

# Status/update $t\bar{t}W$ production in NNLO QCD

*Chiara Savoini*

*University of Zurich*

in collaboration with *L.Buonocore, S.Devoto, M.Grazzini, S.Kallweit, J.Mazzitelli, L.Rottoli*

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**Universität  
Zürich**<sup>UZH</sup>

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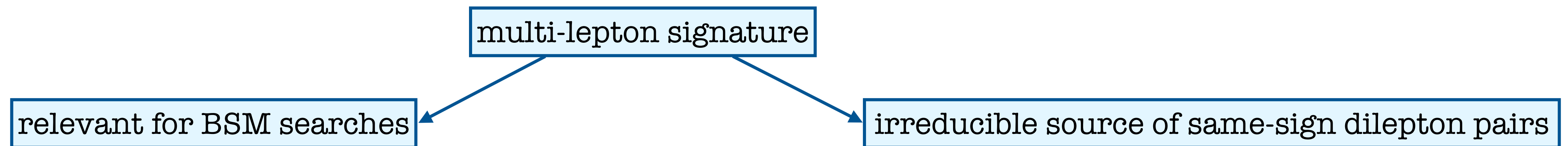
📌 Results **PRELIMINARY!**

# Introduction: why $t\bar{t}W$ production?

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## Motivations :

- ▶ the production of top-quark pairs in association with a  $W$  boson is among the **most massive SM signatures** at hadron colliders
- ▶ since the top quarks rapidly decay, the signature of the process is characterised by **two b-jets and three  $W$  bosons**



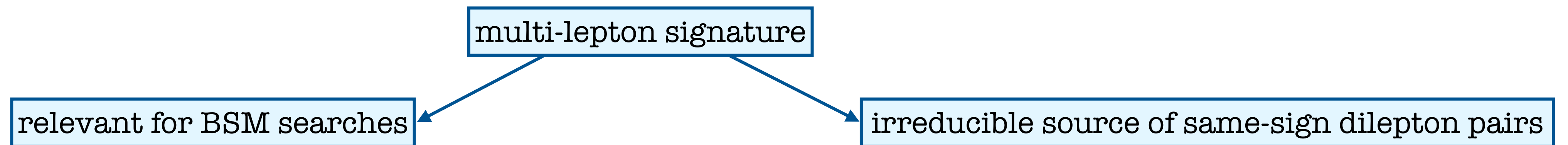
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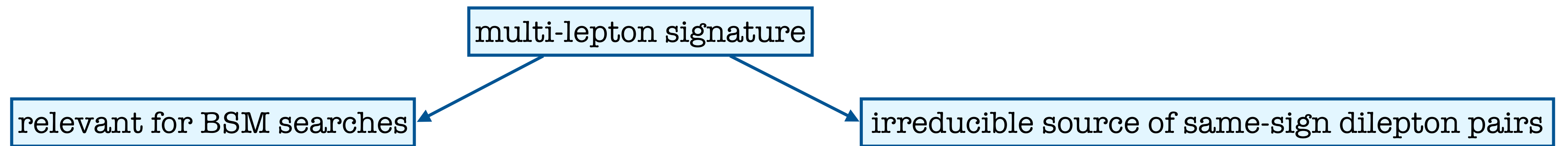


- ▶ it represents a **relevant background** also for SM processes like  $t\bar{t}H$  and  $t\bar{t}t\bar{t}$  production
- ▶ it is “**special**” compared to the other  $t\bar{t}V$  ( $V = \{H, Z, \gamma\}$ ) processes since the  $W$  can only be emitted from the **initial-state light quarks** (i.e. no gluon fusion at LO)

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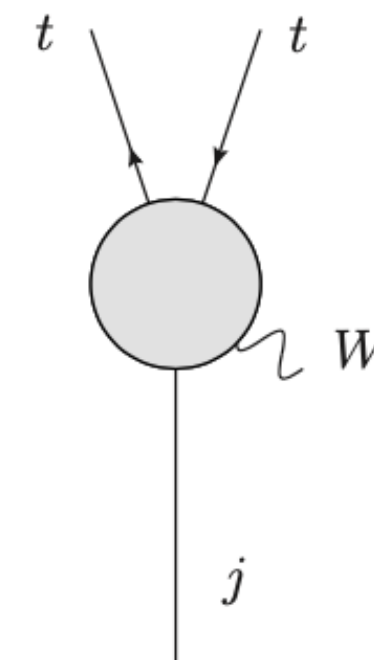
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- ▶ different pattern of radiative corrections: both **QCD and EW corrections are relevant**

dominated by configurations where the  $t\bar{t}$  pair recoils against hard QCD radiation, accompanied by a soft  $W$  boson

[Maltoni, Pagani, Tsinikos (2015)]

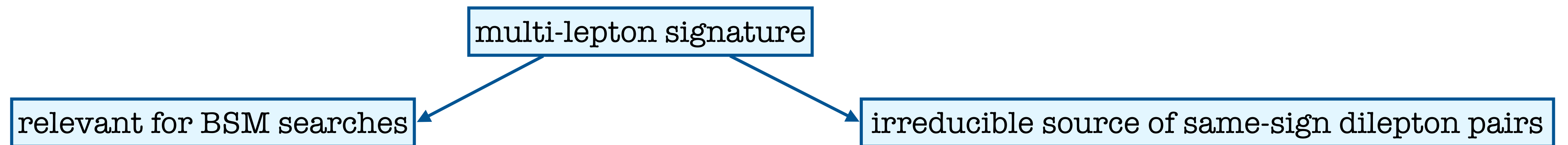




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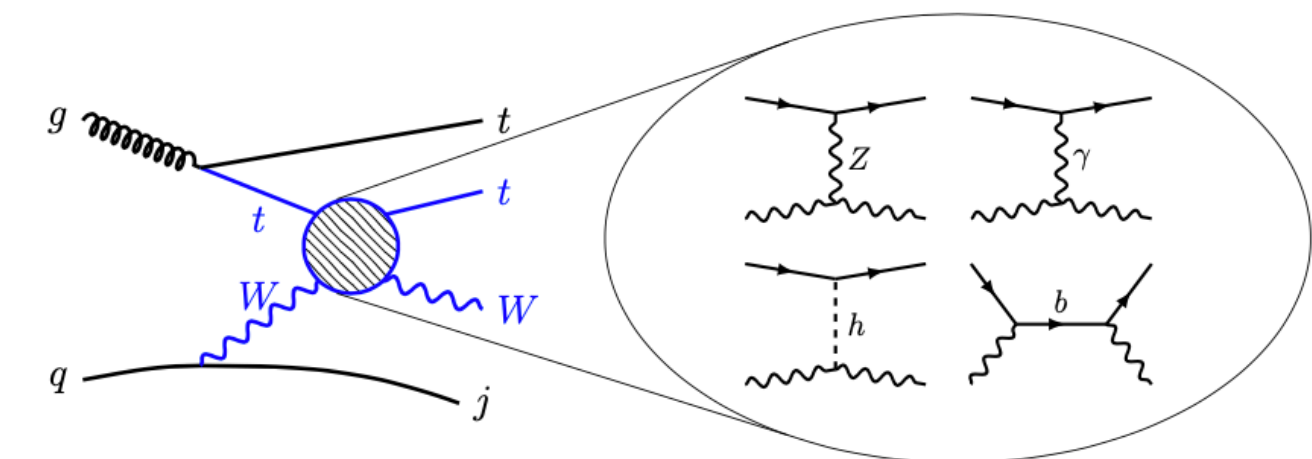
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~ 10 % at the LHC, due to the opening of  $tW \rightarrow tW$  scattering diagrams

[Frederix, Pagani, Zaro (2017)]

