Flavour Physics: A Taster

CERN Summer Student Lecture Programme 2023

Lecture 2 of 3: CP violation and the B factories

17-19 July 2023

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University of Edinburgh
Yesterday we covered the foundations and motivations of the subject

- Quantum loops & indirect searches for new physics
- Discrete symmetries in nature
- Example: Neutral meson oscillations

Today we connect these ideas and examine them in the context of the standard model

- The CKM mechanism and quark mixing
- Complex CKM phases $\Leftrightarrow$ CP violation
- Experimental constraints and the B factory era
Part I: Quark flavour in the SM
Quark mixing

Weak interaction breaks C and P maximally, and CP a bit – how?

In 1960s, list of fundamental fermions was small:

- 4 leptons (e, \(\mu\), \(\nu_e\), \(\nu_\mu\))
- 3 quarks (u, d, s)

From particle lifetimes, can derive weak coupling strengths \(g\) for different decays...

Muon decay \(\ell^- \rightarrow W^- \rightarrow \ell^- \nu_\ell\)

Neutron decay \(d \rightarrow W^- \rightarrow d\)

Kaon decay \(s \rightarrow W^- \rightarrow s\)

Find \(g > g' >> g''\) \(\Rightarrow\) why?
Universal coupling can be recovered if weak interaction ‘sees’ rotated combination of quark flavours

$$\theta_c = 13^\circ$$ from experiments

https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.10.531

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**Quark mixing**

Muon decay

- $g$

Neutron decay

- $g \cos(\theta_c)$

Kaon decay

- $g \sin(\theta_c)$
Quark mixing

Universal coupling can be recovered if weak interaction ‘sees’ rotated combination of quark flavours

\[ \theta_c = 13^\circ \] from experiments

Weak eigenstates are a mixture (superposition) of flavour states:

\[
\begin{pmatrix}
  d' \\
  s'
\end{pmatrix}
= \begin{pmatrix}
  \cos \theta_C & \sin \theta_C \\
  -\sin \theta_C & \cos \theta_C
\end{pmatrix}
\begin{pmatrix}
  d \\
  s
\end{pmatrix}
\]

✔ Saves universality of weak interaction, introduces concept of quark mixing

✘ Predicts additional kaon decays well above observed experimental limits...
“GIM” and the charm quark

Following Cabibbo, questions remain – some apparently allowed decays are never observed.

Process $K^0 \rightarrow \mu^+\mu^-$ apparently highly suppressed (based on exp.) – but why?
“GIM” and the charm quark

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Process $K^0 \rightarrow \mu^+\mu^-$ apparently highly suppressed (based on exp.) – but why?

Add charm quark ⇒ add second diagram (= amplitude)
“GIM” and the charm quark

Following Cabibbo, questions remain – some apparently allowed decays are never observed.

Process $K^0 \rightarrow \mu^+\mu^-$ apparently highly suppressed (based on exp.) – but why?

Add charm quark ⇒ add second diagram (= amplitude)

Two amplitudes ~equal and have opposite sign
⇒ total amplitude highly suppressed!

Cancellation not perfect because $u$ and $c$ quarks have different mass.

⇒ GIM mechanism
Neutral kaon mixing

Same diagrams cause kaon mixing

\[ \begin{align*}
\text{K}^0 & \quad \text{d} \quad W^- \quad \text{s} \\
\text{\overline{K}}^0 & \quad \overline{\text{s}} \quad W^+ \quad \overline{\text{d}}
\end{align*} \]

Mixing rate strongly depends on charm quark mass – if we can observe kaon mixing we can predict this mass.

Kaon mixing experimentally confirmed since 1960s

Measurement of \( \Delta m_k \) (=oscillation frequency) gave prediction \( m_c = 1.5 \text{ GeV} \)

\[
\Delta m_k = \frac{G_F^2}{4\pi} m_K f_K^2 m_c^2 |V_{cs} V_{cd}|^2
\]
"GIM" and the charm quark

We propose a model of weak interactions in which the currents are constructed out of four basic quark fields and interact with a charged massive vector boson. We show, to all orders in perturbation theory, that the leading divergences do not violate any strong-interaction symmetry and the next to the leading divergences respect all observed weak-interaction selection rules. The model features a remarkable symmetry between leptons and quarks. The extension of our model to a complete Yang-Mills theory is discussed.

Leads to remarkable symmetry between quark and lepton sector

\[
\begin{pmatrix}
\nu_e \\
e
\end{pmatrix}_L, \begin{pmatrix}
\nu_\mu \\
\mu
\end{pmatrix}_L
\]

\[
\begin{pmatrix}
u_\mu \\
\mu
\end{pmatrix}_L
\]

\[
\begin{pmatrix}
u_\mu \\
\mu
\end{pmatrix}_L
\]

\[
\begin{pmatrix}
u_\mu \\
\mu
\end{pmatrix}_L
\]

\[
\begin{pmatrix}
u_\mu \\
\mu
\end{pmatrix}_L
\]

Makes testable prediction of existence and mass of charm quark...
“GIM” and the charm quark

Leads to remarkable symmetry between quark and lepton sector

\[
\begin{align*}
\left( \begin{array}{c}
\nu_e \\
e \\
u \mu \\
\mu \\
u \nu_d \\
\nu \nu_s \\
\nu \nu_s'
\end{array} \right)_L, \\
\left( \begin{array}{c}
\nu_e \\
e \\
u \mu \\
\mu \\
u \nu_d \\
\nu \nu_s \\
\nu \nu_s'
\end{array} \right)_L
\end{align*}
\]

J/ψ meson (c̅c bound state) discovered simultaneously at BNL and SLAC in 1974

\[M(\psi) \approx 3 \text{ GeV} \]

Makes testable prediction of existence and mass of charm quark...

