Field Theory and the Electroweak Standard Model

— lecture 2 —

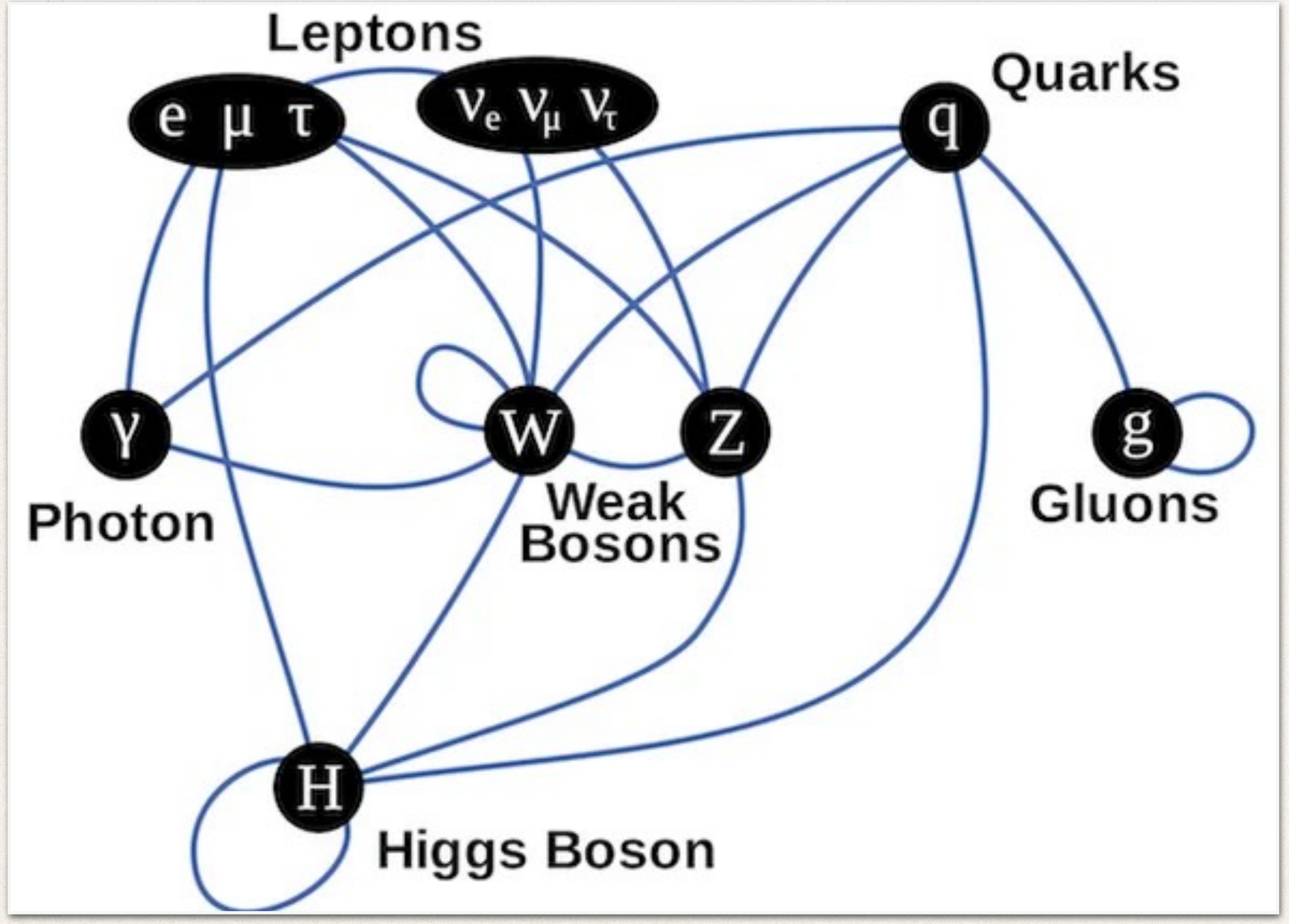
Anna Kulesza (University of Münster)

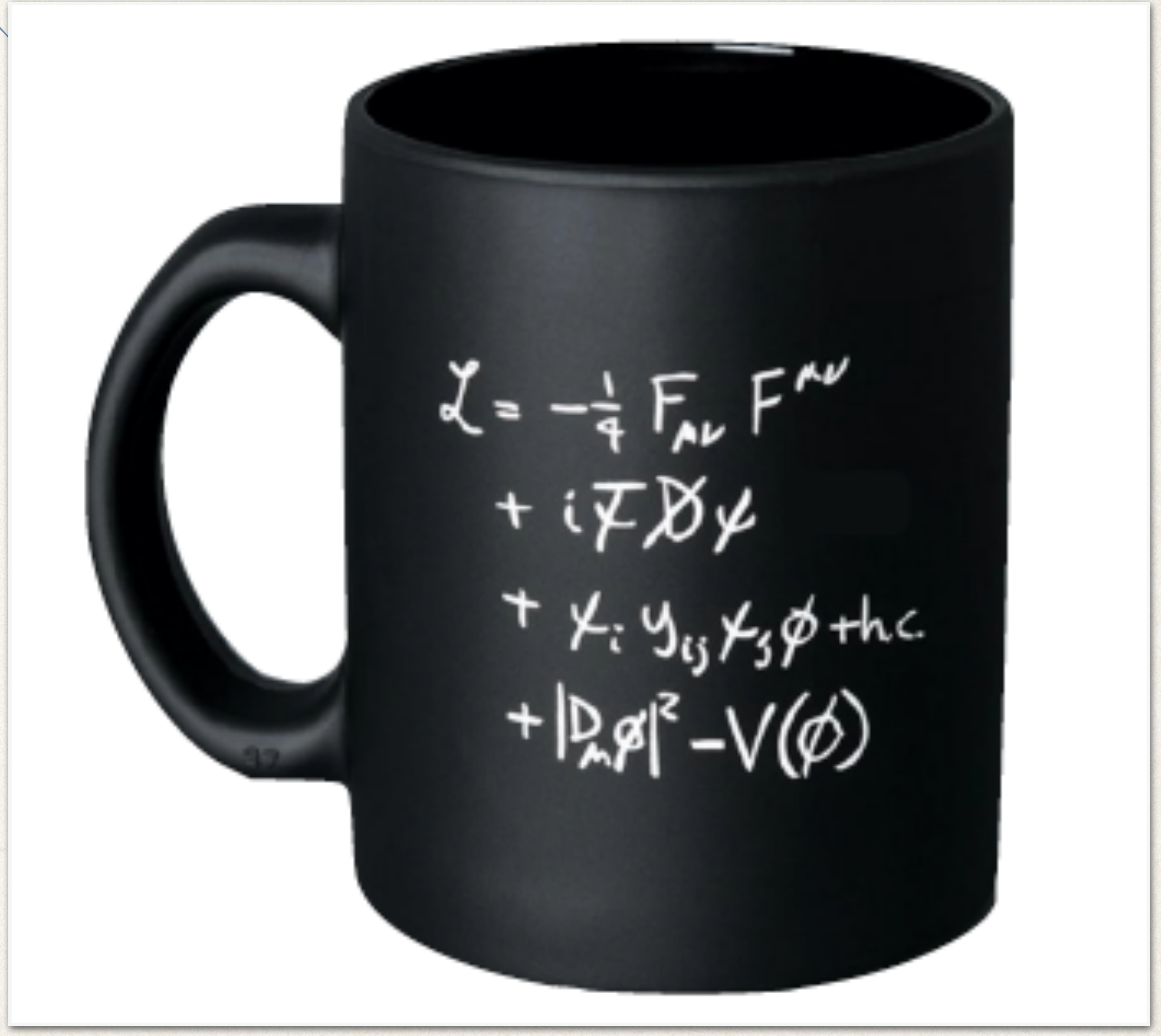
WWU MÜNSTER

The 2023 European School of High Energy Physics

6.-19. September 2023, Grenaa, Denmark

The task ahead ….





The gauge paradigm: QED

- The free Dirac field Lagrangian

$$\mathscr{L}_{\text{Dirac}} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$$

is invariant under global phase U(1) transformations

$$\psi \to e^{i\alpha}\psi \qquad \bar{\psi} \to e^{-i\alpha}\bar{\psi} \qquad (\alpha = \text{ constant phase} \qquad \bar{\psi} = \psi^\dagger \gamma^0)$$

- Under local phase ("gauge") U(1) transformations

$$\psi \to e^{i\alpha(x)}\psi, \quad \bar{\psi} \to e^{-i\alpha(x)}\bar{\psi} \qquad \partial_\mu\psi(x) \to e^{i\alpha(x)}\partial_\mu\psi(x) + ie^{i\alpha(x)}\partial_\mu\alpha(x)\,\psi(x)$$

→ introduce covariant derivative with the transformation rule $D_\mu\psi(x) \to e^{i\alpha(x)}D_\mu\psi(x)$

so that $\mathscr{L} = \bar{\psi}(x)(i\gamma^\mu D_\mu - m)\psi(x)$ is invariant

fulfilled by $D_\mu \equiv \partial_\mu + igA_\mu(x)$ with a new vector field $A_\mu(x)$ transforming as $A_\mu \to A_\mu - \frac{1}{g}\partial_\mu\alpha(x)$

The gauge paradigm: QED (2)

- $\mathscr{L} = \bar{\psi}(x)(i\gamma^\mu D_\mu - m)\psi(x)$ is invariant with $D_\mu = \partial_\mu + igA_\mu(x)$

$$\mathscr{L} = \bar{\psi}(x)(i\gamma^\mu \partial_\mu - m)\psi(x) - \underline{g\bar{\psi}(x)\gamma^\mu\psi(x)A_\mu(x)}$$

interaction piece of the fermion field with a gauge vector (photon) field with g the electric charge of the electron

- Gauge-invariant QED Lagrangian obtained by adding the Maxwell Lagrangian for a vector field $A_\mu(x)$

$$\boxed{\mathscr{L}_{\text{QED}} = \bar{\psi}(x)(i\gamma^\mu D_\mu - m)\psi(x) - \frac{1}{4}F^{\mu\nu}(x)F_{\mu\nu}(x)}$$

where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ is also invariant under the local phase transformation

- Since $A_\mu A^\mu$ not gauge invariant, the term is not allowed → massless photon

The gauge paradigm: QED (2)

- $\mathscr{L} = \bar{\psi}(x)(i\gamma^\mu D_\mu - m)\psi(x)$ is invariant with $D_\mu = \partial_\mu + igA_\mu(x)$

$$\mathscr{L} = \bar{\psi}(x)(i\gamma^\mu \partial_\mu - m)\psi(x) - \underline{g\bar{\psi}(x)\gamma^\mu\psi(x)A_\mu(x)}$$

interaction piece of the fermion field with a gauge vector (photon) field with g the electric charge of the electron

- Gauge-invariant QED Lagrangian obtained by adding the Maxwell Lagrangian for a vector field $A_\mu(x)$

$$\boxed{\mathscr{L}_{\text{QED}} = \bar{\psi}(x)(i\gamma^\mu D_\mu - m)\psi(x) - \frac{1}{4}F^{\mu\nu}(}$$

where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ is also invariant under the local ph

- Since $A_\mu A^\mu$ not gauge invariant, the term is not allowed → ma

Gauge principle: invariance of theory under local symmetry
Promoting global symmetry to local leads to an interacting theory

Non-abelian gauge theories

- Consider now a general case when the local symmetry transformation of fields form a non-abelian group SU(N)

$$\psi(x) \to U(\alpha(x))\psi(x) \quad \text{with} \quad U(\alpha(x)) = \exp\left[ig\alpha^k(x)T^k\right] \qquad k = 1,\ldots,N^2-1$$

- T^k are the generators of the group SU(N) obeying the group algebra $\left[T^i, T^j\right] = if^{ijk}T^k$

- In analogy to QED $\partial_\mu\psi(x) \to \exp\left[ig\alpha^k(x)T^k\right]\partial_\mu\psi(x) + ig(\partial_\mu\alpha^k(x))T^k\exp\left[ig\alpha^k(x)T^k\right]\psi(x)$
  and the Lagrangian $\bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi$ is not invariant under the transformation

- Way out: introduce vector gauge fields $W^\mu = W^{\mu,1}T^1 + W^{\mu,2}T^2 + \ldots = W^{\mu,k}T^k$
  covariant derivative $D^\mu\psi \equiv (\partial^\mu + igW^\mu)\psi$

- Requesting gauge invariance of $\bar{\psi}(i\gamma^\mu D_\mu - m)\psi$ means $D^\mu\psi \to UD^\mu\psi$ and $D^\mu \to UD^\mu U^{-1}$

- It follows $$W^\mu \to UW^\mu U^{-1} - \frac{i}{g}U(\partial^\mu U^{-1})$$

Non-abelian gauge theories (2)

- Transformations: $\psi(x) \to \exp\left[ig\alpha^k(x)T^k\right]\psi(x)$ $\qquad D^\mu \to UD^\mu U^{-1}$ $\qquad W^\mu \to UW^\mu U^{-1} - \frac{i}{g}U(\partial^\mu U^{-1})$

- Generalisation of the QED field strength tensor $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu = -\frac{i}{e}[D^\mu, D^\nu]$ to $W^{\mu\nu} \equiv -\frac{i}{g}[D^\mu, D^\nu]$

  Since $D^\mu\psi = (\partial^\mu + igW^\mu)\psi$ it follows $W^{\mu\nu} = \partial^\mu W^\nu - \partial^\nu W^\mu + ig\left[W^\mu, W^\nu\right]$

  and from $W^\mu = W^{\mu,k}T^k$ $\qquad \Rightarrow W^{\mu\nu,k} = \partial^\mu W^{\nu,k} - \partial^\nu W^{\mu,k} - gf^{ijk}W^{\mu,i}W^{\nu,j}$