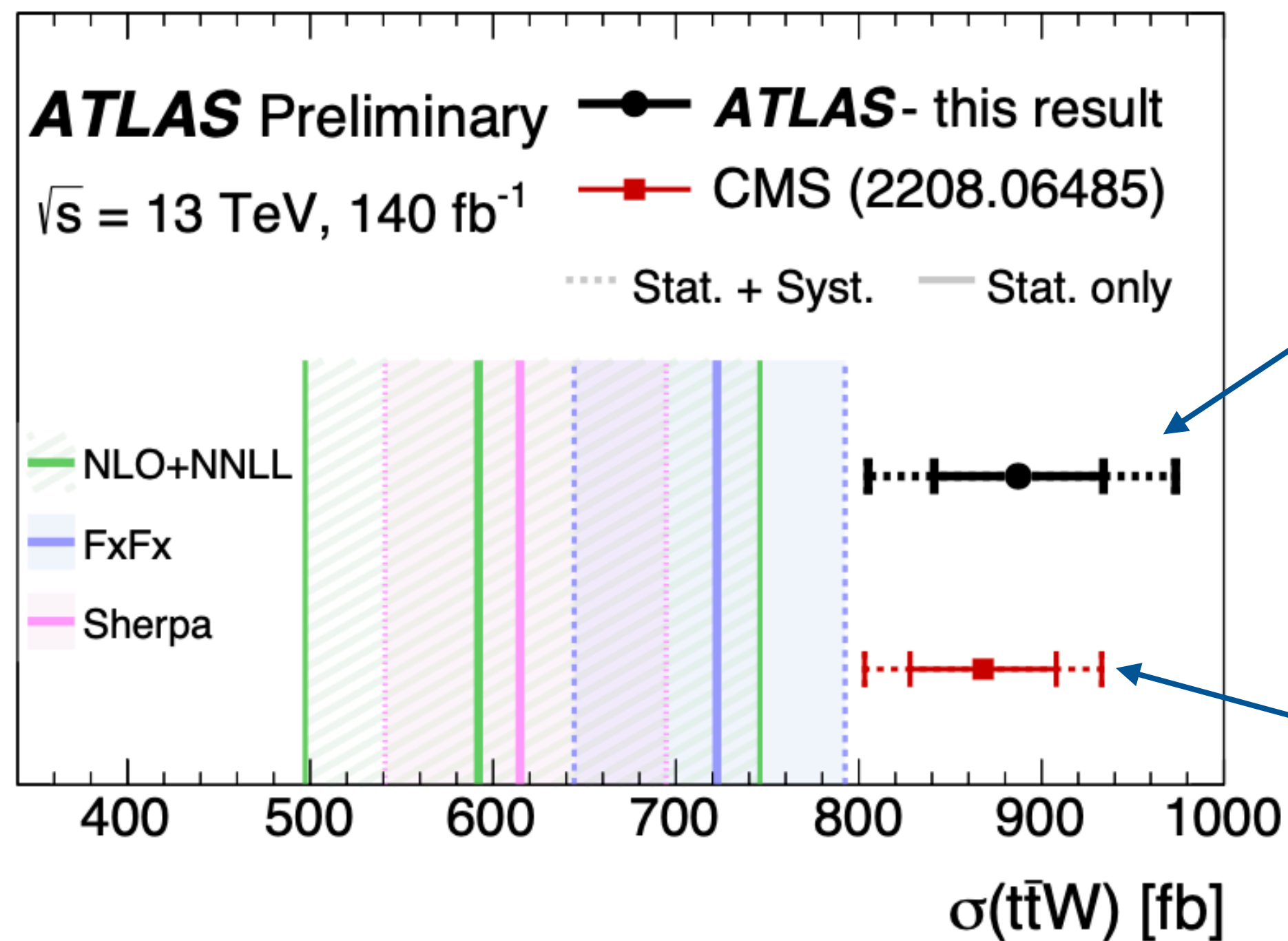


# Introduction: $t\bar{t}W$ production

State of the art : experimental measurements

- ▶ measurements by ATLAS and CMS at  $\sqrt{s} = 8$  TeV and  $\sqrt{s} = 13$  TeV → rates consistently higher compared to the SM prediction
- ▶ discrepancy confirmed also by indirect measurements of  $t\bar{t}W$  in the context of  $t\bar{t}H$  and  $t\bar{t}t\bar{t}$  analyses
- ▶ most recent measurements, based on an integrated luminosity of  $140 \text{ fb}^{-1}$ , confirmed this picture

slight excess at 1-2  $\sigma$  level



$$\sigma(t\bar{t}W^+) = 585^{+35}_{-34} \text{ (stat.) } ^{+47}_{-44} \text{ (syst.)} = 585^{+58}_{-55} \text{ (tot.) fb}$$

$$\sigma(t\bar{t}W^-) = 301^{+28}_{-27} \text{ (stat.) } ^{+35}_{-31} \text{ (syst.)} = 301^{+45}_{-41} \text{ (tot.) fb}$$

[ATLAS-CONF-2023-019]

$\sigma_{t\bar{t}W}$	$868 \pm 40 \text{ (stat)} \pm 51 \text{ (syst) fb}$
$\sigma_{t\bar{t}W^+}$	$553 \pm 30 \text{ (stat)} \pm 30 \text{ (syst) fb}$
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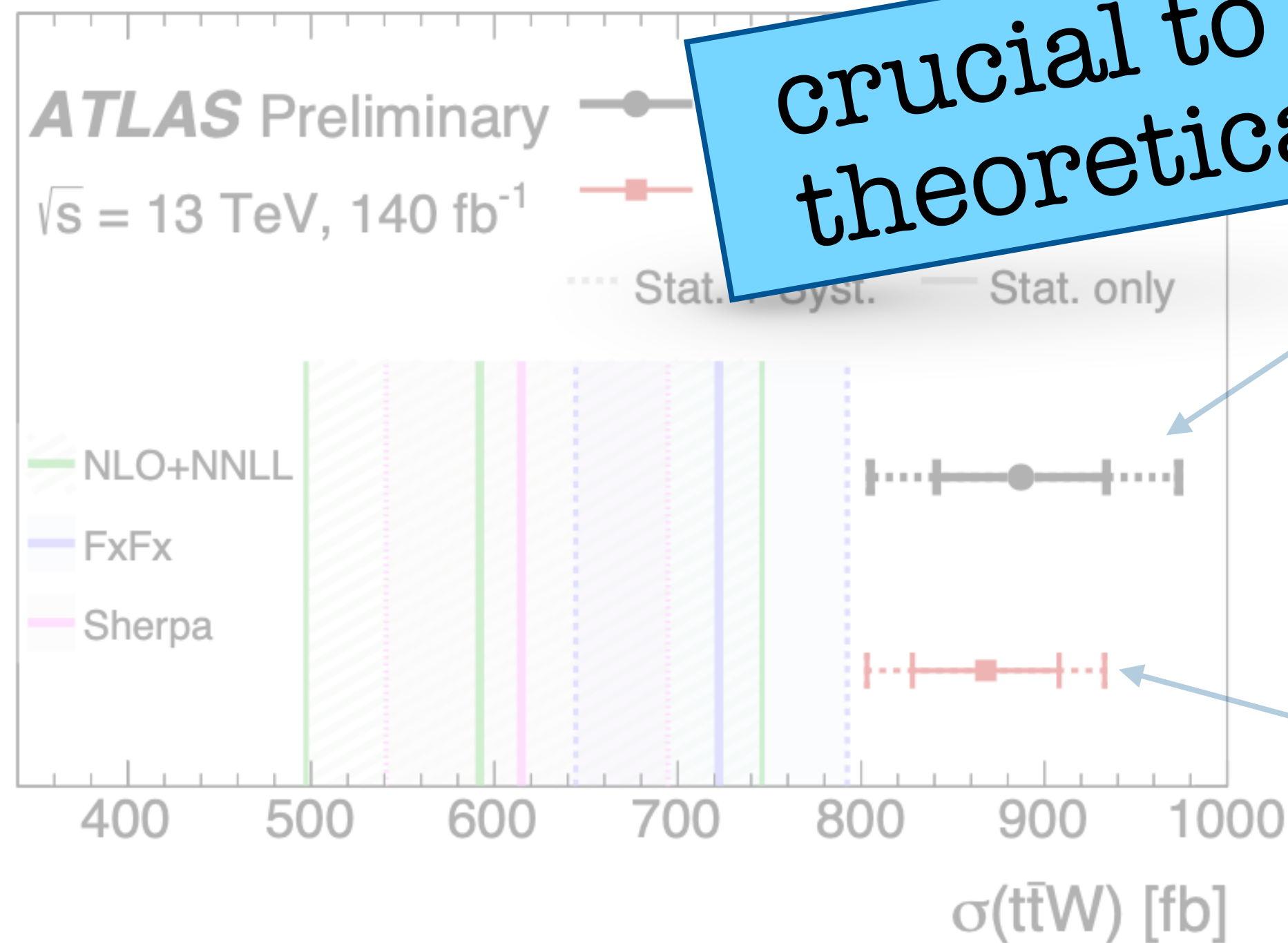
[CMS: arXiv 2208.06485]

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**crucial to rely on precise theoretical predictions!!**



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light excess at 1-2  $\sigma$  level



# Introduction: $t\bar{t}W$ production

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State of the art : theoretical predictions

- ☑ **NLO QCD** corrections (*on-shell top quarks*) [Badger, Campbell, Ellis (2010-2012)]
- ☑ **NLO QCD + EW** corrections (*on-shell top quarks and W*) [Frixione, Hirschi, Pagani, Shao, Zaro (2015)]
- ☑ inclusion of **soft gluon resummation at NNLL** [Broggio et al. (2016)] [Kulesza et al. (2019)]
- ☑ **NLO QCD** corrections (*full off-shell process, three charged lepton signature*) [Bevilacqua et al. (2020)] [Denner, Pelliccioli (2020)]
- ☑ combined **NLO QCD + EW** corrections (*full off-shell process, three charged lepton signature*) [Denner, Pelliccioli (2021)]
- ☑ current experimental measurements are compared with **NLO QCD + EW** (*on-shell*) predictions supplemented with **multi-jet merging** [Frederix, Tsiniikos (2021)]