Weak Interactions with Lepton-Hadron Symmetry*

S. L. Glashow, J. Iliopoulos, and L. Maiani†

Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02139

(Received 5 March 1970)

We propose a model of weak interactions in which the currents are constructed out of four basic quark fields and interact with a charged massive vector boson. We show, to all orders in perturbation theory, that the leading divergences do not violate any strong-interaction symmetry and the next to the leading divergences respect all observed weak-interaction selection rules. The model features a remarkable symmetry between leptons and quarks. The extension of our model to a complete Yang-Mills theory is discussed.

Where’s the CP violation?

CP violation experimentally verified in weak interaction, but couldn’t fit into existing theory...

KM realised that **we need 3 generations** to allow CP violation...

\[
\begin{bmatrix}
    d' \\
    s' \\
    b'
\end{bmatrix} =
\begin{bmatrix}
    \cos \theta_c & \sin \theta_c \\
    -\sin \theta_c & \cos \theta_c
\end{bmatrix}
\begin{bmatrix}
    d \\
    s \\
    b
\end{bmatrix}
\]

**Cabibbo**

1 (real) parameter: mixing angle $\theta_c$

**Cabibbo Kobayashi Maskawa (CKM)**

\[
\begin{bmatrix}
    d' \\
    s' \\
    b'
\end{bmatrix} =
\begin{bmatrix}
    V_{ud} & V_{us} & V_{ub} \\
    V_{cd} & V_{cs} & V_{cb} \\
    V_{td} & V_{ts} & V_{tb}
\end{bmatrix}
\begin{bmatrix}
    d \\
    s \\
    b
\end{bmatrix}
\]

4 parameters: 3 real mixing angles

**1 complex phase!**
Where’s the CP violation?

CP violation experimentally verified in weak interaction, but couldn’t fit into existing theory...

KM realised that we need 3 generations to allow CP violation...

\[
\begin{align*}
[d'] &= \begin{bmatrix} 
\cos \theta_c & \sin \theta_c \\
-\sin \theta_c & \cos \theta_c 
\end{bmatrix} \begin{bmatrix} d \\
 s 
\end{bmatrix} \\
[s'] &= \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\
 s \\
b \end{bmatrix}
\end{align*}
\]

Prediction of another 2 new quarks even before charm was discovered!

⇒ b (t) quark not discovered until 1977 (1994)!
[Discovering beauty/bottom]

Observation of a Dimuon Resonance at 9.5 GeV in 400-GeV Proton-Nucleus Collisions

S. W. Herb, D. C. Hom, L. M. Lederman, J. C. Sens,(a) H. D. Snyder, and J. K. Yoh
Columbia University, New York, New York 10027

and

Fermi National Accelerator Laboratory, Batavia, Illinois 60510

and

A. S. Ito, H. Jöstlein, D. M. Kaplan, and R. D. Kephart
State University of New York at Stony Brook, Stony Brook, New York 11794
(Received 1 July 1977)

1977, Lederman et al (proton beam on fixed target)

2018, LHCb (pp collisions)

‘bottomonium’
(b⁻b⁺ bound state)
\( \Upsilon \rightarrow \mu^+\mu^- \)
\( M \approx 9.5 \text{ GeV} \)
CKM structure

Current experimental status:

\[
\begin{pmatrix}
|V_{ud}| & |V_{us}| & |V_{ub}| \\
|V_{cd}| & |V_{cs}| & |V_{cb}| \\
|V_{td}| & |V_{ts}| & |V_{tb}|
\end{pmatrix} =
\begin{pmatrix}
0.97435 \pm 0.00016 & 0.22500 \pm 0.00067 & 0.00369 \pm 0.00011 \\
0.22486 \pm 0.00067 & 0.97349 \pm 0.00016 & 0.04182^{+0.00085}_{-0.00074} \\
0.00857^{+0.00020}_{-0.00018} & 0.04110^{+0.00083}_{-0.00072} & 0.999118^{+0.000031}_{-0.000036}
\end{pmatrix}
\]

Magnitudes \(|V_{ij}|^2\) appear in probabilities (=rates) of decays.

Magnitudes have suggestive pattern
No known reason!

Transitions within same generation: “Cabibbo Favoured” (CF)

Processes with 1 (2) off-diagonal elements: “Singly (doubly) Cabibbo Suppressed” (SCS / DCS)
CKM and CP violation

CP operator
⇒ complex conjugation of amplitudes

With 3 generations, CKM elements $V_{ij}$ can be complex

A universe with 2 (or 1) generations could not have CP violation this way!

Highly predictive (= good theory!)

• Can make many independent measurements of $V_{ij}$ from different systems
• Test if these are self-consistent

Next job: measure the magnitudes and phases of these complex parameters $V_{ij}$
Decompose into three rotation matrices:

\[
V_{\text{CKM}} = \begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix}
\begin{pmatrix}
c_{13} & 0 & s_{13} e^{-i\delta} \\
0 & 1 & 0 \\
-s_{13} e^{i\delta} & 0 & c_{13}
\end{pmatrix}
\begin{pmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

Parameters:
- 3 rotation angles \(\theta_{12}, \theta_{13}, \theta_{23}\)
- CP-violating phase \(\delta\)

Observed hierarchy motivates an alternative parameterisation...
Expand CKM matrix elements in powers of $\lambda \approx 0.22$ (i.e. $\sin \theta_c$)

Here shown to order $\lambda^3$

Parameters: $A, \lambda, \rho, \eta$

Quantify CP violation
Part II: Testing the CKM mechanism

a. Magnitudes
Testing the CKM mechanism

How to measure CKM matrix elements?
⇒ magnitudes control rates of particle decays
⇒ Ratio of decay rates proportional to ratio of $|\text{amplitude}|^2$

