Neural Network enhanced phase space tomography for AWAKE experiment

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AWAKE Collaboration Meeting
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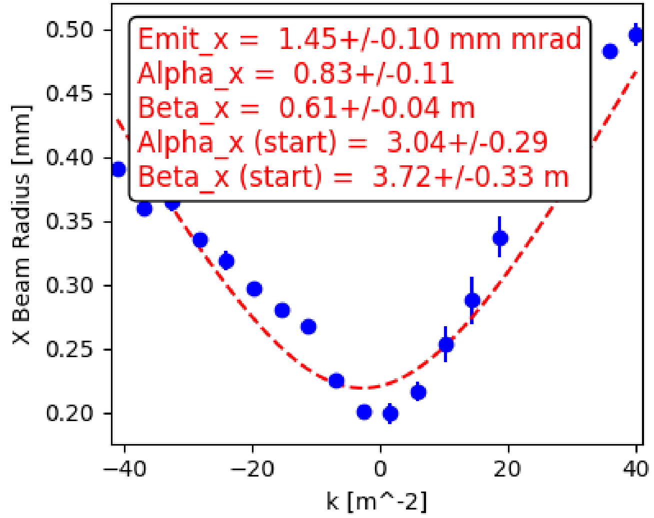
Emittance measurement

- Emittance measurement is fundamental to obtain good agreement between simulations and measurements

How it was done:

- Presently emittance is measured using classical quad scan
  1. Fit gaussian to measured beam profile
  2. Fit proper parabolic function
  3. Extract the Twiss parameters

- Main limitations to accuracy:
  - ❌ Beam is not gaussian!
  - ❌ Parabolic curve does not fit measurements (in x plane at least)

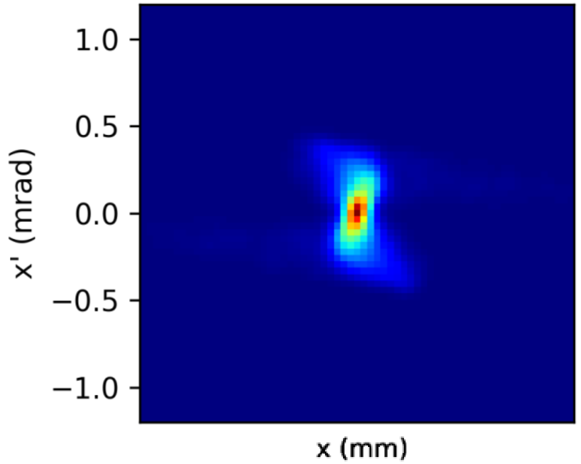


Solution (under development)

- Use phase space tomographic reconstruction.

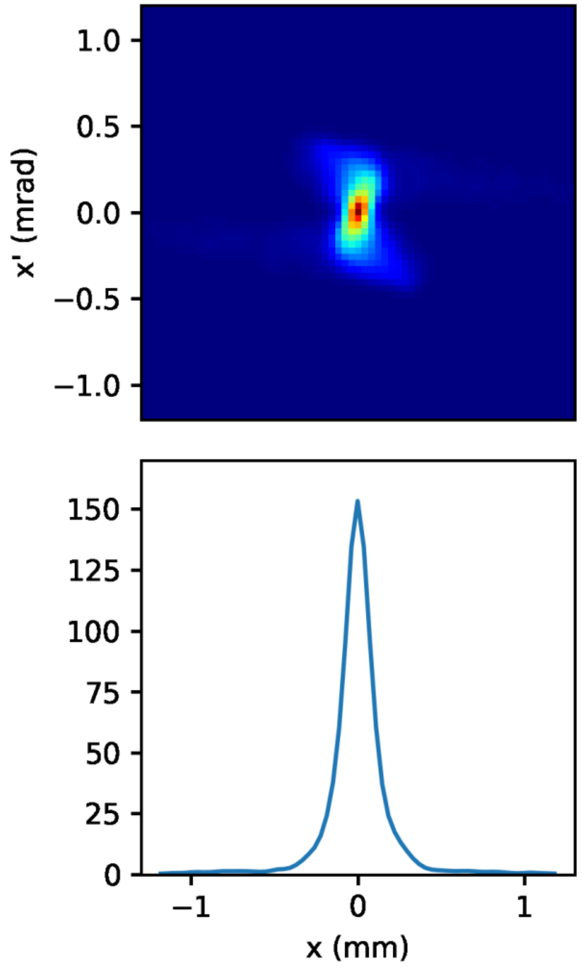
Tomography in a nutshell

- Reconstruct n-dimensional image from projections taken at different angles in n-1 dimensional space



Tomography in a nutshell

- Reconstruct n-dimensional image from projections taken at different angles in n-1 dimensional space
- In 2D we use 1D projections taken from different angles around the object



Tomography in a nutshell

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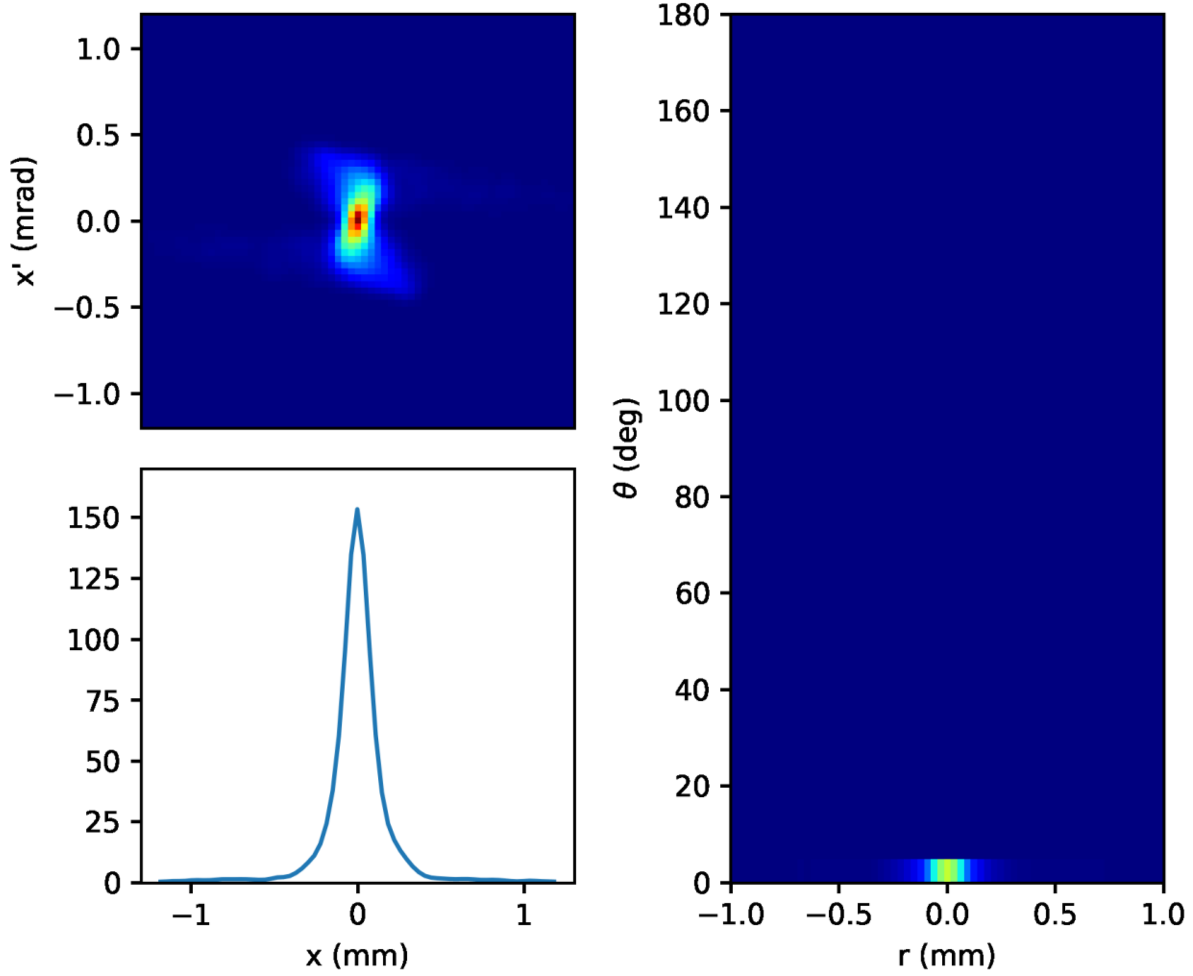
- The projections are stacked in a 2D image called **sinogram**



