On the determination of the W mass at hadron colliders

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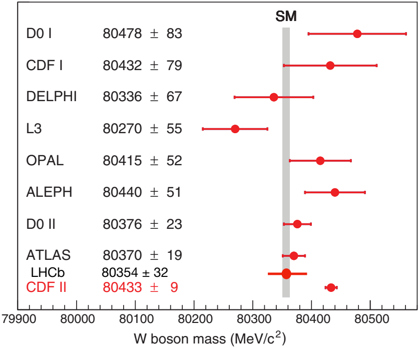
Based on: Rottoli, PT, Vicini, hep-ph/2301.04059

Role of M_W



- M_W fundamental SM parameter, important input to global EW fits.
- Quantum corrections to M_W sensitive to $M_{\rm top}$, M_H, allowing stringent consistency tests on the SM.
- Heading for 10^{-4} relative accuracy on its determination.
- High time to assess accuracy of tools and methodologies employed for M_W extraction from experimental data.

M_W determination at hadron colliders

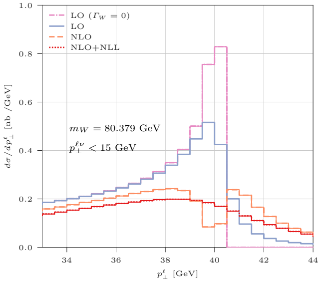
- In charged-current DY (CCDY), neutrino reconstruction possible in the transverse plane: p_t^ν inferred from missing $\not{E}_t$.
- Define relevant transverse observables: p_t^ℓ, and $M_t^{\ell\nu} = \sqrt{2\, p_t^\ell\, p_t^\nu (1 - \cos\Delta\phi^{\ell\nu})}$.
- p_t^ℓ and $M_t^{\ell\nu}$ spectra display a kinematical jacobian peak whose position is related to M_W: e.g.

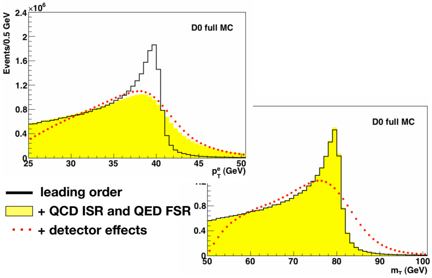
$$\frac{d\sigma}{dp_t^\ell} \propto \frac{1}{\sqrt{1-(2\,p_t^\ell/\hat{s})^2}} \sim \frac{1}{\sqrt{1-(2\,p_t^\ell/M_W)^2}}$$

- Enhanced sensitivity to M_W variations: $\Delta M_W/M_W \sim 10^{-4}$ modifies p_t^ℓ at the 10^{-3} level.

Jacobian-peak description

- Description of the jacobian-peak shape is sensitive to a variety of theoretical and experimental effects.
- Soft radiation causes integrable singularity (Sudakov shoulder [Catani,Webber '97]) in the fixed-order description beyond LO.
- QCD resummation required.

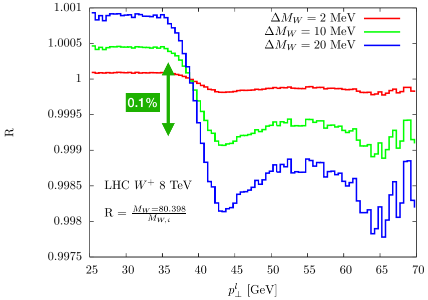




- QCD ISR + QED FSR dominate smearing of the jacobian peak in p_t^ℓ: need for excellent theoretical predictions.
- Detector effects dominate smearing of the jacobian peak in $M_t^{\ell\nu}$: need excellent experimental control on neutrino reconstruction, challenging at the LHC.

