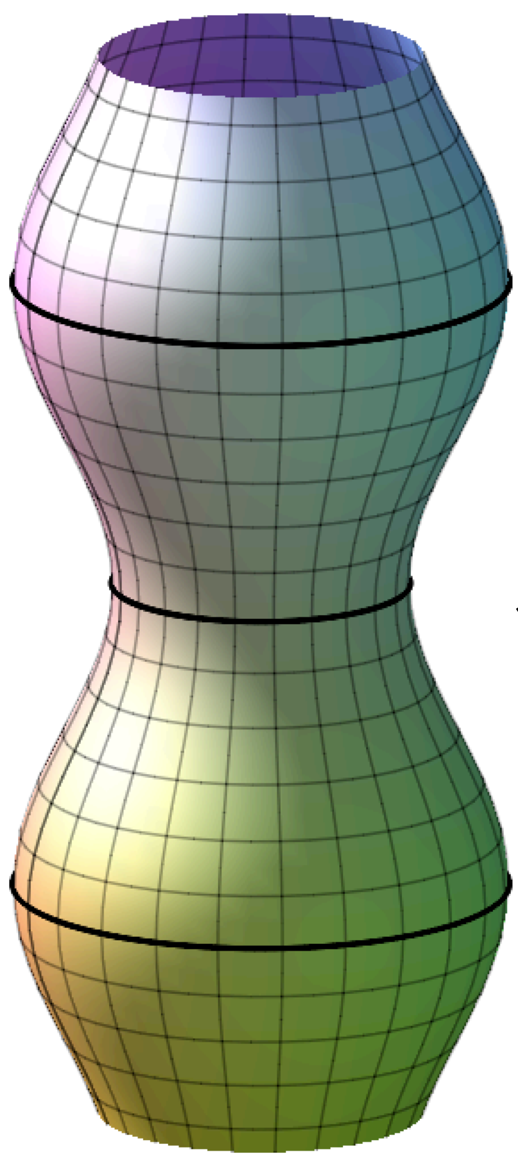
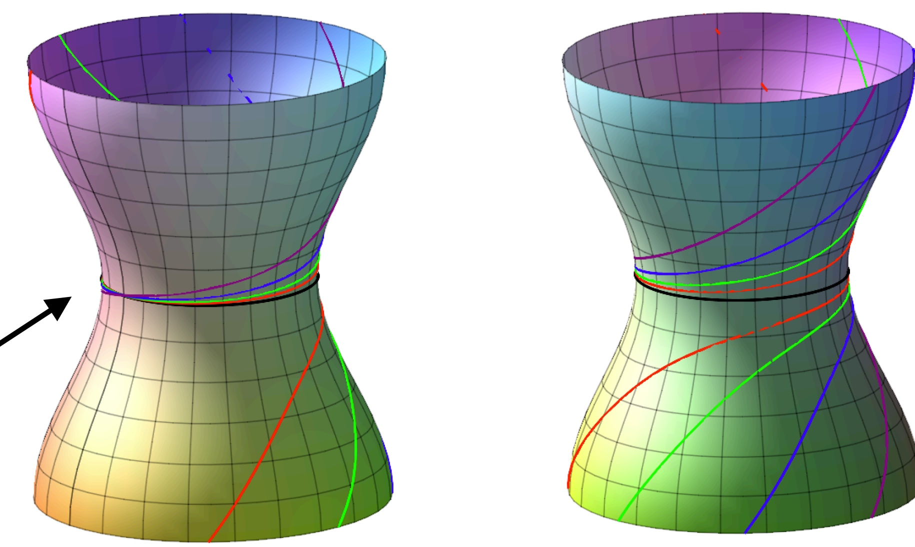
Exotic MOTSs are unstable

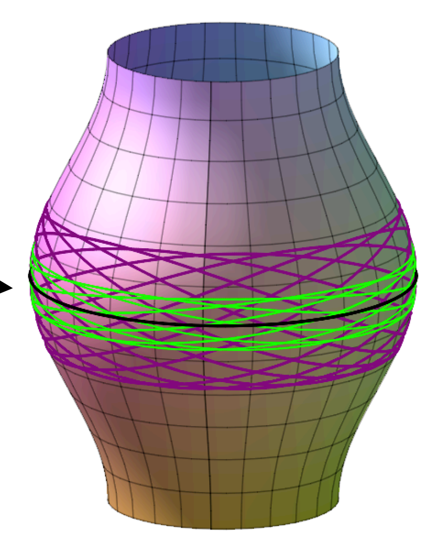


Stable



Should you care?

Unstable

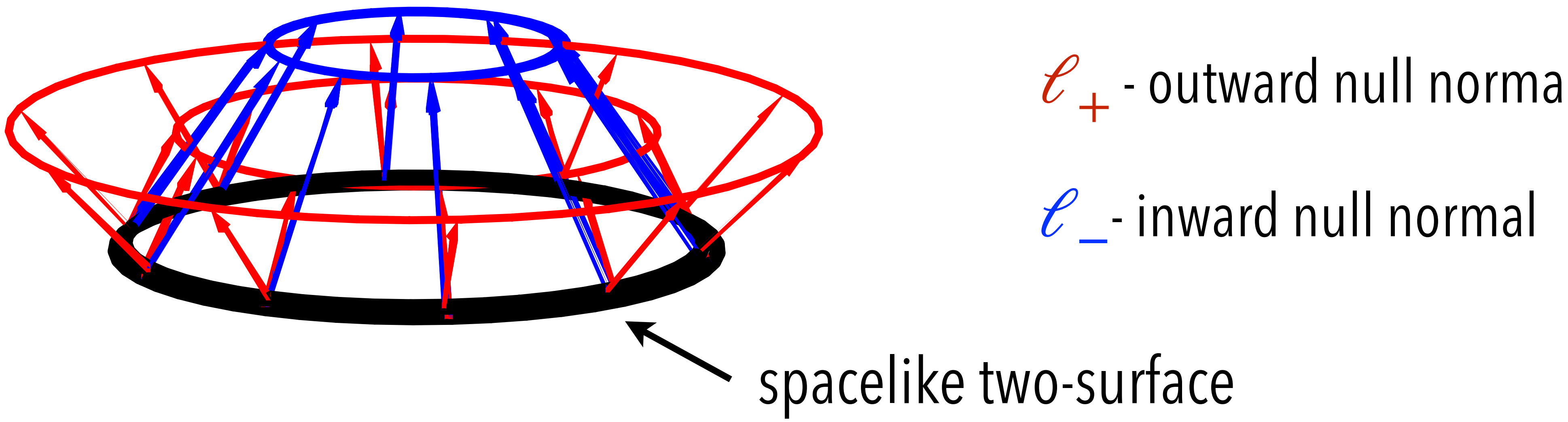


Ivan Booth (Memorial)
Graham Cox (Memorial)
Juan Margalef (Memorial)

MOTSs

Trapped Surfaces

How do you know if you are inside a black hole?

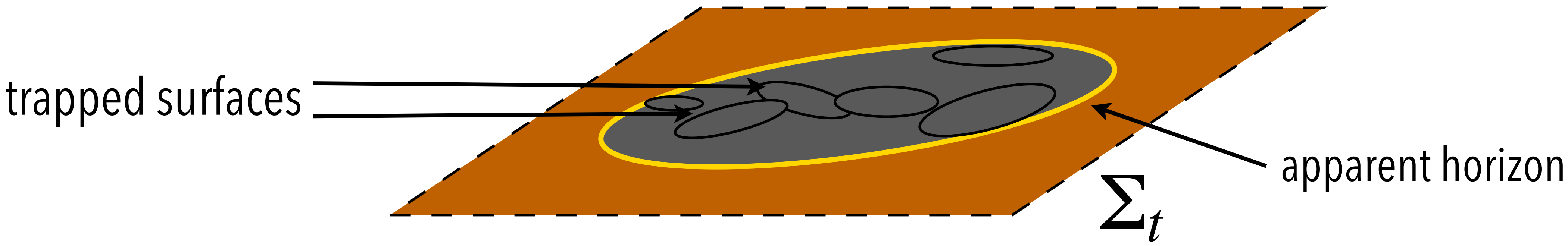


- "Regular" convex surface (ie sphere): $\theta_+ > 0$ and $\theta_- < 0$
- Trapped surface: $\theta_+ < 0$ and $\theta_- < 0$ (everything falls inwards!)
- Trapped surfaces imply the existence of singularities "inside" and event horizons "outside" (Penrose 65)