Reconstruction techniques

Analytical methods

- BP - Back projection
- FBP - Filtered back projection

Pros

- Very fast

Cons

- Performs badly when the number of projections is low
- Performs badly when angles are missing

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Iterative methods

- SART – Simultaneous Algebraic Reconstruction Tomography
- MENT – Maximum Entropy Tomography

Pros
- Performs well when the number of projections is low
- Performs well when angles are missing

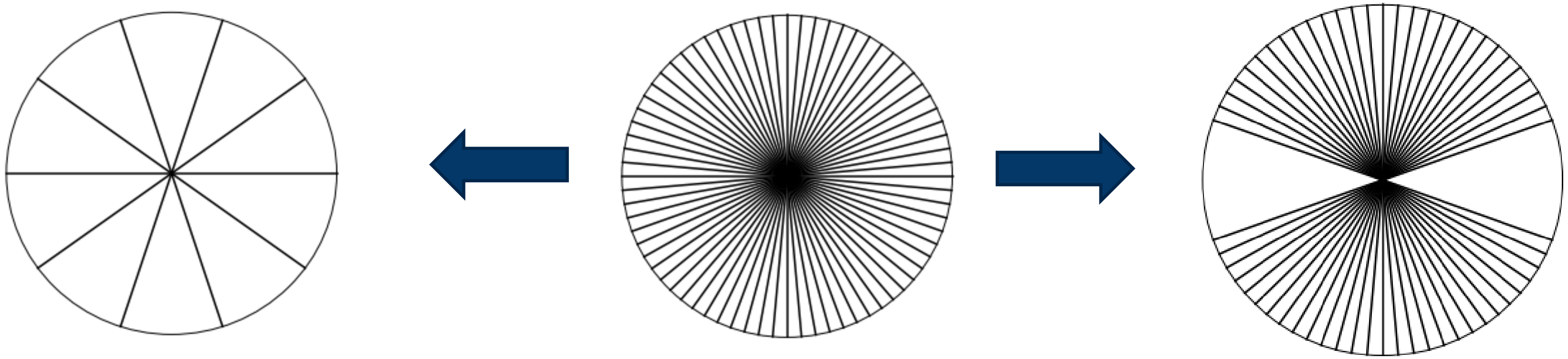
Cons
- Can be slow
- Reconstruction quality depends on number of iterations

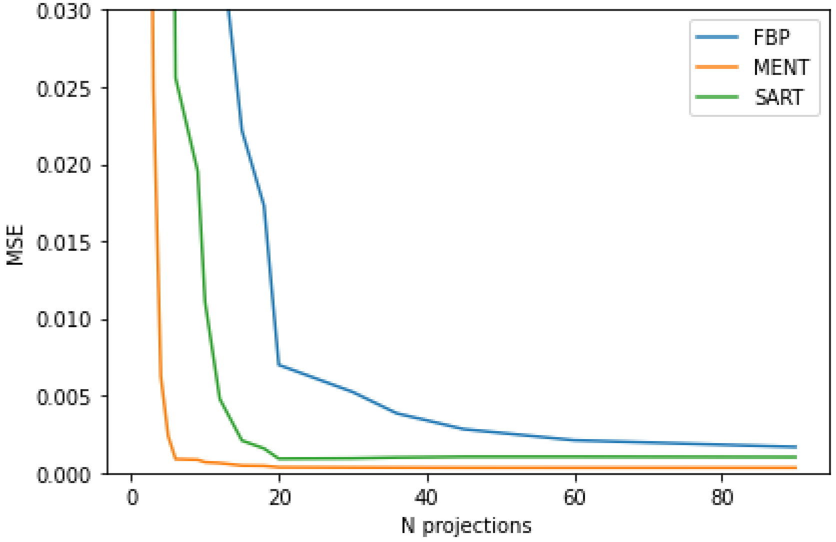
A very comprehensive comparison between algorithms can be found in here

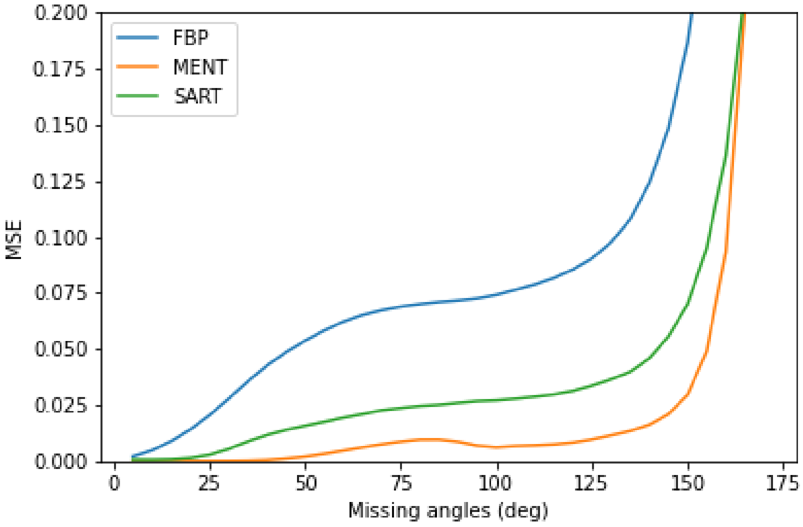
Reconstruction techniques

Low number of projections

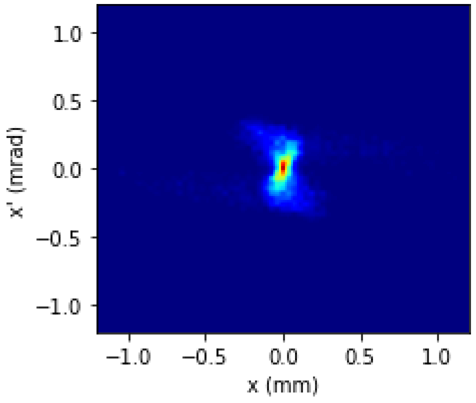
Missing angles



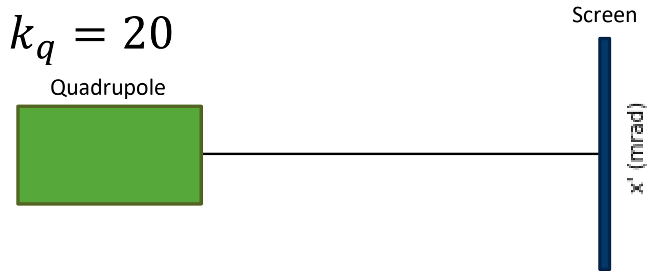


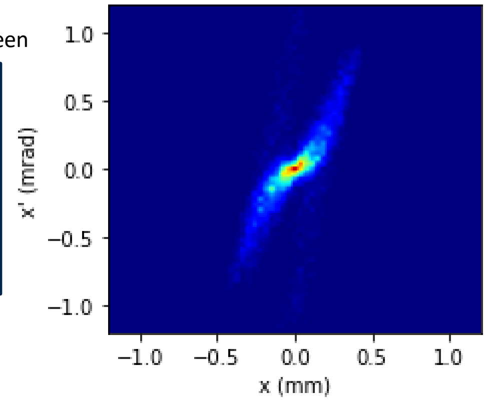


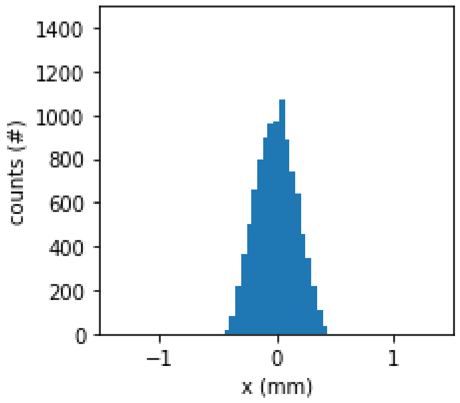
How this applies to beam physics?



