}V^{*}_{cs})$



(Wolfenstein parameters)

J = 2× area

But… if any quark masses are degenerate, CPV vanishes – and small differences suppress it….

Multiply by terms

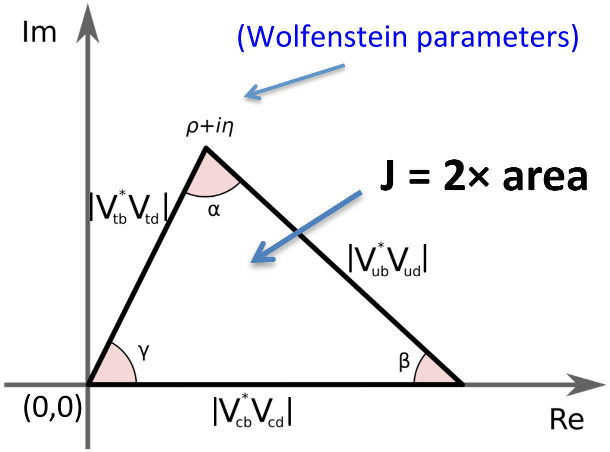
$$P_u = (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)$$
$$P_d = (m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)$$

And divide by electroweak mass scale… M_W^{12}

SM CP violation and the universe

Jarlskog parameter J: Convention-invariant measure of CPV in quark sector

$J = \pm \mathrm{Im}(V_{us}V_{cb}V^*_{ub}V^*_{cs})$



But… if any quark masses are degenerate, CPV vanishes – and small differences suppress it….

Multiply by terms
$$P_u = (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)$$
$$P_d = (m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)$$

And divide by electroweak mass scale… M_W^{12}

$$\frac{n_B - n_{\bar{B}}}{n_\gamma} \approx \frac{n_B}{n_\gamma} \sim \frac{J \times P_u \times P_d}{M^{12}}$$

= $O(10^{-20})$ from SM

= $O(10^{-10})$ Observed!

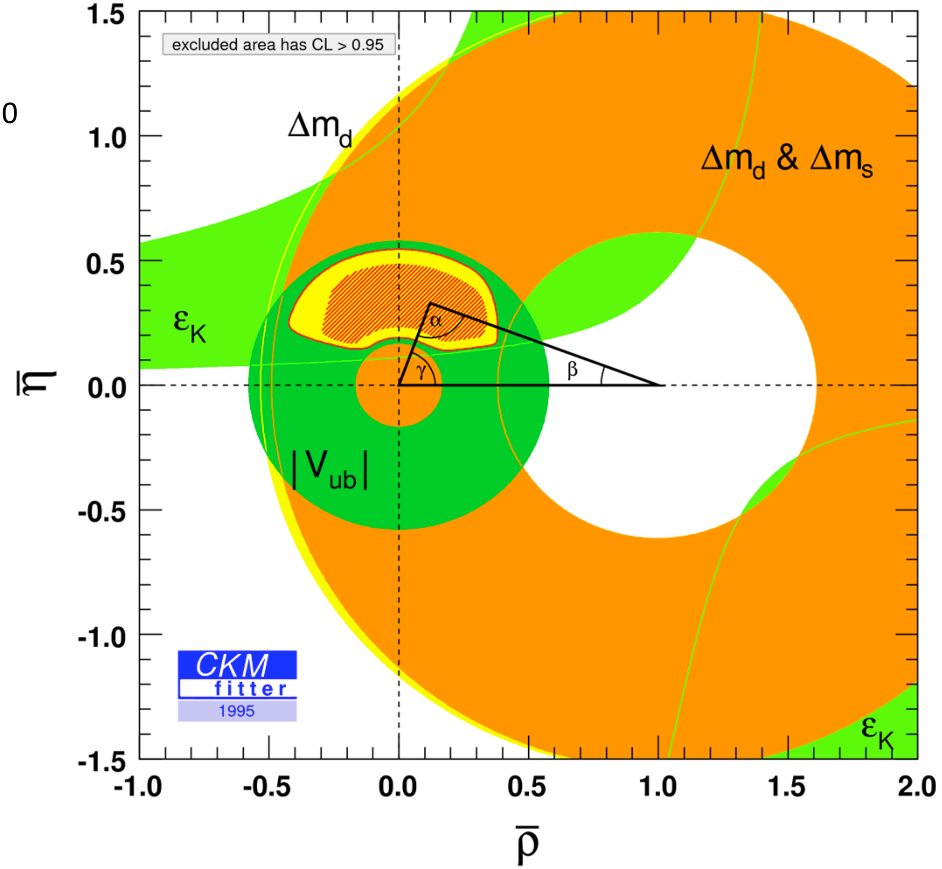
⇒ Need to identify new sources of CPV associated with **high energy scales**

Unitarity triangle in 1995…

Top quark just discovered
⇒ CKM constraint can be derived from B^0 meson mixing measurements (ΔM_d)

First constraints on $|V_{ub}|$ from from LEP, ARGUS, CLEO experiments

Minimum number of measurements needed to locate apex, and large uncertainties – **no measurements of angles**



Lots of work ahead! Sets the stage for the next phase in flavour physics…

The era of the B factories!

Part II: Testing the CKM mechanism

b. Phases

How to measure angles α, β, γ?

Observables are rates, i.e. $|A|^2 \Rightarrow$ not sensitive to phases **$|Ae^{i\phi}|^2 = A^2$**

Need two amplitudes with different phases
– then rate sensitive to their difference…

$|A_1e^{i\phi_1} + A_2e^{i\phi_2}|^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\delta\phi)$

$\delta\phi = \phi_1 - \phi_2$

Unitarity triangle angles are phase differences between CKM elements

e.g. β is angle between $V_{cd}V_{cb}^*$ and $V_{td}V_{tb}^*$

$$\begin{aligned}
\beta &= \phi_1 = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) \\
\alpha &= \phi_2 = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) \\
\gamma &= \phi_3 = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)
\end{aligned}$$

top quark – must be in loop!

Need >1 amplitudes to reach same final state (interference)
One of these must include a top quark loop…

B^0 mixing?

[3 types of CP violation]

Three ways to satisfy the criteria for CPV:

>1 amplitudes with different strong and weak phases:

CP violation in decay:

$$\Gamma(i\rightarrow f) \neq \Gamma(\bar{i}\rightarrow \bar{f})$$

Possible for any decay

CP violation in meson mixing:

$$\Gamma(M^0\rightarrow \bar{M}^0) \neq \Gamma(\bar{M}^0\rightarrow M^0)$$

i.e. $|q/p| \neq 1$

CP violation in interference between mixing and decay:

(to common final state f)

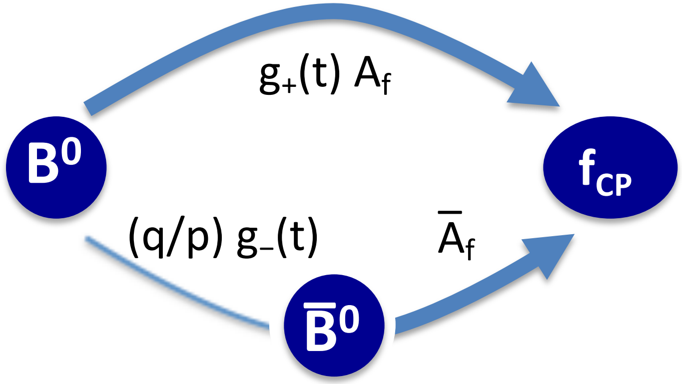
$$\Gamma(M^0\rightarrow \bar{M}^0\rightarrow f) \neq \Gamma(\bar{M}^0\rightarrow M^0\rightarrow f)$$

requires $\arg(q/p) \neq 0$

Only possible for neutral mesons that mix

CP violation in interference

Consider the process $B^0 \rightarrow \bar{B}^0 \rightarrow f_{CP}$



From last lecture, for B^0 at time t=0

$$|B^0(t)\rangle \;=\; g_+(t)|B^0\rangle + \left(\frac{q}{p}\right) g_-(t)|\bar{B^0}\rangle$$

⇒ Total amplitude = $\langle f_{CP}|B^0(t)\rangle$

$$= A_{f_{CP}}\left[g_+(t) + \frac{q}{p}\frac{\bar{A}_{f_{CP}}}{A_{f_{CP}}}\, g_-(t)\right]$$

where $\overset{(-)}{A}_{CP} = \langle f_{CP}|\overset{(-)}{B}{}^0\rangle$

$$= A_{f_{CP}}\left[\, g_+(t) + \lambda_{f_{CP}}\; g_-(t)\,\right]$$

$$\lambda_{f_{CP}} \equiv \frac{q}{p}\frac{\bar{A}_{f_{CP}}}{A_{f_{CP}}}$$

Now plug-in $g_\pm(t)$ terms (see last lecture) and $|\,|^2$ to get rate…

Reminder:

$$\begin{aligned} g_+(t) &= e^{-imt}e^{-\Gamma/2t}\left[\cosh\frac{\Delta\Gamma t}{4}\cos\frac{\Delta M t}{2} - i\sinh\frac{\Delta\Gamma t}{4}\sin\frac{\Delta M t}{2}\right], \\ g_-(t) &= e^{-imt}e^{-\Gamma/2t}\left[-\sinh\frac{\Delta\Gamma t}{4}\cos\frac{\Delta M t}{2} + i\cosh\frac{\Delta\Gamma t}{4}\sin\frac{\Delta M t}{2}\right] \end{aligned}$$

CP violation in interference

B^0 at t=0: $\Gamma(B(t) \to f) \propto e^{-\Gamma t}$

$$\times \left[\cosh(\Delta\Gamma t/2) + A^{dir}_{CP} \cos(\Delta m t) + A_{\Delta\Gamma} \sinh(\Delta\Gamma t/2) + A^{mix}_{CP} \sin(\Delta m t) \right]$$

$\bar{B}^0$ at t=0: $\Gamma(\bar{B}(t) \to f) \propto e^{-\Gamma t}$

$$\times \left[\cosh(\Delta\Gamma t/2) - A^{dir}_{CP} \cos(\Delta m t) + A_{\Delta\Gamma} \sinh(\Delta\Gamma t/2) - A^{mix}_{CP} \sin(\Delta m t) \right]$$

where:

$$A^{dir}_{CP} = C_{CP} = \frac{1-|\lambda_{CP}|^2}{1+|\lambda_{CP}|^2}$$

CPV in decay

$$A_{\Delta\Gamma} = \frac{2\,\Re(\lambda_{CP})}{1+|\lambda_{CP}|^2}$$

CP conserving part

$$A^{mix}_{CP} = S_{CP} = \frac{2\,\Im(\lambda_{CP})}{1+|\lambda_{CP}|^2}$$

CPV in interference between mixing & decay

CP violation in interference

B^0 at t=0: $\Gamma(B(t) \to f) \propto e^{-\Gamma t}$

$$\times \left[\underset{1}{\cancel{\cosh(\Delta\Gamma t/2)}} + A_{CP}^{dir} \cos(\Delta m t) + \cancel{A_{\Delta\Gamma} \sinh(\Delta\Gamma t/2)} + A_{CP}^{mix} \sin(\Delta m t) \right]$$

$\overline{B}^0$ at t=0: $\Gamma(\overline{B}(t) \to f) \propto e^{-\Gamma t}$

$$\times \left[\underset{1}{\cancel{\cosh(\Delta\Gamma t/2)}} - A_{CP}^{dir} \cos(\Delta m t) + \cancel{A_{\Delta\Gamma} \sinh(\Delta\Gamma t/2)} - A_{CP}^{mix} \sin(\Delta m t) \right]$$