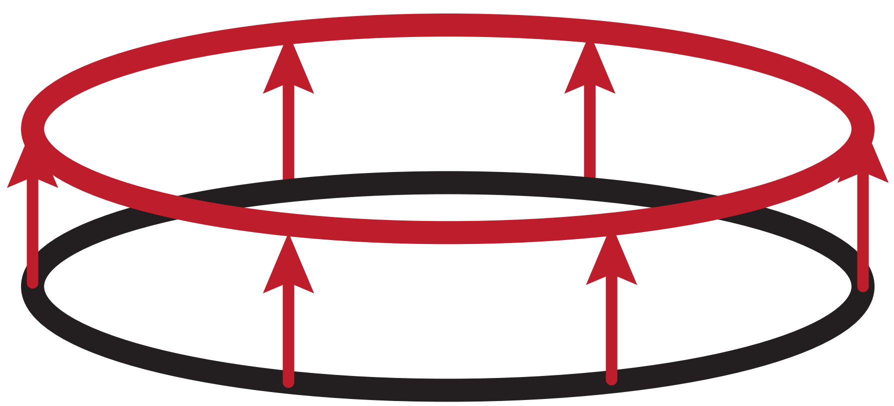
Trapped surfaces are inside black holes

MOTSs

Apparent horizons and MOTS



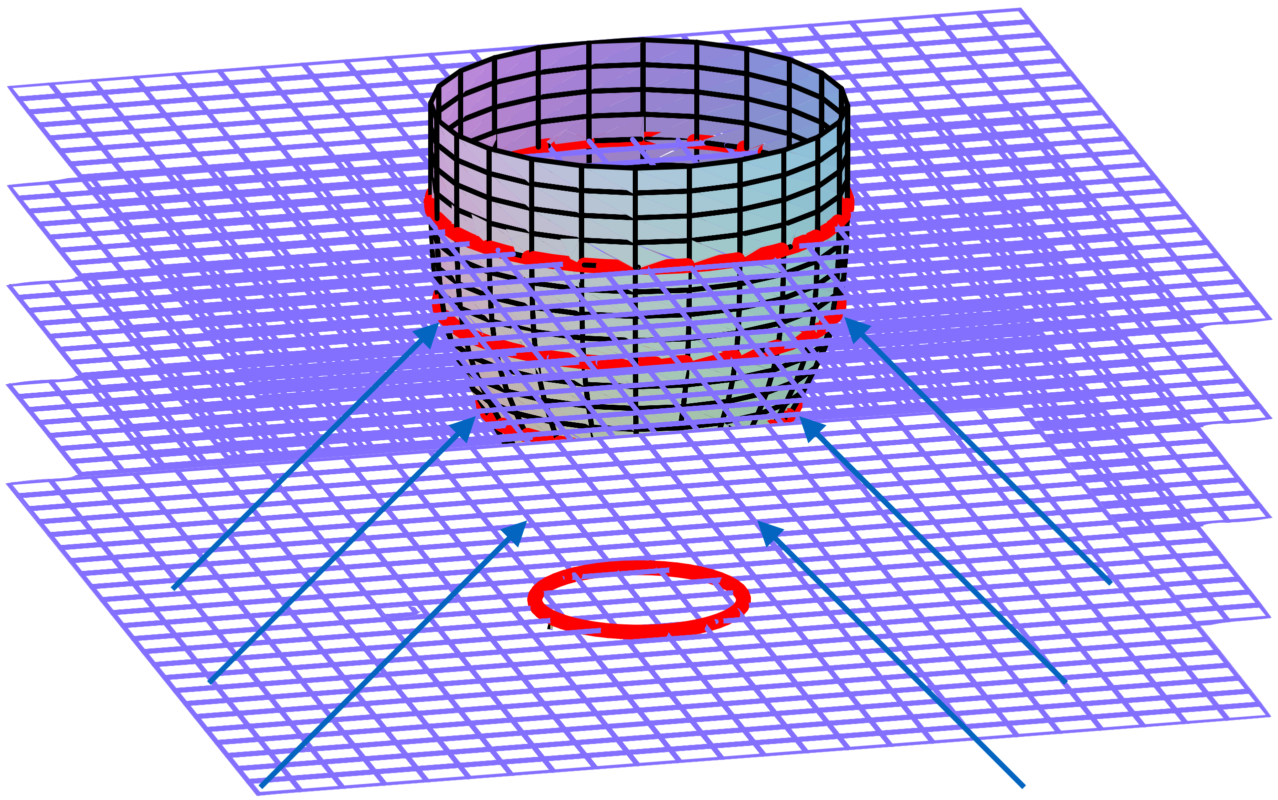
- The apparent horizon is a marginally outer trapped surface (MOTS)



- In practice mathematical and numerical relativists study *outermost MOTS* (and often call them apparent horizons)

MOTSs

Time Evolution of Black Holes



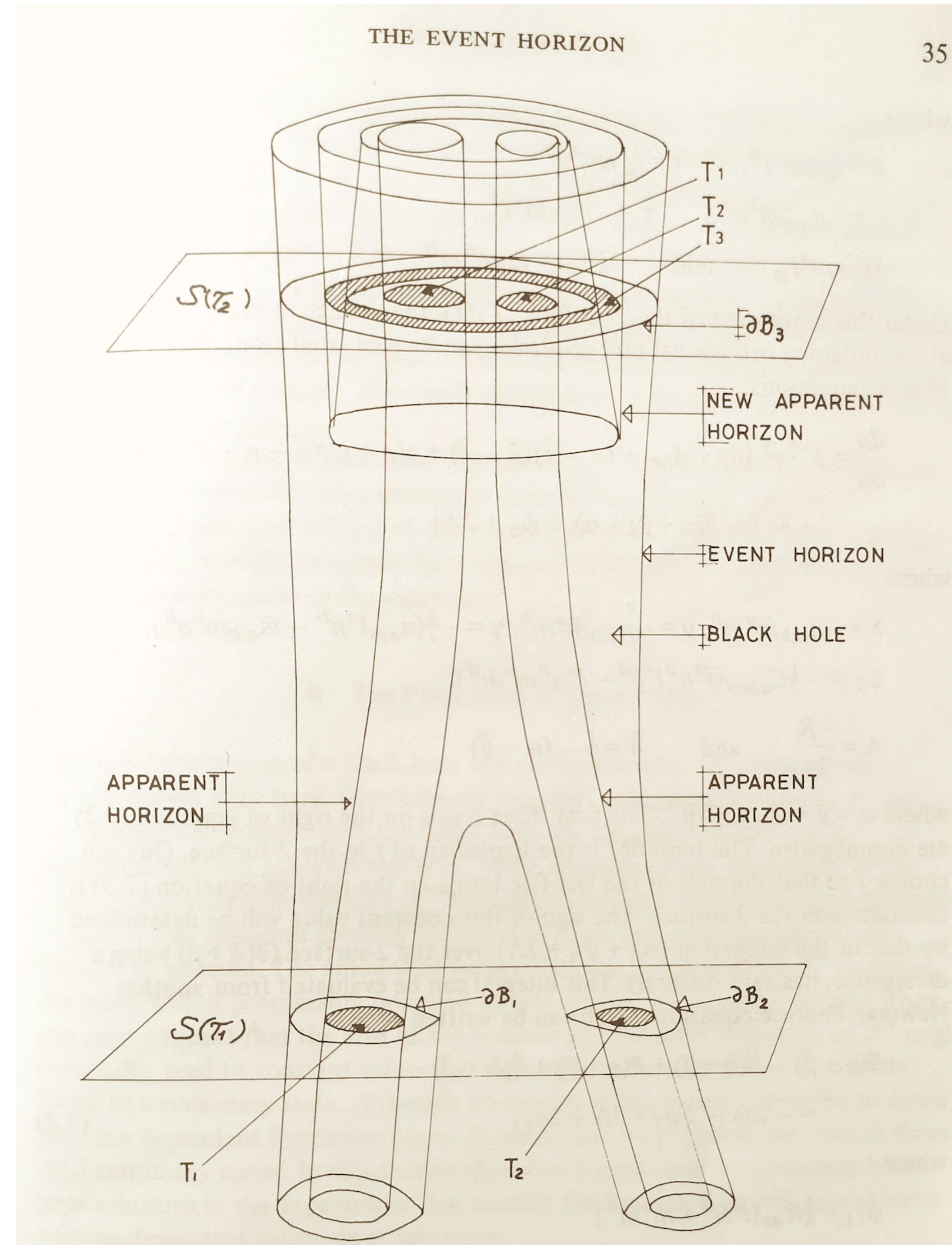
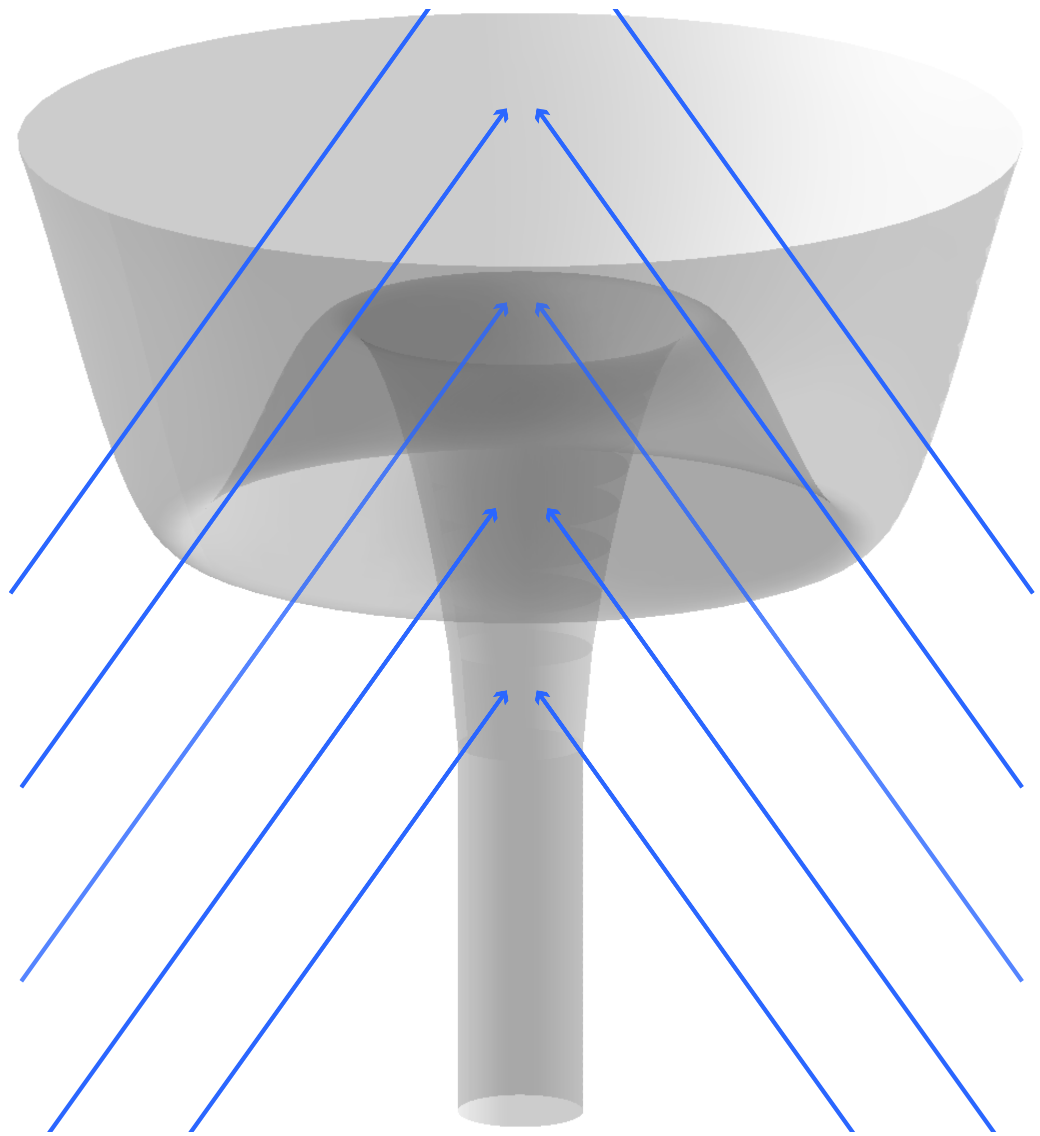


Figure 17. The collision of two Black Holes. The event horizons ∂B_1 and ∂B_2 merge to form the event horizon ∂B_3. The apparent horizons ∂T_2 do not merge but are enveloped by a new apparent horizon ∂T_3.

