Making Predictions at Hadron Colliders

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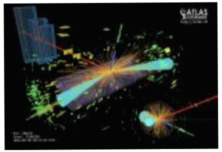
CERN Summer Student Lecture Programme 2023

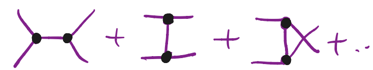
Previously on "Making Predictions" ...



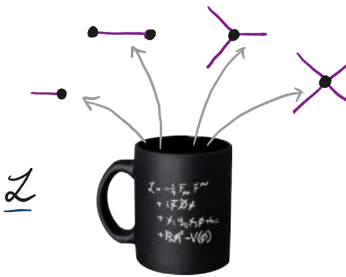
Event rates:

$$N = L \underbrace{\sigma}_{\text{cross section}}$$

$$d\sigma_{2\to n} = \frac{1}{F} \langle \underbrace{|M|}_{\text{scattering amplitudes}}{}^2 \rangle \, d\Phi_n$$

 $\longleftrightarrow \mathcal{L}$

Feynman diagrams & rules



* another important case: decay rates $(\tau = 1/\Gamma)$

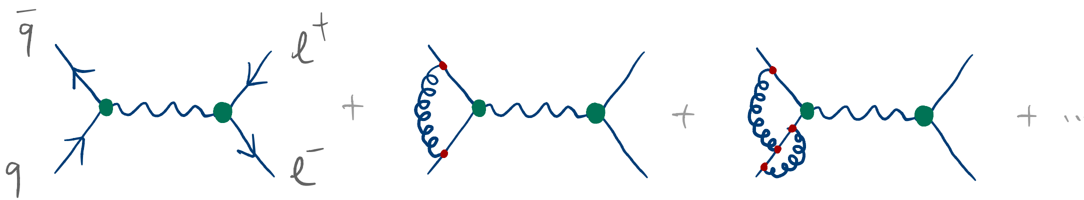
$$d\Gamma_{1\to n} = \frac{1}{2M} \langle |M|^2 \rangle \, d\Phi_n$$

The Drell-Yan process

We saw last time that a leading order (LO) prediction in QCD is often not sufficient for precision phenomenology

$\Rightarrow$ we want to go to the next order! $\rightsquigarrow$ diagrams with <u>loops</u>!

$\mathcal{M}_{2\to 2} =$



$\mathcal{O}(\alpha)\ \mathcal{M}^{(0)}_{2\to 2}$ $\qquad$ $\mathcal{O}(\alpha\alpha_s)\ \mathcal{M}^{(1)}_{2\to 2}$ $\qquad$ $\mathcal{O}(\alpha\alpha_s^2)\ \mathcal{M}^{(2)}_{2\to 2}$

$$\Rightarrow |\mathcal{M}_{2\to 2}|^2 = \underset{\mathcal{O}(\alpha^2)}{|\mathcal{M}^{(0)}_{2\to 2}|^2} + \underset{\mathcal{O}(\alpha^2\alpha_s)}{2\,\mathrm{Re}\left\{\left(\mathcal{M}^{(0)}_{2\to 2}\right)^* \mathcal{M}^{(1)}_{2\to 2}\right\}} + \underbrace{|\mathcal{M}^{(1)}_{2\to 2}|^2 + 2\,\mathrm{Re}\left\{\left(\mathcal{M}^{(0)}_{2\to 2}\right)^* \mathcal{M}^{(2)}_{2\to 2}\right\}}_{\mathcal{O}(\alpha^2\alpha_s^2)} + \dots$$

"virtual"

Divergences in Loop Diagrams

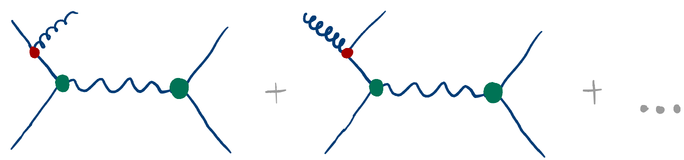
QM tells us that we have to sum over all intermediate configurations

$\Rightarrow$ need to integrate over the unconstained loop momentum $\rightsquigarrow \int \frac{d^4k}{(2\pi)^4}$

① ultraviolet (UV) $\leftrightsquigarrow$ large loop momentum $\Rightarrow$ treated by renormalization $\alpha_s(\mu_R)$

② infrared (IR) $\leftrightsquigarrow$ soft and/or collinear $\Rightarrow$ requires real emisson contribution & PDF renormalization

