Flavour-physics

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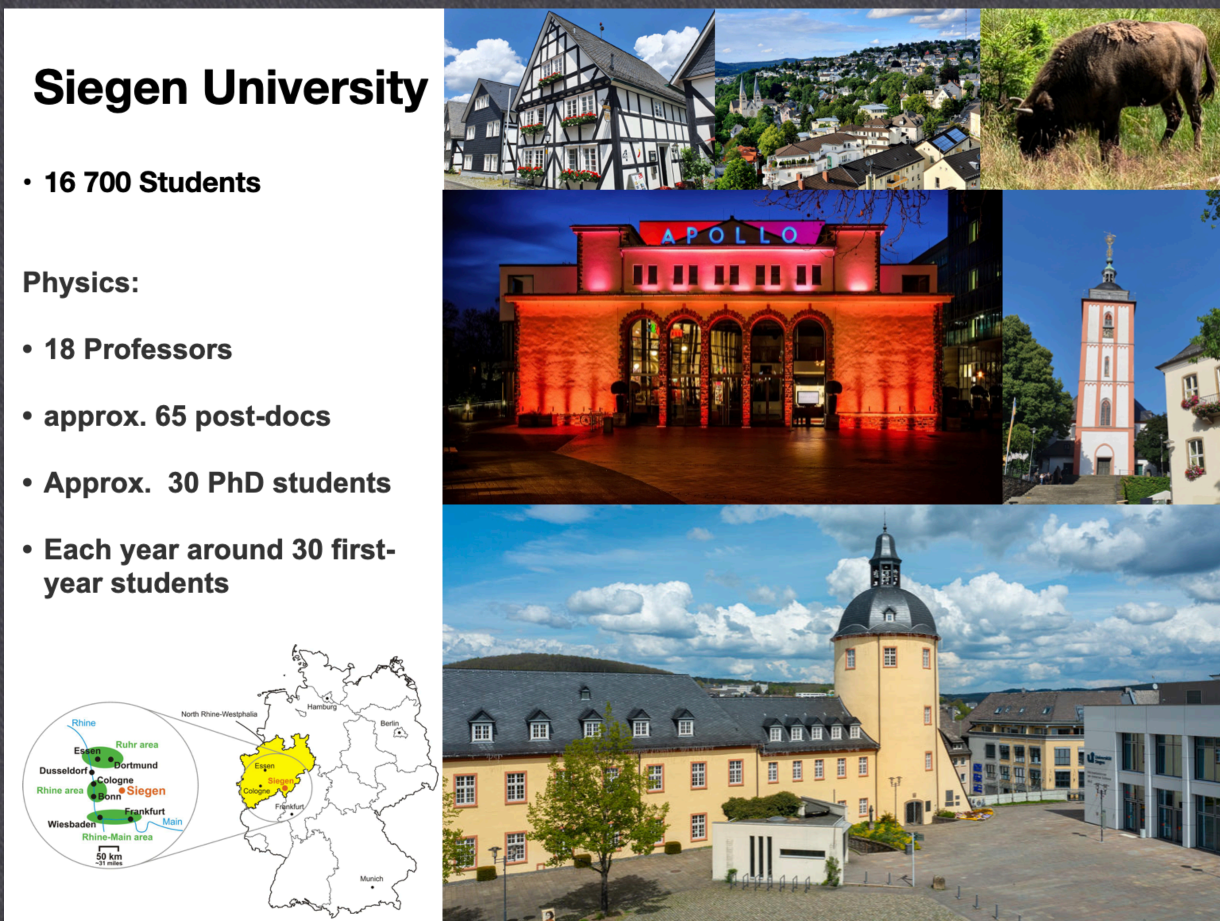
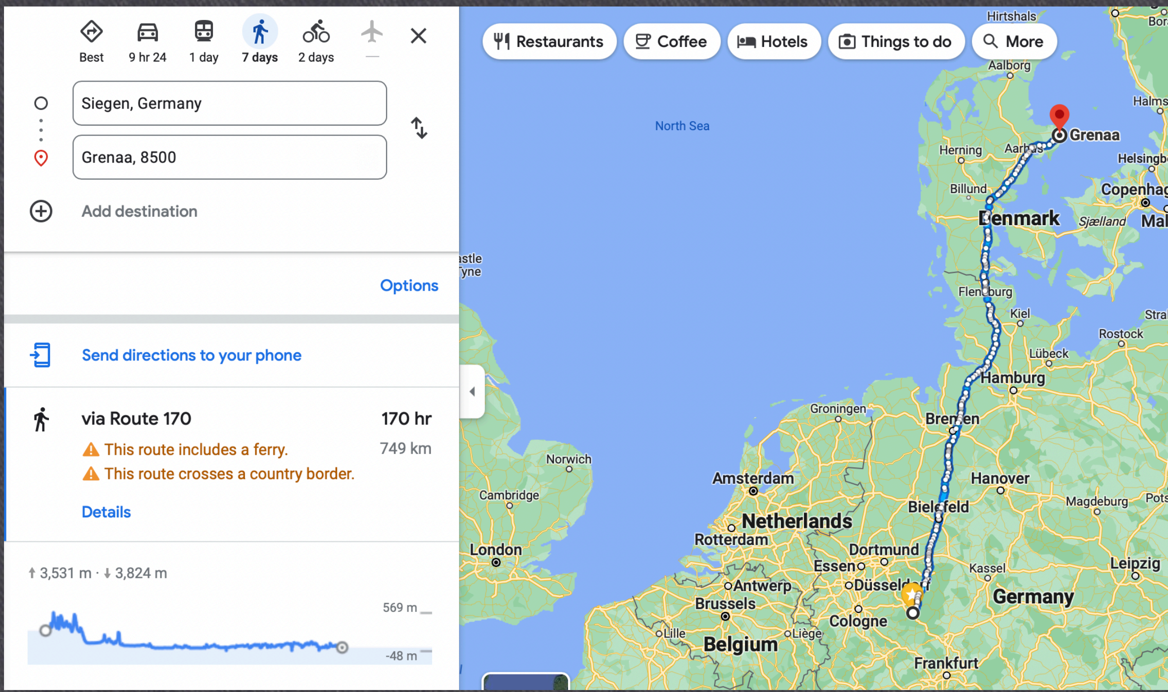