- Transformation of the field tensor: $W^{\mu\nu} \to UW^{\mu\nu}U^{-1}$

- The kinetic term $-\frac{1}{4}W^k_{\mu\nu}W^{\mu\nu,k} = -\frac{1}{2}\mathrm{Tr}\left[W_{\mu\nu}W^{\mu\nu}\right]$ is then gauge invariant and hence the Lagrangian

$$\mathcal{L}_{YM} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{2}\mathrm{Tr}\left[W_{\mu\nu}W^{\mu\nu}\right]$$

is also gauge invariant

General features of non-abelian gauge theories

- $N^2 - 1$ generators of the SU(N) symmetry group → $N^2 - 1$ gauge fields
- Similarly to QED, the interaction of gauge fields with fermion fields is given by the $-g\,\bar{\psi}\gamma^{\mu}T^{k}W_{\mu}^{k}\psi$ term in the Lagrangian
- New types of interaction in comparison with an abelian theory: from $-\frac{1}{4}W_{\mu\nu}^{k}W^{\mu\nu,k}$ with $W^{\mu\nu,k} = \partial^{\mu}W^{\nu,k} - \partial^{\nu}W^{\mu,k} - gf^{ijk}W^{\mu,i}W^{\nu,j}$ follow terms that are cubic and quartic in gauge boson fields → gauge bosons interact with each other
- Gauge bosons are massless since the term $W_{\mu}^{k}W^{\mu,k}$ is not invariant under local gauge transformations
- Gauge invariance fixes the strength of the gauge boson self-interactions and interactions with the fermion fields in terms of a single parameter g

QCD Lagrangian

- The kinetic part for the gluon field

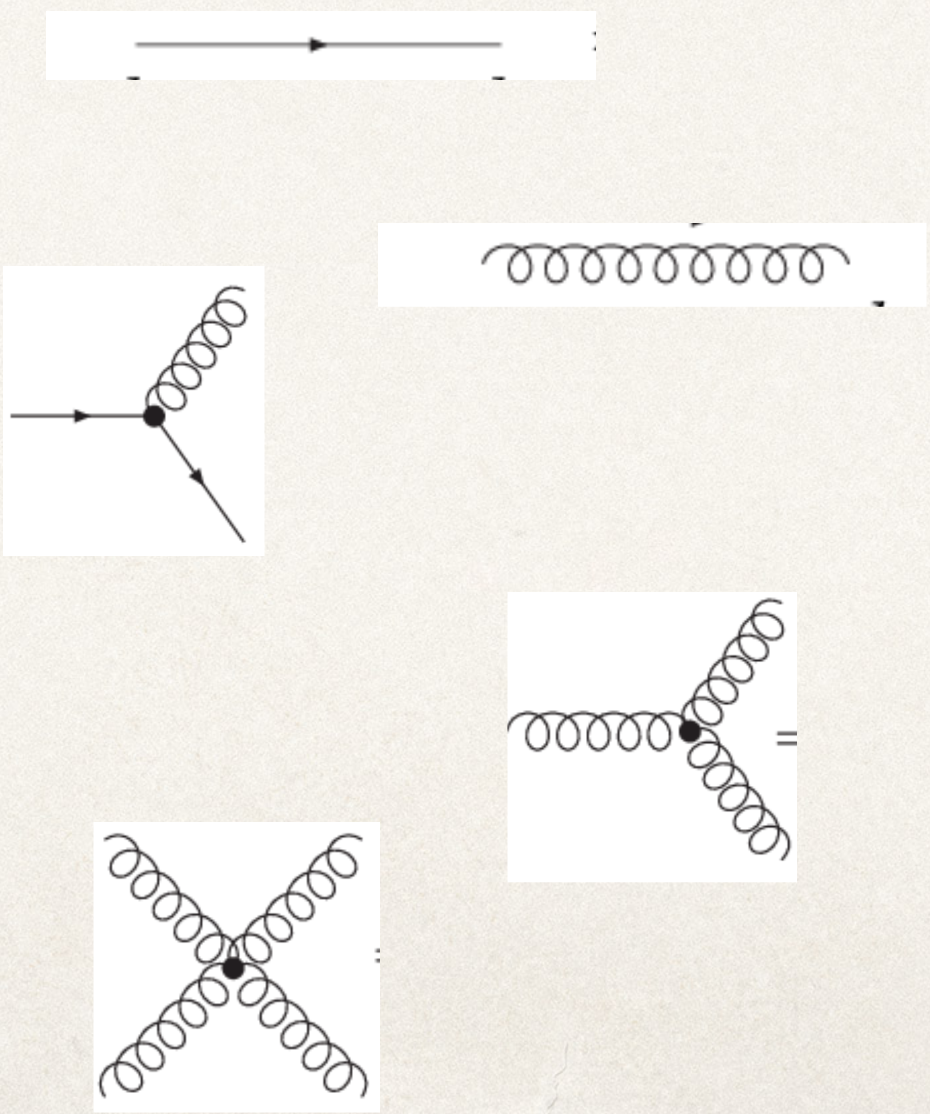
$$\mathcal{L}_G = -\frac{1}{4} F^k_{\mu\nu} F^{\mu\nu,k} \qquad\qquad F^{\mu\nu,k} = \partial^\mu A^{\nu,k} - \partial^\nu A^{\mu,k} - g_s f^{ijk} A^{\mu,i} A^{\nu,j}$$

carries information about triple and quartic gluon self-interactions.

- Altogether, summing over flavours

Feynman rules

$$\begin{aligned}\mathcal{L}_{QCD} = \sum_f \bar{\psi}^{(f)} &(i\gamma^\mu \partial_\mu - m_f)\, \psi^{(f)} \\ &-(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \\ &-g_s\, \bar{\psi}(f) \gamma^\mu T^a A^a_\mu \psi(f) \\ &-\frac{1}{2} g_s\, (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu) f_{abc} A^{\mu,b} A^{\nu,c} \\ &-\frac{1}{4} g_s^2\, f_{abc}\, A^{\mu,b} A^{\nu,c} f_{ade}\, A^d_\mu A^e_\nu\end{aligned}$$



QCD Lagrangian

- The kinetic part for the gluon field

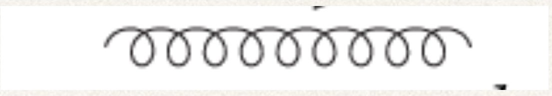
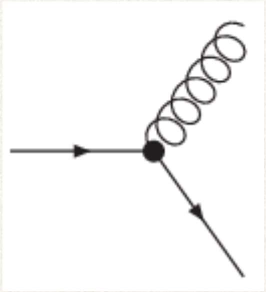
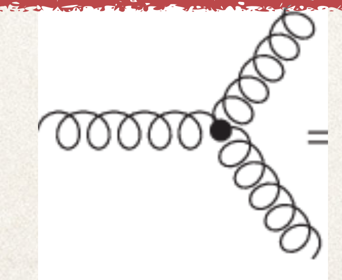
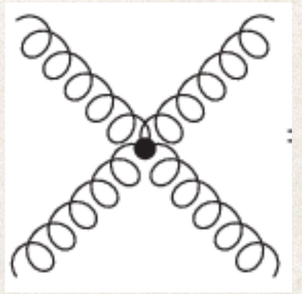
$$\mathcal{L}_G = -\frac{1}{4}F^k_{\mu\nu}F^{\mu\nu,k} \qquad\qquad F^{\mu\nu,k} = \boxed{\partial^\mu A^{\nu,k} - \partial^\nu A^{\mu,k}}\boxed{- g_s f^{ijk}A^{\mu,i}A^{\nu,j}}$$

carries information about triple and quartic gluon self-interactions.

- Altogether, summing over flavours

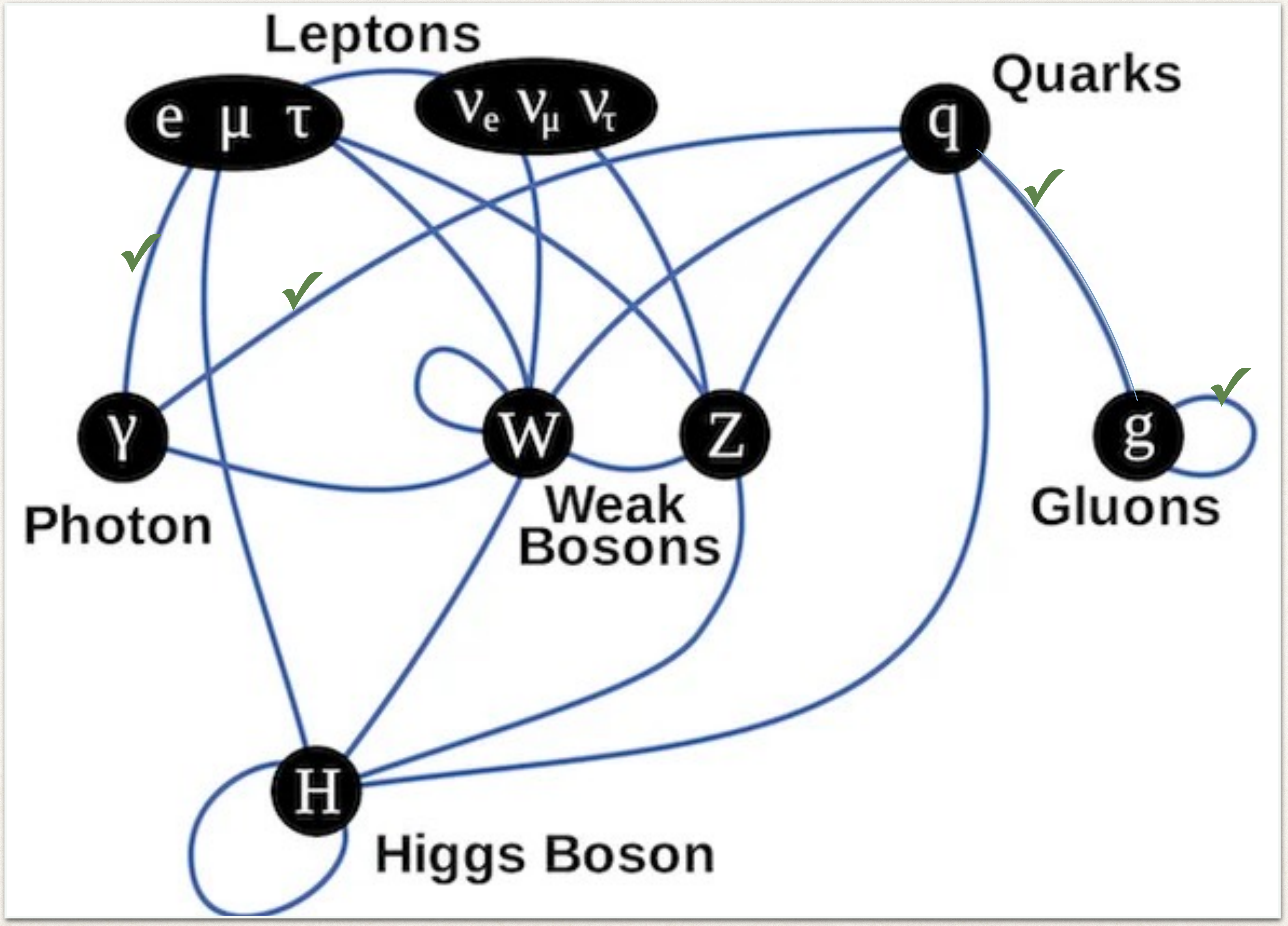
Feynman rules

$$\begin{aligned}\mathcal{L}_{QCD} = &\sum_f \bar{\psi}^{(f)}\left(i\gamma^\mu\partial_\mu - m_f\right)\psi^{(f)} \\ &-(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \\ &-g_s\,\bar{\psi}(f)\gamma^\mu T^a A^a_\mu \psi(f) \\ &-\frac{1}{2}g_s\,(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu) f_{abc} A^{\mu,b}A^{\nu,c} \\ &-\frac{1}{4}g_s^2\, f_{abc}\,A^{\mu,b}A^{\nu,c} f_{ade}\,A^d_\mu A^e_\nu\end{aligned}$$

QED-like

non-abelian



Leptons
e μ τ
νe νμ ντ
Quarks
q
γ
W
Z
g
Photon
Weak Bosons
Gluons
H
Higgs Boson

Electroweak (EW) theory

- Quantum field theory of electromagnetic and weak interactions
  - based on principle of gauge symmetry
  - with massive weak gauge bosons (weak interactions ~ short range) but massless photons, as well as massive fermions
  - able to describe flavour-changing processes
    - β-decay (where weak interactions discovered)
      $n \to p^+ + e^- + \bar{\nu}_e$ -> at the quark level $d \to u + e^- + \bar{\nu}_e$
  - with weak interactions chiral and maximally parity violating *(Lee and Young'56, Wu'57)*: charged currents only involving left-handed particles (right-handed antiparticles)  (Nobel Prize 1957)
  - neutral current weak processes (discovered after the EW Standard Model was proposed -> prediction of the theory)

Chiral fermions

- Chirality operator γ_5

$$\gamma_5 = -\frac{i}{4}\epsilon_{\mu\nu\lambda\rho}\gamma^\mu\gamma^\nu\gamma^\lambda\gamma^\rho = i\gamma^0\gamma^1\gamma^2\gamma^3 \qquad \gamma_5^2 = 1 \quad \gamma_5^\dagger = \gamma_5 \quad \{\gamma_5, \gamma_\mu\} = 0$$