- ✗ For B^0 case, ΔΓ small – can be neglected…

CP violation in interference

B^0 at t=0:

$$\Gamma(B(t) \to f) \propto e^{-\Gamma t}$$

$$\times \left[\, \underset{1}{\cancel{\cosh(\Delta\Gamma t/2)}} + \cancel{A_{CP}^{dir}\cos(\Delta m t)} + \cancel{A_{\Delta\Gamma}\sinh(\Delta\Gamma t/2)} + A_{CP}^{mix}\sin(\Delta m t) \,\right]$$

$\overline{B}^0$ at t=0:

$$\Gamma(\overline{B}(t) \to f) \propto e^{-\Gamma t}$$

$$\times \left[\, \underset{1}{\cancel{\cosh(\Delta\Gamma t/2)}} - \cancel{A_{CP}^{dir}\cos(\Delta m t)} + \cancel{A_{\Delta\Gamma}\sinh(\Delta\Gamma t/2)} - A_{CP}^{mix}\sin(\Delta m t) \,\right]$$

✗ For ‘golden mode’ **$B^0 \to J/\psi\, K_S^0$**: No direct CPV ($A_{CP}^{dir} = 0$, a = 0)

and **$A_{CP}^{mix} = -\sin(2\beta)$**

CP violation in interference

B^0 at t=0: $\Gamma(B(t) \rightarrow f) \propto e^{-\Gamma t} \times [\, 1 - \sin(2\beta) \sin(\Delta m t) \,]$

$\overline{B}^0$ at t=0: $\Gamma(\overline{B}(t) \rightarrow f) \propto e^{-\Gamma t} \times [\, 1 + \sin(2\beta) \sin(\Delta m t) \,]$

⇒ By time-dependent analysis, can extract β from amplitude of oscillations

CP violation in interference

B^0 at t=0: $\Gamma(B(t) \rightarrow f) \propto e^{-\Gamma t} \times [\, 1 - \sin(2\beta) \sin(\Delta m t) \,]$

$\bar{B}^0$ at t=0: $\Gamma(\bar{B}(t) \rightarrow f) \propto e^{-\Gamma t} \times [\, 1 + \sin(2\beta) \sin(\Delta m t) \,]$

⇒ By time-dependent analysis, can extract β from amplitude of oscillations

⇒ Even cleaner using CP asymmetry:

$$\frac{\Gamma(t)\,[B^0 \rightarrow J/\psi K_S^0] - \Gamma(t)\,[\bar{B}^0 \rightarrow J/\psi K_S^0]}{\Gamma(t)\,[B^0 \rightarrow J/\psi K_S^0] + \Gamma(t)\,[\bar{B}^0 \rightarrow J/\psi K_S^0]} = -\sin(2\beta)\sin(\Delta m t)$$

Hence,
“Golden mode”

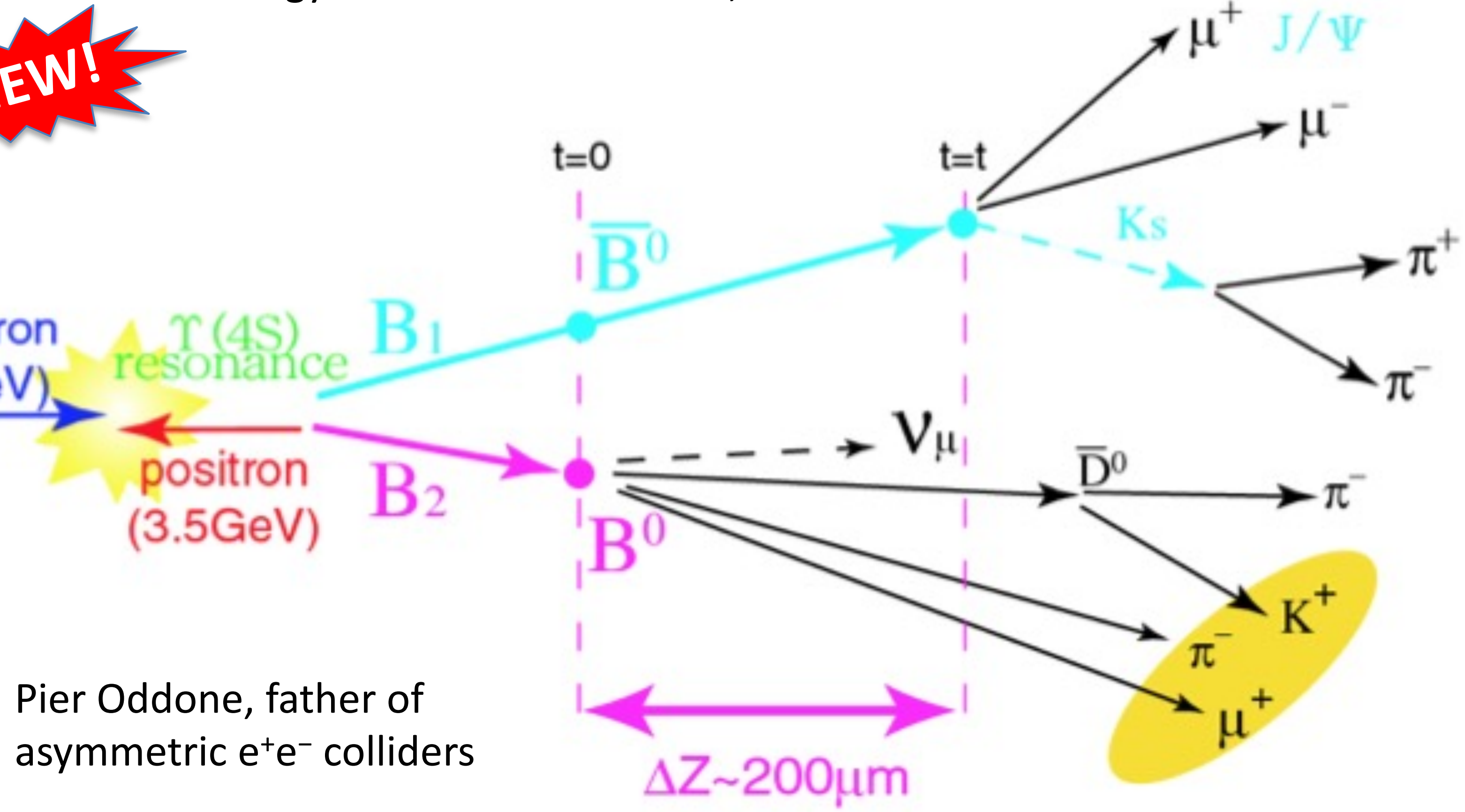
But note: asymmetry integrates to zero over time

Part III: The B factories

The B Factories: BaBar and Belle

- Collide e^+e^- at $\Upsilon(4S)$ resonance energy ⇒ $\Upsilon(4S) \rightarrow B^{(0,\pm)}\overline{B}^{(0,\pm)}$
- B hadrons quantum correlated – can determine initial state from ‘other B’
- Asymmetric beam energy ⇒ B hadrons boosted, so can measure ‘t’

NEW!



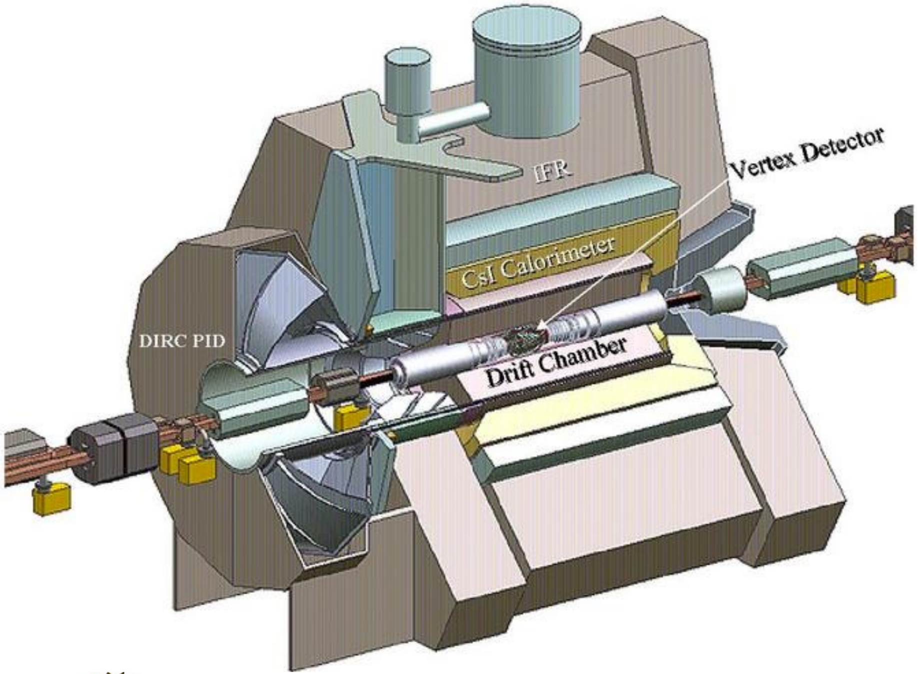
Pier Oddone, father of asymmetric e^+e^- colliders

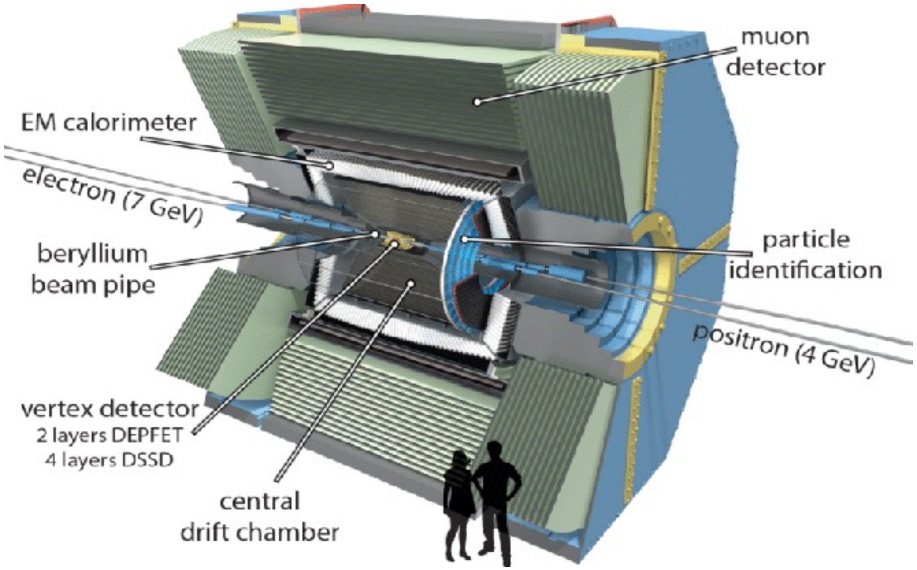
The B Factories: BaBar and Belle

BaBar: on PEP-II @ SLAC, USA
9 GeV e^- ⟷ 3.1 GeV e^+
433 fb^{-1} (1999 – 2008)

Different detectors, same ideas:

- Vertex + tracking detectors
- Particle ID
- Calorimetry





BABAR
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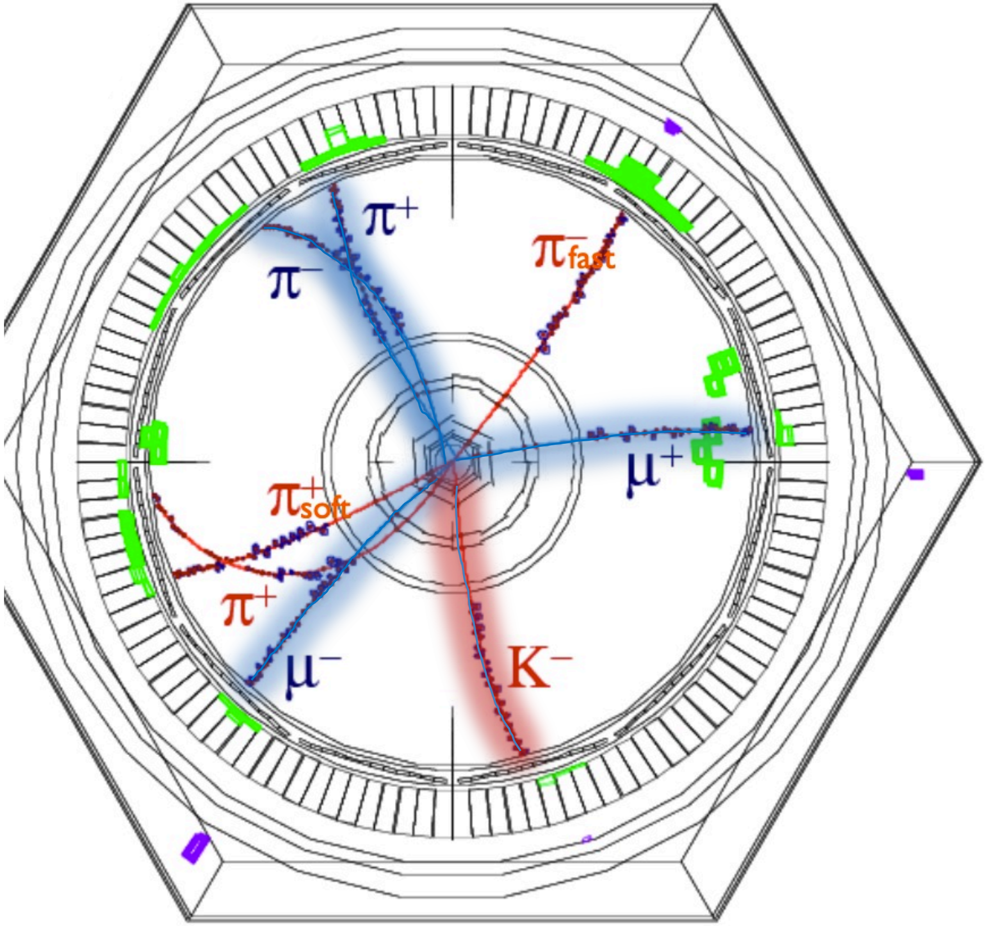
Belle: on KEKB accelerator (Japan)
8 GeV e^- ⟷ 3.5 GeV e^+
711fb^{-1} (1999 – 2010)

BELLE

Example event



Tagging side: $\bar{B}^0 \to D^{*+} \pi^-_{fast}$

↳ $D^0 \pi^+_{soft}$

↳ $K^- \pi^+$

K^- tags initial flavor as $\bar{B}^0$

⇒ Signal must be B^0 at “t=0”

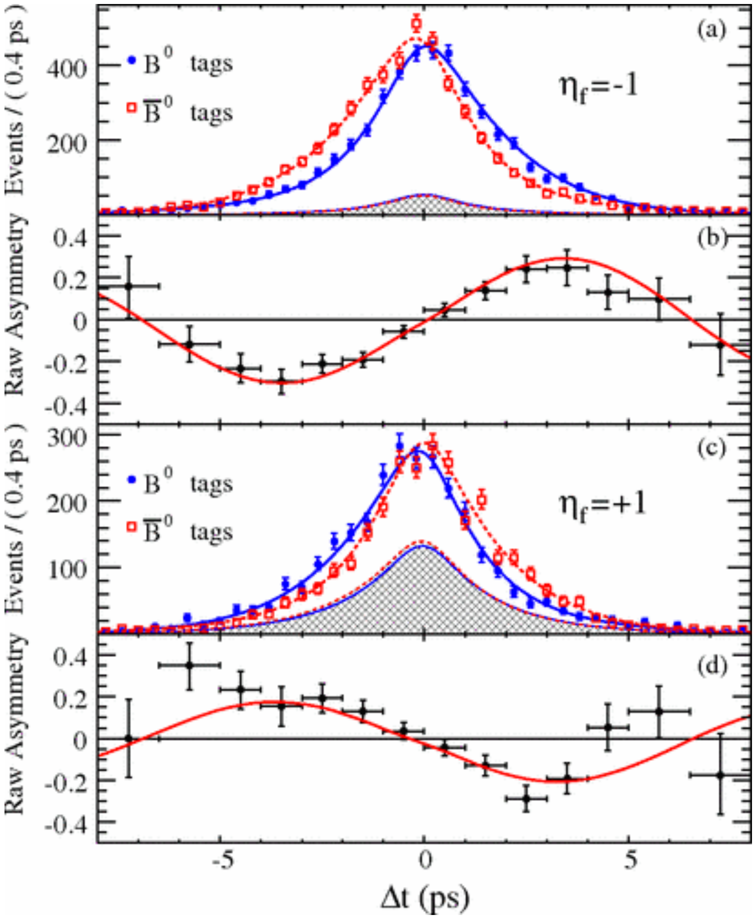
$B^0 \to J/\psi\, K_S^0$

↳ $\pi^+\pi^-$

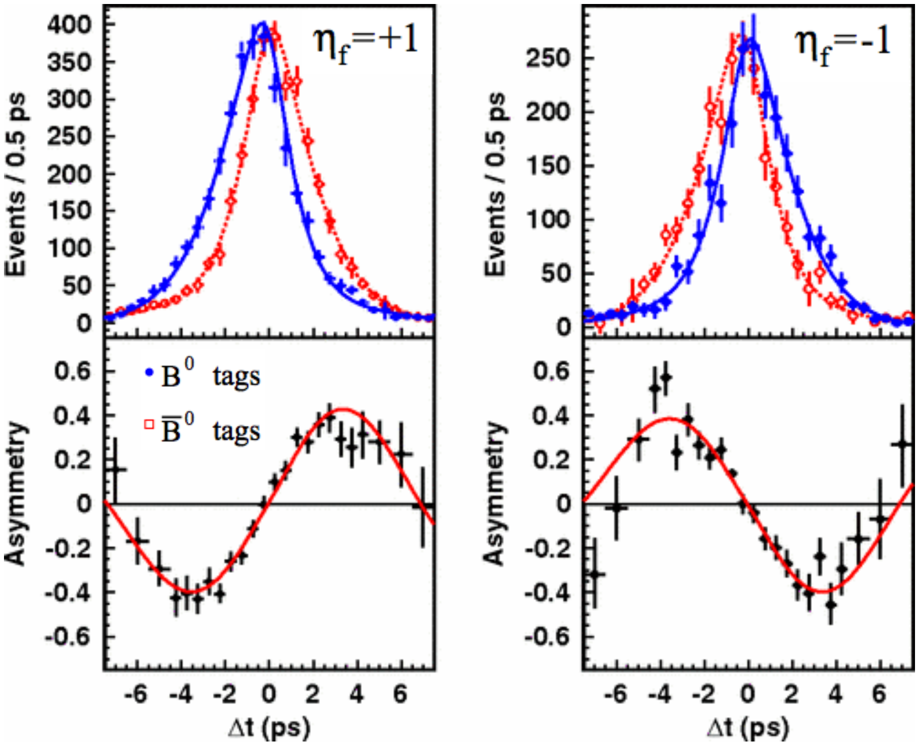
↳ $\mu^+\mu^-$

Golden mode results: sin(2β)

BaBar https://arxiv.org/abs/0902.1708



Belle https://arxiv.org/abs/1201.4643



⇒ **Clear CP-asymmetry! Measure sin(2β)**

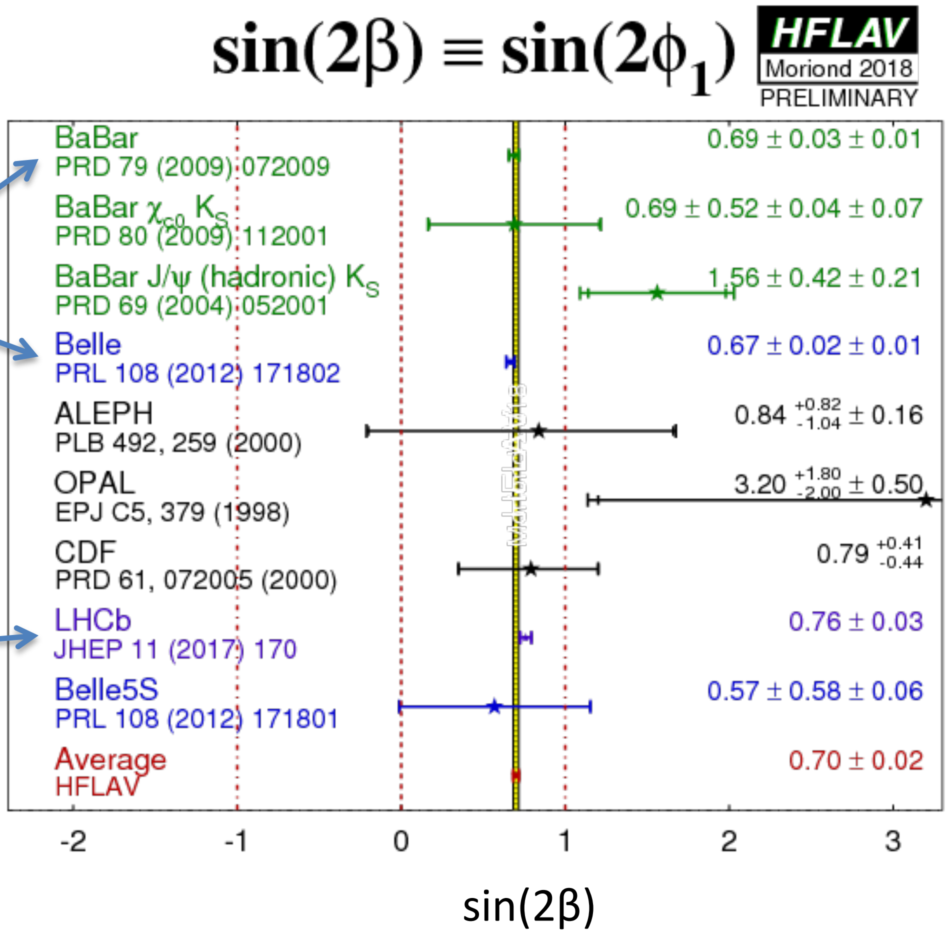
Actually use many different channels (both CP-odd and CP-even, η_f = ±1)

