# Inflation

- Problems of initial conditions of expanding Universe
  - horison problem, why universe is homogeneous and isotropic?
  - flatness problem: why $\Omega_M + \Omega_\Lambda = 1$, i.e. $\rho = \rho_c$?
- Idea of inflation
- scalar field and chaotic inflation

# Horizon problem

Particle horizons.

Flat static universe:

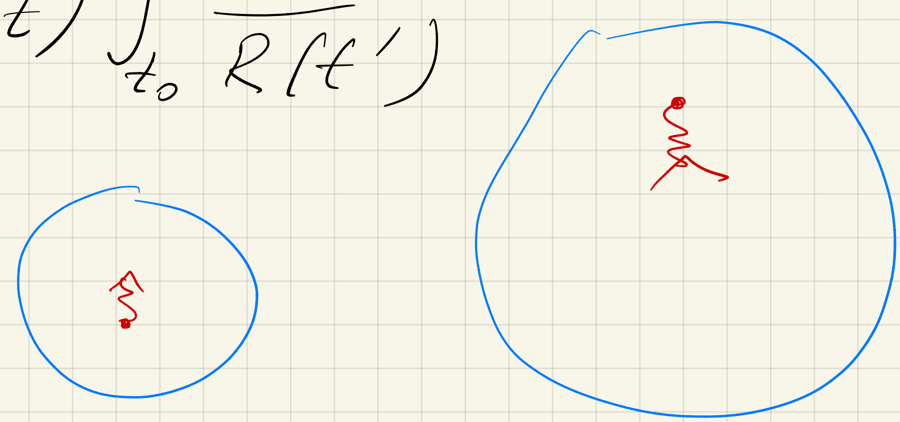
events are independent if
$\Delta l$ (distance) $> \Delta t$ (time)

$x=0$, $t=0$ $\qquad \Delta l \qquad$ $t$

Expanding universe, light propagation

$$\frac{dl}{dt} = 1 + \frac{\dot{R}}{R}\, l$$

$$l(t) = R(t) \int_{t_0}^{t} \frac{dt'}{R(t')}$$



Radiation dominated Universe,

$R(t) \sim t^{1/2}$

matter dominated Universe,

$R(t) \sim t^{2/3}$

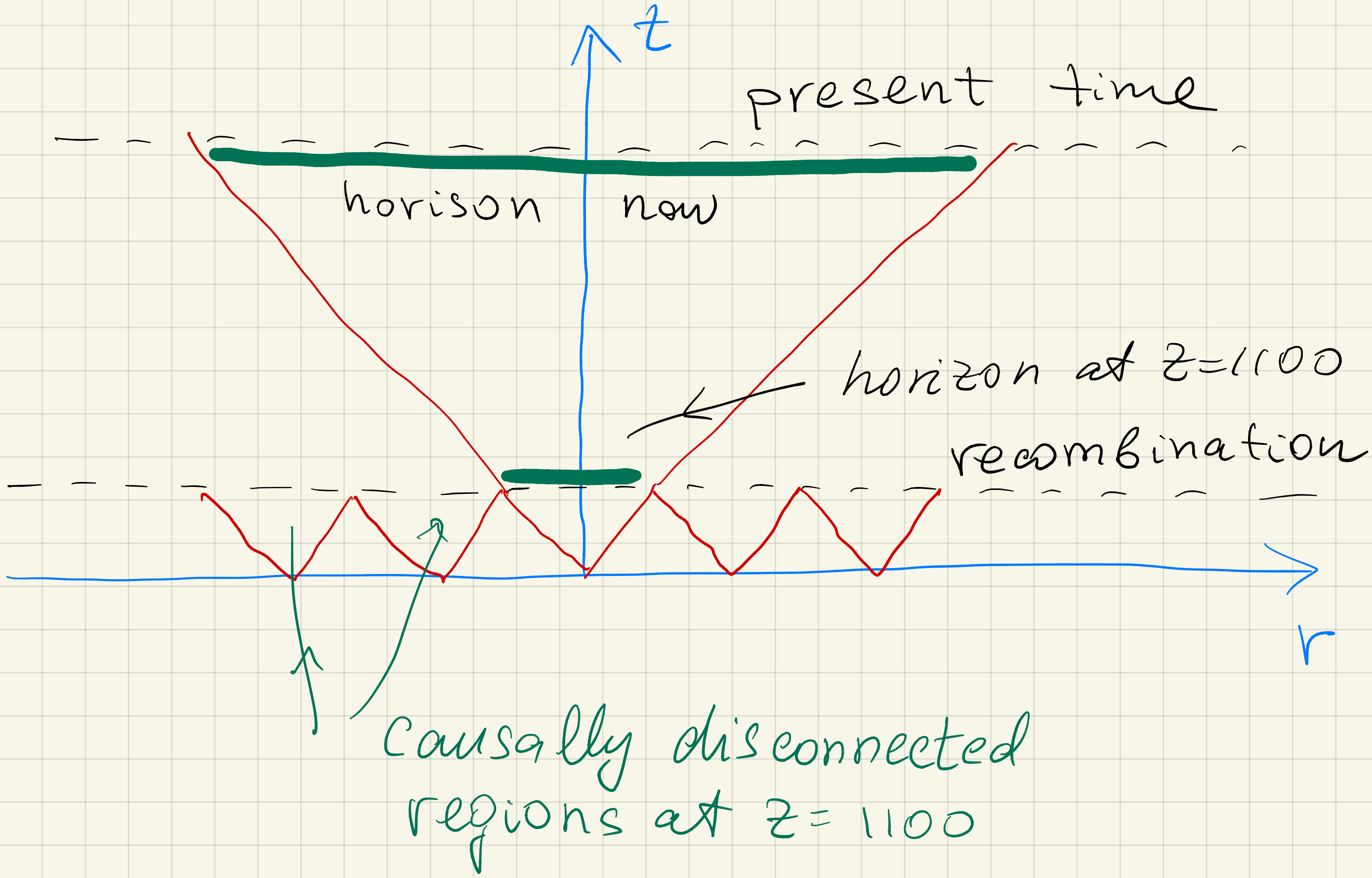
$\ell(t)$ exists (integral converges)

even if $t_0 = 0$:

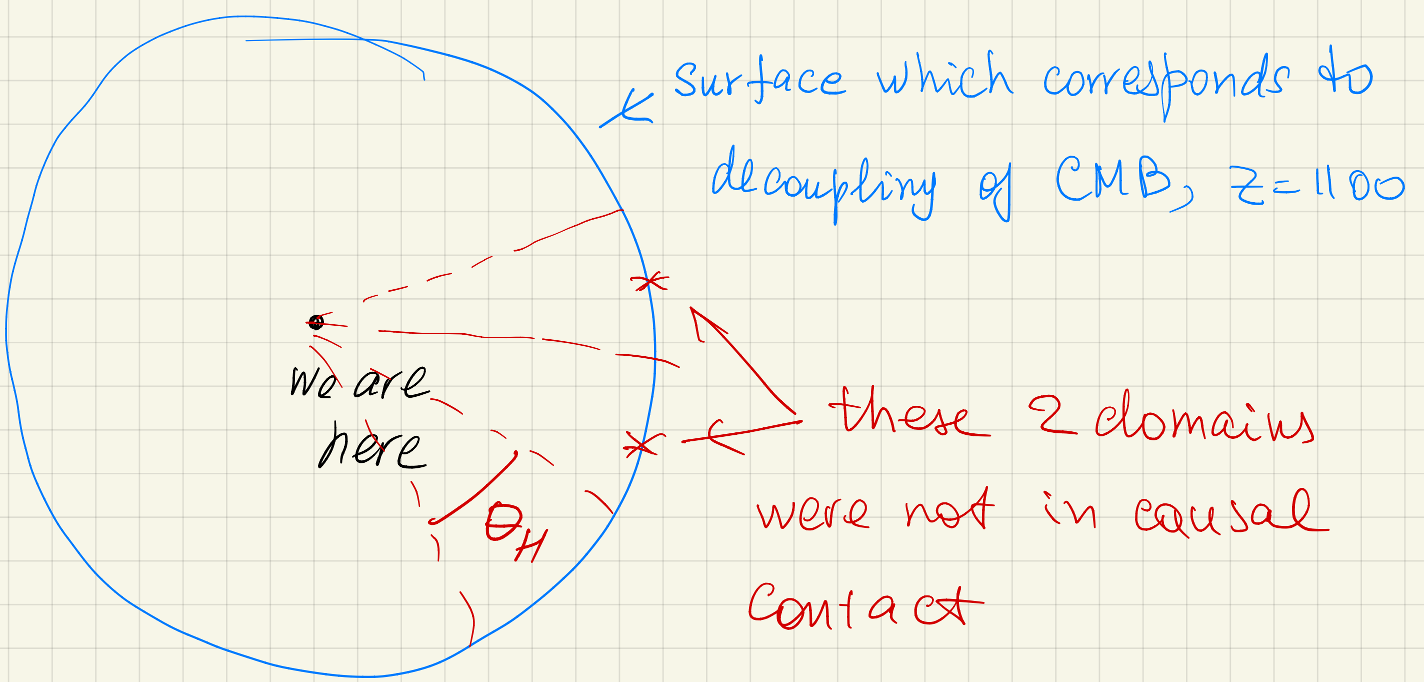
$$\ell_H = \begin{cases} 2t, & \text{radiation} \\ 3t, & \text{matter} \end{cases}$$

If the Universe were dominated by matter or radiation, then the points at distances $\ell > \ell_H$ never „talked" to each other $\Rightarrow$ we should expect to have inhomogeneities

Use CMB as a snapshot of the Universe at $z = 1100$




The same picture from different perspective:



Estimate of the angular size of causally connected domain at $z=1100$

horizon at $z=1100$

$$\Theta \sim \frac{t_d}{t_{now}} \left(\frac{t_{now}}{t_d}\right)^{2/3} \sim$$

expansion in matter dominated era

horizon @ $z=0$

$$\sim \left(\frac{1}{z}\right)^{2/3} \sim 2^\circ .$$

We should expect to have 100% fluctuations in CMB for $\Theta \gtrsim 2^\circ$, but $\frac{\delta T}{T} \sim 10^{-5}$ !

This is horizon problem

## Flatness problem

Consider $\Omega = \Omega_M + \Omega_\Lambda + \Omega_\gamma$

Einstein equations tell:

$$\Omega - 1 = \frac{\varkappa}{R^2 H^2}$$

$\varkappa = 0, \pm 1$ – flat, open or closed Universe

For radiation or matter dominated Universe:

$R \sim t^{1/2}$ or $t^{2/3}$; $H \sim \frac{1}{t} \Rightarrow$

$\Omega - 1 \sim t$ or $t^{2/3}$ –

radiation matter

increases with $t$. Now $\Omega - 1 \lesssim 10^{-2}$

at BBN: $|\Omega - 1| \sim 10^{-15} \ll 1$. Why?

Summary:

The experimental facts that the present universe is flat, homogeneous and isotropic are very bizarre: if the universe was dominated by matter or radiation and expanded like $R \sim t^{\alpha}$, $\alpha < 1$, then the initial conditions for expantion must be highly fine tuned: universe must be super-duper flat, homogeneous and isotropic at the beginning.

