

# Scalar cosmological perturbations from full quantum gravity

In collaboration with: D. Oriti, E. Wilson-Ewing, A. Pithis, A. Jercher, P. Fischer

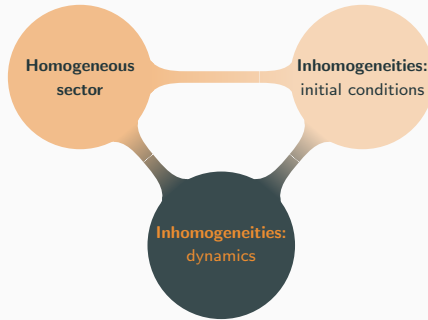
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**Luca Marchetti**

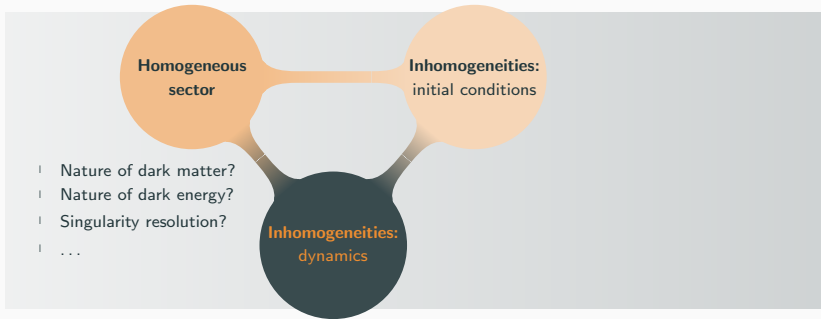
Theory Canada 15  
Mount Allison University  
17 June 2023

Department of Mathematics and Statistics  
UNB Fredericton

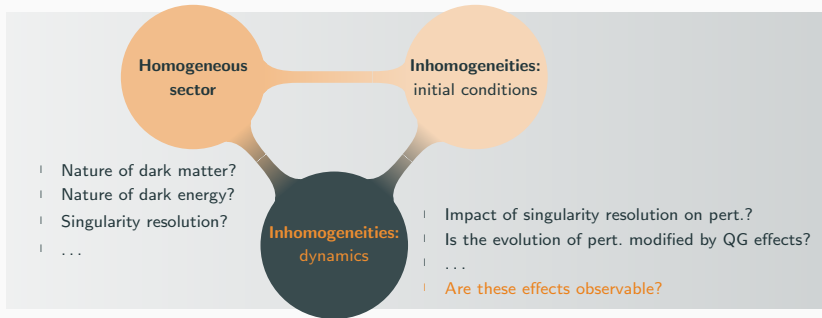
# The QG perspective on Cosmology



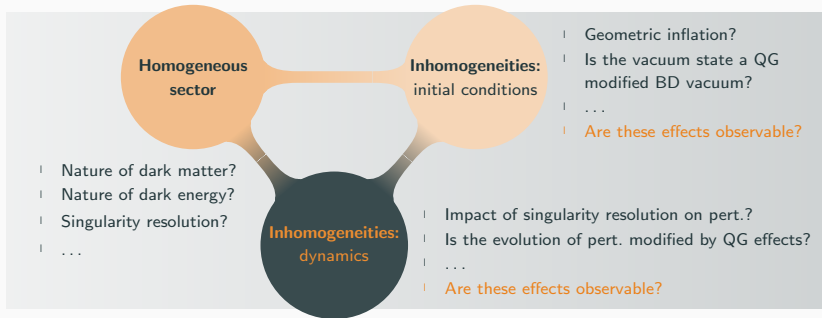
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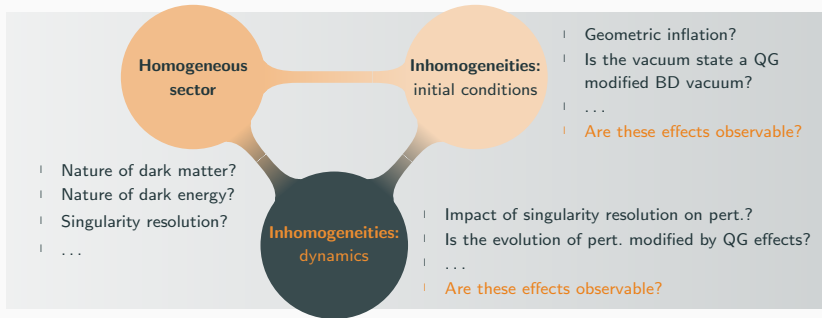
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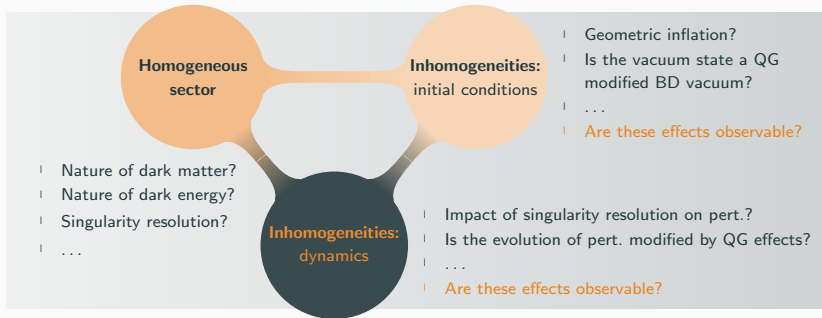
# The QG perspective on Cosmology



## Challenges in background independent and emergent QG:

- | How to define (in)homogeneity?
- | How to extract macroscopic dynamics?
- | How to construct cosmological geometries?
- | ...

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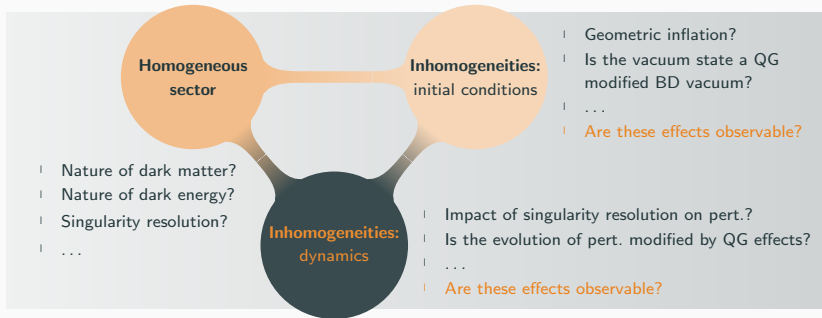


