The Known Unknowns

★ Next generation Long-Baseline experiments (such as DUNE) can address three of these questions:

- Are neutrinos Dirac or Majorana?
  - Is there a connection to the GUT scale?

- Are there light sterile neutrino states?
  - No clear theoretical guidance on mass scale, M, ...

- What is the neutrino mass hierarchy?
  - An important question in flavor physics, e.g. CKM vs. PMNS

- Is CP violated in the leptonic sector?
  - Are vs key to understanding the matter-antimatter asymmetry?
We determined that $m(K_L) > m(K_S)$ by

- Passing kaons through matter (regenerator)

- Beating the unknown sign $[m(K_L) - m(K_S)]$ against the known sign [reg. ampl.]

We will determine the sign $(\Delta m^2_{31})$ by

- Passing neutrinos through matter (Earth)

- Beating the unknown sign $(\Delta m^2_{31})$ against the known sign [forward $\nu_e e \rightarrow \nu_e e$ ampl]

$$L \approx \frac{2\pi}{G_F n_e} \approx 1.16 \times 10^4 \text{ km} \left( \frac{1.69 \times 10^{24} \text{ cm}^3}{n_e} \right)$$
In principle, it is straightforward

\[ P(\nu_\mu \to \nu_e) - P(\bar{\nu}_\mu \to \bar{\nu}_e) = 4 s_{12} s_{13} c_{13}^2 s_{23} c_{23} \sin \delta \times \left[ \sin \left( \frac{\Delta m_{21}^2 L}{4E} \right) \times \sin \left( \frac{\Delta m_{23}^2 L}{4E} \right) \times \sin \left( \frac{\Delta m_{31}^2 L}{4E} \right) \right] \]

\( \text{vacuum osc.} \)

\[ \star \text{Requires } \{\theta_{12}, \theta_{13}, \theta_{23}\} \neq \{0, \pi\} \]

- now know that this is true, \( \theta_{13} \approx 9^\circ \)
- but, despite hints, don’t yet know “much” about \( \delta \)

\[ \star \text{So “just” measure } P(\nu_\mu \to \nu_e) - P(\bar{\nu}_\mu \to \bar{\nu}_e) ? \]

\[ \star \text{Not quite, there is a complication...} \]
Neutrino Oscillations in Matter

Accounting for this potential term, gives a Hamiltonian that is not diagonal in the basis of the mass eigenstates

\[
\mathcal{H} \left( \begin{array}{c} |v_1\rangle \\ |v_2\rangle \\ |v_3\rangle \end{array} \right) = i \frac{d}{dt} \left( \begin{array}{c} |v_1\rangle \\ |v_2\rangle \\ |v_3\rangle \end{array} \right) = \begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{pmatrix} \left( \begin{array}{c} |v_1\rangle \\ |v_2\rangle \\ |v_3\rangle \end{array} \right) + V |v_e\rangle
\]

Complicates the simple picture !!!!

\[
P(v_\mu \rightarrow v_e) - P(\overline{v}_\mu \rightarrow \overline{v}_e) =
\]

**ME**

\[
\frac{16A}{\Delta m^2_{31}} \sin^2 \left( \frac{\Delta m^2_{31} L}{4E} \right) c_{13}^2 s_{13}^2 s_{23}^2 (1 - 2 s_{13}^2)
\]

**ME**

\[
- \frac{2AL}{E} \sin \left( \frac{\Delta m^2_{31} L}{4E} \right) c_{13}^2 s_{13}^2 s_{23}^2 (1 - 2 s_{13}^2)
\]

**CPV**

\[
- \frac{\Delta m^2_{21} L}{2E} \sin^2 \left( \frac{\Delta m^2_{31} L}{4E} \right) \sin \delta \cdot s_{13} c_{13}^2 c_{23} s_{23} c_{12} s_{12}
\]

with \( A = 2 \sqrt{2} G_F n_e E = 7.6 \times 10^{-5} \text{eV}^2 \cdot \frac{\rho}{\text{g/cm}^3} \cdot \frac{E}{\text{GeV}} \)
Experimental Strategy

**EITHER:**
- Keep $L$ small ($\sim 200$ km): so that matter effect is significant.
  - First oscillation maximum:
    $$\frac{\Delta m^2_{31} L}{4E} \sim \frac{\pi}{2} \Rightarrow E_\nu < 1 \text{ GeV}$$
  - Want high flux at oscillation maximum
    - **Off-axis beam:** narrow range of neutrino energies