- Chirality projectors

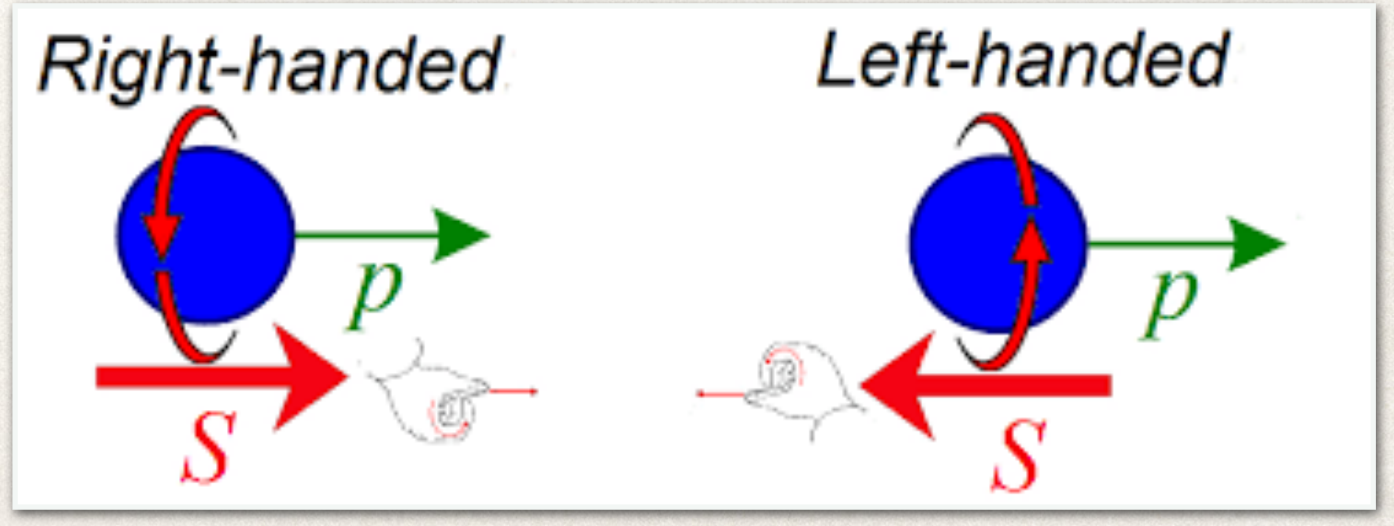
$$P_L = \frac{1}{2}(1-\gamma_5) \qquad P_R = \frac{1}{2}(1+\gamma_5)$$

$$P_{L/R}^2 = P_{L/R} \qquad P_R P_L = P_L P_R = 0 \qquad P_L + P_R = 1$$

- Left- (right-) handed fermions

$$\psi_L = P_L\,\psi \qquad \psi_R = P_R\,\psi \qquad \psi = \psi_L + \psi_R \qquad \bar\psi_{L/R} = \bar\psi\, P_{R/L}$$

- For massless particles chirality is equivalent to helicity (projection of direction of spin on the direction of motion)



Gauge group structure

- At the time (beginning ’60s), only weak charged currents and EM current known → 3 particles as force carriers → 3 generators of SU(2)?
  - Problem: the generators corresponding to these currents do not form a closed algebra
  - Solution: close the SU(2) algebra with an additional generator, corresponding to a new gauge field, mediating neutral currents, and add an extra U(1) group (*Glashow’61*)

SU(2) x U(1)

SU(2)xU(1): fermion field transformations

- Matter content (only 1st generation leptons for now):

Left-handed fermions: $\psi_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$ $\quad \bar{\psi}_L = (\bar{\nu}_L, \bar{e}_L)$ weak isospin doublet

Right-handed fermions: $e_R,\ \bar{e}_R$ singlet under SU(2)

  - In the original Standard Model only ν_L (in accordance with observations) and neutrinos massless (though it is known now they are massive ➜ see lectures by G. Barenboim)

  - SU(2)xU(1) transformations

$$\psi_L \to \exp\left(i\theta^k T^k + i\beta Y\right) \psi_L \qquad e_R \to \exp(i\beta Y)\, e_R$$

SU(2) generator U(1) generator

under SU(2) $e_R \to e_R$

SU(2)xU(1): covariant derivatives

- Covariant derivatives

$$D_\mu \psi_L = \left(\partial_\mu + igT^k W_\mu^k + i\frac{g'}{2} Y B_\mu\right)\psi_L \qquad D_\mu e_R = \left(\partial_\mu + i\frac{g'}{2} Y B_\mu\right) e_R$$

W_μ^k: three gauge vector bosons of SU(2)
g: coupling constant of SU(2)

B_μ: gauge vector boson of U(1)
g' : coupling constant of U(1)

- Fermionic part of the SU(2)xU(1) Lagrangian for the 1st generation leptons

$$\mathscr{L}_{\text{lep},1} = \bar{\psi}_L i\gamma^\mu \left(D_\mu \psi_L\right) + \bar{e}_R i\gamma^\mu \left(D_\mu e_R\right)$$

- Currents

SU(2): $J_\mu^k = \bar{\psi}_L \gamma_\mu T^k \psi_L$

U(1): $J_\mu = \bar{e}_R \gamma_\mu Y e_R + \bar{\psi}_L \gamma_\mu Y \psi_L$

SU(2)xU(1): currents

- Currents SU(2): $J^k_\mu = \bar{\psi}_L \gamma_\mu T^k \psi_L$ U(1): $J_\mu = \bar{e}_R \gamma_\mu Y e_R + \bar{\psi}_L \gamma_\mu Y \psi_L$

$T^k = \dfrac{\sigma^k}{2}$, hence $J^1_\mu = \dfrac{1}{2}\left(\bar{\nu}_L \gamma^\mu e_L + \bar{e}_L \gamma_\mu \nu_L\right)$ $J^2_\mu = \dfrac{i}{2}\left(-\bar{\nu}_L \gamma^\mu e_L + \bar{e}_L \gamma_\mu \nu_L\right)$

$$J^3_\mu = \frac{1}{2}\left(\bar{\nu}_L \gamma^\mu \nu_L - \bar{e}_L \gamma_\mu e_L\right)$$

- Observe $J^+_\mu \equiv J^1_\mu + iJ^2_\mu = \bar{\nu}_L \gamma_\mu e_L$ $J^-_\mu \equiv J^1_\mu - iJ^2_\mu = \bar{e}_L \gamma_\mu \nu_L$ physical charged currents
- Additionally $J^{\text{EM}}_\mu = -\bar{e}\gamma_\mu e = -\bar{e}_L \gamma_\mu e_L - \bar{e}_R \gamma_\mu e_R$
- Note $2(J^{\text{EM}}_\mu - J^3_\mu) = -\bar{e}_L \gamma_\mu e_L - \bar{\nu}_L \gamma_\mu \nu_L - 2\bar{e}_R \gamma_\mu e_R$ and identify it as a current corresponding to U(1) symmetry → weak hypercharge current

$$J^Y_\mu \equiv 2(J^{\text{EM}}_\mu - J^3_\mu)$$

SU(2)xU(1): quantum numbers

- EW SM symmetry group

$$SU(2)_L \times U(1)_Y$$

weak isospin   weak hypercharge

- Weak isospin and hypercharge quantum numbers are related by $Q = T^3 + \frac{1}{2}Y$

	T	T^3	Q	Y
ν_L	1/2	1/2	0	-1
e_L	1/2	-1/2	-1	-1
e_R	0	0	-1	-2

- The definition of J^Y_μ in terms of J^{EM}_μ - J^3_μ and the resulting relation $Q = T^3 + \frac{1}{2}Y$ are not unique; the factor of 2 can be rescaled with the assigned Y values rescaled accordingly

Charged current interactions

- Covariant derivative with $T^k = \dfrac{\sigma^k}{2}$

$$D_\mu \psi_L = \left(\partial_\mu + igT^k W^k_\mu + i\frac{g'}{2} Y B_\mu\right)\psi_L = \left[\partial_\mu + i\frac{g}{\sqrt{2}}\begin{pmatrix} 0 & W^-_\mu \\ W^+_\mu & 0\end{pmatrix} + \frac{i}{2}\begin{pmatrix} gW^3_\mu + g' Y B_\mu & 0 \\ 0 & -gW^3_\mu + g' Y B_\mu\end{pmatrix}\right]\psi_L$$

where $W^\pm_\mu = \dfrac{1}{\sqrt{2}}\left(W^1_\mu \pm iW^2_\mu\right)$

- $\mathscr{L}_{\text{lep},1} = \bar{\psi}_L i\gamma^\mu \left(D_\mu \psi_L\right) + \bar{e}_R i\gamma^\mu \left(D_\mu e_R\right)$ $\qquad D_\mu e_R = \left(\partial_\mu + i\dfrac{g'}{2} Y B_\mu\right) e_R$

will then contain the charged current part $\mathscr{L}_{\text{lep,CC}} = -\dfrac{g}{\sqrt{2}} W^-_\mu \bar{\nu}_L \gamma^\mu e_L - \dfrac{g}{\sqrt{2}} W^+_\mu \bar{e}_L \gamma^\mu \nu_L = -\dfrac{g}{\sqrt{2}} W^-_\mu J^{+,\mu} - \dfrac{g}{\sqrt{2}} W^+_\mu J^{-,\mu}$

and the neutral current part

$$\begin{aligned}\mathscr{L}_{\text{lep,NC}} &= -\frac{g}{2} W^3_\mu \left(\bar{\nu}_L \gamma^\mu \nu_L - \bar{e}_L \gamma^\mu e_L\right) - \frac{g'}{2} B_\mu \left[Y_L\left(\bar{\nu}_L \gamma^\mu \nu_L + \bar{e}_L \gamma^\mu e_L\right) + Y_R \bar{e}_R \gamma^\mu e_R\right] \\ &= -gW^3_\mu J^{3,\mu} - \frac{g'}{2} B_\mu J^{Y,\mu}\end{aligned}$$

- Unlike the photon, W^3_μ and B_μ both couple to neutrinos

Neutral current interactions

- One can rotate the fields W^3_μ and B_μ using the weak mixing angle

$$W^3_\mu = \sin\theta_W A_\mu + \cos\theta_W Z_\mu \qquad\qquad B_\mu = \cos\theta_W A_\mu - \sin\theta_W Z_\mu$$

- After rotation

$$\begin{aligned}\mathscr{L}_{\text{lep,NC}} &= \left(-g\sin\theta_W J^{3,\mu} - \frac{g'}{2}\cos\theta_W J^{Y,\mu}\right)A_\mu + \left(-g\cos\theta_W J^{3,\mu} + \frac{g'}{2}\sin\theta_W J^{Y,\mu}\right)Z_\mu \\ &= \left(-\frac{g}{2}\sin\theta_W + \frac{g'}{2}\cos\theta_W\right)\bar{\nu}_L\gamma^\mu\nu_L A_\mu + \left(\frac{g}{2}\sin\theta_W + \frac{g'}{2}\cos\theta_W\right)\bar{e}_L\gamma^\mu e_L A_\mu + \dots\end{aligned}$$

hence $\frac{g}{2}\sin\theta_W - \frac{g'}{2}\cos\theta_W = 0$ and $\frac{g}{2}\sin\theta_W + \frac{g'}{2}\cos\theta_W = e$

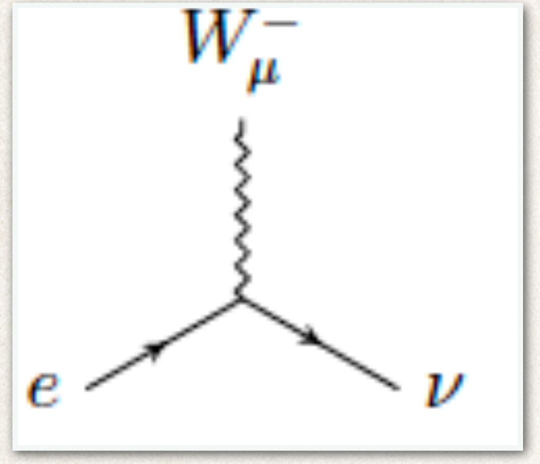
$$\boxed{\tan\theta_W = \frac{g'}{g} \qquad\qquad g\sin\theta_W = e}$$

- With these relations and $J^3_\mu + \frac{1}{2}J^Y_\mu = J^{EM}_\mu$

$$\mathscr{L}_{\text{lep,NC}} = -eJ^{\text{EM},\mu}A_\mu - \frac{g}{\cos\theta_W}\left(J^{3,\mu} - \sin^2\theta_W J^{\text{EM},\mu}\right)Z_\mu = \text{QED inter.} - \frac{g}{2\cos\theta_W}\left[\bar{\nu}\gamma^\mu\left(\frac{1}{2} - \frac{1}{2}\gamma_5\right)\nu - \bar{e}\gamma^\mu\left(-\frac{1}{2} + 2\sin^2\theta_W + \frac{1}{2}\gamma_5\right)e\right]Z_\mu$$