Hawking, Les Houches 1972

MOTSs

Outermost MOTS can ``jump''



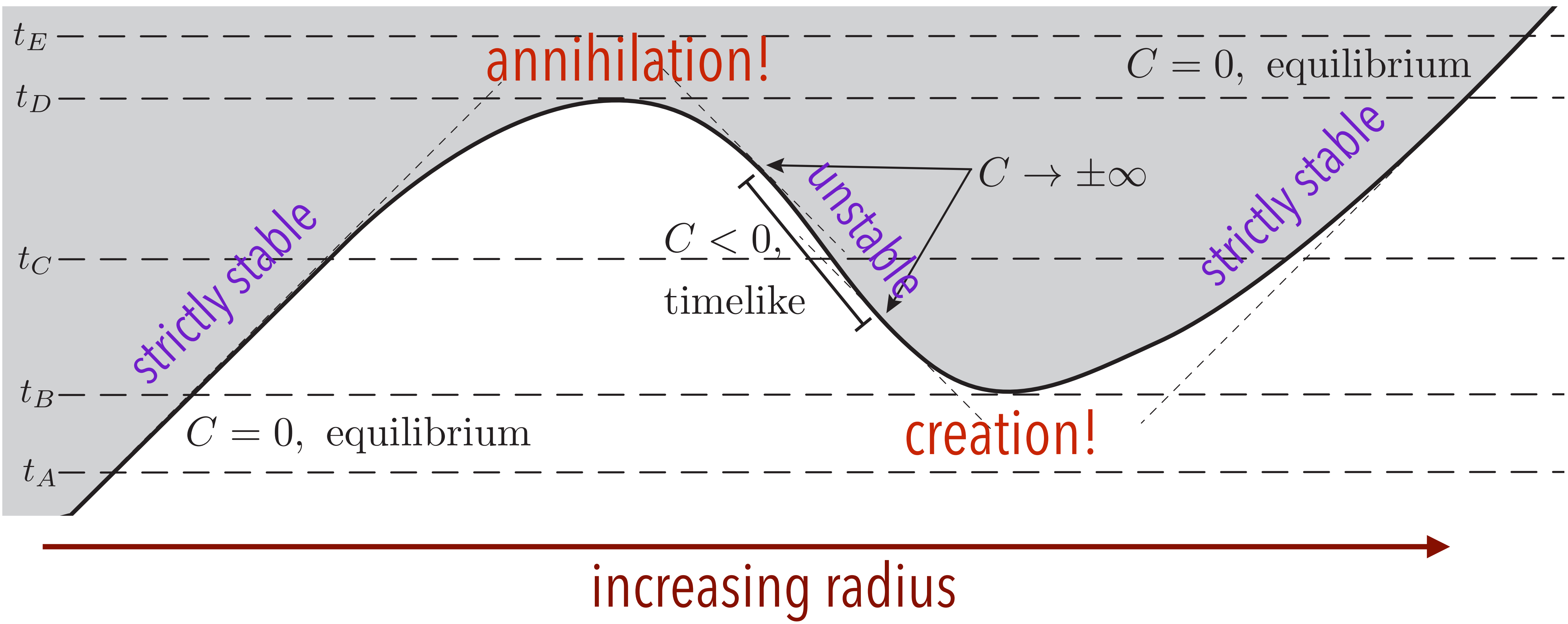
- There are exact solutions showing this behaviour

I Bendov
(FLRW + Schwarz)
PRD 2004

IB, Brits, Gonzalez,
Van Den Broeck
CQG 2006

MOTSs

MOTS “creation” and “annihilation”



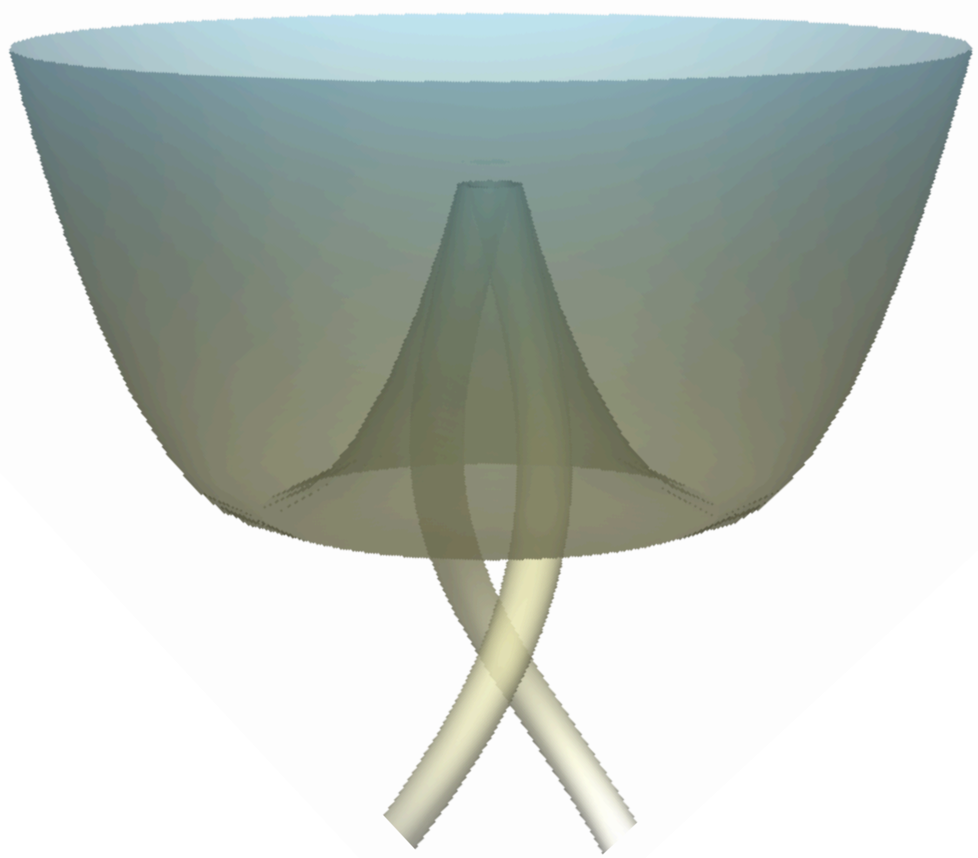
or weaving through time…

MOTSs A pair of pants for apparent horizons? (pre 2018)

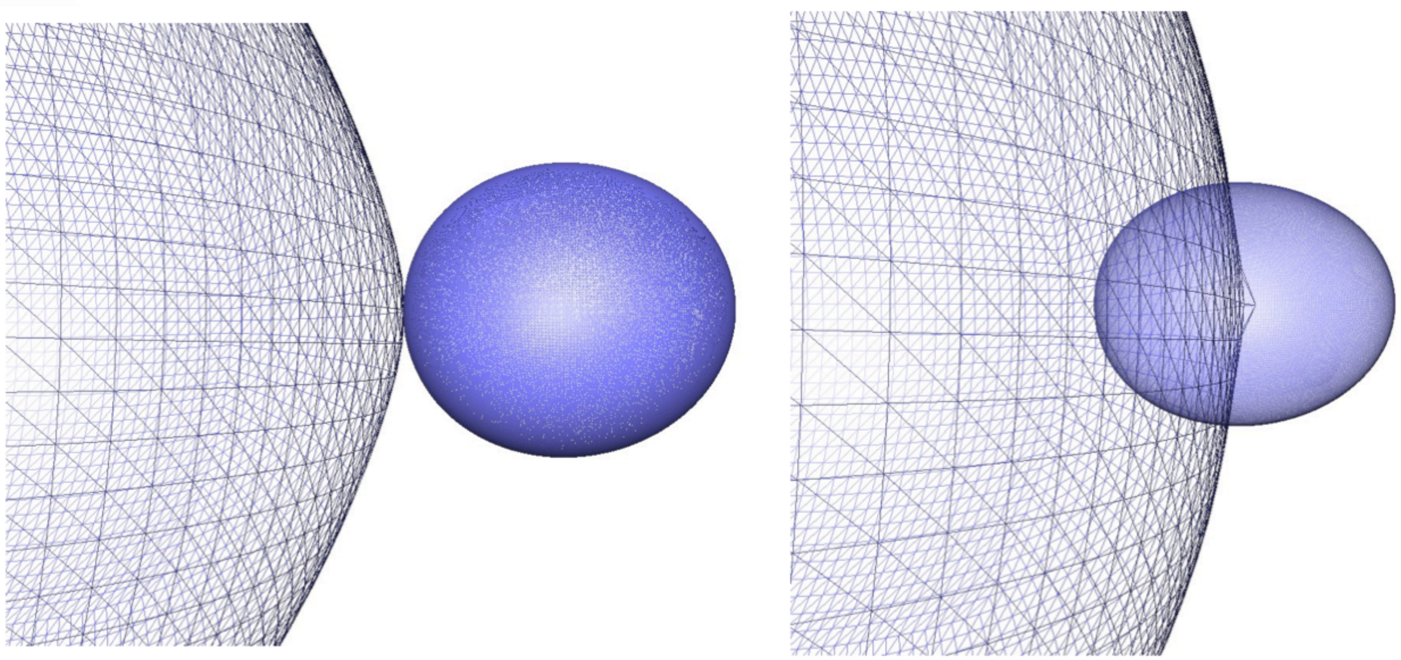
Since at least 2000: one continuous ``horizon''

(S. Hayward, Proceedings MG9, gr-qc/0008071)

Saggy pair of pants?



(Gupta, Krishnan, Nielsen, Schnetter, 2018, PRD97, 084028)



(Mösta, Andersson, Metzger, Szilágyi, Winicour, 2015, CQG 32, 235003)

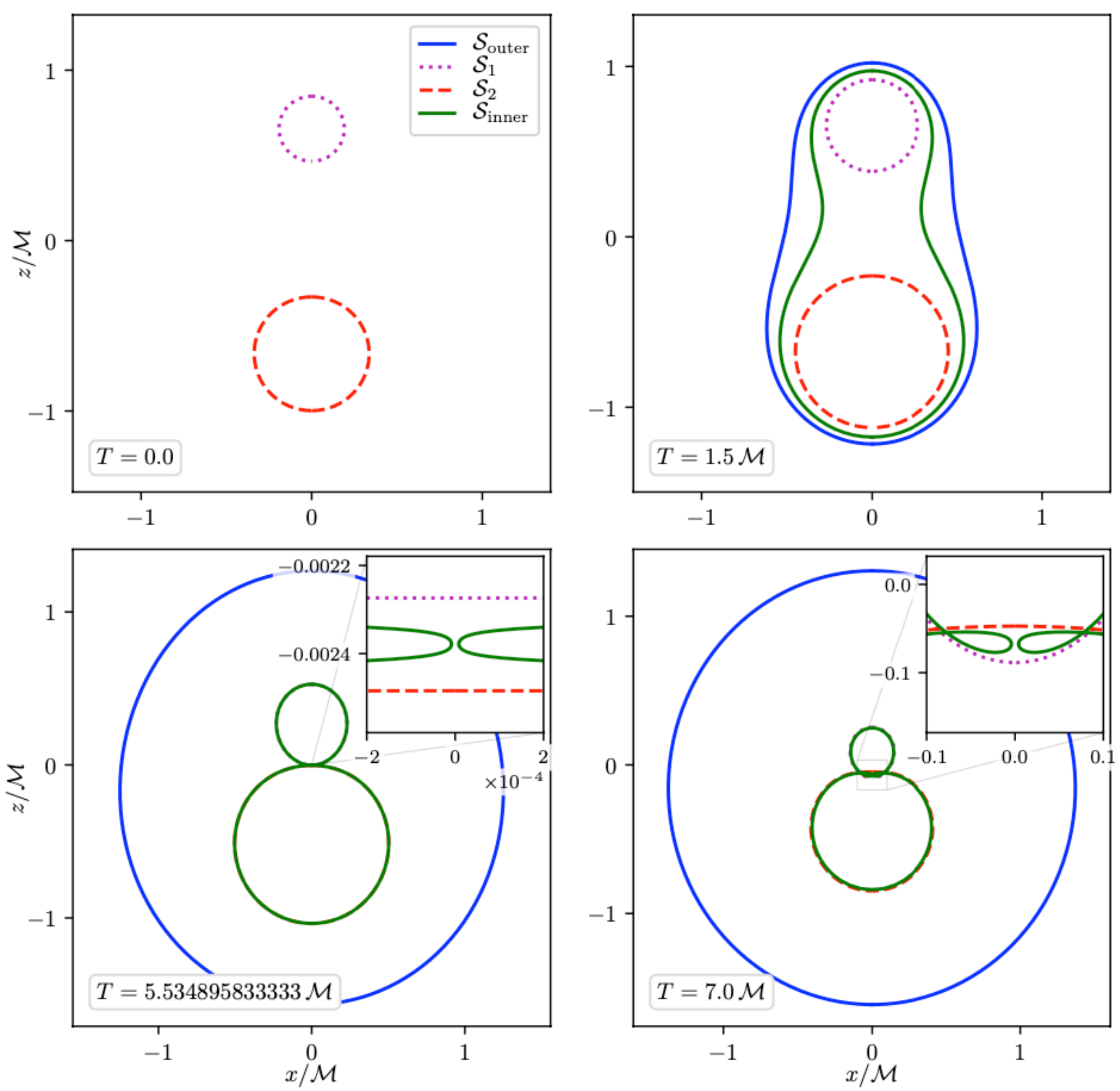
AH finders lost track of the original (and inner) horizons