For $V_{ud}$, compare neutron ($\beta$ decay) and muon decay rates

$V_{ud}$

$V_{cd}$

$V_{td}$

$V_{us}$

$V_{cs}$

$V_{ts}$

$V_{ub}$

$V_{cb}$

$V_{tb}$

$n \{d, d, d, u\} \rightarrow p u u u$
Testing the CKM mechanism

\[\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}\]
Testing the CKM mechanism

\[
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\]

Semileptonic (or leptonic) Kaon decays
Testing the CKM mechanism

- β-decays
- Kaon decays
- Semileptonic (or leptonic)

\[
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\]

B meson decays to charm mesons
Testing the CKM mechanism

- Semileptonic (or leptonic) Kaon decays
- Semileptonic Charm meson decays
- B meson decays to light mesons
- B meson decays to charm mesons
- B$_s^0$ mixing
- B$^0$ mixing
- top decays

Often require theory inputs to relate hadron measurements to quark-level CKM
Unitarity triangle(s)

CKM matrix is unitary: $V_{\text{CKM}}V_{\text{CKM}}^\dagger = I$

Provides 9 constraints relating elements, e.g.

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

Sum of three complex numbers = 0
⇒ triangle on Argand plane

There are in fact 6 triangles
(one per quark pair)
– this one (‘bd’) is most insightful
Unitarity triangle(s)

CKM matrix is unitary: $V^\text{CKM}V^{\dagger\text{CKM}} = I$

Provides 9 constraints relating elements, e.g.

$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$

Sum of three complex numbers = 0
⇒ triangle on Argand plane

Rescale by dividing all sides by $|V_{cd}V_{cb}^*|$

\[
\begin{align*}
\beta &= \phi_1 = \arg \left( -\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*} \right) \\
\alpha &= \phi_2 = \arg \left( -\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*} \right) \\
\gamma &= \phi_3 = \arg \left( -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right)
\end{align*}
\]

(modified Wolfenstein parameters)
Unitarity triangle(s)

CKM matrix is unitary: \( \mathbf{V}_{\text{CKM}} \mathbf{V}_{\text{CKM}}^\dagger = \mathbf{I} \)

Provides 9 constraints relating elements, e.g.

\[ V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \]

Sum of three complex numbers = 0
⇒ triangle on Argand plane

Rescale by dividing all sides by \( |V_{cd}V_{cb}^*| \)

Now experimental measurements form constraints of various shape on the position of the apex
• Length of sides (x2)
• Angles (x3)
**SM CP violation and the universe**

**Jarlskog** parameter $J$: Convention-invariant measure of CPV in quark sector

$$J = \pm \text{Im}(V_{us} V_{cb} V_{ub}^* V_{cs}^*)$$

Expressed as Wolfenstein parameters:

$$J = A^2 \lambda^6 \eta(1 - \lambda^2/2) + O(\lambda^{10}) \approx 3 \times 10^{-5}$$

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[Image of Cecilia Jarlskog with colleagues at the Nordic Institute of Theoretical Physics (NORDITA) in Copenhagen, in the early 1980s.]

[Hypertext link to the original publication](https://doi.org/10.1103/PhysRevLett.55.1039) (1985)
**SM CP violation and the universe**

**Jarlskog** parameter $J$: Convention-invariant measure of CPV in quark sector

$$J = \pm \text{Im}(V_{us}V_{cb}^*V_{ub}^*V_{cs}^*)$$

But... if any quark masses are degenerate, CPV vanishes – and small differences suppress it....

Multiply by terms

$$P_u = (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)$$

$$P_d = (m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)$$

And divide by electroweak mass scale... $M_W^{12}$

(Wolfenstein parameters) $J = 2 \times \text{area}$
**SM CP violation and the universe**

**Jarlskog** parameter $J$: Convention-invariant measure of CPV in quark sector

$$J = \pm \text{Im}(V_{us}V_{cb}V_{ub}^*V_{cs}^*)$$

But... if any quark masses are degenerate, CPV vanishes – and small differences suppress it....

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$$P_u = (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)$$

$$P_d = (m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)$$

And divide by electroweak mass scale... $M_W^{12}$

$$\frac{n_B - n_{\bar{B}}}{n_Y} \approx \frac{n_B}{n_Y} \sim \frac{J \times P_u \times P_d}{M^{12}} = O(10^{-20}) \quad \text{from SM}$$

$$= O(10^{-10}) \quad \text{Observed!}$$

⇒ Need to identify new sources of CPV associated with **high energy scales**
Top quark just discovered
⇒ CKM constraint can be derived from $B^0$ meson mixing measurements ($\Delta M_d$)

First constraints on $|V_{ub}|$ from LEP, ARGUS, CLEO experiments

Minimum number of measurements needed to locate apex, and large uncertainties – no measurements of angles

Lots of work ahead! Sets the stage for the next phase in flavour physics...
The era of the B factories!
Part II: Testing the CKM mechanism
b. Phases
How to measure angles $\alpha$, $\beta$, $\gamma$?

Observables are rates, i.e. $|A|^2 \Rightarrow$ not sensitive to phases

Need two amplitudes with different phases – then rate sensitive to their difference...

$|A_1e^{i\phi_1} + A_2e^{i\phi_2}|^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\delta\phi)$

$\delta\phi = \phi_1 - \phi_2$

Unitarity triangle angles are phase differences between CKM elements
e.g. $\beta$ is angle between $V_{cd}V_{cb}^*$ and $V_{td}V_{tb}^*$

top quark – must be in loop!

Need >1 amplitudes to reach same final state (interference)
One of these must include a top quark loop...