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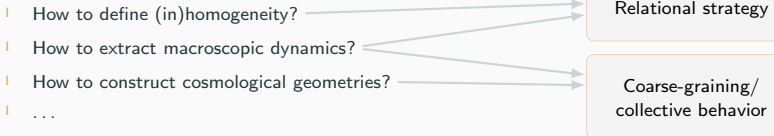
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Relational strategy

# The QG perspective on Cosmology

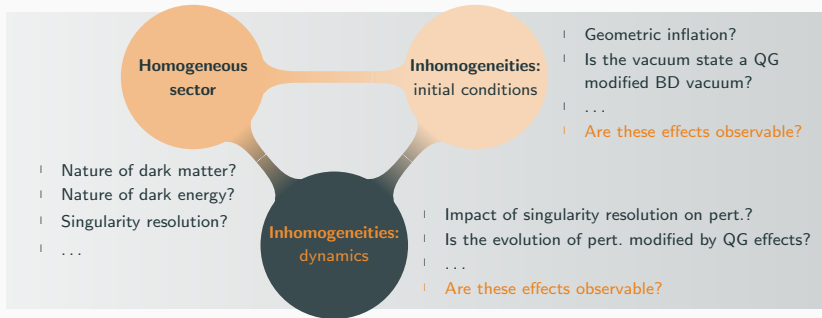


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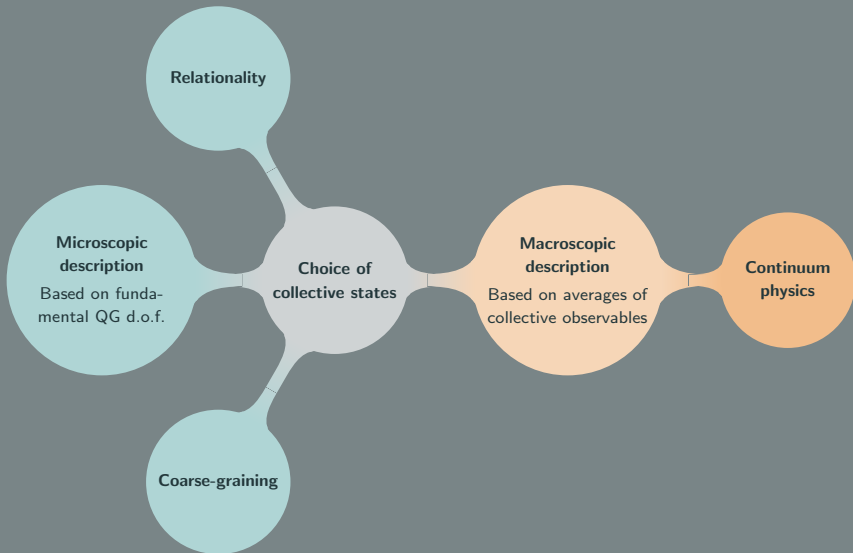
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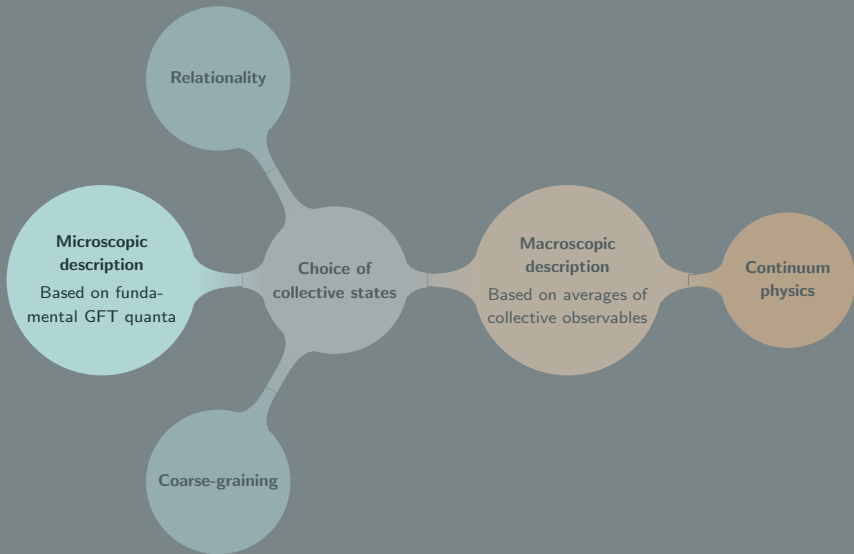
- | How to define (in)homogeneity?
- | How to extract macroscopic dynamics?
- | How to construct cosmological geometries?
- | ...

Relational strategy

Coarse-graining/  
collective behavior

Approximate only





# Introduction to Group Field Theory

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# The (T)GFT approach to quantum gravity

(Tensorial) Group Field Theories:  
theories of a field  $\varphi : G^d \rightarrow \mathbb{C}$  defined  
on  $d$  copies of a group manifold  $G$ .

$d$  is the dimension of the "spacetime to be" ( $d = 4$ )  
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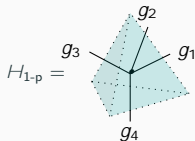
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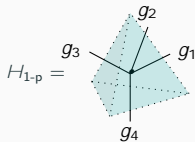
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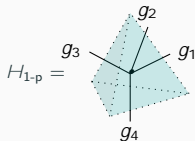
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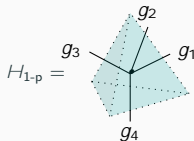
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## Processes: Discrete spacetimes

$S_{\text{GFT}}$  obtained by comparing  $Z_{\text{GFT}}$  with simplicial gravity path integral.

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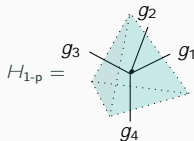
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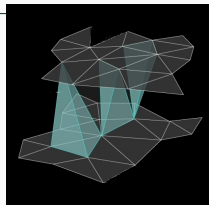


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$Z_{\text{GFT}}$  obtained by comparing  $Z_{\text{GFT}}$  with simplicial gravity path integral.

