What is String Theory?

(in 55 minutes)

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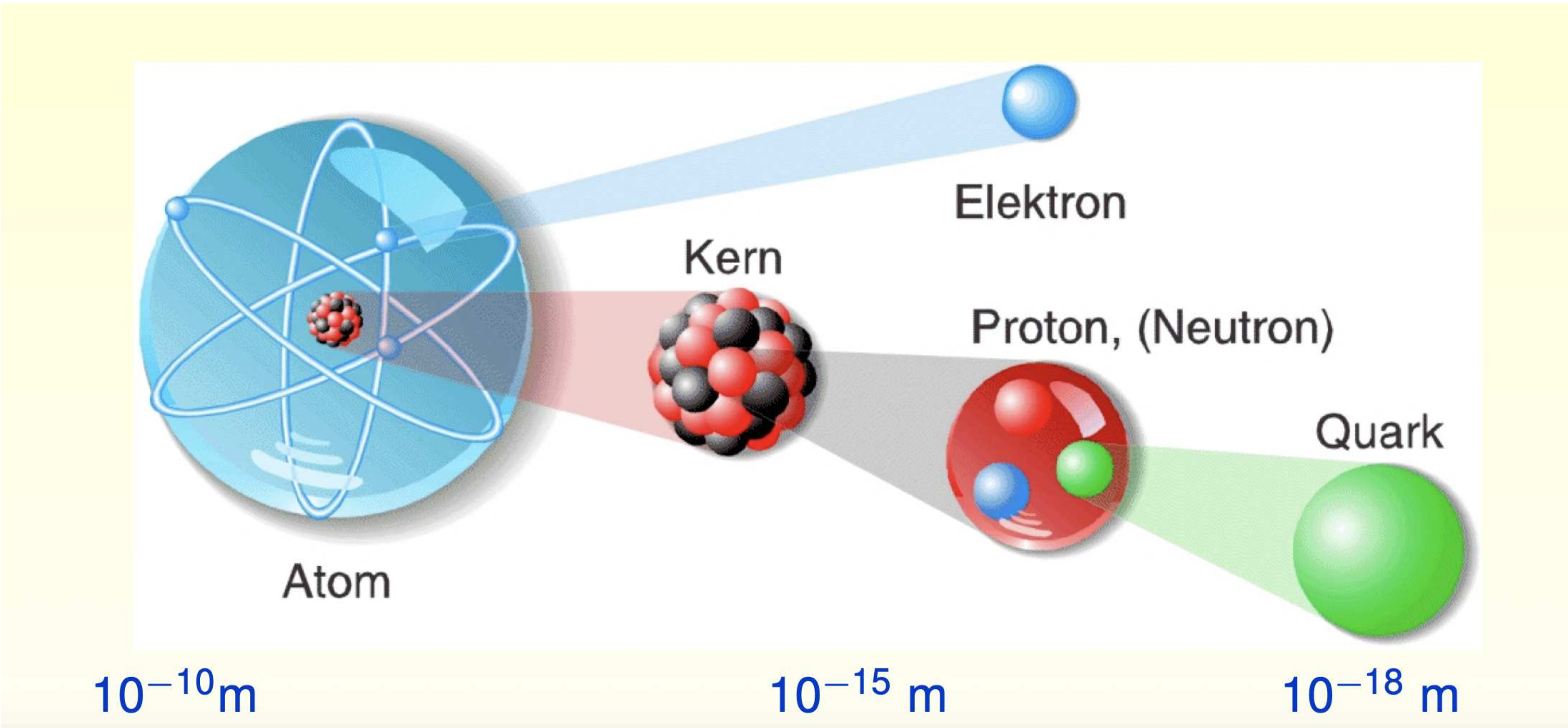
What is String Theory?

If only we knew for sure....

But it aims to answer an important question:

What makes up the Universe?

The visible part of the Universe is made of elementary particles (electrons, quarks, . . .)



But what makes up, say, an electron?

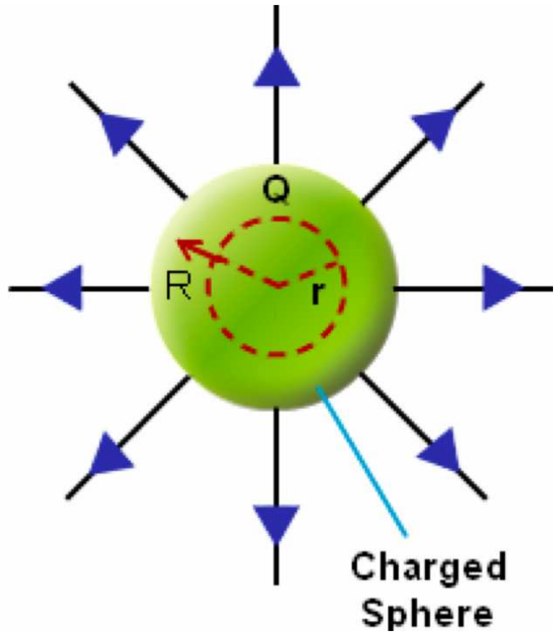
What makes up an electron?

Working hypothesis:

Think of electrons are pointlike objects.

As it stands, this leads to problems.

1) Naive picture:
Classical Electrodynamics



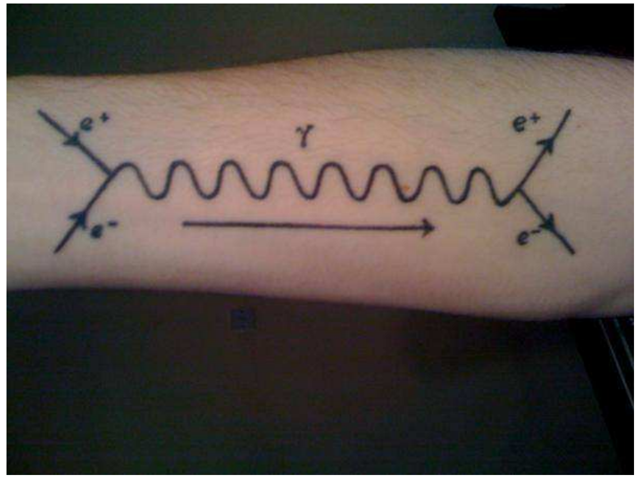
- If the electron were literally a point, its mass should be infinite.
- Model electron as a ball of radius r_e.

$$m_e c^2 = \text{energy} = \frac{3}{5}\frac{1}{4\pi\epsilon_0}\frac{e^2}{r_e} \to \infty \qquad \text{as} \qquad r_e \to 0.$$

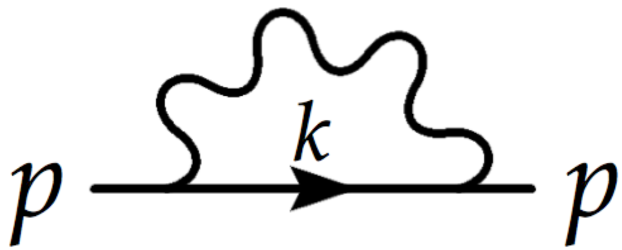
Ultraviolet Divergences - I

2) Quantum Electrodynamics (QED)

- describes electromagnetism at subatomic scales
- Interaction mediated by photons = U(1) gauge bosons = spin 1 particle .