**OR:**
- Make $L$ large ($>1000$ km): measure the matter effects (i.e. MH)
  - First oscillation maximum:
    $$\frac{\Delta m^2_{31} L}{4E} \sim \frac{\pi}{2} \Rightarrow E_\nu > 2 \text{ GeV}$$
  - Unfold CPV from Matter Effects through $E$ dependence
    - **On-axis beam:** wide range of neutrino energies
Non unitarity
Non standard neutrino interactions

They can be described by effective four-fermion operators of the form

$$2\sqrt{2}G_F \mathcal{E}_{\alpha\beta} \left( \bar{\nu}_\beta \gamma^\mu P_L l_\alpha \right) \left( \bar{f} \gamma_\mu P_{L,R} f' \right)$$

normalizing the operator with the Fermi constant

$$\mathcal{E}_{\alpha\beta} = \frac{M_w^2}{M_{NSNI}^2}$$
NSNI can appear at every step. It is therefore necessary to break down the analysis in three stages

- the production process
- the time evolution
- the detection process
We are left “only” with neutral current NSNI

\[
2\sqrt{2}G_F \varepsilon_{\alpha\beta} \left( \bar{\nu}_\beta \gamma^\mu P_L \nu_\alpha \right) \left( \bar{f} \gamma_\mu P_{L,R} f \right)
\]

\[
\frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \frac{1}{2E} \begin{bmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{bmatrix} U^\dagger + a \begin{pmatrix} 1 + \varepsilon_{ee} & \varepsilon_{e\mu} & \varepsilon_{e\tau} \\ \varepsilon_{e\mu}^* & \varepsilon_{\mu\mu} & \varepsilon_{\mu\tau} \\ \varepsilon_{e\tau}^* & \varepsilon_{\mu\tau}^* & \varepsilon_{\tau\tau} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}
\]

\[
H = \frac{1}{2E} \left[ U \begin{pmatrix} 0 \\ \Delta m_{32}^2 \end{pmatrix} U^\dagger + a \begin{pmatrix} \varepsilon_{\mu\mu} & \varepsilon_{\mu\tau} \\ \varepsilon_{\mu\tau}^* & \varepsilon_{\tau\tau} \end{pmatrix} \right]
\]

\[a = 2\sqrt{2}G_F n_e E\]
\[ P(v_\mu \rightarrow v_\mu) = 1 - \sin^2(2\theta)\sin^2(1.27\Delta m^2 L / E) \]
$\epsilon_{\mu \tau}$ changes the dissapearence probability at large energies shifts the position of the minimum in energy

$\Delta m^2$

$\epsilon_{\tau \tau}$ modifies the dissapearence probability near the first oscillation minimum, especially the depth of the minimum

$\sin^2(2\theta_{23})$
CPT violation
\[ \left| \frac{m(K_0) - m(K_0)}{m_{K-av}} \right| < 10^{-18} \]

\[ m_{K-av} \approx \frac{1}{2} \times 10^9 \text{ eV} \]

\[ (m(K_0) - m(K_0))(m(K_0) + m(K_0)) < 2 \times 10^{-18} m_{K-av}^2 \]

\[ |m^2(K_0) - m^2(K_0)| \approx \frac{1}{2} \text{ eV}^2 \]
CPT tests

CPT invariance tested in several matter-antimatter systems:

- neutral kaons
- electron/positron
- proton/antiproton
- H/anti-H

Several experiments at the Antiproton Decelerator and ELENA (Extra Low Energy Antiproton) @CERN

E. Widmann, arXiv:2111.04056 [hep-ex]
Current bounds

- We can use data of various experiments to calculate the neutrino and antineutrino oscillation parameters:
  - Solar neutrino data: $\theta_{12}, \Delta m^2_{21}, \theta_{13}$
  - Neutrino mode in LBL: $\theta_{23}, \Delta m^2_{31}, \theta_{13}$
  - KamLAND data: $\bar{\theta}_{12}, \Delta m^2_{21}, \bar{\theta}_{13}$
  - SBL reactors: $\bar{\theta}_{13}, \Delta m^2_{31}$
  - Antineutrino mode in LBL: $\bar{\theta}_{23}, \Delta m^2_{31}, \bar{\theta}_{13}$