~ K_L^0 **~K_S^0**

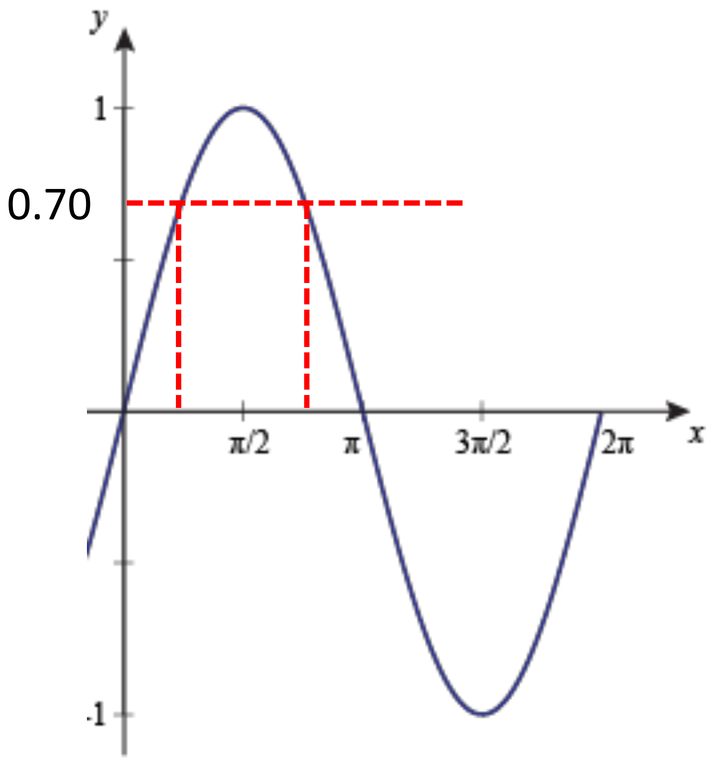
Golden mode results: sin(2β)

Results on previous slide

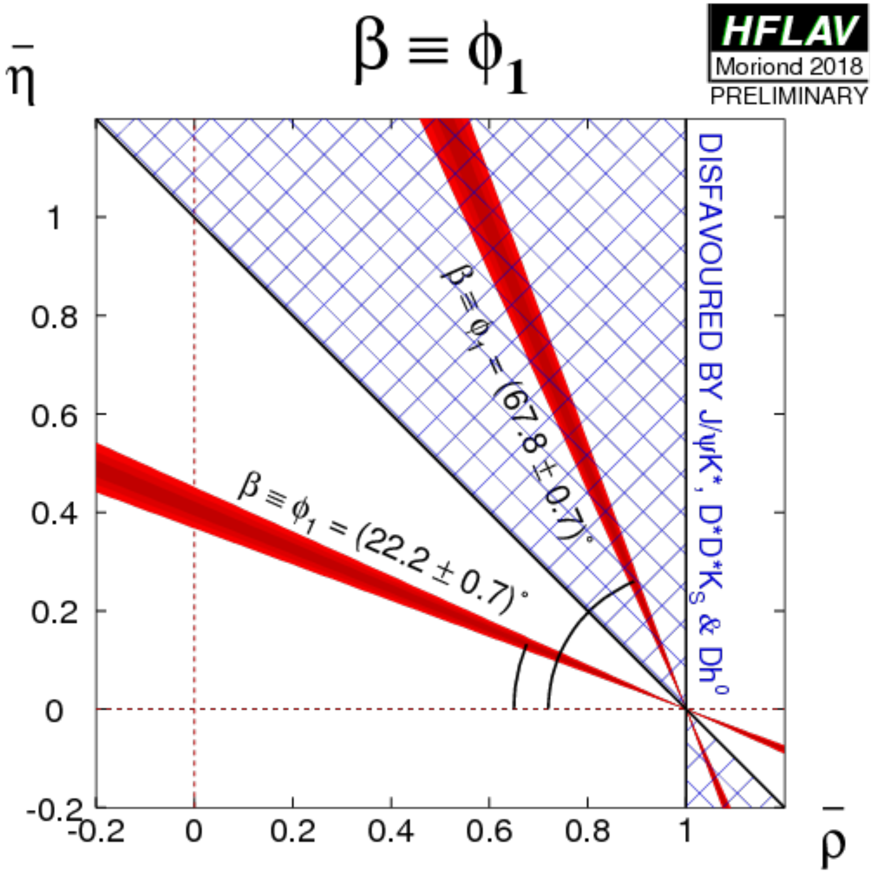
LHCb now competitive with B-factories!



Golden mode results: sin(2β)



Two-fold ambiguity on β, but second solution ruled-out by other inputs



Other angles: α and γ

Similar approach to measure other angles…

$$\beta = \phi_1 = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

$$\alpha = \phi_2 = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$$

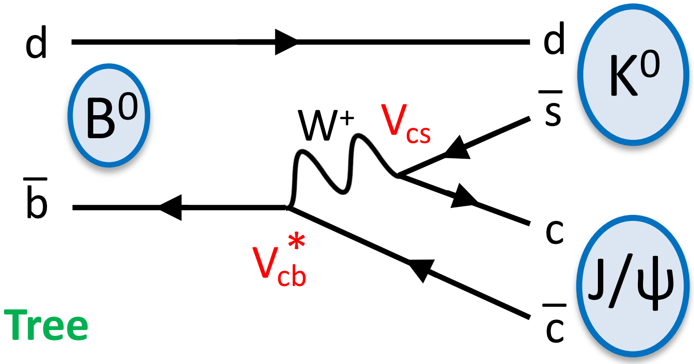
$$\gamma = \phi_3 = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

b→cW transitions, with B mixing
(e.g. $B^0 \rightarrow J/\psi\, K_S^0$)

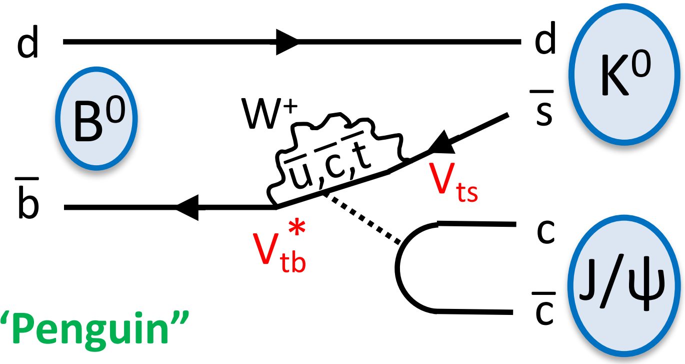
b→uW transitions, with B mixing
(e.g. $B^0 \rightarrow \pi^+\pi^-$)
Messy – many interfering processes, and direct CPV

Penguin pollution

Beyond tree-level…



Tree

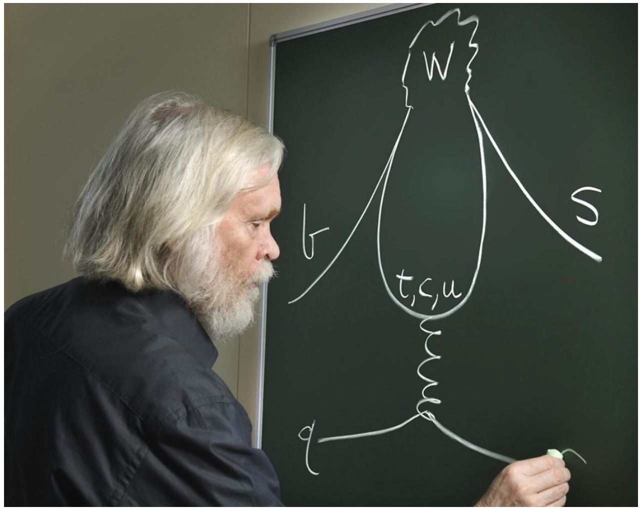


“Penguin”

Can have penguin diagrams with different weak phase

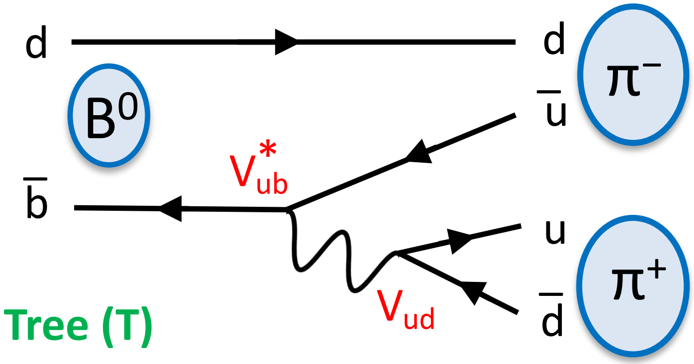
For $B^0 \rightarrow J/\psi\, K_S^0$, tree-level process dominates
⇒ penguin can be ignored (<1% effect)

With sufficient experimental precision, these penguin contributions must be included.

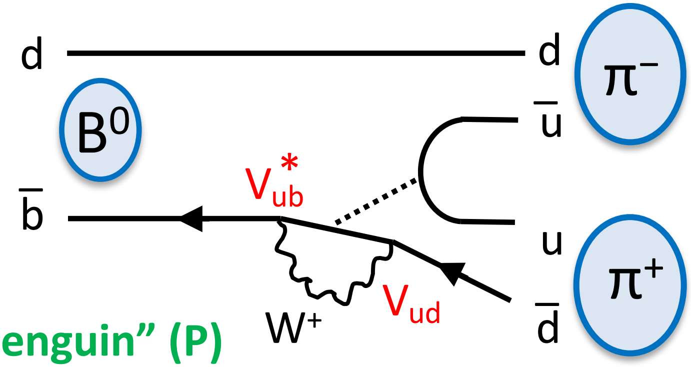


Measuring CKM angle α

Similar process allows α to be measured , **BUT** cannot ignore penguin pollution here



Tree (T)



"Penguin" (P)

$$A_{CP}^{dir} = 2\frac{|P|}{|T|}\sin\alpha\,\sin(\delta_P - \delta_T)$$

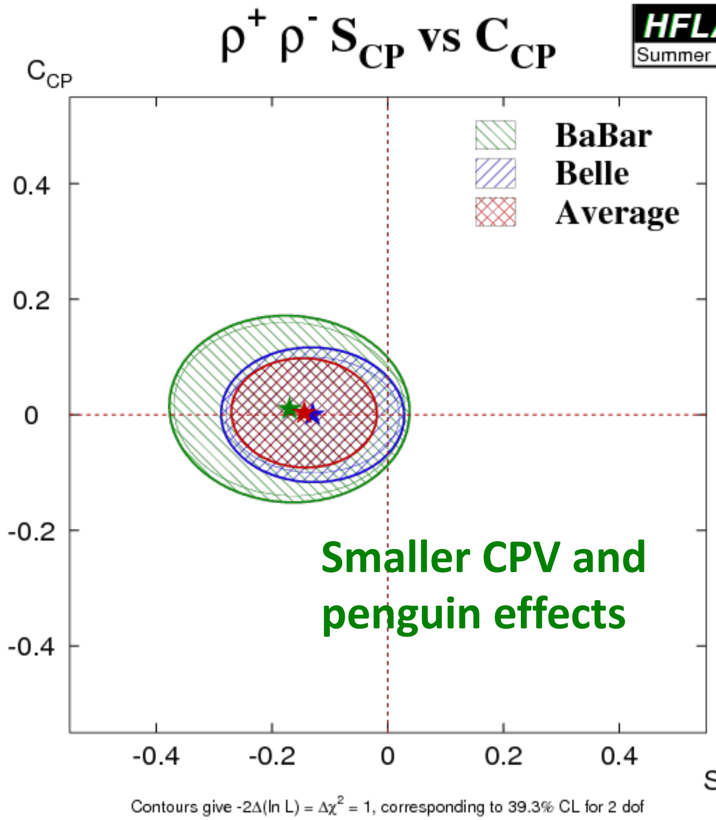
$$A_{CP}^{mix} = \sin 2\alpha - \underbrace{2\frac{|P|}{|T|}}_{\sim 20\%}\sin\alpha\cos 2\alpha\cos(\delta_P - \delta_T) = \sin 2\alpha_{eff}$$

Several proposed techniques to reduce sensitivity to penguin pollution, e.g.