Standard extraction of M_W from jacobian-peak shape: template fitting

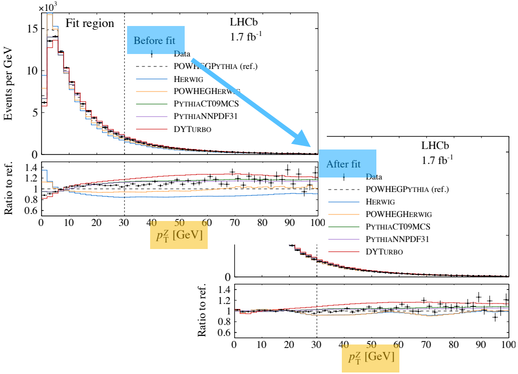
- Compute theoretical distributions for p_t^ℓ (or $M_t^{\ell\nu}$) with different hypotheses $M_{W,i}$ for the W mass (template distributions).
- Compare templates with experimental measurements in a given fit window; calculate χ_i^2 for each $M_{W,i}$ hypothesis.
- Extract M_W as the $M_{W,i}$ hypothesis associated to the smallest χ_i^2.



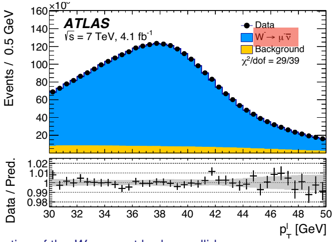
- Theory prediction must be sufficiently close to experimental data to get reasonable minimum χ^2.
- Need to control shapes at the permille level to resolve $\Delta M_W/M_W \sim 10^{-4}$.
- But: even state-of-the-art N^{3}LO+N^{3}LL QCD predictions for p_t^ℓ have uncertainties at the percent level (see e.g. [Chen, Gehrmann, Glover, Huss, Monni, Re, Rottoli, PT, 2203.01565]).

The role of tuning in template fits

- Procedure can be restored leveraging high-precision p_t^Z data in neutral-current Drell Yan (NCDY).
- Leaving formal accuracy aside, flexible enough parton shower can be tuned to perfectly describe NCDY data.
- Tuning performed on the parameters of a non-perturbative (NP) QCD model.



- Same tuning parameters are used to prepare CCDY template distributions.
- After tuning, χ^2 of template distributions under control.



Potential concern about template fitting

Conceptual

- Heavily reliant on tuning to data. Dominated by NP physics, the least understood.
- Potential BSM effects absorbed in the tuning parameters.

Practical

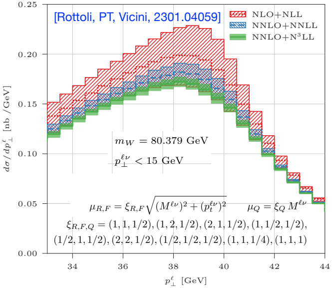
- Based on tools with low formal accuracy, typically NLO+(N)LL: higher-order perturbative information mimicked by NP tuning parameters.
  Significant progress in perturbative understanding of the DY process not exploited.
- The definition of χ^2 does not include theoretical uncertainties, owing to non-statistical nature of scale variations.
- Tuning does not include theoretical uncertainties: one should assess how tuning parameters depend on scale choices, and propagate the dependence to CCDY.
- Assumes universality of NP model: parameters extracted from NCDY applied to CCDY.
- Does not assess uncertainty on information transfer from NCDY p_t^Z to CCDY p_t^ℓ (or $M_t^{\ell\nu}$).

Robust assessment of theoretical uncertainties is lost in data-driven approach.

A new observable for M_W determination

p_t^ℓ distribution in CCDY

- p_t^ℓ spectrum at $\pm 2\%$ accuracy (9-point scale variation) using state-of-the-art QCD predictions. N^{3}LL from RadISH [Bizon, Monni, Re, Rottoli, PT, '17,'18,'19,'21], NNLO from MCFM [Campbell, Neumann, '19].
- Including resummation cures integrable singularity: physical description of the jacobian peak.
- Peak position shifted from $M_W/2$ to $\sim$ 38.5 GeV by QCD resummation and Γ_W effects.