$\mathcal{M}_{2\to3} =$



$\mathcal{O}(\alpha\sqrt{\alpha_s})\ \mathcal{M}^{(0)}_{2\to3}$

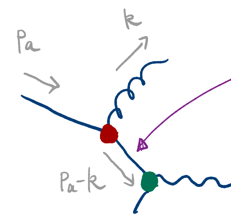
$$|\mathcal{M}_{2\to3}|^2 = |\mathcal{M}^{(0)}_{2\to3}|^2 + \dots$$

$\mathcal{O}(\alpha^2\alpha_s)$ "real"

technically, a different process <u>but</u> necessary to include (at least when unresolved)

Cancellations of Divergences

Let's look at this sub-diagram
(appears both in the virtual & real contribution)



this propagator contains

$$\frac{1}{(p_a-k)^2} = \frac{1}{E_a E\,(1-\cos\theta)}$$

$\Rightarrow$ potential divergence when

(a) gluon "soft": $E \to \emptyset$

(b) gluon collinear: $\vec{k} \parallel \vec{p}_a$

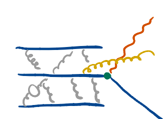
(a) the "virtual" (loop) corrections give

$$\hat{\sigma}_{LO}(p_a,p_b) \cdot \frac{\alpha_s}{2\pi} C_F \frac{(4\pi)^{\epsilon}}{\Gamma(1-\epsilon)} \times \left\{ \frac{1}{\epsilon^2} + \frac{3}{2}\frac{1}{\epsilon} + \text{finite} \right\}$$

(b) the "real" corrections give

$$\hat{\sigma}_{LO}(p_a,p_b) \cdot \frac{\alpha_s}{2\pi} C_F \frac{(4\pi)^{\epsilon}}{\Gamma(1-\epsilon)} \times \left\{ -\frac{1}{\epsilon^2} - \frac{3}{2}\frac{1}{\epsilon} + \text{finite} \right\}$$

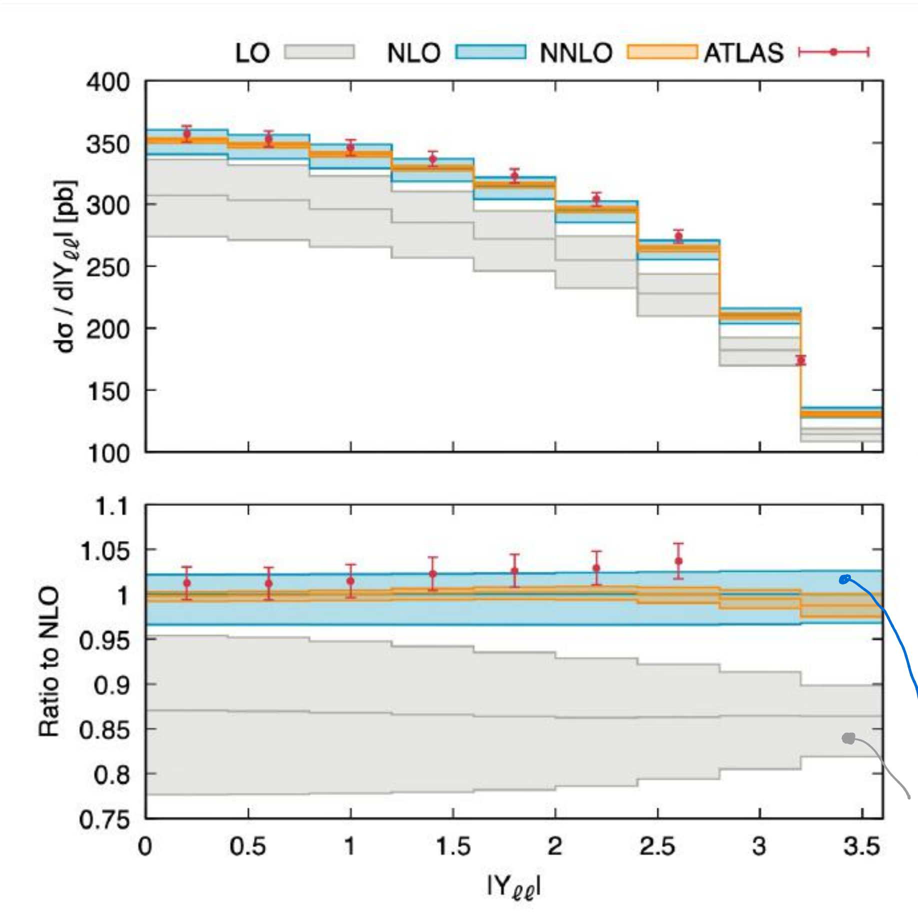
$$+ \int_0^1 dz_a \, \hat{\sigma}_{LO}(z_a p_a, p_b) \frac{\alpha_s}{2\pi} C_F \frac{(4\pi)^{\epsilon}}{\Gamma(1-\epsilon)} \times \left\{ \underbrace{-\frac{1}{\epsilon}}_{\text{this is absorbed as part of the "NLO PDF"}} - \ln\left(\mu_F^2/Q^2\right) \right\} P_{qq}(z_a) + \left(\begin{array}{c} z_a \leftrightarrow z_b \\ q_{in} \to g_{in} \end{array} \right)$$