- Non-local and combinatorial interactions mimic the gluing of  $d-1$ -simplices into  $d$ -simplices.
- $\Gamma$  are dual to spacetime triangulations.

$$Z_{\text{GFT}} = \prod_{\Gamma} w_{\Gamma}(f, g) A_{\Gamma} = \text{discrete gravity path-integral.}$$



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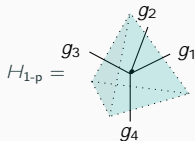
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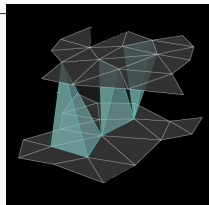


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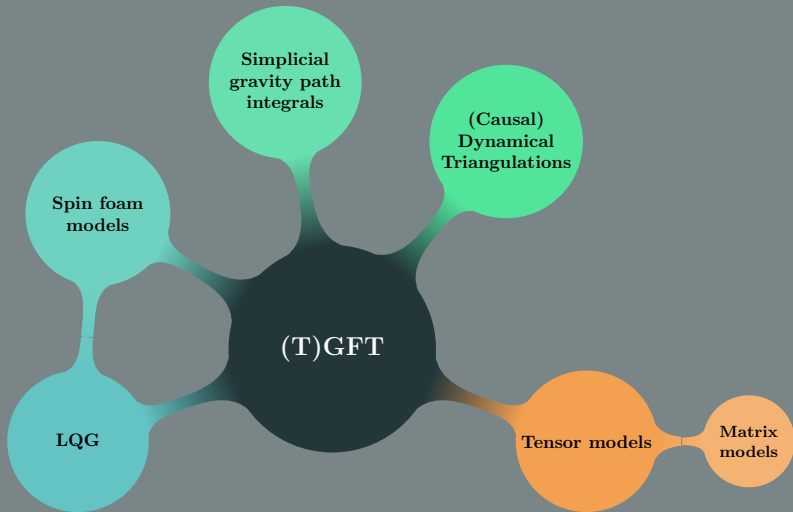
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GFTs are QFTs of atoms of spacetime.



# Group Field Theory and matter: scalar fields

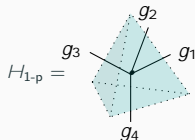
**Group Field Theories:** theories of a field  $\psi : G^d \rightarrow \mathbb{C}$  defined on the product  $G^d$ .

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Quanta are  $d - 1$ -simplices decorated with quantum geometric and scalar data:

- | **Geometricity constraints** imposed analogously as before.



## Processes: Discrete spacetimes

$S_{\text{GFT}}$  obtained by comparing  $Z_{\text{GFT}}$  with simplicial gravity + scalar fields path integral.

- | Geometric data enter interactions in a **non-local and combinatorial** fashion.

# Group Field Theory and matter: scalar fields

**Group Field Theories:** theories of a field  $\phi : G^d \rightarrow \mathbb{R}^d / \mathbb{C}$  defined on the product of  $G^d$  and  $\mathbb{R}^d$ .

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Quanta are  $d$  1-simplices decorated with quantum geometric and scalar data:

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$$H_{1-p} =$$

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- Geometric data enter interactions in a **non-local and combinatorial** fashion.
- Scalar field data are **local** in interactions.
- For minimally coupled, free, massless scalars:

$$K(g_a; g_b; \dots; \phi) = K(g_a; g_b; (\dots)^{\phi})^2$$
$$V_5(g_a^{(1)}; \dots; g_a^{(5)}; \dots) = V_5(g_a^{(1)}; \dots; g_a^{(5)})$$

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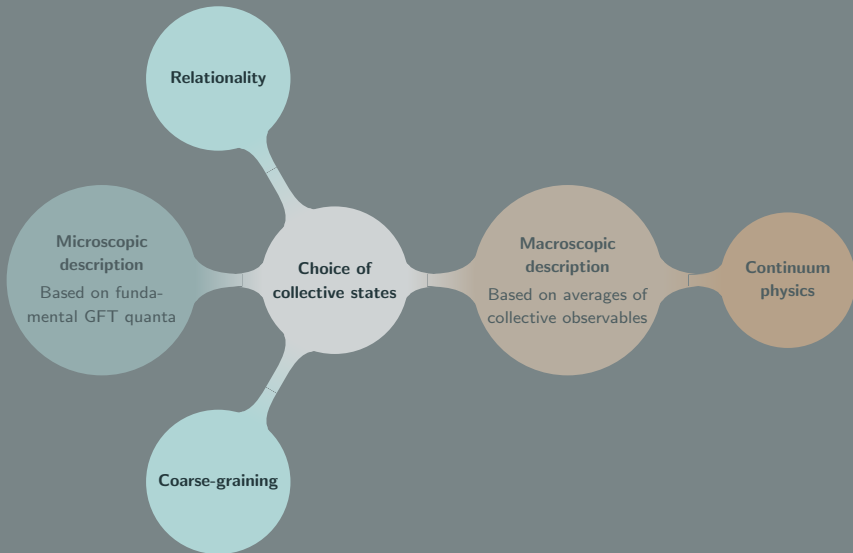
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Domain of GFTs is the space of (discretized) continuum fields





# The main ingredients

## GFT condensates

- | From the GFT perspective, continuum geometries are associated to large number of quanta.
- | The simplest states that can accommodate infinite number of quanta are condensate states:

$$|j\rangle = N \exp \left( \int d^d x \int dg_a (g_a) \right) |j_0\rangle$$

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## Effective dynamics

### Mean-field approximation

- When interactions are small (certainly satisfied in an appropriate regime) the dynamics of  $|j\rangle$  is:

$$\frac{\delta S[\tilde{g}; \tilde{h}]}{\delta \tilde{g}_I(x)} = \int dh_a \int dK (g_a; h_a(x))^2 (h_a(x)) + \frac{\delta V[\tilde{g}; \tilde{h}]}{\delta \tilde{g}_I(x)} = 0$$

- Equivalent to **mean-field** (saddle-point) approx. of  $Z_{\text{GFT}}$  (reliable for physical models).

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Relationality

## Condensate Peaked States

- Constructing relational observables in full QG is difficult (QFT with no continuum intuition).

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- From the GFT perspective, continuum geometries are associated to large number of quanta.
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$$|j\rangle = N \exp \left[ \sum_i d^d_i \phi_{g_a}(g_a; \dots)^{j_i} \right]$$

## E effective dynamics

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- When interactions are small (certainly satisfied in an appropriate regime) the dynamics of  $|j\rangle$  is:

$$\frac{S[\phi; \dots]}{\phi(g_i; x)} = \sum_a \frac{d}{d\phi_a} K(g_a; \phi_a; (x \dots)^2) (\phi_a; \dots) + \frac{V[\phi; \dots]}{\phi(g_a; x)} = 0$$

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## Relationality

### Condensate Peaked States

- Constructing relational observables in full QG is difficult (QFT with no continuum intuition).
- Relational localization implemented at an **effective** level on observable **averages** on condensates.
- If  $|j\rangle$  constitute a physical reference frame, this can be achieved by assuming  $\langle \phi \rangle = (\text{fixed peaking function})$  (dynamically determined reduced wavefunction  $\rightarrow$ )

Relationality

Microscopic  
description  
Based on funda-  
mental GFT quanta

Choice of  
collective states

Macroscopic  
description  
Based on averages of  
collective observables

Cosmological  
physics

Coarse-graining

# Group Field Theory Cosmology

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Homogeneous sector



# Effective relational homogeneous volume dynamics

## Assumptions

### (Relational) Homogeneity

| depends on a single clock MCFM field  $\phi$ .

