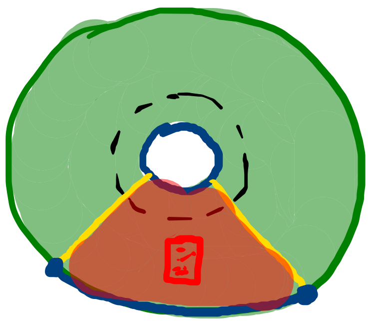
HOW TO RECOVER YOUR HOMEWORK FROM A BLACK HOLE



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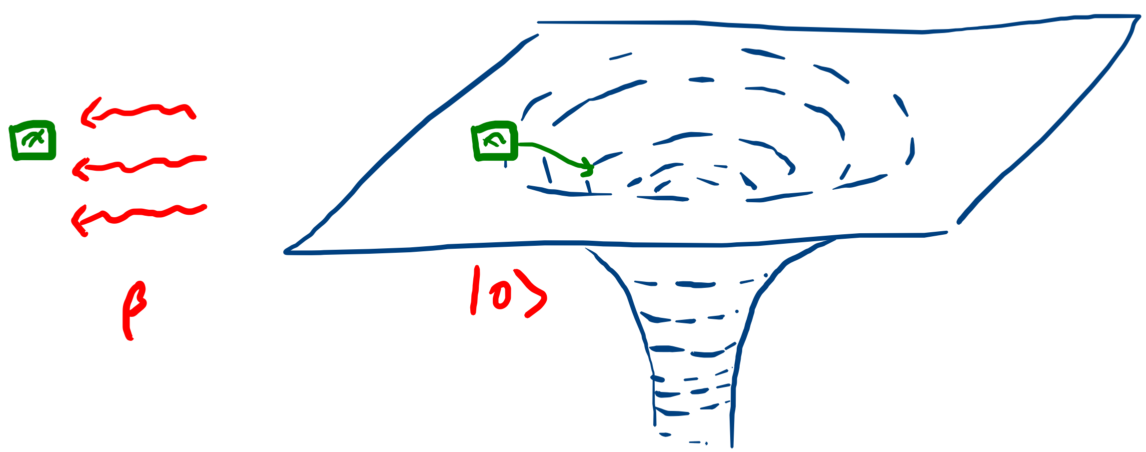
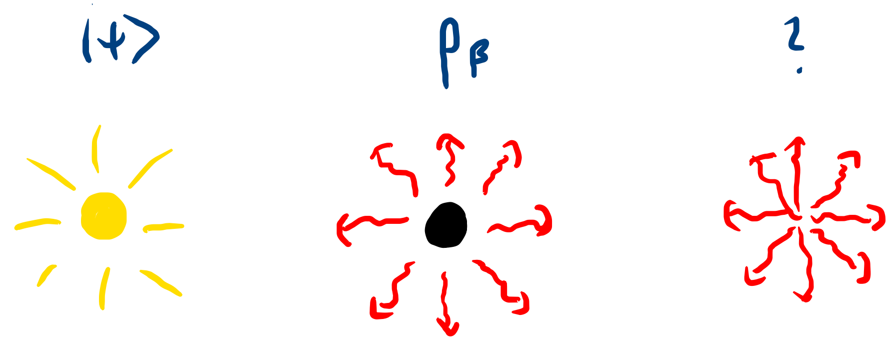
Theory Canada, 2023

- Alice takes a course on QFT in curved spacetime. Her midterm goes over the standard (Boguliovov) derivation of Hawking radiation.



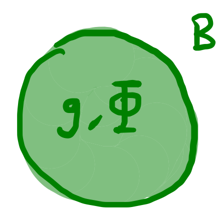
- An infalling observer sees vacuum $|0\rangle$.
- A stationary observer sees a thermal spectrum at $\beta \propto M$.

- This suggests information is turned into thermal noise.