part of the proton if its scale $< \mu_F$ $\Big\}$ $f_a(x) \longmapsto f_a(x, \mu_F^2)$

UNIVERSAL $\leftrightarrow$ PDF evolution

The Drell-Yan process at higher orders



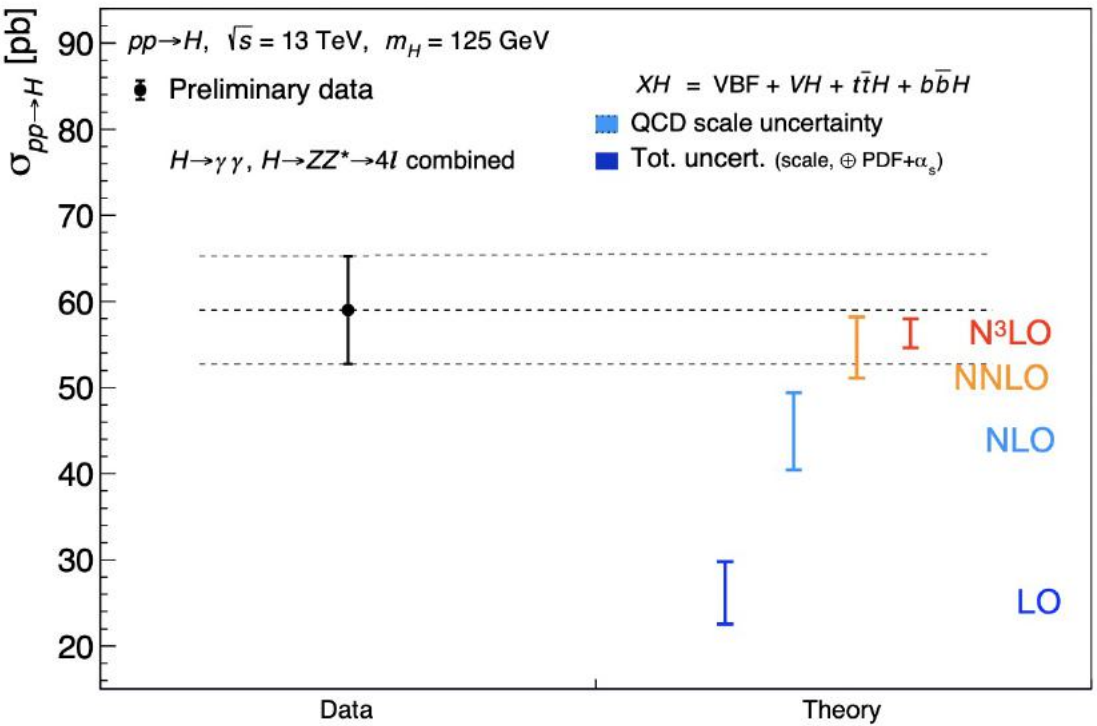
on repository

last lecture

theory uncertainties

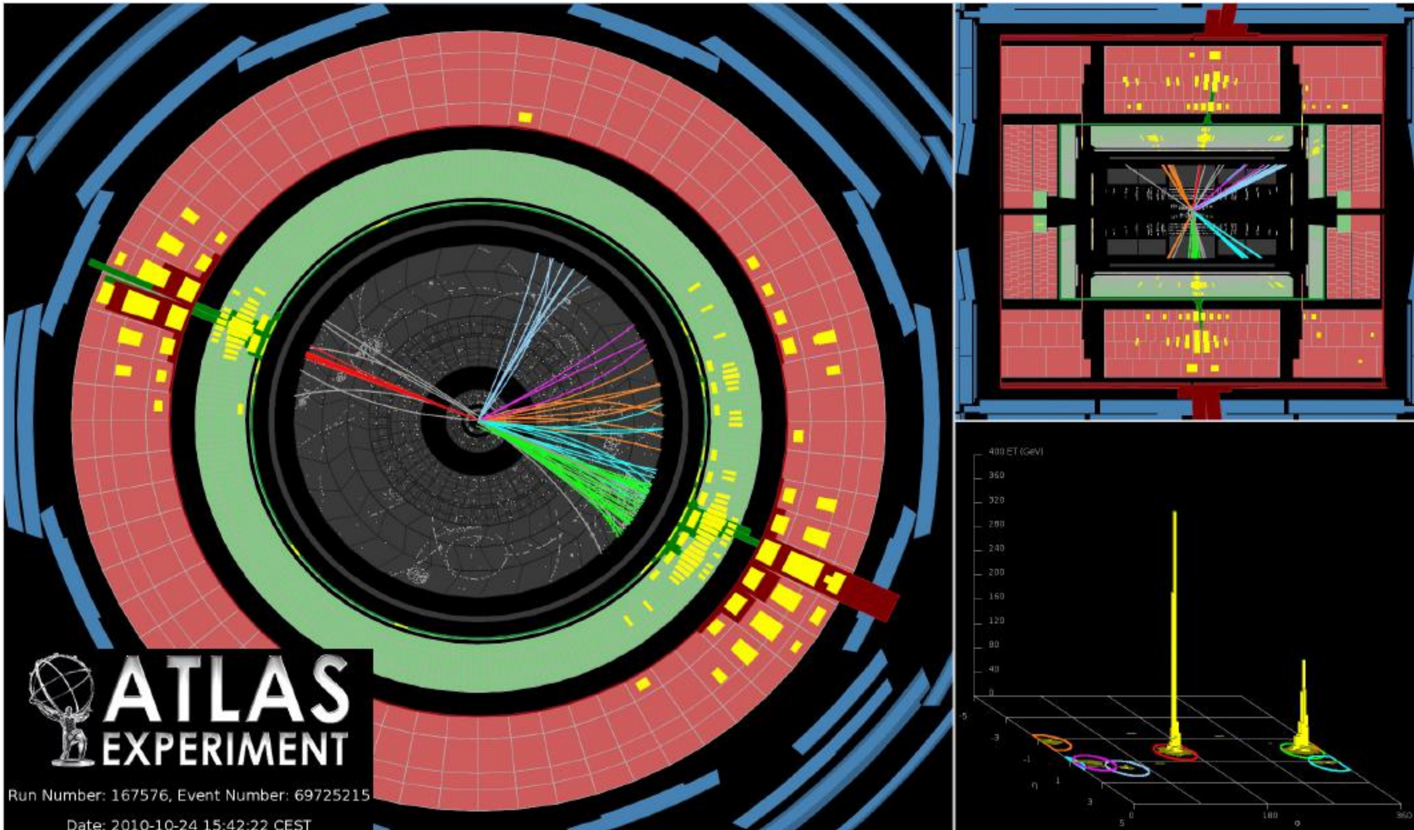
* missing higher orders
  we have (in principle) arbitrary scales in our predictions:
  $$\alpha_s(\mu_R) \;\&\; f_a(x, \mu_F)$$
  varying them induces logarithmic terms beyond the order we computed
  typically varied by factors $[\frac{1}{2}, 2]$
* parametric $(\alpha_s(M_Z), \dots)$ (not included)
  $\mathcal{O}(1\%)$
* PDF uncertainties (not included)
  $\mathcal{O}(1\%)$

The state of the art in Fixed Order



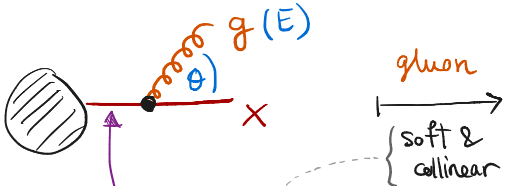
* LO
  almost any process
* NLO
  most processes
  (up to $2\to 8$)
* NNLO
  $2\to 2$ done,
  first $2\to 3$
* N^3LO
  $2\to 1$