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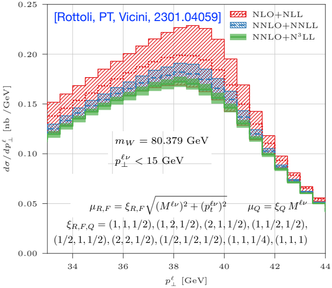
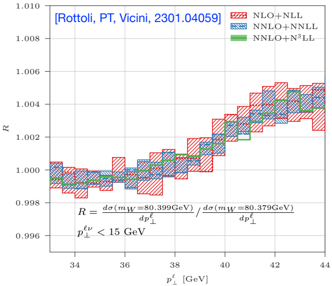


- Ratio of p_t^ℓ spectra with different M_W hypotheses is largely independent of QCD approximation (for resummed predictions). Mild dependence only in uncertainty band.
- Sensitivity to M_W variations stems from W propagation and decay, factorised from QCD ISR.
- Sensitivity to $\Delta M_W/M_W \sim 10^{-4}$ well resolvable beyond theoretical-uncertainty band.



Covariance matrix w.r.t. M_W variations (I)

- Given N bins σ_i around the p_t^ℓ jacobian peak, their sensitivity to M_W can be quantified by constructing the covariance matrix

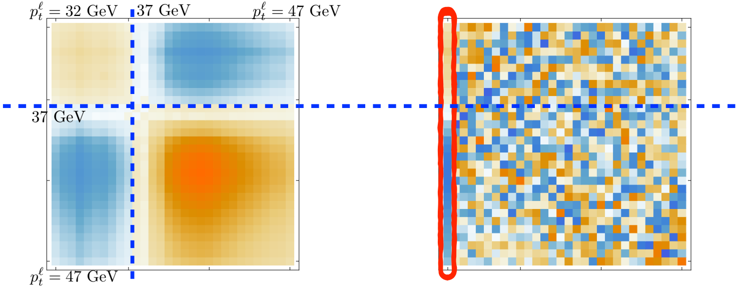
$$\mathcal{C}_{ij}^{(M_W)} = \langle \sigma_i \, \sigma_j \rangle - \langle \sigma_i \rangle \langle \sigma_j \rangle \,, \qquad \langle x \rangle = \frac{1}{p} \sum_{k=1}^{p} x_{(k)} \,,$$

  where p is the number of M_W hypotheses considered.

- Diagonalisation of $\mathcal{C}^{(M_W)}$ gives N orthogonal p_t^ℓ-bin combinations (eigenvectors).

- Corresponding eigenvalues represent the sensitivity of eigenvectors to M_W variations.

Covariance matrix w.r.t. M_W variations (II)



Covariance @ NNLO+N3LL central scales

Covariance eigenvectors

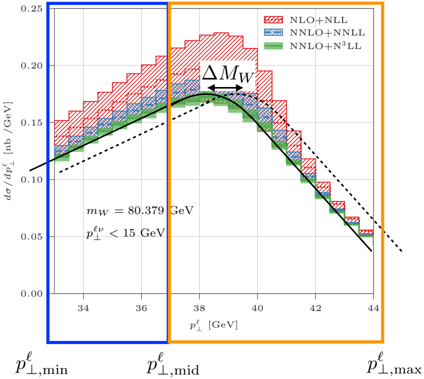
- First eigenvalue dominates ($\sim$ 99%): bulk of M_W sensitivity captured by a single bin combination.
- Stemming from the fact that the dominant effect of ΔM_W is a rigid shift of the spectrum by $\Delta M_W/2$.
- Coefficients of the dominant eigenvector change sign around $p_t^\ell \sim 37$ GeV.
- Define a simple (theoretically and experimentally) observable mimicking dominant covariance eigenvector: jacobian asymmetry.

