BSM Lecture 2

Tuesday, 25 July 2023 22:14

Yesterday

- How we got here: lessons from building the SM
- Shortcomings of the SM:
  - Aesthetic
  - Unnatural: Higgs mass, cosmological constant, strong-CP problem
  - Inconsistent: experimental, theory

Today

- The SM is an Effective Field Theory (EFT)
- The EFT "totalitarian principle": understanding the structure of the SM through symmetry
- EFT as a gateway to BSM: captures indirect effects of physics at heavier scales
  ↳ Neutrino mass and the Weinberg operator, GUTs and proton decay, accidental symmetries

> Almost every particle in the SM first appeared indirectly in data before being discovered directly.

What is an EFT? (Recap)

- Given the particle content of all experimentally accessible degrees of freedom at low energies, write down all the terms allowed by the symmetries of the theory, including higher-dimensional operators.

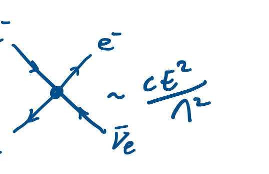
↳ EFT "Totalitarian Principle"
"Anything that is not forbidden is compulsory" – Gell-Mann

Operator dimension = mass dimension in natural units

$$E = mc^2,\quad E = hf,\quad E = \frac{hc}{\lambda} \quad \overset{\hbar=c=1}{\Longrightarrow} \quad [E]=[m]\equiv M,\quad [E]=[T^{-1}] \Rightarrow [T]=M^{-1},\quad [E]=[L^{-1}] \Rightarrow [L]=M^{-1}$$

e.g. Fermi theory

$$\mathcal{L}^{EFT}_{Fermi} = \frac{c}{\Lambda^2}(\bar\Psi\gamma^\mu\Psi)(\bar\Psi\gamma_\mu\Psi) \Rightarrow \text{Feynman diagram} \sim \frac{cE^2}{\Lambda^2}$$



dimensions
$[c]=0 \quad [\Lambda]=M \quad [\Psi]=M^{3/2} \quad [\mathcal{L}]=M^4$

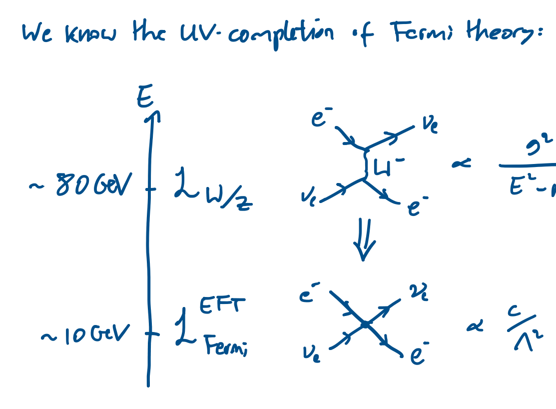
Why? $S=\int d^4x\,\mathcal{L}$ $\quad [S]=0$ since exponent is dimensionless, $[d^4x]=M^{-4}$ $\Rightarrow [\mathcal{L}]=M^4$

e.g. This four-fermion operator is a dimension-6 operator.

$2\to2$ scattering amplitude grows like $\sim \frac{cE^2}{\Lambda^2}$ ⇒ EFT breaks down at $E\sim\Lambda$

Λ = cut-off scale of EFT
c = Wilson coefficient

We know the UV-completion of Fermi theory: the SM weak gauge bosons



$\mathcal{L}_{W/Z}$: $\propto \frac{g^2}{E^2-M_W^2} = -\frac{g^2}{M_W^2}\frac{1}{(1-E^2/M_W^2)} \approx -\frac{g^2}{M_W^2}$

$\mathcal{L}^{EFT}_{Fermi}$: $\propto \frac{c}{\Lambda^2}$

$$\left[\frac{c}{\Lambda^2}\sim\frac{g^2}{M_W^2}\right]$$

The SM is an EFT

E
- $\Lambda\sim 1\,\text{TeV}$? — $\mathcal{L}_{UV}$?
- ~125 GeV — $\mathcal{L}^{EFT}_{SM}$
- ~80 GeV — $\mathcal{L}^{EFT}_{W/Z}$
- ~10 GeV — $\mathcal{L}^{EFT}_{Fermi}$

- Follow the totalitarian principle recipe:

Local gauge symmetries: $SU(3)_c\times SU(2)_L\times U(1)_Y$

Particle content: $\Psi\equiv\{Q_L, L_L, u_R, d_R, e_R\}\times 3$ generations $+\ H$

Quantum numbers: $(3,2,\frac16)$ $(1,2,-\frac12)$ etc.