Exotic MOTS

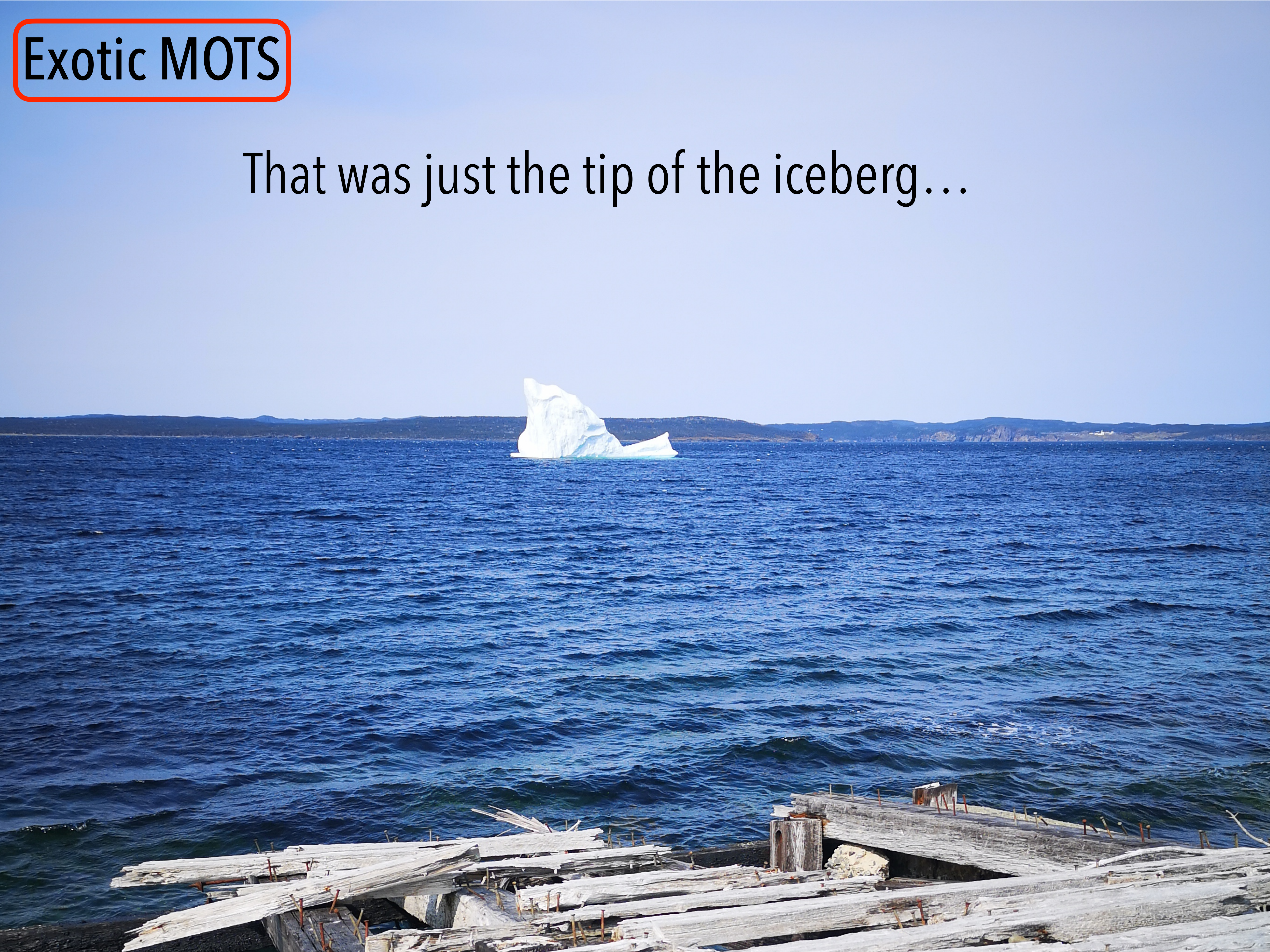
Self-intersecting marginally outer trapped surfaces

Daniel Pook-Kolb,[1,2] Ofek Birnholtz,[3] Badri Krishnan,[1,2] and Erik Schnetter[4,5,6]

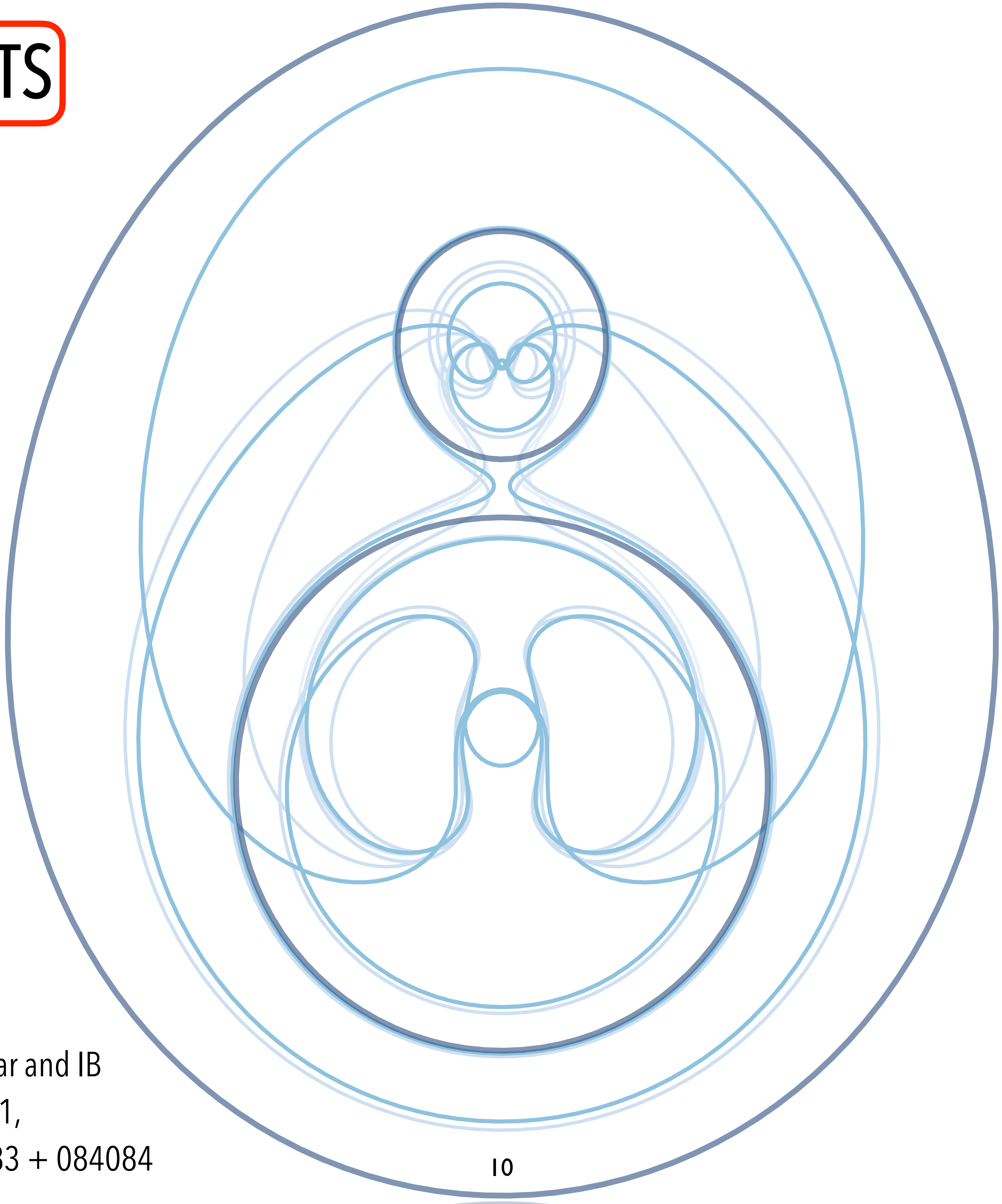
Phys. Rev. D 100, 084044 (2019)



- There are self-intersecting MOTS


Exotic MOTS
That was just the tip of the iceberg…

Exotic MOTS



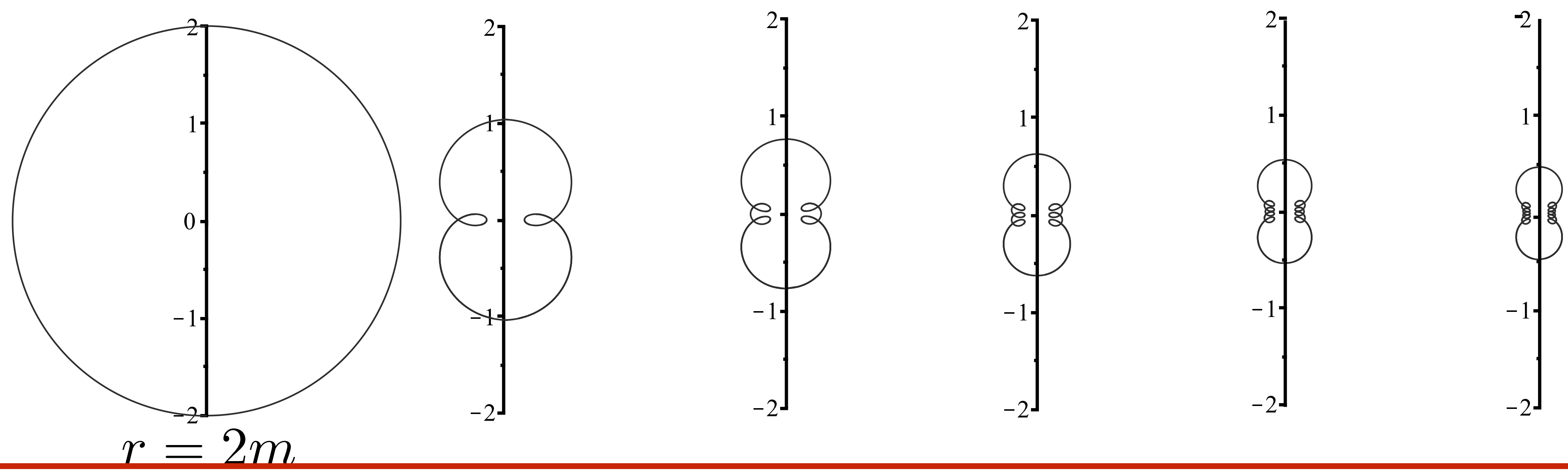
D.Pook-Kolb, R.Hennigar and IB
PRL 127 (2021) 181101,
PRD 104 (2021) 084083 + 084084

Exotic MOTS

Exact Solutions

Marginally outer trapped surfaces in the Schwarzschild spacetime: Multiple self-intersections and extreme mass ratio mergers

PHYSICAL REVIEW D **102,** 044031 (2020) IB, Robie Hennigar, Saikat Mondal



How generic are these?