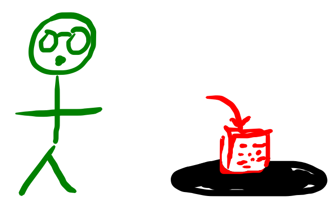
- Alice's midterm ignored metric fluctuations. But she learns a Euclidean hack for the calculation which sums over metrics:

$$Z = \int_{\partial M = B} Dg \, D\Phi \, e^{-I(g,\Phi)}$$

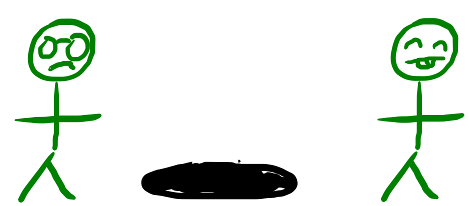


- The entropy $S = (1 - \beta \partial_\beta) \ln Z \neq 0$, so even with fluctuations the state is mixed. Semiclassical fluctuations don't help!

- Alice takes a shortcut through a condensed matter lab to give her midterm to the TA. She passes an artificial black hole.

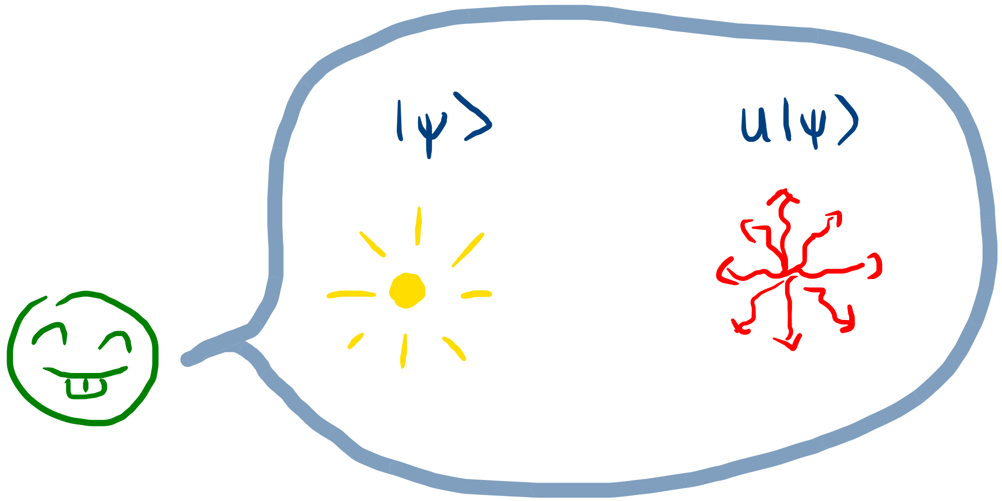


- Distracted, she accidentally drops her midterm in! Her course has become unexpectedly (and self-referentially) relevant.

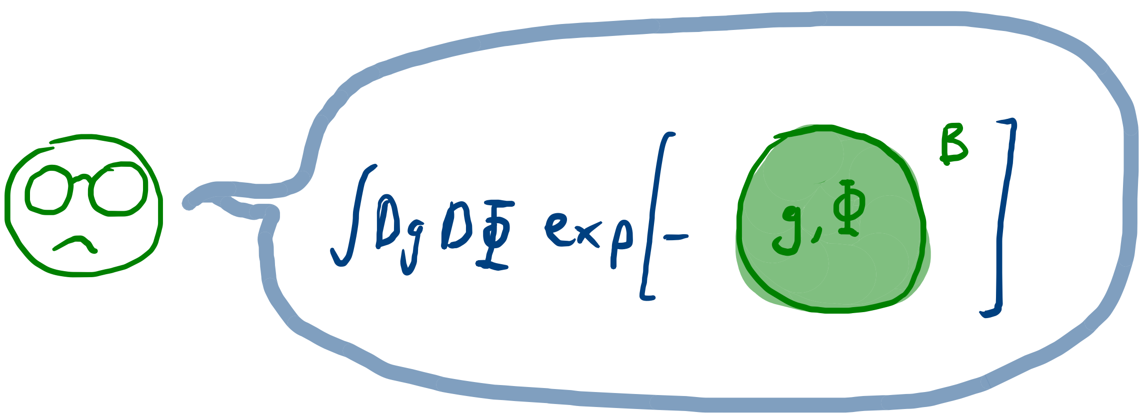


- From Hawking's calculation, Alice thinks it is lost forever. Bob disagrees!