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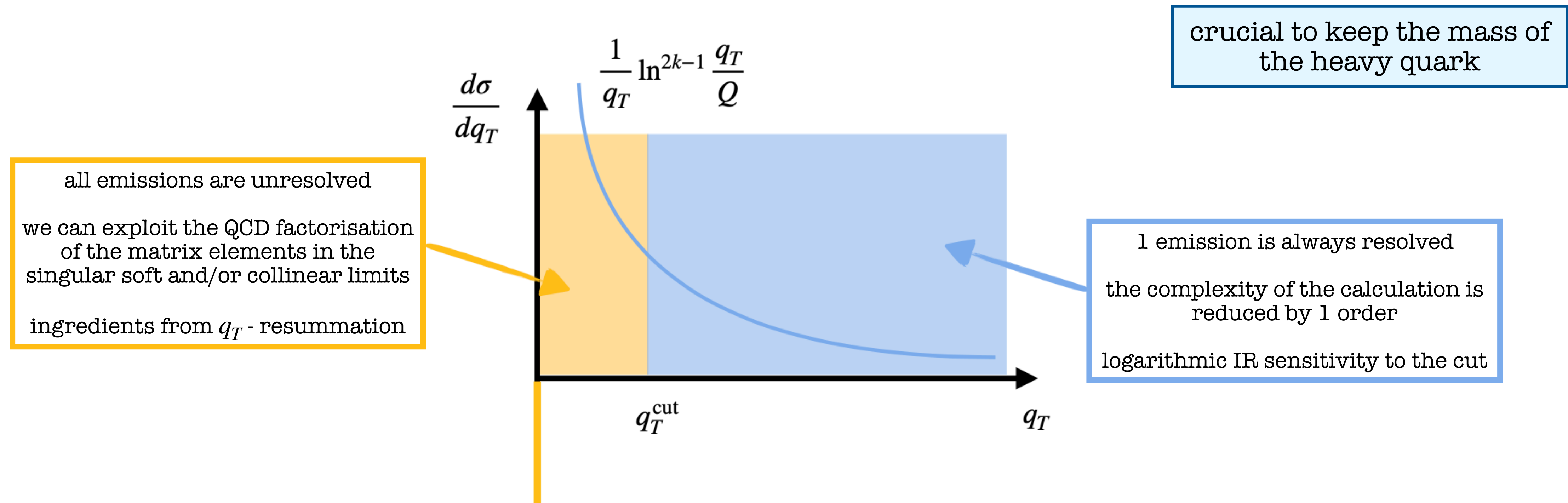
↓  
still affected by relatively large uncertainties

- ✓ **complete NNLO QCD + NLO EW** (*on-shell*) with approximated two-loop amplitudes in this talk!

# The framework: $q_T$ -subtraction

[Catani, Grazzini (2007)]

- cross section for the production of a triggered final state at  $N^k LO$  (in our case the triggered final state is  $t\bar{t}W$ )



$$d\sigma_{N^k LO} = \mathcal{H}_{N^k LO} \otimes d\sigma_{LO} + [d\sigma_{N^{k-1} LO}^R - d\sigma_{N^k LO}^{CT}]_{q_t > q_t^{\text{cut}}} + \mathcal{O}((q_t^{\text{cut}})^p)$$

# The framework: $q_T$ -subtraction [Catani, Grazzini (2007)]

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► master formula at NNLO

$$d\sigma_{NNLO} = \mathcal{H}_{NNLO} \otimes d\sigma_{LO} + [d\sigma_{NLO}^R - d\sigma_{NNLO}^{CT}]_{q_t > q_t^{\text{cut}}} + \mathcal{O}((q_t^{\text{cut}})^p)$$

- ✓ the required matrix elements can be computed with automated tools like OpenLoops2
- ✓ the remaining NLO-type singularities can be removed by applying a local subtraction method



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[Catani, Devoto, Grazzini, Mazzitelli (2023)]

- ✓ non trivial ingredient: **two-loop soft function** for an arbitrary kinematics of the heavy quarks [Devoto, Mazzitelli (in preparation)]
- ✓ all ingredients are known except for the **two-loop virtual amplitudes** contributing to the the hard-collinear coefficient

$$\mathcal{H}_{NNLO} = H^{(2)} \delta(1 - z_1) \delta(1 - z_2) + \delta \mathcal{H}^{(2)}(z_1, z_2)$$

where

$$H^{(2)} = \frac{2\Re(\mathcal{M}_{fin}^{(2)}(\mu_{IR}, \mu_R) \mathcal{M}^{(0)*})}{|\mathcal{M}^{(0)}|^2} \Bigg|_{\mu_R=Q}$$

Remark: analogous definition for the hard-collinear coefficient at NLO

$$H^{(1)} = \frac{2\Re(\mathcal{M}_{fin}^{(1)}(\mu_{IR}, \mu_R) \mathcal{M}^{(0)*})}{|\mathcal{M}^{(0)}|^2} \Bigg|_{\mu_R=Q}$$

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[Catani, Grazzini (2007)]

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main bottleneck:  
2 → 3 two-loop amplitudes  
with internal and external  
massive legs are currently out  
of reach!

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crucial to find one (or more) reasonable approximation

main bottleneck:

two-loop amplitudes  
 internal and external  
 lines are currently out  
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# Two-loop amplitudes: soft approximation

soft approximation in a nutshell:

**bottleneck:** the two-loop amplitudes are at the frontier of the current techniques  
**solution:** development of a soft boson approximation

► for a **soft scalar Higgs** radiated off a **heavy** quark  $i$ , we have that

$$\lim_{k \rightarrow 0} \mathcal{M}(\{p_i\}, k) = F(\alpha_s(\mu_R); m/\mu_R) J^{(0)}(k) \mathcal{M}(\{p_i\})$$

renormalised mass of the heavy quark

$$J^{(0)}(k) = \sum_i \frac{m}{v} \frac{m}{p_i \cdot k}$$

- overall normalisation, finite, gauge-independent and perturbatively computable
- **effective coupling** accounting for effects due to the renormalisation of the heavy quark mass and wave function

we assume that all heavy quarks involved in the process have the same mass



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► for a **soft gauge W boson** radiated off a **massless** quark  $i$ , we have that

$$\lim_{k \rightarrow 0} \mathcal{M}(\{p_i\}, k) = J^{(0)\mu}(k) \cdot \epsilon_\mu(k) \mathcal{M}(\{p_i\})$$

valid at all perturbative orders

$$J^{(0)\mu}(k) = \frac{g_W}{\sqrt{2}} \sum_i \left( \sigma_i \frac{p_i^\mu}{p_i \cdot k} \right) \frac{1 - \gamma_5}{2}$$

$$\sigma_i = \begin{cases} +1 & \text{incoming } \bar{q}, \text{ outgoing } q \\ -1 & \text{incoming } q, \text{ outgoing } \bar{q} \end{cases}$$

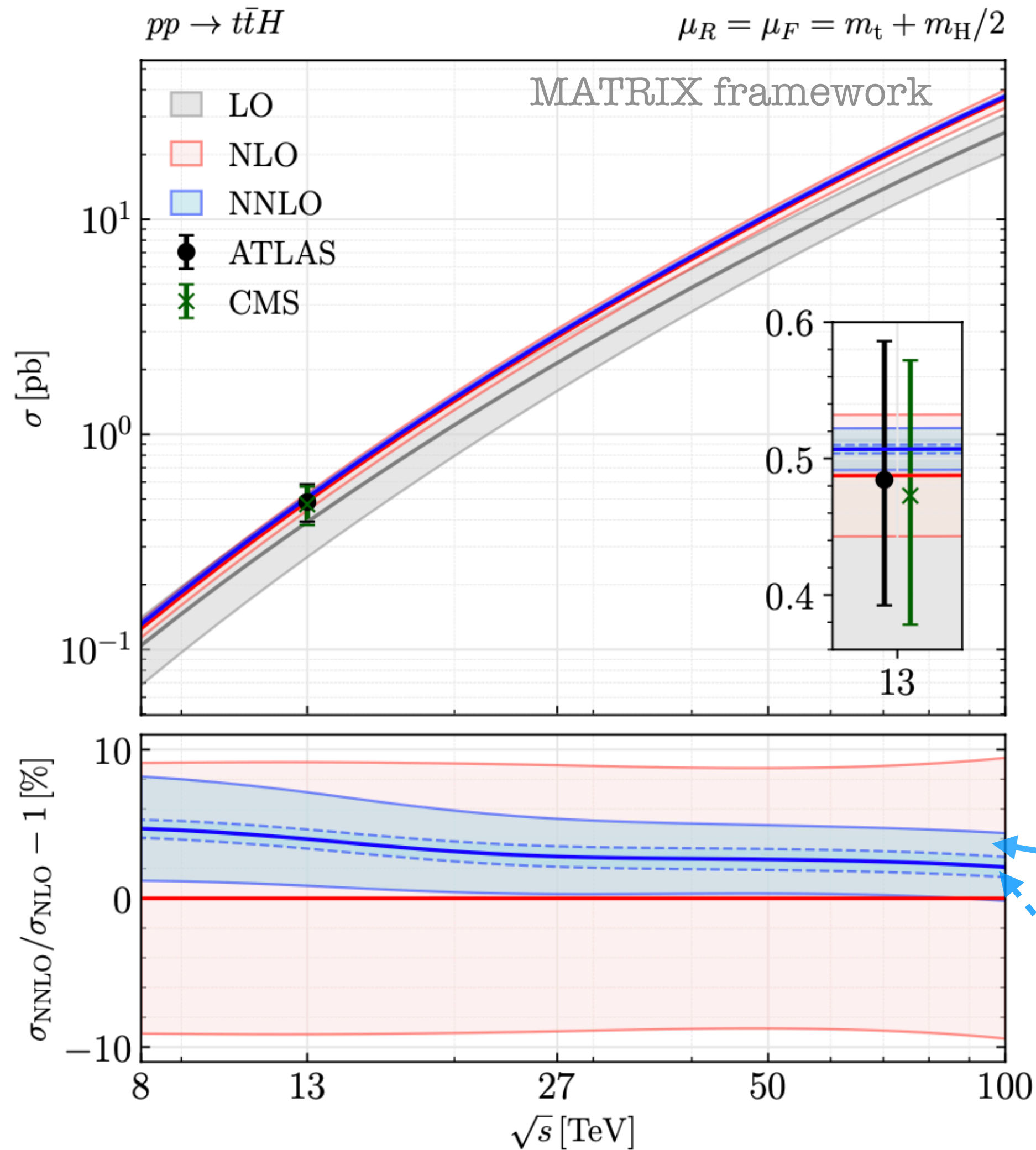
**main differences** between Higgs and W:

- scalar vs vectorial current
- massless vs massive emitters
- no renormalisation effects
- selection of the polarisation state of the emitter

# Soft approximation in $t\bar{t}H$

[Catani et al. (2022)]

setup: NNLO NNPDF31,  $m_H = 125\text{GeV}$ ,  $m_t = 173.3\text{GeV}$ ,  $\mu_R = \mu_F = (2m_t + m_H)/2$



- ▶ **all** NNLO ingredients are computed **exactly** except for the double virtual contribution
- ▶ at NNLO, the hard contribution is about **1%** of the LO cross section in  $gg$  and **2-3%** in  $q\bar{q}$
- ▶ the observed deviation at NLO is used to estimate the uncertainty at NNLO
- ▶ it is clear that the quality of the final result depends on the size of the contribution we are approximating

FINAL UNCERTAINTY:

$\pm 0.6\%$  on  $\sigma_{NNLO}$ ,  $\pm 15\%$  on  $\Delta\sigma_{NNLO}$

symmetrised 7-point scale variation

systematic + soft-approximation

# Two-loop amplitudes: massification

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## massification in a nutshell:

**bottleneck:** the two-loop amplitudes are at the frontier of the current techniques

**solution:** exploit the massification of available massless two-loop 5-point amplitudes

- ▶ the idea is to exploit the leading-colour massless two-loop 5-point amplitudes for  $q\bar{q}' \rightarrow WQ\bar{Q}$  production [Abreu et al. (2021)]  
[Badger et al. (2021)]
- ▶ we apply the **massification** technique [Moch, Mitov (2007)] to reconstruct the corresponding massive amplitudes up to power corrections  $m_Q/Q \ll 1$  [Becher, Melnikov (2007)]
- ▶ massification relies on the factorisation properties of QCD amplitudes (into jet, hard and soft functions)
- ▶ **basic idea:** the mass acts as a physical regulator of collinear singularities 1/ε poles are traded into log  $m_Q$

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change in the renormalisation scheme

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change in the renormalisation scheme

### ▶ master formula:

$$\mathcal{M}^{(m_Q)}(\{p_i\}; \mu, \epsilon) = Z_{[q]}^{(m_Q|0)}(\alpha_s(\mu), m_Q/\mu, \epsilon) \mathcal{M}^{(m_Q=0)}(\{p_i\}; \mu, \epsilon) + \mathcal{O}(m_Q^2/Q^2)$$

- universal factor, perturbatively computable
- ratio between massive and massless quark form factors



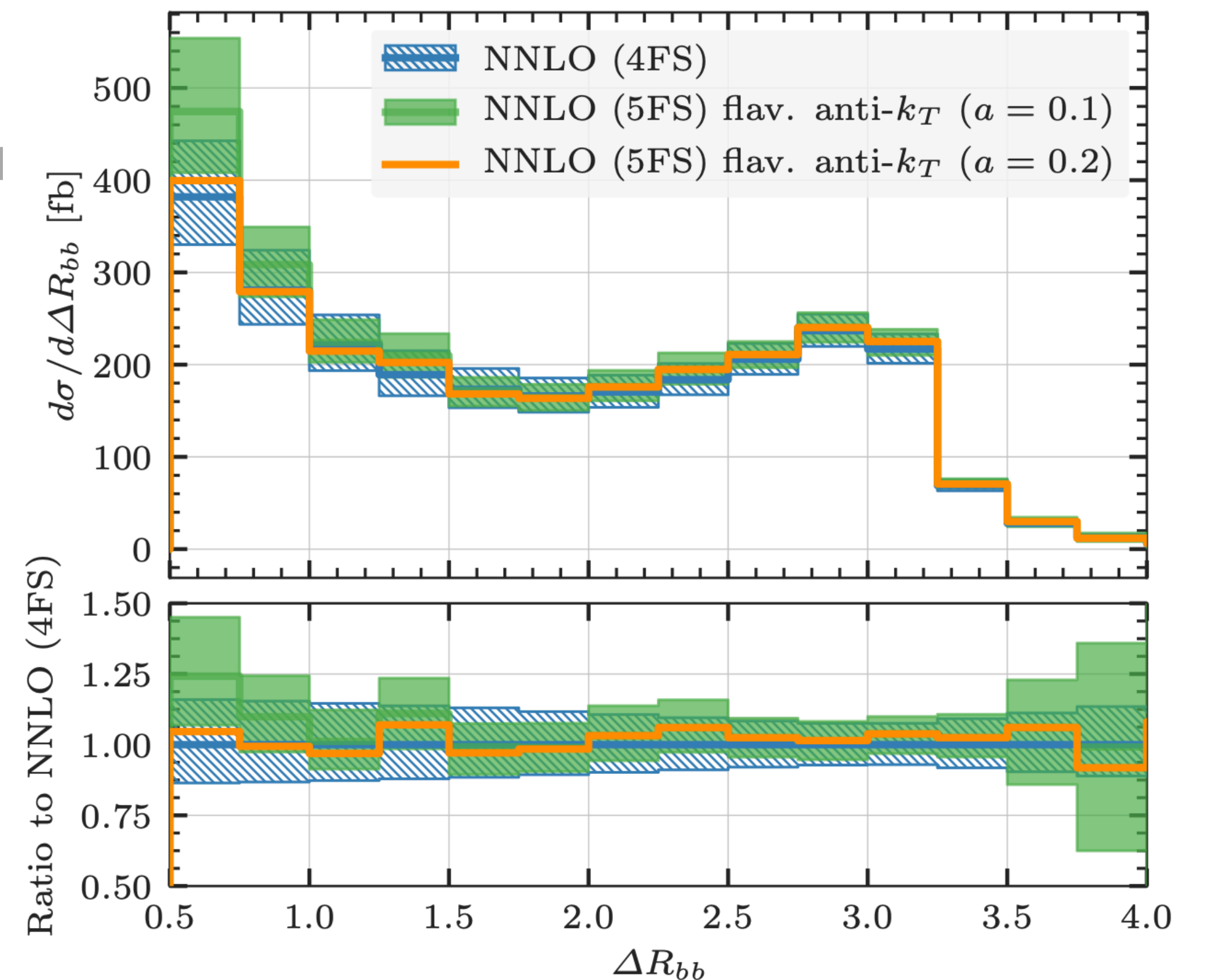
# Massification in $Wb\bar{b}$

[Buonocore et al. (2022)]

[CMS: arXiv 1608.07561]

setup: NNLO NNPDF31 4F,  $\sqrt{s} = 8 \text{ TeV}$ ,  $\mu_R = \mu_F = E_T(\nu) + p_T(b_1) + p_T(b_2)$   
 $p_{T,l} > 30 \text{ GeV}$   $|\eta_l| < 2.1$ ,  $p_{T,b} > 25 \text{ GeV}$   $|\eta_l| < 2.4$ ,  $p_{T,j} > 25 \text{ GeV}$   $|\eta_l| < 2.4$