- No bounds on CP-phases since all values are allowed
Current bounds

- We use the same data (except atmospheric neutrinos) as for the global fit to obtain

\[
\begin{align*}
|\Delta m_{21}^2 - \Delta \overline{m}_{21}^2| &< 4.7 \times 10^{-5} \text{ eV}^2, \\
|\Delta m_{31}^2 - \Delta \overline{m}_{31}^2| &< 2.5 \times 10^{-4} \text{ eV}^2, \\
|\sin^2 \theta_{12} - \sin^2 \overline{\theta}_{12}| &< 0.14, \\
|\sin^2 \theta_{13} - \sin^2 \overline{\theta}_{13}| &< 0.029, \\
|\sin^2 \theta_{23} - \sin^2 \overline{\theta}_{23}| &< 0.19.
\end{align*}
\]
T2K results, a hint?

- T2K studied neutrino and anti-neutrino oscillations separated

\[
\sin^2 \theta_{23} = 0.51, \quad \Delta m^2_{32} = 2.53 \times 10^{-3} \text{eV}^2
\]
\[
\sin^2 \bar{\theta}_{23} = 0.42, \quad \Delta \bar{m}^2_{32} = 2.55 \times 10^{-3} \text{eV}^2
\]

- Results are consistent with
- CPT-conservation
• In experiments and in fits normally you assume CPT-conservation

• If CPT is not conserved this leads to impostor (fake) solutions in the fits

• To perform the standard fit you would calculate

\[ \chi^2_{\text{total}} = \chi^2(\nu) + \chi^2(\bar{\nu}) \]

and then minimize this function
\[ h(x, y) = f(x) + g(y) \]

\[ \partial_x f(x) = 0 \quad \partial_y g(y) = 0 \]

\[ x = y \]

\[ h(x) = f(x) + g(x) \]

\[ \partial_x f(x) = \partial_x g(x) = 0 \]

\[ \partial_x f(x) = -\partial_x g(x) \]
Obtaining impostor solutions

- This was done for $\sin^2(\theta_{23}) = 0.5$, $\sin^2(\bar{\theta}_{23}) = 0.43$

Combined best fit value is now
$\sin^2(\theta_{23}^{\text{comb}}) = 0.467$

Real true values are disfavored at close to 3$\sigma$ and more 5$\sigma$ confidence levels
This can also happen

\[ \delta = \begin{cases} \pi/2 & \text{and} \\ 0 & \end{cases} \] and \[ \bar{\delta} = \begin{cases} 0 & \text{and} \\ \pi/2 & \end{cases} \]

G.B., C. Ternes and M. Tortola, JHEP 07 (2020) 155
$\theta_{13} \neq \theta_{13}$ can account for different behavior in neutrino and antineutrino channels

\begin{align*}
\text{Tension between NOvA, T2K and SK atm. and } \delta_{bf} = 1.08 \pi \\
\text{• Disfavours:} \\
\delta = \pi/2 \text{ at } 4.0\sigma \\
\delta = 0 \text{ at } 3.0\sigma \\
\delta = 3\pi/2 \text{ with } \Delta \chi^2 = 4.9
\end{align*}

all values of $\delta$ and $\bar{\delta}$ remain allowed at $\sim 1\sigma$
Distinguishing CPT violation from NSNI

The muon neutrino survival probability in matter can be written as:

$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^22\theta_\nu \sin^2\left(\frac{\Delta m^2_{\nu} L}{4E}\right).$$

in matter

$$\Delta m^2_{\nu}\cos2\theta_\nu = \Delta m^2\cos2\theta + \epsilon_{\tau\tau}A, \quad \Delta m^2\cos2\theta_\nu = \Delta m^2\cos2\theta - \epsilon_{\tau\tau}A,$$

$$\Delta m^2_{\nu}\sin2\theta_\nu = \Delta m^2\sin2\theta + 2\epsilon_{\mu\tau}A, \quad \Delta m^2\sin2\theta_\nu = \Delta m^2\sin2\theta - 2\epsilon_{\mu\tau}A.$$
Violations of Lorentz invariance

\[ (h_{\text{eff}})_{ab} = \frac{m_{ab}^2}{2E} + \frac{1}{E} \left[ (a_L)_{\alpha} p_{\alpha} - (c_L)^{\alpha\beta} p_{\alpha} p_{\beta} \right]_{ab}. \]
Violations of Lorentz invariance

\[
(h_{\text{eff}})_{ab} = \frac{m_{ab}^2}{2E} + \frac{1}{E} \left[ (a_L)^\alpha p_\alpha - (c_L)^{\alpha\beta} p_\alpha p_\beta \right]_{ab}
\]