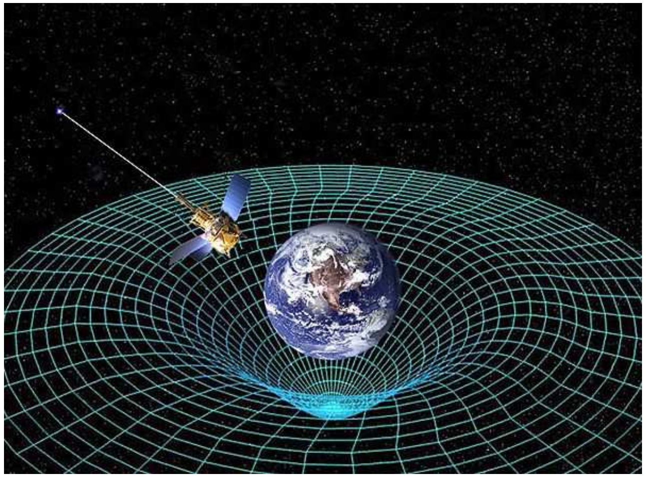
This same ultraviolet (UV) divergence remains.



Wilsonian renormalisation:

- Powerful techniques hide this infinity for all practical purposes.
- QED is a perfectly working effective theory.
- Something's not right at the fundamental level.

Ultraviolet Divergences - II

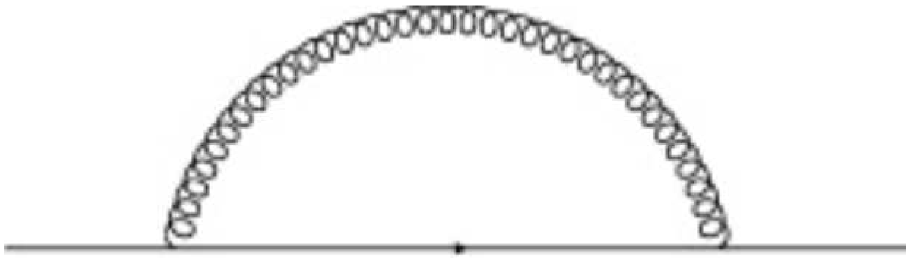


3) General Relativity

- Gravitational physics at astronomical scales
- Classical gravity mediated by curvature of spacetime.

Similar issues arise in **perturbative quantum treatment of gravity**:

- Carrier of force is graviton = spin 2 particle.
- The divergences are worse and the same (perturbative) techniques fail.



Physics at a crossroads

Two alternative conclusions are possible:

1) Stick to gravity and pointlike particles, but change quantization.

(Loop Quantum Gravity, Asymptotic Safety, . . .)

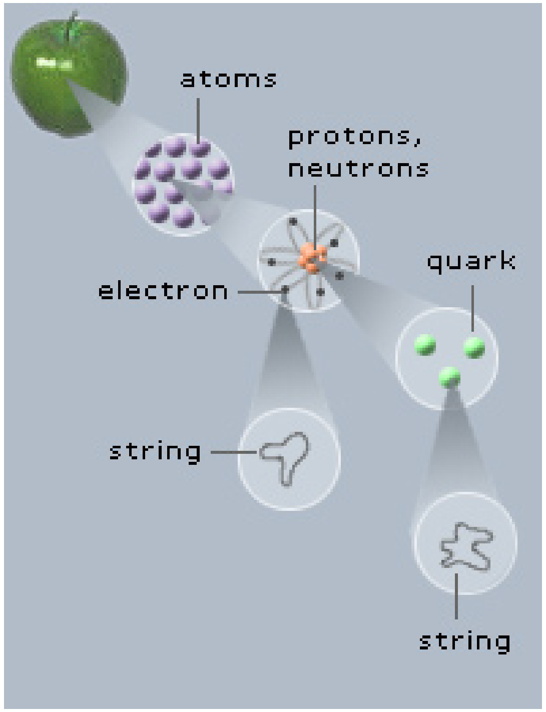
2) Stick to usual quantisation, but rethink pointlike nature of particles.

(That's what we do in string theory.)

If it's not a point - what is it?

The simplest object with substructure is 1-dimensional → **String theory**

- **Dynamical input**: The fundamental objects in Nature are not pointlike, but 1-dimensional strings
- **Kinematical input**: Describe these strings via the familiar rules of quantum theory and general covariance



Claim: This next simplest option inevitably leads to

- a consistent theory of gauge interactions and gravity
- free of UV divergences (order by order in pert. theory)
- with no free dimensionless parameters.

A symphony from 1 string

✓ A string can vibrate just like the string of a violin does.

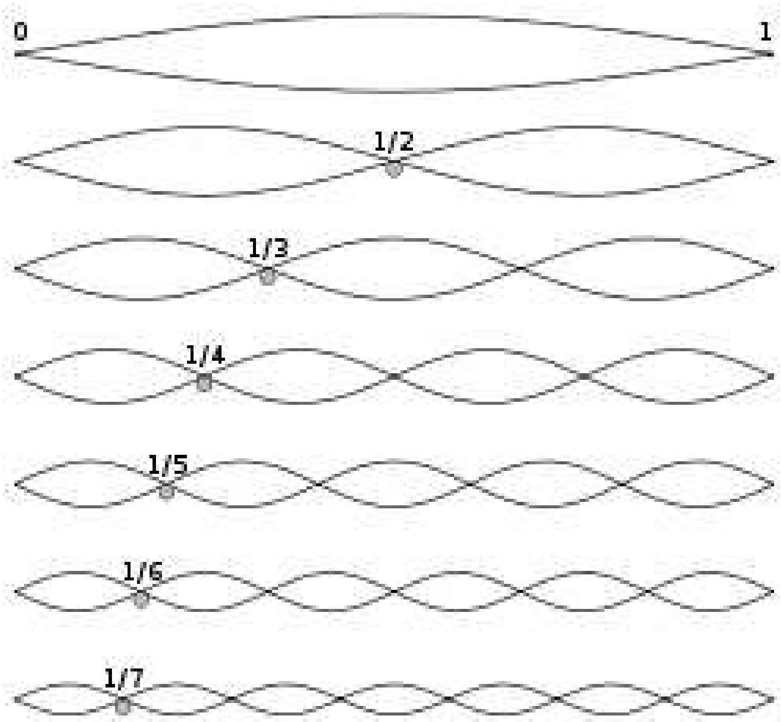
✓ The different oscillation modes correspond to different particles.

⇒ **Maximal unification:**

- There is only one kind of "stuff" - the string.
- All physics is captured by its excitations.