Very!

Observed in (IB, KTB.Chan, R.Hennigar, H.Kunduri, S.Mondal, S.Muth, L.Newhook, M.Tavayef, Z.Hoyles) PRD 105 (2022) 4, 044024; CQG 40 (2023) 9, 095010 + preparation):

assorted Schwarzschild coordinates, Reissner-Nordström, Kerr, Gauss-Bonnet, LQG-inspired, 5D (Schwarzschild + Myers Perry), Tolman-Bondi, deSitter and SdS

Exotic MOTS

Should you care?

No:

- This is all inside the event horizon (unobservable)
- Not a source of gravitational waves

Exotic MOTS

Should you care?

No:

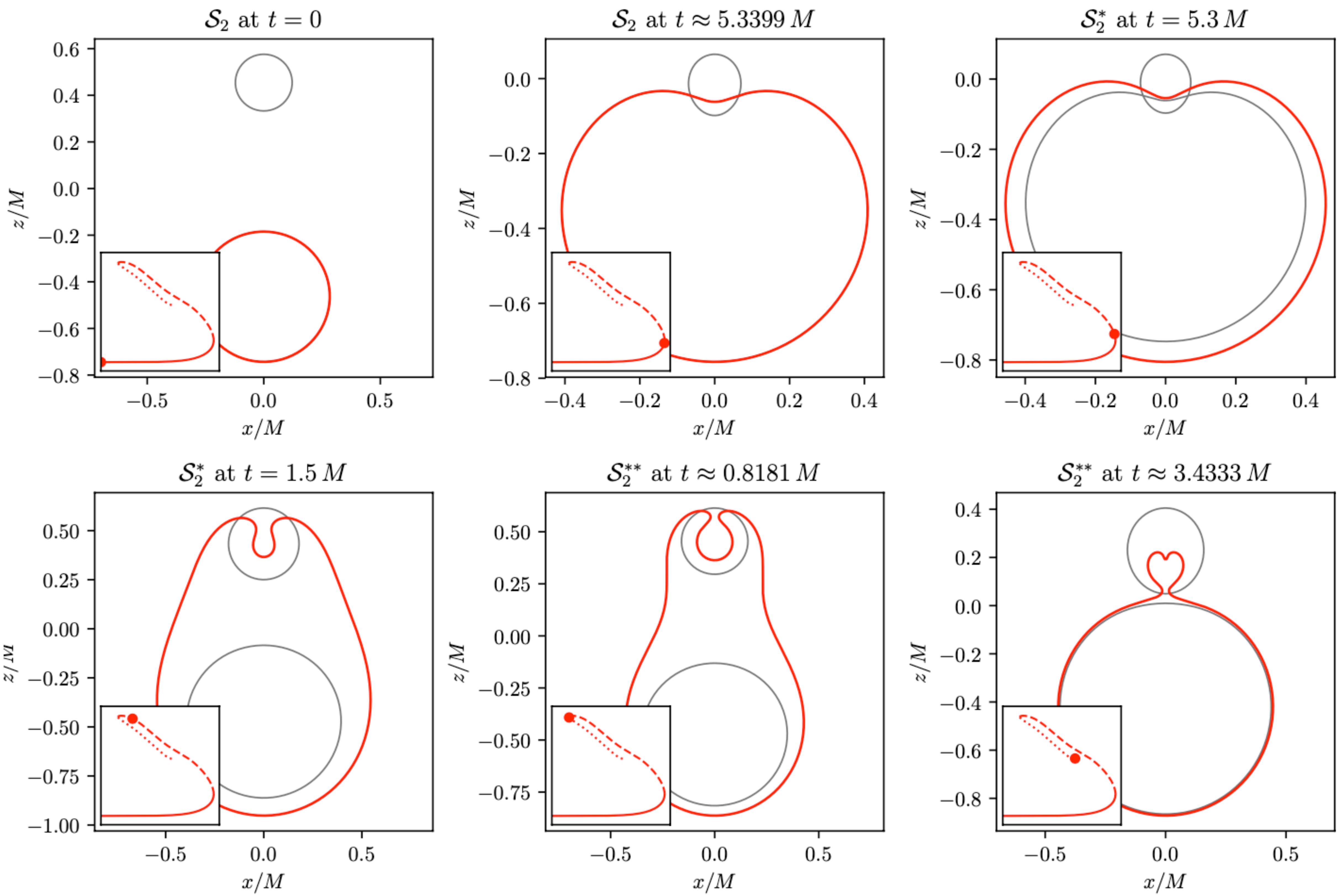
- This is all inside the event horizon (unobservable)
- Not a source of gravitational waves

Yes:

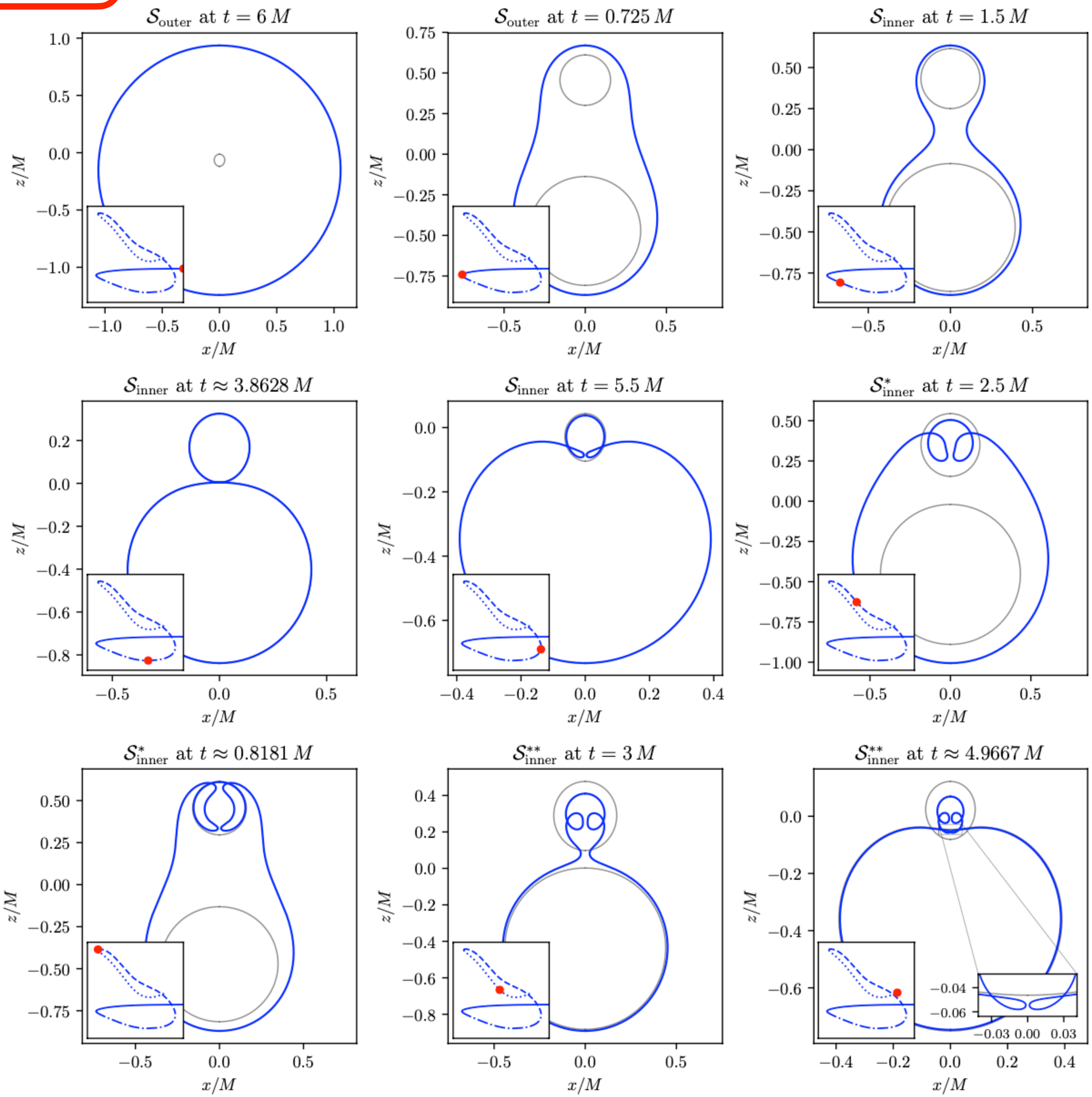
- They play a core role in horizon mergers

Exotic MOTS

Annihilation of original apparent horizons



Exotic MOTS Outer/inner split: one of many



Should you care?

No:

- This is all inside the event horizon (unobservable)
- Not a source of gravitational waves

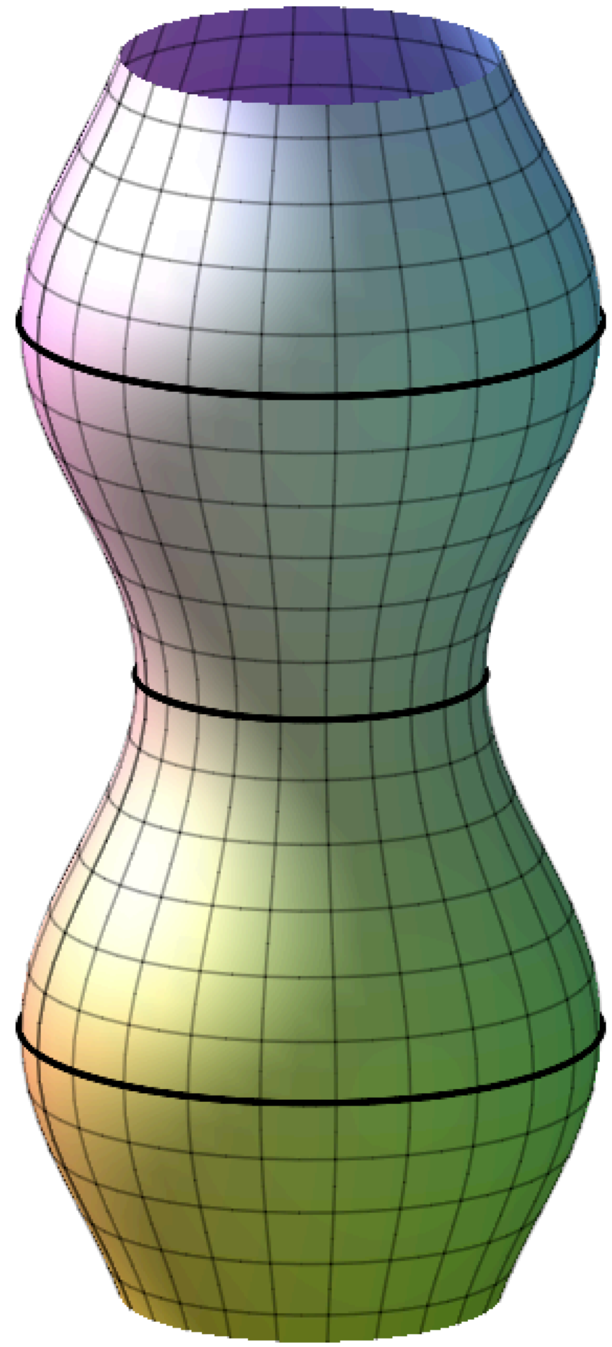
Yes:

- They play a core role in horizon mergers
- Need tools to interpret numerical simulations.
  How black hole boundaries evolve is a tool.
- Physicists like to understand things…

Exotic MOTS

Should you be surprised?