|  $D = \text{minisuperspace} + \text{clock}$ :

Volume operator captures the relevant physics:

### Isotropy

| depends only on a single rep. label  $\lambda$ .

|  $2 N=2$  (EPRL-like) or  $2 R$  (ext. BC).

$$V_{\text{h}}(\hat{\phi})_{x^0} = V_{\lambda}^2(x^0); \quad j \sim -j.$$



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$$V = \int d^3x \sqrt{|g|} \hat{V}_i = V^2(x^0); \quad j \sim -j.$$

### E effective relational Friedmann dynamics

$$\frac{V^0}{3V} = \frac{2^R V \operatorname{sgn}(\phi)^p E}{3^R V^2} Q^2 = \frac{2^R V \operatorname{sgn}(\phi)^p E}{3^R V^2} + \frac{2^R V \operatorname{sgn}(\phi)^p E}{3^R V^2} \dot{\phi}^2; \quad \frac{V^{00}}{V} = \frac{2^R V \operatorname{sgn}(\phi)^p E}{3^R V^2} + \frac{2^R V \operatorname{sgn}(\phi)^p E}{3^R V^2} \dot{\phi}^2$$

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## Effective relational Friedmann dynamics

$$\frac{\dot{V}^0}{3V} = \frac{2^R V \text{sgn}(\dot{\phi})^p E}{3^R V^2} Q^2 = \frac{2 + 2^2}{2^2}; \quad \frac{V^{00}}{V} = \frac{2^R V^R E + 2^2}{V^2}$$

## Classical limit (large $N$ , late times)

- If  $\dot{\phi}^2$  is mildly dependent on  $\phi$  (or one is dominating) and equal to  $3G$

$$(V^0 = 3V)^2 \cdot 4G = 3 \longrightarrow \text{at FLRW}$$

- Quantum fluctuations on clock and geometric variables are under control.

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$$(V^0=3V)^2 \cdot 4G=3 \longrightarrow \text{at FLRW}$$

- Quantum fluctuations on clock and geometric variables are under control.

### Bounce

- A non-zero volume bounce happens for a large range of initial conditions (at least one  $Q \neq 0$  or one  $E < 0$ ).
- The average singularity resolution may still be spoiled by quantum effects on geometric and clock variables.

## Inhomogeneous sector (dynamics)

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# Scalar perturbations from GFT condensates

Simplest (slightly) relationally inhomogeneous system

# Scalar perturbations from GFT condensates

Simplest (slightly) relationally inhomogeneous system

Classical

- | 4 MCMF **reference** elds ( $^0; ^i$ ),
- | 1 MCMF **matter** eld dominating the e.m. budget and **relationally inhomog.** wrt.  $^i$ .

# Scalar perturbations from GFT condensates

Simplest (slightly) relationally inhomogeneous system

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## Quantum

- ' ( $g_a; \dots$ ) depends on 5 discretized scalar variables and is associated to **spacelike** tetrahedra.
- S<sub>GFT</sub> respecting the classical matter symmetries.

# Scalar perturbations from GFT condensates

Simplest (slightly) relationally inhomogeneous system

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- 4 MCMF **reference** elds  $(\gamma^0; \gamma^i)$ ,
- 1 MCMF **matter** eld dominating the e.m. budget and **relationally inhomog.** wrt.  $\gamma^i$ .

Quantum

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- $\mathcal{S}_{\text{GFT}}$  respecting the classical matter symmetries.

Observables

notation:  $(\gamma; \gamma) = \int d^4x \gamma^0 \gamma^i \gamma^j \gamma^k$

$$\hat{X} = (\gamma^0; \gamma^i) \quad \hat{Y} = i(\gamma^0; \gamma^i)$$

Only isotropic info:  $\hat{V} = (\gamma^0; \gamma^i)$

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Mat. Vol. Frame



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$$\hat{X} = (\gamma^y; \gamma^x) \quad \hat{\Lambda} = i(\gamma^y; @ \gamma^x)$$

Only isotropic info:  $\hat{V} = (\gamma^y; V[\gamma^x])$

$$\hat{\Lambda} = (\gamma^y; \gamma^x) \quad \hat{\Lambda} = i(\gamma^y; @ \gamma^x)$$

Mat. Vol. Frame

States

- CPSs around  $\gamma = x$ , with
  - $\hat{\Lambda}$ : **Isotropic** peaking on rods;
  - $\hat{\Lambda} \sim$ : **Isotropic** distribution of geometric data.
- Small relational  $\sim$ -inhomogeneities ( $\sim = e^i$ ):
  - $= (\gamma; \gamma^0) + (\gamma; \gamma^i)$ ,  $= (\gamma; \gamma^0) + (\gamma; \gamma^i)$ .

# Scalar perturbations from GFT condensates

Mat. Vol. Frame

## Observables

notation:  $(; ) = \int d^4x \int d^3g_a$

$$\hat{\chi} = (\hat{\gamma}; \hat{\gamma}) \quad \hat{\gamma} = \int d^3x (\hat{\gamma}; @ \hat{\gamma})$$

Only isotropic info:  $\hat{\gamma} = (\hat{\gamma}; V[\hat{\gamma}])$

$$\hat{\gamma} = (\hat{\gamma}; \hat{\gamma}) \quad \hat{\gamma} = \int d^3x (\hat{\gamma}; @ \hat{\gamma})$$

## States

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  - $\hat{\gamma}$ : **Isotropic** peaking on rods;
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- Small relational  $\sim$ -inhomogeneities ( $\sim = e^i$ ):
  - $\hat{\gamma} = (\hat{\gamma}; \hat{\gamma}) + (\hat{\gamma}; \hat{\gamma})$ ,  $\hat{\gamma} = (\hat{\gamma}; \hat{\gamma}) + (\hat{\gamma}; \hat{\gamma})$ .

## Late times volume and matter dynamics

- Averaged q.e.o.m. (no interactions) ! coupled eqs. for  $(; )$ .
  - Decoupling for a range of values of CPSs and large  $\Omega$  (late times).
- $\xrightarrow[\text{label}]{\text{single}}$  Dynamic equations for  $\langle \hat{\gamma} \rangle_{x^0}, \langle \hat{\gamma} \rangle_{x^0}$ .