The jacobian asymmetrty $\mathcal{A}_{p_t^\ell}$

$$\mathcal{A}(p^\ell_{\perp,\min}, p^\ell_{\perp,\mathrm{mid}}, p^\ell_{\perp,\max}) \equiv \frac{L-U}{L+U}$$

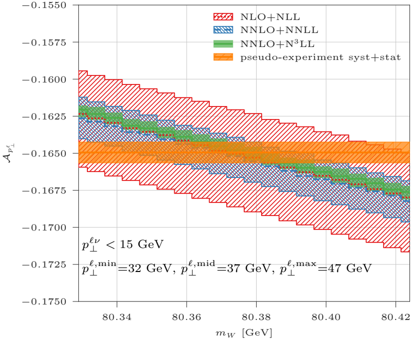
$$L \equiv \int_{p^\ell_{\perp,\min}}^{p^\ell_{\perp,\mathrm{mid}}} dp_t^\ell \, \frac{d\sigma}{dp_t^\ell}$$

$$U \equiv \int_{p^\ell_{\perp,\mathrm{mid}}}^{p^\ell_{\perp,\max}} dp_t^\ell \, \frac{d\sigma}{dp_t^\ell}$$



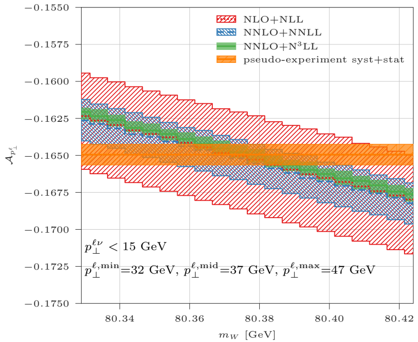
- Simple observable constructed as combination of fiducial rates in relatively wide p_t^ℓ bins.
- A single scalar number depending only on the bin edges, measurable via counting.
- At fixed bin edges, $+\Delta M_W$ shifts p_t^ℓ spectrum by $\sim +\Delta M_W/2$, depleting L and populating U: asymmetry decreases linearly if $p^\ell_{\perp,\mathrm{mid}}$ is at the left of the peak.

$\mathcal{A}_{p_t^\ell}$ sensitivity to M_W (I)



- Linear sensitivity to M_W stems from linear dependence of peak position.
- Sensitivity (slope) largely independent of QCD approximation/scale: reflecting factorisation of QCD production from M_W-sensitive propagation/decay.
- Expected to carry over to NP QCD.
- Slope related to the magnitude of the first covariance eigenvalue for the considered range.
- Slope depends on the value of the bin edges $[\,p^\ell_{\perp,\min}, p^\ell_{\perp,\mathrm{mid}}, p^\ell_{\perp,\max}\,]$.

$\mathcal{A}_{p_t^\ell}$ sensitivity to M_W (II)



$\mathcal{A}_{p_t^\ell}$ = combination of fiducial rates.

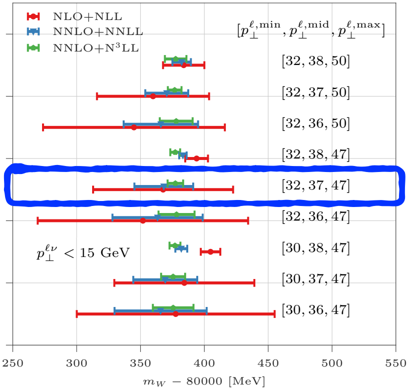
- Excellent perturbative QCD convergence.
- Importance of higher-order results for high-accuracy prediction.

$\mathcal{A}_{p_t^\ell}$ = based on wide p_t^ℓ bins $\mathcal{O}$(5-10 GeV).

- Small statistical/systematic errors.
- Viability to unfold detector effects: combination of different experimental M_W determinations.

