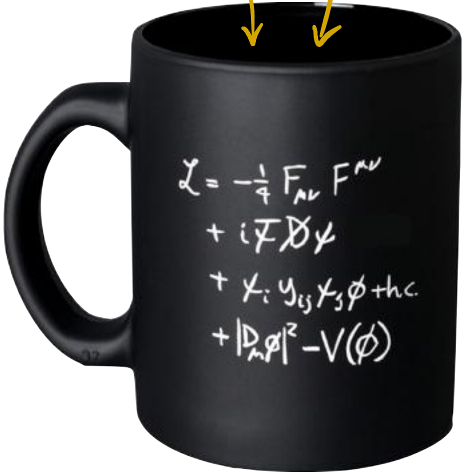
Making Predictions at Hadron Colliders

Alexander Huss

alexander.huss@cern.ch

CERN Summer Student Lecture Programme 2023

$SU(3)_c \times SU(2)_L \times U(1)_Y$



$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$
$$+ i\bar{\psi}\not{D}\psi$$
$$+ \psi_i y_{ij} \psi_j \phi + h.c.$$
$$+ |D_\mu \phi|^2 - V(\phi)$$

THEORY

framework of QFT

$\hat{=}$ QM ⚭ SR

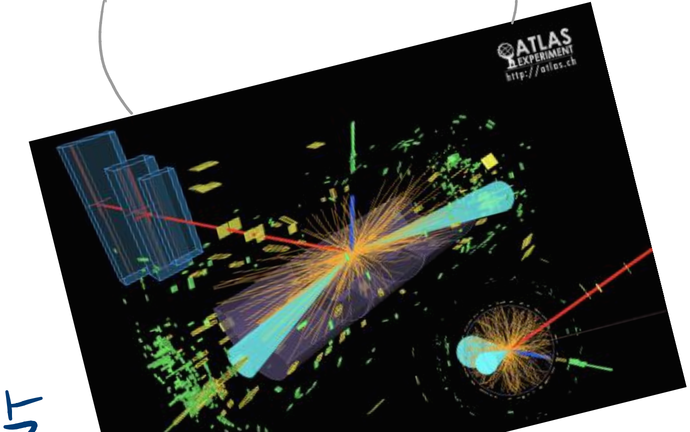
PHENOMENOLOGY

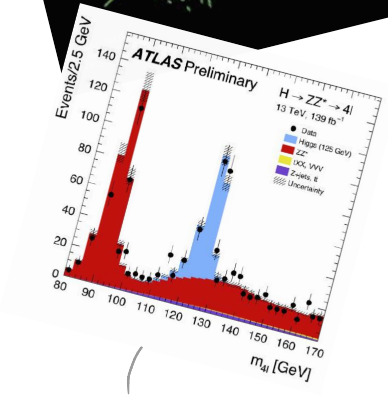
this lecture

EXPERIMENT

Accelerators

Detectors





data aquisition/analysis

statistics

Sources & Conventions

- all examples on: github.com/aykhuss/ssl23

"making predictions"

(mainly python in org notebooks)

- Conventions $[\hbar] = [c] = 1$

$\Rightarrow [\text{length}] = [\text{time}] = eV^{-1}$

$[\text{mass}] = [\text{energy}] = [\text{momentum}] = eV$

(remember: $m_{\text{proton}} \sim 1\,\text{GeV}$)

- four vectors: $x^\mu = (t, x, y, z)^T$

$p^\mu = (E, p_x, p_y, p_z)^T$

$\Rightarrow$ energy-momentum conservation:

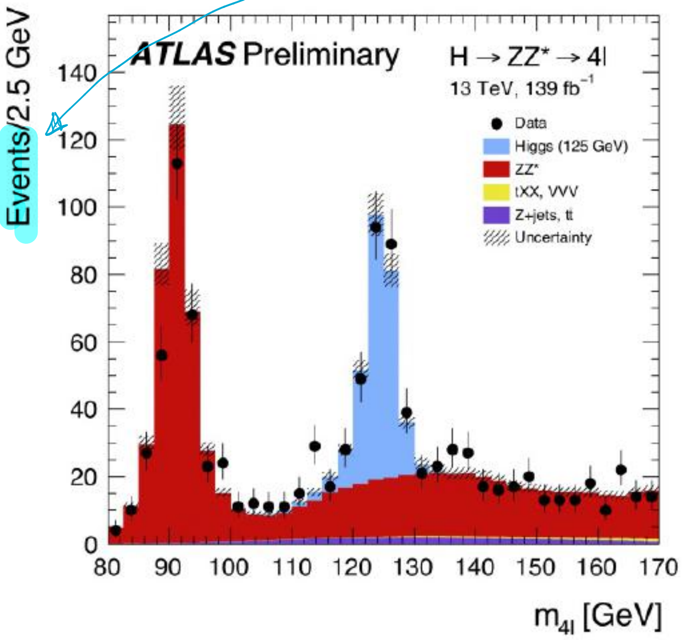
$$\delta^{(4)}(p_1 + p_2 - (p_a + p_b)) = \delta(E_a + E_b - E_1 - E_2)\ \delta^{(3)}(\vec{p}_1 + \vec{p}_2 - \vec{p}_a - \vec{p}_b)$$

