Some thoughts on theory uncertainties (3) and tools for electroweak corrections

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Including photon exchange and photon form factor estimate:

(neglecting boxes and s-dependence of Z form factors)

$$A_4 = \frac{\sum_q X_q \, 4\left(\frac{v_\ell}{a_\ell}\frac{v_q}{a_q} + \frac{v_{\ell q}(s)}{a_\ell a_q}\right)}{\sum_q X_q \left(1 + \frac{v_\ell^2}{a_\ell^2} + \frac{v_q^2}{a_q^2} + \frac{v_{\ell q}^2(s)}{a_\ell^2 a_q^2}\right)}$$

$$X_q = f_q(x_1) f_{\bar{q}}(x_2) + f_{\bar{q}}(x_1) f_q(x_2)$$

$$v_{\ell q}(s) = v_\ell v_q + \frac{s - M_Z^2 - i M_Z \Gamma_Z}{s} e^2 \, e_q \, (1 + \overline{\Delta}_q)$$

$$\frac{v_\ell}{a_\ell} = 1 - 4 s_\ell^2, \qquad s_\ell^2 \equiv \sin^2 \theta_{\rm eff}^\ell$$

$$\frac{v_q}{a_q} = 1 - 4|e_q|(s_\ell^2 + \Delta_q)$$

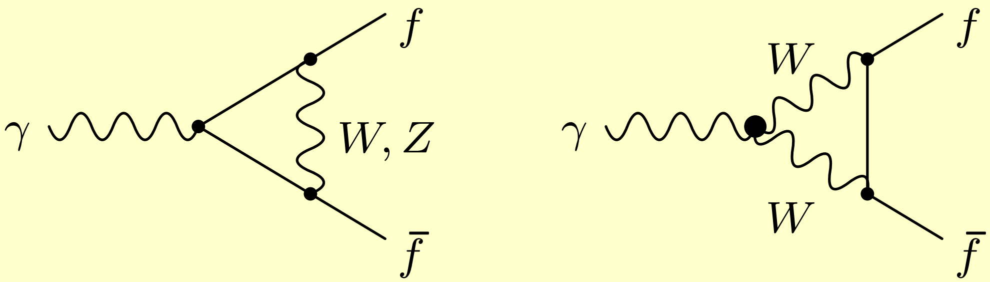
$$\Delta_q = \Delta_{q(1)} + \Delta_{q(2)}$$

$$\Delta_q = \underbrace{\overline{\Delta}_{q(1)}}_{\text{known}} + \underbrace{\overline{\Delta}_{q(2)}}_{\text{unknown}}$$

$\Delta_{q(2)}$ is known (in SM) for leading Z pole term

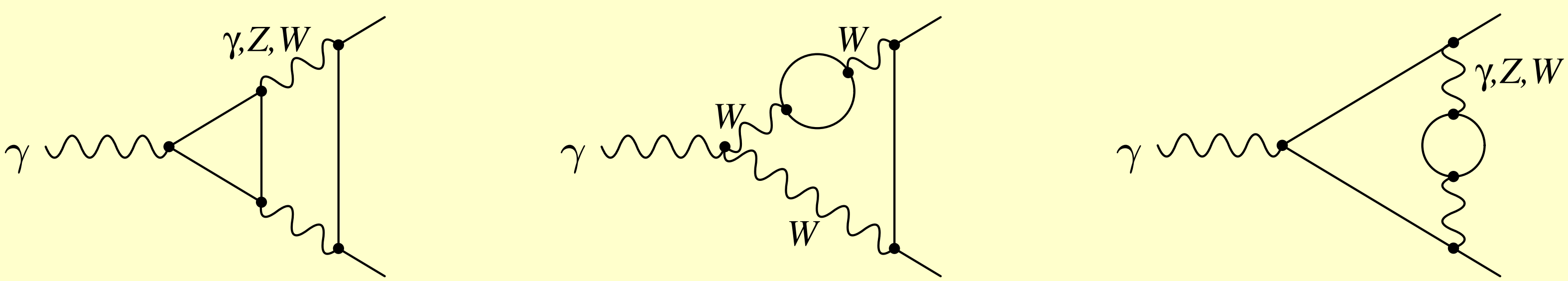
$\overline{\Delta}_{q(2)} = \pm \overline{\Delta}_{q(1)} \times \frac{g^2}{16\pi^2} n_f, \qquad n_f = 6 + 6 N_c$ (maybe underestimate?)