$k_q = 20$

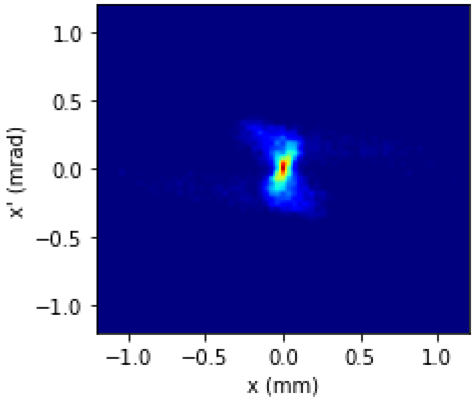






$$\begin{bmatrix} x_s \\ x'_s \end{bmatrix} = \begin{bmatrix} M_{1,1}(k_q) & M_{1,2}(k_q) \\ M_{2,1}(k_q) & M_{2,2}(k_q) \end{bmatrix} \begin{bmatrix} x_0 \\ x'_0 \end{bmatrix}$$

THANKS TO THIS TRANSFORMATION WE TREAT IT AS A TOMOGRAPHY PROBLEM!



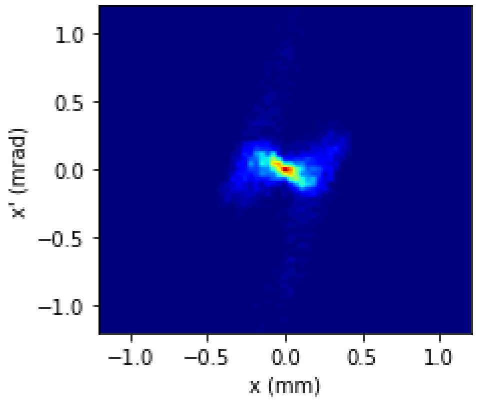
ROTATE

$$\theta = \mathrm{atan}\left(\frac{M_{1,1}(k_q)}{M_{1,2}(k_q)}\right)$$

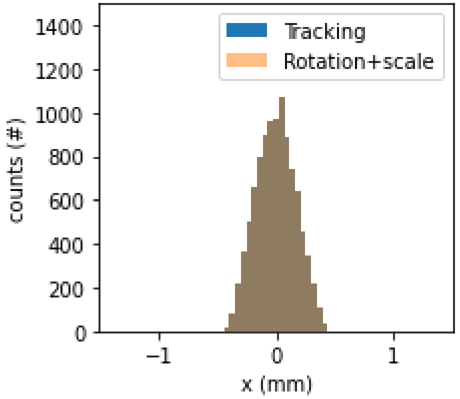
SCALE

$$s = \sqrt{M_{1,1}^2(k_q) + M_{1,2}^2(k_q)}$$

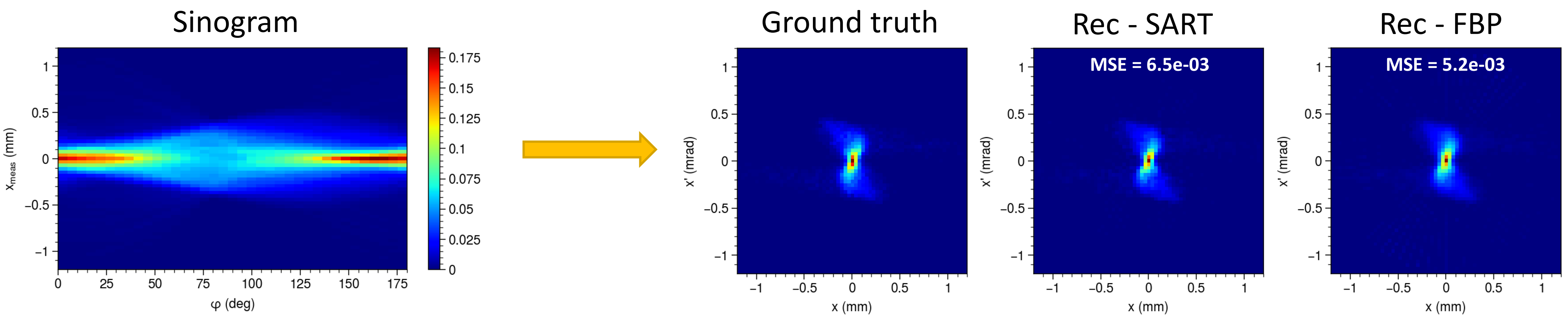
$\theta = 101°$



$s = 0.84$

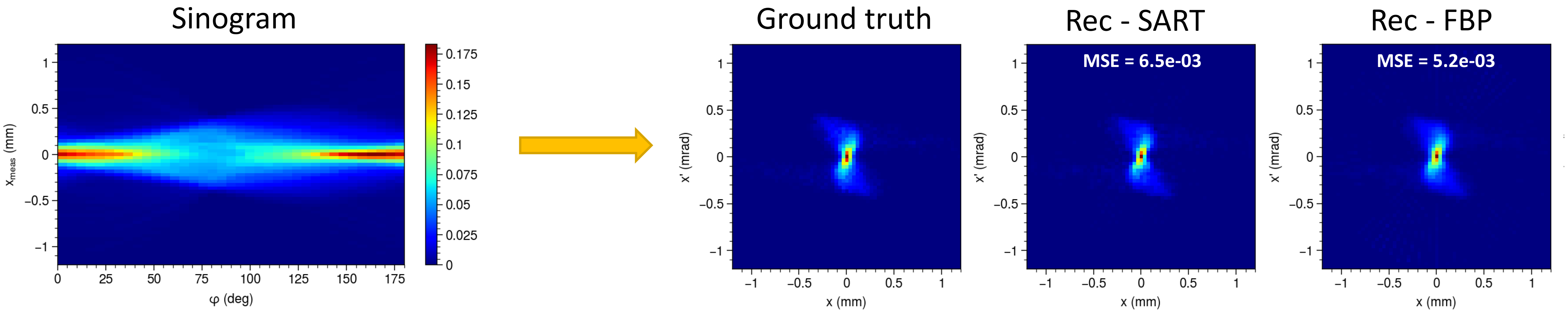


It works!



Sinogram
Ground truth
Rec - SART
Rec - FBP
MSE = 6.5e-03
MSE = 5.2e-03
x_{meas} (mm)
φ (deg)
x' (mrad)
x (mm)

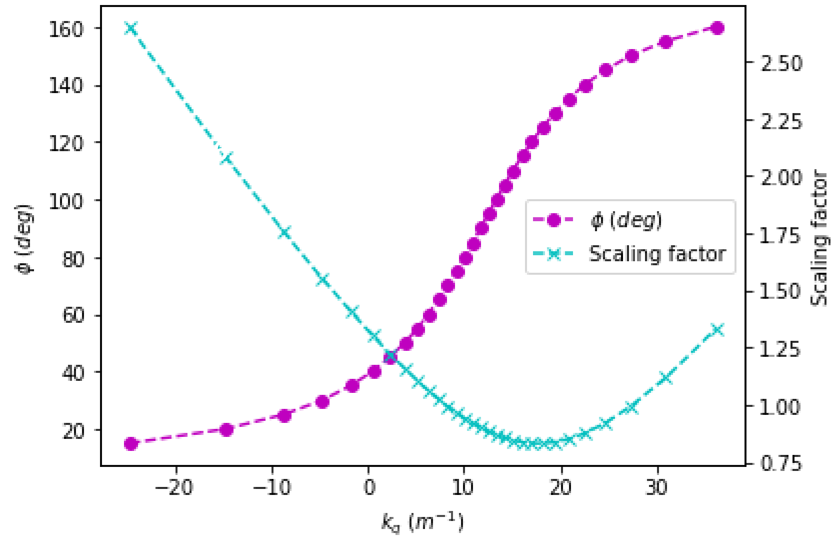
Not that easy



- Scaling factor and angle can be obtained from transport matrix.
- The range of accessible angles is limited by the system geometry and the quads strengths limit.