Analogy:

- Suppose your favorite violin has only one string.
- Many different oscillations still allow for a full symphony of different tones.



Roadmap

Part I: Basic Principles of String Theory

1) Classical strings in pics and formulae

2) Quantisation and spectrum

Part II: String Compactification

3) The concept of compactification

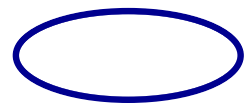
4) Brane Worlds

5) The String Landscape

Classical Strings

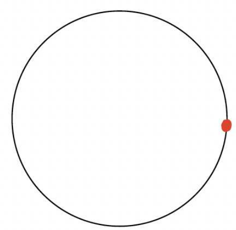
2 types of strings:

closed strings



open strings

- Position along string parametrized by $0 \leq \sigma \leq \ell$

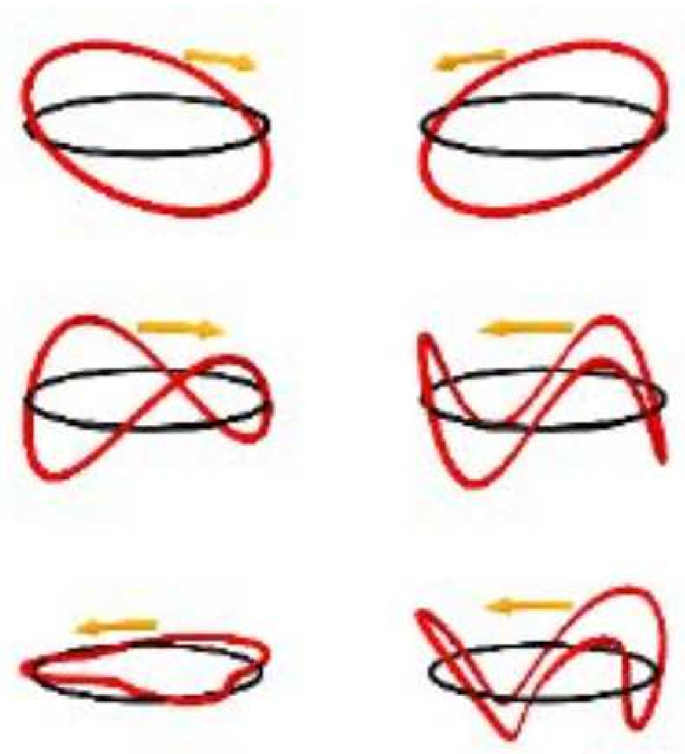


- String coordinates in spacetime: $X^\mu(\tau, \sigma), \quad \mu = 0, \ldots, d-1$

Equations of motion

- free point particle: $(\frac{\partial}{\partial \tau})^2 X^\mu(\tau) = 0$
- free string: $\left((\frac{\partial}{\partial \tau})^{\mathbf{2}} - (\frac{\partial}{\partial \sigma})^{\mathbf{2}} \right) \mathbf{X}^\mu(\tau, \sigma) = \mathbf{0} \leftrightarrow$ **2D wave equation**

Classical Strings - Dynamics I



Strings carry energy

- c.o.m. momentum
- oscillations along string

Strings carry spin

$\leftrightarrow$ polarisation of oscillation

Energy scale set by **string length**

$$\ell_s \equiv 2\pi\sqrt{\alpha'}$$

Typical string scale $M_s = \ell_s^{-1}$ is the only 'free parameter' of the theory:

$$6.8\mathrm{TeV} \simeq \tfrac{1}{2} M_{\mathrm{Run\,3}} \leq M_s \leq M_{\mathrm{Pl.}} \simeq 10^{18}\mathrm{GeV}$$

Classical Strings - Dynamics II

- 2D wave equation: $\left(\left(\frac{\partial}{\partial\tau}\right)^2 - \left(\frac{\partial}{\partial\sigma}\right)^2\right) X^\mu(\tau,\sigma) = 0$
- Ansatz: $X^\mu(\tau,\sigma) = \underbrace{X_R^\mu(\tau-\sigma)}_{\text{right−moving wave}} + \underbrace{X_L^\mu(\tau+\sigma)}_{\text{left−moving wave}}$
- Boundary conditions for **closed string**

$$X^\mu(\tau,\sigma) = X^\mu(\tau,\sigma+\ell) \qquad \ell : \text{circumference of string}$$

Most general solution: Fourier expansion

$$X_L^\mu = \frac{1}{2}x^\mu + \frac{\pi\alpha'}{\ell}p^\mu(\tau+\sigma) + i\sqrt{\frac{\alpha'}{2}}\sum_{m\in\mathbb{Z}\neq 0}\frac{1}{m}\tilde{\alpha}_m^\mu e^{-i\frac{2\pi}{\ell}m(\tau+\sigma)}$$

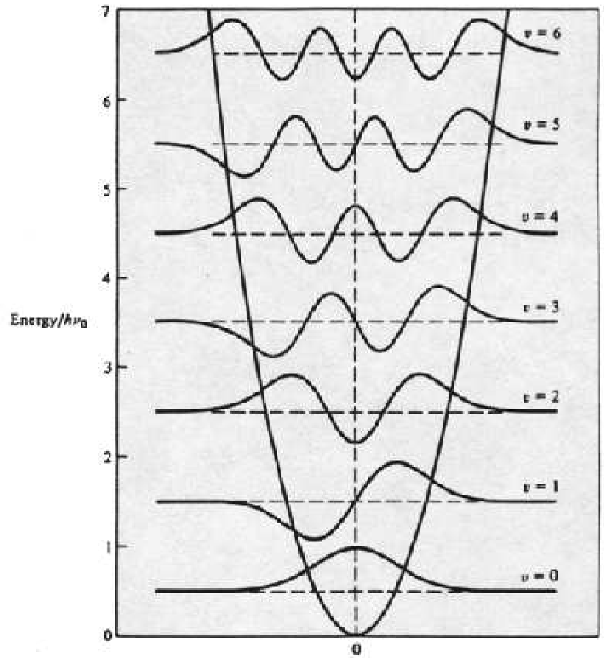
$$X_R^\mu = \frac{1}{2}x^\mu + \frac{\pi\alpha'}{\ell}p^\mu(\tau-\sigma) + i\sqrt{\frac{\alpha'}{2}}\sum_{m\in\mathbb{Z}\neq 0}\frac{1}{m}\alpha_m^\mu e^{-i\frac{2\pi}{\ell}m(\tau-\sigma)}$$

- Frequencies: $\frac{2\pi}{\ell}m$ Amplitudes: α_m^μ (Right) $\tilde{\alpha}_m^\mu$ (Left)
- c.o.m momentum p^μ and positition x^μ

String Quantisation - I

= quantisation of waves along the string

Each excitation mode $\alpha_m^\mu, \tilde{\alpha}_m^\mu$ represents a **harmonic oscillator**



States:

- c.o.m. momentum p: $|0, p\rangle$
- Excite each left/right oscillation frequency $\frac{2\pi}{\ell} m$ arbitrarily often:

$$\prod_{m>0,\mu} (\alpha_{-m}^\mu)^{n_{m,\mu}} \prod_{m>0,\mu} (\tilde{\alpha}_{-m}^\mu)^{\tilde{n}_{m,\mu}} |0; p\rangle$$

(Special technicality here: equal number of left/rightmoving quanta)

String Quantisation - II

Tower of string excitations - characterized by oscillation number $N_L = N_R$

- $N_L = 0 = N_R$: $|0, p\rangle$: momentum eigenstate with zero oscillations
- $N_L = 1 = N_R : \zeta_{\mu\nu}\alpha^{\mu}_{-1}\tilde{\alpha}^{\nu}_{-1}|0; p\rangle$: first mode excited
- $\ldots$

Mass of string excitations: (for bosonic string)

$$M^2 = 4M_s^2 \times (N - a) \qquad a = 1 \qquad N = N_L = N_R$$

$M_s \simeq \ell_s^{-1}$: string scale $\leftrightarrow$ sets scale of oscillations

$N_L = 0 = N_R$: tachyon - removed in superstring theory

$N_L = 1 = N_R$: massless excitations

$N = 2, 3, \ldots$: massive states of mass-squared set by M_s

Each oscillation = object with mass and spin = particle.

Gravitons from closed strings

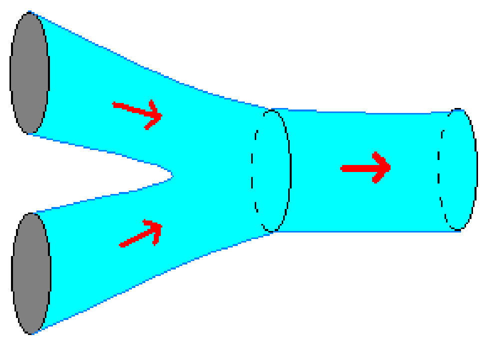
Low-energy regime ($E << M_s$): only massless modes relevant

$$\text{Closed massless} : \zeta_{\mu\nu}\alpha^{\mu}_{-1}\tilde{\alpha}^{\nu}_{-1}|0;p\rangle, \quad \zeta_{\mu\nu} : \text{polarisation tensor}$$

- This object contains a spin-2 mode $=$ 2-index symmetric tensor.
- **This must be the graviton $h_{\mu\nu}$.**
  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$: fluctuation around background

Direct check:

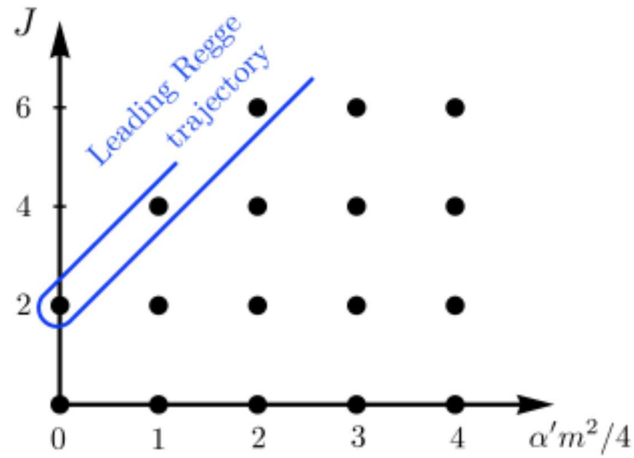
- Compute interactions in string perturbation theory
- Find same interactions as for perturbative graviton



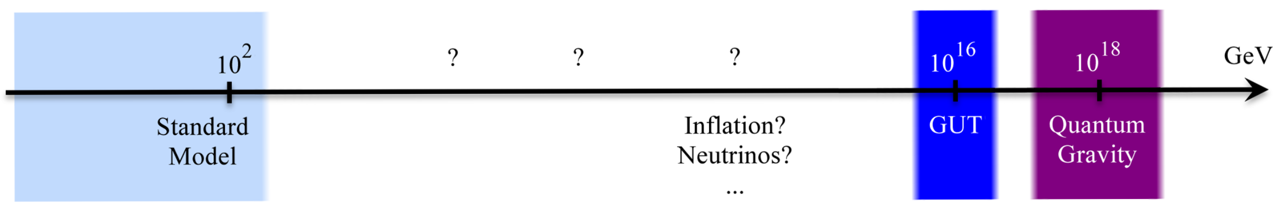
String spectrum

string spectrum:

- mass $M^2 \simeq n \times M_s^2$,
  $n = 0, 1, 2, 3, \ldots$
- maximal spin
  $J \simeq M^2/M_s^2 \sim n, \quad n \in \mathbb{N}$



$$7{\times}10^3\,\mathrm{GeV} \;\leq M_s \leq\; M_{\mathrm{Pl.}} \simeq 10^{18}\,\mathrm{GeV}$$

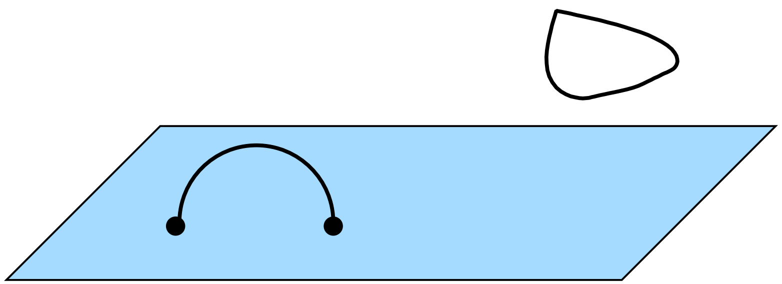


Prediction: excited strings can be directly detected at colliders
- but their mass depends on the value of M_s

Photons from open strings

- An open string has two endpoints at $\sigma = 0$ and $\sigma = \ell$
- Repeat program of classical solutions and quantisation with suitable boundary conditions
- Result: String endpoints can move freely along an object called a **Dp-brane** = (p+1)-dimensional hypersurface of spacetime

Polchinski 1996

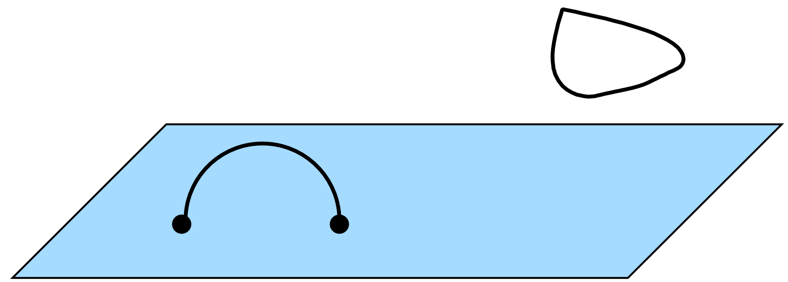


- Boundary conditions relate left/rightmoving waves
- Massless level: $\zeta_\mu \alpha^\mu_{-1}|0;p\rangle$: spin-1 particle
- Interpretation as vector boson responsible for a U(1) gauge theory

