Motivating Semiclassical Gravity: An Approximation for Bipartite Quantum Systems

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work done with Viqar Husain, Irfan Javed and Sanjeev S. Seahra (arXiv:2306.01060)

Motivation

- QFT in curved spacetime (cosmology, black holes...)

$$G_{\alpha\beta}(g) = 0, \quad i\,\partial_t|\psi\rangle = \hat{H}_\psi(g)|\psi\rangle$$

- Including backreaction (pathologies in curved spacetime, superposition is lost...)

$$G_{\alpha\beta}(g) = M_{\mathrm{Pl}}^{-2}\langle\psi|\,\hat{T}_{\alpha\beta}(g)|\psi\rangle$$

Page and Geilker (1981), Ford (1997)

- Can this be derived from WdW using Born-Oppenheimer approximation? - No

Singh and Padmanabhan (1989), Kiefer and Singh (1991)

Assumptions and What to Expect

1. The quantum system starts in product state $|\varphi_1\rangle \otimes |\varphi_2\rangle$
2. One subsystem is in a semiclassical state.
3. The coupling between the subsystems is weak.

- Timescales: Scrambling time, Ehrenfest time
- Example: Non-linearly coupled oscillators
- Regimes: Quantum-Quantum (QQ), Classical-Quantum (CQ)
- Results: Dynamics, energy exchange, entanglement...

Framework

Quantum Hamiltonian: $\hat{H} = \hat{H}_1 \otimes \mathbb{I}_2 + \mathbb{I}_1 \otimes \hat{H}_2 + \lambda \hat{V}_1 \otimes \hat{V}_2$

QQ dynamics: $|n,\mu\rangle = |n\rangle \otimes |\mu\rangle \to |\psi(t)\rangle = \sum_{n\mu} z_{n,\mu}(t)|n,\mu\rangle$, then Schrodinger eq. is

$$i\partial_t Z = H_1 Z + Z H_2 + \lambda V_1 Z V_2^{\mathsf{T}}$$

E.O.M. for reduced density matrices:

$$i\partial_t \rho_1 = [H_1, \rho_1] + \lambda [V_1, Z V_2^{\mathsf{T}} Z^\dagger],$$
$$i\partial_t \rho_2 = [H_2, \rho_2] + \lambda [V_2, Z^{\mathsf{T}} V_1^{\mathsf{T}} Z^*]$$

where $\rho_1 = Z\,Z^\dagger, \quad \rho_2 = Z^{\mathsf{T}} Z^*$

Product State Approximation

Measures of Entanglement: $1 - \gamma = S_{\text{lin}} \ll 1 - \frac{1}{d^2}, \quad S_{\text{VN}} \ll \ln d$

Nearly product state: If $Z = \mathbf{u}_1 \mathbf{u}_2^{\mathsf{T}} + \lambda\, \delta Z$ then

$$i\partial_t \rho_1 = [H_1^{\text{eff}}, \rho_1] + \mathcal{O}(\lambda^2), \quad H_1^{\text{eff}} = H_1 + \lambda \langle \varphi_2 | \hat{V}_2 | \varphi_2 \rangle \hat{V}_1$$

$$i\partial_t \rho_2 = [H_2^{\text{eff}}, \rho_2] + \mathcal{O}(\lambda^2), \quad H_2^{\text{eff}} = H_2 + \lambda \langle \varphi_1 | \hat{V}_1 | \varphi_1 \rangle \hat{V}_2$$

Evolution is non-linear in the state, unitary

CQ Approximation

Subsystem 1 behaves ‘classically’:

$$\mathcal{H}^{\text{eff}} = \mathcal{H}_1 + \lambda \mathcal{V}_1 \langle \varphi_2 | \hat{V}_2 | \varphi_2 \rangle$$

where $\mathcal{V}_1(q_1(t), p_1(t)) = \langle \varphi_1(t) | \hat{V}_1 | \varphi_1(t) \rangle [1 - \varepsilon(t)]$

$$\hat{H}^{\text{eff}} = \hat{H}_2 + \lambda \mathcal{V}_1 \hat{V}_2$$

CC and Classical-background approximation are special cases

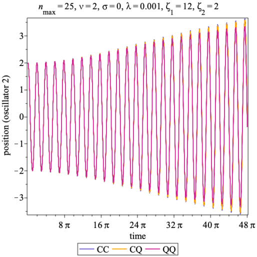
Note: Energy is conserved in all cases except classical-background