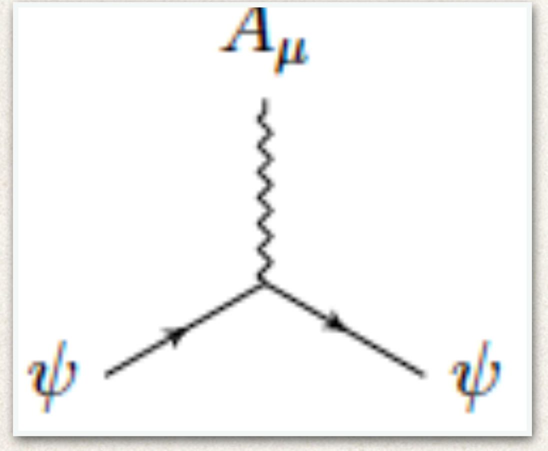
Lepton interactions, Feynman rules

- Charged current

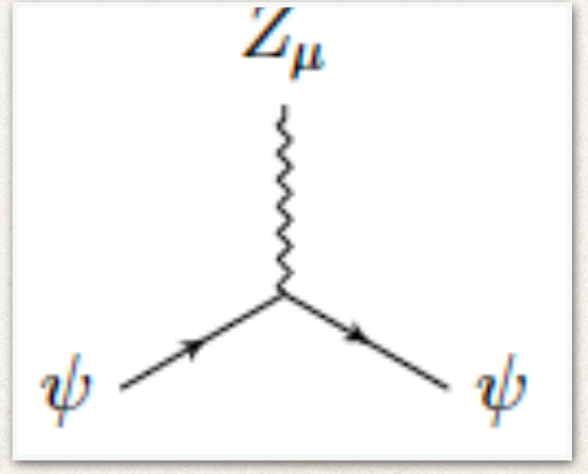


$$-\frac{ig}{2\sqrt{2}}\gamma^\mu(1-\gamma_5)$$

- Neutral current



$$-ie\gamma^\mu$$



$$-\frac{ig}{2\cos\theta_W}\gamma^\mu(c_V^l - c_A^l\gamma_5)$$

	ν	e
c_V^l	1/2	$-1/2 + 2\sin^2\theta_W$
c_A^l	1/2	-1/2

Gauge fields interactions

- Lagrangian of the gauge bosons

$$\mathscr{L}_{\text{gauge}} = -\frac{1}{4}W^k_{\mu\nu}W^{\mu\nu,k} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

with the field strength tensors $F^{\mu\nu} = \partial^\mu B^\nu - \partial^\nu B^\mu$ and $W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\epsilon^{ijk}W^j_\mu W^k_\nu$

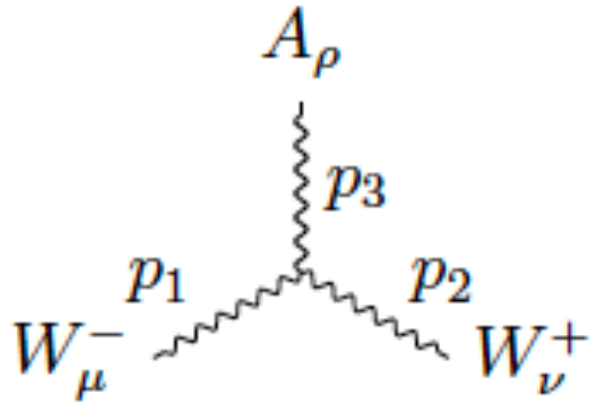
Non-abelian structure of SU(2) → W^i interactions

- Express the Lagrangian in terms of physical fields

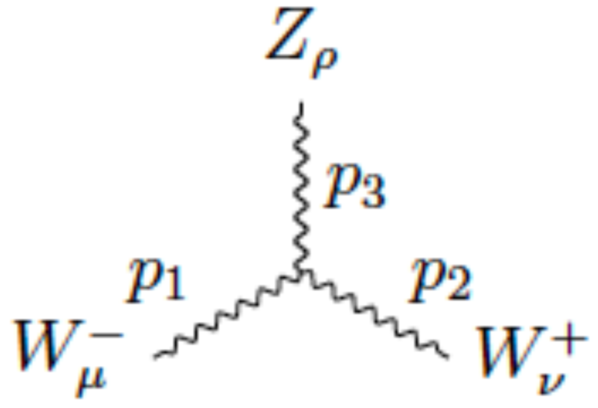
$$W^3_\mu = \sin\theta_W A_\mu + \cos\theta_W Z_\mu \qquad\qquad B_\mu = \cos\theta_W A_\mu - \sin\theta_W Z_\mu$$

  - cubic gauge boson self couplings: $A\,W^+W^-$, ZW^+W^-
  - quartic couplings: $AA\,W^+W^-$, $AZ\,W^+W^-$, ZZW^+W^-, $W^+W^-\,W^+W^-$

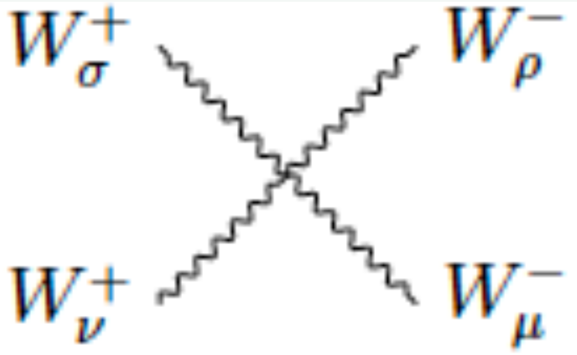
Gauge boson self-interactions, Feynman rules



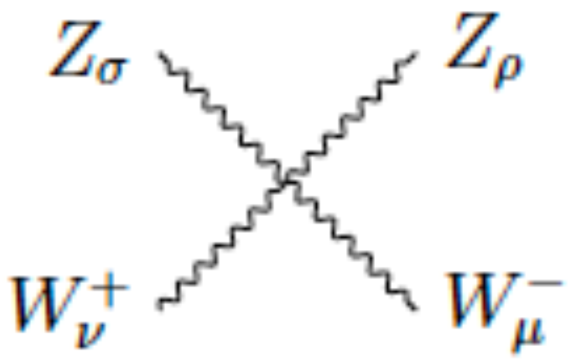
$$ig\sin\theta_W((p_1-p_2)_\rho g_{\mu\nu}+(p_2-p_3)_\mu g_{\nu\rho}+(p_3-p_1)_\nu g_{\rho\mu})$$



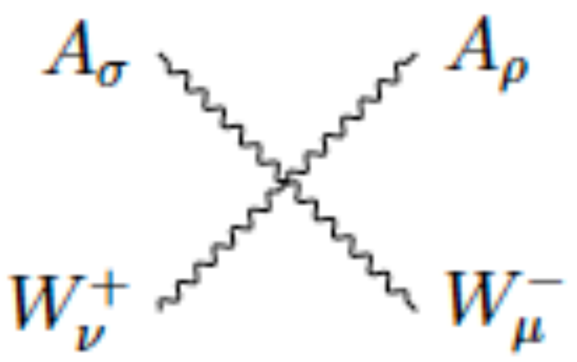
$$ig\cos\theta_W((p_1-p_2)_\rho g_{\mu\nu}+(p_2-p_3)_\mu g_{\nu\rho}+(p_3-p_1)_\nu g_{\rho\mu})$$



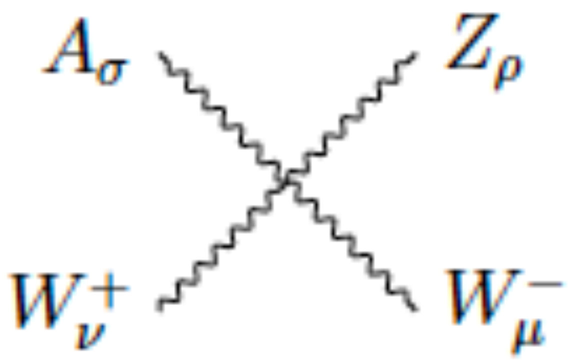
$$ig^2(2g_{\mu\rho}g_{\nu\sigma}-g_{\mu\nu}g_{\rho\sigma}-g_{\mu\sigma}g_{\nu\rho})$$



$$ig^2\cos^2\theta_W(2g_{\mu\nu}g_{\rho\sigma}-g_{\mu\rho}g_{\nu\sigma}-g_{\mu\sigma}g_{\nu\rho})$$



$$ig^2\sin^2\theta_W(2g_{\mu\nu}g_{\rho\sigma}-g_{\mu\rho}g_{\nu\sigma}-g_{\mu\sigma}g_{\nu\rho})$$



$$ig^2\cos\theta_W\sin\theta_W(2g_{\mu\nu}g_{\rho\sigma}-g_{\mu\rho}g_{\nu\sigma}-g_{\mu\sigma}g_{\nu\rho})$$

Towards EW SM

- So far, we have built an SU(2) x U(1) theory, BUT with massless gauge bosons and massless fermions — both $W^i_\mu W^{i,\mu}$ and $\bar{\psi}\psi = \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L$ terms are not gauge invariant, so cannot be present in the Lagrangian

- Solution (*Brout, Englert'64, Higgs'64, Guralnik, Hagen, Kibble'64*): spontaneous symmetry breaking -> Higgs, or Brout-Englert-Higgs (BEH), mechanism (Nobel Prize 2013) 

  - application (*Weinberg'67, Salam'68*) to the SU(2)xU(1) model (*Glashow'61*) renders EW SM (Nobel Prize 1979) 

- Generally speaking, the equations (Lagrangian) obey a symmetry while the solutions (ground state of the system) don't -> "symmetry broken by vacuum"

Abelian Higgs model

- A simpler model with U(1) local gauge symmetry with one complex scalar field

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^*(D^\mu\phi) - V(\phi) \qquad D_\mu\phi = \partial_\mu + igA_\mu$$

$$V(\phi) = -\mu^2\phi^*\phi + \lambda(\phi^*\phi)^2 \qquad \lambda > 0 \quad \text{(potential bounded from below)}$$

invariant under $\phi(x) \to e^{i\alpha(x)}\phi(x) \qquad A_\mu(x) \to A_\mu(x) + \frac{1}{g}\partial_\mu\alpha(x)$

Potential $V(\phi)$ as a function of the field $\phi(x) = \frac{1}{\sqrt{2}}\left(\phi_1(x) + i\phi_2(x)\right)$:

Abelian Higgs model

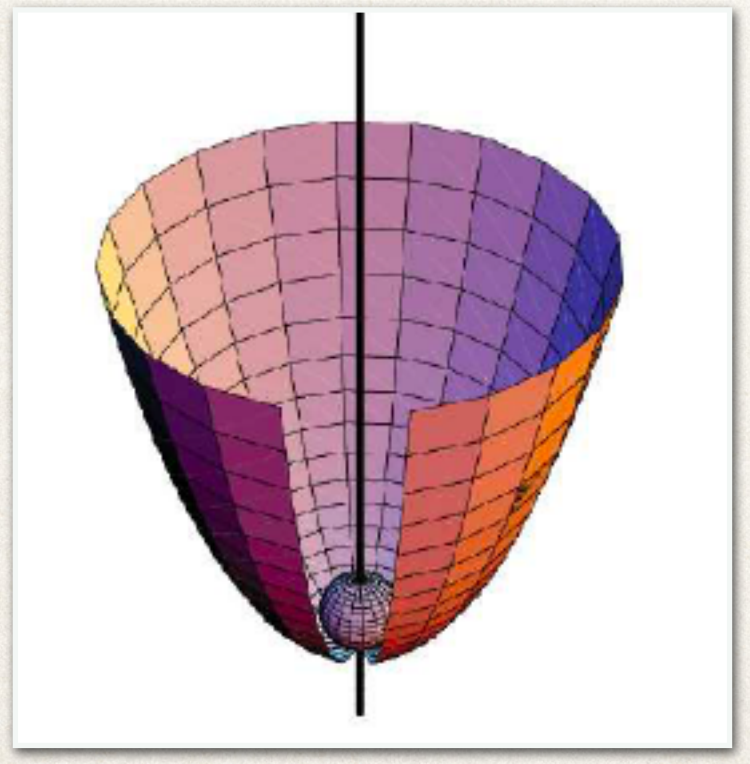
- A simpler model with U(1) local gauge symmetry with one complex scalar field

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^*(D^\mu\phi) - V(\phi) \qquad D_\mu\phi = \partial_\mu + igA_\mu$$

$$V(\phi) = -\mu^2\phi^*\phi + \lambda(\phi^*\phi)^2 \qquad \lambda > 0 \quad \text{(potential bounded from below)}$$

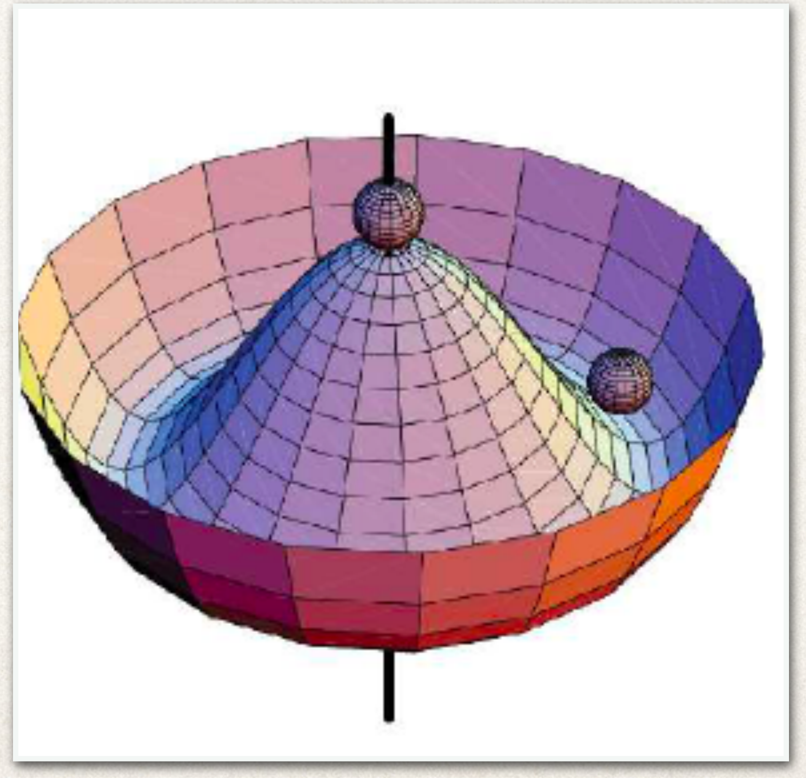
invariant under $\phi(x) \to e^{i\alpha(x)}\phi(x) \qquad A_\mu(x) \to A_\mu(x) + \frac{1}{g}\partial_\mu\alpha(x)$