As usual, the oscillation probability is governed by the difference of the eigenvalues of the effective Hamiltonian.

\[
\sin^2(\Delta_{ab} \frac{L}{2})
\]

\[
m_{ab}^2 \frac{L}{E}
\]

\[
(a^\alpha)_{ab} \frac{L}{E}
\]

\[
(c^{\alpha\beta})_{ab} \frac{L}{E}
\]

Lorentz violation

standard Lorentz covariant term

violates both CPT and Lorentz invariance
\[ P(v_\mu \rightarrow v_\mu) = 1 - \sin^2(2\theta)\sin^2(1.27 \Delta m^2 L / E) \]
Neutrinos,
In and Beyond the Standard Model:

NEUTRINO MASS:
\[ \delta m^2_{atm} = 2.7^{+0.4}_{-0.3} \times 10^{-3} eV^2 \quad \text{L/E} = 500 \text{ km/GeV} \]

\[ \delta m^2_{solar} = 8.0 \pm 0.4 \times 10^{-5} eV^2 \quad \text{L/E} = 15 \text{ km/MeV} \]

\[ m^H_{\nu > \sqrt{\delta m^2_{atm}}} = 50 \text{ meV} \]

**Normal mass hierarchy**

\[ \nu_1 \quad \Delta m^2_{atm} \quad \Delta m^2_{sun} \quad \nu_2 \quad \nu_3 \]

**Inverted mass hierarchy**

\[ \nu_2 \quad \Delta m^2_{sun} \quad \nu_3 \quad \Delta m^2_{atm} \]
Masses:

States 1 and 2 are $\nu_e$ rich.
**KATRIN Task:**
Investigate Tritium endpoint with sub-eV precision

**KATRIN Aim:**
Improve $m_e$ sensitivity 10 x (2eV $\rightarrow$ 0.2eV)

**Requirements:**
- Strong source
- Excellent energy resolution
- Small endpoint energy $E_0$
- Long term stability
- Low background rate
Decay Rate:

\[ |\langle \text{He} + e^- + \bar{\nu} | T | \text{H}\rangle|^2 \sim pE(E_0 - E) \sum_k |U_{ek}|^2 \sqrt{(E_0 - E)^2 - m_{k}^2} \]

if \( \nu \)'s quasi-degenerate: \( m_1 \approx m_2 \approx m_3 \)

\[ |\langle \text{He} + e^- + \bar{\nu} | T | \text{H}\rangle|^2 \sim pE(E_0 - E) \sqrt{(E_0 - E)^2 - m_{\nu}^2} \]
Los Alamos (91)
Tokyo (91)
Zürich (92)
Mainz (93)
Beijing (93)
Livermore (95)
Troitsk (95)
Mainz (99)
Troitsk (99)
Mainz (05)
Troitsk (11)
KATRIN (19)
KATRIN (21)
KATRIN (comb.)
\[ \sum m_i \text{ in eV} \]

\[ \sum m_\nu < 0.13 \text{ eV (Planck 2018, TT,TE,EE+lowE+BAO)} \]
\[ \sum m_\nu < 0.11 \text{ eV (+lensing+Pantheon)} \]

Si could exclude \( m_{\nu_e} > \frac{1}{30} \text{ eV} \), t.

- General Relativity applies
- Neutrinos only interact weakly (no non-standard interactions)
- Universe reached thermal equilibrium before \( T \sim \text{few MeV} \)
**CMB: neutrino mass**

**Spherical harmonics decomposition:**

\[
T(\hat{n}) = \sum_{\ell=0}^{\ell_{\text{max}}} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\hat{n})
\]

**With expansion coefficients:**

\[
a_{\ell m} = \int_{4\pi} T(\hat{n}) Y_{\ell m}^*(\hat{n}) d\Omega
\]

The angular power spectrum measures the amplitude of the expansion coefficients as a function of the wavelength:

\[
C_\ell = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2
\]
CMB: a lot to learn about....