- 'Gronau London' (https://doi.org/10.1103/PhysRevLett.65.3381, 1990)
- 'Snyder-Quinn' (https://doi.org/10.1103/PhysRevD.48.2139, 1993)

CKM angle α: state-of-the-art

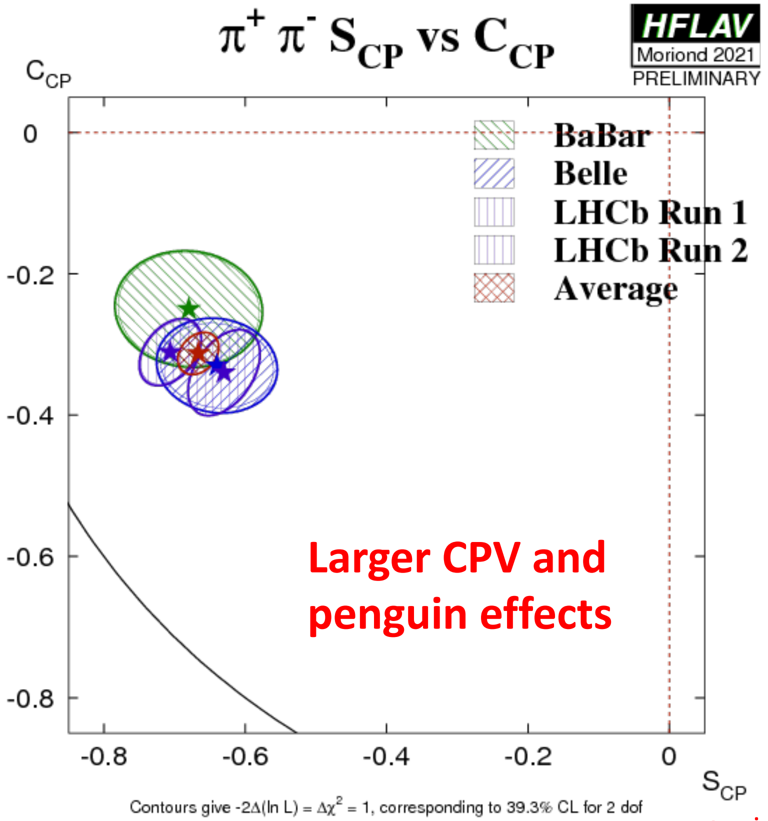
Current measurements from different channels not in perfect agreement – need more precision!



A^{dir}_{CP}

A^{mix}_{CP}

Smaller CPV and penguin effects



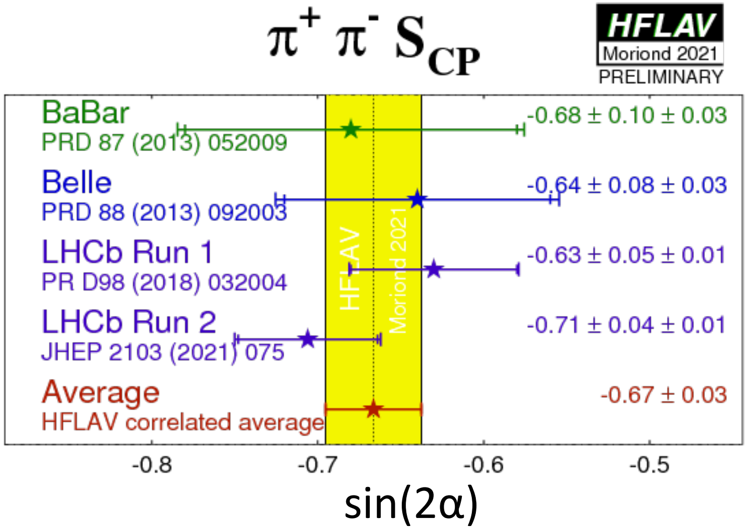
A^{dir}_{CP}

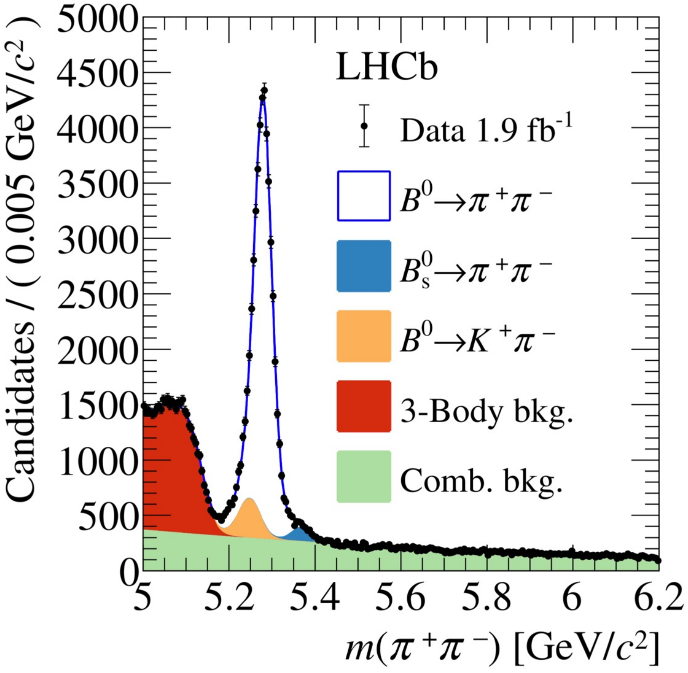
A^{mix}_{CP}

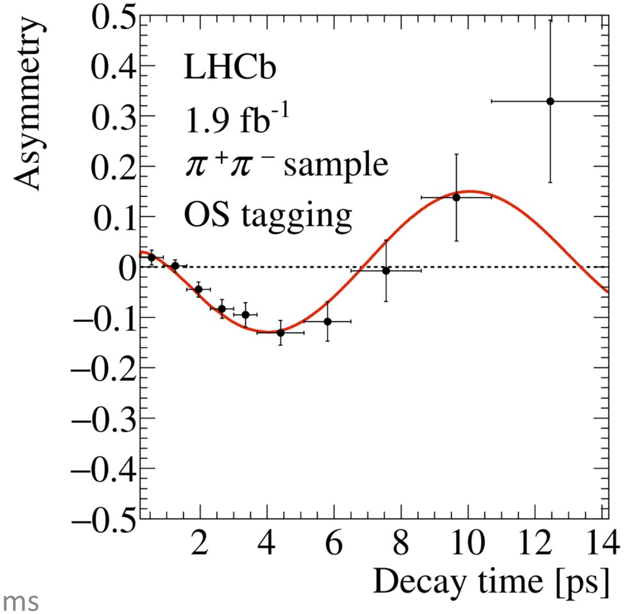
Larger CPV and penguin effects

CKM angle α: state-of-the-art

LHCb measurements now most precise



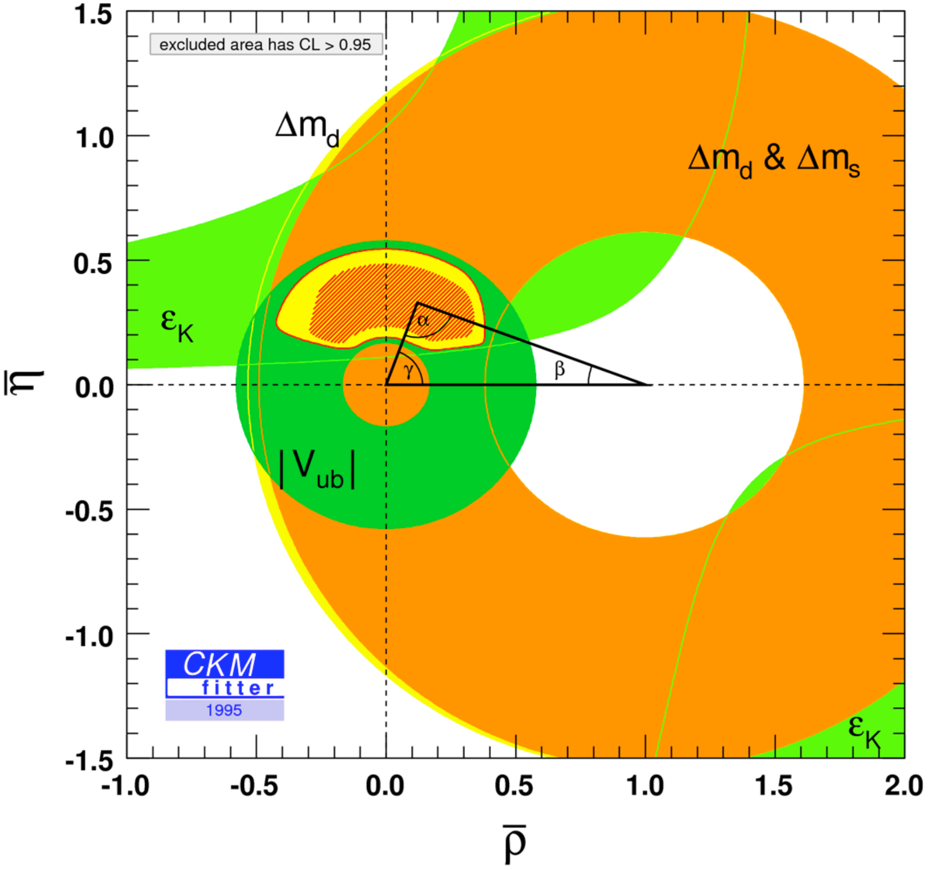




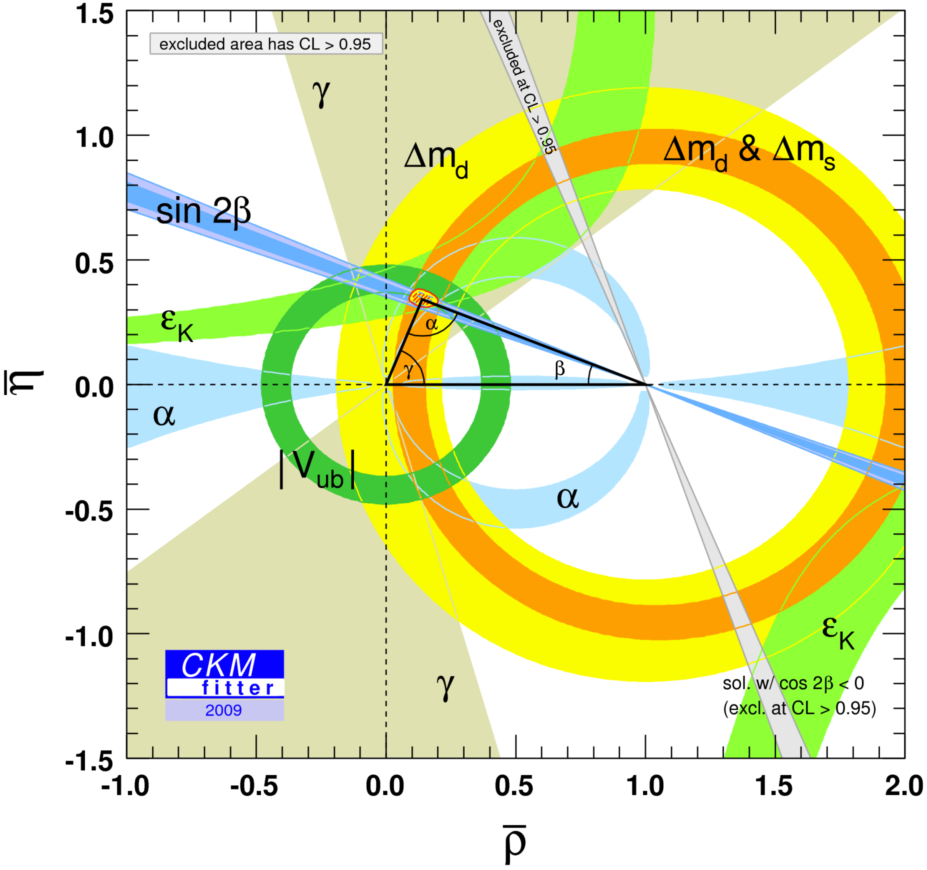
https://doi.org/10.1007/JHEP03(2021)075

Impact of B-factories

1995



2009



On the eve of the LHC…

All constraints consistent with single point for apex

Direct measurements of angles:

$\beta = (21.15 \pm 0.90)°$

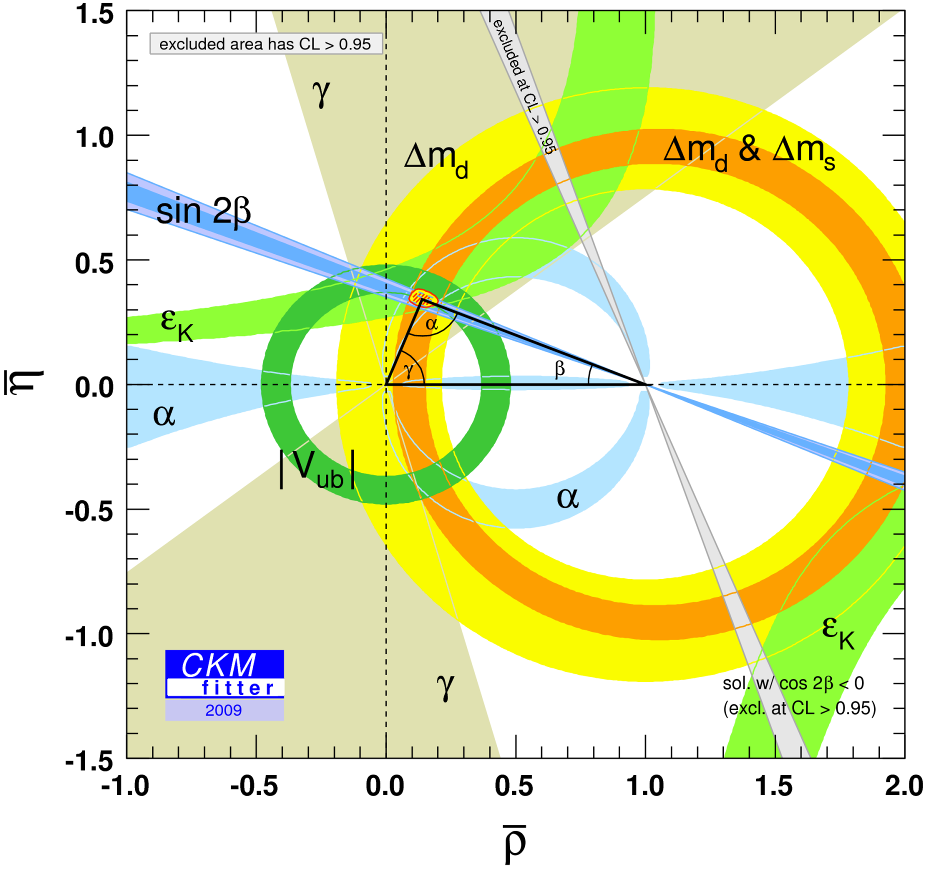
$\alpha = (89.0^{+4.4}_{-4.2})°$

$\gamma = (73^{+22}_{-25})°$

⇒ Need to improve γ measurement!

Brings us to the LHC era of flavour

2009



Summary

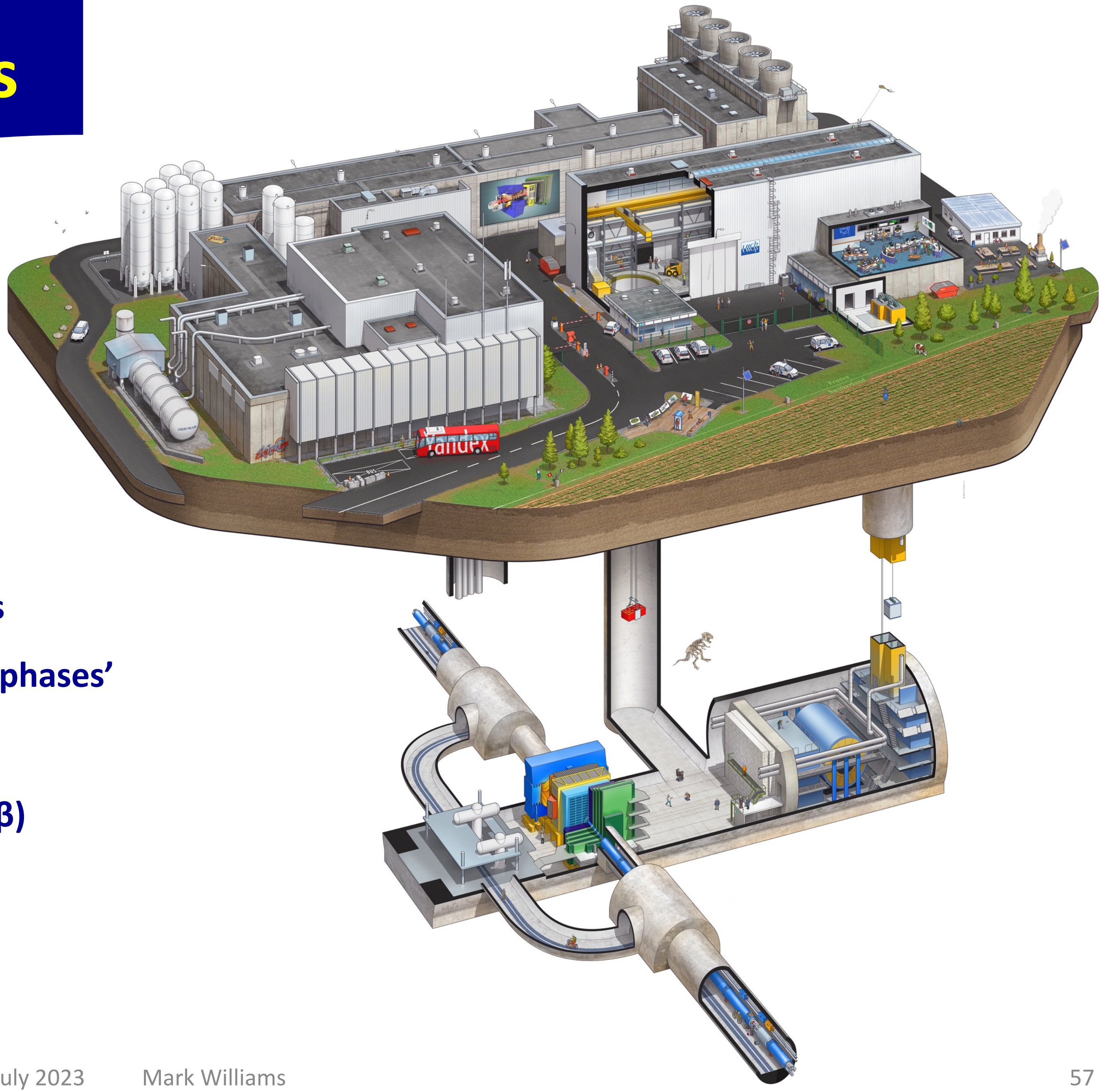
Today we covered the foundations of b physics:

- CP violation in the SM (quark sector)
- Unitarity triangle(s)
- Measuring CKM phases
- B-factory measurements of β and α

Next time – we will cover b (and c) physics in the LHC era:

- Hadron colliders vs B-factories
- Mixing and CP violation in B_s^0 and D^0 mesons
- CKM angle gamma
- Rare decays and lepton universality

Extra Slides



- **CKM parameters**
- **CPV and ‘strong phases’**
- **Measuring $|V_{ub}|$**
- **Measuring sin(2β)**

CKM matrix: Why 4 parameters?

Why does a **3×3** CKM matrix only have **3 real** and **1 complex** parameters?

Most general N×N complex matrix would have $2N^2$ = **18 parameters**

- Must be unitary, i.e. $V_{CKM}V_{CKM}^* = I$ ⇒ N^2 constraints, leaving N^2=**9 parameters**
  (in physics: t→d + t→s + t→b = 1)
- We can readily change conventions which describe phases between quark fields
  ⇒ 6 quarks, so 5 phase differences, leaving **4 free parameters**
- $N(N-1)/2$ = **3 are rotation angles**
- Remaining parameter is **irreducible phase**

Note: For N=2 (Cabibbo), we have **8 – 4 – 3 = 1** free parameter (must be rotation angle)

Conditions for CPV

Consider a process with two interfering amplitudes – can it violate CP symmetry?

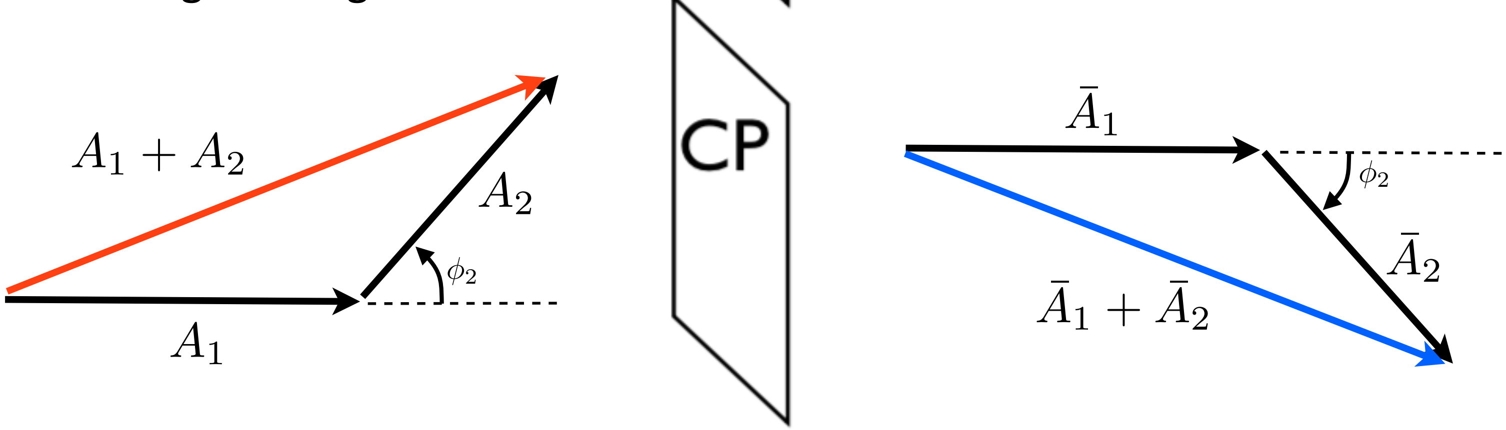
Amplitude $A = A_1e^{i\phi_1} + A_2e^{i\phi_2}$

Rate = $|A_1e^{i\phi} + A_2e^{i\phi'}|^2$
$= A_1^2 + A_2^2 + 2A_1A_2\cos(\delta\phi)$