# Scalar perturbations from GFT condensates

Mat. Vol. Frame

## Observables

notation:  $(; ) = \int d^4x \int d^3g_a$

$$\hat{\chi} = (\hat{\gamma}; \hat{\gamma}) \quad \hat{\gamma} = \int d^3x (\hat{\gamma}; @ \hat{\gamma})$$

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## States

- CPSs around  $\gamma = x$ , with
  - $\hat{\gamma}$ : **Isotropic** peaking on rods;
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- Small relational  $\sim$ -inhomogeneities ( $\sim = e^i$ ):
  - $\hat{\gamma} = (\hat{\gamma}; \hat{\gamma}) + (\hat{\gamma}; \hat{\gamma})$ ,  $\hat{\gamma} = (\hat{\gamma}; \hat{\gamma}) + (\hat{\gamma}; \hat{\gamma})$ .

## Late times volume and matter dynamics

- Averaged q.e.o.m. (no interactions) ! coupled eqs. for  $(; )$ .
  - Decoupling for a range of values of CPSs and large  $\mathcal{N}$  (late times).
- single label  $\rightarrow$  Dynamic equations for  $h^i_{x^0}, h^i_{x^0}$ .

## Background

- ✓ Matching with GR possible.
- Macro. couplings defined in terms of GFT ones.

# Scalar perturbations from GFT condensates

Mat. Vol. Frame

## Observables

notation:  $(; ) = \int d^4x \mathcal{L}_{\text{GFT}}$

$$\hat{\mathcal{X}} = (\delta\gamma_{ij}; \delta\chi) \quad \hat{\mathcal{A}} = \int d^3x (\delta\chi; @ \delta\chi)$$

Only isotropic info:  $\hat{\mathcal{V}} = (\delta\gamma_{ij}; V[\delta\chi])$

$$\hat{\mathcal{A}} = (\delta\gamma_{ij}; \delta\chi) \quad \hat{\mathcal{A}} = \int d^3x (\delta\chi; @ \delta\chi)$$

## States

- CPSs around  $\gamma_{ij} = x_{ij}$ , with
  - $\hat{\mathcal{A}}$ : **Isotropic** peaking on rods;
  - $\hat{\mathcal{A}} \sim$ : **Isotropic** distribution of geometric data.
- Small relational  $\sim$ -inhomogeneities ( $\sim = e^i$ ):
  - $\hat{\mathcal{A}} = (\delta\gamma_{ij}^0) + (\delta\gamma_{ij})$ ,  $\hat{\mathcal{A}} = (\delta\gamma_{ij}^0) + (\delta\gamma_{ij})$ .

## Late times volume and matter dynamics

- Averaged q.e.o.m. (no interactions) ! coupled eqs. for  $(; )$ .
  - Decoupling for a range of values of CPSs and large  $\mathcal{N}$  (late times).
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- ✓ Matching with GR possible.
- Macro. couplings defined in terms of GFT ones.

### Perturbations

- ✓ Large scales ("super-horizon") GR matching.

# Scalar perturbations from GFT condensates

Mat. Vol. Frame

## Observables

notation:  $(; ) = \int d^4x \int d^3g_a$

$$\hat{\chi} = (\hat{\chi}^y; \hat{\chi}^x) \quad \hat{\chi} = \int d^3x (\hat{\chi}^y; @ \hat{\chi}^x)$$

Only isotropic info:  $\hat{\chi} = (\hat{\chi}^y; V[\hat{\chi}])$

$$\hat{\chi} = (\hat{\chi}^y; \hat{\chi}^x) \quad \hat{\chi} = \int d^3x (\hat{\chi}^y; @ \hat{\chi}^x)$$

## States

- CPSs around  $\chi = x$ , with
  - $\hat{\chi}$ : **Isotropic** peaking on rods;
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- Small relational  $\sim$ -inhomogeneities ( $\sim = e^i$ ):
  - $= (; ^0) + (; )$ ,  $= (; ^0) + (; )$ .

## Late times volume and matter dynamics

- Averaged q.e.o.m. (no interactions) ! coupled eqs. for  $(; )$ .
  - Decoupling for a range of values of CPSs and large  $\chi$  (late times).
- single label  $\rightarrow$  Dynamic equations for  $h^i_{x^0}, h^i_{x^0}$ .

### Background

- ✓ Matching with GR possible.
- Macro. couplings defined in terms of GFT ones.

### Perturbations

- ✓ Large scales ("super-horizon") GR matching.
- ! **Unphysical behavior** of spatial derivative terms.

# Super-horizon scalar perturbations

Mat. Vol. Frame

## Observables

notation:  $(; ) = \int d^4x \mathcal{L}_{\text{obs}}$

$$\hat{\chi} = (\delta^i_j; \delta^i_j) \quad \hat{\chi} = \int d^3x (\delta^i_j; \delta^i_j)$$

Only isotropic info:  $\hat{\chi} = (\delta^i_j; V[\delta^i_j])$

$$\hat{\chi} = (\delta^i_j; \delta^i_j) \quad \hat{\chi} = \int d^3x (\delta^i_j; \delta^i_j)$$

## States

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  - $\hat{\chi}$ : Isotropic peaking on rods;
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## Super-horizon volume and matter dynamics

- Averaged q.e.o.m. (no interactions) ! coupled eqs. for  $(; )$ .
- Restrict to super-horizon modes but study also early times.

single spin  $\rightarrow$  Dynamic equations for  $h^i_j(x^0), h^i_j(x^0)$

# Super-horizon scalar perturbations

Mat. Vol. Frame

## Observables

notation:  $(; ) = \int d^4x \mathcal{L}_{\text{obs}}$

$$\hat{\chi} = (\delta^{\text{y}}; \delta^{\text{h}}) \quad \hat{\chi} = \int d^3x (\delta^{\text{y}}; \delta^{\text{h}})$$

Only isotropic info:  $\hat{\chi} = (\delta^{\text{y}}; V[\delta^{\text{h}}])$

$$\hat{\chi} = (\delta^{\text{y}}; \delta^{\text{h}}) \quad \hat{\chi} = \int d^3x (\delta^{\text{y}}; \delta^{\text{h}})$$

## States

- CPSs around  $\chi = x$ , with
  - $\delta^{\text{h}}$ : Isotropic peaking on rods;
  - $\delta^{\text{h}} \sim$ : Isotropic distribution of geometric data.
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  - $\delta^{\text{h}} = (\delta^{\text{h}}; \delta^{\text{h}}) + (\delta^{\text{h}}; \delta^{\text{h}})$ ,  $\delta^{\text{h}} = (\delta^{\text{h}}; \delta^{\text{h}}) + (\delta^{\text{h}}; \delta^{\text{h}})$ .

