Exercise 2: **Maximum Likelihood Fit**

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You are given experimental data from lifetime measurements of a newly created element. Data is stored in a file "decay.txt" and simply consists of N lifetime measurements measured in seconds.

Problem 1

Draw the histogram distribution of the data. Discuss what would be the best choice for the histogram bin size. Explain your answer. What theoretical PDF would you choose to describe the data and why?

Problem 2

Draw the chosen theoretical PDF for different values of the parameter of interest θ and discuss how the shape changes. Calculate the probability to measure value $t = 1$ s or less under one theoretical hypothesis of your choice.

Problem 3

Given the data you have to pick a correct theoretical PDF that describes it well. Don't forget to use your physics knowledge! After you have defined the PDF, use it to derive a Likelihood function for all the measured values of t. Can you find the maximum? What problem you encounter? Just for practice, let's pretend there is only a single measured value $t = 1$ s in the experiment. Draw the likelihood function for that single measurement. Where is the maximum?

Problem 4

Starting from the likelihood function define a Log-Likelihood function. Draw a $-2 \ln L$ function for all the measured values t. Using it calculate the expected value for the mean lifetime and corresponding uncertainties $\hat{\tau} \pm \sigma_{\hat{\tau}}$. Did you perform a binned or unbinned ML fit?

Problem 5*

Perform the maximum likelihood fit three different ways:

- analytically solving the problem of finding the maximum of the log-likelihood function and calculating the uncertainty inverting the correlation matrix ;
- numerically solving the problem of finding the maximum of the log-likelihood curve and determining uncertainties from the lnL curve;
- using already available modules for python.

Compare the 3 results you obtain. Which one is the most correct one to be used for you scientific publication?