$B^0$ mixing?
Three ways to satisfy the criteria for CPV:

1. Amplitudes with different strong and weak phases:
   \[ \Gamma(i \rightarrow f) \neq \Gamma(i \rightarrow \bar{f}) \]

2. CP violation in meson mixing:
   \[ \Gamma(M^0 \rightarrow \bar{M}^0) \neq \Gamma(\bar{M}^0 \rightarrow M^0) \]
   i.e. \(|q/p| \neq 1\)
   Only possible for neutral mesons that mix

3. CP violation in interference between mixing and decay:
   \[ \Gamma(M^0 \rightarrow \bar{M}^0 \rightarrow f) \neq \Gamma(\bar{M}^0 \rightarrow M^0 \rightarrow f) \]
   requires \(\text{arg}(q/p) \neq 0\)
   Possible for any decay
Consider the process $B^0 \rightarrow \bar{B}^0 \rightarrow f_{CP}$

From last lecture, for $B^0$ at time $t=0$

$$|B^0(t)\rangle = g_+(t)|B^0\rangle + \left(\frac{q}{p}\right) g_-(t)|\bar{B}^0\rangle$$

⇒ Total amplitude = $A_{f_{CP}} \left[ g_+(t) + \frac{q}{p} \frac{\bar{A}_{f_{CP}}}{A_{f_{CP}}} g_-(t) \right]$ where $\bar{A}_{f_{CP}} = \langle f_{CP}|B^0\rangle$

= $A_{f_{CP}} \left[ g_+(t) + \lambda_{f_{CP}} g_-(t) \right]$

= $A_{f_{CP}} \left[ e^{-i\lambda_{f_{CP}} g_-(t)} \right]$ where

$$\lambda_{f_{CP}} = \frac{q}{p} \frac{\bar{A}_{f_{CP}}}{A_{f_{CP}}}$$

Now plug-in $g_\pm(t)$ terms (see last lecture) and $| |^2$ to get rate...

Reminder: 

$$g_+(t) = e^{-imt} e^{-\Gamma t/2t} \left[ \cosh \frac{\Delta \Gamma t}{4} \cos \frac{\Delta M t}{2} - i \sinh \frac{\Delta \Gamma t}{4} \sin \frac{\Delta M t}{2} \right],$$

$$g_-(t) = e^{-imt} e^{-\Gamma t/2t} \left[ -\sinh \frac{\Delta \Gamma t}{4} \cos \frac{\Delta M t}{2} + i \cosh \frac{\Delta \Gamma t}{4} \sin \frac{\Delta M t}{2} \right]$$
### CP violation in interference

#### B^0 at t=0:
\[
\Gamma(B(t) \rightarrow f) \propto e^{-\Gamma t} \\
\times \left[ \cosh(\Delta \Gamma t/2) + A_{CP}^{dir} \cos(\Delta m t) + A_{\Delta \Gamma} \sinh(\Delta \Gamma t/2) + A_{CP}^{mix} \sin(\Delta m t) \right]
\]

#### \(\bar{B}^0\) at t=0:
\[
\Gamma(\bar{B}(t) \rightarrow f) \propto e^{-\Gamma t} \\
\times \left[ \cosh(\Delta \Gamma t/2) - A_{CP}^{dir} \cos(\Delta m t) + A_{\Delta \Gamma} \sinh(\Delta \Gamma t/2) - A_{CP}^{mix} \sin(\Delta m t) \right]
\]

where:

\[
A_{CP}^{dir} = C_{CP} = \frac{1 - |\lambda_{CP}|^2}{1 + |\lambda_{CP}|^2} \\
A_{\Delta \Gamma} = \frac{2 \Re(\lambda_{CP})}{1 + |\lambda_{CP}|^2} \\
A_{CP}^{mix} = S_{CP} = \frac{2 \Im(\lambda_{CP})}{1 + |\lambda_{CP}|^2}
\]

- **CPV in decay**
- **CP conserving part**
- **CPV in interference between mixing & decay**
CP violation in interference

\( B^0 \text{ at } t=0: \quad \Gamma(B(t) \to f) \propto e^{-\Gamma t} \times \left[ \cosh(\Delta \Gamma t/2) + A_{\text{dir}}^{\text{CP}} \cos(\Delta m t) + A_{\text{dir}}^{\text{mix}} \sinh(\Delta \Gamma t/2) + A_{\text{CP}}^{\text{mix}} \sin(\Delta m t) \right] \)

\( \bar{B}^0 \text{ at } t=0: \quad \Gamma(\bar{B}(t) \to f) \propto e^{-\Gamma t} \times \left[ \cosh(\Delta \Gamma t/2) - A_{\text{dir}}^{\text{CP}} \cos(\Delta m t) + A_{\text{dir}}^{\text{mix}} \sinh(\Delta \Gamma t/2) - A_{\text{CP}}^{\text{mix}} \sin(\Delta m t) \right] \)

\( \times \) For \( B^0 \) case, \( \Delta \Gamma \) small – can be neglected...
CP violation in interference

\[ \Gamma(B(t) \rightarrow f) \propto e^{-\Gamma t} \times \left[ \cosh(\Delta \Gamma t/2) + A_{\text{dir}}^{\text{CP}} \cos(\Delta m t) + A_{\Delta \Gamma} \sinh(\Delta \Gamma t/2) + A_{\text{mix}}^{\text{CP}} \sin(\Delta m t) \right] \]

\[ \Gamma(\bar{B}(t) \rightarrow f) \propto e^{-\Gamma t} \times \left[ \cosh(\Delta \Gamma t/2) - A_{\text{dir}}^{\text{CP}} \cos(\Delta m t) + A_{\Delta \Gamma} \sinh(\Delta \Gamma t/2) - A_{\text{mix}}^{\text{CP}} \sin(\Delta m t) \right] \]

\[ \times \]