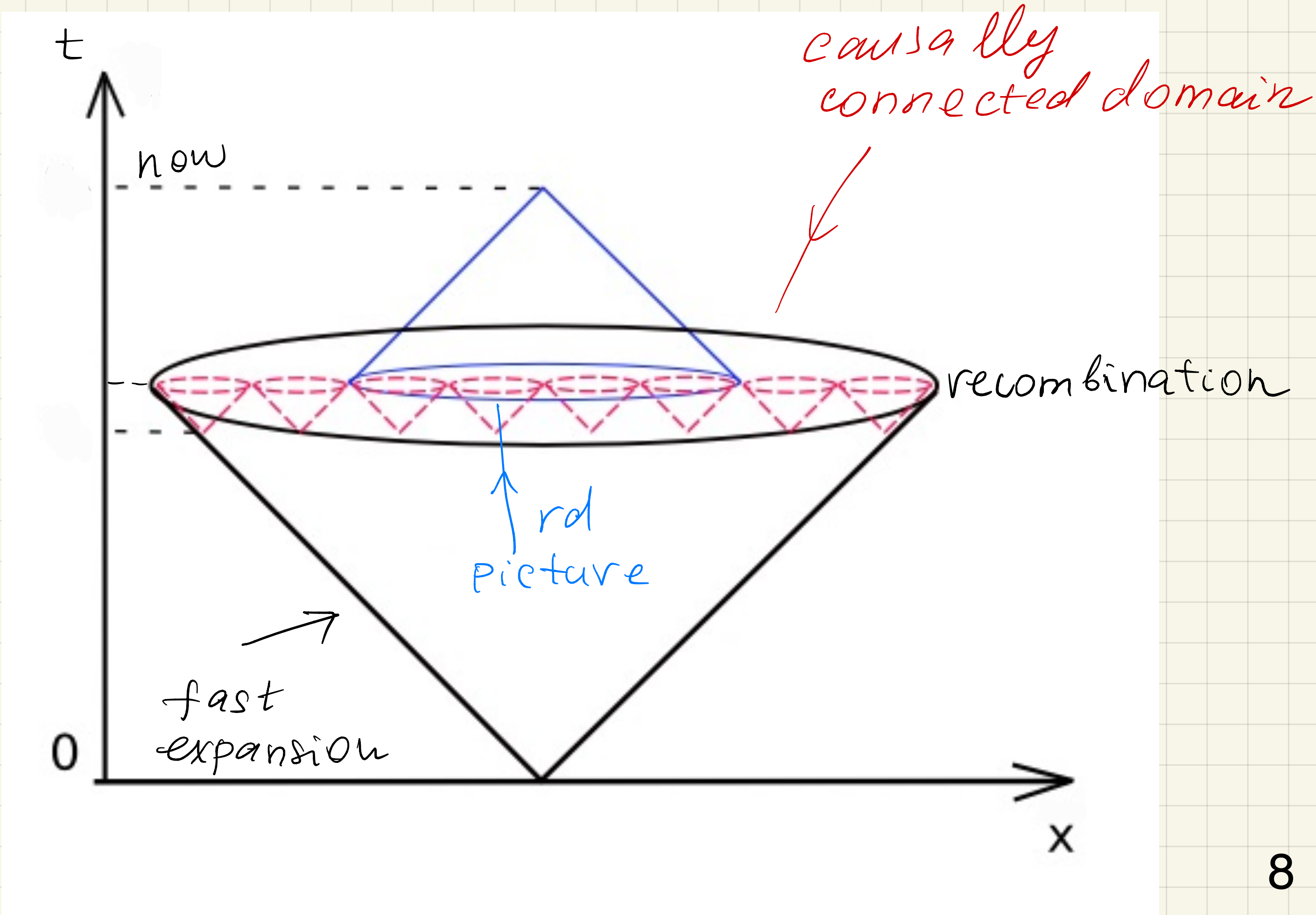
Is there any rational behind that?

Probable answer – yes.

Inflationary theory.

Main point: assuming that Universe was always dominated by matter or radiation in the past is groundless. We only know that for BBN (perhaps baryogenesis). What was before is unknown.

Let's make the Universe expand faster!



How can we change the expansion low?

- $t$ from $0$ to $t_0$ : unknown equation of state

- $t$ from $t_0$ to $t_1$: cosmological constant dominates, $p=-\rho$ exponential expansion

- $t$ from $t_1$ to the end of radiation-domination stage: $p = \rho/3$

Then:

$$R(t) = \begin{cases} R_0 \exp(H(t-t_0)), & t_0 < t < t_1 \\ R_0 \exp(H(t_1 - t_0)) \left(\frac{t}{t_1}\right)^{1/2}, & t > t_1 \end{cases}$$

horizon at recombination:

$$\ell_H \approx \frac{1}{H} \exp(H(t_1 - t_0))$$

Numerical example:

take „vaccum" energy $\sim (10^{15}\,\text{GeV})^4$

To solve horizon and flatness problems it is enough to have $H(t_1 - t_0) \gtrsim 65$,

65 – „number of e-foldings".

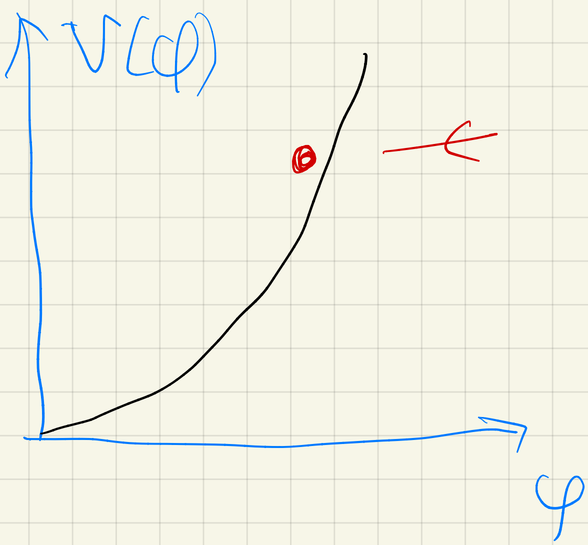
# Chaotic inflation (Linde)

Simplest theory leading to inflation

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{1}{2} m^2 \varphi^2 -$$

non-interacting real scalar field.

Potential:

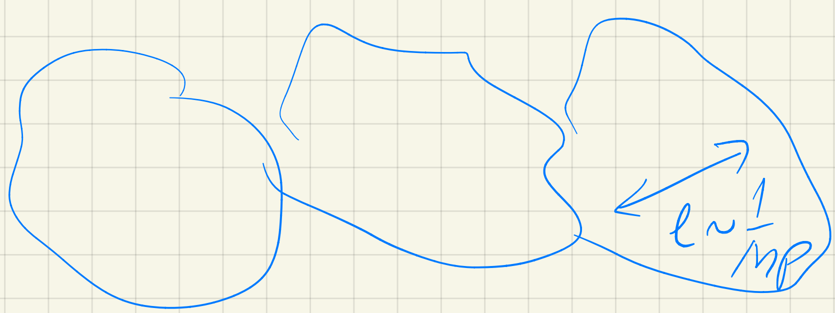



Chaotic initialy conditions:

$$\dot{\varphi}^2 \sim M_p^4$$

$$(\nabla \varphi)^2 \sim M_p^4$$

$$m^2 \varphi^2 \sim M_p^4$$




⟸ „space-time foam"