Example contributions to $\Delta_{q(1)}$, $\overline{\Delta}_{q(1)}$:



Note: $\overline{\Delta}_{q(1)}$ also gets contributions from box diagrams and the s-dependence of Z vertex form factors (new: all included now)

Example contributions to $\Delta_{q(2)}$, $\overline{\Delta}_{q(2)}$:



Z-pole 2-loop flavor dependence:

Assume: all EW 2-loop corrections are a source theory uncertainties

Schemes:

- α' : Use α, M_W, M_Z as inputs, perturb. exp. in α
- α : Use α, G_μ, M_Z as inputs, perturb. exp. in α
- G_μ : Use G_μ, M_W, M_Z as inputs, perturb. exp. in G_μ

Scheme:	α'	α	G_μ
$\Delta_{u(\alpha^2)}$ $[10^{-5}]$	-1.74	-1.82	-1.45
$\Delta_{d(\alpha^2)}$ $[10^{-5}]$	-1.49	-1.67	-0.88

Inputs: $M_Z = 91.1876$ GeV, $M_W = 80.385$ GeV, $M_H = 125.7$ GeV

$m_t = 173.5$ GeV, $\Delta\alpha = 0.059$, $\alpha_s = 0.1184$, $G_\mu = 1.16638 \times 10^{-5}$ GeV^{-2}

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Scheme:	α'	α	G_μ
$\Delta_{u(\alpha^2)}$ $[10^{-5}]$	-1.74	-1.82	-1.37
$\Delta_{d(\alpha^2)}$ $[10^{-5}]$	-1.49	-1.67	-0.88
including non-factorizable EW$\times$QCD corrections:			
$\Delta_{u(\alpha^2+\alpha\alpha_s)}$ $[10^{-5}]$	$+1.46$	$+1.38$	$+1.52$
$\Delta_{d(\alpha^2+\alpha\alpha_s)}$ $[10^{-5}]$	$+2.33$	$+2.14$	$+2.46$

Czarnecki, Kühn '96

Harlander, Seidensticker, Steinhauser '97

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$m_t = 173.5$ GeV, $\Delta\alpha = 0.059$, $\alpha_s = 0.1184$, $G_\mu = 1.16638 \times 10^{-5}$ GeV^{-2}

Combine $\Delta_{q(2)}$ numbers with $\overline{\Delta}_{q(2)}$ estimate as sources of th. unc.

Impact of missing EW 2-loop contributions (including EW$\times$QCD):

$\delta A_4/A_4$: [10^{-3}]

$m_{\ell\ell}$ [GeV]	Scheme:	α'	α	G_μ
60		4.2	1.44	1.24
70		2.1	0.80	0.65
80		9.9	3.1	3.02
M_Z-2		38.6	17.1	12.9
M_Z-1		6.8	3.1	2.5
M_Z		0.41	0.43	0.43
M_Z+2		2.4	0.68	0.56
M_Z+1		3.9	1.3	1.1
100		6.3	2.3	1.9
110		5.5	2.0	1.6
130		2.9	1.0	0.80
150		1.1	0.33	0.23

- Dominated by photon form factor unc. $\overline{\Delta}_q$
- New: Error estimate for A_4 is larger than what I showed before (due to including all NLO contributions)
- New: Schemes that use G_μ have smaller corrections/ uncertainties

New: only EW 2-loop corrections beyond $\Delta\rho$ are a source of th. unc.

Impact of missing EW 2-loop contributions (including EW$\times$QCD):

$\delta A_4/A_4$: [10^{-3}]

$m_{\ell\ell}$ [GeV]	Scheme:	α'	α	G_μ
60		3.0	1.8	1.76
70		1.5	0.89	0.89
80		7.1	4.3	4.25
M_Z-2		27.6	19.2	17.2
M_Z-1		5.0	3.3	3.2
M_Z		0.42	0.43	0.43
M_Z+2		1.6	0.92	0.89
M_Z+1		2.7	1.6	1.6
100		4.5	2.7	2.7
110		3.9	2.4	2.3
130		2.0	1.2	1.2
150		0.75	0.38	0.37

- Error estimate for α' scheme reduced by $\sim$30%
- Error estimate for other schemes increased (probably coincidence – limitation of method)