All allowed terms by Lorentz and gauge invariance:

$$\mathcal{L}^{EFT}_{SM} = \underbrace{\mathcal{L}_M+\mathcal{L}_G+\mathcal{L}_H+\mathcal{L}_Y}_{\mathcal{L}_{SM}} + \mathcal{L}_{dim\text{-}5}+\mathcal{L}_{dim\text{-}6}+\dots$$

($\mathcal{L}_{SM}$: SM usually refers to dim ≤ 4 Lagrangian)

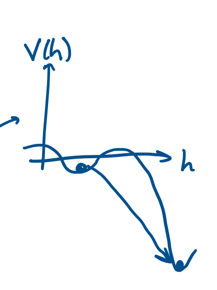
$$\mathcal{L}_M = \bar\Psi i\gamma^\mu D_\mu\Psi$$

$$\mathcal{L}_G = -\frac14 F_{\mu\nu}F^{\mu\nu} \quad \boxed{+\ \theta\frac{g_s^2}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}G^{\mu\nu}G^{\rho\sigma}\ ?}$$

Absence of CP violation in nEDM ⇒ $\theta\lesssim10^{-9}$
Strong-CP problem

$$\mathcal{L}_H = (D_\mu H)^\dagger(D^\mu H) - V(H)$$

$$V(H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2$$
c.c. problem, hierarchy problem, vacuum metastability



$$\mathcal{L}_Y = -y\bar\Psi_L H\Psi_R + \text{h.c.}$$

$$\mathcal{L}_{dim\geq5} = \sum_{n\geq5}\sum_i \frac{c_i}{\Lambda^{n-4}}\mathcal{O}_i^n$$

(Could also have light new physics, in which case we must add the NP particle content to the SM EFT as well)

- EFT cut-off scale Λ can be different for operators with different symmetries

- Global $U(1)_B$ and $U(1)_L$ are accidental symmetries: they were not imposed but just happen to be conserved by $\mathcal{L}_{dim\leq4}$.

- $U(1)_L$ Lepton number violated at dim-5 by Weinberg operator:

$$\mathcal{L}_{dim\text{-}5} = \frac{c_5}{\Lambda}(\bar L\tilde H)(\tilde H^T L^c) \Rightarrow m_\nu\sim\frac{c_5 v^2}{\Lambda} \Rightarrow c_5\sim\mathcal{O}(1),\ \Lambda\sim10^{14}\,\text{GeV}$$

Y: $-\frac12$, $\frac12$ and Lorentz invariance $\bar\psi_L(\psi_L^c)$, $\psi^c = C\bar\psi^T$, $C=\begin{pmatrix} i\sigma^2 & 0\\ 0 & -i\sigma^2\end{pmatrix}$

A possible UV-completion of the dim-5 Weinberg operator:

Add ν_R with quantum numbers $(1,1,0)$

$$\mathcal{L}\supset \underbrace{-y_\nu\bar L\tilde H\nu_R}_{m\bar\nu_L\nu_R,\ m=y_\nu v} + \text{h.c.}\ \boxed{-M\bar\nu^c_R\nu_R}$$

However, nothing forbids Majorana mass term! Apply totalitarian principle:

$$\mathcal{L}_{see\text{-}saw}\supset -m\bar\nu_L\nu_R - M\bar\nu_R^c\nu_R + \text{h.c.}$$
$$= -(\bar\nu_L, \bar\nu_R^c)\begin{pmatrix}0 & m\\ m & M\end{pmatrix}\begin{pmatrix}\nu_L^c\\ \nu_R\end{pmatrix}$$
$$= -(\bar\nu, \bar N)\begin{pmatrix}m_\nu & 0\\ 0 & M_N\end{pmatrix}\begin{pmatrix}\nu\\ N\end{pmatrix}$$

where
$$m_\nu = \frac12\left(M-\sqrt{M^2+4m^2}\right) \simeq -\frac{m^2}{M} \qquad M_N = \frac12\left(M+\sqrt{M^2+4m^2}\right) \simeq M$$

when $M\gg m$.