Take a region where potential energy dominates,

$$V(\varphi) \gg (\nabla\varphi)^2, \quad V(\varphi) \gg \dot{\varphi}^2$$

$$m^2\varphi^2 \approx M_p^4, \quad \varphi \simeq \frac{M_p^2}{m} \gg M_p$$

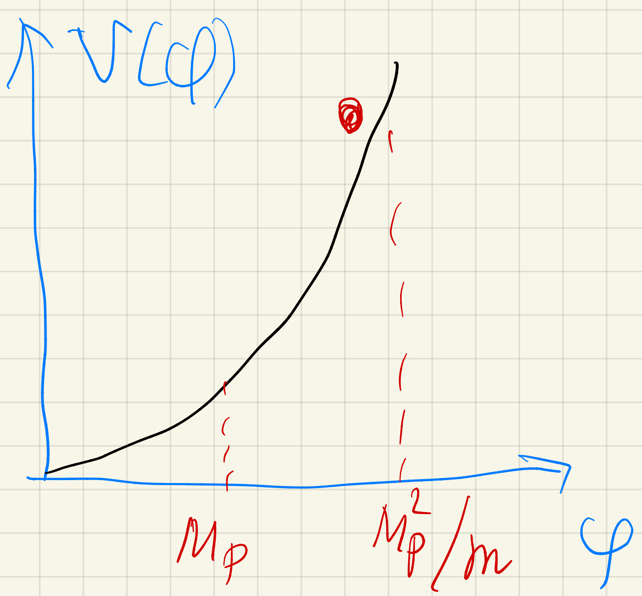
for $m \ll M_p$

Consider dynamics of field $\varphi$ in this domain

"friction"

$$\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = 0$$

$$H^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\varphi}^2 + V(\varphi)\right)$$




damped oscillator

„Slow-roll“ inflation : $H \gg \dfrac{\ddot{\varphi}}{\dot{\varphi}}$

$$H \approx \frac{4\pi m^2 \varphi^2}{3 M_P^2}$$

Slow-roll equation:

$$\frac{\sqrt{12\pi}\, m \varphi \dot{\varphi}}{M_P} + m^2 \varphi = 0$$

Solution:

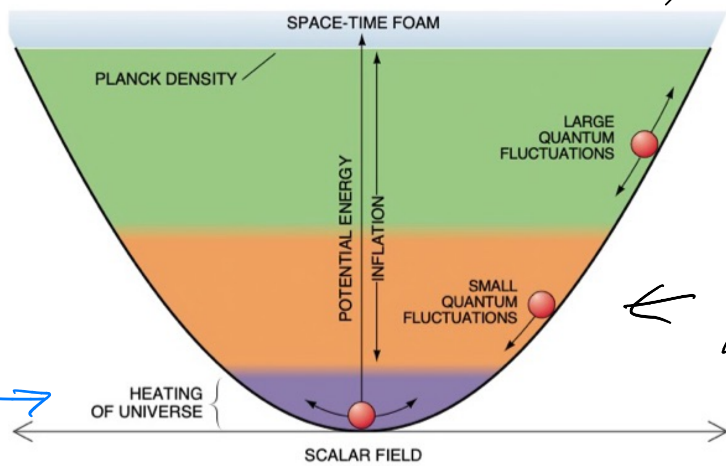
$$\varphi = \varphi_0 - \frac{m M_P}{\sqrt{12\pi}}\, t$$

Slow-roll approximation breaks down at $\varphi \simeq M_P$, after that $\varphi$ oscillates near zero.

Universe inflates by a factor

$$e^{Ht} \sim \exp\left(\sim M_P^2/m^2\right) \gg 1$$

Quantum gravity




"slow roll", "inflation"

energy transfer to all particles, Big Bang.

small quantum fluctuations, small distances. Inflation – make them long-ranged:

$\frac{\delta\varphi}{\varphi} \neq 0$ Inflation $\frac{\delta\varphi}{\varphi} \neq 0$

quantum Small size

macroscopical size

Inflation imprint: density perturbations,

$$\left\langle \frac{\delta\rho(\vec{x})}{\rho} \frac{\delta\rho(0)}{\rho} \right\rangle = f(|\vec{x}|) \neq 0$$

$\langle \dots \rangle$ quantum average.

Power low parametrisation:

$$f(x) = \text{Const} \cdot |\vec{x}|^{1-n_s}$$

„spectral" index

If $n_s = 1$: „scale-invariant" spectrum

called „Harisson-Zeldovich spectrum

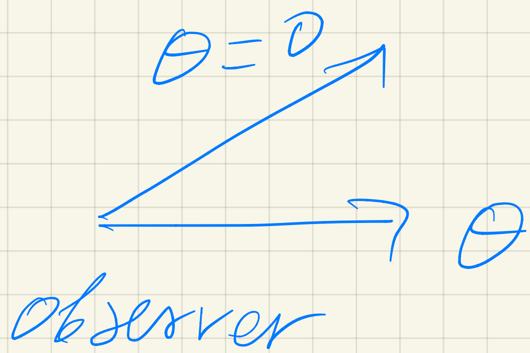
$n_s$ can be computed in inflationary model

Another parameter of inflation:

$$r = \frac{\text{energy in grav. waves}}{\text{energy in scalar perturbations}}$$

Density perturbations $\Rightarrow$ temperature fluctuations of CMB:

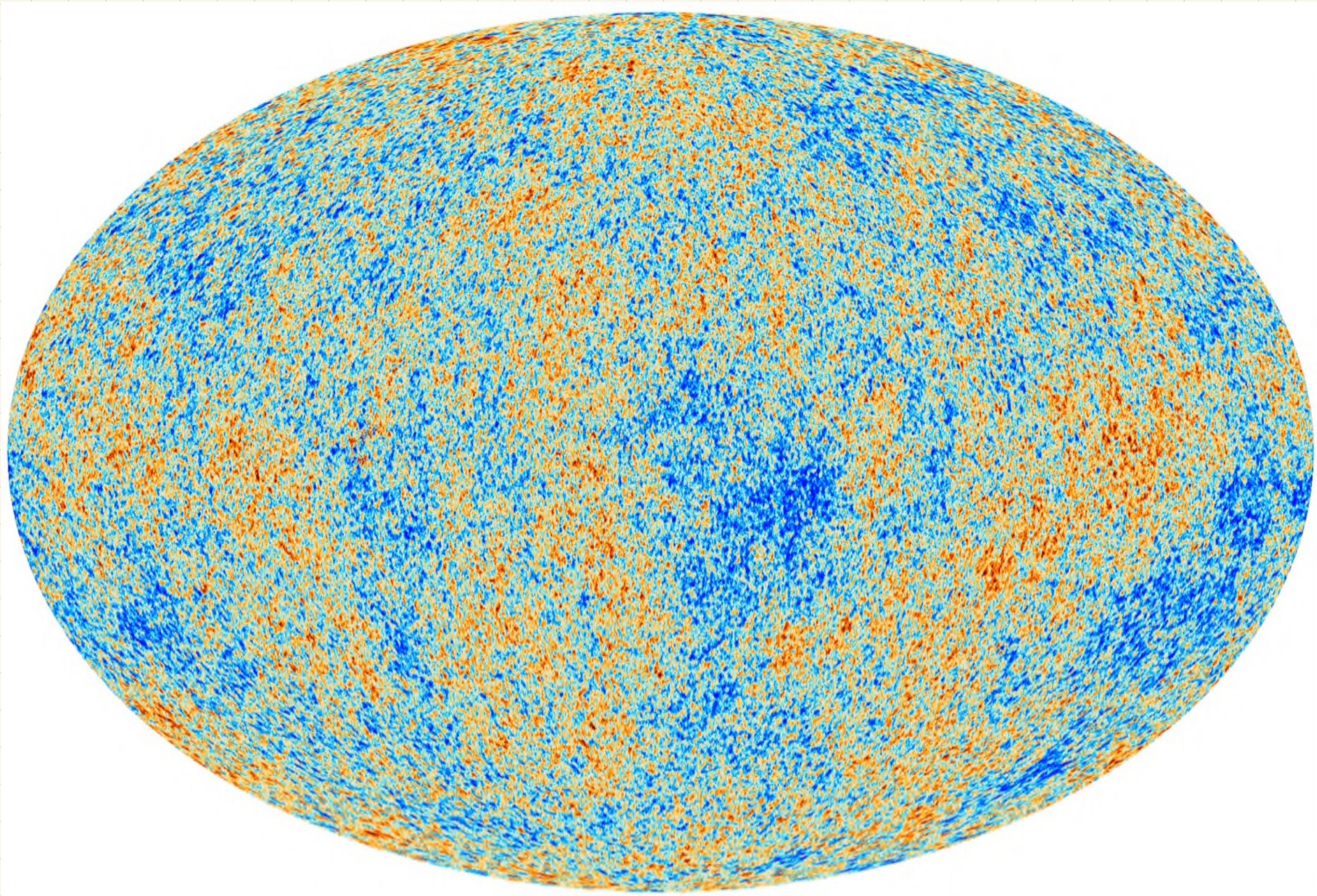
$$\langle \delta T(\theta)\, \delta T(0) \rangle = F(\theta) \neq 0$$