- ▶ **all** NNLO ingredients are computed **exactly** except for the double virtual contribution
- ▶ comparison against the 5F massless computation [Poncelet et al. (2022)]
  - overall **good agreement** within the scale uncertainties
  - the uncertainties due to variation of  $m_b \in [4.2, 4.92] \text{ GeV}$  are at **2%** level (smaller than the ones due to the variation of  $a$ )
- ▶ **reliability** of the procedure: the discrepancy between the exact and massified virtual contribution @NLO is only **3%** of the NLO correction



# How to deal with $Wt\bar{t}$ ?

---

► good news! we have **two rather different** and **complementary approximations** of the exact two-loop virtual amplitudes

► **soft approximation:**

- it works nicely in the case of  $t\bar{t}H$ , mainly due to the smallness of the approximated  $H^{(2)}$  contribution
- formally it is valid in the limit  $E_W \rightarrow 0$ ,  $m_W \ll m_t$  (which is not true for a physical  $W$  boson ...)

$$\mathcal{M}_{Wt\bar{t}}(\{p_i\}, p_W; \mu, \epsilon) = J^{(0)\mu}(p_W) \cdot \epsilon_\mu(p_W) \mathcal{M}_{t\bar{t}}(\{p_i\}; \mu, \epsilon) + \mathcal{O}(m_W/m_t, E_W/Q_{t\bar{t}})$$

[Bärnreuther, Czakon, Fiedler (2013)]

► **massification:**

- it works nicely in the case of  $Wb\bar{b}$ , mainly due to the smallness of the bottom mass (negligible power corrections)
- formally it is valid in the limit  $m_t \ll Q_{t\bar{t}W}$  (which is not true ...)

$$\mathcal{M}_{Wt\bar{t}}(\{p_i\}, p_W; \mu, \epsilon) = Z_{[q]}^{(m_t|0)}(\alpha_s(\mu), m_t/\mu, \epsilon) \mathcal{M}_{Wt\bar{t}}^{(m_t=0)}(\{p_i\}, p_W; \mu, \epsilon) + \mathcal{O}(m_t^2/Q_{Wt\bar{t}}^2)$$

[Abreu et al. (2021)]

how do these approximations perform for  $Wt\bar{t}$ ?

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$$\mathcal{M}_{Wt\bar{t}}(\{p_i\}, p_W; \mu, \epsilon) = \mathcal{M}_{Wt\bar{t}}^{(0)}(\{p_i\}, p_W; \mu, \epsilon) + \mathcal{O}(m_W/m_t, E_W/Q_{t\bar{t}})$$

**we will try to answer this question in the following !!**

▶ **massification:**

- it works nicely in the case of  $wbb$ , mainly due to the smallness of the bottom mass (negligible power corrections)
- formally it is valid in the limit  $m_t \ll Q_{t\bar{t}W}$  (which is not true ...)

$$\mathcal{M}_{Wt\bar{t}}(\{p_i\}, p_W; \mu, \epsilon) = Z_{[q]}^{(m_t|0)}(\alpha_s(\mu), m_t/\mu, \epsilon) \mathcal{M}_{Wt\bar{t}}^{(m_t=0)}(\{p_i\}, p_W; \mu, \epsilon) + \mathcal{O}(m_t^2/Q_{Wt\bar{t}}^2)$$

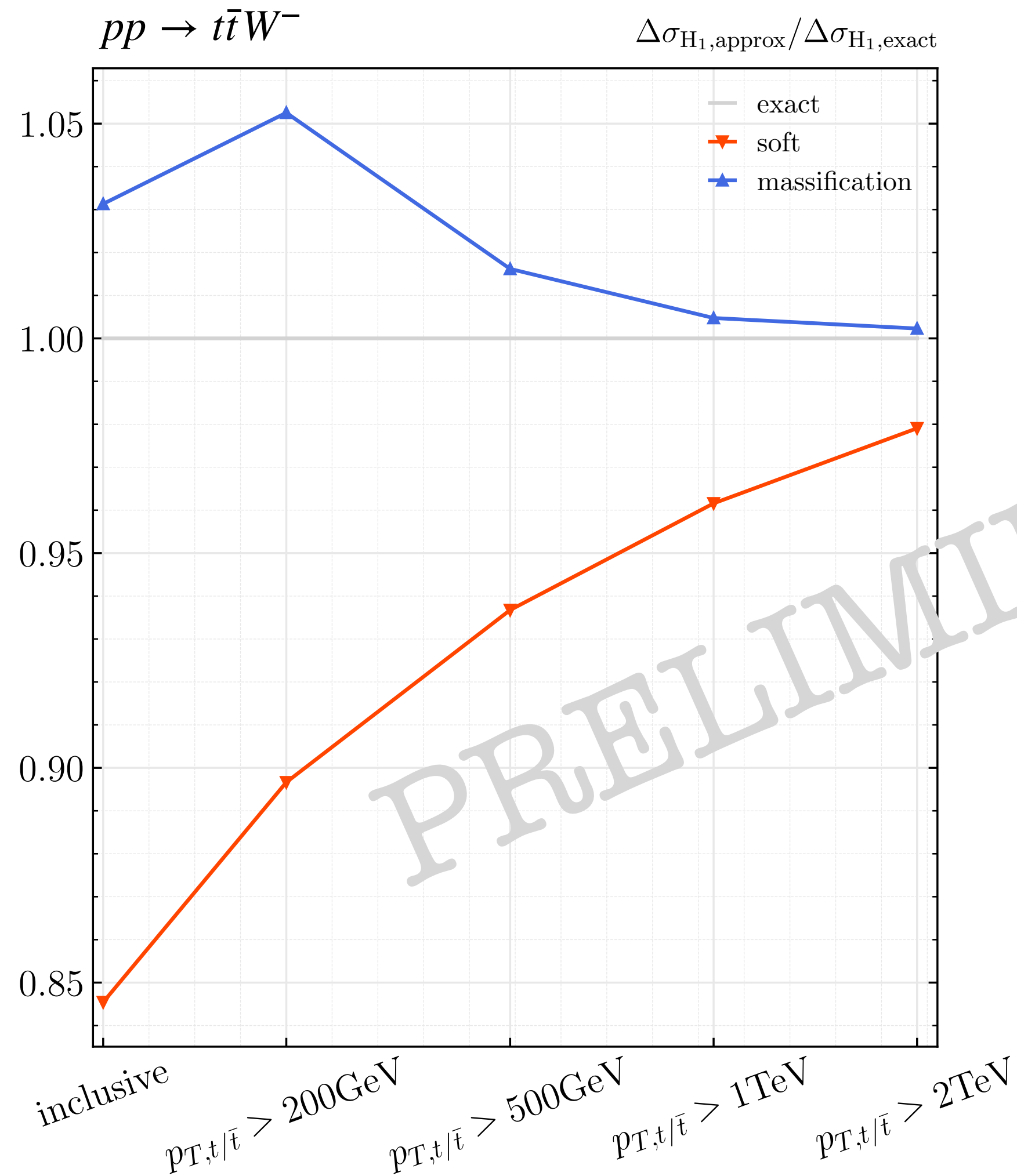
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how do these approximations perform for  $Wt\bar{t}$ ?



# Preliminary results: validation @NLO

setup: NNLO LUXPDF4LHC15,  $\sqrt{s} = 13 \text{ TeV}$ ,  $m_W = 80.385 \text{ GeV}$ ,  $m_t = 173.2 \text{ GeV}$ ,  $\mu_R = \mu_F = (2m_t + m_W)/2$