Potential $V(\phi)$ as a function of the field $\phi(x) = \frac{1}{\sqrt{2}}\left(\phi_1(x) + i\phi_2(x)\right)$:



$\mu^2 < 0$

exact symmetry
unique minimum
$\phi^*\phi = 0 \Rightarrow |\phi| = 0$



$\mu^2 > 0$

broken, or "hidden" symmetry
circle of degenerate minima

$$\phi^*\phi = \frac{\mu^2}{2\lambda} \Rightarrow |\phi| = \sqrt{\frac{\mu^2}{2\lambda}}$$

symmetry is broken by the system choosing one of the ground states

Abelian Higgs model

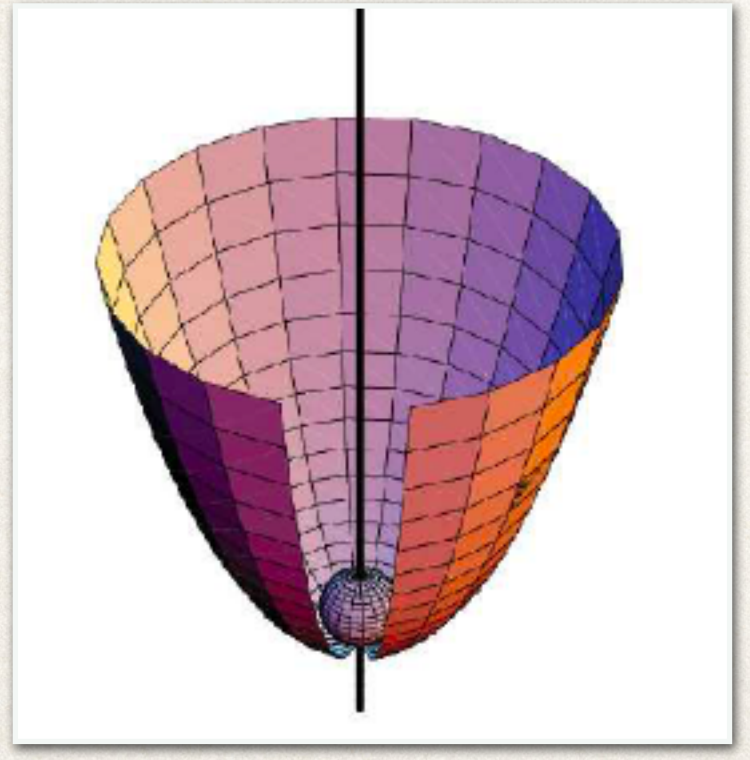
- A simpler model with U(1) local gauge symmetry with one complex scalar field

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^*(D^\mu\phi) - V(\phi) \qquad D_\mu\phi = \partial_\mu + igA_\mu$$

$$V(\phi) = -\mu^2\phi^*\phi + \lambda(\phi^*\phi)^2 \qquad \lambda > 0 \quad \text{(potential bounded from below)}$$

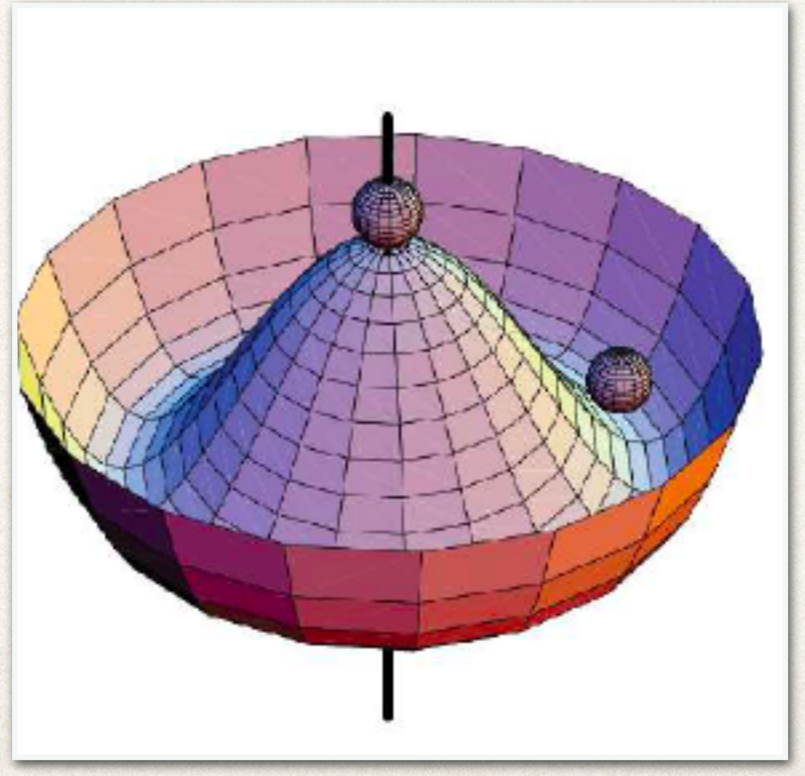
invariant under $\phi(x) \to e^{i\alpha(x)}\phi(x)$ $\qquad A_\mu(x) \to A_\mu(x) + \frac{1}{g}\partial_\mu\alpha(x)$

Potential $V(\phi)$ as a function of the field $\phi(x) = \frac{1}{\sqrt{2}}\left(\phi_1(x) + i\phi_2(x)\right)$



$\mu^2 < 0$

exact symmetry
unique minimum
vacuum expectation value $\langle\phi\rangle = 0$



$\mu^2 > 0$

broken, or "hidden" symmetry
circle of degenerate minima

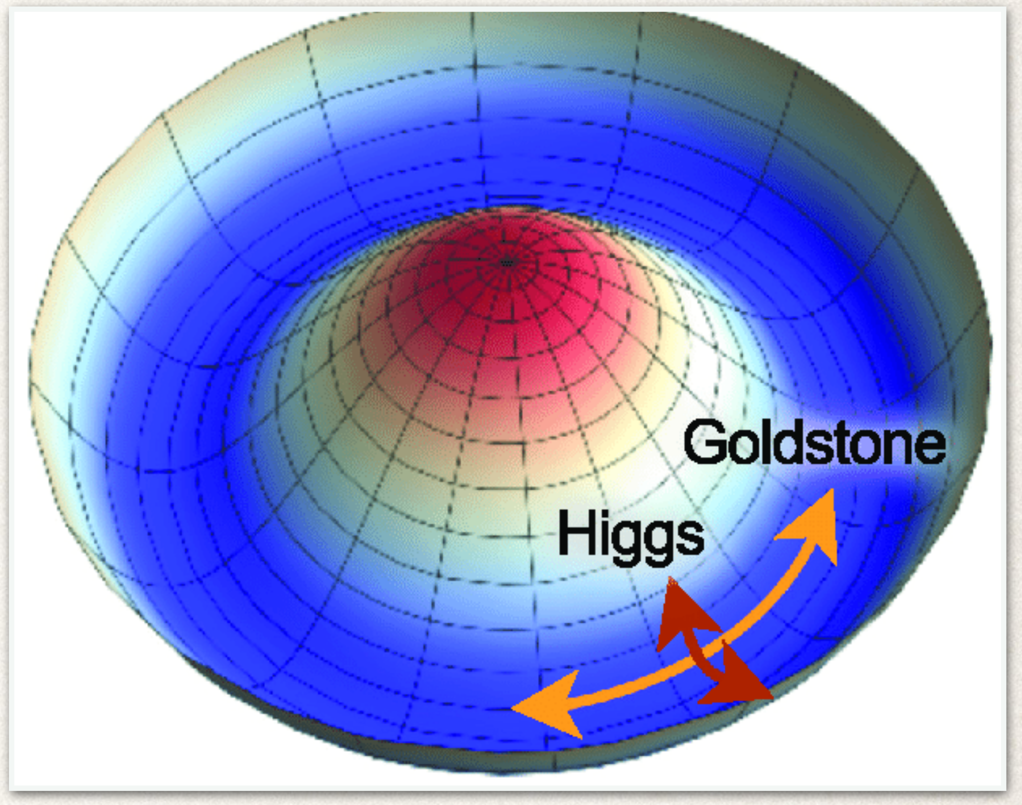
$$|\langle\phi\rangle| = \sqrt{\frac{\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}}$$

Abelian Higgs model (2)

- Field redefinition: expansion around (chosen, without loss of generality) minimum $\phi_0 = \frac{v}{\sqrt{2}}$

$$\phi(x) = \frac{1}{\sqrt{2}}\left(v + \rho(x)\right) e^{i\xi(x)/v} = \frac{1}{\sqrt{2}}\left(v + \rho(x) + i\xi(x) + \ldots\right)$$

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^*(D^\mu\phi) - V(\phi) \qquad V(\phi) = -\mu^2\phi^*\phi + \lambda(\phi^*\phi)^2$$

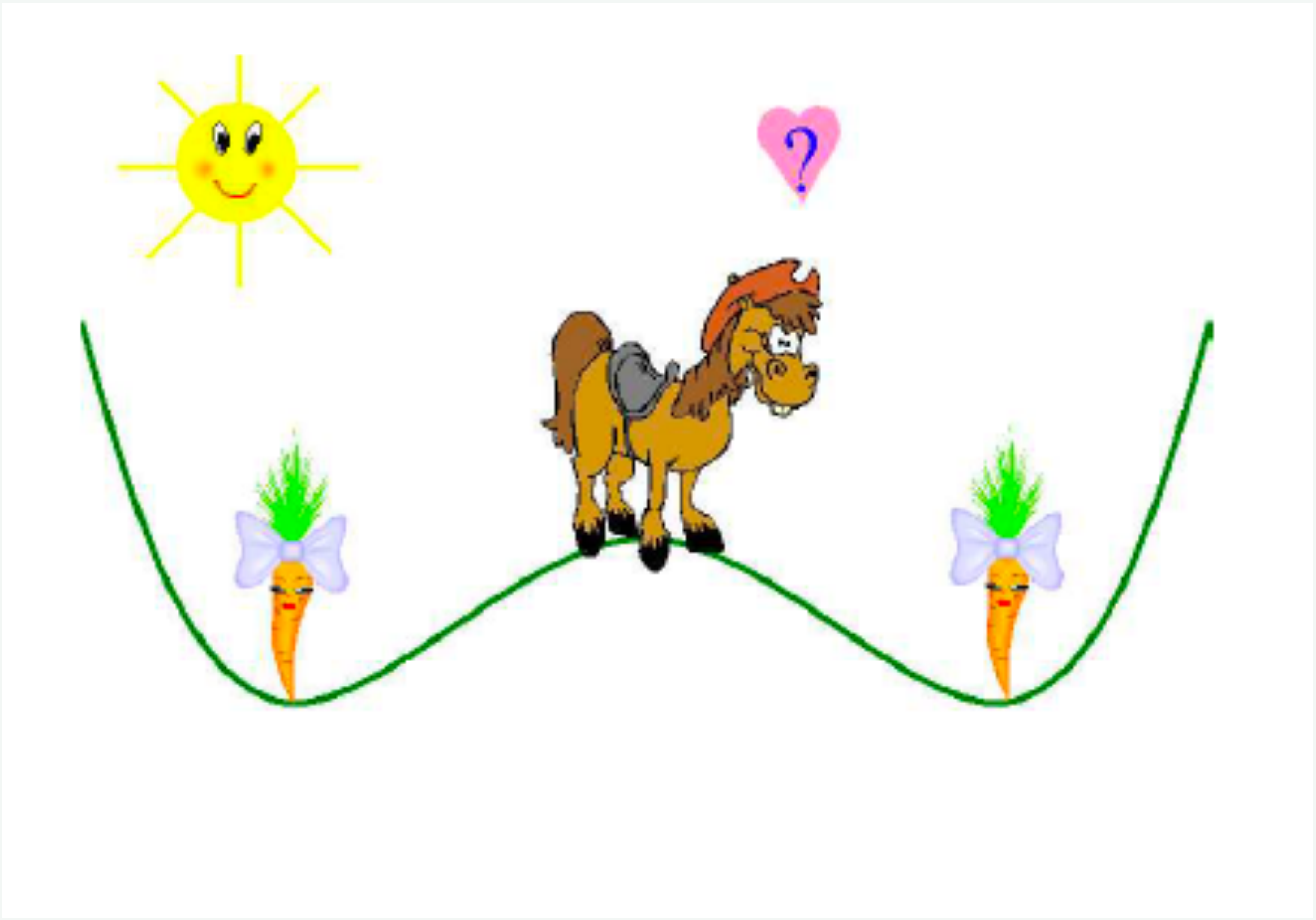


- Potential becomes $V(\phi) = \frac{-\mu^4}{4\lambda} + \mu^2\rho^2 + \mathcal{O}(\rho^3)$
  - mass term for the scalar ρ with $m_\rho^2 = 2\mu^2 = 2\lambda v^2$, no mass term for the scalar ξ