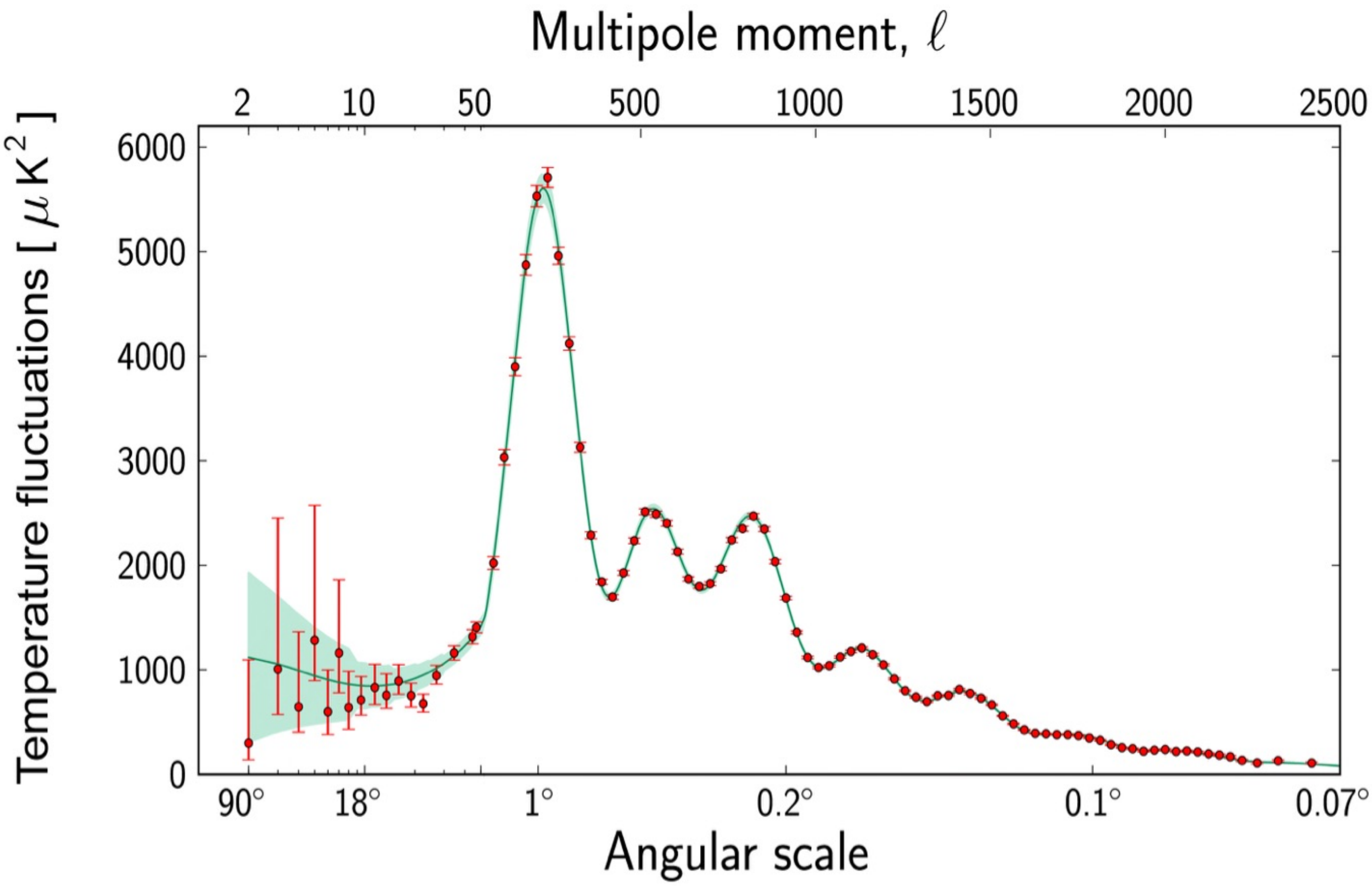



$F(\theta)$ is related to $f(|\vec{x}|)$, and depends also on cosmological parameters:

$\Omega$, $\Omega_M$, $\Omega_{baryon}$, $\Omega_\nu$, etc: initial simple spectrum is modified by processes @ recombination.

How to convert this picture to numbers: $r, n_s, \Omega, \Omega_B \ldots$



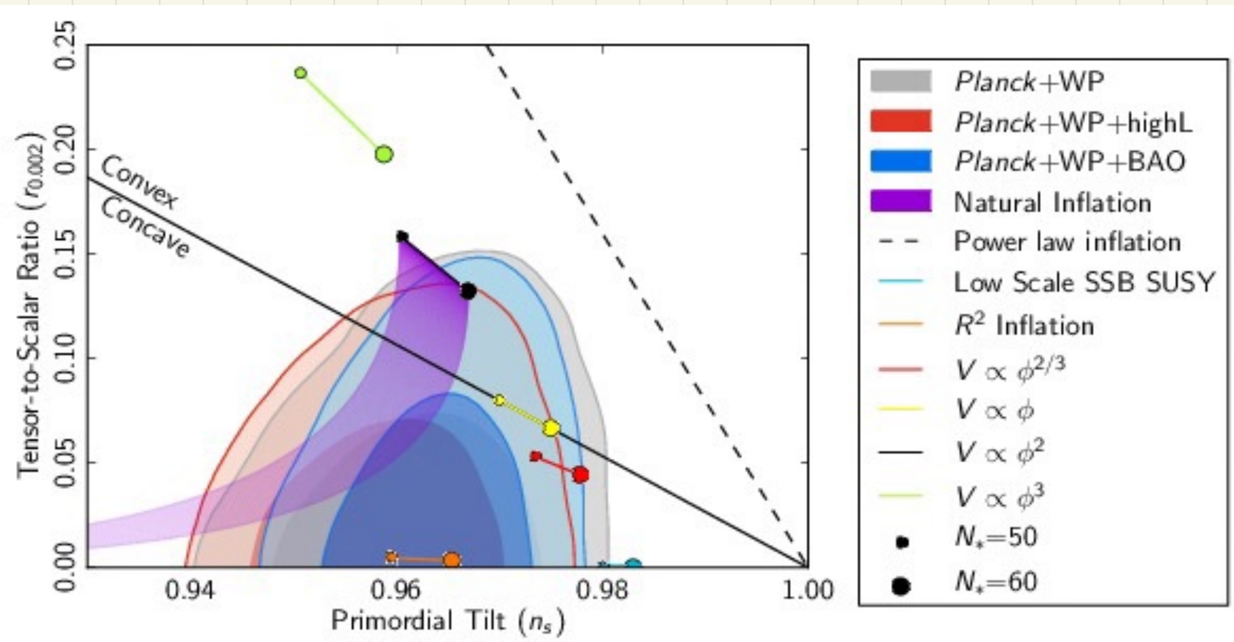


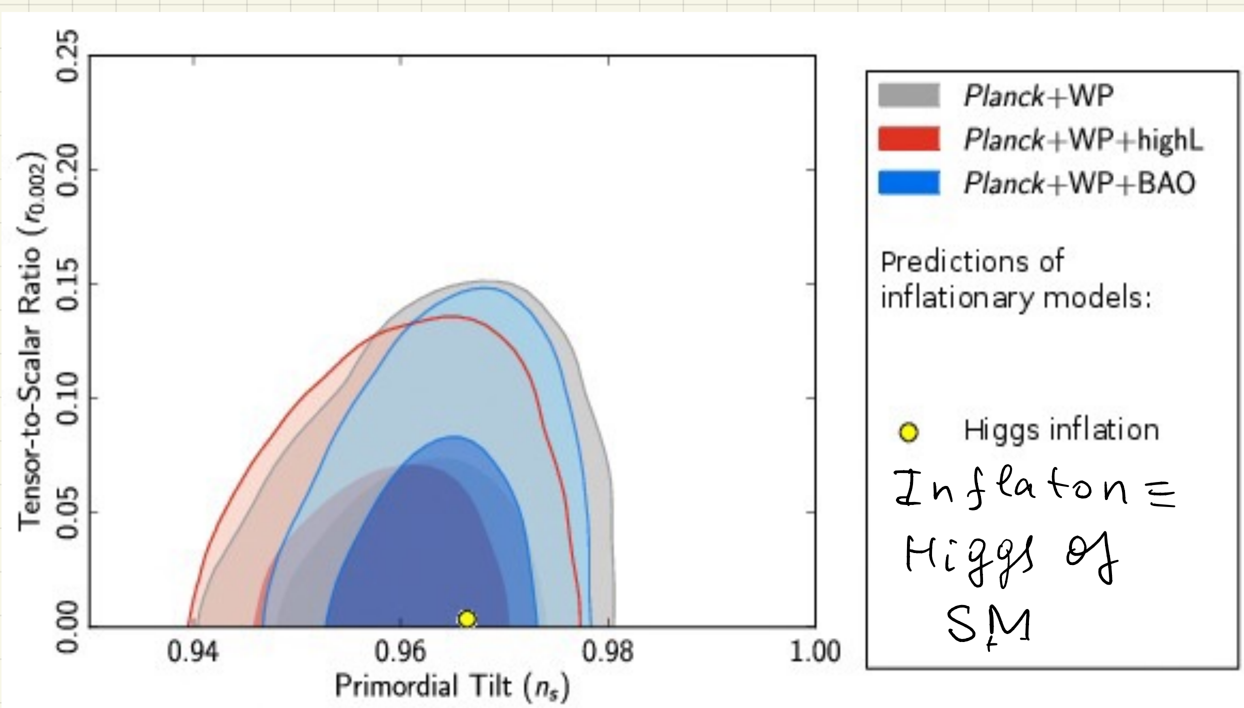

This picture is for a particular choice of cosmological parameters.