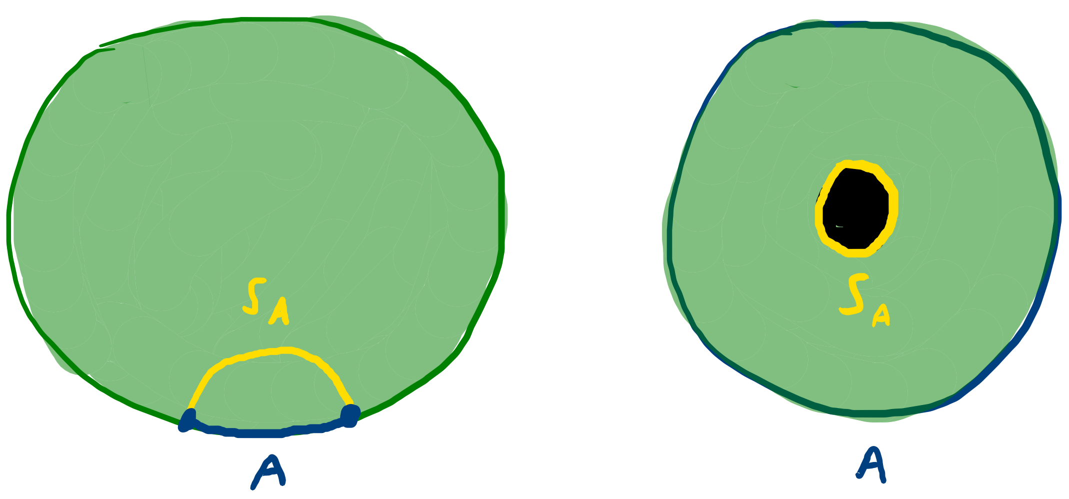
- "It can't be gone," he argues. "Nature is unitary. It only looks thermal."



- Alice wants to understand what goes wrong in the entropy calculation.



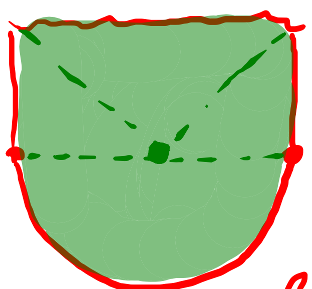
- "The partition function gives coarse-grained entropy. We need to compute fine-grained quantum entropy," Bob argues. "But how?" Alice asks.
- "Well, in AdS/CFT, fine-grained entropy of a boundary region is computed by the area of a homologous minimal surface."



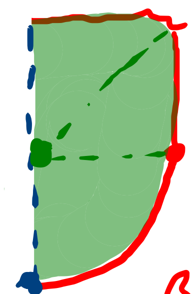
- Alice immediately notices a problem. "In Schwarzschild, the horizon is the minimal surface homologous to the whole boundary."

- Bob pauses, then suggests: "The Schwarzschild geometry must be the result of averaging over geometries. We need pure state geometries!"

- In the Euclidean formalism, black holes are periodic in imaginary time.



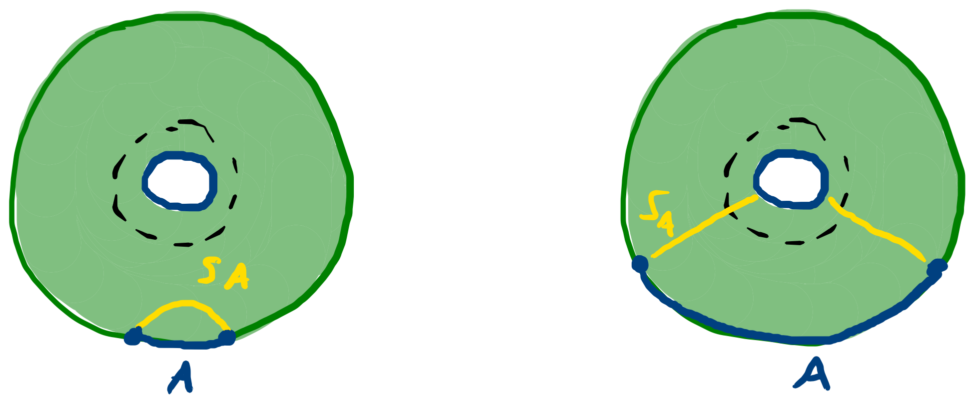
$\beta/2$



$|B\rangle$ $\beta/4$

- Averaging over the half-disk prepares a Schwarzschild black hole. We can fold in half and impose boundary conditions.