Events at hadron colliders look more complex

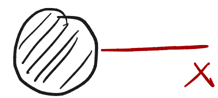


Why? Any chance to compute this with what we did so far? [demo: diags]

The QCD emission pattern



enhanced (divergent) $\sim \dfrac{1}{E_x E\,(1-\cos\theta)}$



$\times \; d\omega_{x\to x+g}$

$$\boxed{2\,\frac{\alpha_s}{\pi}\, C_x\, \frac{dE}{E}\, \frac{d\theta}{\theta}}$$

$$C_x \begin{cases} = C_F = 4/3 & \text{if } x = q \\ = C_A = 3 & \text{if } x = g \end{cases}$$

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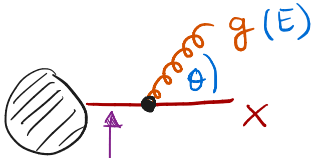
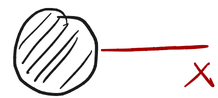


gluon

soft & collinear

enhanced (divergent) $\sim \dfrac{1}{E_x E(1-\cos\theta)}$



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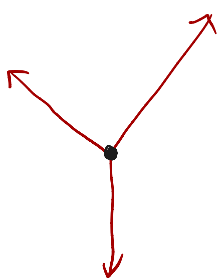
⇒ jets are an emergent feature of QCD

① high energetic partons

↪ hard scattering



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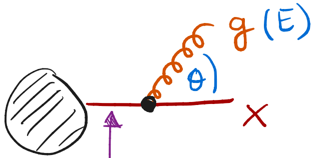

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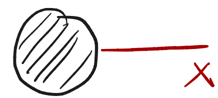

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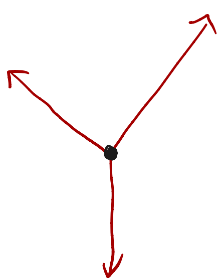
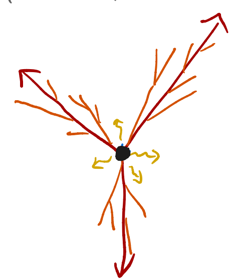
② asymptotic freedom & $d\omega$

↳ pert. parton shower





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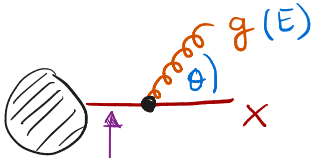
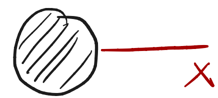


gluon $\longmapsto$ $\left\{ \begin{array}{l} \text{soft \&} \\ \text{collinear} \end{array} \right.$

enhanced (divergent) $\sim \dfrac{1}{E_x E(1-\cos\theta)}$



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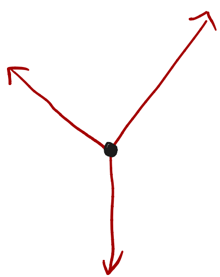
1 high energetic partons

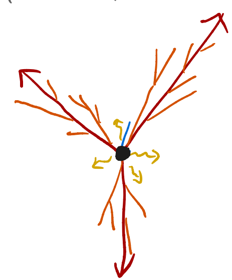
↪ hard scattering

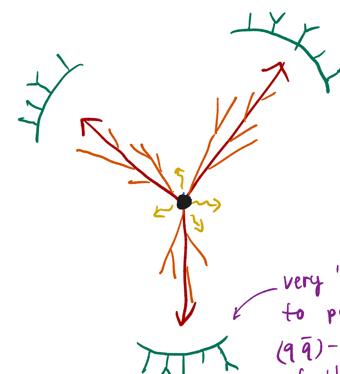
2 asymptotic freedom & $d\omega$

↪ pert. parton shower

3 hadronization



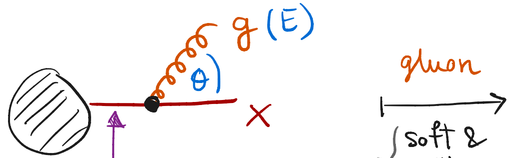




very "cheap" to pop a $(q\bar{q})$-pair out of the vacuum

$m_{u,d} \ll \Lambda_{QCD}$

The QCD emission pattern



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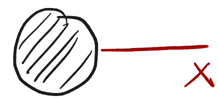