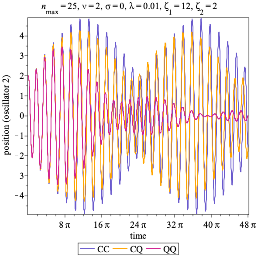
Example: Coupled Non-linear oscillators

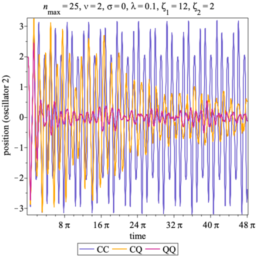
$$\mathcal{H} = \frac{1}{2}\Omega_1(q_1^2 + p_1^2) + \frac{1}{2}\Omega_2(q_2^2 + p_2^2) + \lambda\bar{\Omega} q_1^\nu q_2^\nu$$

- Initial data consistent with the Bohr correspondence principle (i.e. $\exists$ states such that expectation values follow classical trajectories)
- Coherent states as initial data
- Results: Trajectories, energy exchange, QQ-CQ-CC comparison, Entanglement
- Parameters: $\nu, \lambda, n_{\max}, \sigma, \zeta_1, \zeta_2$

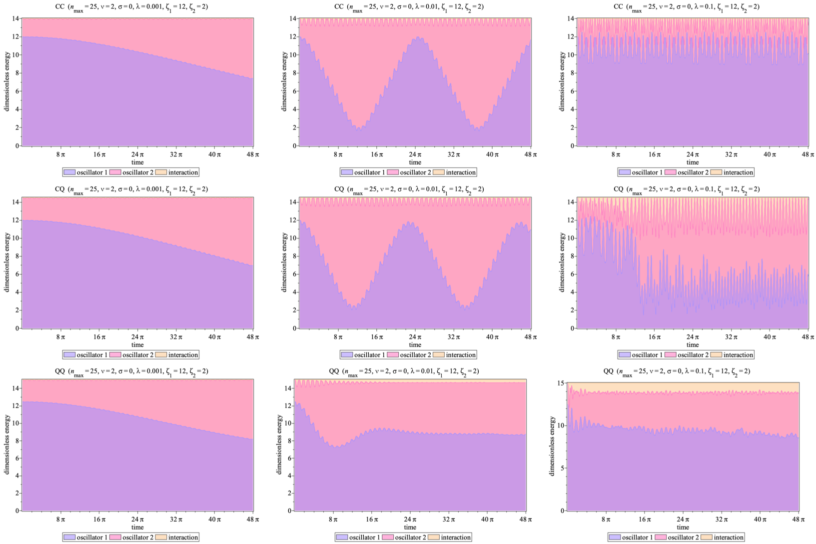
Results: Position of oscillator 2



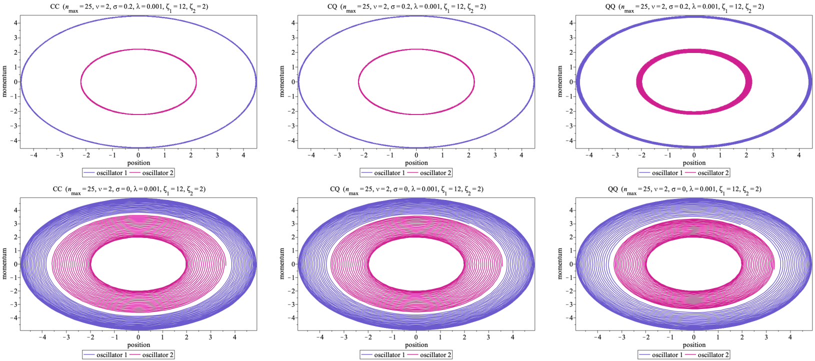


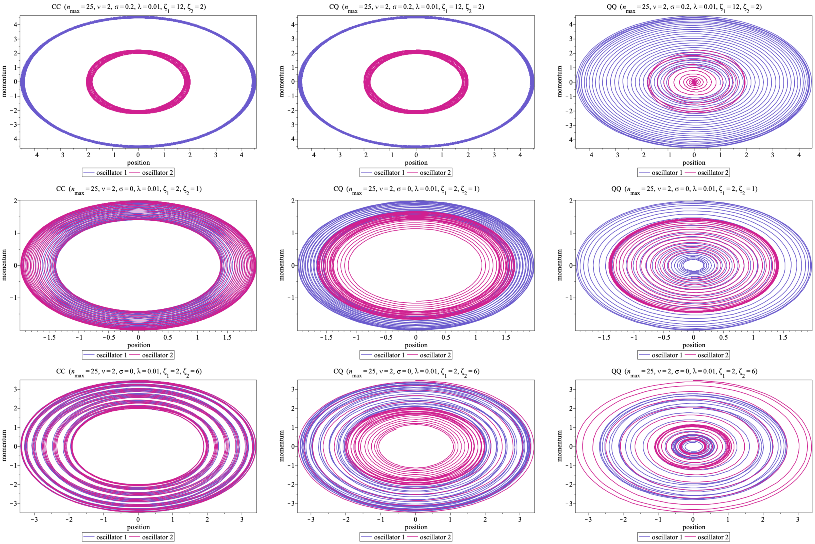


Results: Distribution of energy



Results: Phase space trajectories





CC (n_{max} = 25, ν = 2, σ = 0.2, λ = 0.01, ζ_1 = 12, ζ_2 = 2)
CQ (n_{max} = 25, ν = 2, σ = 0.2, λ = 0.01, ζ_1 = 12, ζ_2 = 2)
QQ (n_{max} = 25, ν = 2, σ = 0.2, λ = 0.01, ζ_1 = 12, ζ_2 = 2)
CC (n_{max} = 25, ν = 2, σ = 0, λ = 0.01, ζ_1 = 2, ζ_2 = 1)
CQ (n_{max} = 25, ν = 2, σ = 0, λ = 0.01, ζ_1 = 2, ζ_2 = 1)
QQ (n_{max} = 25, ν = 2, σ = 0, λ = 0.01, ζ_1 = 2, ζ_2 = 1)
CC (n_{max} = 25, ν = 2, σ = 0, λ = 0.01, ζ_1 = 2, ζ_2 = 6)
CQ (n_{max} = 25, ν = 2, σ = 0, λ = 0.01, ζ_1 = 2, ζ_2 = 6)
QQ (n_{max} = 25, ν = 2, σ = 0, λ = 0.01, ζ_1 = 2, ζ_2 = 6)
momentum
position
oscillator 1
oscillator 2

Results: Comparisons between schemes

- For small