- No! (in retrospect)
- Defining equations are second order
- Expect solutions for good initial data
- There are an infinite number of (open) MOTS through every point in space
- But do they close?



Geodesic "analogy"

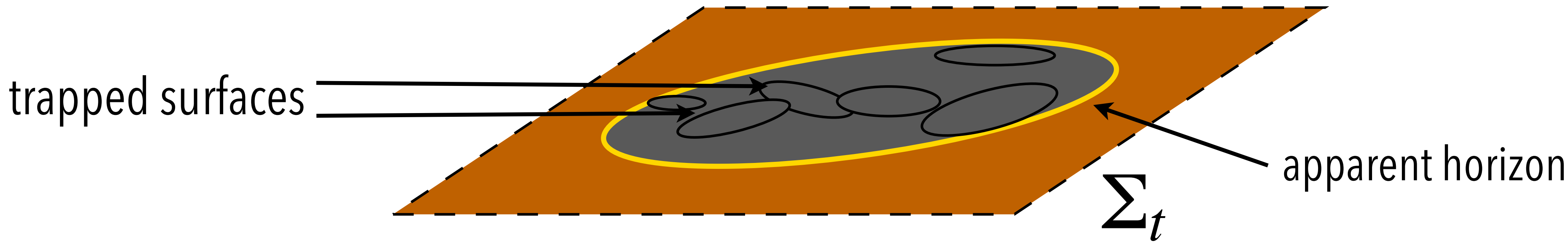
Closed geodesics (MOTS)

+ ``exotic'' closed

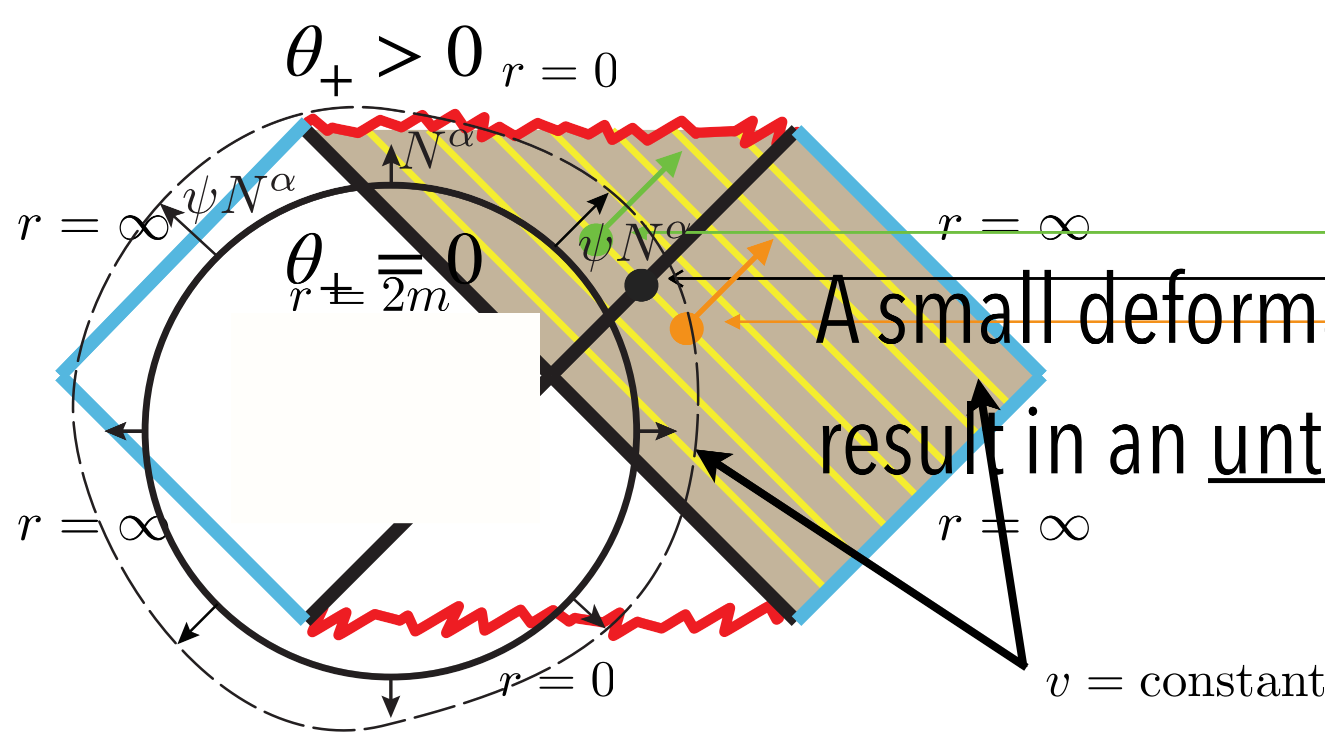
+ ``open'' geodesics
= infinite number of marginally outer trapped **open** surfaces (MOT**O**S)

Stability

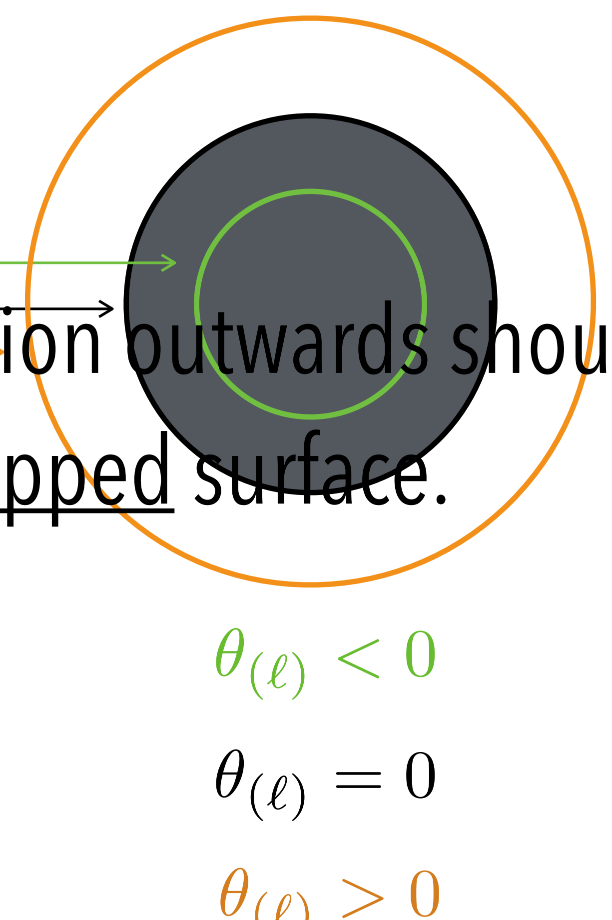
Which MOTS are black hole boundaries?



It should be a boundary between trapped and untrapped regions:



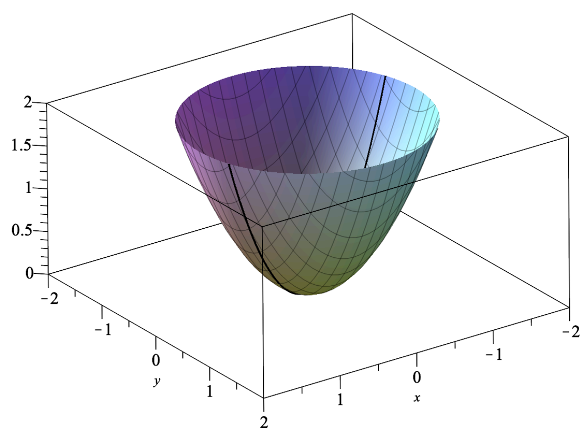
A small deformation outwards should result in an untrapped surface.

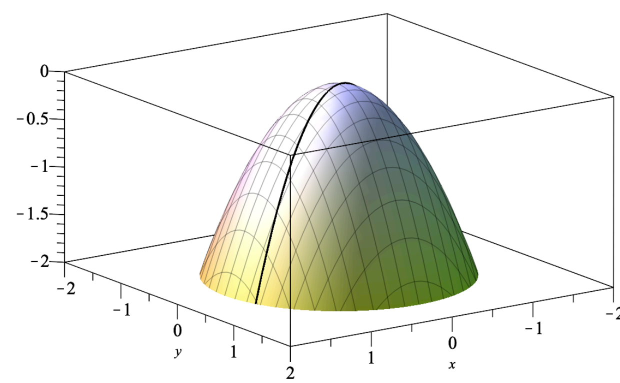


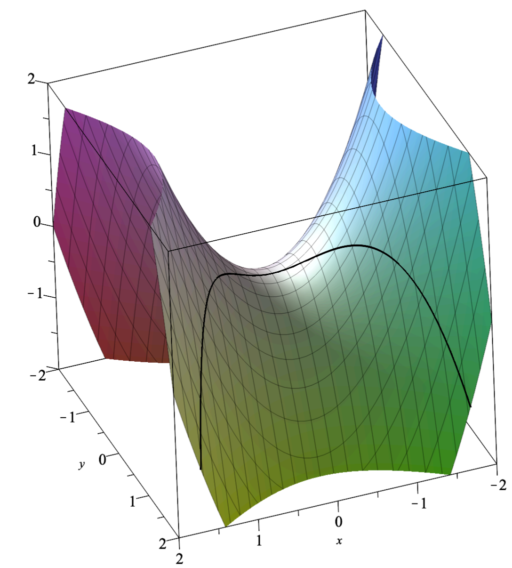
Stability

Max/Min Problems: $z = f(x, y)$

- Consider a curve $x = X(t), y = Y(t) \Longrightarrow z(t) = f(X(t), Y(t))$







Stable/Isolated: $\lambda_1\lambda_2 > 0,$ no nearby points with same value

Marginally stable/inconclusive: $\lambda_1\lambda_2 = 0$

Unstable/Not Isolated: $\lambda_1\lambda_2 < 0,$ nearby points with same value

Stability Operator (Geodesics)