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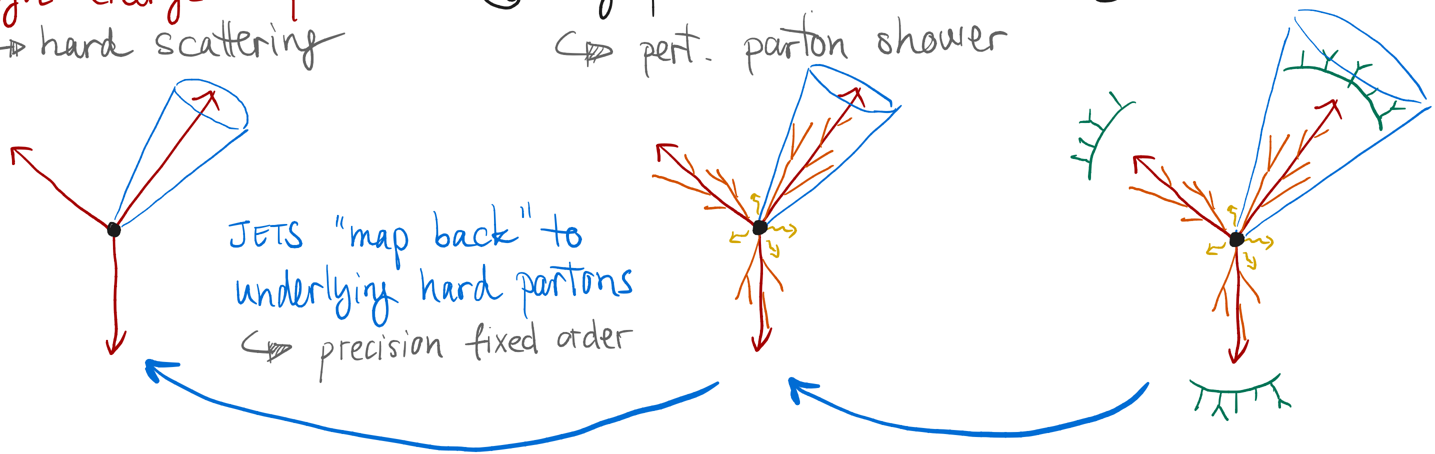
⇒ jets are an emergent feature of QCD

① high energetic partons ↪ hard scattering

② asymptotic freedom & $d\omega$ ↪ pert. parton shower

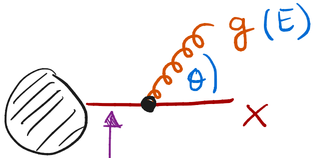
③ hadronization



JETS "map back" to underlying hard partons

↪ precision fixed order

The QCD emission pattern



gluon $\longmapsto$ soft & collinear

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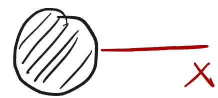

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$$C_x \begin{cases} = C_F = 4/3 & \text{if } x = q \\ = C_A = 3 & \text{if } x = g \end{cases}$$

$\Rightarrow$ emission factorizes!

Integral over E & θ diverges $\rightsquigarrow$ introduce a scale $q^2 > Q_0^2$

$\leftrightarrow$ emission "resolved"

$$\Rightarrow \quad P_x \simeq \frac{\alpha_s C_x}{2\pi} \underbrace{\ln^2\left(Q^2/Q_0^2\right)} + \mathcal{O}\left(\alpha_s \ln Q^2, \alpha_s^2\right)$$

probability to emit a resolved gluon

potentially a very large log $\longrightarrow$ will want to resum these to all orders

$$\left(\left. \begin{aligned} Q_0 &= \Lambda_{QCD} = 0.2\ \text{GeV} \\ Q &= 100\ \text{GeV} \end{aligned} \right\} \Rightarrow \ln(\dots) = \mathcal{O}(10) \right)$$

Parton Showers

* We wish to account for an arbitrary number of emissions ordered in our resolution variable $Q^2 > q_1^2 > q_2^2 > \dots > Q_0^2$ (strong ordering)
* current scale $q_n^2 \rightsquigarrow$ probability to have next emission @ q_{n+1}^2 ?