Comments and discussion points

- New: Dependence of form factors on $s = m_{\ell\ell}$ and box contributions are included, resulting in larger error estimate

- New: Error estimate is not systematically reduced by including h.o. $\Delta\rho$ corrections

- Coherent treatment of $\mathcal{O}(\alpha\alpha_\mathsf{s})$ for full process $q\bar{q} \to \ell^+\ell^-$ missing: include in analysis, or use available results for error estimate?
  [should be added in quadr. to $\mathcal{O}(\alpha^2)$ estimate]

- **Note:** Evaluation carried out with GRIFFIN library Chen, Freitas '22

Motivation:

- Exisiting tools (ZFITTER/DIZET, TOPAZ0, ...) developed for LEP era contain many SM resuls, including QED radiation
  Bardin et al. '99
  Montagna et al. '98
- Difficult to expand and maintain
  (Fortran77, not fully gauge-invariant framework, ...)
- QED more effectively handled with MC generators
- For future applications / colliders: need EW library that is ...
  - ... modular / object-oriented
  - ... based on formally gauge-invariant setup
  - ... can be extended to include BSM phsyics (also SMEFT), new processes, etc.

Setup

Framework for $f\bar{f} \to Z^*/\gamma* \to f'\bar{f}'$:

- Laurent expansion about Z-pole + regular matrix element off-resonance

$$M_{ij} = M_{ij}^{\text{exp},s_0} + M_{ij}^{\text{noexp}} - M_{ij}^{\text{exp},M_Z^2},$$

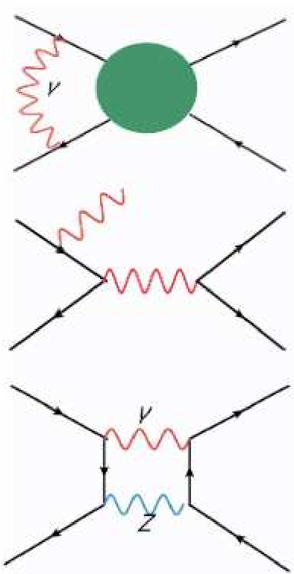
@NLO (for M_{ij}^{noexp} and $M_{ij}^{\text{exp},M_Z^2}$); avoid double counting

$$M_{ij}^{\text{exp},s_0} = \frac{R_{ij}}{s - s_0} + S_{ij} + (s - s_0)S'_{ij} + \ldots \qquad s_0 \equiv M_Z^2 - iM_Z\Gamma_Z$$

@NNLO (for R_{ij}) @NLO (for S_{ij})