For ‘golden mode’ \( B^0 \rightarrow J/\psi K_s^0 \): No direct CPV \((A_{\text{CP}}^{\text{dir}} = 0, \ a = 0)\)

\[ \text{and } A_{\text{mix}}^{\text{CP}} = -\sin(2\beta) \]
\[ \Gamma(B(t) \rightarrow f) \propto e^{-\Gamma t} \times [1 - \sin(2\beta) \sin(\Delta m t)] \]

\[ \Gamma(\bar{B}(t) \rightarrow f) \propto e^{-\Gamma t} \times [1 + \sin(2\beta) \sin(\Delta m t)] \]

⇒ By time-dependent analysis, can extract \( \beta \) from amplitude of oscillations
CP violation in interference

\[ \Gamma(B(t) \rightarrow f) \propto e^{-\Gamma t} \times [1 - \sin(2\beta) \sin(\Delta m t)] \]

\[ \Gamma(\bar{B}(t) \rightarrow f) \propto e^{-\Gamma t} \times [1 + \sin(2\beta) \sin(\Delta m t)] \]

\[ \Rightarrow \text{By time-dependent analysis, can extract } \beta \text{ from amplitude of oscillations} \]

\[ \Rightarrow \text{Even cleaner using CP asymmetry:} \]

\[ \frac{\Gamma(t) [B^0 \rightarrow J/\psi K_S^0] - \Gamma(t) [\bar{B}^0 \rightarrow J/\psi K_S^0]}{\Gamma(t) [B^0 \rightarrow J/\psi K_S^0] + \Gamma(t) [\bar{B}^0 \rightarrow J/\psi K_S^0]} = -\sin(2\beta)\sin(\Delta m t) \]

Hence, “Golden mode”

But note: asymmetry integrates to zero over time
Part III: The B factories
The B Factories: BaBar and Belle

- Collide $e^+e^-$ at $\Upsilon(4S)$ resonance energy $\Rightarrow \Upsilon(4S) \rightarrow B^{(0,\pm)}\overline{B}^{(0,\pm)}$
- B hadrons quantum correlated – can determine initial state from ‘other B’
- Asymmetric beam energy $\Rightarrow$ B hadrons boosted, so can measure ‘t’

NEW!

Pier Oddone, father of asymmetric $e^+e^-$ colliders
The B Factories: BaBar and Belle

BaBar: on PEP-II @ SLAC, USA
9 GeV $e^-$ $\leftrightarrow$ 3.1 GeV $e^+$
433 fb$^{-1}$ (1999 – 2008)

Different detectors, same ideas:
• Vertex + tracking detectors
• Particle ID
• Calorimetry

Belle: on KEKB accelerator (Japan)
8 GeV $e^-$ $\leftrightarrow$ 3.5 GeV $e^+$
711 fb$^{-1}$ (1999 – 2010)
Example event

Tagging side: \[ \bar{B}^0 \rightarrow D^{*+} \pi^-_{\text{fast}} \]
\[ \downarrow \]
\[ D^0 \pi^+_{\text{soft}} \]
\[ \downarrow \]
\[ K^- \pi^+ \]

K^- tags initial flavor as \( \bar{B}^0 \)

⇒ Signal must be \( B^0 \) at “t=0”

\[ B^0 \rightarrow J/\psi K_{S}^0 \]
\[ \downarrow \]
\[ \mu^+\mu^- \]
Golden mode results: $\sin(2\beta)$

**BaBar**  

**Belle**  

- Actually use many different channels (both CP-odd and CP-even, $\eta_f = \pm 1$)
  - $\sim K_L^0$
  - $\sim K_S^0$

⇒ Clear CP-asymmetry! Measure $\sin(2\beta)$
Golden mode results: $\sin(2\beta)$

$\sin(2\beta) \equiv \sin(2\phi_1)$

Results on previous slide

LHCb now competitive with B-factories!

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Reference</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>BaBar</td>
<td>PRD 79 (2009):072009</td>
<td>0.69 ± 0.03 ± 0.01</td>
</tr>
<tr>
<td>BaBar $\chi^0 K_S$</td>
<td>PRD 80 (2009):112001</td>
<td>0.69 ± 0.52 ± 0.04 ± 0.07</td>
</tr>
<tr>
<td>BaBar $J/\psi$ (hadronic) $K_S$</td>
<td>PRD 69 (2004):052001</td>
<td>1.56 ± 0.42 ± 0.21</td>
</tr>
<tr>
<td>Belle</td>
<td>PRL 108 (2012) 171802</td>
<td>0.67 ± 0.02 ± 0.01</td>
</tr>
<tr>
<td>ALEPH</td>
<td>PLB 492, 259 (2000)</td>
<td>0.84 ± 0.32 ± 0.16</td>
</tr>
<tr>
<td>OPAL</td>
<td>EPJ C5, 379 (1998)</td>
<td>3.20 ± 1.30 ± 0.50</td>
</tr>
<tr>
<td>CDF</td>
<td>PRD 61, 072005 (2000)</td>
<td>0.79 ± 0.41</td>
</tr>
<tr>
<td>LHCb</td>
<td>JHEP 11 (2017) 170</td>
<td>0.76 ± 0.03</td>
</tr>
<tr>
<td>BelleSS</td>
<td>PRL 108 (2012) 171801</td>
<td>0.57 ± 0.58 ± 0.06</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>0.70 ± 0.02</td>
</tr>
</tbody>
</table>

HFLAV
Moriond 2018
PRELIMINARY

$\sin(2\beta)$
Golden mode results: \( \sin(2\beta) \)

Two-fold ambiguity on \( \beta \), but second solution ruled-out by other inputs.

\[
\beta = \phi_1 = (22.2 \pm 0.7)^\circ
\]

\[
\eta
\]

\[
\rho
\]
Other angles: $\alpha$ and $\gamma$

Similar approach to measure other angles...

$$\beta = \phi_1 = \arg \left( -\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*} \right)$$

$$\alpha = \phi_2 = \arg \left( -\frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*} \right)$$

$$\gamma = \phi_3 = \arg \left( -\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \right)$$

b→cW transitions, with B mixing (e.g. $B^0 \rightarrow J/\psi K_{S0}$)

b→uW transitions, with B mixing (e.g. $B^0 \rightarrow \pi^+\pi^-$)

Messy – many interfering processes, and direct CPV
Beyond tree-level...