Amplitude $A^* = A_1e^{-i\phi_1} + A_2e^{-i\phi_2}$

Rate = $|A_1e^{-i\phi} + A_2e^{-i\phi'}|^2$
$= A_1^2 + A_2^2 + 2A_1A_2\cos(-\delta\phi)$
$= A_1^2 + A_2^2 + 2A_1A_2\cos(\delta\phi)$

No! Obvious in Argand diagram…



There is a second condition to allow CP violation…

Conditions for CPV

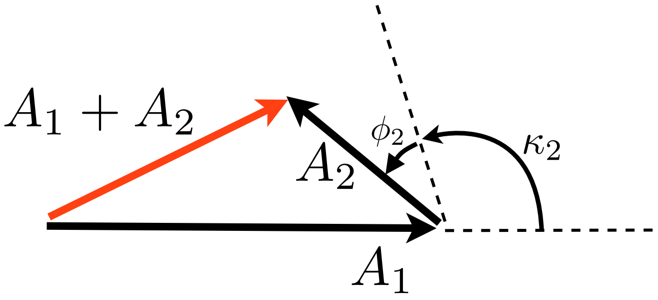
There is a second condition to allow CP violation…

Different strong phase (i.e. CP conserving – no sign change) between amplitudes

$A = A_1 e^{i\phi_1} e^{i\kappa_1} + A_2 e^{i\phi_2} e^{i\kappa_2}$

Rate

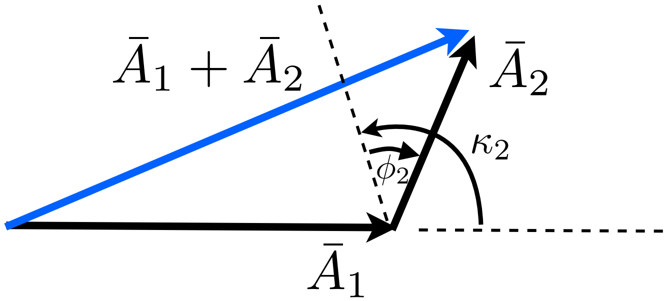
$= \mathbf{A_1^2 + A_2^2 + 2A_1A_2\cos(\delta\phi + \delta\kappa)}$



$A = A_1 e^{-i\phi_1} e^{i\kappa_1} + A_2 e^{-i\phi_2} e^{i\kappa_2}$

Rate

$= \mathbf{A_1^2 + A_2^2 + 2A_1A_2\cos(\delta\phi - \delta\kappa)}$

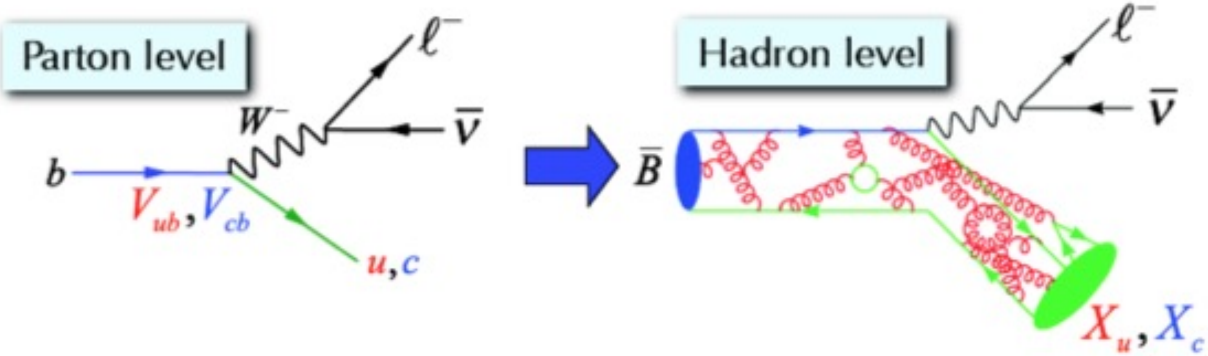


CP violation!

Difference in rates: $\mathbf{\Gamma(i \to f) - \Gamma(\bar{i} \to \bar{f}) = -4A_1A_2 \sin(\delta\phi) \sin(\delta\kappa)}$

Measuring $|V_{ub}|$

$|V_{ub}|$ determined from semileptonic b→u decays:



Two different approaches:

	Experiment	Theory
“**Exclusive**” semileptonic decays (i.e. a known set of particular decays, e.g. $B^0 \rightarrow \pi^- e^+ \nu$)	✔ Easier	✗ Less clean – requires understanding of form factors (Lattice QCD)
“**Inclusive**” semileptonic decays (i.e. $B^0 \rightarrow X_u e^+ \nu$ where X_u includes all possible hadrons)	✗ Harder – need to reject background from b→c	✔ Cleaner – can use Operator Product Expansion (OPE)

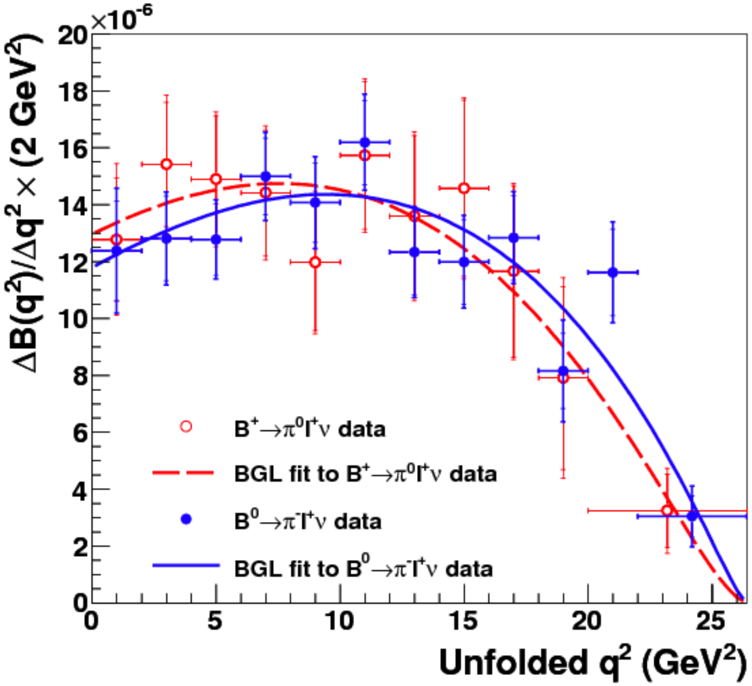
Measuring $|V_{ub}|$

Exclusive approach

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} |p_\pi|^3 |f_+(q^2)|^2$$

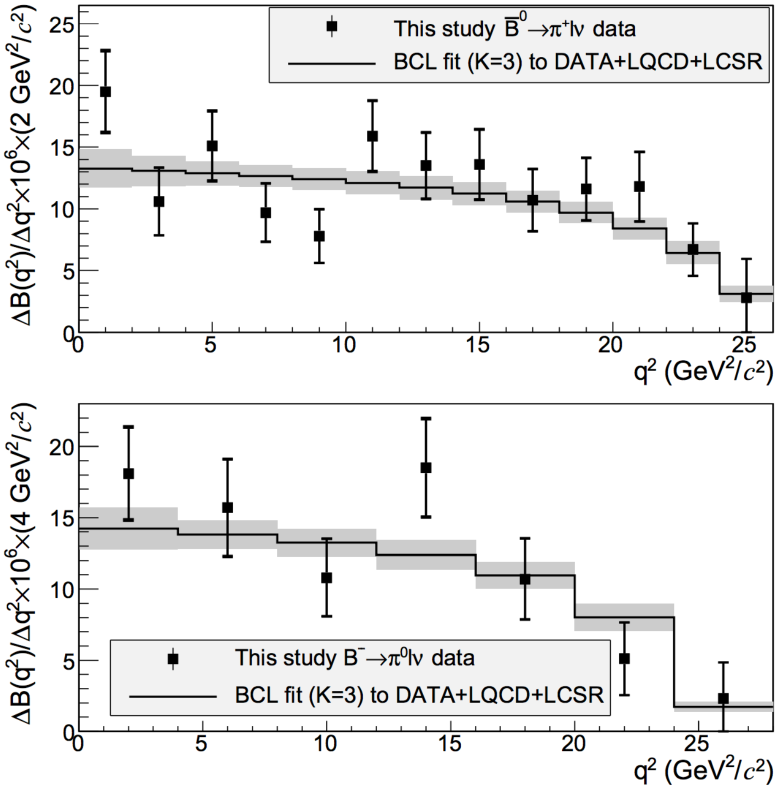
⇒ $B^0 \rightarrow \pi^- e^+ \nu$ rate versus q^2 is sensitive to $|V_{ub}|$, but requires theory input $|f_+(q^2)|$

BaBar (2012) https://arxiv.org/abs/1208.1253



$|V_{ub}| = (3.25 \pm 0.31)\times 10^{-3}$

Belle (2013) https://arxiv.org/abs/1306.2781



$|V_{ub}| = (3.52 \pm 0.29)\times 10^{-3}$

Measuring $|V_{ub}|$

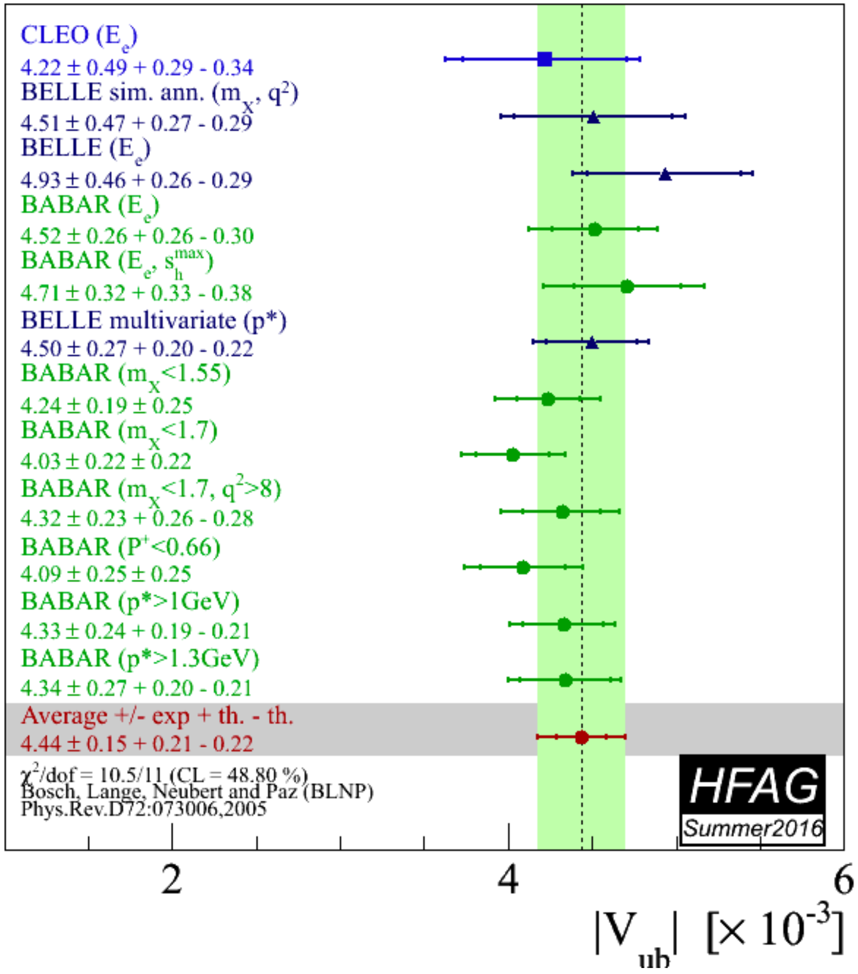
Inclusive approach

Total decay rate to all Xu is easier to calculate – don't care about details of hadronisation

But – large contamination from $b \rightarrow c$ needs to be rejected.