- The fold terminates spacetime in an end-of-the-world (EOW) brane. These offer an alternative place for minimal surfaces to end!

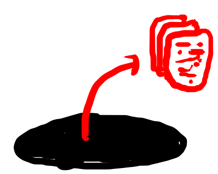


- Since the brane doesn't contribute to the minimal surface, the empty surface is now homologous to the whole boundary. The state is pure!
- A neat thing about AdS/CFT is that you can explicitly check this weird rule agrees with a microscopic calculation of entropy.

- To recap, we have a family of black hole pure states with branes, a way to compute fine-grained entropy of subregions, & consistency checks.

- This is all well and good, but as Alice points out to Bob:

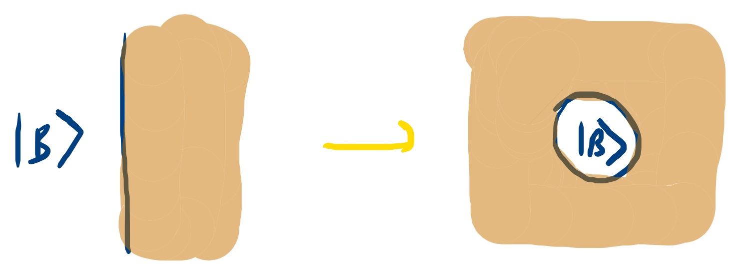
"The Hawking radiation bounces back, so there's no evaporation."

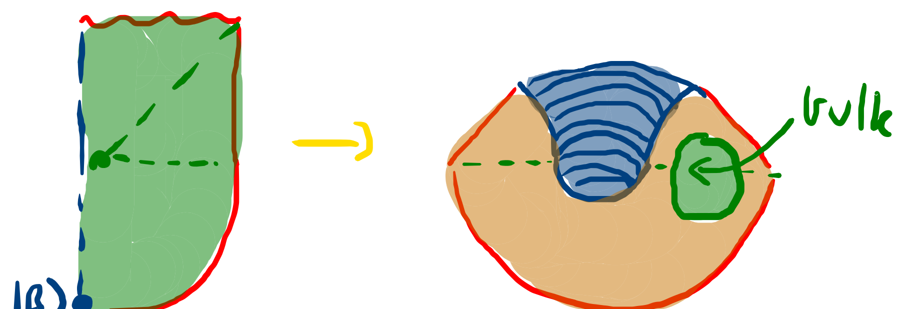


- Alice still needs her midterm!

- However, by a sneaky change of coordinates, Alice and Bob see how to make the spacetime dynamic.

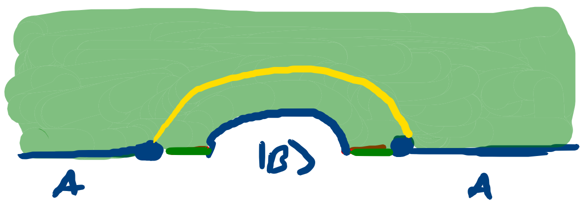
- On the boundary the fold is a line. We turn it into a circle.



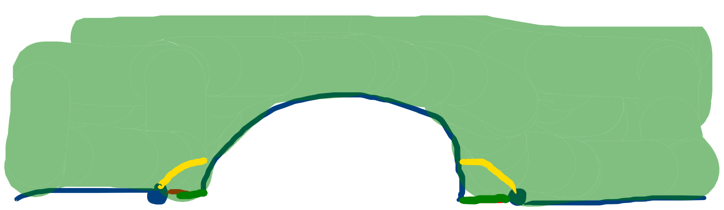


- This turns a static bulk black hole with spherical brane into a dynamic boundary black hole with an accelerating Rindler brane.

- The time-dependent behaviour of subregion entropy is of most interest.
- At early times, the minimal surface for a neighborhood of infinity (analogous to an asymptotic particle detector) is connected.

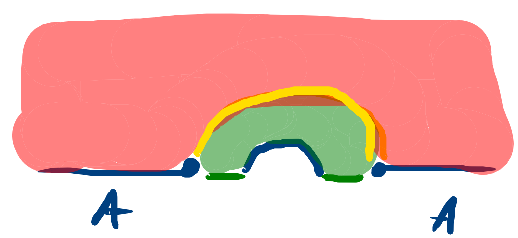


- At late times, it transitions to a disconnected surface. This looks like information escaping from the black hole!