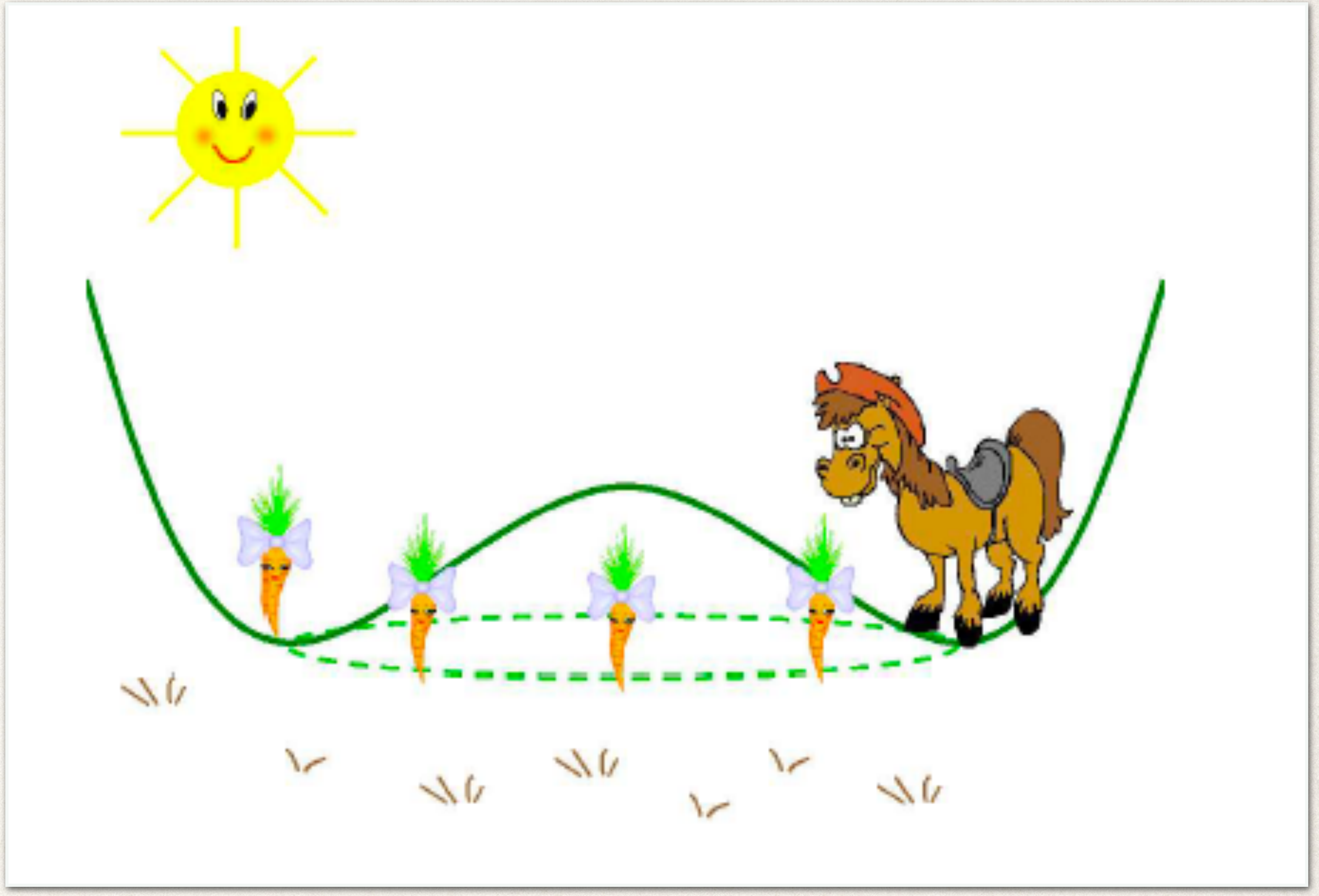
- Interpretation: ρ corresponds to radial excitations → curvature of potential → massive particle
  ξ corresponds to tangential excitations → flat direction → no mass term for the would-be Goldstone boson mode
  (massless Goldstone bosons appear as a result of spontaneous breaking of continuous global symmetries)

….or alternatively…

(picture/idea credit: A. Pich)



symmetric food configuration:
both carrots are identical
but one needs to be chosen first…



… thankfully other carrots can be
reached with no effort…

Abelian Higgs model (3)

- Field redefinition: expansion $\phi(x) = \frac{1}{\sqrt{2}}\left(v + \rho(x)\right) e^{i\xi(x)/v} = \frac{1}{\sqrt{2}}\left(v + \rho(x) + i\xi(x) + \ldots\right)$
- Kinetic term $(D_\mu\phi)^*(D^\mu\phi) = \frac{1}{2}(\partial_\mu\rho)^2 + \frac{1}{2}(\partial_\mu\xi)^2 + \frac{1}{2}g^2v^2A_\mu A^\mu + gvA_\mu\partial^\mu\xi + \text{interaction terms}$
    - suggests massive gauge boson A with $m_A^2 = g^2v^2$!
    - quadratic mixing term $gvA_\mu\partial^\mu\xi$: quadratic terms not diagonalized, cannot read off particle spectrum
- Degrees of freedom:
    - 4 for unbroken symmetry (2 scalars + 2 polarisation of a massless photon) so apparent mismatch after symmetry breaking (3 polarisations of a massive photon + 2 scalars)
    - one field must be unphysical such that it is not counted as an independent d.o.f. -> would-be Goldstone boson mixes with photon, giving rise to photon's longitudinal polarisation

Abelian Higgs model (4)

- In fact, the field ξ can be transformed away using the following gauge transformation, called unitary gauge

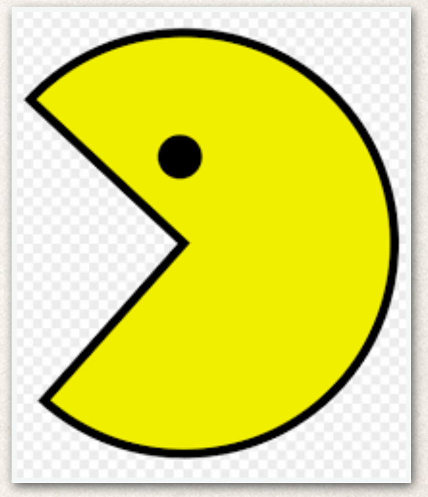
$$\phi(x) \to \phi'(x) = e^{(-i\xi(x)/v)}\phi(x) = \frac{1}{\sqrt{2}}\left(v + \rho(x)\right)$$

$$\phi(x) = \frac{1}{\sqrt{2}}\left(v + \rho(x)\right) e^{i\xi(x)/v}$$

$$A_\mu(x) \to A'_\mu(x) = A_\mu(x) - \frac{1}{gv}\partial_\mu\xi(x)$$

- In this gauge (dropping primes)

$$\mathscr{L} = -\frac{1}{4}F\mu\nu F^{\mu\nu} + \frac{1}{2}(\partial_\mu\rho)^2 + \frac{1}{2}g^2v^2A_\mu A^\mu - \mu^2\rho^2 + \frac{1}{2}g^2A\mu A^\mu\rho^2 + g^2vA_\mu A^\mu\rho - \lambda v\mu\rho^3 - \frac{\lambda}{4}\rho^4 + \frac{1}{4}\mu^2v^2$$



  - ρ is a massive scalar field with $m_\rho^2 = 2\mu^2 = 2\lambda v^2$ $\to$ BEH field
  - Photon acquired mass $m_A^2 = g^2v^2$. No mixing term, no other terms containing ξ .
  - In a spontaneously broken gauge theory gauge bosons acquire mass and the would-be Goldstone bosons' degrees of freedom are used for transition from massless to massive gauge bosons -> they are “eaten” by gauge bosons

BEH mechanism for SU(2)xU(1)

- Introduce an SU(2) doublet of complex scalar fields

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + \phi_4 \end{pmatrix} \quad \text{transforming as} \quad \Phi \to \exp\left(i\theta^k T^k + i\beta Y\right)\Phi$$

construct

$$\mathscr{L}_\Phi = (D_\mu\Phi)^\dagger D^\mu\Phi - V(\Phi)$$

$$D_\mu\Phi = \left(\partial_\mu + igT^kW_\mu^k + \frac{i}{2}g'B_\mu\right)\Phi \qquad V(\Phi) = -\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2 \quad (\lambda > 0)$$

$$\mathscr{L}_\Phi = (D_\mu\Phi)^\dagger D^\mu\Phi + \mu^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2$$

BEH mechanism for SU(2)xU(1)

- Introduce an SU(2) doublet of complex scalar fields

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + \phi_4 \end{pmatrix} \quad \text{transforming as} \quad \Phi \to \exp\left(i\theta^k T^k + i\beta Y\right)\Phi$$

construct

$$\mathscr{L}_\Phi = (D_\mu \Phi)^\dagger D^\mu \Phi - V(\Phi)$$

$$D_\mu \Phi = \left(\partial_\mu + igT^k W_\mu^k + \frac{i}{2} g' B_\mu\right)\Phi$$

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda(\Phi^\dagger \Phi)^2 \quad (\lambda > 0)$$

$$\mathscr{L}_\Phi = (D_\mu \Phi)^\dagger D^\mu \Phi + \mu^2 \Phi^\dagger \Phi - \lambda(\Phi^\dagger \Phi)^2$$

- Spontaneous symmetry breaking when $\mu^2 > 0$, then minima of the potential at $\Phi^\dagger \Phi = \dfrac{\mu^2}{2\lambda} = \dfrac{v^2}{2}$

- Selecting a particular vacuum state breaks the symmetry. Choose $\langle \Phi \rangle = \dfrac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$.

BEH mechanism for SU(2)xU(1)

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \qquad \Rightarrow \text{ with } Q = T^3 + \frac{1}{2}Y, \quad Y(\phi^+) = Y(\phi^0) = 1$$

and

$$Q = \frac{1}{2}\sigma^3 + \frac{1}{2}I = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

- Under $U(1)_{\text{EM}}$

$$\langle \Phi \rangle \to e^{(i\alpha Q)} \langle \Phi \rangle \simeq \langle \Phi \rangle + i\alpha Q \langle \Phi \rangle$$

- For $\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$ $\qquad Q \langle \Phi \rangle = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\qquad$ and $\qquad \langle \Phi \rangle \to \langle \Phi \rangle$

- Invariance of the vacuum under U(1) of electromagnetism $\Rightarrow U(1)_{\text{EM}}$ symmetry preserved

$$\boxed{SU(2)_L \times U(1)_Y \to U(1)_{\text{EM}}}$$

BEH mechanism for SU(2)xU(1)

- Parametrize Φ around chosen minimum $\Phi = \frac{1}{\sqrt{2}} \exp\left(\frac{i}{2}\theta^k T^k\right) \begin{pmatrix} 0 \\ v+H \end{pmatrix}$

- In the unitary gauge $\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+H \end{pmatrix}$

$$D_\mu \Phi = \left(\partial_\mu + igT^k W^k_\mu + \frac{i}{2} g' B_\mu\right) \Phi = \frac{1}{\sqrt{2}} \left[\partial_\mu + i\frac{g}{\sqrt{2}} \begin{pmatrix} W^3_\mu/\sqrt{2} & W^-_\mu \\ W^+_\mu & -W^3_\mu/\sqrt{2} \end{pmatrix} + \frac{i}{2} g' B_\mu\right] \begin{pmatrix} 0 \\ v+H \end{pmatrix}$$

$$(D_\mu \Phi)^\dagger (D_\mu \Phi) = \frac{1}{2}\partial_\mu H \partial^\mu H + \frac{g^2 v^2}{4} W^{+,\mu} W^-_\mu + \frac{v^2}{8}(gW^3_\mu - g' B_\mu)(gW^3_\mu - g' B^\mu) + \text{interaction terms}$$

- Remember mixing $W^3_\mu = \sin\theta_W A_\mu + \cos\theta_W Z_\mu$ $\quad B_\mu = \cos\theta_W A_\mu - \sin\theta_W Z_\mu$ $\quad \tan\theta_W = \frac{g'}{g}$

$$(D_\mu \Phi)^\dagger (D_\mu \Phi) = \frac{1}{2}\partial_\mu H \partial^\mu H + \frac{g^2 v^2}{4} W^{+,\mu} W^-_\mu + \frac{v^2}{8}(g^2 + g'^2) Z_\mu Z^\mu + \text{interaction terms}$$

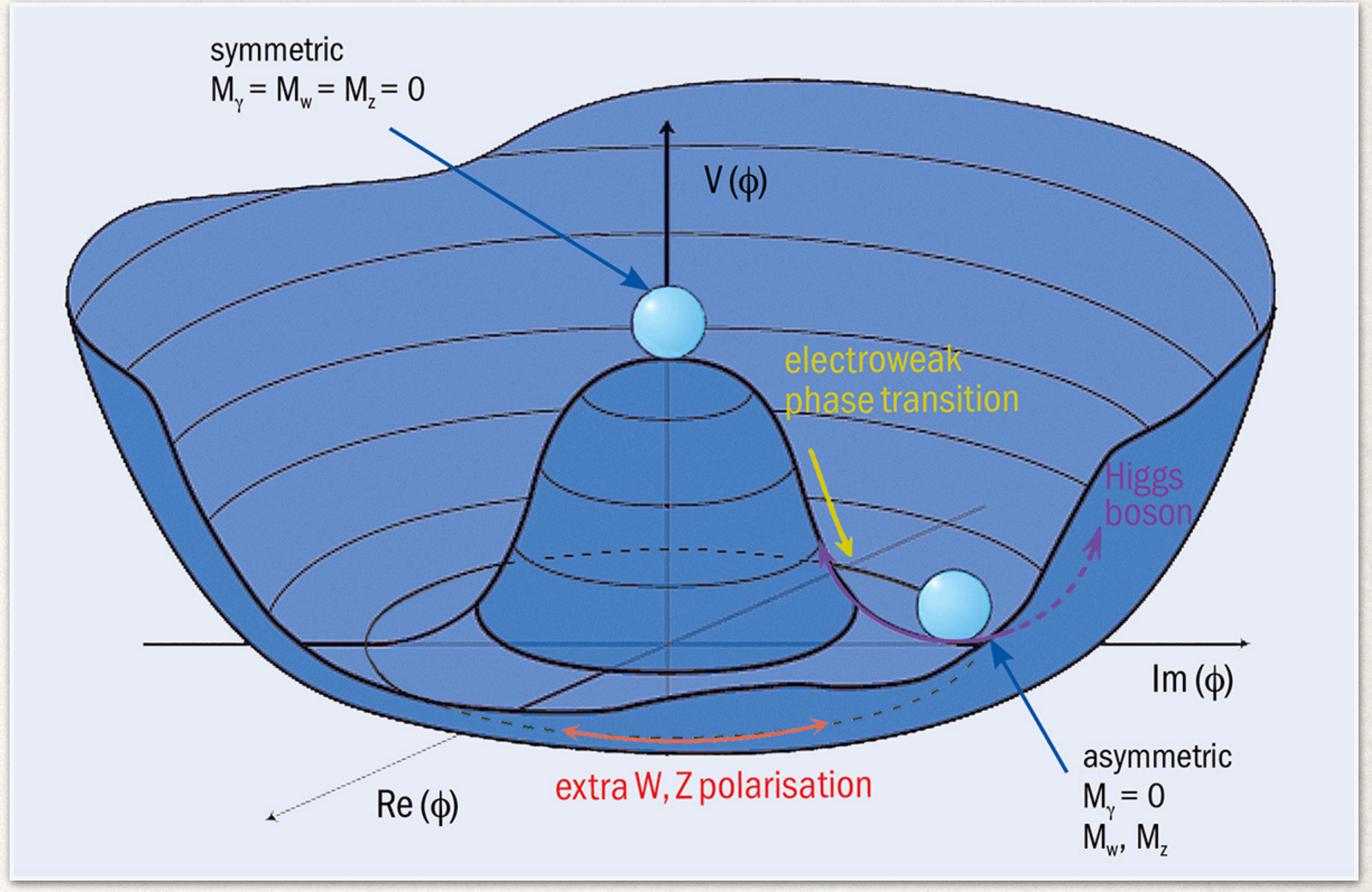
BEH mechanism for SU(2)xU(1)

$$(D_\mu\Phi)^\dagger(D_\mu\Phi) = \frac{1}{2}\partial_\mu H\partial^\mu H + \frac{g^2v^2}{4}W^{+,\mu}W^-_\mu + \frac{v^2}{8}(g^2+g'^2)Z_\mu Z^\mu + \text{interaction terms}$$