Variation of these parameters make the theory curve different.

What kind of particle inflated the Universe?

Many different models of inflation




Planck: ESA Space observatory
WP: NASA Wilkinson Anisotropy Probe, WMAP

highL: large angular momenta L
BAO: baryon acoustic oscillations

# Higgs inflation

Higgs boson coupling to gravity:

$$\xi R \cdot H^2$$

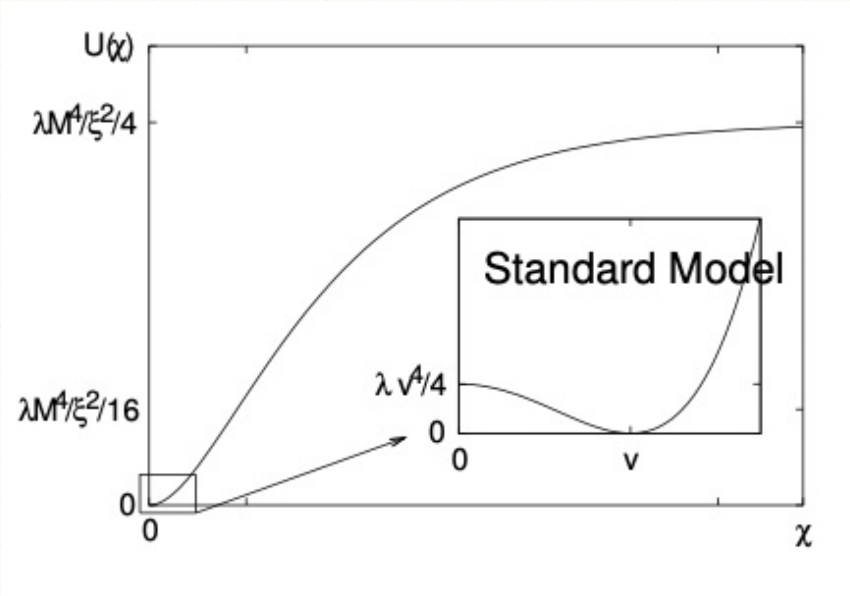
called „non-minimal coupling"

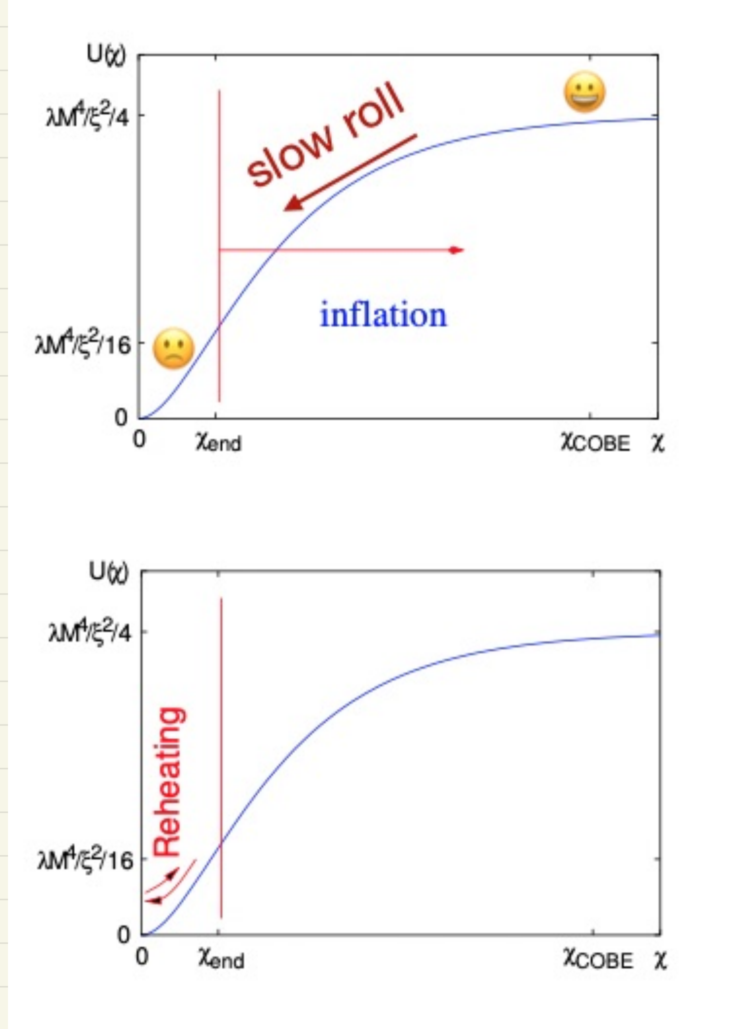
$$S_G = -\frac{M_P^2}{2} R - \xi \frac{H^2}{2} R$$