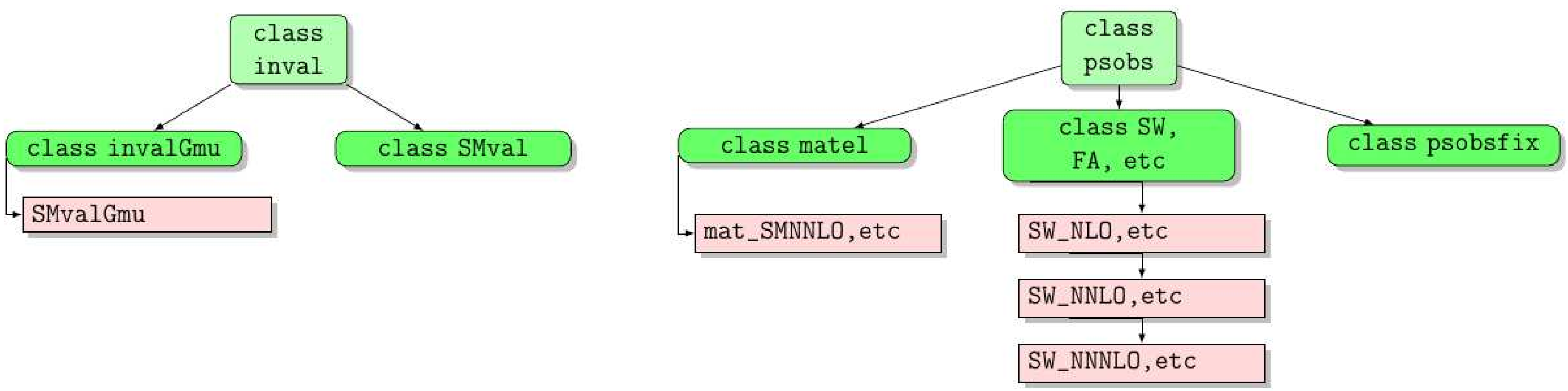
Stuart '91; Veltman '94

- QED contributions have been subtracted



Structure of the library

`class inval`		`class psobs`	
input parameters (in the SM)		output observables	
Boson masses and widths	$M_{\mathrm{W,Z,H}}$ $\Gamma_{\mathrm{W,Z}}$	pesudo-observables defined at Z-peak	$F_{V,A}$, $\sin^2\theta^f_{eff}$ $\Gamma_{Z\to f\bar{f}}$, Δr, etc.
Fermion masses	$m^{\mathrm{OS}}_{\mathrm{e},\mu,\tau}$ $m^{\overline{\mathrm{MS}}}_{\mathrm{d,u,s,c}}(M_{\mathrm{Z}})$ $m^{\mathrm{OS}}_{\mathrm{t}}$	amplitude coefficients under pole scheme	R , S , and S'
Couplings	$\alpha(0)$ $\Delta\alpha \equiv 1-\alpha(0)/\alpha(M^2_{\mathrm{Z}})$ $\alpha^{\overline{\mathrm{MS}}}_s(M^2_{\mathrm{Z}})$, G_μ	(polarized) matrix element square near Z-peak	$\mathrm{Re}\, M_{ij}M^*_{kl}$



Implementation of higher-order corrections

Corrections entering through $\delta\rho$:			
	`drho2aas`	$\mathcal{O}(\alpha_t\alpha_s)$	[3, 4]
	`drho2a2`	$\mathcal{O}(\alpha_t^2)$	[5–9]
*	`drho3aas2`	$\mathcal{O}(\alpha_t\alpha_s^2)$	[10, 11]
*	`drho3a2as`	$\mathcal{O}(\alpha_t^2\alpha_s)$	[12, 13]
*	`drho3a3`	$\mathcal{O}(\alpha_t^3)$	[12, 13]
*	`drho4aas3`	$\mathcal{O}(\alpha_t\alpha_s^3)$	[14–16]
Full corrections to F_A^f, $\sin^2\theta_{\rm eff}^f$:			
*	`res2ff`	$\mathcal{O}(\alpha_f^2)$	[17–19]
*	`res2fb`	$\mathcal{O}(\alpha_f\alpha_b)$	[17–20]
*	`res2bb`	$\mathcal{O}(\alpha_b^2)$	[21–25]
*	`res2aas`	$\mathcal{O}(\alpha\alpha_s)$	[26–28] (correction to internal gauge-boson self-energies)
*	`res2aasnf`	$\mathcal{O}(\alpha\alpha_s)$	[29–34] (non-factorizable final-state corrections for $f = q$)
*	`res3fff`	$\mathcal{O}(\alpha_f^3)$	[35]
*	`res3ffa2as`	$\mathcal{O}(\alpha_f^2\alpha_s)$	[36]

Sample program


```
#include <iostream>
using namespace std;

#include "EWPOZ2.h"
#include "xscnnlo.h"
#include "SMval.h"

int main()
{
  SMval myinput;  // convert masses from PDG values to complex pole scheme
  myinput.set(al, 1/137.03599976);
  myinput.set(MZ, 91.1876);
  myinput.set(MW, 80.377);
  myinput.set(GamZ, 2.4952);
  myinput.set(GamW, 2.085);
  myinput.set(MH, 125.1);
  myinput.set(MT, 172.5);
  myinput.set(MB, 2.87);
  myinput.set(Delal, 0.059);
  myinput.set(als, 0.1179);

  cout << endl << "Complex-pole masses: MW=" << myinput.get(MWc) << ", MZ="
    << myinput.get(MZc) << endl << endl;
```

Sample program (2)


```
// compute matrix element for ee->dd with vector coupling in initial
// state and vector coupling in final state
int ini = ELE, fin = DQU, iff = VEC, off = VEC;

cout << "=== Matrix element for ee->dd (i=e, f=d) ===" << endl << endl;

// compute vertex form factors:
FA_SMNNLO FAi(ini, myinput), FAf(fin, myinput);
SW_SMNNLO SWi(ini, myinput), SWf(fin, myinput);
cout << "F_A^i (NNLO+) = " << FAi.result() << endl;
cout << "F_A^f (NNLO+) = " << FAf.result() << endl;
cout << "sineff^i (NNLO+) = " << SWi.result() << endl;
cout << "sineff^f (NNLO+) = " << SWf.result() << endl;
cout << endl;
```