The Plan

1. Event Rates, Cross Sections & Scattering Amplitudes
2. Warmup: Lepton Collider
3. Hadron Colliders – Parton Distribution Functions
4. The Drell-Yan Process
5. Higher-Order Corrections
6. QCD Jets, Parton Showers & MC Simulations

Event Rates

We ultimately measure # Events for a specific process: $a+b \longrightarrow 1+2+\dots+n$



Luminosity
~ # collisions

cross section

$$dN = L \, d\sigma$$

* σ_H (13 TeV) ≈ 50 pb

$\int_{\text{Run 2}} dt\, \mathcal{L} \approx 150\ \text{fb}^{-1}$

~ 7 million Higgs bosons produced

* σ_Z (13 TeV) ≈ 50 nb

σ_W (13 TeV) ≈ 200 nb

$\mathcal{L}$ (instantaneous) ≈ $0.02\ \text{pb}^{-1}\,\text{s}^{-1}$

~ 1000 Z's
~ 4000 W's
every second!

Calculating Cross Sections

Fermi's golden rule: $a+b \to 1+2+\dots+n$

$$d\sigma = \underbrace{\frac{1}{F}}_{\text{flux}} \underbrace{\langle |\mathcal{M}|^2 \rangle}_{|\text{amplitude}|^2} \underbrace{d\Phi}_{\text{phase space}}$$

(LIPS)

$$d\Phi_n(p_1,\dots,p_n;\, p_a+p_b) = \prod_{i=1}^{n} \frac{d^4p_i}{(2\pi)^4}\,(2\pi)\,\underbrace{\delta(p_i^2-m_i^2)}_{\text{on shell}}\,\underbrace{\Theta(p_i^0)}_{E>0}$$

$$\times (2\pi)^4\, \underbrace{\delta^{(4)}(p_1+\dots+p_n-(p_a+p_b))}_{\text{energy-momentum conservation}}$$

flux:

$$= \frac{1}{4(p_a\cdot p_b)} = \frac{1}{2E_{cm}^2}$$

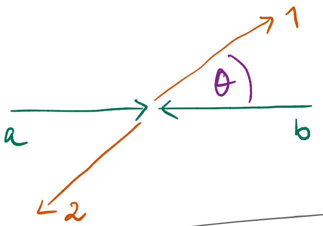
|amplitude|²:

$$= \frac{1}{n_a^{d.o.f.}\, n_b^{d.o.f.}} \sum_{d.o.f.} |\mathcal{M}|^2$$

(degrees of freedom: spin, colour)

Special case $a+b \to 1+2$

$$d\Phi_2 = \frac{d\cos\theta}{16\pi} \quad \text{(massless)}$$



The Scattering Amplitude

↔ evaluation of the path integral $Z[J] = \int \mathcal{D}[\Phi]\, e^{i\int d^4x\,[\mathcal{L}(\Phi,\partial\Phi) + J(x)\Phi(x)]}$

(analogy to QM)