SUBATOMIC HEROES

Lecture 1: SM, CKM, weak decay

Lecture 2: Theoretical Framework

Lecture 3: Mixing & $\cancel{CP}$

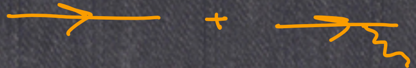
Lecture 1:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

γ, W, Z

$$+ i \bar{\psi} \not{D} \psi$$

$D_\mu = \partial_\mu + i g A_\mu$



$$+ (D_\mu \phi)^2 - V(\phi)$$

$$+ y \bar{\psi} \phi \psi$$

DM

$+ \mathcal{L}_{BSM}$ ← Baryon asymmetry

indirect new physics searches

Exp ⟷ SM + BSM

$\psi \to e^{+i\alpha(x)} \psi$

$\bar{\psi} \to e^{-i\alpha(x)} \bar{\psi}$

$\Rightarrow m \bar{\psi} \psi$ invariant?

no!

$\psi = \psi_L + \psi_R$, with $\psi_L = \frac{1-\gamma_5}{2}\psi$, $\psi_R = \frac{1+\gamma_5}{2}\psi$

$\psi_L \to e^{+i\alpha(x)} \psi_L$

$\psi_R \to 1 \;\psi_R$

Exp. of WW: only ψ_L transforms

$m \bar{\psi} \psi = m \underbrace{\bar{\psi}_L \psi_R}_{\text{not inv.!}} + m \bar{\psi}_R \psi_L$

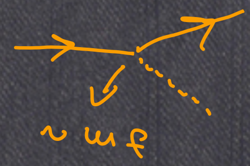
but $\bar{\psi} \phi \psi = \bar{\psi}_L \phi \psi_R + \ldots$

$e^{-i\alpha} \quad e^{+i\alpha} \to 1$

☺

Yukawa: SSB $\quad \phi(x) \longrightarrow \frac{v}{\sqrt{2}} + \varphi(x)$

$$y\bar{\psi}\phi\psi \longrightarrow \underbrace{\frac{vy}{\sqrt{2}}}_{m_{\text{fermion}}}\bar{\psi}\psi + y\bar{\psi}\varphi\psi$$



in reality 3 generations:

$$\underbrace{(\bar{u},\bar{c},\bar{t})}_{\bar{\psi}} \underbrace{\begin{pmatrix} \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \end{pmatrix}}_{\hat{Y}} \underbrace{\phi}_{\phi} \underbrace{\begin{pmatrix} u \\ c \\ t \end{pmatrix}}_{\psi}$$

↳ not necessarily diagonal

Basis transformation to get mass eigenstates

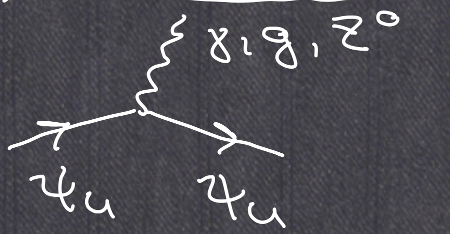
$$\psi_u = \begin{pmatrix} u \\ c \\ t \end{pmatrix} \longrightarrow U\psi_u = \begin{pmatrix} u' \\ c' \\ t' \end{pmatrix}$$

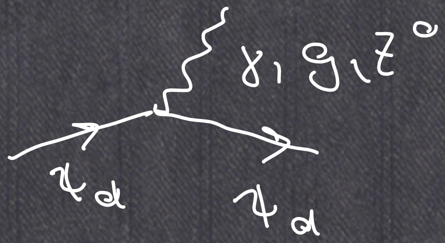
unitary ↳ mass eigenstate

$$\psi_d = \begin{pmatrix} d \\ s \\ b \end{pmatrix} \longrightarrow D\psi_d = \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}$$

Does this have further consequences?