$$-40 \leq k_q \leq 40 \Rightarrow 11 \leq \theta \leq 162$$

- Access to limited angles means a lower reconstruction accuracy



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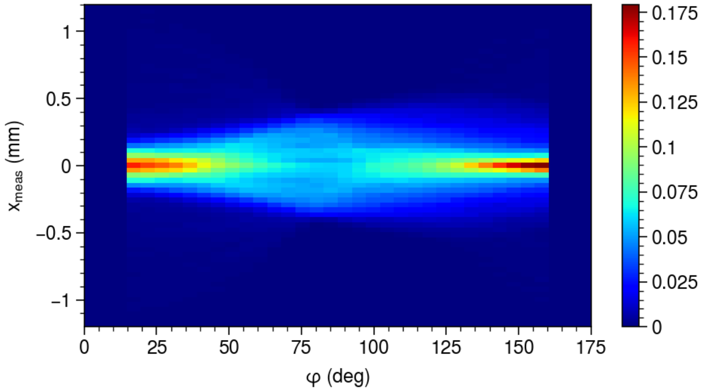

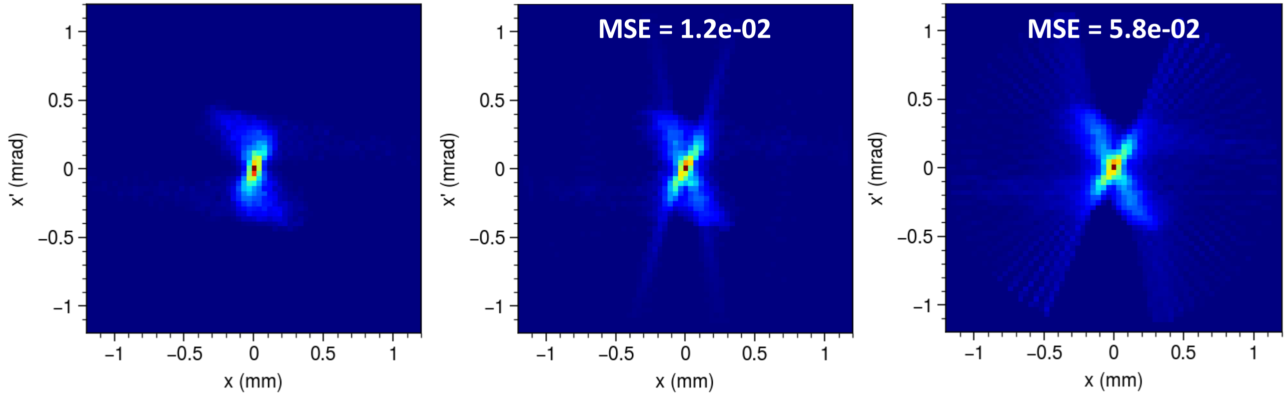

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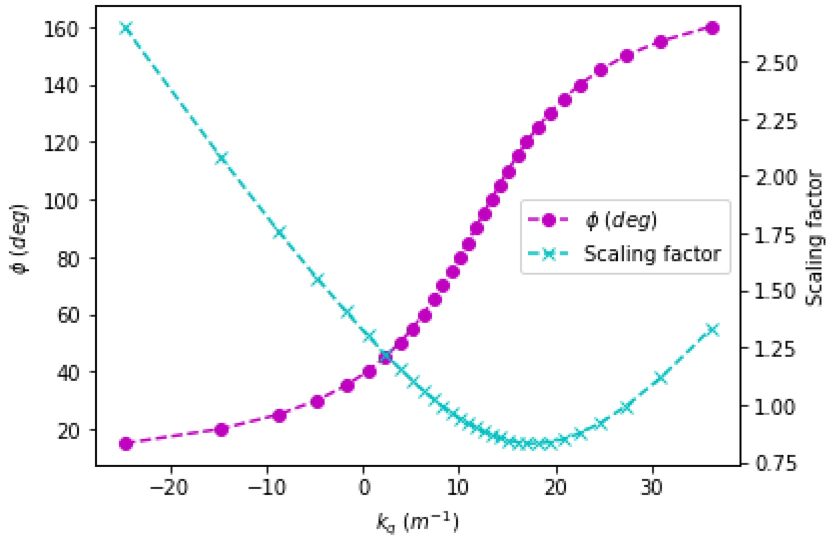
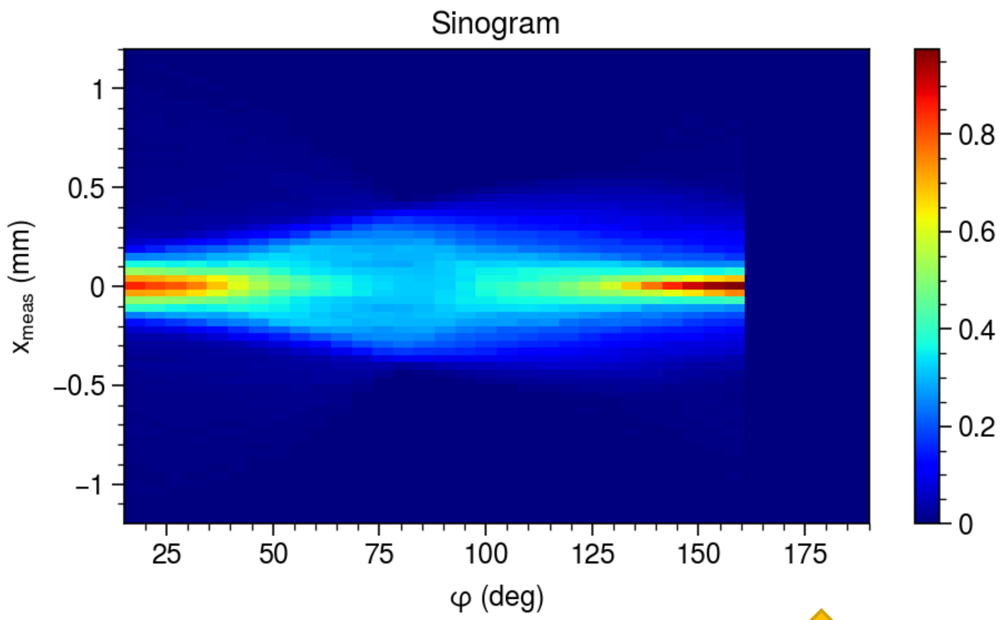


Can we use AI to solve this issue?