$\Phi \in \{\psi, \phi, A_\mu, \dots\}$

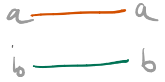
↪ extremely difficult to solve

except a free theory ↔ $\mathcal{L}$ only has terms with at most two fields Φ

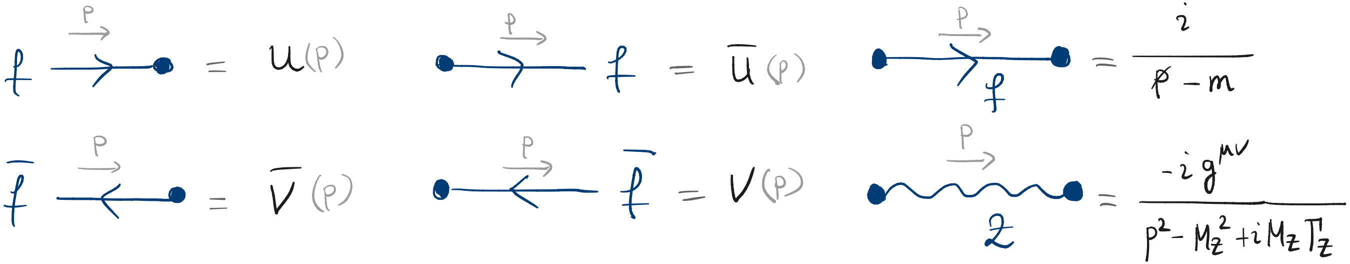
e.g. $\bar\psi(x)(i\not\partial - m)\psi(x)$

very boring "scattering"

$a+b \to a+b$



Feynman rules for the free theory



incoming

$f \;\; \xrightarrow{p} \;\; = u(p)$

$\bar f \;\; \xrightarrow{p} \;\; = \bar v(p)$

outgoing

$\xrightarrow{p} \;\; f = \bar u(p)$

$\xrightarrow{p} \;\; \bar f = v(p)$

propagators

$f:\;\; = \dfrac{i}{\not p - m}$

$Z:\;\; = \dfrac{-i g^{\mu\nu}}{p^2 - M_Z^2 + i M_Z \Gamma_Z}$

Perturbation Theory

when interaction strength is small ($\alpha_{em} \sim 1/137$, $\alpha_s \sim 0.118$)

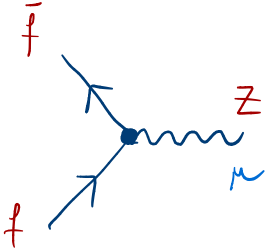
$\Rightarrow$ compute $\mathcal{M}$ perturbatively by expanding around free theory

Feynman rules for interactions — vertices

direct correspondence with terms in $\mathcal{L}$

$$-e \, Z_\mu \, \bar{\psi}_f \, \gamma^\mu (v_f - a_f \gamma^5) \psi_f \in \mathcal{L}$$

$\rightsquigarrow$



$$= -ie \gamma^\mu (v_f - a_f \gamma^5)$$

* more subtle than just "dropping" the fields when derivatives (∂_μ) and/or identical particles

Warmup: Lepton Collider

Consider the process $e^+ e^- \to \mu^+ \mu^-$

At lowest order (tree level) there are two diagrams

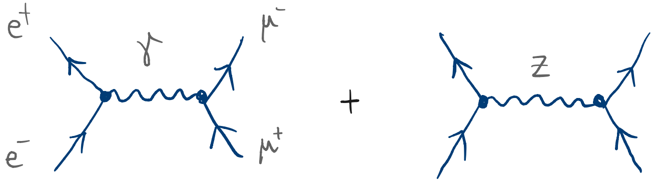
What are they? [demo: FeynGames]

Warmup: Lepton Collider

[demo: e^+e^-]

Consider the process $e^+ e^- \to \mu^+ \mu^-$

At lowest order (tree level) there are two diagrams



$$= i\mathcal{M}^{\text{tree}}_{e^+e^-\to\mu^+\mu^-}$$

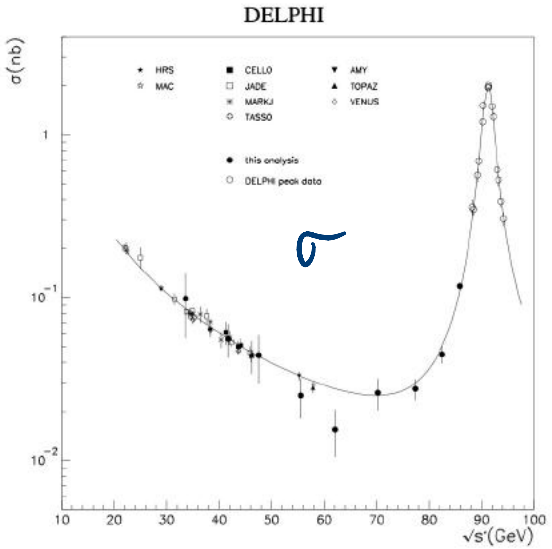
$\Rightarrow$ inserting into Fermi's golden rule $\left[s = E_{cm}^2; \quad p_a \cdot p_1 = p_a^\mu\, p_{1,\mu} = E_{cm}^2\,(1-\cos\theta)\right]$