Sample program (3)


```
  double cme,         // center-of-mass energy
         cost = 0.5; // scattering angle
  Cplx res1, res2;

  cout << "SM matrix element M_VV for cos(theta)=" << cost << ": " << endl;
  // compute matrix element for ee->dd using SM form factors:
  mat_SMNNLO M(ini, fin, iff, off, FAi, FAf, SWi, SWf, cme*cme, cost,
   myinput);
  cout << "sqrt(s)\t\ttot. result\t\toff-resonance contrib." << endl;
  for(cme = 10.; cme <= 190.; cme += 20.)
  {
    M.setkinvar(cme*cme, cost);
    res1 = M.result();
    res2 = M.resoffZ();
    cout << cme << " \t" << res1 << " \t" << res2 << endl;
  }
  cout << endl;

  return 0;
}
```

Sample program (output)


```
Complex-pole masses: MW=80.35, MZ=91.1535

=== Matrix element for ee->dd (i=e, f=d) ===

F_A^i (NNLO+) = (0.034499,0)
F_A^f (NNLO+) = (0.0345443,0)
sineff^i (NNLO+) = (0.231172,0)
sineff^f (NNLO+) = (0.230985,0)

SM matrix element M_VV for cos(theta)=0.5:
sqrt(s)         tot. result                   off-resonance contrib.
10       (0.000316739,-5.58082e-06)      (0.000309429,-5.53734e-06)
30       (3.53793e-05,-5.99317e-07)      (2.84458e-05,-5.59139e-07)
50       (1.25851e-05,-1.90789e-07)      (6.4247e-06,-1.59184e-07)
70       (6.07798e-06,-5.97311e-08)      (1.19433e-06,-4.81728e-08)
90       (-7.31188e-07,-3.55673e-06)     (8.7104e-09,-1.80673e-09)
110      (3.14635e-06,-1.62001e-07)      (4.59289e-07,1.10821e-08)
130      (2.12596e-06,-7.90095e-08)      (1.82894e-06,1.92144e-08)
150      (1.5668e-06,-5.34561e-08)       (3.83515e-06,2.49419e-08)
170      (1.20884e-06,-3.97403e-08)      (6.35319e-06,2.97998e-08)
190      (9.60973e-07,-3.33532e-08)      (9.31833e-06,3.12732e-08)
```

Comparison GRIFFIN 1.0 vs. DIZET 6.45

Numerical Results:

$$|\rho_Z^f| = \frac{2\sqrt{2}F_A^f}{G_\mu M_Z^2}$$

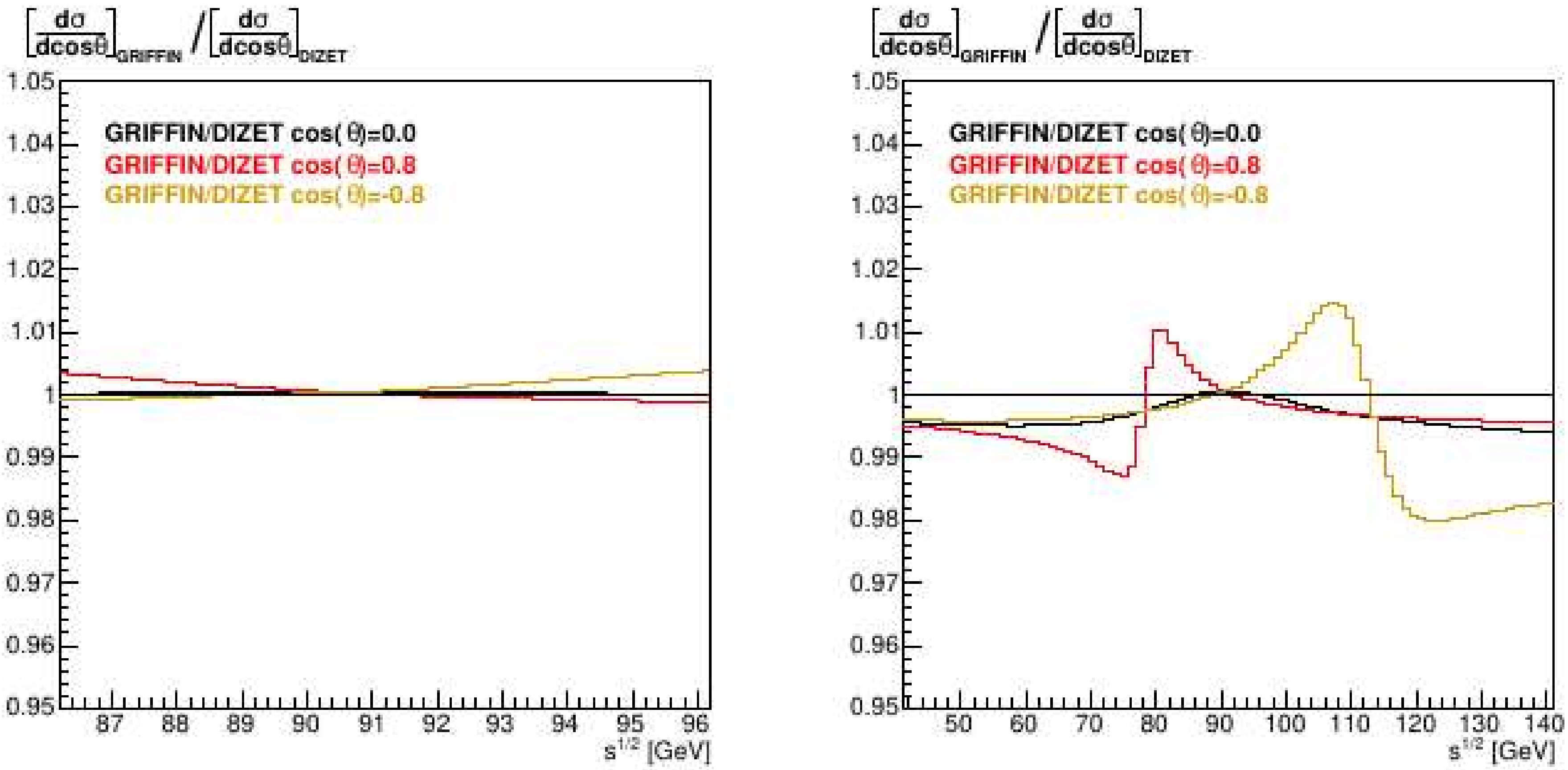
	$\lvert\rho_Z^f\rvert$		$\sin^2\theta_{\rm eff}^f$		$\Gamma_{Z\to f\bar{f}}$	
	DIZET 6.45	GRIFFIN	DIZET 6.45	GRIFFIN	DIZET 6.45	GRIFFIN
$\nu\bar{\nu}$	1.00800	1.00814	0.231119	NAN	0.167206	0.167197
$\ell\bar{\ell}$	1.00510	1.00519	0.231500	0.231534	0.083986	0.083975
$u\bar{u}$	1.00578	1.00573	0.231393	0.231420	0.299938	0.299958
$d\bar{d}$	1.00675	1.00651	0.231266	0.231309	0.382877	0.382846
$b\bar{b}$	0.99692	0.99420	0.232737	0.23292	0.376853	0.377432

	DIZET 6.45	GRIFFIN all orders	GRIFFIN $\mathcal{O}(\alpha, \alpha^2, \alpha_t\alpha_s, \alpha_t\alpha_s^2)$
Δr	3.63947×10^{-2}	3.68836×10^{-2}	3.63987×10^{-2}

- Not a **one-one-one match.** (no leading N3LO implemented in dizet v.6.45)
- most numbers are in agreement up to at least **4-digit**. The actual discrepancy is in the realm of missing N3(4)LO.
- fictitious discrepancies stem from the input scheme/definition of the form factors/EWPOs.

Comparison GRIFFIN 1.0 vs. DIZET 6.45

Ratios of differential cross-sections for $e^+e^- \to \mu^+\mu^-$ for different θ:



- $\lesssim \mathcal{O}(10^{-3})$ agreement near Z-pole (∼NNLO precision)
- %-level agreement away from Z pole (NLO prec., different implementations)

[Note: enhanced corrections when tree-level matrix element is small]