Geometry

$\nu$

Baryonmeter

$N_{\text{eff}}$

Diffusion
How structures form...

Initial fluctuations seeded by, e.g., inflation.

Primordial fluctuations in the dark matter density field
How structures form...

Photons freestream: Inhomogeneities turn into anisotropies

Collapsed objects (e.g., halos, galaxies, etc.)

Primordial fluctuations in the dark matter density field

Fluctuations at some later time

Initial fluctuations seeded by, e.g., inflation.

Fluctuations grow with time.

Overdense regions eventually collapse and form gravitationally bound objects.

Voids (regions of low density)
Large scale structure

Matter power spectrum suppression

$$\frac{\Delta P}{P} = -8 f_\nu = -8 \frac{\Omega_\nu}{\Omega_{DM}}$$

$$\frac{\Delta P}{P} = -(\sum m_\nu / 0.01 \text{ eV})\%$$

(From M. Tegmark)
\[ \sum m_\nu \]  
Planck  
TTTEEE+lowT+lowE+lensing  
\[ \sum m_\nu < 0.24 \text{ eV} \] 95\% CL  
+ BAO  
\[ \sum m_\nu < 0.12 \text{ eV} \] 95\% CL  
+ BAO + SNIa  
\[ \sum m_\nu < 0.11 \text{ eV} \] 95\% CL  
+ SDSS-IV (BAO + RSD) + SNIa  
+ BAO + SNIa + H_0=73.45 \pm 1.66 \text{ km/s/Mpc}  
\[ \sum m_\nu < 0.0970 \text{ eV} \] 95\% CL
What is Fermion Mass ???
A mass can be thought of as a $L \leftrightarrow R$ transition:

$$m \bar{\psi}_L \psi_R + h.c.$$ 

In the SM fermion masses originate in the interaction with the Higgs field:

$$\lambda_f \bar{\psi}_L \Phi \psi_R + h.c. \rightarrow m_f = \lambda_f v$$
Fermion Masses:

<table>
<thead>
<tr>
<th>Electron</th>
<th>Positron</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_L$</td>
<td>$\bar{e}_R$</td>
</tr>
<tr>
<td>$e_R$</td>
<td>$\bar{e}_L$</td>
</tr>
</tbody>
</table>

SU(2) \times U(1) 
U(1)

CPT: $e_L \leftrightarrow \bar{e}_R$ and $e_R \leftrightarrow \bar{e}_L$

Mass couples L to R:

$e_L$ to $e_R$ AND also $\bar{e}_R$ to $\bar{e}_L$ Dirac Mass terms.
Mass couples L to R:

\[ P^2 = M^2, \quad P \cdot S = 0 \quad \text{and} \quad S^2 = -1 \]

\[ u(P, S) = \frac{(1 + \gamma_5)}{2} u \left( \frac{P + MS}{2} \right) + e^{i\phi} \frac{(1 - \gamma_5)}{2} u \left( \frac{P - MS}{2} \right) \]

right massless \hspace{1cm} \text{left massless}

A coupling of \( e_L \) to \( \bar{e}_R \) OR \( e_R \) to \( \bar{e}_L \) would be (Majorana) mass term but this violates conservation of electric charge!
Seesaw / Dirac Neutrinos / Light Sterile Neutrinos

<table>
<thead>
<tr>
<th>Nu</th>
<th>CPT:</th>
<th>Anti-Nu</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left Chiral</td>
<td>νₗ</td>
<td>⇔</td>
</tr>
<tr>
<td></td>
<td>$\updownarrow$</td>
<td></td>
</tr>
<tr>
<td>Right Chiral</td>
<td>νᵢᵣ</td>
<td>⇔</td>
</tr>
</tbody>
</table>

Coupling of

- $\nu_L$ to $\nu_R$ AND $\bar{\nu}_R$ to $\bar{\nu}_L$ are the Dirac masses.
- $\nu_L$ to $\bar{\nu}_R$ forbidden by weak isospin.
- $\nu_R$ to $\bar{\nu}_L$ allowed and coefficient is unprotected. ($\rightarrow M$)
Two Majorana neutrinos with masses $\frac{m_D^2}{M}$ and $M$

- Coupling of $\nu_R$ to $\bar{\nu}_L$ allowed and coefficient is unprotected. ($\rightarrow M$)

Also applies to sterile neutrinos.