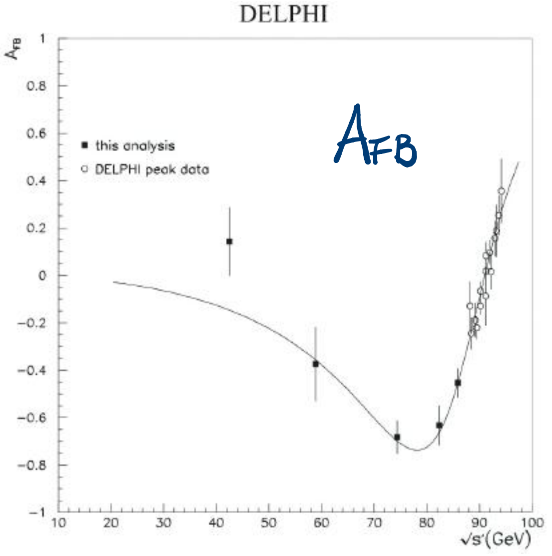
$$\frac{d\sigma}{d\cos\theta} = \frac{\alpha^2 \pi}{2s}\left[(1+\cos^2\theta)\; G_1(s) + 2\cos\theta\; G_2(s)\right]$$

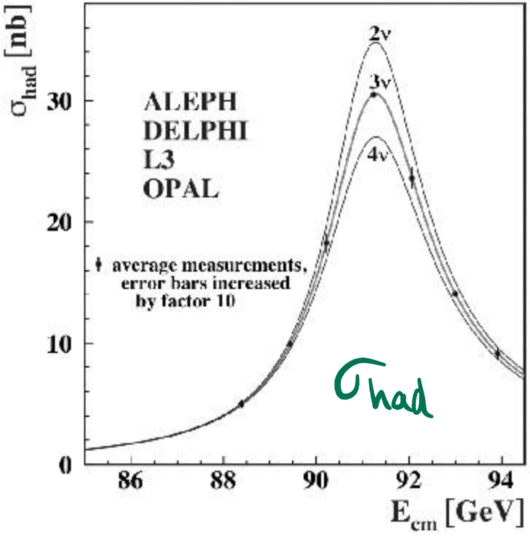
$$G_1(s) = 1 + 2V_e^2\,\mathrm{Re}\left\{\frac{s}{s-M_Z^2+iM_Z\Gamma_Z}\right\} + (V_e^2+a_e^2)^2\left|\frac{s}{s-M_Z^2+iM_Z\Gamma_Z}\right|^2$$

$$G_2(s) = 0 + 2a_e^2\,\mathrm{Re}\left\{\frac{s}{s-M_Z^2+iM_Z\Gamma_Z}\right\} + 4V_e^2\cdot a_e^2\left|\frac{}{s-M_Z^2+iM_Z\Gamma_Z}\right|^2$$

"Comparison" with data







* In principle, you now can use the predictions to fit M_Z & $\sin^2\theta_W$ from the data (at leading order)

* σ_{had} is the hadronic cross section: @ LO: $e^+e^- \to q\bar{q}$ (what changes w.r.t. $\mu^+\mu^-$?)

Hadron Colliders: The parton model

At the LHC we collide protons ≠ elementary

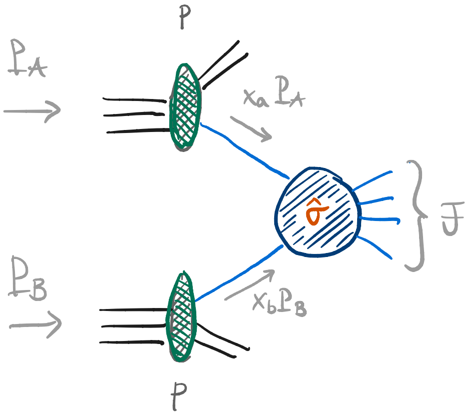
⇝ hadronic cross section = sum over scattering of partons (q & g)

fractional momentum

$$d\sigma_{A+B\to \mathcal{J}+X}(P_A, P_B) = \sum_{a,b} \int_0^1 dx_a \int_0^1 dx_b \, f_a(x_a) \, f_b(x_b) \, d\hat{\sigma}_{a+b\to \mathcal{J}+X}(x_a P_A, x_b P_B)$$