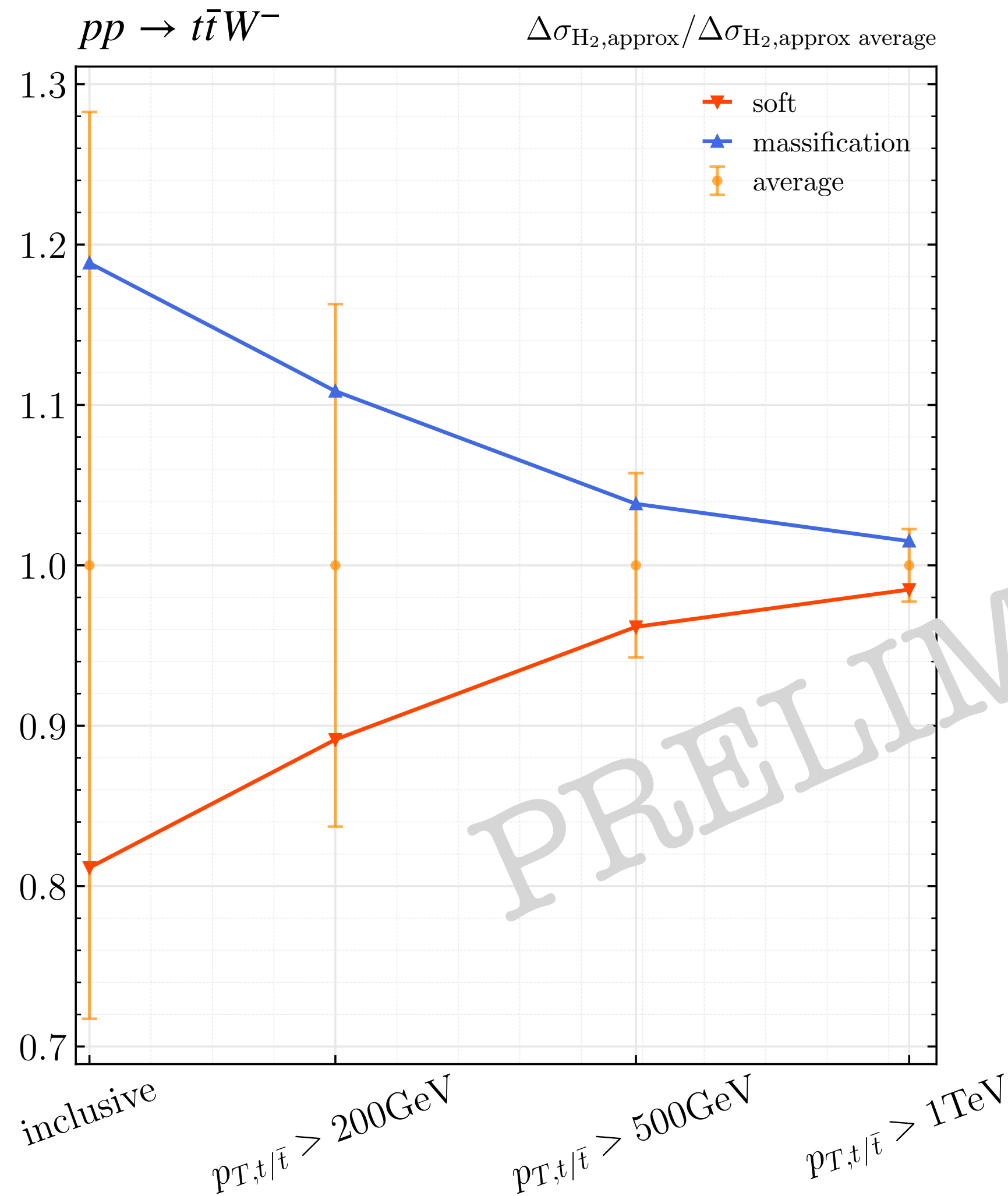
$\Delta\sigma_{\text{NLO}}^{H_1}$ [fb]	exact	soft	massification
inclusive	52.6	$44.47^{+3.5}_{-3.5}$	$54.25^{+4.3}_{-4.4}$
$p_T > 200 \text{ GeV}$	14.745	$13.22^{+0.74}_{-0.73}$	$15.519^{+0.12}_{-0.13}$
$p_T > 500 \text{ GeV}$	1.5162	$1.42^{+0.04}_{-0.04}$	$1.541^{+0.0008}_{-0.0008}$
$p_T > 1000 \text{ GeV}$	0.0823	$0.0792^{+0.08}_{-0.01}$	$0.0826^{+0.00002}_{-0.00006}$

- ▶ both approaches provide a **good quantitative approximation** of the exact virtual coefficient (discrepancy of 5-15%)
- ▶ the soft approximation tends to **undershoot** the exact result while the massification **overshoots** it
- ▶ clear **asymptotic behaviour** towards the exact result for high  $p_{T,t}$  where both approximations are expected to perform better (faster convergence of the massification)



# Preliminary results: performance @NNLO

setup: NNLO LUXPDF4LHC15,  $\sqrt{s} = 13 \text{ TeV}$ ,  $m_W = 80.385 \text{ GeV}$ ,  $m_t = 173.2 \text{ GeV}$ ,  $\mu_R = \mu_F = (2m_t + m_W)/2$



$\Delta\sigma_{\text{NNLO}}^{H_2}$ [fb]	soft	massification
inclusive	$15.5 \pm 4.8$	$22.7 \pm 4.9$
$p_T > 200 \text{ GeV}$	$4.72 \pm 0.97$	$5.87 \pm 0.89$
$p_T > 500 \text{ GeV}$	$0.577 \pm 0.073$	$0.623 \pm 0.055$
$p_T > 1000 \text{ GeV}$	$0.0359 \pm 0.0028$	$0.037 \pm 0.0016$

▶ similar behaviour as @NLO: massified two-loop result systematically higher than the one from soft approximation

▶ **our best prediction:** average of the two approximated results

▶ **conservative uncertainty:** semi-difference multiplied by a tolerance factor 1.5

▶ the two-loop contribution turns out to be **6-7%** of the NNLO cross section (both for  $t\bar{t}W^+$  and  $t\bar{t}W^-$ )

much better control than in  $t\bar{t}H$

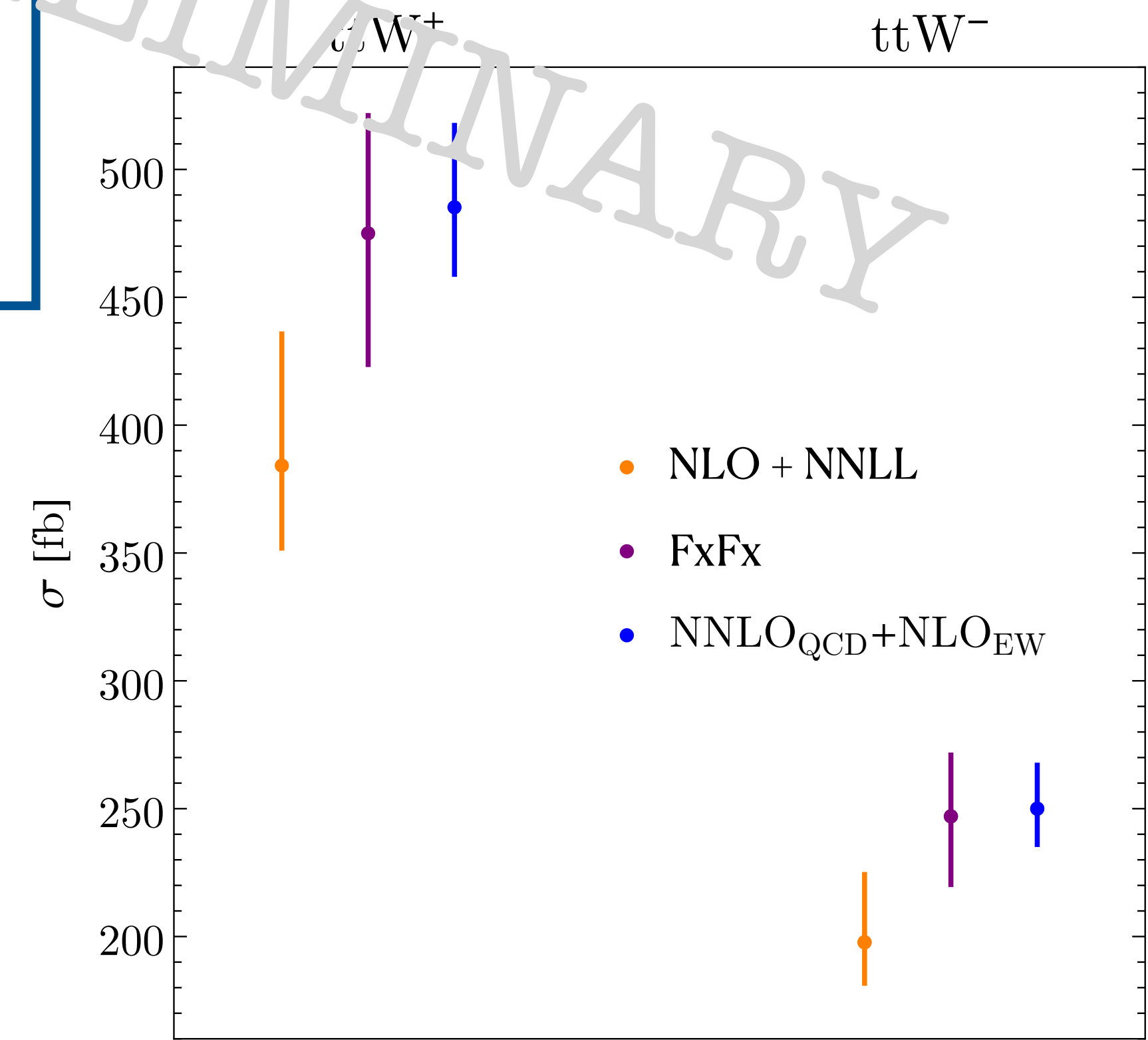
# Preliminary results: total XS

setup: NNLO LUXPDF4LHC15,  $\sqrt{s} = 13 \text{ TeV}$ ,  $m_W = 80.385 \text{ GeV}$ ,  $m_t = 173.2 \text{ GeV}$ ,  $\mu_R = \mu_F = (2m_t + m_W)/2$