Intersecting Brane Worlds - I

String excitations along 1 Dp-brane:

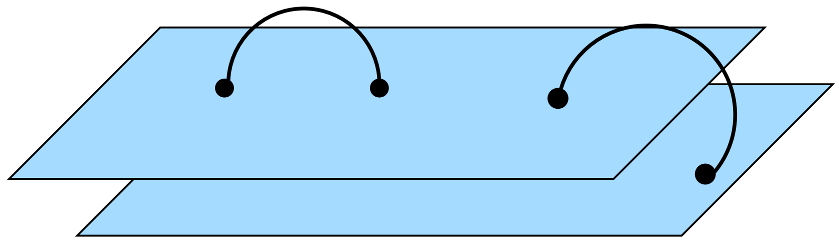
$U(1)$ gauge $\quad A^i, i = 0, \ldots p$



N coincident Dp-branes:

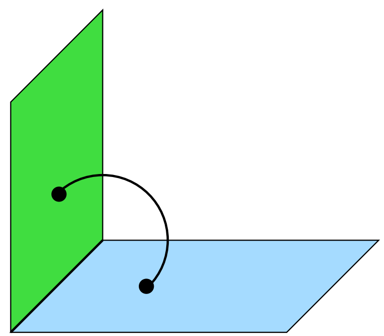
$U(N)$ gauge symmetry

$N \times N$ gauge bosons



Dp-branes at intersection:

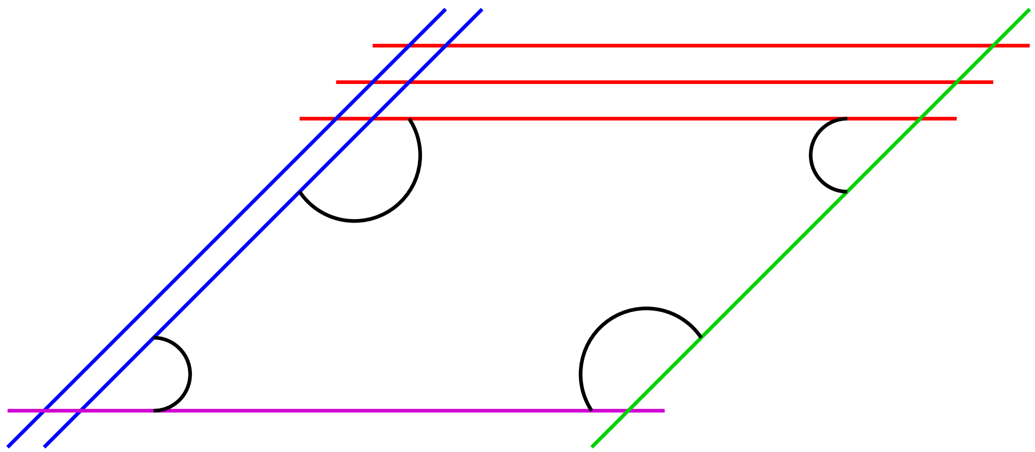
Matter fields (chiral fermions) in bifundamental resprentation $(\overline{N}_a, N_b)$



Intersecting Brane Models

- Simple realisation of gauge groups of the type $\prod_i U(N_i)$ with chiral matter in bifundamental representations
- Basic ingredients of the Standard Model $SU(3) \times SU(2) \times U(1)_Y$

Direct implementations of Standard Model gauge interactions and matter via "Intersecting Brane Worlds"