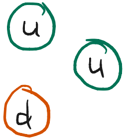
parton distribution function

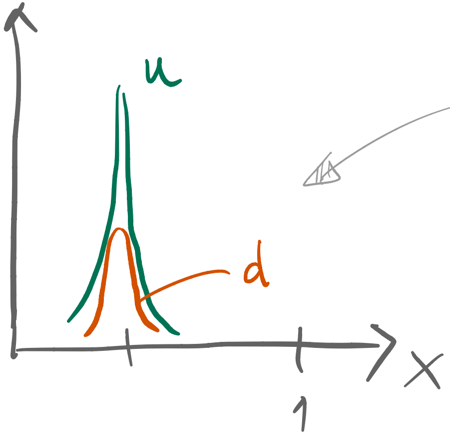
≙ number density for parton a to carry momentum fraction $[x, x+dx]$ of parent hadron



Parton Distribution Functions

* just free quarks? ($p = uud$)



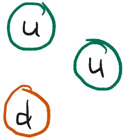


$$f_a(x) = \delta_{au}\, 2\, \delta(x - 1/3) + \delta_{ad}\, 1\, \delta(x - 1/3)$$

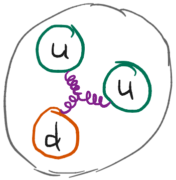
+ some smearing

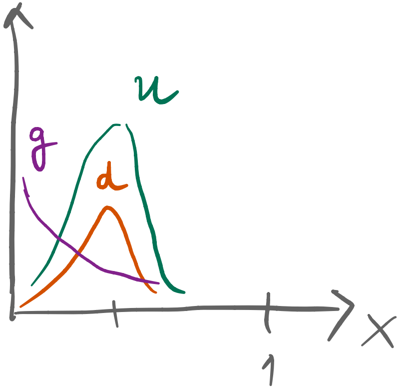
Parton Distribution Functions

- * just free quarks? (p = uud)



- * bound by gluons?





naive parton model:

↪ composition of point particles

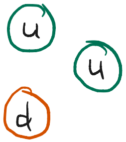
⇝ zoom in ($Q^2 \uparrow$) ↔ same composition

scaling (PDFs independent on scale
at which it is probed,
as long as $Q^2 \gg m_p^2$)

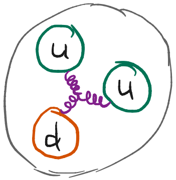
Parton Distribution Functions

[demo: PDF]

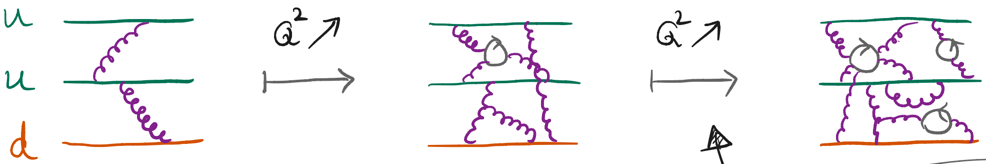
- just free quarks? (p = uud)



- bound by gluons?



- QCD-improved parton model



evolution perturbatively calculable!

⇒ predominantly shifts partons from high-x to low-x

The Drell-Yan process

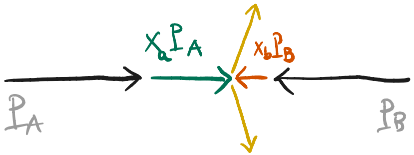
[demo: DY]

$$d\sigma_{DY} = \sum_{a,b} \int_0^1 dx_a \int_0^1 dx_b \, f_a(x_a) \, f_b(x_b) \, d\hat{\sigma}_{ab\to\ell^+\ell^-} \qquad \left(\text{only } (a,b) \in \{ (q,\bar{q}), (\bar{q},q) \} \text{ @ LO} \right)$$

* After integrating out $Z\to\ell^+\ell^-$ decay we'll look at the observables of the intermediate gauge boson $q^\mu = (p_1+p_2)^\mu = (p_a+p_b)^\mu$

$$M_{\ell\ell} = \sqrt{q^2} \quad ; \quad Y_{\ell\ell} = \frac{1}{2} \ln\left(\frac{q^0+q^3}{q^0-q^3} \right)$$

rapidity: $Y \to Y + \frac{1}{2}\ln(x_1/x_2)$



$$\Rightarrow \boxed{ \frac{d^2\sigma_{DY}}{dM_{\ell\ell}\, dY_{\ell\ell}} = f_a(x_a) \, f_b(x_b) \, \frac{2M_{\ell\ell}}{E_{cm}^2} \, \hat{\sigma}_{ab} \quad \Big| \quad x_{a/b} = \frac{M_{\ell\ell}}{E_{cm}} e^{\pm Y_{\ell\ell}} }$$