Light Sterile Neutrinos and/or Dirac Neutrinos Unexpected!!!
The consequences of this alternative are profound:

- **Physics beyond the SM** at a scale $M$!

- Majorana fermions carry no conserved charge: $L$ is violated!

$$\nu_L \rightarrow e^{i\alpha} \nu_L$$

does not leave the Majorana mass term invariant.

→ Most welcome for **baryogenesis**: a mechanism to understand the matter-antimatter asymmetry in the Universe emerges naturally

→ Most welcome by **string theory**: it is difficult to get global $U(1)$ charges conserved
Leptogenesis

Baryon Asymmetry is created by a Lepton Asymmetry produced by the decays of super heavy Majorana Neutrinos.

\[
\frac{\Gamma(N \rightarrow l^+ \phi^-) - \Gamma(N \rightarrow l^- \phi^+)}{\Gamma(N \rightarrow l^+ \phi^-) + \Gamma(N \rightarrow l^- \phi^+)}
\]

\(\Gamma(N \rightarrow l^\pm \phi^{\mp})\) depends on the Majorana Phases in the MNS mixing matrix.

\[
B_{\text{now}} = \frac{1}{2}(B - L) + \frac{1}{2}(B + L) = \frac{1}{2}(B - L)_{\text{ini}} = -\frac{1}{2}L_{\text{ini}}
\]
Types of see-saw mechanism

Type I see-saw mechanism

\[ m_{LL}^I \approx -m_{LR} M_{RR}^{-1} m_{LR}^T \]

Type II see-saw mechanism

\[ m_{LL}^{II} \equiv -v_u^2 \frac{Y_\Delta}{M_\Delta} \]

Heavy triplet
How Can We Demonstrate That $\bar{\nu}_i = \nu_i$?

We assume neutrino **interactions** are correctly described by the SM. Then the **interactions** conserve L ($\nu \rightarrow \ell^- ; \bar{\nu} \rightarrow \ell^+$).

**An Idea that Does Not Work**

[and illustrates why most ideas do not work]

Produce a $\nu_i$ via—

![Diagram of pion rest frame with neutrino and muon transitions]

**Pion Rest Frame**

Give the neutrino a Boost:

$\beta_\pi(\text{Lab}) > \beta_\nu(\pi \text{ Rest Frame})$

![Diagram of lab frame with pion and neutrino transitions]

Lab. Frame
The SM weak interaction causes —

$\bar{\nu}_i \rightarrow \mu^+$

Target at rest

Recoil

$\nu_i = \bar{\nu}_i$ means that $\nu_i(h) = \bar{\nu}_i(h)$.

If $\nu_i \Rightarrow \bar{\nu}_i \Rightarrow \mu^+$,

our $\nu_i \Rightarrow \mu^+$ too.
Minor Technical Difficulties

\[ \beta_\pi (\text{Lab}) > \beta_\nu (\pi \text{ Rest Frame}) \]

\[ \Rightarrow \frac{E_\pi (\text{Lab})}{m_\pi} > \frac{E_\nu (\pi \text{ Rest Frame})}{m_\nu} \]

\[ \Rightarrow E_\pi (\text{Lab}) > 10^4 \text{ TeV} \quad \text{if} \quad m_\nu \sim 1 \text{ eV} \]

Fraction of all \(\pi\)-decay that get helicity flipped

\[ \approx \left( \frac{m_\nu}{E_\nu (\pi \text{ Rest Frame})} \right)^2 \sim 10^{-16} \quad \text{if} \quad m_\nu \sim 1 \text{ eV} \]
How we can find out?

SM double weak process

4 body decay: continuous spectrum for the $e$ energy sum

Only allowed for Majorana $\nu$

2 body decay: $e$ energy sum is a delta
$\bar{\nu}_i$ is emitted (RH + $O(m_i/E)$ LH)