Can have penguin diagrams with different weak phase

For $B^0 \rightarrow J/\psi K^0_s$, tree-level process dominates

$\Rightarrow$ penguin can be ignored (<1% effect)

With sufficient experimental precision, these penguin contributions must be included.
Measuring CKM angle $\alpha$

Similar process allows $\alpha$ to be measured, **BUT** cannot ignore penguin pollution here.

\[
A_{CP}^{\text{dir}} = 2 \frac{|P|}{|T|} \sin \alpha \sin(\delta_P - \delta_T)
\]

\[
A_{CP}^{\text{mix}} = \sin 2\alpha - 2 \frac{|P|}{|T|} \sin \alpha \cos 2\alpha \cos(\delta_P - \delta_T)
\]

~20% = $\sin 2\alpha_{\text{eff}}$

Several proposed techniques to reduce sensitivity to penguin pollution, e.g.

CKM angle $\alpha$: state-of-the-art

Current measurements from different channels not in perfect agreement – need more precision!

- Smaller CPV and penguin effects
- Larger CPV and penguin effects

Contours give $-2\Delta \ln L = \Delta \chi^2 = 1$, corresponding to 90.3% CL for 2 dof.
CKM angle $\alpha$: state-of-the-art

LHCb measurements now most precise

https://doi.org/10.1007/JHEP03(2021)075
Impact of B-factories

1995

2009

excluded area has CL > 0.95

$\Delta m_d$ & $\Delta m_s$

$\varepsilon_K$

$|V_{ub}|$

$\rho$

$\sin 2\beta$

$\Delta m_d$

$\Delta m_d & \Delta m_s$

$\gamma$

$\varepsilon_K$

$\alpha$

$\beta$

solved w/ cos $2\beta < 0$
(excl. at CL > 0.95)
On the eve of the LHC...

All constraints consistent with single point for apex

Direct measurements of angles:

\[ \beta = (21.15 \pm 0.90) \]°
\[ \alpha = (89.0^{+4.4}_{-4.2}) \]°
\[ \gamma = (73^{+22}_{-25}) \]°

⇒ Need to improve \( \gamma \) measurement!

Brings us to the LHC era of flavour
Today we covered the foundations of b physics:

- CP violation in the SM (quark sector)
- Unitarity triangle(s)
- Measuring CKM phases
- B-factory measurements of $\beta$ and $\alpha$

Next time – we will cover b (and c) physics in the LHC era:

- Hadron colliders vs B-factories
- Mixing and CP violation in $B_s^0$ and $D^0$ mesons
- CKM angle gamma
- Rare decays and lepton universality
• CKM parameters
• CPV and ‘strong phases’
• Measuring $|V_{ub}|$
• Measuring $\sin(2\beta)$
CKM matrix: Why 4 parameters?

Why does a $3 \times 3$ CKM matrix only have 3 real and 1 complex parameters?

Most general $N \times N$ complex matrix would have $2N^2 = 18$ parameters

- Must be unitary, i.e. $V_{\text{CKM}}V_{\text{CKM}}^* = I$  \(\Rightarrow\) $N^2$ constraints, leaving $N^2=9$ parameters  
  (in physics: $t \rightarrow d + t \rightarrow s + t \rightarrow b = 1$)

- We can readily change conventions which describe phases between quark fields  \(\Rightarrow\) 6 quarks, so 5 phase differences, leaving 4 free parameters

- $N(N-1)/2 = 3$ are rotation angles

- Remaining parameter is irreducible phase

Note: For $N=2$ (Cabibbo), we have $8 - 4 - 3 = 1$ free parameter (must be rotation angle)
Consider a process with two interfering amplitudes – can it violate CP symmetry?

Amplitude \( A = A_1 e^{i\phi_1} + A_2 e^{i\phi_2} \)

Rate = \( |A_1 e^{i\phi} + A_2 e^{i\phi'}|^2 \)
= \( A_1^2 + A_2^2 + 2A_1A_2\cos(\delta\phi) \)

No! Obvious in Argand diagram...

Amplitude \( A^* = A_1 e^{-i\phi_1} + A_2 e^{-i\phi_2} \)

Rate = \( |A_1 e^{-i\phi} + A_2 e^{-i\phi'}|^2 \)
= \( A_1^2 + A_2^2 + 2A_1A_2\cos(-\delta\phi) \)
= \( A_1^2 + A_2^2 + 2A_1A_2\cos(\delta\phi) \)

There is a second condition to allow CP violation...
There is a second condition to allow CP violation...

Different strong phase (i.e. CP conserving – no sign change) between amplitudes

\[ A = A_1 e^{i\phi_1} e^{i\kappa_1} + A_2 e^{i\phi_2} e^{i\kappa_2} \]

Rate

\[ = A_1^2 + A_2^2 + 2A_1 A_2 \cos(\delta\phi + \delta\kappa) \]

\[ \bar{A} = A_1 e^{-i\phi_1} e^{i\kappa_1} + A_2 e^{-i\phi_2} e^{i\kappa_2} \]

Rate

\[ = A_1^2 + A_2^2 + 2A_1 A_2 \cos(\delta\phi - \delta\kappa) \]

CP violation!