- W and Z bosons acquire mass! $(g' = g\tan\theta_W)$

$$M_W = \frac{gv}{2} \qquad M_Z = \frac{v}{2}\sqrt{g^2+g'^2} = \frac{gv}{2\cos\theta_W} = \frac{M_W}{\cos\theta_W} \qquad M_A = 0$$

- Ratio of M_W to M_Z is the prediction of the EWSM



- Degrees of freedom

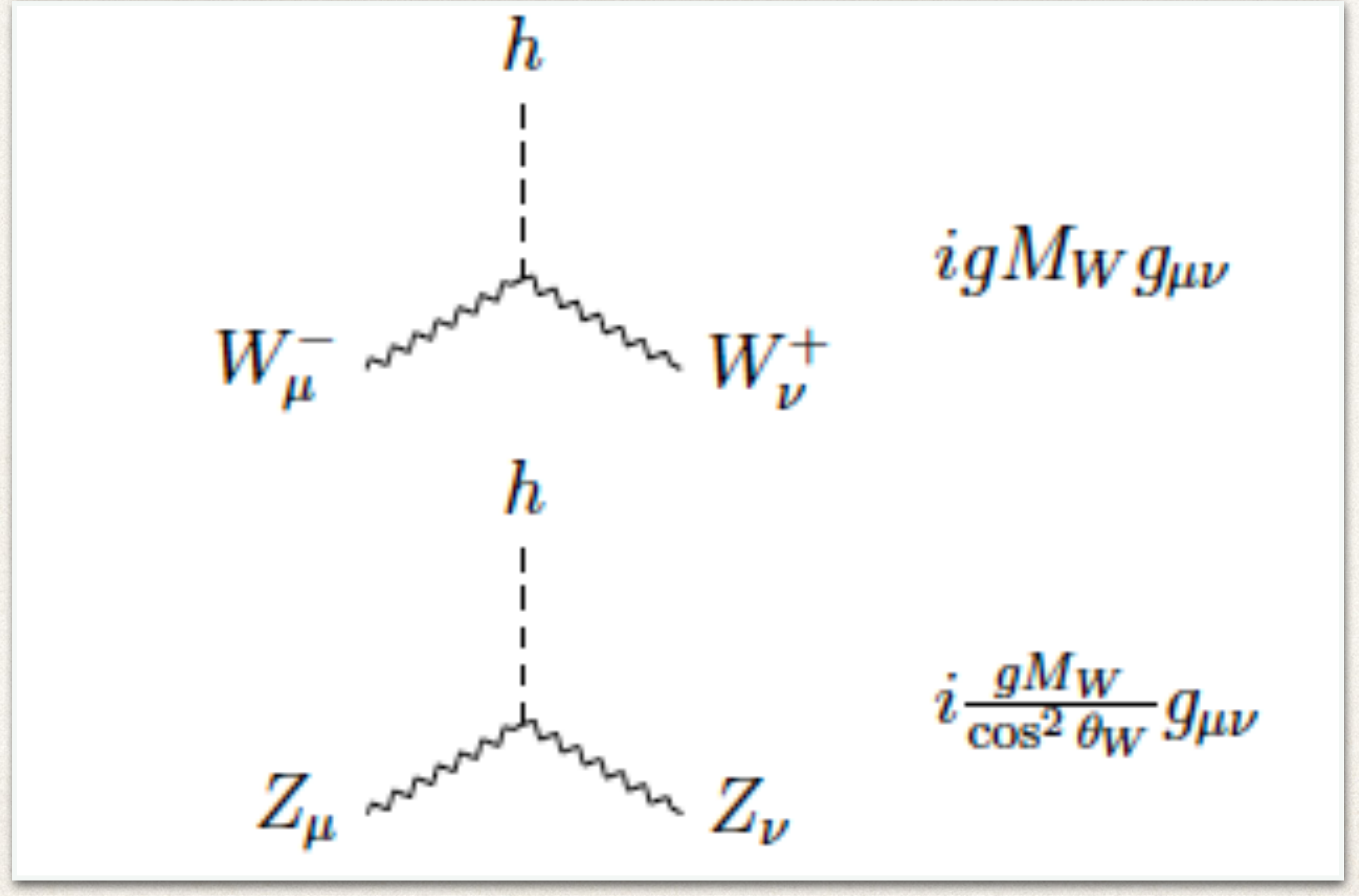
Before SSB	4×2 +	2×2	= 12 =	3×3 +	2 + 1	After SSB
	$W^{1,2,3}, B$	ϕ^+, ϕ^0		W^+, W^-, Z	A H	

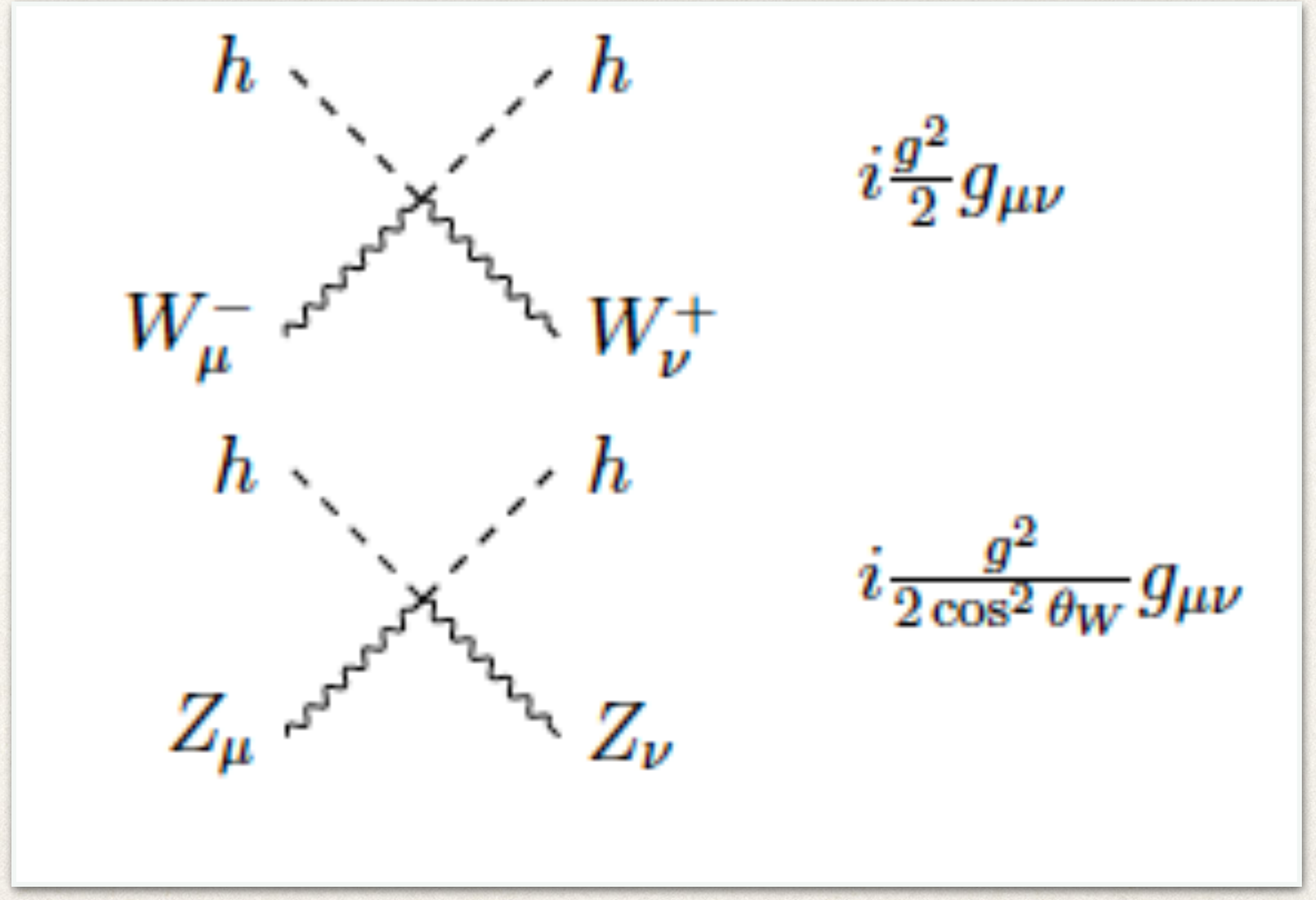
Gauge boson - Higgs interactions

- $(D_\mu \Phi)^\dagger D^\mu \Phi$ also provides trilinear and quadric couplings of the Higgs boson to gauge bosons

$$(D_\mu\Phi)^\dagger(D_\mu\Phi) = \frac{1}{2}\partial_\mu H \partial^\mu H + \left[\frac{g^2 v^2}{4} W^{+,\mu} W^-_\mu + \frac{v^2}{8}(g^2 + g'^2) Z_\mu Z^\mu\right]\left(1 + \frac{H}{v}\right)^2$$

- Feynman rules





Higgs self-interactions

$$V(\Phi) = -\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2 \qquad \Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+H \end{pmatrix}$$

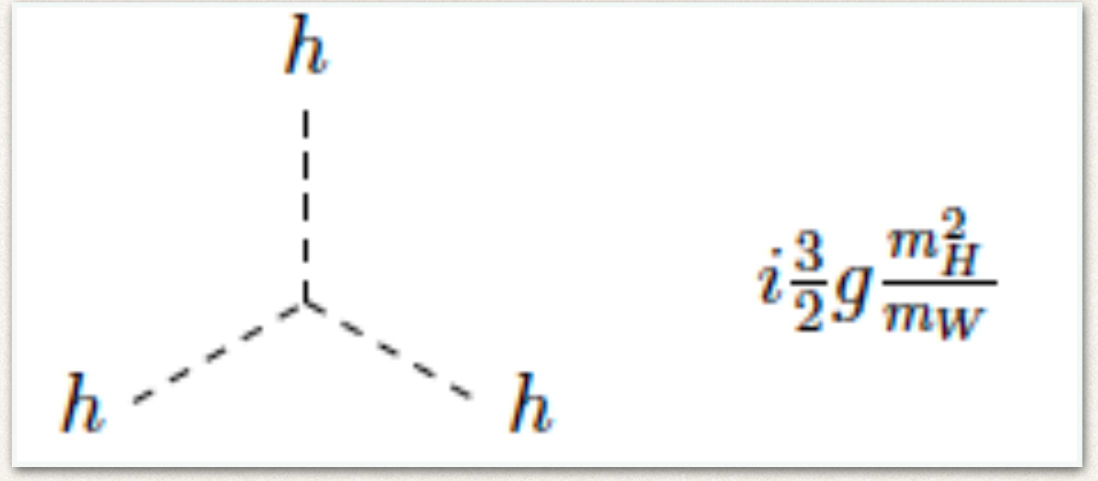
$$\Rightarrow V(\Phi) = \mu^2 H^2 + \lambda v H^3 + \frac{\lambda}{4} H^4 + \text{constant}$$