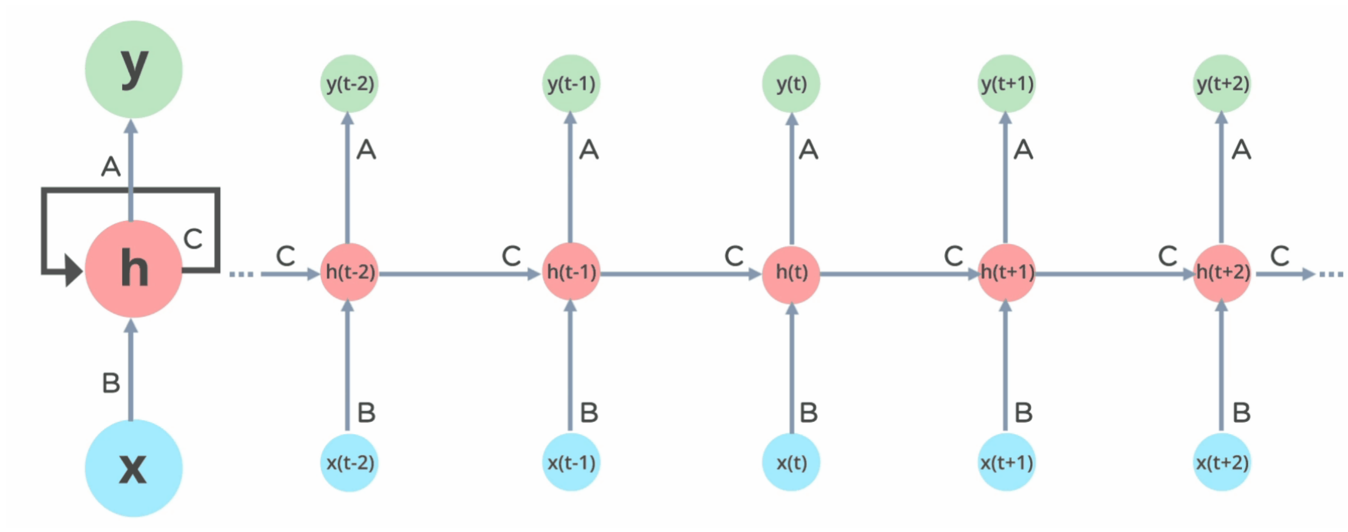
Can we fill this gap?

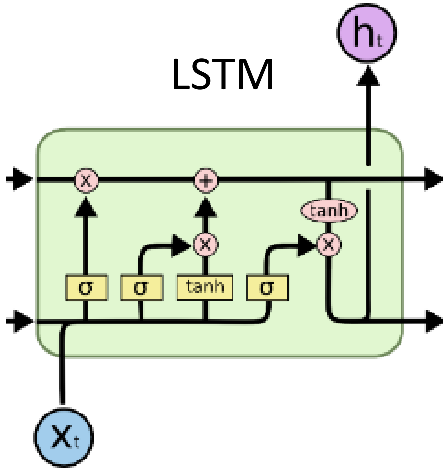
- Neural networks can be a powerful tool to complete the information about missing angles
- Important dependence between each projection and the one that precedes
- The ideal model would take into account the sequential structure of the data

- Recurrent Neural Networks are a good fit!

Using LSTM neural nets

- Long Short Term Memory (LSTM) neural networks are a type of recurrent neural network.
- They help capture long-term dependencies in data that other models cannot.
- The memory block within an LSTM cell helps maintain information over time.





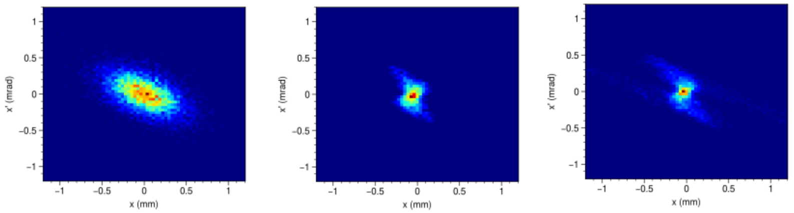
[Credit for the animation]

Plots from https://colah.github.io/posts/2015-08-Understanding-LSTMs/

Setting up the problem

1. Create particle distributions

- Sampling $\alpha, \beta, \varepsilon$ in a given range
- Select among 6 possible distribution types
- Generate 10000 distributions



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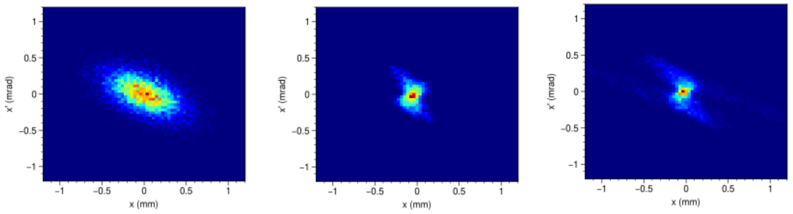
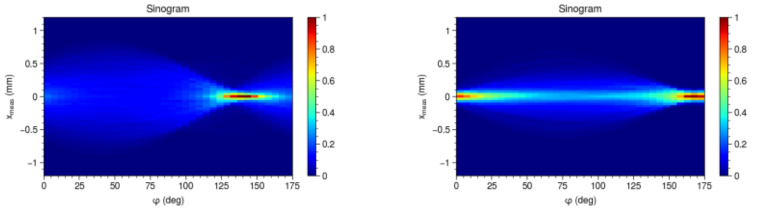


2. Create sinograms in the full angle range

- Track each distribution in the full angle range with 5 deg spacing
- Produce the sinograms



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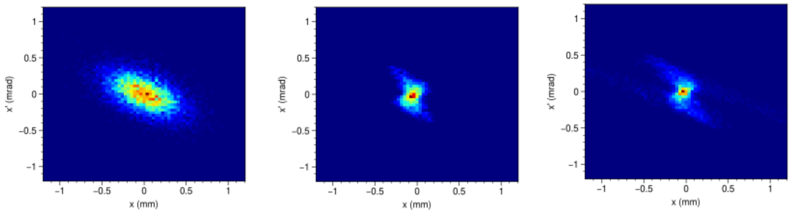

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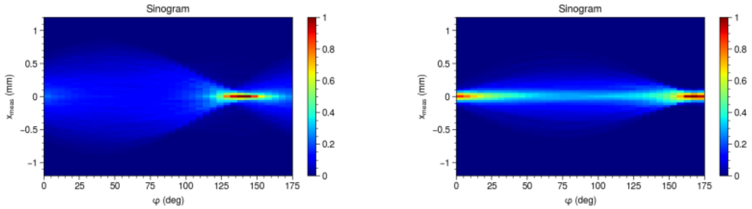
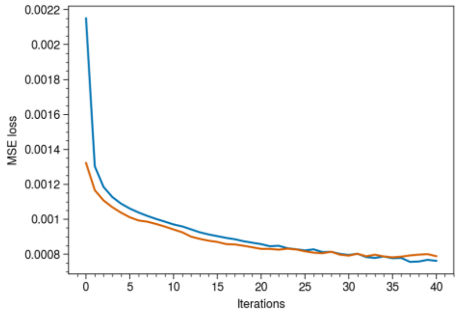


3. Train the LSTM network

- 65 features, 29 “time-steps”, 7 to predict
- LSTM with 2 layers of 128 and 64 neurons each



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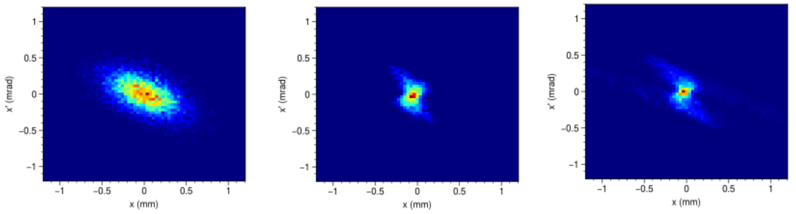

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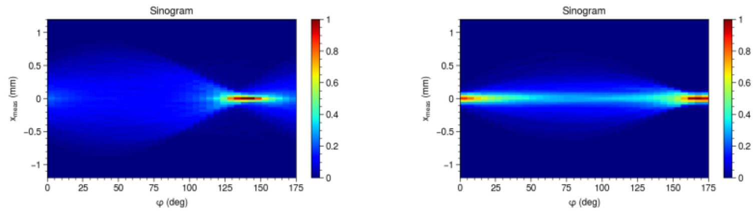