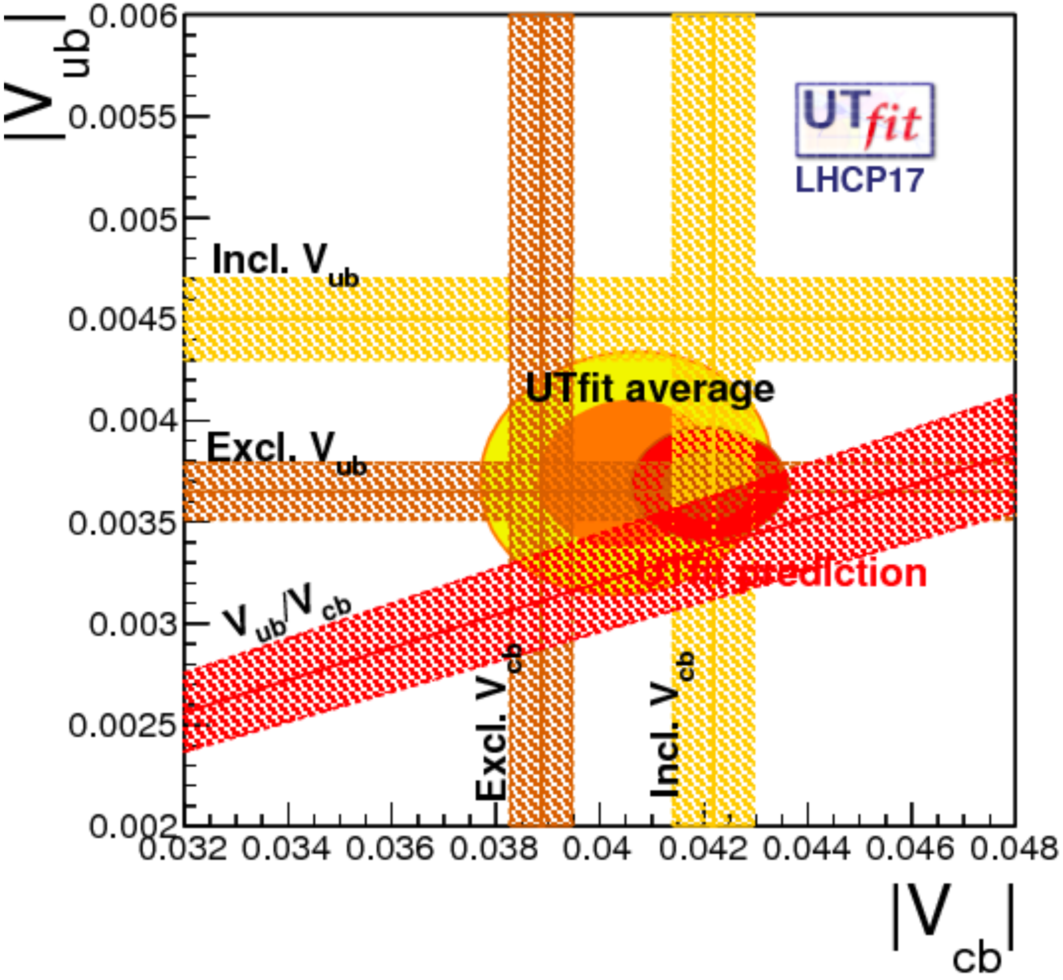
$\Rightarrow$Cut on lepton energy or q^2 – charm hadrons more massive

Several theoretical approaches – this is a summary of one of them (from Heavy Flavour Averaging Group, HFlav)



Measuring $|V_{ub}|$

Exclusive vs Inclusive



‘Golden Mode’ $B^0 \rightarrow J/\psi K_S^0$

$$\beta = \phi_1 = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

Why is $A_{CP}^{mix} = -\sin(2\beta)$ for $B^0 \rightarrow J/\psi K_S^0$?

(1) remember: $A_{CP}^{mix} = S_{CP} = \dfrac{2\,\Im(\lambda_{CP})}{1+|\lambda_{CP}|^2}$

so this is satisfied if $\lambda_{CP} = -e^{-2i\beta}$
$= -\cos(2\beta) - i\sin(2\beta)$

(2) remember: $\lambda_{f_{CP}} \equiv \dfrac{q}{p}\dfrac{\overline{A}_{f_{CP}}}{A_{f_{CP}}}$

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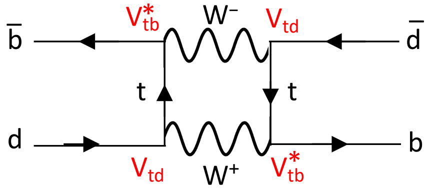

$$\frac{V_{tb}{}^*V_{td}}{V_{tb}V_{td}{}^*}$$

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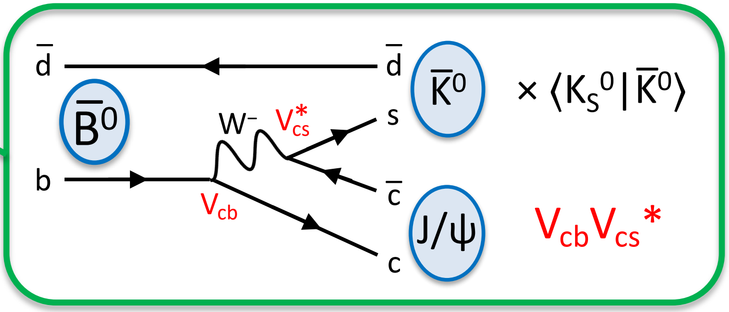
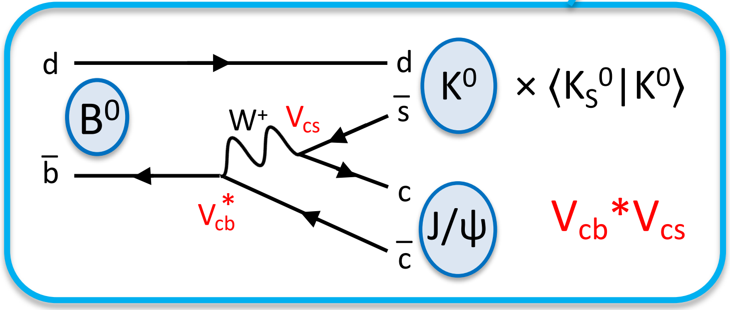


$\times \langle K_S^0 | \overline{K}^0 \rangle$

$V_{cb}V_{cs}^*$



$\times \langle K_S^0 | K^0 \rangle$

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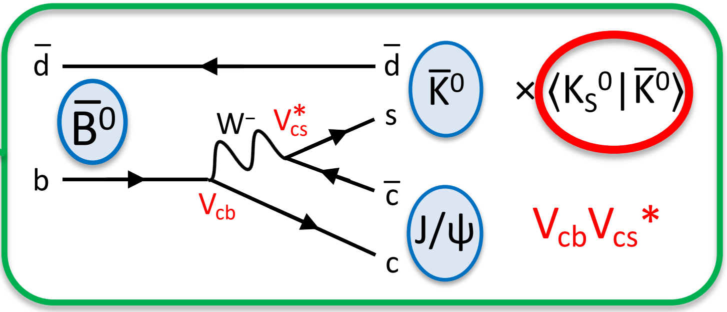

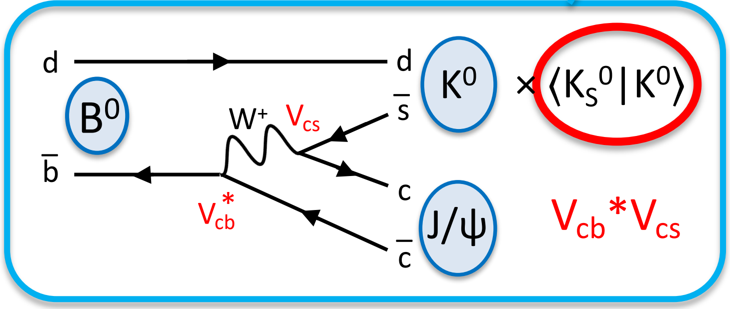

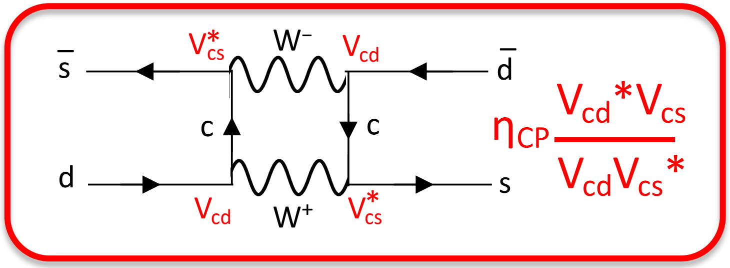

‘Golden Mode’ $B^0 \rightarrow J/\psi K_S^0$

Why is $A_{CP}^{mix} = -\sin(2\beta)$ for $B^0 \rightarrow J/\psi\ K_S{}^0$?

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‘Golden Mode’ $B^0 \to J/\psi K_S^0$

$$\beta = \phi_1 = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

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$$= -\frac{V_{tb}{}^*V_{td}}{V_{tb}V_{td}{}^*}\;\frac{V_{cb}V_{cd}{}^*}{V_{cb}{}^*V_{cd}}$$

Cancel terms, and
$\eta_{CP} = -1$ for $J/\psi K_S^0$

‘Golden Mode’ $B^0 \rightarrow J/\psi K^0_S$

Why is $A^{mix}_{CP} = -\sin(2\beta)$ for $B^0 \rightarrow J/\psi\ K_S{}^0$?

$$\beta \quad = \quad \phi_1 = \arg\left(-\frac{V_{cd}V^*_{cb}}{V_{td}V^*_{tb}}\right)$$

(1) remember: $$A^{mix}_{CP} = S_{CP} = \frac{2\,\Im(\lambda_{CP})}{1+|\lambda_{CP}|^2}$$

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$$= -\ \frac{V_{tb}{}^*V_{td}}{V_{tb}V_{td}{}^*}\ \frac{V_{cb}V_{cd}{}^*}{V_{cb}{}^*V_{cd}}$$

Cancel terms, and
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$$= \ -\ \frac{V_{cb}V_{cd}{}^*}{V_{tb}V_{td}{}^*}\ \frac{V_{tb}{}^*V_{td}}{V_{cb}{}^*V_{cd}}$$

Rearrange

‘Golden Mode’ $B^0 \rightarrow J/\psi K_S^0$

Why is $A^{mix}_{CP} = -\sin(2\beta)$ for $B^0 \rightarrow J/\psi K_S^0$?

$$\beta = \phi_1 = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

$$\Rightarrow Ae^{i\beta} = \left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

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Cancel terms, and
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$$= \boxed{-\frac{V_{cb}V_{cd}{}^*}{V_{tb}V_{td}{}^*}}\;\boxed{\frac{V_{tb}{}^*V_{td}}{V_{cb}{}^*V_{cd}}}$$

Rearrange

$= [Ae^{i\beta}]^*$
$= Ae^{-i\beta}$

$= [-Ae^{i\beta}]^{-1}$
$= -A^{-1}e^{-i\beta}$

$\Rightarrow \lambda_{J/\psi KS0} = -e^{-2i\beta}$ Q.E.D