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- Averaged q.e.o.m. (no interactions) ! coupled eqs. for  $(; )$ .
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single spin  $\rightarrow$  Dynamic equations for  $h^{\text{h}}_{\text{x}^0}, h^{\text{h}}_{\text{x}^0}$

### Modified gravity

- Dynamics of super-horizon scalar perturbations can be obtained generically for **any** MG theory.
- No matching** at early times with effective GFT volume dynamics.

# Super-horizon scalar perturbations

Mat. Vol. Frame

## Observables

notation:  $(; ) = \int d^4x \mathcal{L}_{\text{obs}}$

$$\hat{\chi} = (\delta^i_j; \delta^i_j) \quad \hat{\chi} = \int d^3x (\delta^i_j; \delta^i_j)$$

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- Averaged q.e.o.m. (no interactions) ! coupled eqs. for  $(; )$ . single spin  $\rightarrow$  Dynamic equations for  $h^i_j(x^0), h^i_j(x^0)$
- Restrict to super-horizon modes but study also early times.

### Modified gravity

- Dynamics of super-horizon scalar perturbations can be obtained generically for **any** MG theory.
- No matching** at early times with effective GFT volume dynamics.

### Perturbing background dynamics

- Study super-horizon scalar perturbations by perturbing background QG volume equation.
- No matching** at early times with full effective GFT volume dynamics.



# Scalar perturbations from quantum correlations

## Two-body correlations

Including timelike tetrahedra allows to better couple the physical frame: **two-sector** (+; ) GFT!

$$j_i = N \exp(\wedge l + l_+ b + c l + c + l_+ c) j_{0i}$$

# Scalar perturbations from quantum correlations

## Two-body correlations

Including timelike tetrahedra allows to better couple the physical frame: **two-sector** (+; ) GFT!

$$|j\rangle\langle i| = N \exp(\hat{L} + \hat{L}_+ + \hat{b} + \hat{c} + \hat{L} + \hat{c} + \hat{L}_+ + \hat{c}) |j\rangle\langle i|$$

### Background

- |  $\hat{L} = ( ; \hat{L}_+^y)$ : spacelike condensate.
- |  $\hat{L} = ( ; \hat{L}^y)$ : timelike condensate.
- | , peaked;  $\sim$  ~ homogeneous.

# Scalar perturbations from quantum correlations

## Two-body correlations

Including timelike tetrahedra allows to better couple the physical frame: **two-sector** (+; ) GFT!

$$|j\rangle\langle i| = N \exp(\Lambda \quad | \quad + \quad |_+ \quad b + c \quad | \quad + \quad c + \quad |_+ \quad c) |j\rangle\langle i|$$

### Background

- |  $\Lambda = ( ; \Lambda^Y_+)$ : spacelike condensate.
- |  $\Lambda = ( ; \Lambda^Y)$ : timelike condensate.
- | , peaked;  $\sim \sim$  homogeneous.

### Perturbations

- |  $C = ( ; \Lambda^Y_+ \Lambda^Y_+)$ ,  $C = ( ; \Lambda^Y_+ \Lambda^Y)$ ,  $C = ( ; \Lambda^Y \Lambda^Y)$ .
- | , and small and relationally inhomogeneous.
- | Perturbations = nearest neighbour 2-body **correlations**.

# Scalar perturbations from quantum correlations

## Two-body correlations

Including timelike tetrahedra allows to better couple the physical frame: **two-sector** (+; ) GFT!

$$|j\rangle_i = N \exp(\Lambda | + |_+ \quad b + c \quad | + c + |_+ \quad c) |j\rangle_i$$

### Background

- $\Lambda = ( ; \Lambda_+^y)$ : spacelike condensate.
- $\Lambda = ( ; \Lambda^y)$ : timelike condensate.
- $\Lambda$  peaked;  $\Lambda_+ \sim$  homogeneous.

### Perturbations

- $c = ( ; \Lambda_+^y \Lambda_+^y)$ ,  $c = ( ; \Lambda_+^y \Lambda^y)$ ,  $c = ( ; \Lambda^y \Lambda^y)$ .
- $c$ , and  $c$  small and relationally inhomogeneous.
- Perturbations = nearest neighbour 2-body **correlations**.

## Scalar perturbations

- 2 mean-eld eqs. for 3 variables ( ; ; ):

$$h S = \Lambda_+^y i = 0 = h S = \Lambda^y i$$

- Late times and single (spacelike) rep. label.

# Scalar perturbations from quantum correlations

## Two-body correlations

Including timelike tetrahedra allows to better couple the physical frame: **two-sector** (+; ) GFT!

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## Scalar perturbations

- 2 mean-eld eqs. for 3 variables ( ; ; ):
 
$$h S = \Lambda_+^y i = 0 = h S = \Lambda^y i$$
- Late times and single (spacelike) rep. label.

- Physical behavior of spatial derivative terms xes dynamical freedom (e.g. in  $V / \text{Re}( ; \sim \sim) + \text{Re}( ; \sim^2)$ ).

# Scalar perturbations from quantum correlations

Collective states

## Two-body correlations

Including timelike tetrahedra allows to better couple the physical frame: **two-sector** (+; ) GFT!

$$|j\rangle\langle i| = N \exp(\Lambda |l| + l_+ |b| + c |l| + c + l_+ |c|) |j\rangle\langle i|$$

### Background

- $\Lambda = (\dots; \Lambda_+^y)$ : spacelike condensate.
- $\Lambda = (\dots; \Lambda^y)$ : timelike condensate.
- $\dots$ , peaked;  $\sim$  homogeneous.

### Perturbations

- $c = (\dots; \Lambda_+^y \Lambda_+^y)$ ,  $c = (\dots; \Lambda_+^y \Lambda^y)$ ,  $c = (\dots; \Lambda^y \Lambda^y)$ .
- $\dots$ , and  $\dots$  small and relationally inhomogeneous.
- Perturbations = nearest neighbour 2-body **correlations**.

Effective dynamics

## Scalar perturbations

- 2 mean-field eqs. for 3 variables ( $\dots; \dots; \dots$ ):  

$$h S = \Lambda_+^y i = 0 = h S = \Lambda^y i$$
- Late times and single (spacelike) rep. label.
- Physical behavior of spatial derivative terms crosses dynamical freedom (e.g. in  $\dots$ ).

### Late times volume perturbations

- QG corrections to trans-Planckian modes dynamics.
- GR matching at larger scales.

Relationality  
via peaking

Microscopic  
description  
Based on funda-  
mental GFT quanta

Collective states  
(condensates)

Macroscopic  
description  
Based on averages of  
collective observables

Cosmological  
physics

Coarse-graining  
via mean-eld







## Results

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- ✓ Super-horizon analysis with MCMF scalar fields:
- ✓ Scalar pert. dynamics differs from any MG model.
- ✓ Full QG scalar pert. dynamics differs from QG perturbed background one.