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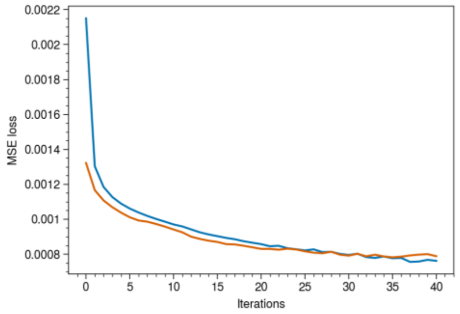
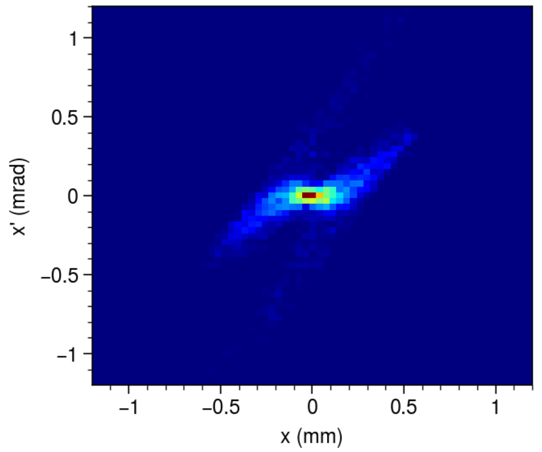


4. Evaluate the model

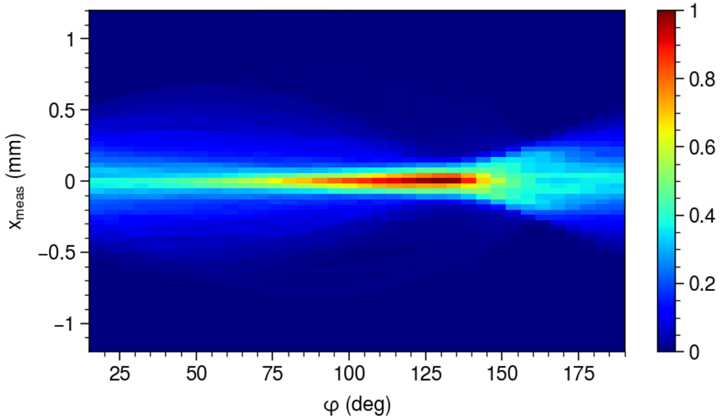
- Evaluate capacity of reproducing sinograms
- Compare impact on reconstruction quality for different methods

Example from test set

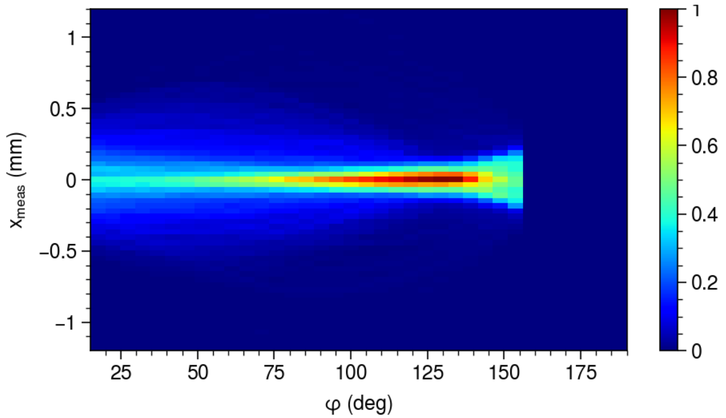
Select a beam from the test set



All angles



Measurement range



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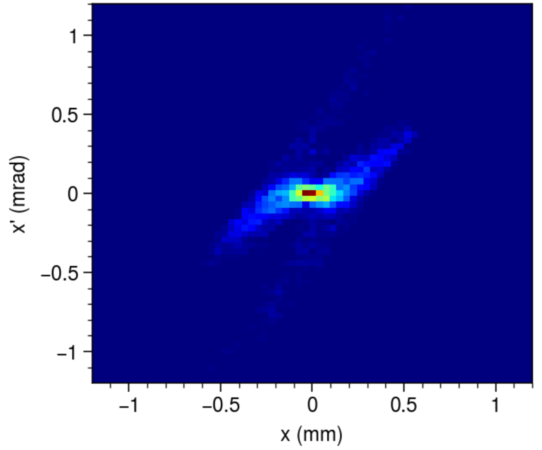

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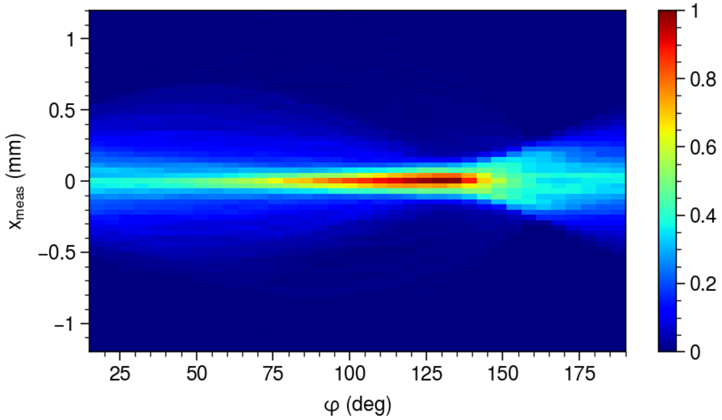

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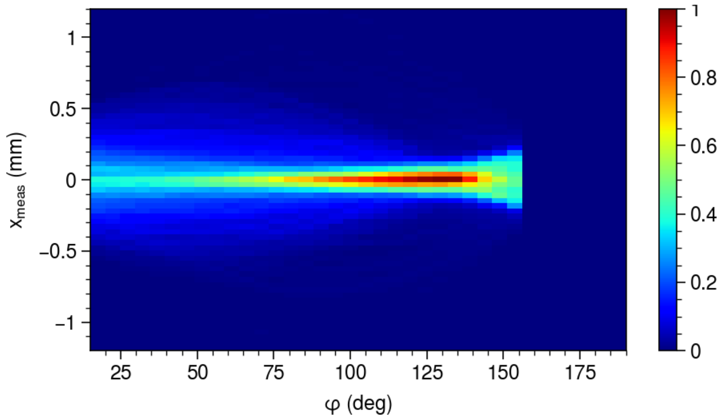
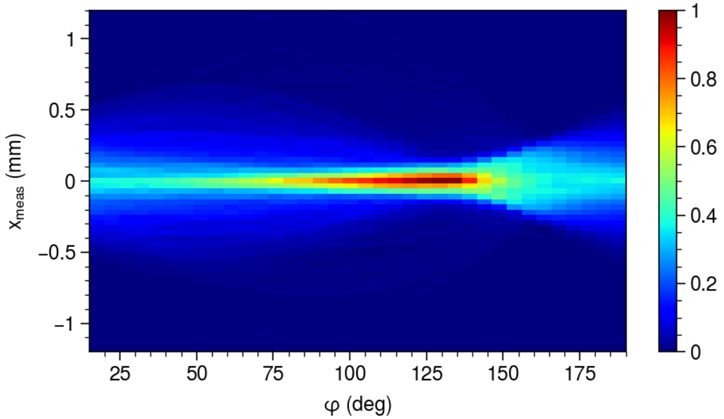
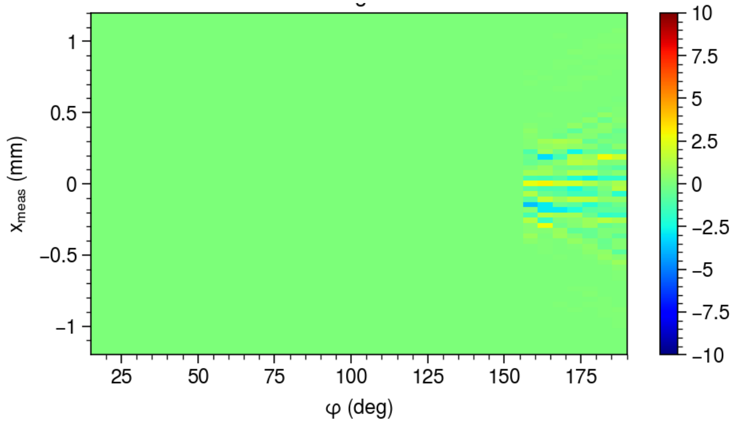