- M_W simply extracted as the intersection of theoretical and experimental lines.
- $\Delta M_W \sim \pm 15$ MeV from asymmetry measurement seems feasible experimentally. Experimental error band obtained assuming 0.1% error on the measurement of L and U and no correlation. Statistical error $\sim$ 10 times smaller already with $\mathcal{L} = 140$ fb^{-1}.

$\mathcal{A}_{p_t^\ell}$ dependence on p_t^ℓ bin edges



- Perturbative convergence generally very well behaved.
- Importance of including N^3LL to assess quality of perturbative convergence. Perturbative stability checked beyond mere scale variation.
- Some trade-off between sensitivity (improving at higher $p^\ell_{\perp,\mathrm{mid}}$) and perturbative convergence (improving at lower $p^\ell_{\perp,\mathrm{mid}}$).
- $\Delta M_W \sim \pm 5$ MeV achievable from perturbative QCD based on CCDY alone.

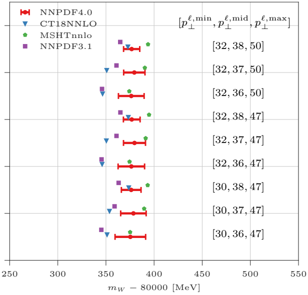
Including further effects

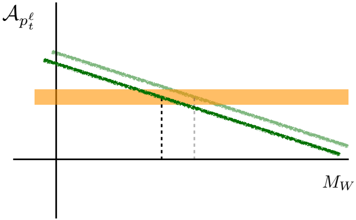
So far only dealt with perturbative QCD.

This is the starting point for a complete classification and quantitative assessment of all effects sensitive to M_W variations.

- Impact of PDFs and profiling.
- Impact of NP QCD modelling.
- QED and mixed QCD-EW perturbative corrections.
- Systematic covariance studies: beyond asymmetry.

Effect on asymmetry from PDF choice

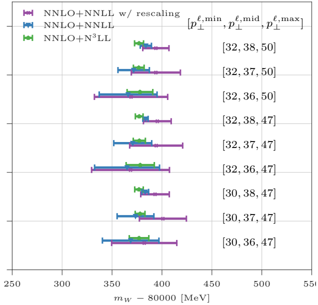


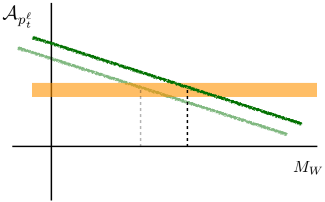


- Variations from 100 NNPDF4.0 NNLO replicas on NLO+NLL result: $\Delta M_W \sim \pm 12$ MeV.
- Spread from 3 other PDF sets (central replica) on N3LL+NNLO result: $\Delta M_W \sim 30$ MeV.
- Asymmetry slope unaffected: factorisation of initial-state effects from W propagation/decay.
- PDF spread can be reduced to few MeV using additional p_t^ℓ bins, combination of different rapidity windows (forward and central rapidities anti-correlated) [Bozzi, Citelli, Vesterinen, Vicini, '15; Bagnaschi, Vicini, '19], combination of results from W^+ and W^-.