coupling, the errors in CC, CQ $\propto \lambda$

- For most parameters,

the CQ errors < CC errors. Exceptions occur when $\sigma \to 1$ or when $\zeta_2 > \zeta_1$

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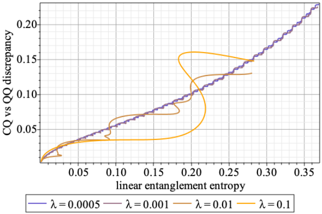
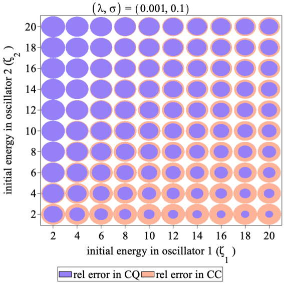
CQ gets worse with increasing σ, while the error in CC is fairly insensitive to it

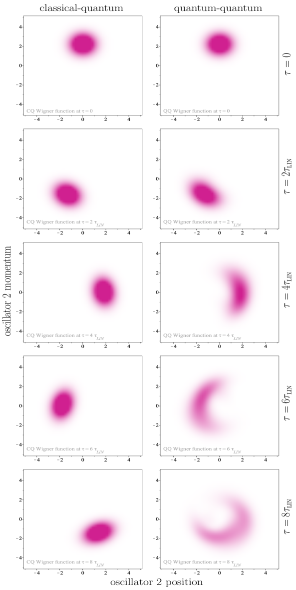
- CQ improves when $\zeta_2 < \zeta_1$
- CC improves when ζ_2, ζ_1 become large





Wigner Distributions

- small coupling ($\lambda = 0.001$) upto τ_s
- moderate coupling ($\lambda = 0.01$) over $[0, 6\,\tau_{lin}]$
- moderate coupling ($\lambda = 0.01$) over $[0, 2\,\tau_{lin}]$
- large coupling ($\lambda = 1$) over $[0, 18\,\tau_{lin}]$



Discussion

- Approximations valid for a broad class of bipartite systems and define a regime where CQ works
- Oscillator system exhibits parametric resonance at the fully quantum level, a result in agreement with classical parametric resonance with a suitable choice of initial data.
- In the CQ case, this calculation bears some resemblance to the phenomenon of particle creation in dynamical spacetimes where subsystem 1 plays the role of a classical geometry driving creation of quanta in subsystem 2.
- Qualitatively distinct from Born-Oppenheimer (no large mass parameter)
- Comparison with the Lindblad formalism
- Might be possible to apply in simple gravity-matter systems