Apply LSTM model to predict the missing projections

LSTM model

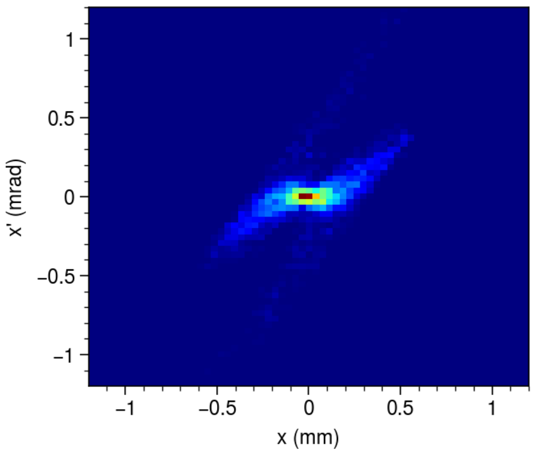


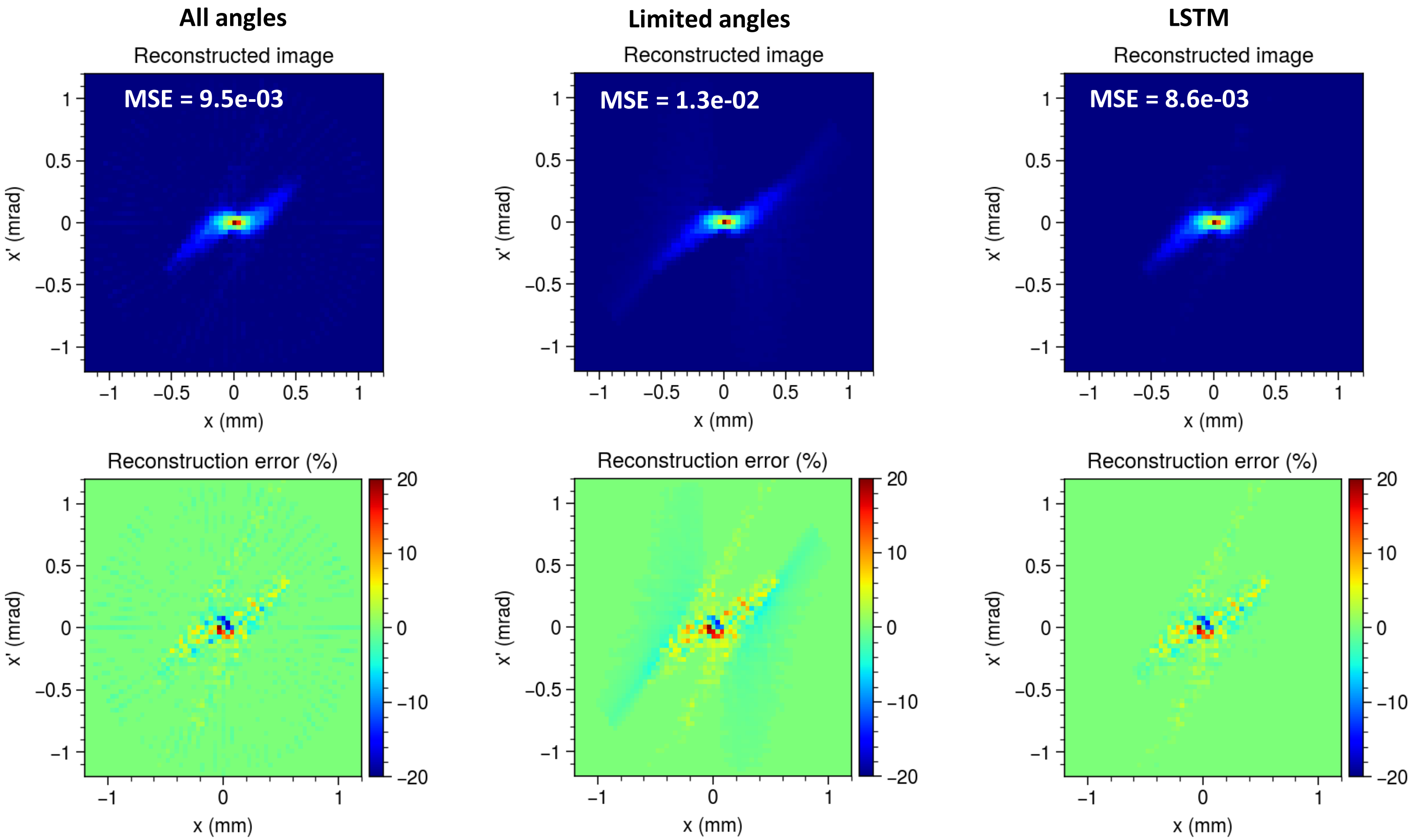
Error (%)



Example from test set

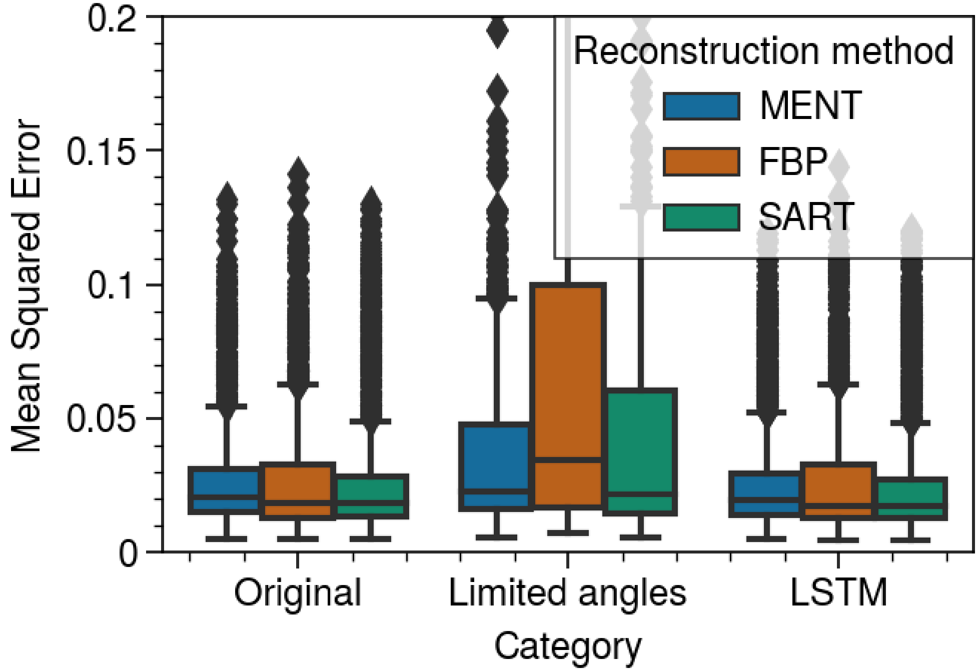
Reconstructing the beam distribution using the LSTM completed sinogram gives an accuracy equal to the one with full angle range.