order	$\sigma_{t\bar{t}W^+}$ [fb]	$\sigma_{t\bar{t}W^-}$ [fb]	ratio
LO <sub>QCD</sub>	272.4 <sup>+25.1%</sup> <sub>-18.7%</sub>	136.5 <sup>+25.0%</sup> <sub>-18.7%</sub>	1.996
NLO <sub>QCD</sub>	404.4 <sup>+12.8%</sup> <sub>-11.5%</sub>	206.0 <sup>+13.4%</sup> <sub>-11.8%</sub>	1.963
NNLO <sub>QCD</sub>	462.2 <sup>+6.2%</sup> <sub>-4.8%</sub> ± 2.3%	237.0 <sup>+6.7%</sup> <sub>-5.1%</sub> ± 2.5%	1.950
NNLO <sub>QCD</sub> + NLO <sub>EW</sub>	485.2 <sup>+6.6%</sup> <sub>-5.4%</sub> ± 2.2%	250.0 <sup>+6.8%</sup> <sub>-5.6%</sub> ± 2.4%	1.941

systematic uncertainty due to the approximation

- ▶ @NLO QCD: large corrections (+50%)
- ▶ @NNLO QCD: moderate corrections (+15%)
- ▶ inclusion of **all subdominant LO and NLO** contributions ( $\mathcal{O}(\alpha^3)$ ,  $\mathcal{O}(\alpha_s^2\alpha^2)$ ,  $\mathcal{O}(\alpha_s\alpha^3)$ ,  $\mathcal{O}(\alpha^4)$ ) labelled as NLO EW
- ▶ the ratio  $\sigma(t\bar{t}W^+)/\sigma(t\bar{t}W^-)$  is slightly reduced (very stable perturbative behaviour)



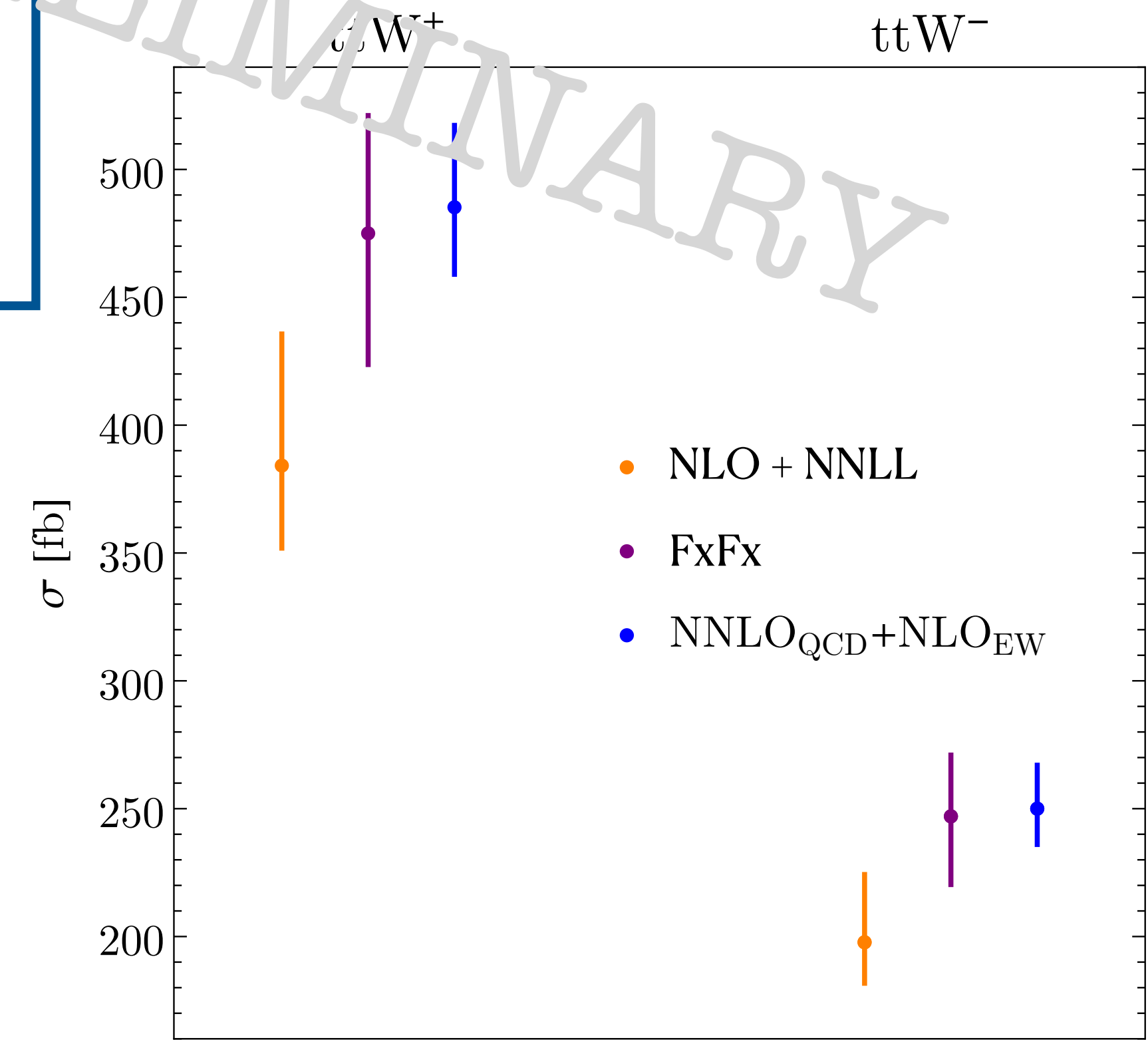
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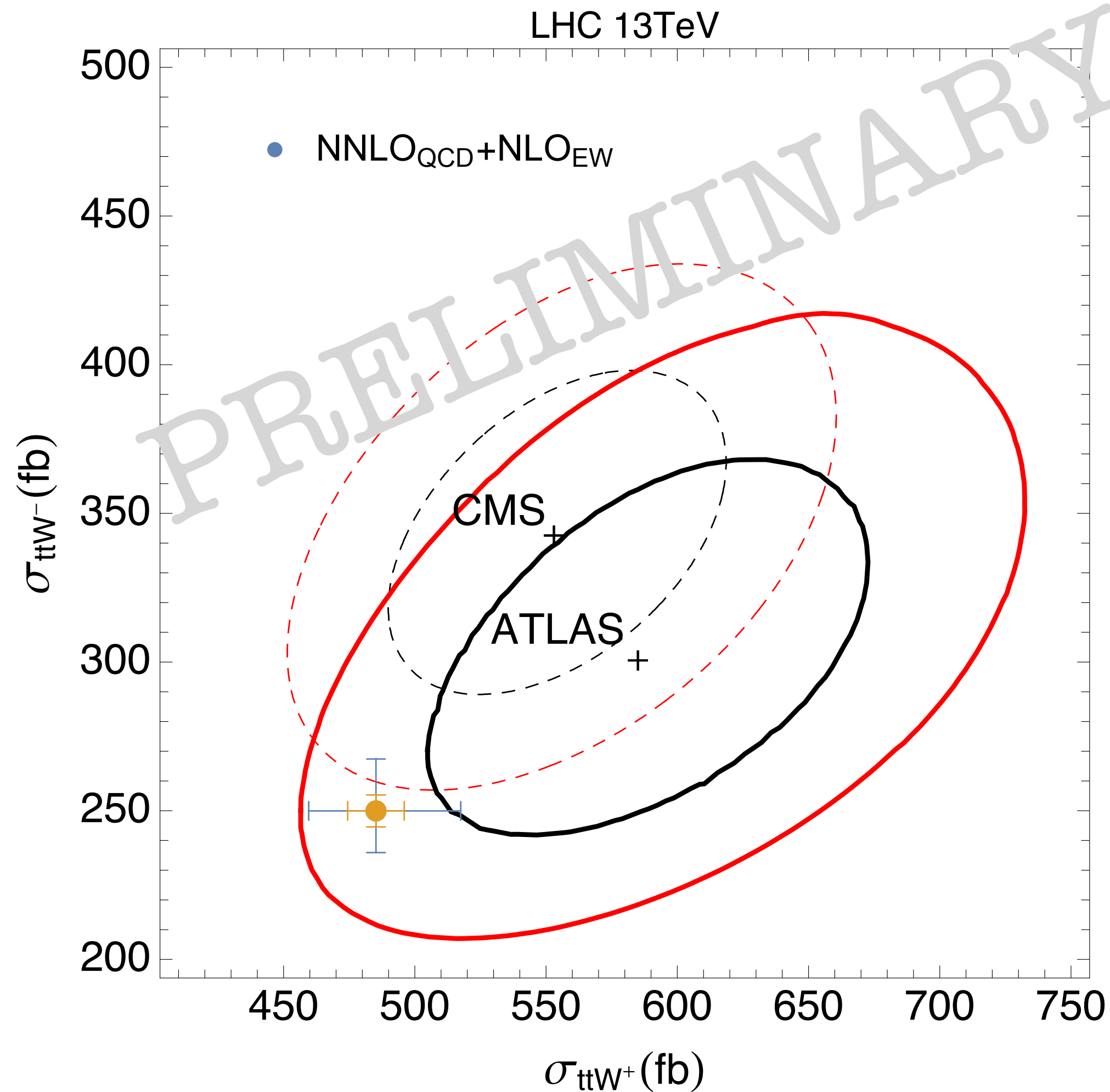
our result is fully compatible with FxFx with smaller perturbative uncertainties !!