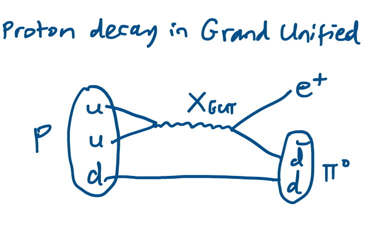
E
- $M\sim10^{14}\,\text{GeV}$ — $\mathcal{L}_{\nu_R}$
- $v\sim246\,\text{GeV}$ — $\mathcal{L}_{dim\text{-}5}$

- $U(1)_B$ Baryon number violated at dimension-6.

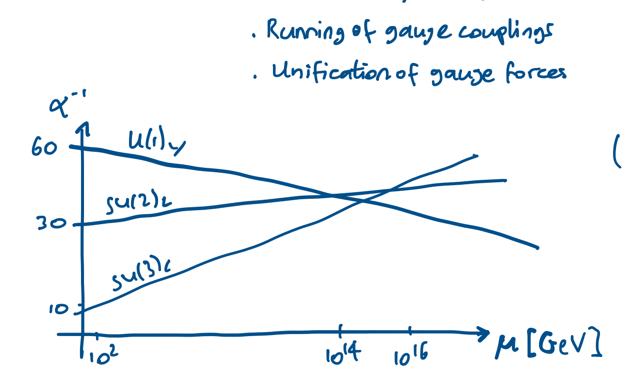
$$\mathcal{L}_6\supset\frac{c}{\Lambda^2}\bar Q^c Q_L \bar u^c_R e_R \quad \text{(violates B+L, conserves B−L)}$$

No proton decay seen in Super-Kamiokande ⇒ $\Lambda\gtrsim10^{15}\,\text{GeV}$

Proton decay in Grand Unified Theories (GUTs):



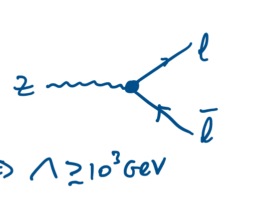
Motivation for GUTs: SM multiplets fit neatly into GUT multiplets
- Quantised hypercharges
- Running of gauge couplings
- Unification of gauge forces



(Don't quite meet in SM, but expect new physics to modify running)

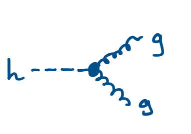
- All kinds of other dimension-6 operators

e.g. $\frac{c_R^\ell}{\Lambda^2}iH^\dagger\overleftrightarrow{D}_\mu H\bar\ell_R\gamma^\mu\ell_R \Rightarrow Z_\mu\bar\ell_R\gamma^\mu\ell_R$



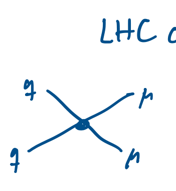
LEP constraints ⇒ $\Lambda\gtrsim10^3\,\text{GeV}$

$\frac{c_g}{\Lambda^2}|H|^2G_{\mu\nu}G^{\mu\nu} \Rightarrow hG_{\mu\nu}G^{\mu\nu}$



LHC constraints ⇒ $\Lambda\gtrsim10^2\,\text{GeV}$

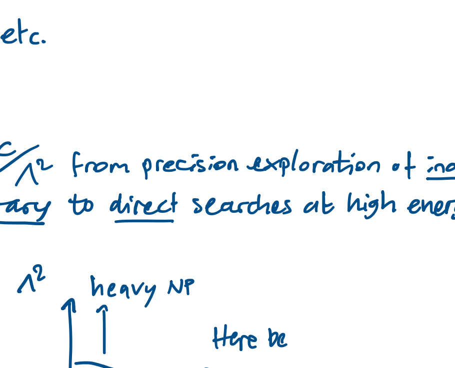
$\frac{c_{4f}}{\Lambda^2}\bar Q_L Q_L\bar L_L L_L$



flavour constraints ⇒ $\Lambda\gtrsim10^5\,\text{GeV}$

etc.

- Constraints on c/Λ^2 from precision exploration of indirect effects of BSM is complementary to direct searches at high energies.



> SM EFT is the Fermi theory of the 21st Century

- To go beyond the SM means finding signs of a non-zero Wilson coefficient or a new particle.