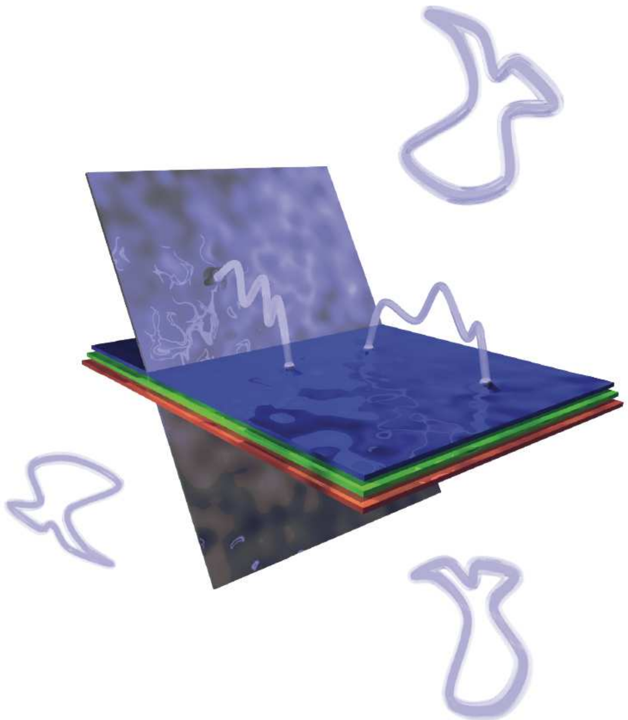


Intersecting Brane Models

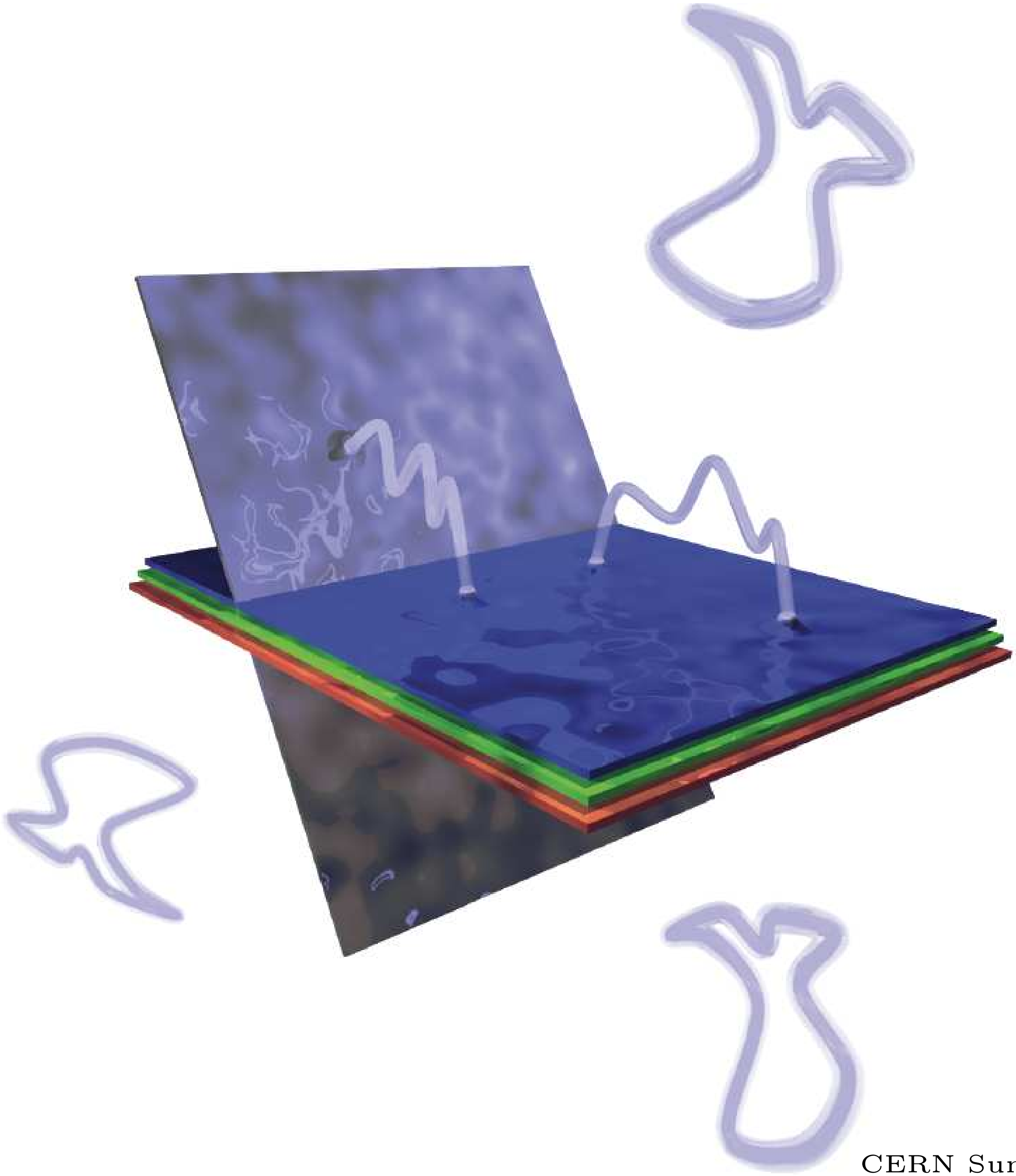
- Simple realisation of gauge groups of the type $\prod_i U(N_i)$
  with chiral matter in bifundamental representations
- Basic ingredients of the Standard Model
  $$SU(3) \times SU(2) \times U(1)_Y$$

"Intersecting Brane Worlds"

- All SM matter comes from massless string excitations.
- Masses then generated by field theoretic Higgs mechanism (brane recombination)
- Massive string states have not (yet) been observed



Gravity in bulk - EM on brane

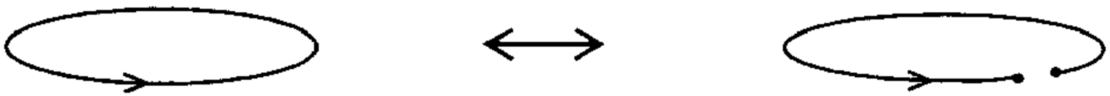


A crucial consistency check

In string theory, gauge theory implies gravity.

Reason:

- Strings interact by joining and splitting.



- Open string endpoints can join to form a stable closed string.
  (The converse is not always true)

✓ Behaviour consistent with universality of gravity:
photons $\Longrightarrow$ energy $\Longrightarrow$ gravity

✓ In string theory, gauge interactions and gravity are not independent.
They are linked by the internal consistency of the theory.

String theory is the only known theory with this property.

UV finiteness

General picture: **String as intrinsic UV regulator**

- High energy scattering probes string length $\leftrightarrow$ non-local behaviour

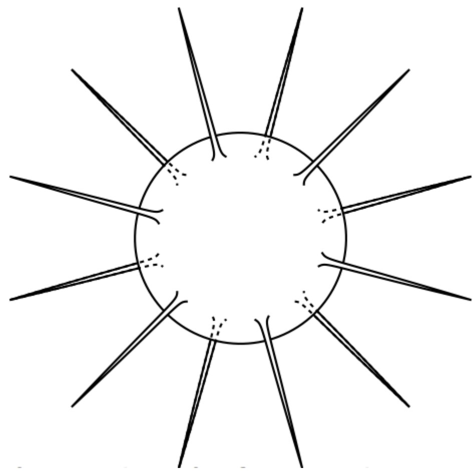
- Point-like interaction vertex is smoothened out.

This can be made very precise quantitatively, and finiteness of loop diagrams in perturbation theory can be checked.

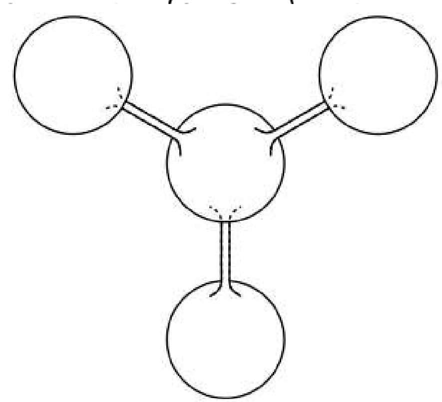
Why strings are special

Can a particle have even higher-dimensional substructure?

Model particle as a **membrane** -
2 spatial dimensions



Tubes of length L and radius R have spatial volume $\simeq L \times R$.



Quantum fluctuations:

- Long, thin tubes can form without energy cost.
- Membranes automatically describe multi-particle states.

No first quantisation of higher-branes à la strings possible.

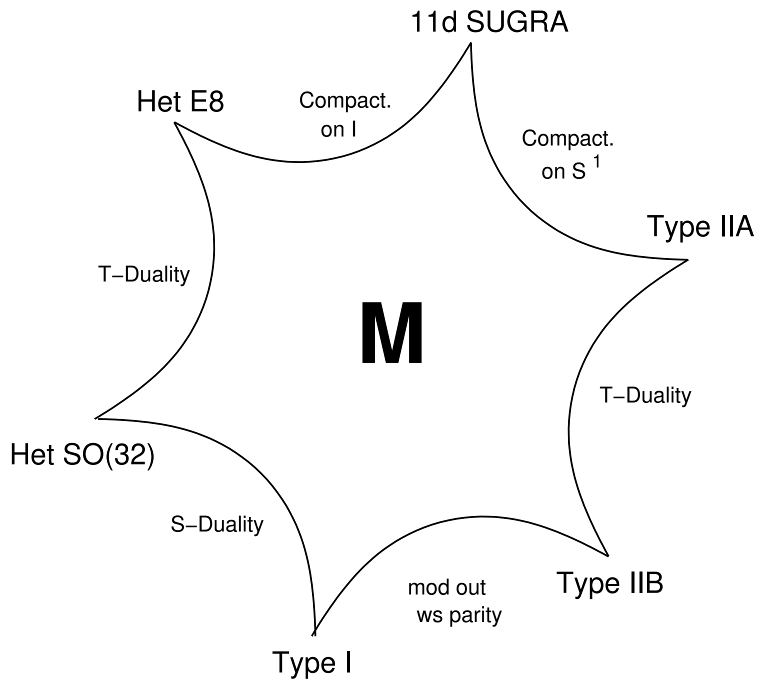
Consequences of string theory

Internal consistency conditions make further predictions:

- Spacetime is not 4-dimensional, but 10-dimensional.