Dynamical Planck scale,

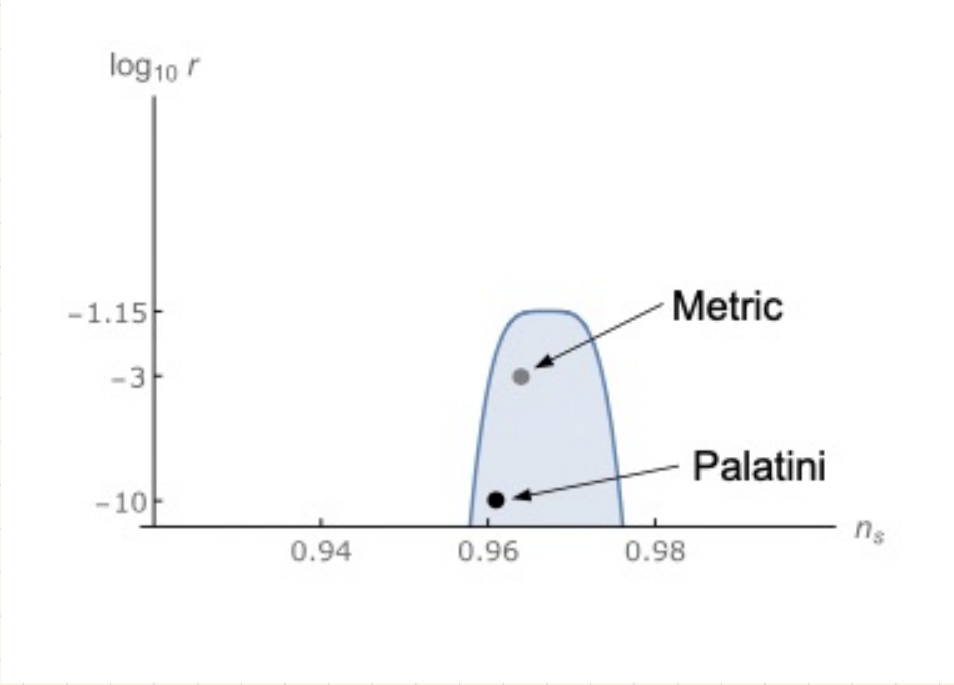
$$M_P^2 + \xi H^2$$

If $\xi H^2 \gtrsim M_P^2$ Higgs gives masses to everebody, and determines the strength of gravity.

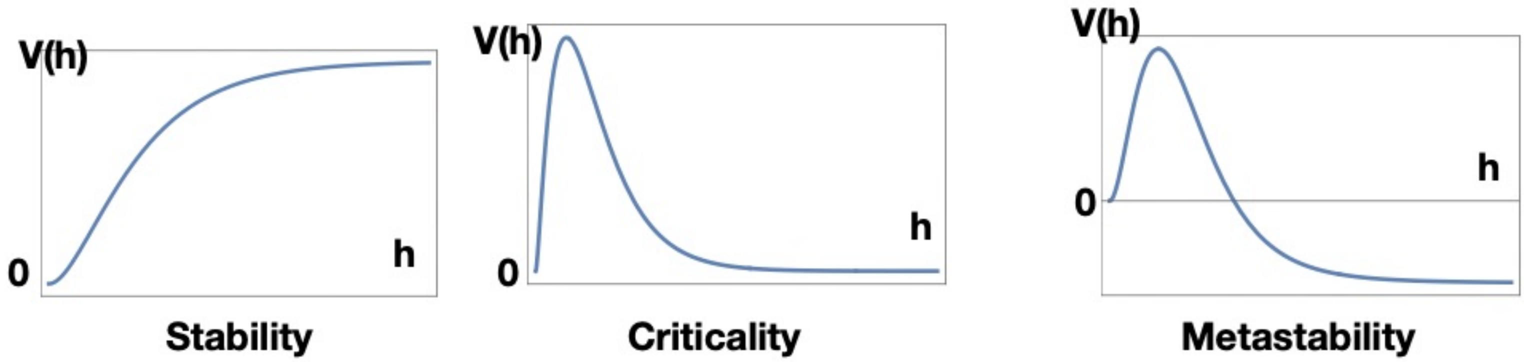


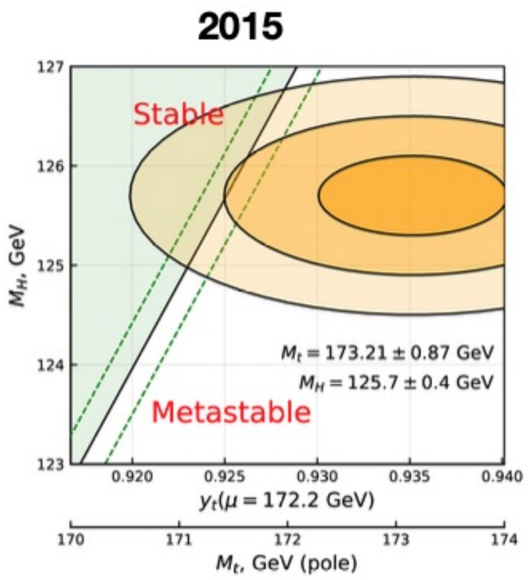

tensor to scalar ratio depends on formulation of gravity - metric, palatini, Einstein-Cartan, ...



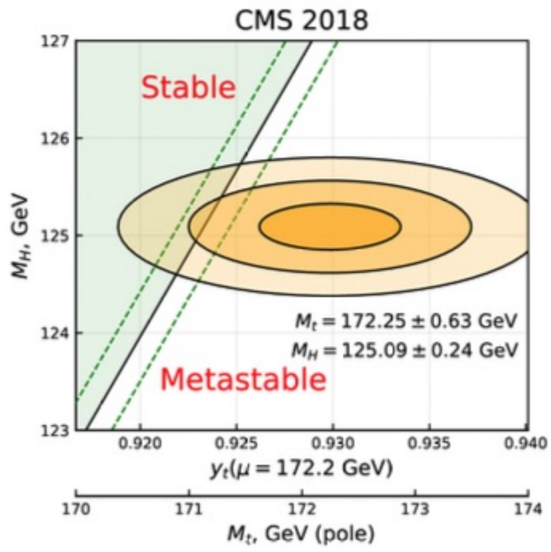
# Vacuum metastability in the SM?




- Marginal evidence (less than 2σ) for the SM vacuum metastability given uncertainties in relation between Monte-Carlo top mass and the top quark Yukawa coupling




**Time evolution of SM vacuum metastability**



## Conclusions for inflation

- the problem of initial conditions of the Universe – horizon, flatness – can be solved by inflation

- inflation & Big Bang can be realised in dynamics of scalar field

- Inflationary predictions:
  - $\Omega = 1$
  - quantum fluctuations of scalar field – density fluctuations in the Universe

- analysis of CMB spectrum can fix Universe parameters $\Omega, \Omega_\Lambda, \Omega_M$, etc & inflationary predictions $n_s$, $r$, etc

- Who is inflaton is an open question, Higgs boson is a possibility.

## General conclusions

Interplay between particle physics and cosmology is an exciting topic.

Expect new discoveries in the future!