- First variation: $\delta L = 0$
- Stability from second variation of length

$$\delta^2_{\psi N} L = \delta_{\psi N} k_N = -\int_{s_1}^{s_2} \psi(\mathscr{L}_\gamma \psi)\mathrm{d}s$$

where the Jacobi/stability operator is

$$\mathscr{L}_\gamma \psi = \left(\frac{\mathrm{d}^2}{\mathrm{d}s^2} + K\right)\psi$$

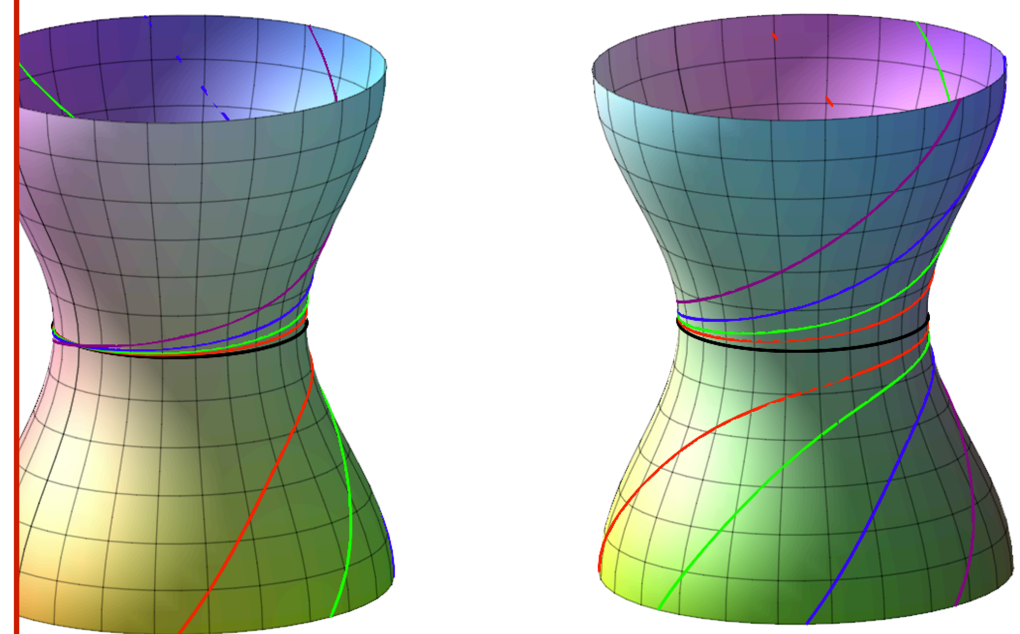
- Eigenvalue spectrum of $\mathscr{L}_\gamma$ determines stability:

$\lambda_n > 0 \implies$ stable (minimum length)

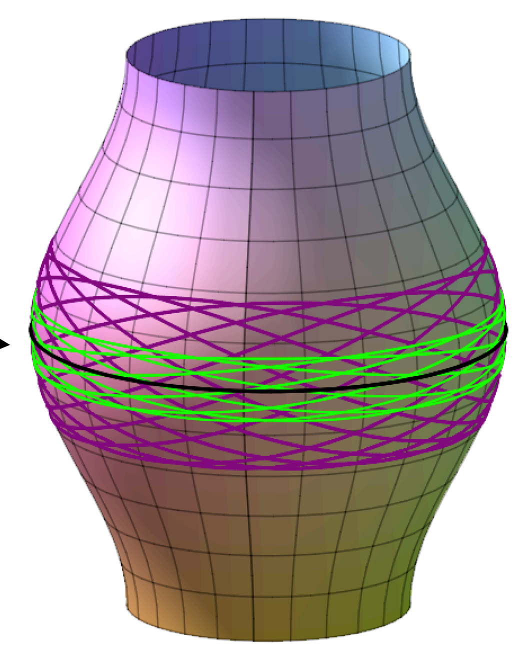
$\exists \lambda_n < 0 \implies$ unstable, conjugate points for nearby geodesics

- Eigenfunctions = basis for deformations

Stable (no nearby close geodesics)



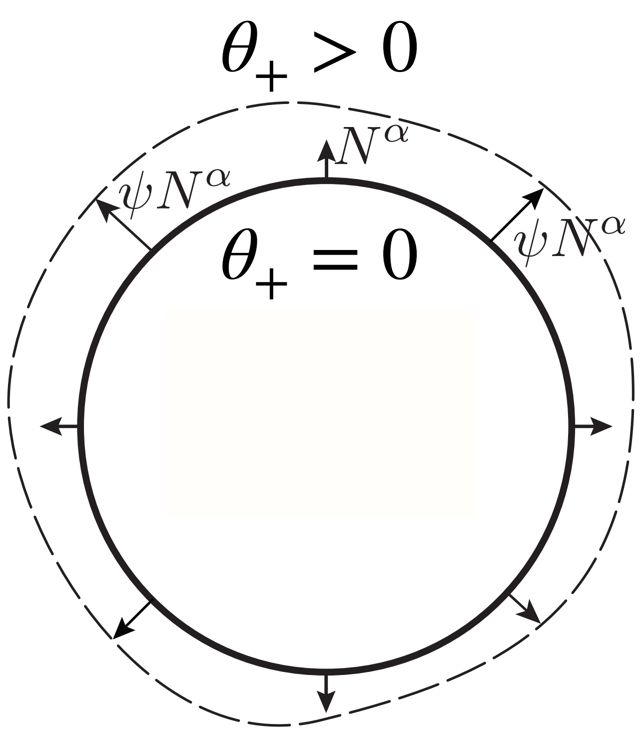
Unstable (nearby closed geodesics)



Stability

Stability Operator (MOTS)

- There is a similar stability operator for MOTS



$$\delta_\psi \theta_+ = \mathscr{L}\psi = -\triangle\psi - 2\tilde{\omega}^A d_A \psi + (K - 2\|\sigma_+\|^2 + \|\omega\|^2)\psi$$

- There is a principal (real, smallest) eigenvalue λ_0
- $\lambda_0 > 0 \implies$ (real parts of) all eigenvalues positive

  $\implies$ a boundary between trapped and untrapped
- $\lambda_0 < 0 \implies$ there are modes for which θ_+ increases

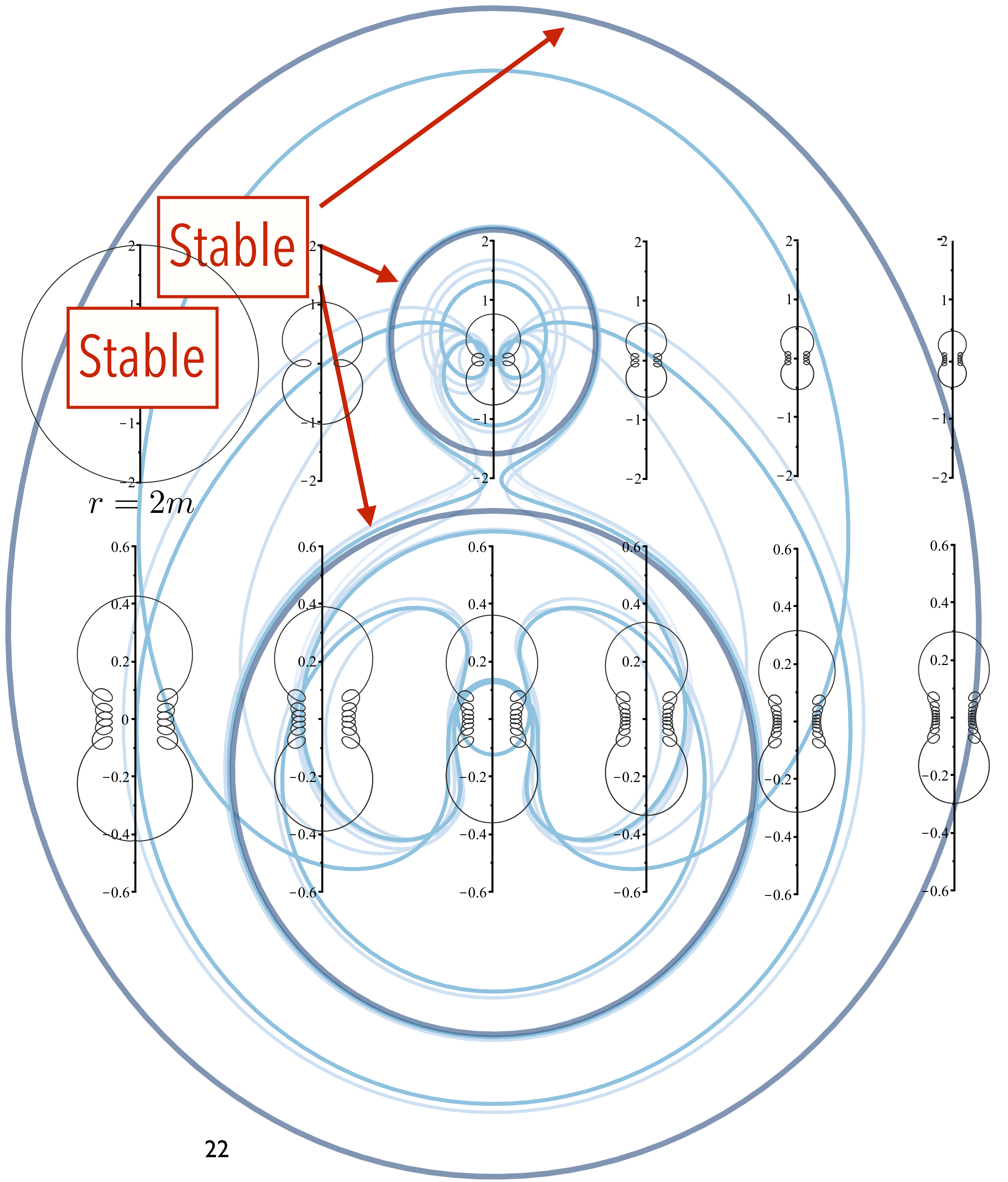
  $\implies$ there are modes for which θ_+ decreases

  $\implies$ not a boundary

(Introduced by Andersson, Mars, Simon PRL 2005 / Hayward PRD 2005)