### Results

- ✓ Super-horizon analysis with MCMF scalar fields:
  - ✓ Scalar pert. dynamics differs from any MG model.
  - ✓ Full QG scalar pert. dynamics differs from QG perturbed background one.

### Results

- ✓ All scales analysis with MCMF scalar fields:
  - ✓ Manifest causal properties of quanta allow for a careful coupling of the physical ref. frame.
  - ✓ Scalar pert. ! quantum correlations!
  - ✓ Late-times volume pert. dynamics matches GR at large scales. . .
  - ✓ . . .but receives corrections for trans-Planckian modes!



### Perspectives

- ⚠ Di erent fundamental d.o.f. ! di erent perturbation dynamics?
- ⚠ Scalar eld perturbations? EFT description?
  - | Generalization to physically interesting uids.
  - | Extension to VT modes: more observables!
  - | Initial conditions and power spectra?
    - ^ Fock quantization of early-times dynamics.
    - ^ Can we derive it from full QG?

### Perspectives

- ⚠ Physical (perhaps observable) consequences of trans-Planckian mismatch?
- ⚠ Scalar eld perturbations? EFT description?
  - | Generalization to physically interesting uids.
  - | Extension to VT modes: more observables!
  - | How do quantum perturbations classicalize?

Backup

---

# Group Field Theory and spinfoam models

Definition

**Group Field Theories:** theories of a field  $\phi : G^d \rightarrow \mathbb{C}$  defined on  $d$  copies of a group manifold  $G$ .

$d$  is the dimension of the spacetime to be" (d = 4) and  $G$  is the local gauge group of gravity,  $G = SL(2 ; \mathbb{C})$  or, in some cases,  $G = SU(2)$ .

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Action

$$S[\Psi] = \sum_{\text{vertices}} \int dg_a \Psi(g_a) K[\Psi](g_a) + \sum_{\text{edges}} \frac{1}{n} \text{Tr}_V[\Psi] + \text{c.c.} :$$

- Interaction terms are **combinatorially non-local**.
- Field arguments convoluted pairwise following the combinatorial pattern dictated by the graph  $\Gamma$ :

$$\text{Tr}_V[\Psi] = \sum_{i=1}^Z \int dg_a \Psi(g_a^{(i)}; g_b^{(j)}) \Psi(g_a^{(i)}) :$$

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Action

$$S[\phi] = \sum_{\text{graphs}} \int \prod_{\text{edges}} dg_a \phi(g_a) K[\phi](g_a) + \sum_{\text{vertices}} \frac{1}{n} \text{Tr}_V[\phi] + \text{c.c.} :$$

- Interaction terms are **combinatorially non-local**.
- Field arguments convoluted pairwise following the combinatorial pattern dictated by the graph :

$$\text{Tr}_V[\phi] = \sum_{i=1}^Z \int \prod_{(a;i;b;j)} dg_a \phi(g_a^{(i)}; g_b^{(j)})$$

Partition function

$$Z[\phi] = \sum_{\text{graphs}} \int \prod_{\text{edges}} dg \phi(g) A$$

- $\text{Tr}_V[\phi]$  = stranded diagrams dual to  $d$ -dimensional cellular complexes of arbitrary topology.
- Amplitudes  $A$  = sums over group theoretic data associated to the cellular complex.

# Group Field Theory and spinfoam models

Definition

**Group Field Theories:** theories of a field  $\Psi : G^d \rightarrow \mathbb{C}$  defined on  $d$  copies of a group manifold  $G$ .

$d$  is the dimension of the spacetime to be" ( $d = 4$ ) and  $G$  is the local gauge group of gravity,  $G = \text{SL}(2; \mathbb{C})$  or, in some cases,  $G = \text{SU}(2)$ .

Action

$$S[\Psi] = \sum_{i=1}^Z dg_a \Psi(g_a) K[\Psi](g_a) + \sum_{i=1}^X \frac{1}{n} \text{Tr}_V[\Psi] + \text{c.c.} :$$

- Interaction terms are **combinatorially non-local**.
- Field arguments convoluted pairwise following the combinatorial pattern dictated by the graph :

$$\text{Tr}_V[\Psi] = \sum_{i=1}^Z \Psi_{(a;i;b;j)} dg_a \sum_{i=1}^Y V(g_a^{(i)}; g_b^{(j)}) \Psi(g_a^{(i)}) :$$

Partition function

$$Z[\Psi] = \sum_w (f, g) A = \text{complete spinfoam model.}$$

- $w$  = stranded diagrams dual to  $d$ -dimensional cellular complexes of arbitrary topology.
- Amplitudes  $A$  = sums over group theoretic data associated to the cellular complex.
- $K$  and  $V$  chosen to match the desired spinfoam model.



# Group Field Theory and Loop Quantum Gravity

## One-particle Hilbert space

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The one-particle Hilbert space is  $H_{\text{tetra}} = \bigoplus_{a=1}^4 H_a$  (subset defined by the imposition of constraints)

# Group Field Theory and Loop Quantum Gravity

## One-particle Hilbert space

The one-particle Hilbert space is  $H_{\text{tetra}} = \prod_{a=1}^4 H_{\Delta_a}$  (subset defined by the imposition of constraints)

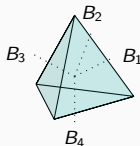
### Lie algebra (metric)

$$H_{\Delta_a} = L^2(\mathfrak{g})$$

### Constraints

Geometricity constraints (appropriately encoded in  $K$  and  $V$ ) allow for a  $d = 1$ -simplicial interpretation of the fundamental quanta:

Closure	Simplicity
$\prod_a B_a = 0$	$\prod_a (B_a \wedge B_a) = 0$ (EPRL);
(faces of the tetrahedron close).	$\prod_a B_a = 0$ (BC).



# Group Field Theory and Loop Quantum Gravity

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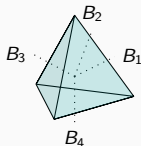
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 $\prod_a B_a = 0$   
(faces of the tetrahedron close).

**Simplicity**  
 $\sum_a (B_a \wedge ? B_a) = 0$  (EPRL);  
 $\sum_a B_a = 0$  (BC).



# Group Field Theory and Loop Quantum Gravity

## One-particle Hilbert space

The one-particle Hilbert space is  $H_{\text{tetra}} = \prod_{a=1}^4 H_{\Delta_a}$  (subset defined by the imposition of constraints)

**Lie algebra (metric)**

$$H_{\Delta_a} = L^2(\mathfrak{g})$$

Non-comm.

**Lie group (connection)**

$$H_{\Delta_a} = L^2(G)$$

FT

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**Closure**

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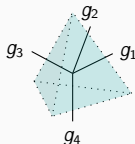
(faces of the tetrahedron close).