Difference in rates:

\[ \Gamma(i \rightarrow f) - \Gamma(\bar{i} \rightarrow \bar{f}) = -4A_1 A_2 \sin(\delta\phi) \sin(\delta\kappa) \]
Measuring $|V_{ub}|$

$|V_{ub}|$ determined from semileptonic $b \rightarrow u$ decays:

Two different approaches:

- **“Exclusive”** semileptonic decays (i.e. a known set of particular decays, e.g. $B^0 \rightarrow \pi^- e^+ \nu$)
  - **Experiment**: Easier
  - **Theory**: Less clean – requires understanding of form factors (Lattice QCD)

- **“Inclusive”** semileptonic decays (i.e. $B^0 \rightarrow X_u e^+ \nu$ where $X_u$ includes all possible hadrons)
  - **Experiment**: Harder – need to reject background from $b \rightarrow c$
  - **Theory**: Cleaner – can use Operator Product Expansion (OPE)
Measuring $|V_{ub}|$

Exclusive approach

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} |p_\pi|^3 |f_+(q^2)|^2$$

$B^0 \rightarrow \pi^- e^+ \nu$ rate versus $q^2$ is sensitive to $|V_{ub}|$, but requires theory input $|f_+(q^2)|$

**BaBar (2012)**

$|V_{ub}| = (3.25 \pm 0.31) \times 10^{-3}$


**Belle (2013)**

$|V_{ub}| = (3.52 \pm 0.29) \times 10^{-3}$

Inclusive approach

Total decay rate to all \( X \) is easier to calculate – don’t care about details of hadronisation.

But – large contamination from \( b \to c \) needs to be rejected.

\[ \Rightarrow \text{Cut on lepton energy or } q^2 \text{ – charm hadrons more massive} \]

Several theoretical approaches – this is a summary of one of them (from Heavy Flavour Averaging Group, HFlav)

**Table: Measuring \( |V_{ub}| \)**

<table>
<thead>
<tr>
<th>Source</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLEO ((E_\gamma))</td>
<td>(4.22 \pm 0.49 \pm 0.29 - 0.34)</td>
</tr>
<tr>
<td>BELLE sim. ann. ((m_X, q^2))</td>
<td>(4.51 \pm 0.47 \pm 0.27 - 0.29)</td>
</tr>
<tr>
<td>BELLE ((E_\gamma))</td>
<td>(4.93 \pm 0.46 \pm 0.26 - 0.29)</td>
</tr>
<tr>
<td>BABAR ((E_\gamma))</td>
<td>(4.52 \pm 0.26 \pm 0.26 - 0.30)</td>
</tr>
<tr>
<td>BABAR ((E_\gamma \text{ min}))</td>
<td>(4.71 \pm 0.32 \pm 0.33 - 0.38)</td>
</tr>
<tr>
<td>BELLE multivariate ((p^*))</td>
<td>(4.50 \pm 0.27 \pm 0.20 - 0.22)</td>
</tr>
<tr>
<td>BABAR ((m_X &lt; 1.55))</td>
<td>(4.24 \pm 0.19 \pm 0.25)</td>
</tr>
<tr>
<td>BABAR ((m_X &lt; 1.7))</td>
<td>(4.03 \pm 0.22 \pm 0.22)</td>
</tr>
<tr>
<td>BABAR ((m_X &lt; 1.7, q^2 &gt; 8))</td>
<td>(4.32 \pm 0.23 \pm 0.26 - 0.28)</td>
</tr>
<tr>
<td>BABAR ((p^* &lt; 0.66))</td>
<td>(4.09 \pm 0.25 \pm 0.25)</td>
</tr>
<tr>
<td>BABAR ((p^* &gt; 1 \text{GeV}))</td>
<td>(4.33 \pm 0.24 \pm 0.19 - 0.21)</td>
</tr>
<tr>
<td>BABAR ((p^* &gt; 1.3 \text{GeV}))</td>
<td>(4.34 \pm 0.27 \pm 0.20 - 0.21)</td>
</tr>
</tbody>
</table>

\[ \chi^2/\text{dof} = 10.5/11 \text{ (CL = 48.80 %)} \]

Bosch, Lange, Neubert and Paz (BLNP)


\[ |V_{ub}| [\times 10^{-3}] \]
Measuring $|V_{ub}|$

Exclusive vs Inclusive
'Golden Mode' $B^0 \rightarrow J/\psi K_S^0$

Why is $A_{CP}^{mix} = -\sin(2\beta)$ for $B^0 \rightarrow J/\psi K_S^0$?

(1) remember: $A_{CP}^{mix} = S_{CP} = \frac{2 \Im(\lambda_{CP})}{1 + |\lambda_{CP}|^2}$

so this is satisfied if $\lambda_{CP} = -e^{-2i\beta} = -\cos(2\beta) - i \sin(2\beta)$

(2) remember: $\lambda_{f_{CP}} \equiv \frac{q A_{f_{CP}}}{p A_{f_{CP}}}$

$\beta = \phi_1 = \text{arg} \left( \frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*} \right)$
‘Golden Mode’ $B^0 \rightarrow J/\psi K^0_S$

Why is $A_{CP}^{mix} = -\sin(2\beta)$ for $B^0 \rightarrow J/\psi K^0_S$?

(1) remember: $A_{CP}^{mix} = S_{CP} = \frac{2}{1 + |\lambda_{CP}|^2} \Im(\lambda_{CP})$ so this is satisfied if $\lambda_{CP} = e^{-2i\beta}$

(2) remember: $\lambda_{fCP} = \frac{q}{p} \frac{A_{fCP}}{A_{fCP}} = \frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*}$
‘Golden Mode’ $B^0 \to J/\psi K_S^0$

Why is $A_{CP}^{\text{mix}} = -\sin(2\beta)$ for $B^0 \to J/\psi K_S^0$?