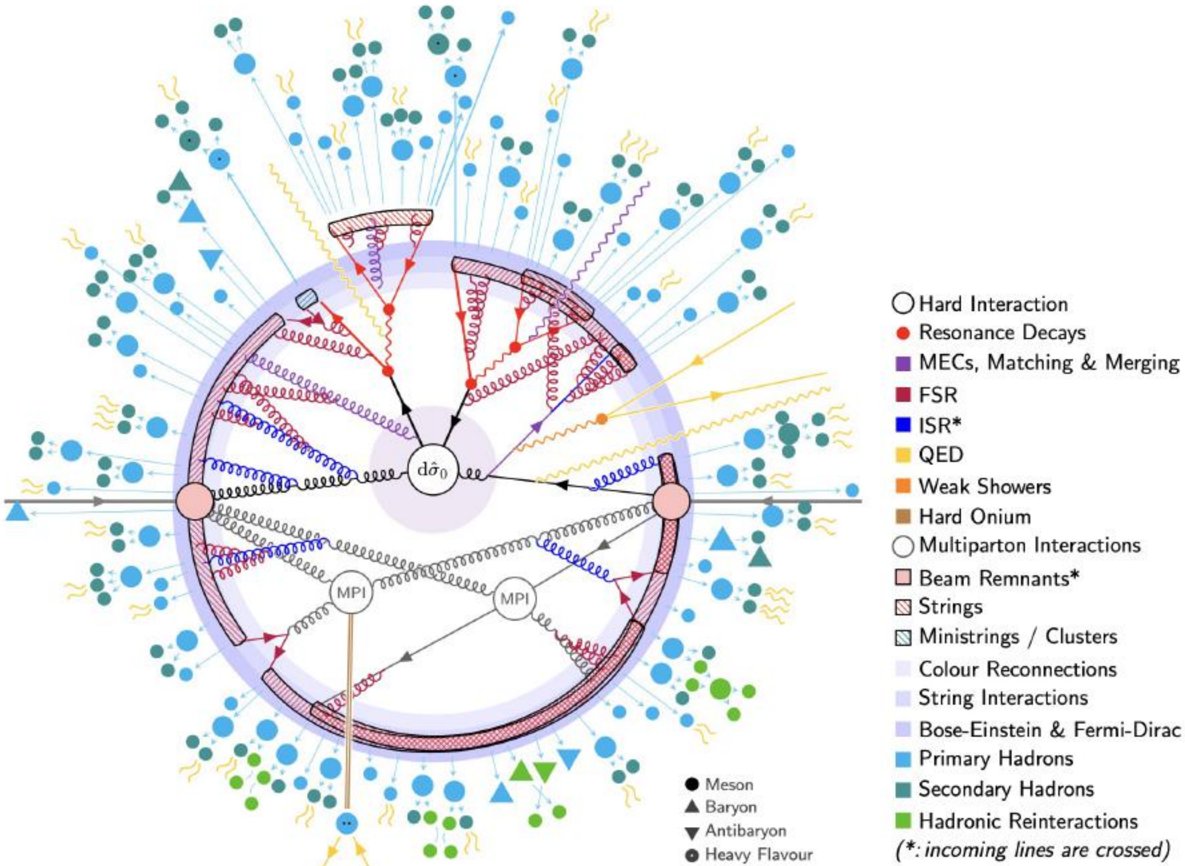
$$\Leftrightarrow \underbrace{\begin{pmatrix}\text{pobability of having} \\ \underline{\text{no}} \text{ emissions } q_n^2 \to q_{n+1}^2\end{pmatrix}}_{\text{Sudakov from factor } \Delta(q_n^2, q_{n+1}^2)} \times \underbrace{\begin{pmatrix}\text{emission} \\ @\ q_{n+1}^2\end{pmatrix}}_{\left.\frac{d\omega_{X\to X+g}}{dq^2}\right|_{q^2=q_{n+1}^2}}$$

$$\Leftrightarrow \frac{d\Delta(Q^2,q^2)}{dq^2} = \Delta(Q^2,q^2)\frac{d\omega}{dq^2}$$

$$\left(\Delta(Q^2, q^2-dq^2) = \Delta(Q^2,q^2)\underbrace{\Delta(q^2,q^2-dq^2)}_{\left(1-\frac{d\omega}{dq^2}\right)}\right)$$

[demo: PS]

Full event generator



Conclusions

* Covered basic ingredients that goes into hadron-collider predictions
  ↪ a key idea: separation of scales ("factorization")

* Moment of comparing your predictions to data always exciting
  ↪ learn to play with the tools; break them (often interesting physics)

* Hope was able to lower the fear of entry for some of you, as it is sometimes percieved as very technical
  ↪ pushing the frontiers in precision can become arbitrarily complex
  new ideas needed (maybe one of you?)

Thank you for your attention & participation!