# Preliminary results: comparison with data

setup: NNLO LUXPDF4LHC15,  $\sqrt{s} = 13 \text{ TeV}$ ,  $m_W = 80.385 \text{ GeV}$ ,  $m_t = 173.2 \text{ GeV}$ ,  $\mu_R = \mu_F = (2m_t + m_W)/2$



[ATLAS-CONF-2023-019]

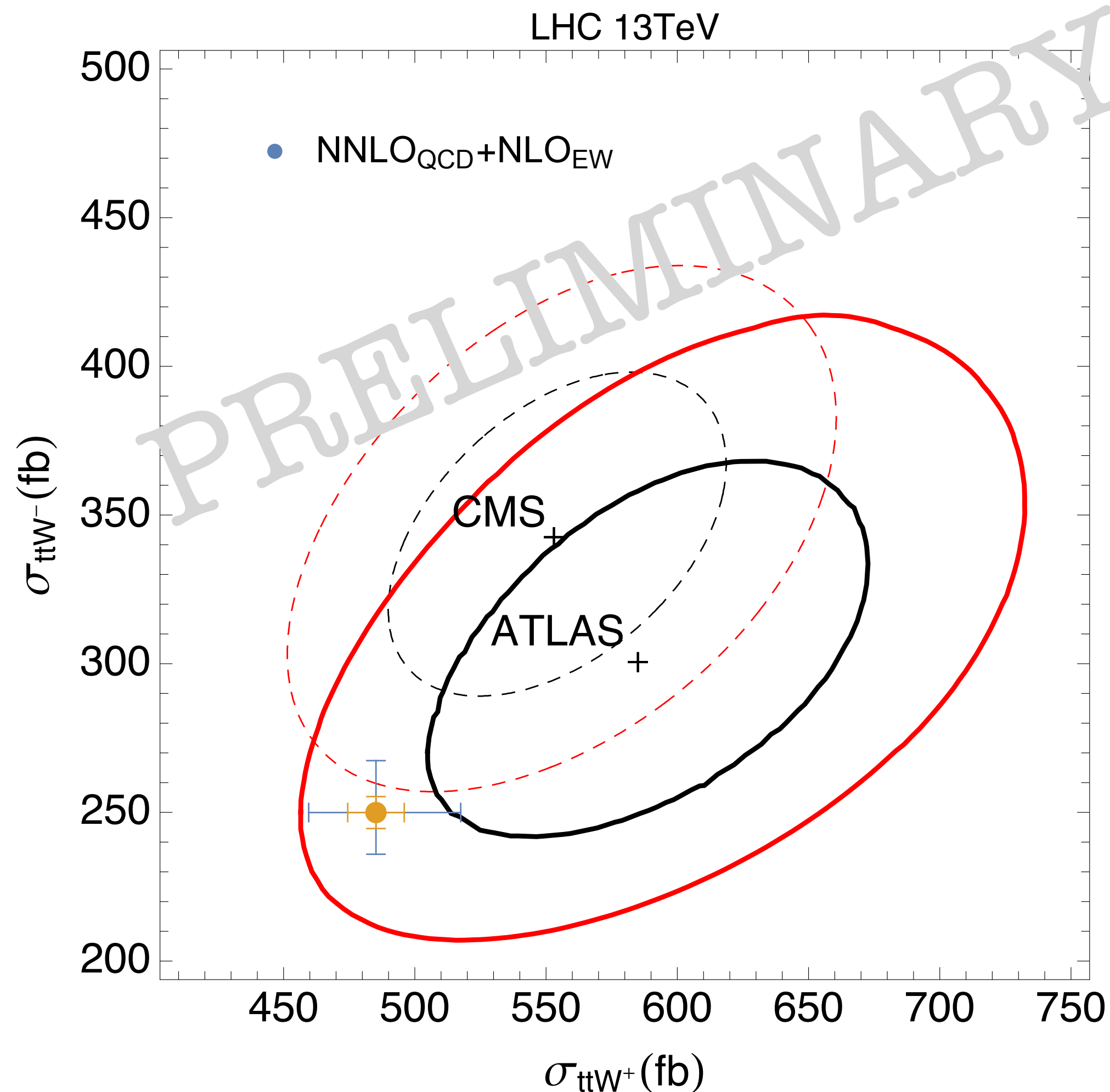
[CMS: arXiv 2208.06485]

- comparison against the most recent ATLAS and CMS data:
  - the **agreement is at the  $1\sigma$  and  $2\sigma$  level** respectively
  - reduction of the perturbative scale uncertainties
  - systematic uncertainties due the two-loop approximation are under control and much smaller than the scale uncertainties



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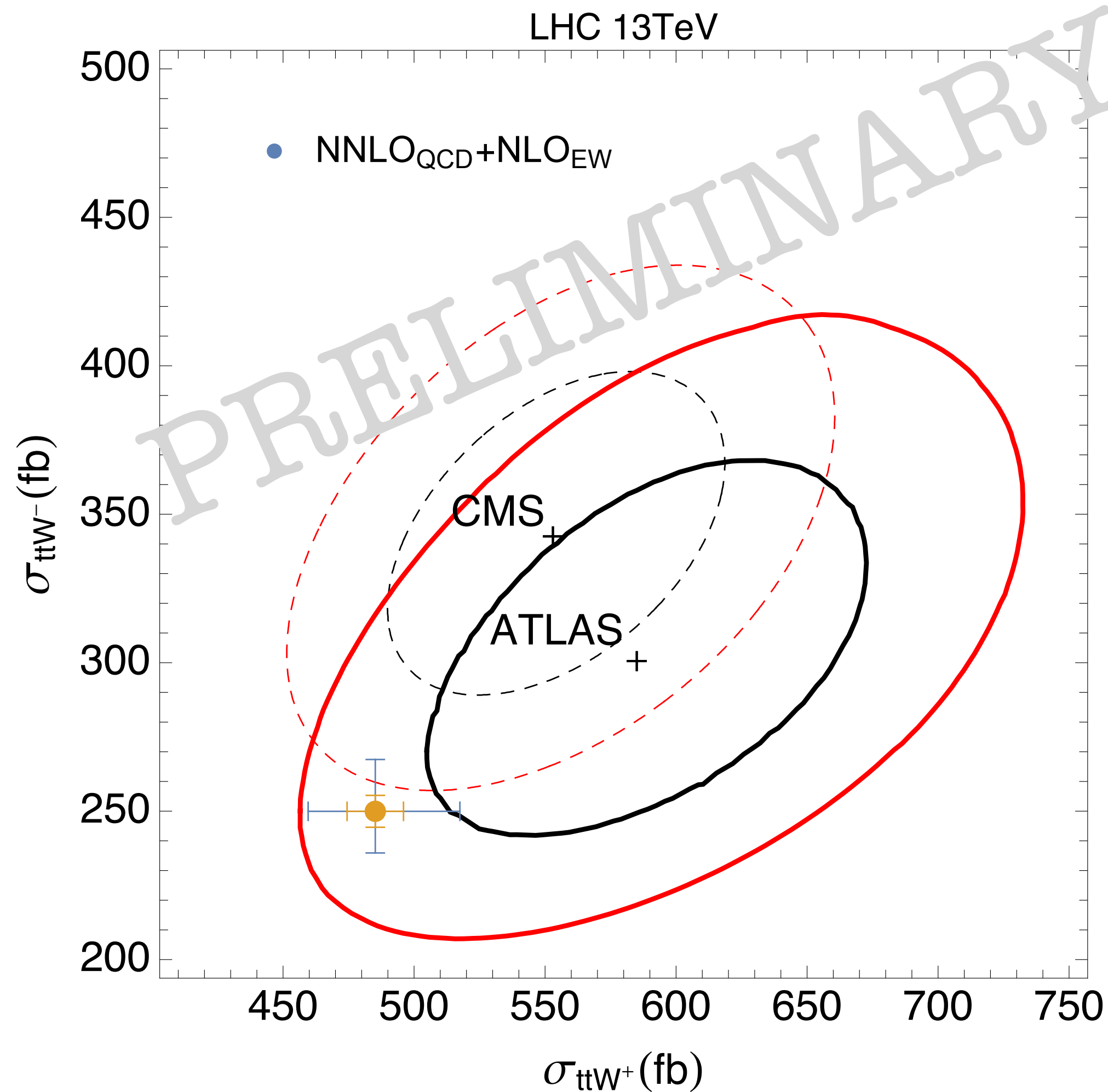
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### take-home message:

two completely different approximations which lead to compatible results for the missing two-loop virtual contribution!!

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STAY TUNED FOR UPDATES in the following weeks !!