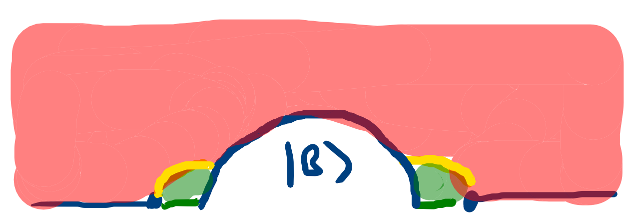


- In fact, this transition is exactly what's needed to show information is conserved, from the viewpoint of this lower-dimensional black hole.

- The pure state information is in the boundary condition $|B\rangle$.
  The green region is unknown to an observer with access to A.

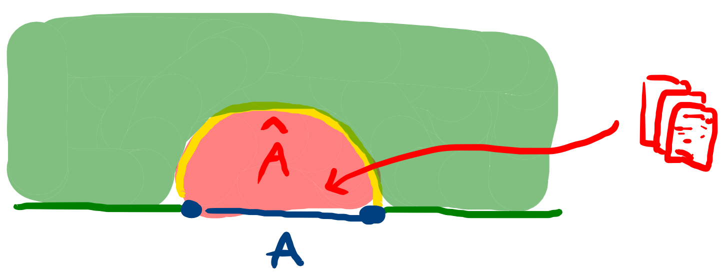


- After the transition, the red region pinches off, and part of the brane becomes known. This is sufficient to determine $|B\rangle$!



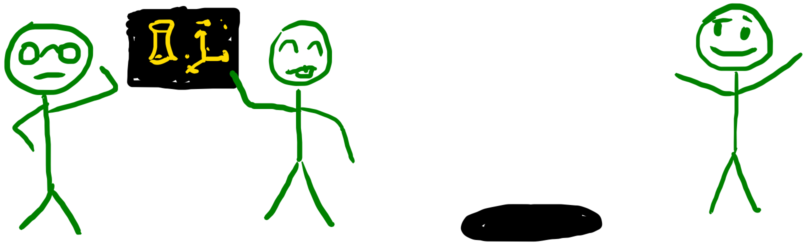
- But what does it mean to have "access" to a bulk region? And how does Alice finally get her midterm?

- The basic insight is that a boundary subregion A contains the same information as the bulk region $\hat{A}$ between A and its minimal surface:

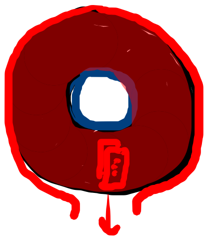


- The region $\hat{A}$ is called the entanglement wedge, and in principle we can reconstruct midterms in $\hat{A}$ from knowledge of A.

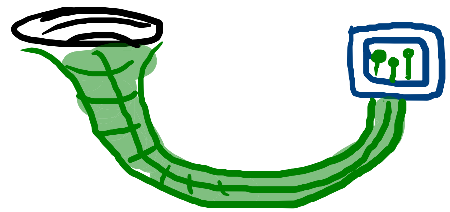
- At this point in their deliberations, Charlotte joins Alice and Bob. She is the architect of the artificial black hole.



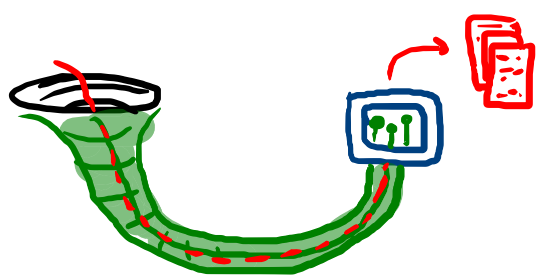
- "What are you kvetching about?" They explain their plan to use entanglement wedge reconstruction to recover the midterm.



- Charlotte sighs. "First, the black hole isn't in a boundary state. It's entangled with my quantum computer. It's a wormhole!"



- "Usually, that wormhole is non-traversable but luckily I was testing a double-trace deformation library which made it briefly traversable."



- "Your midterm popped up in my quantum inbox!"

- Alice breathes a sigh of relief. Her GPA is out of danger! Still, she regrets not being able to use her newfound knowledge.

- "I wouldn't worry too much," says Charlotte. "Sounds like you have the material for a pretty cool thesis".

∎

THANKS!