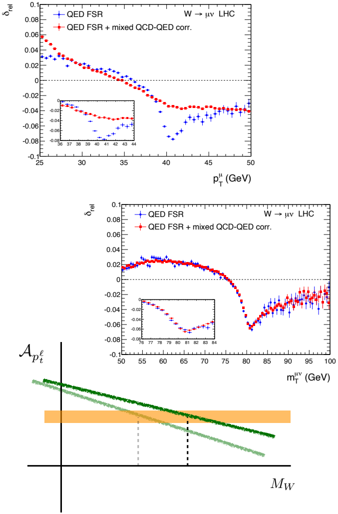
Toy study on effect of NP modelling

- From NNLO+NNLL NCDY predictions, compute p_t^Z reweighing factors to match NCDY 'data' (NNLO+N^{3}LL central) $\rightarrow$ mimic tuning. One factor per scale choice.
- Apply rewgeighing factors to NNLO+NNLL CCDY $p_t^{\ell\nu}$ spectrum; compare with CCDY 'data' (NNLO+N^{3}LL central).
- $p_t^{\ell\nu}$ and p_t^{ℓ} distribution after reweighing agree better with CCDY 'data', but maintain some shape difference.
- QCD uncertainty on reweighting robustly estimated only using one reweighing factor per scale choice.
- Uncertainty on M_W of same size (or larger) as that of the starting NNLO+NNLL distribution, not of the target NNLO+N^{3}LL 'data': importance of accurate perturbative starting point for assessing NP effects.
- NP = additional effect to precisely calculate asymmetry value (slope unaffected), not the central ingredient of M_W extraction, as for template fitting.



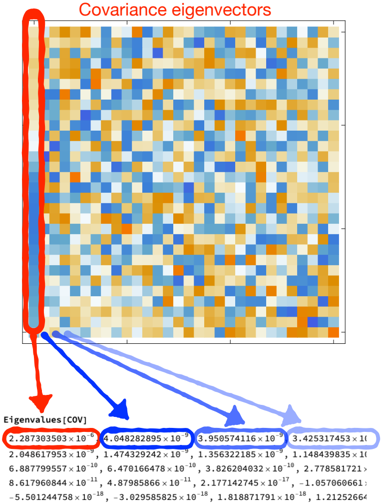


Importance of EW effects



- Significant effects from QED FSR and from mixed QCD-EW corrections at the jacobian peak for p_t^ℓ (see e.g. [Carloni, Chiesa, Martinez, Montagna, Nicrosini, Piccinini, Vicini, 1612.02841]).
- Effects only from QED FSR at the jacobian peak for $M_t^{\ell\nu}$.
- QED FSR affect both asymmetry value and slope.
- Smearing of p_t^ℓ distribution expected, leading to slight M_W-sensitivity loss.

Covariance studies beyond jacobian asymmetry



- Effect of ΔM_W on p_t^ℓ is a shift: sensitivity to M_W mainly encoded in primary eigenvector ($\sim$ asymmetry) representing translations (Eigen[1]/Tr[COV] $\sim$ 0.99).
- Secondary eigenvalues extremely suppressed
- Covariance allows systematic classification of eigenvectors according to M_W sensitivity.
- Secondary eigenvectors can be separately analysed (if need be) and included once their perturbative stability is established.
- Not possible with template fitting, all secondary eigenvectors lumped with the first: very little gain in sensitivity at the price of much more noisy analysis.
- Under study: refinement of asymmetry definition to better match dominant covariance eigenvector.

Outlook

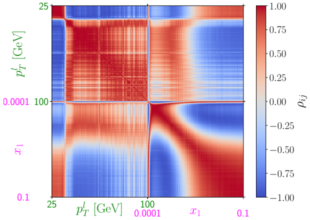
Jacobian asymmetry $\mathcal{A}_{p_t^\ell}$ for M_W determination.

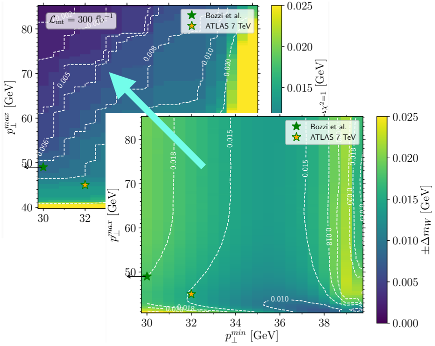
- Combination of fiducial rates $\rightarrow$ excellent QCD perturbative stability and accuracy, precision measurement.
- Based on large p_t^ℓ bins, large linear dependence on M_W $\rightarrow$ experimental statistics and systematics under control.
- Possibility to unfold data to particle level $\rightarrow$ global experimental combination.
- Allows systematic inclusion and assessment of all sources of sensitivity to M_W: PDF, NP, EW, ...