**Simplicity**

$$\prod_a (B \cdot ? B)_a = 0 \text{ (EPRL);}$$

$$\prod_a B_a = 0 \text{ (BC).}$$

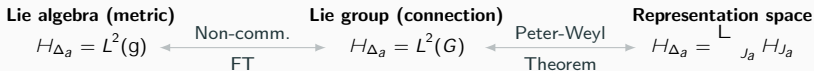
THIS TALK



# Group Field Theory and Loop Quantum Gravity

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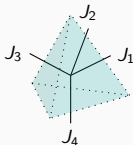


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Geometricity constraints (appropriately encoded in  $K$  and  $V$ ) allow for a  $d = 3$ -simplicial interpretation of the fundamental quanta:

**Closure**  
 $\prod_a B_a = 0$   
 (faces of the tetrahedron close).

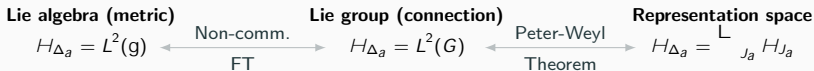
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# Group Field Theory and Loop Quantum Gravity

## One-particle Hilbert space

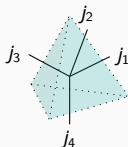
The one-particle Hilbert space is  $H_{\text{tetra}} = \prod_{a=1}^4 H_{\Delta_a}$  (subset defined by the imposition of constraints)



## Constraints

Geometricity constraints (appropriately encoded in  $K$  and  $V$ ) allow for a  $d = 1$ -simplicial interpretation of the fundamental quanta:

- |  |  |
|--|--|
| <p style="text-align: center;"><b>Closure</b></p> $\prod_a B_a = 0$ <p>(faces of the tetrahedron close).</p> | <p style="text-align: center;"><b>Simplicity</b></p> <p style="text-align: center; opacity: 0.5; font-size: 1.2em;">THIS TALK</p> $\prod_a (B_a - \epsilon B_a) = 0 \text{ (EPRL);}$ $\prod_a B_a = 0 \text{ (BC).}$ |
|--|--|

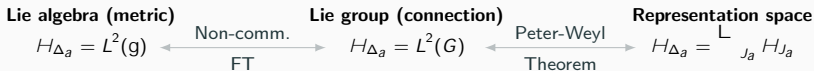


- | Impose simplicity and reduce to  $G = \text{SU}(2)$ .
- | Impose closure (gauge invariance).

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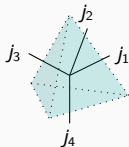
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- 1. Impose closure (gauge invariance).

$$H_{\text{tetra}} = \prod_f \text{Inv} \prod_{a=1}^4 H_{J_a}$$

= open spin-network vertex space

# The Group Field Theory Fock space

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$\psi(g_1; \dots; g_4)$   
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$$F_{\text{GFT}} = \sum_{V=0}^{\infty} \text{sym}^h H_{\text{tetra}}^{(1)} H_{\text{tetra}}^{(2)} \dots H_{\text{tetra}}^{(V)}$$

- |  $F_{\text{GFT}}$  generated by action of  $\hat{\psi}(g_a)$  on  $|j0i\rangle$ , with  $[\hat{\psi}(g_a); \hat{\psi}(g_a^0)] = |_G(g_a; g_a^0)$ .
- |  $H_{\Gamma} \subset F_{\text{GFT}}$ ,  $H_{\Gamma}$  space of states associated to connected simplicial complexes  $\Gamma$ .
- | Generic states **do not** correspond to connected simplicial lattices nor classical simplicial geometries.
- | Similar to  $H_{\text{LQG}}$  but also different: no continuum intuition, orthogonality wrt nodes, not graphs.

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Operators

**Volume operator**  $\hat{V} = \int dg_a^{(1)} dg_a^{(2)} V(g_a^{(1)}; g_a^{(2)}) \hat{\psi}(g_a^{(1)}) \hat{\psi}(g_a^{(2)}) = \prod_{j_a; m_a} V_{j_a; m_a} \hat{\psi}_{j_a; m_a} \hat{\psi}_{j_a; m_a} \dots$

- Generic second quantization prescription to build a  $m + n$ -body operator: sandwich matrix elements between spin-network states between  $m$  powers of  $\hat{\psi}$  and  $n$  powers of  $\hat{\psi}$ .

**Spatial relational homogeneity:**  
depends on a MCMF “clock” scalar field <sup>0</sup>  
( $D = \text{minisuperspace} + \text{clock}$ )

# Macroscopic cosmological variables and effective relationality

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## Collective Observables

**Number**, **volume** (determined e.g. by the mapping with LQG) and **matter** operators (notation:  $(; ) = d^0 dg_a$ ):

$$\hat{N} = (\gamma^Y; \gamma)$$

$$\hat{V} = (\gamma^Y; V[\gamma])$$

$$\hat{X}^0 = (\gamma^Y; \phi^0)$$

$$\hat{\rho}^0 = i(\gamma^Y; @_0 \gamma)$$

Observables  $\mathcal{S}$  collective operators on Fock space.

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|  $\hat{O}_i = O[\gamma]_{x^0}$ :  
functionals of  $\gamma$   
localized at  $x^0$ .

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$\hat{H}^0_{x^0} = O[\gamma]_{\phi^0=x^0}$ :  
 functionals of  $\gamma$   
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## Relationality

Averaged evolution wrt  $x^0$  is physical:

$$\text{Intensive} \longleftarrow \hbar \hat{X}^0_{x^0} \quad \hbar \hat{X}^0_{x^0} = \hbar \hat{N}_{x^0} \quad \phi^0 = x^0$$

Emergent effective relational description:

- Small clock quantum fluctuations.
- Effective Hamiltonian  $H_{x^0} = \hbar \hat{\pi}^0_{x^0}$ .

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$\langle \hat{O} i_{x^0} = O[\tilde{r}]_{j^0=x^0}$ :  
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Wavefunction  
 $\xrightarrow{\text{isotropy}}$

$$\langle \hat{V} i_{x^0} = \int \tilde{r}^Y V[\tilde{r}] \rangle$$

$$\langle \hat{N} i_{x^0} = \int \tilde{r}^Y \tilde{r} \rangle$$

$$V \int \tilde{r}^Y V[\tilde{r}]$$

$$\int \tilde{r}^Y \tilde{r}$$

$$\int \tilde{r}^Y \tilde{r} = \int \tilde{r}^Y \tilde{r} \quad \int \tilde{r}^Y \tilde{r} = \int \tilde{r}^Y \tilde{r}$$

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