(1) remember: $A_{CP}^{\text{mix}} = S_{CP} = \frac{2 \Im(\lambda_{CP})}{1 + |\lambda_{CP}|^2}$

(2) remember: $\lambda_{fCP} = \frac{q}{p} A_{fCP}$

so this is satisfied if

$\lambda_{CP} = -e^{-2i\beta}$

$= -\cos(2\beta) - i \sin(2\beta)$

For neutral $B$ mesons, $g^-(t)$ has a $90^\circ$ phase difference wrt. $g^+(t)$

$g^+\pm(t) = e^{\pm i!} H_{t}$
‘Golden Mode’ $B^0 \rightarrow J/\psi K_s^0$ 

Why is $A_{CP}^{mix} = -\sin(2\beta)$ for $B^0 \rightarrow J/\psi K_s^0$?

(1) remember: 

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so this is satisfied if 

$$\lambda_{CP} = -e^{-2i\beta} = -\cos(2\beta) - i \sin(2\beta)$$

(2) remember: 

$$\lambda_{fCP} \equiv \frac{q}{p} A_{fCP}$$
Why is $A_{\text{mix}}^{\text{CP}} = -\sin(2\beta)$ for $B^0 \rightarrow \psi K^{0}_S$?

(1) remember: $A_{\text{CP}}^{\text{mix}} = S_{\text{CP}} = \frac{2 \Im(\lambda_{\text{CP}})}{1 + |\lambda_{\text{CP}}|^2}$ so this is satisfied if $\lambda_{\text{CP}} = -e^{-2i\beta}$

(2) remember: $\lambda_{f\text{CP}} \equiv \frac{q }{p} \frac{A_{f\text{CP}}}{A_{f\text{CP}}} = \frac{V_{tb}^*V_{td}^*}{V_{tb}V_{td}^*} \frac{V_{cb}^*V_{cs}^*}{V_{cb}V_{cs}^*} \eta_{\text{CP}} \frac{V_{cd}^*V_{cs}}{V_{cd}V_{cs}^*}$
Why is $A_{\text{mix}} = -\sin(2\beta)$ for $B^0 \rightarrow J/\psi K_S^0$?

1) remember: $A_{\text{mix}} = S_{CP} = \frac{2 \Im(\lambda_{CP})}{1+|\lambda_{CP}|^2}$

2) remember: $\lambda_{f_{CP}} \equiv \frac{q A_{f_{CP}}}{p A_{f_{CP}}} = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \frac{V_{cb}^* V_{cs}}{V_{cb} V_{cs}^*} \eta_{CP} \frac{V_{cd}^* V_{cs}}{V_{cd} V_{cs}^*}$

$= -\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \frac{V_{cb} V_{cd}^*}{V_{cb}^* V_{cd}}$

Cancel terms, and

$\eta_{CP} = -1$ for $J/\psi K_S^0$
‘Golden Mode’ $B^0 \rightarrow J/\psi K^0_S$

Why is $A_{CP}^{\text{mix}} = -\sin(2\beta)$ for $B^0 \rightarrow J/\psi K^0_S$?

1) remember: $A_{CP}^{\text{mix}} = S_{CP} = \frac{2 \Im(\lambda_{CP})}{1 + |\lambda_{CP}|^2}$ so this is satisfied if $\lambda_{CP} = -e^{-2i\beta} = -\cos(2\beta) - i \sin(2\beta)$

2) remember: $\lambda_{fCP} \equiv \frac{q A_{fCP}}{p A_{fCP}} = \frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*} \frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}} \eta_{CP} \frac{V_{cd}^*V_{cs}}{V_{cd}V_{cs}^*}$

$= -\frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*} \frac{V_{cb}V_{cd}^*}{V_{cb}^*V_{cd}}$

Cancel terms, and $\eta_{CP} = -1$ for $J/\psi K^0_S$

$= -\frac{V_{cb}^*V_{cd}}{V_{tb}V_{td}^*} \frac{V_{tb}V_{td}^*}{V_{cb}^*V_{cd}}$

Rearrange

[Equation for $\beta = \phi_1 = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$]
‘Golden Mode’ $B^0 \rightarrow J/\psi K_S^0$

Why is $A_{CP}^{mix} = -\sin(2\beta)$ for $B^0 \rightarrow J/\psi K_S^0$?

(1) remember:  
\[ A_{CP}^{mix} = S_{CP} = \frac{2 \Im(\lambda_{CP})}{1 + |\lambda_{CP}|^2} \]  
so this is satisfied if  
\[ \lambda_{CP} = -e^{-2i\beta} \]  
\[ = -\cos(2\beta) - i \sin(2\beta) \]

(2) remember:  
\[ \lambda_{f_{CP}} \equiv \frac{q}{p} \frac{A_{f_{CP}}}{A_{f_{CP}}} = \frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*} \frac{V_{cb}V_{cd}^*}{V_{cb}^*V_{cd}} \eta_{CP} \frac{V_{cd}^*V_{cs}}{V_{cd}V_{cs}^*} \]

\[ = - \frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*} \frac{V_{cb}V_{cd}^*}{V_{cb}^*V_{cd}} \]

Cancel terms, and  
\[ \eta_{CP} = -1 \text{ for } J/\psi K_S^0 \]

Rearrange  
\[ = [Ae^{i\beta}]^* \]
\[ = Ae^{-i\beta} \]
\[ = [-Ae^{i\beta}]^{-1} \]
\[ = -A^{-1}e^{-i\beta} \]

\[ \Rightarrow \lambda_{J/\psi K_S^0} = -e^{-2i\beta} \quad \text{Q.E.D} \]