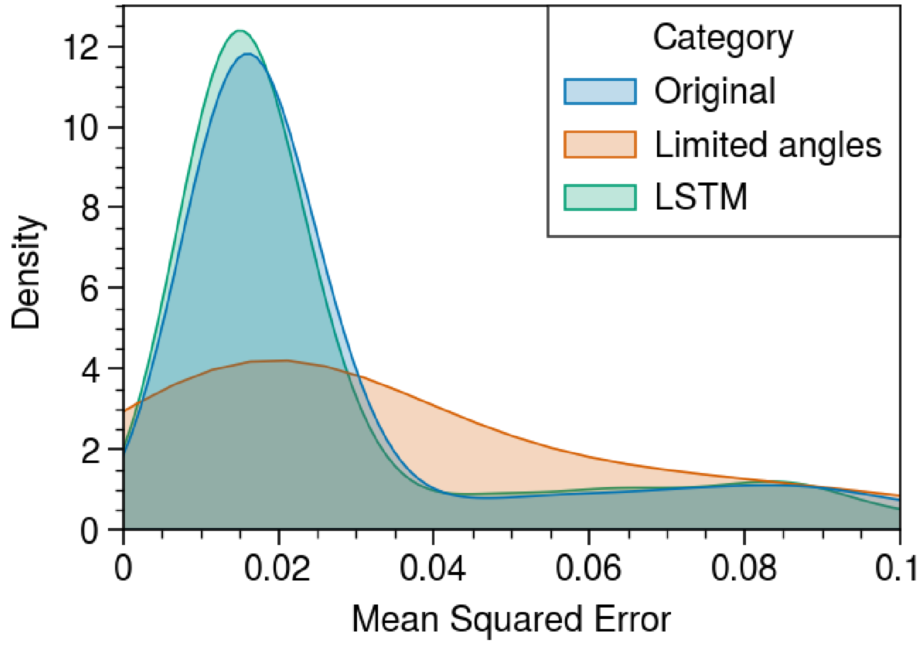




Model performance

- Comparing different algorithms on the full test set.
- With the help of LSTMs the original accuracy can be fully restored.





Conclusions and next steps

Conclusions

- Simulated different algorithms for phase space tomography to **reconstruct initial beam distribution** from **beam profile measurements.**
- **Developed a new approach** that uses LSTM networks to address the issue of missing angles, leading to reconstruction accuracy equal to the case with full angle range.
- The method (and the code) can be applied to other systems. Creating the dataset and training the network takes few hours and has to be done only once!

Next Steps

- Study the sensitivity of the reconstruction to errors in quadrupole strength or momentum.
- Development of a customized loss function for the network to consider physics constraint to the problem (constant density in projection)
- Test method on real data in the next AWAKE electron run.

Thank you for your attention!

How does it work in practice?

Model set-up

1. Create particle distributions
2. Create sinograms in the full angle range
3. Train the LSTM network
4. Evaluate the model

How does it work in practice?

Preparation

1. Define reference Twiss parameters (can be done with a conventional quad scan)
   - Reference to define the Twiss range to create the dataset
   - Used to normalize the beam coordinates
2. Specify characteristics of the transport section
3. Assume set of distributions to create the dataset

Model set-up

1. Create particle distributions
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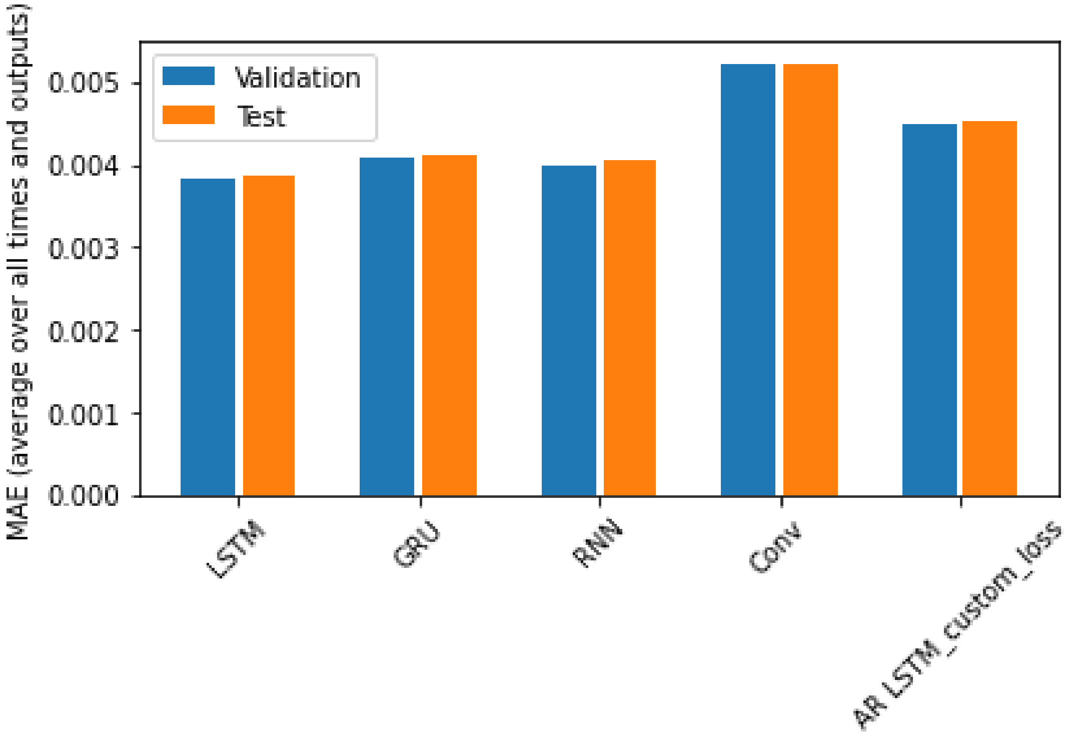
Use on real data

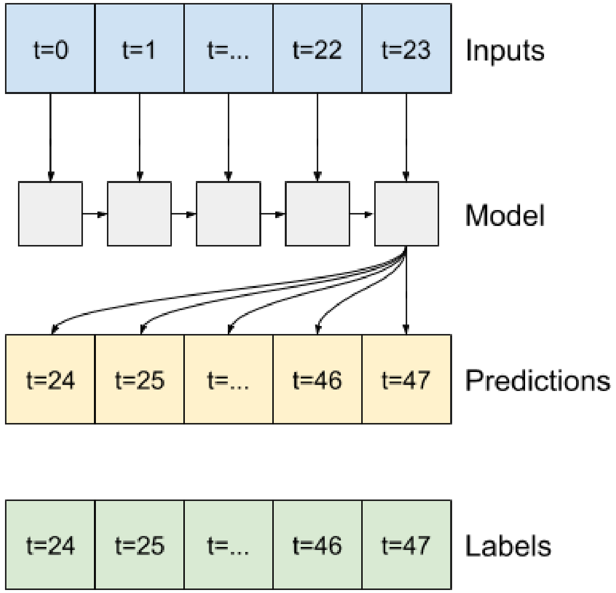
1. Measure profiles at given angles (converted to equivalent quad strengths)
2. Apply scaling to projections
3. Re-binning of projections to match number of bins used in the model
4. Perform reconstruction applying LSTM model

Compare networks



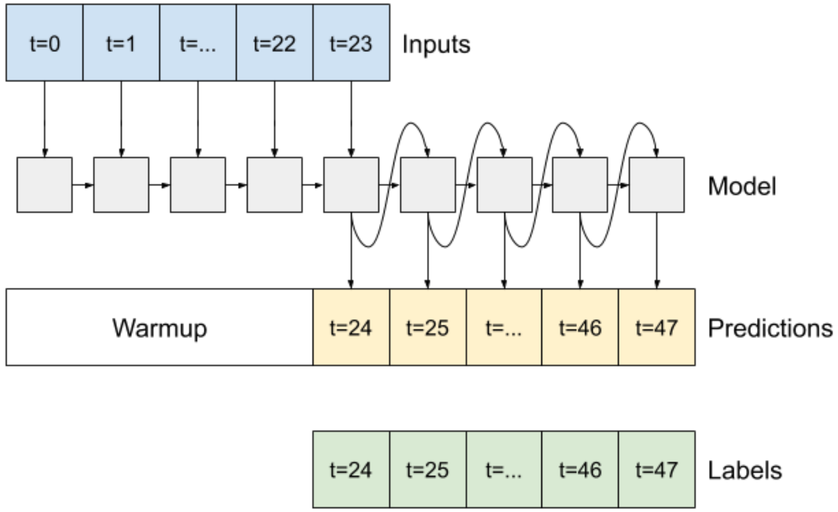
Compare networks

Classic architecture architecture



Fixed length of inputs and predictions

Autoregressive architecture



Number of prediction can be modified