- In 10 dimensions there is only one unique type of string theory.
  It has many equivalent formulations which are dual to each other.

  [Witten 1995, ...]



- The 10-dim. theory ist supersymmetric:
  Every boson has a fermionic superpartner.

  This does NOT imply that supersymmetry must be found at LHC.

Summary so far

Superstring theory is well-defined and unique (up to dualities) in 10d.

1) Low energy regime $E \ll M_s$

Theory **predicts** Einstein gravity and gauge theory.

- Within the full 10d bulk a graviton propagates.
- Along lower dimensional D-branes a gauge boson propagates.

2) High energy regime $E \geq M_s$

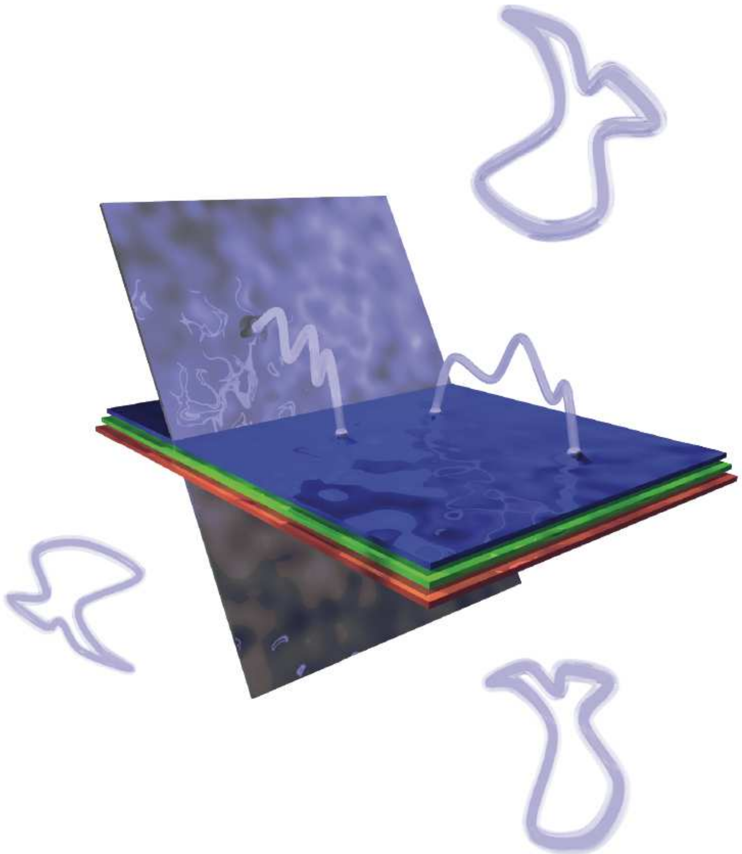
- Characteristic tower of massive string excitations
  $\rightarrow$ measurable (in principle) as resonances!
- Energy dependence of interactions differs from field theory.
- Scattering amplitudes are ultra-violet finite without the need for renormalisation.
  $\Longrightarrow$ **truly fundamental (as opposed to effective) theory**

Part II: String Compactification

3) Compactification

4) Brane Worlds

5) The String Landscape



Extra dimensions

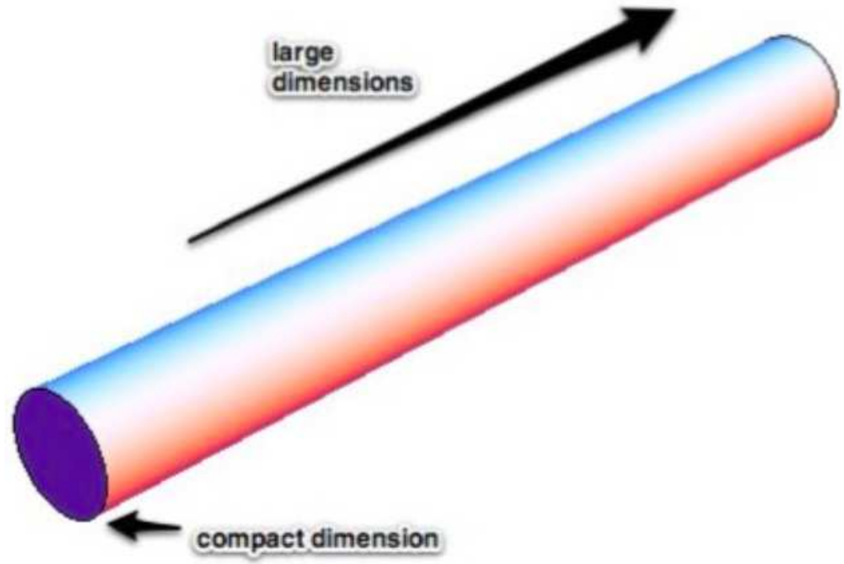
Superstring Theory is well-defined only if spacetime is 10 dimensional.

It is thus an example of a theory of extra dimensions:

- Such theories are considered also in point particle framework.
- Extra dimensions are compact and very small.

Toy example:

- Consider theory in 5 dimensions: $x^M, M = \underbrace{0, 1, \ldots 3}_{\mu}, 4$
- Compactify direction x^4 along circle S^1 of radius R



- As radius $R \to 0$ this becomes an effectively 4D theory

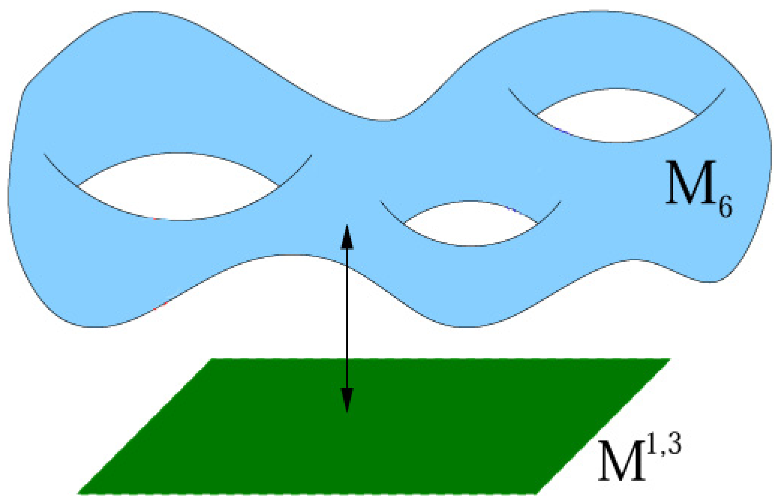
String compactification

Back to strings:

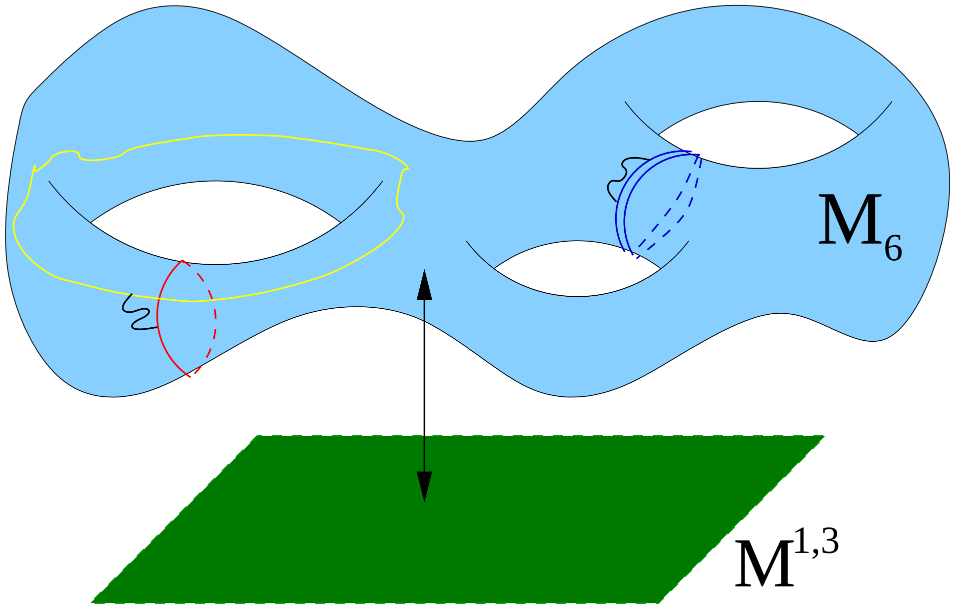
To arrive at 4 large extra dimensions we need to compactify 6 dimensions.

- Simplest solution:
  Each dimension is a circle S^1, i.e.
  internal space is a six-dimensional torus $T^6 = S^1 \times \ldots \times S^1$
  x_0, x_1, x_2, x_3: macroscopic $x_4, x_5, \ldots, x_9$: rolled up on T^6

- More general 6-dimensional spaces allowed.
- Each consistent compactification yields a solution to string equation of motions with specific physics in 4D.



Intersecting Brane Models



- Configuration of multiple branes $\leftrightarrow$ gauge groups
- Intersection pattern $\leftrightarrow$ charged matter
- Specifics of geometry $\leftrightarrow$ interactions (computable!)

String phenomenology:

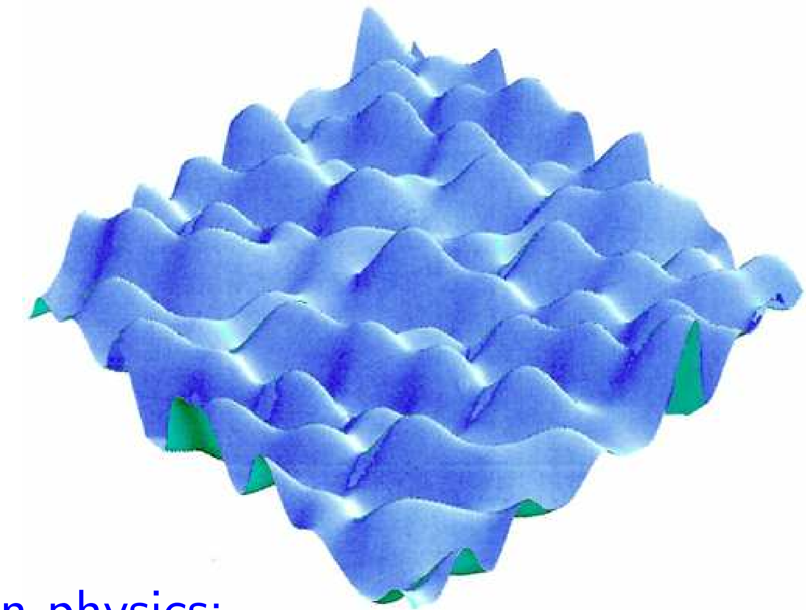
Explore interplay of string geometry and physics in 4 dimensions

The landscape of string vacua

Each consistent compactification is a solution to string equ. of motion.

Each 4d solution is called a **4d string vacuum**.

In 10d: All interactions uniquely determined



In 4d: Plethora of consistent solutions exists - the **landscape of string vacua**

Existence of many solutions is typical in physics:
Einstein gravity is one theory with many solutions!

Pressing question: Consequences for physics in 4D?
Solution to fine-tuning problems (Higgs, Cosmological Constant)?

Swampland versus Landscape

Which EFT can be coupled to a fundamental theory of QG?

Swampland of inconsistent EFTs

$\leftrightarrow$

Landscape of consistent QGs

Image: F.Marchesano

Swampland Conjectures of general scope, but not sharply proven.

String theory as a framework for QG allows to test explicit conjectures

- Quantitative check of swampland conjectures and sharper formulation
- Study manifestations of swampland conjectures in string geometry

String Geometry

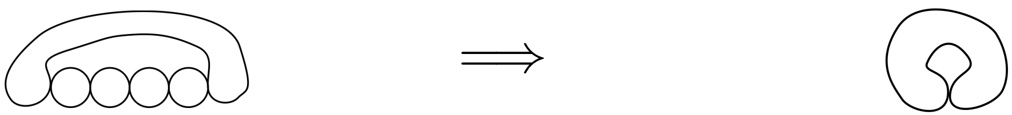
Geometry of compactification space $\leftrightarrow$ Physics in 4d (or higher)

- Strings as extended objects probe geometry differently than points
- Opens door for fascinating interplay between mathematics and physics

$\Rightarrow$ New physics ways to think about geometry by translating into physics

Ex.: Classification of singularities in geometry

- Singularities occur when submanifolds shrink to zero size



- Branes can wrap these vanishing cycles and give rise to massless particles in effective theory
- Gives interpretation for classification of singularities in mathematics and guidelines for new situations unknown to mathematicians

Many more, beautiful examples of this type

Summary

String theory is a maximally economic quantum theory of gravity, gauge interactions and matter.
Assumption of stringlike nature of particles leads to calculable theory without UV divergences.

Challenge for String Phenomenology: understanding the vacuum of this theory

This talk has focused on **String Theory as a fundamental theory**.
What we haven't discussed at all:

- **String Theory as modern mathematical physics**:
  deep interplay with sophisticated mathematics [Witten,Douglas,...]
  (Mirror symmetry, D-brane categories,. . .)
- **String Theory as a tool**: [Maldacena'97,Witten'98,...]
  Holographic principle - AdS/CFT: Insights and applications

String Theory is a framework for modern physics.