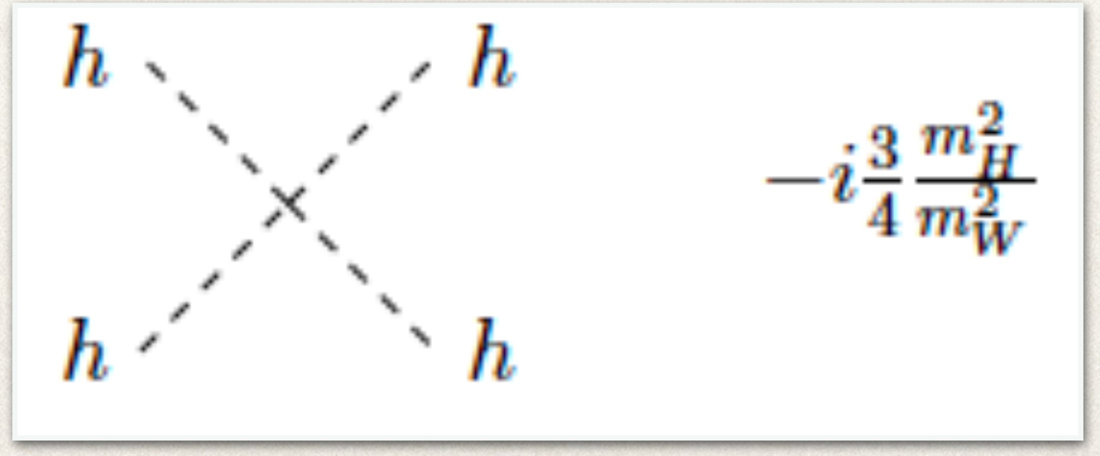
- Mass term for the Higgs boson

$$M_H = \sqrt{2}\mu = \sqrt{2\lambda}v$$

  - v and M_H measured by experiment (v = 246 GeV, M_H = 125 GeV) $\Rightarrow$ Higgs self-coupling λ fixed (λ=0.129)

- Feynman rules





Fermion masses

- One more nut to crack: explicit mass terms for fermions break gauge invariance $\bar{\psi}\psi = \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L$ under SU(2)
- Introduce gauge-invariant Yukawa terms (now only for the electron)

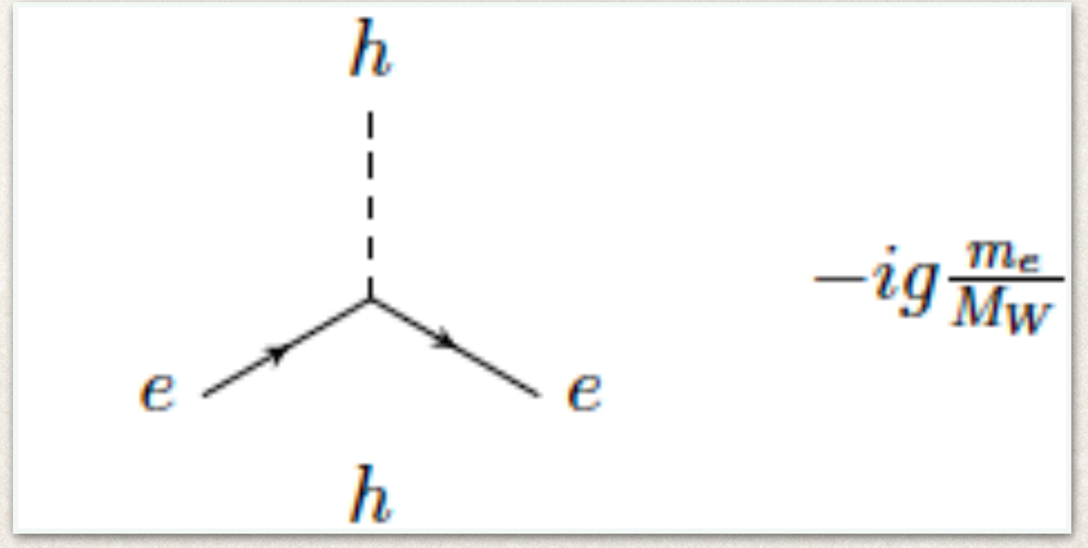
$$\mathscr{L}_{\text{Yukawa,e}} = y_e\left[\bar{\psi}_L\Phi e_R + \bar{e}_R\Phi^\dagger\psi_L\right]$$

"5th force"

- After SSB, in the unitary gauge $\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+H \end{pmatrix}$

$$\mathscr{L}_{\text{Yukawa,e}} = -\,y_e\frac{v+H}{\sqrt{2}}\left(\bar{e}_L e_R + \bar{e}_R e_L\right) = -\frac{y_e}{\sqrt{2}}(v+H)\,\bar{e}e = -\underbrace{\frac{y_e v}{\sqrt{2}}\,\bar{e}e}_{\text{mass term}} - \underbrace{\frac{y_e}{\sqrt{2}}\,\bar{e}e\,H}_{\text{interaction term}}$$

- Mass term for the electron with $m_e = \frac{y_e}{\sqrt{2}}v$
- Yukawa coupling proportional to the electron mass $y_e = \sqrt{2}\,\frac{m_e}{v} = \frac{g}{\sqrt{2}}\frac{m_e}{M_W}$



Weak interactions of quarks

- So far, only 1 generation of leptons considered
- Extension to three lepton generations in the original EWSM (with massless neutrinos) is a trivial threefold copy of the Lagrangian for the 1st generation leptons
- Extending to 1st generation quarks
  - Matter content

$$\psi_q = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \qquad u_R,\, d_R$$

	T	T^3	Q	Y
u_L	1/2	1/2	2/3	1/3
d_L	1/2	-1/2	-1/3	1/3
u_R	0	0	2/3	4/3
d_R	0	0	-1/3	-2/3

  - Quark masses : need an additional Yukawa term to generate up quark mass

$\mathscr{L}_{\text{Yukawa,d}} = -y_d \bar{\psi}_q \Phi d_R + h.c.$ (analogous to electron) $\qquad \mathscr{L}_{\text{Yukawa,u}} = -y_d \bar{\psi}_q^{\dagger} \Phi^c u_R + h.c.$ with $\Phi^c \equiv i\sigma^2 \Phi^*$

After SSB $\quad \Phi^c = \dfrac{1}{\sqrt{2}} \begin{pmatrix} v + H \\ 0 \end{pmatrix}$ and $\quad \mathscr{L}_{\text{Yukawa,u}} = -y_d \dfrac{v+H}{\sqrt{2}} (\bar{u}_L u_R + \bar{u}_R u_L)$

Weak interactions of fermions

- In general, the structure of the Yukawa terms (after SSB) for all generations of quarks $(i, j = 1,2,3)$ is

$$\mathscr{L}_{\text{Yukawa}} = -\, y_u^{ij} \frac{v+H}{\sqrt{2}} \bar{u}_L^i u_R^j - y_d^{ij} \frac{v+H}{\sqrt{2}} \bar{d}_L^i d_R^j + \text{h.c.} = -\sum_f \bar{f}_L\, M_f f_R \left(1 + \frac{H}{v}\right) + \text{h.c.}$$

where $M_f^{ij} = y_f^{ij} \dfrac{v}{2}$ is a non-diagonal mass matrix for quarks

- Introduce unitary transformations U_L^f and U_R^f rotating the vectors

$$f_L = \begin{pmatrix} f_L^1 \\ f_L^2 \\ f_L^3 \end{pmatrix} \text{ and } f_R = \begin{pmatrix} f_R^1 \\ f_R^2 \\ f_R^3 \end{pmatrix} \text{ in the gauge basis to vectors in the mass basis } f'_L = \begin{pmatrix} f_L^{'1} \\ f_L^{'2} \\ f_L^{'3} \end{pmatrix} = U_L^f f_L \quad f'_R = \begin{pmatrix} f_R^{'1} \\ f_R^{'2} \\ f_R^{'3} \end{pmatrix} = U_R^f f_R$$

such that the matrix $M_{f,D} = U_L^f\, M_f\, (U_R^f)^\dagger$ is diagonal

$$\Rightarrow \mathscr{L}_{\text{Yukawa}} = -\sum_f \bar{f}_L\, (U_L^f)^\dagger\, M_{f,D}\, U_R^f\, f_R \left(1 + \frac{H}{v}\right) + \text{h.c.} = -\sum_f m_f^k \left(\bar{f}_L^{'k} f_R^{'k} + \bar{f}_R^{'k} f_L^{'k}\right) \left(1 + \frac{H}{v}\right) + \text{h.c.}$$

Quark sector

- Write the charged quark current in terms of mass eigenstates $u_L^{'k}$ and $d_L^{'k}$

$$\mathscr{L}_{\mathrm{q,CC}} = -\frac{g}{\sqrt{2}} W_\mu^- \bar{u}_L^j \gamma^\mu d_L^j - \frac{g}{\sqrt{2}} W_\mu^+ \bar{d}_L^j \gamma^\mu u_L^j = -\frac{g}{\sqrt{2}} W_\mu^- \bar{u}_L^{'k} (U_L^u)^{kj} \gamma^\mu (U_L^{d\,\dagger})^{jl} d_L^{'l} + \mathrm{h.c.} = -\frac{g}{\sqrt{2}} V_{kl} W_\mu^- \bar{u}_L^{'k} \gamma^\mu d_L^{'l} + \mathrm{h.c.}$$

where $V_{kl} = \left(U_L^u U_L^{d\,\dagger} \right)_{kl}$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix

⇒ physical charged currents mix flavours, known as flavour-changing charged currents (FCCC)

- Neutral currents are diagonal in the mass basis ⇒ no flavour-changing neutral currents (FCNC) in the SM at tree level
- CKM matrix provides a source of CP violation in the SM ➜ see lectures by A. Lenz

Electroweak (EW) theory

- What do we want?
  - Quantum field theory of electromagnetic and weak interactions
    - based on principle of gauge symmetry

      SU(2)xU(1)

    - with massive weak gauge bosons (weak interactions ~ short range) but massless photons, as well as massive fermions

      BEH mechanism

      Yukawa terms

    - able to describe flavour-changing processes, e.g. β-decay (where weak interactions discovered)
      $n \to p^+ + e^- + \bar{\nu}_e$ -> at the quark level $d \to u + e^- + \bar{\nu}_e$

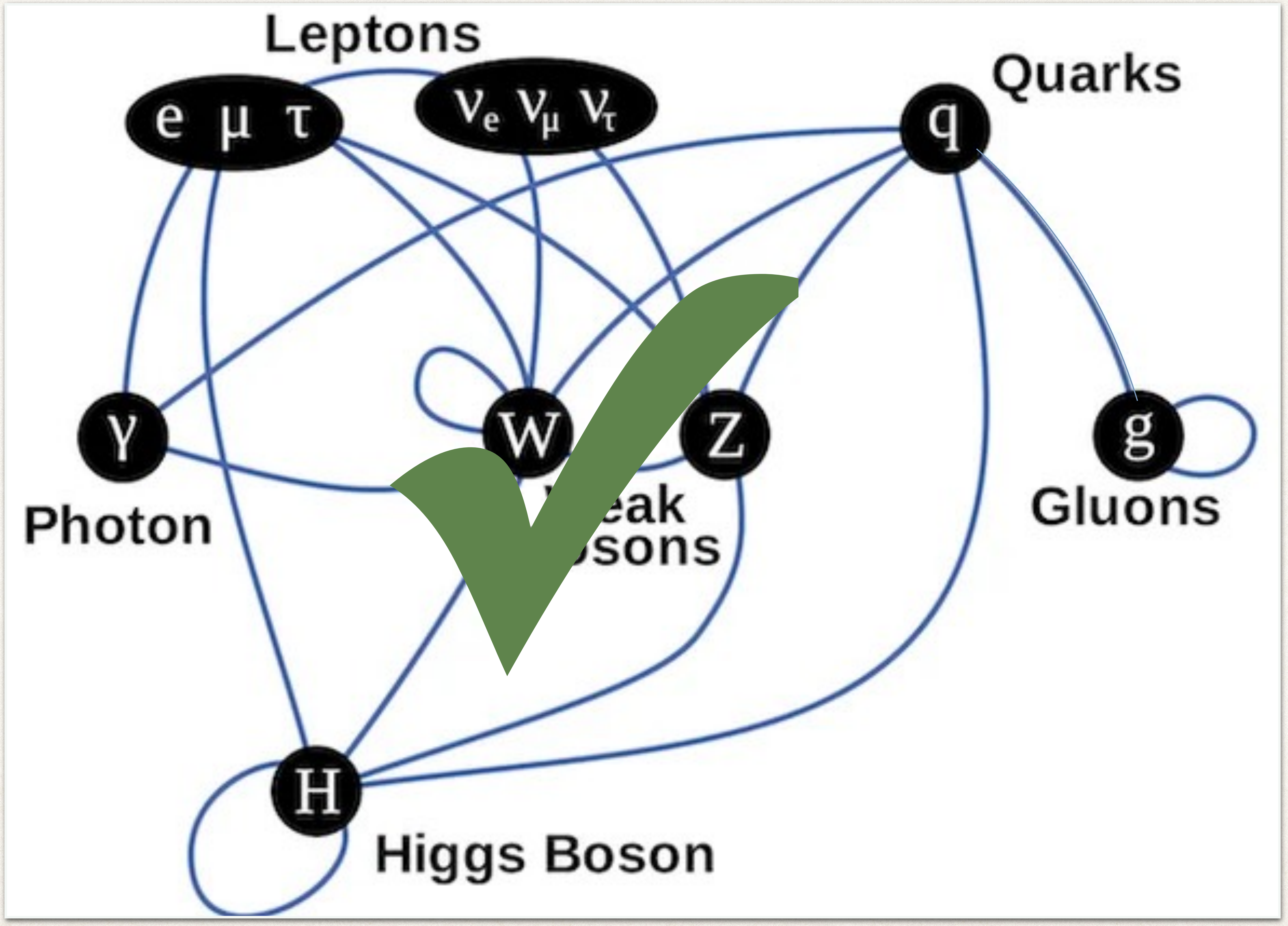
      FCCC

    - with weak interactions chiral and maximally parity violating *(Lee and Young'56, Wu'57)*: charged currents only involving left-handed particles (right-handed antiparticles)

      $\mathscr{L}_{CC}$

    - neutral current weak processes (discovered after the EW Standard Model was proposed -> prediction of the theory)

      $\mathscr{L}_{NC}$


Leptons
e μ τ
ν_e ν_μ ν_τ
Quarks
q
γ
W
Z
g
Photon
Gluons
H
Higgs Boson