Covariance studies to systematically classify and potentially include yet subdominant effects.

Backup

Taming PDF uncertainties with additional p_t^ℓ bins [Bagnaschi, Vicini, 1910.04726]





- Strong anti-correlation under PDF variations between p_t^ℓ regions below/above jacobian peak.
- Take PDF correlations into account directly in the definition of χ^2:

$$\chi_i^2 = \sum_{r,s\,\in \mathrm{bins}} \left(\mathcal{T}^i - \mathcal{D}^{\rm exp}\right)_r \left(C^{-1}\right)_{rs} \left(\mathcal{T}^i - \mathcal{D}^{\rm exp}\right)_s$$

$$C = \Sigma_{\rm pdf} + \Sigma_{\rm stat} + \Sigma_{\rm mc} + \Sigma_{\rm exp}^{\rm syst}$$

$$(\Sigma_{\rm pdf})_{rs} = \langle \mathcal{T}_r^i\, \mathcal{T}_s^i \rangle_{\rm pdf} - \langle \mathcal{T}_r^i \rangle_{\rm pdf}\, \langle \mathcal{T}_s^i \rangle_{\rm pdf}$$

- Correlation leads to profiling of PDF replicas $\rightarrow$ significant reduction of PDF uncertainty w.r.t. the case with no PDF covariance, at the few-MeV level.

EW effects at the jacobian peak [Carloni, Chiesa, Martinez, Montagna, Nicrosini, Piccinini, Vicini, 1612.02841]

$p\bar{p} \to W^+$, $\sqrt{s} = 1.96$ TeV			M_W shifts (MeV)			
Templates accuracy: NLO-QCD+QCD$_{\rm PS}$			$W^+ \to \mu^+\nu$		$W^+ \to e^+\nu$(dres)	
Pseudodata accuracy		QED FSR	M_T	p_T^ℓ	M_T	p_T^ℓ
1	NLO-QCD+(QCD+QED)$_{\rm PS}$	Pythia	-91±1	-308±4	-37±1	-116±4
2	NLO-QCD+(QCD+QED)$_{\rm PS}$	Photos	-83±1	-282±4	-36±1	-114±3
3	NLO-(QCD+EW)-two-rad+(QCD+QED)$_{\rm PS}$	Pythia	-86±1	-291±3	-38±1	-115±3
4	NLO-(QCD+EW)-two-rad+(QCD+QED)$_{\rm PS}$	Photos	-85±1	-290±4	-37±2	-113±3

$pp \to W^+$, $\sqrt{s} = 14$ TeV		M_W shifts (MeV)			
Templates accuracy: LO		$W^+ \to \mu^+\nu$		$W^+ \to e^+\nu$	
Pseudo–data accuracy		M_T	p_T^ℓ	M_T	p_T^ℓ
1	Horace only FSR-LL at $\mathcal{O}(\alpha)$	-94±1	-104±1	-204±1	-230±2
2	Horace FSR-LL	-89±1	-97±1	-179±1	-195±1
3	Horace NLO-EW with QED shower	-90±1	-94±1	-177±1	-190±2
4	Horace FSR-LL + Pairs	-94±1	-102±1	-182±2	-199±1
5	Photos FSR-LL	-92±1	-100±2	-182±1	-199±2

- EW effects induce a smearing of the jacobian peak, shifting the extracted M_W.
- Leading effect from QED FSR, extra few-MeV shifts from subleading EW effects.
- Quantitatively, EW impact depends on the underlying QCD model: importance to include EW effects on top of an accurate QCD prediction.
- Progress in calculation of QCD-EW corrections at fixed order [Buonocore et al. 2102.12539; Bonciani et al., 2106.11953; Armadillo et al. 2201.01754; Buccioni et al. 2203.11237], and in resummation [Cieri et al. 1805.11948; Autieri et al. 2302.05403].