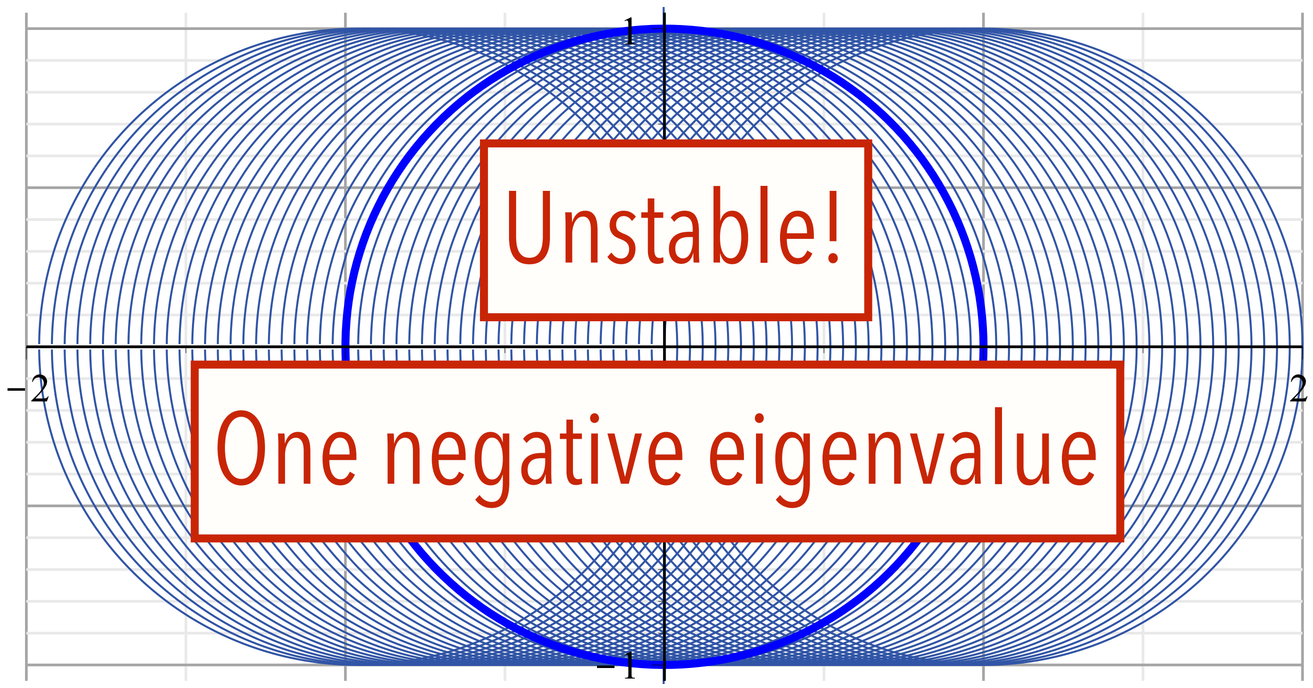
Stability

- <u>Binary mergers:</u>
  only the initial and final outer AHs are stable
- <u>Stationary solutions:</u>
  only the outer AH is stable
- <u>BUT:</u>This is purely computational and case-by-case



MOTS in deSitter Spacetime

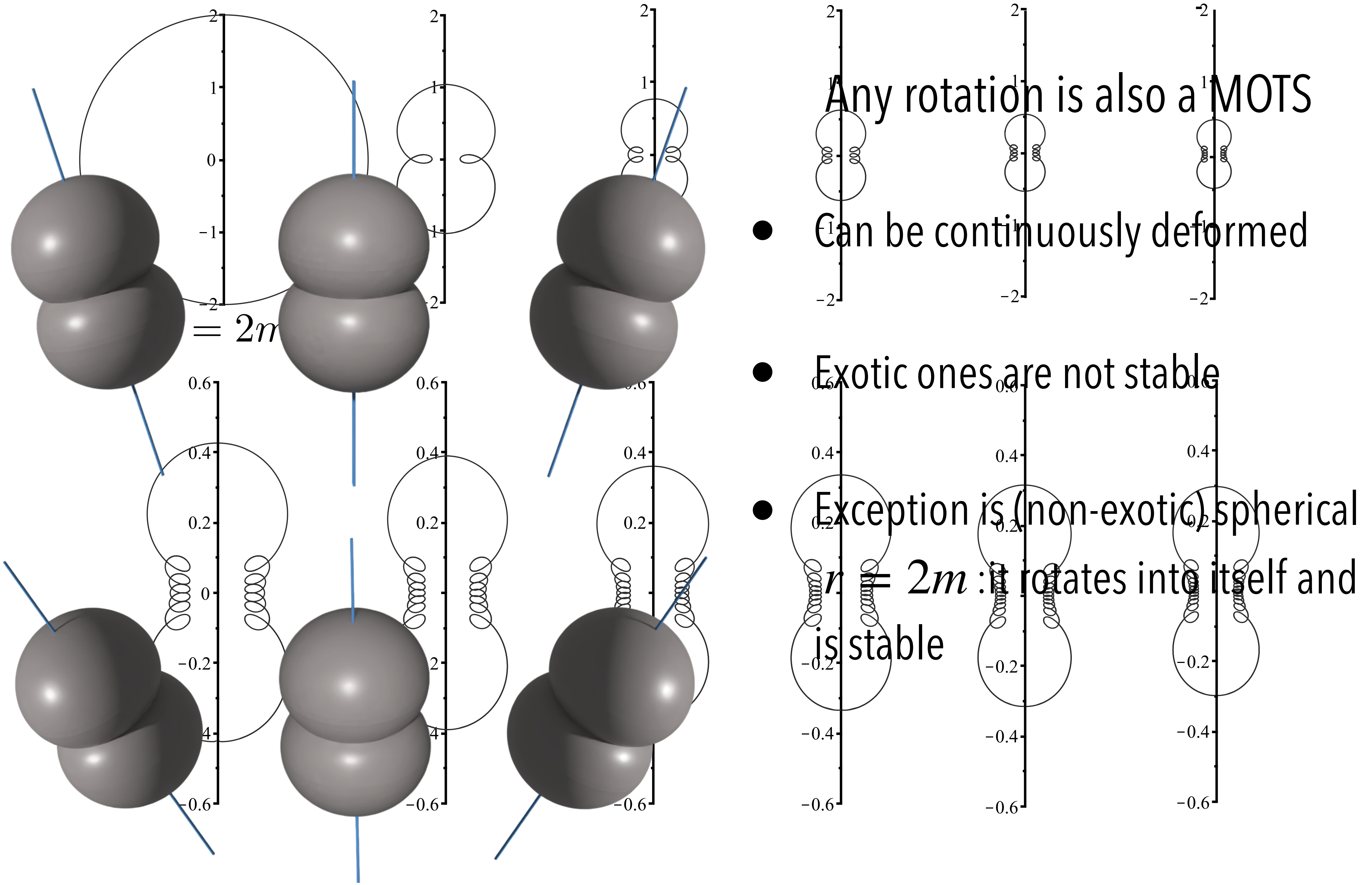
- deSitter is homogeneous and isotropic
- There is a cosmological horizon of areal radius $\sqrt{\frac{3}{\Lambda}}$ centred around <u>every</u> point (this is a MOTS)



IB, H Kunduri, A O'Grady
CQG 2017

- There are continuous "deformations" of the cosmological horizons
- These horizons are not boundaries

Exotic MOTS in Schwarzschild - Revisited

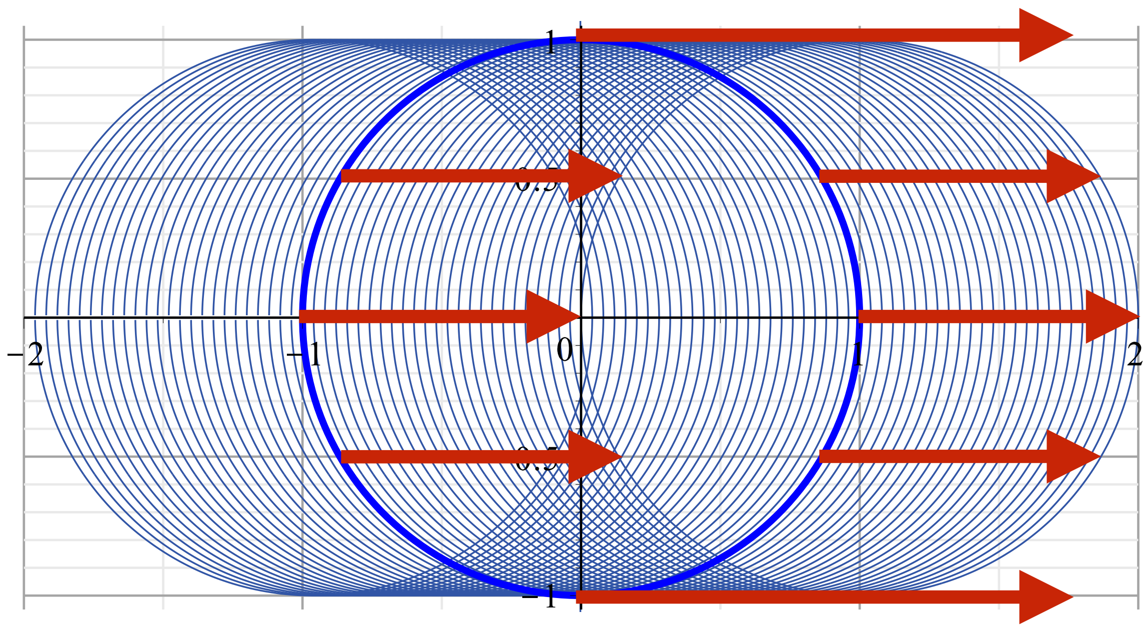


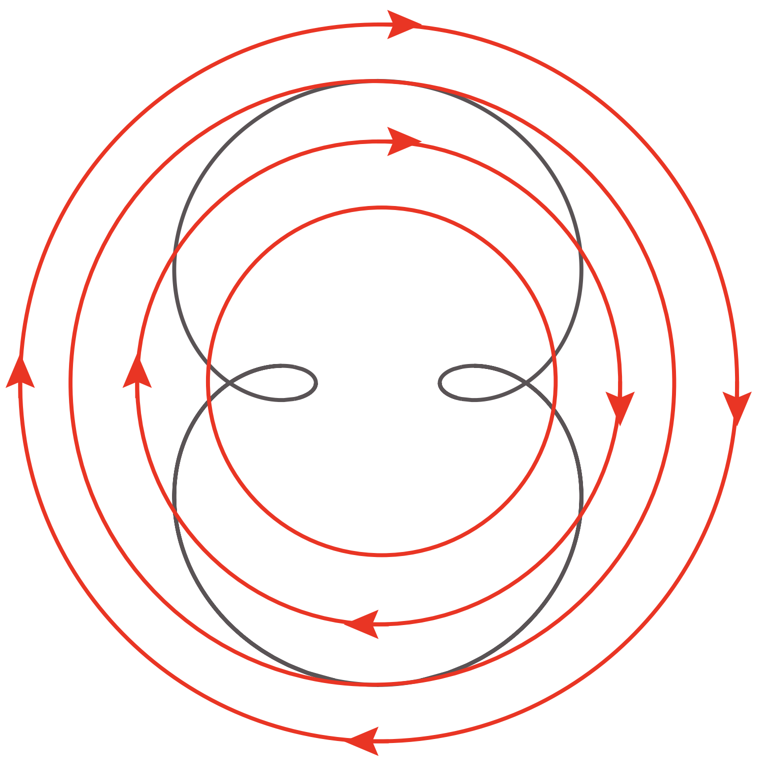
- Any rotation is also a MOTS
- Can be continuously deformed
- Exotic ones are not stable
- Exception is (non-exotic) spherical $r = 2m$: it rotates into itself and is stable

Highlighted Theorem:

Let S be a MOTS and X be a symmetry of (Σ, h_{ij}, K_{ij}) but not S.

Then S is unstable if an only if X is tangent to S at some point.





- All exotic MOTS in spherically symmetric exact solutions are necessarily unstable
- Black hole boundaries must share the symmetries of the spacetime

Conclusions

- Our understanding of black hole boundaries has advanced significantly in the last few years
- There have been big surprises!
- Continues to be useful to import tools from classical DG
- There is rich set of properties still to be fully understood.