$\text{Amp}[\nu_i \text{ contribution}] \sim m_i$

$\text{Amp}[O_{\nu\beta\beta}] \propto \left| \sum m_i U_{ei}^2 \right|$  

effective mass
Neutrinoless double beta decay

- Most sensitive (terrestrial) probe of the absolute neutrino mass
- Unique way of proving Majorana nature of $\nu$
- If Majorana $\nu$ is the only mechanism, $\implies$

\[
\langle m \rangle_{\beta\beta} = \left| \sum_{i=1}^{3} m_i U_{ei}^2 \right|
\]

\[
= \left| m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{2i\beta} + m_3 s_{13}^2 e^{2i(\gamma-\delta)} \right|
\]

\[
T_{1/2}^{0\nu} = \frac{m_e}{G_{\nu} M_{\text{nucl}}^2 m_{ee}^2}
\]
Best bounds from
$^{136}$Xe (KamLAND-ZEN):

\[ T_{1/2}^{0\nu,Xe} > 2.3 \times 10^{26} \text{ yr at 90\%CL} \]

$^{76}$Ge (Gerda):

\[ T_{1/2}^{0\nu,Ge} > 1.8 \times 10^{26} \text{ yr at 90\%CL} \]

$^{130}$Te (Cuore):

\[ T_{1/2}^{0\nu,Te} > 2.2 \times 10^{25} \text{ yr at 90\%} \]

\[ m_{\beta\beta} = |\sum m_i U_{ei}^2| \]
$U_{\alpha i} = \begin{pmatrix} 1 & e^{i\alpha} \\ 0 & e^{i\beta} \end{pmatrix}$

At 0$\nu$\$\beta\beta$ decay

<table>
<thead>
<tr>
<th>parameter</th>
<th>best fit $\pm 1\sigma$</th>
<th>$2\sigma$ range</th>
<th>$3\sigma$ range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta m^2_{21} [10^{-5} \text{ eV}^2]$</td>
<td>$7.50^{+0.22}_{-0.20}$</td>
<td>$7.11$-$7.93$</td>
<td>$6.94$-$8.14$</td>
</tr>
<tr>
<td>$</td>
<td>\Delta m^2_{31}</td>
<td>[10^{-3} \text{ eV}^2]$ (NO)</td>
<td>$2.55^{+0.02}_{-0.03}$</td>
</tr>
<tr>
<td>$</td>
<td>\Delta m^2_{31}</td>
<td>[10^{-3} \text{ eV}^2]$ (IO)</td>
<td>$2.45^{+0.02}_{-0.03}$</td>
</tr>
<tr>
<td>$\sin^2 \theta_{12}/10^{-1}$</td>
<td>$3.18^{+0.16}_{-0.16}$</td>
<td>$2.86$-$3.52$</td>
<td>$2.71$-$3.69$</td>
</tr>
<tr>
<td>$\sin^2 \theta_{23}/10^{-1}$ (NO)</td>
<td>$5.74^{+0.14}_{-0.14}$</td>
<td>$5.41$-$5.99$</td>
<td>$4.34$-$6.10$</td>
</tr>
<tr>
<td>$\sin^2 \theta_{23}/10^{-1}$ (IO)</td>
<td>$5.78^{+0.10}_{-0.17}$</td>
<td>$5.41$-$5.98$</td>
<td>$4.33$-$6.08$</td>
</tr>
<tr>
<td>$\sin^2 \theta_{13}/10^{-2}$ (NO)</td>
<td>$2.200^{+0.069}_{-0.062}$</td>
<td>$2.069$-$2.337$</td>
<td>$2.000$-$2.405$</td>
</tr>
<tr>
<td>$\sin^2 \theta_{13}/10^{-2}$ (IO)</td>
<td>$2.225^{+0.064}_{-0.070}$</td>
<td>$2.086$-$2.356$</td>
<td>$2.018$-$2.424$</td>
</tr>
<tr>
<td>$\delta_{CP}/\pi$ (NO)</td>
<td>$1.08^{+0.13}_{-0.12}$</td>
<td>$0.84$-$1.42$</td>
<td>$0.71$-$1.99$</td>
</tr>
<tr>
<td>$\delta_{CP}/\pi$ (IO)</td>
<td>$1.58^{+0.15}_{-0.16}$</td>
<td>$1.26$-$1.85$</td>
<td>$1.11$-$1.96$</td>
</tr>
</tbody>
</table>
• Light Steriles ???
• Mass Hierarchy $m_3 > m_2 > m_1$ OR $m_2 > m_1 > m_3$
• Is CP violated? $\sin \delta \neq 0$
• Mass of Heaviest Neutrino
• Mass of Lightest Neutrino
• New Interactions, Surprises !!!