<u>neutral current</u>



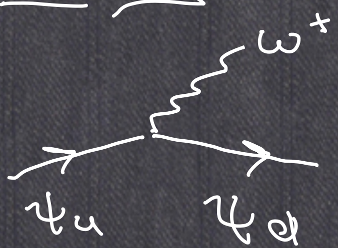


$$g\,\bar\psi_u \gamma^\mu \psi_u \qquad\qquad g\,\bar\psi_d \gamma^\mu \psi_d$$

$$\rightarrow g\,\bar\psi'_u U^\dagger \gamma^\mu U \psi'_u \qquad \rightarrow g\,\bar\psi'_d D^\dagger \gamma^\mu D \psi'_d$$

$$= g\,\bar\psi'_u \gamma^\mu \psi'_u \qquad\qquad = g\,\bar\psi'_d \gamma^\mu \psi'_d$$

<u>charged current:</u>



$$\bar\psi_d \gamma^\mu \psi_u$$

$$\rightarrow \underset{(d,s,b)}{\bar\psi'_d}\; \underbrace{D^\dagger \gamma^\mu U}_{\text{in general } \neq \mathbb{1}}\; \underset{(u,c,t)}{\psi'_u}$$

$V_{CKM} = D^\dagger U$

Cabibbo - Kobayashi - Maskawa

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{matrix} \\ \sim 0.04 \\ \end{matrix}$$

0.2

0.004

$\sim \sim 1$

general 3×3, unitary matrix: 3 real

1 imaginary parameters

$$V_{CKM} = \prod_{i=1}^{3} \text{Rotation matrix } (+ e^{i\varphi})$$

$$= V_{CKM}(\Theta_{12}, \Theta_{23}, \Theta_{13}, \varphi)$$

Taylor expansion: $\Theta_{12} < 1$

$$\sin \Theta_{12} \simeq \Theta_{12}$$

$$\cos \Theta_{12} \simeq 1 - \frac{\Theta_{12}^2}{2}$$

$\Rightarrow$ Wolfenstein parametrisation

$$V_{CKM} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$\lambda \sim 0.2$

$A \sim 1$

$\rho \sim 1$

$\eta \sim 1$

CP violation

(1983: 3389 citations till 13.9.23)

CKM fitting groups: CKMfitter, UTfit

$$\lambda = 0.2250^{+0.00024}_{-0.00022}$$

$$A = 0.8132^{+0.0119}_{-0.0060}$$

$$\rho = 0.1566^{+0.0085}_{-0.0048}$$

$$\eta = 0.3475^{+0.0118}_{-0.0054}$$

under the SM assumptions
- 3 generations
- ⋮

- $\underbrace{V_{ub}}_{\text{Exp}} = 0.003683 = \lambda^{3.8}$ ✓

- $\eta \neq 0 \implies$ imaginary coupling
  $\implies \cancel{CP}$ in the SM

Sakharov 1967

first citation 1975

13.9.23 4552 citations

Unitarity triangle

V_{CKM} is by construction unitary

K^0 B_d

$$V_{CKM}^\dagger V_{CKM} = \sum_{q_u = u,c,t} V^*_{q_u d_1} V_{q_u d_2} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

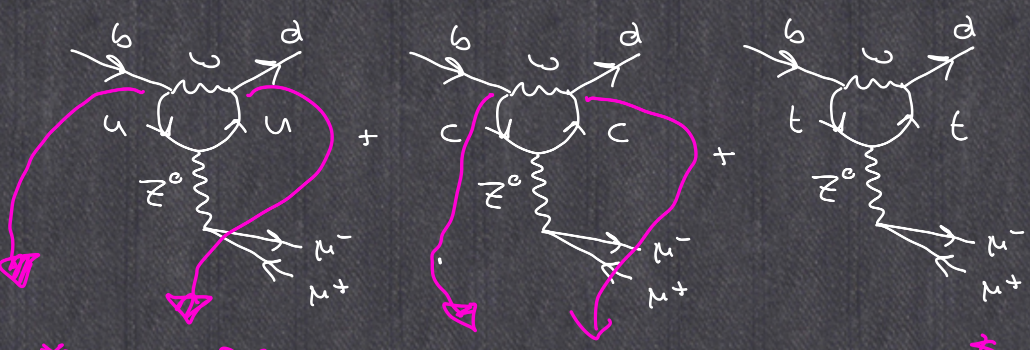
B_s

D^0 T^0

$$V_{CKM} V_{CKM}^\dagger = \sum_{q_d = d,s,b} V_{u_1 q_d} V^*_{u_2 q_d} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

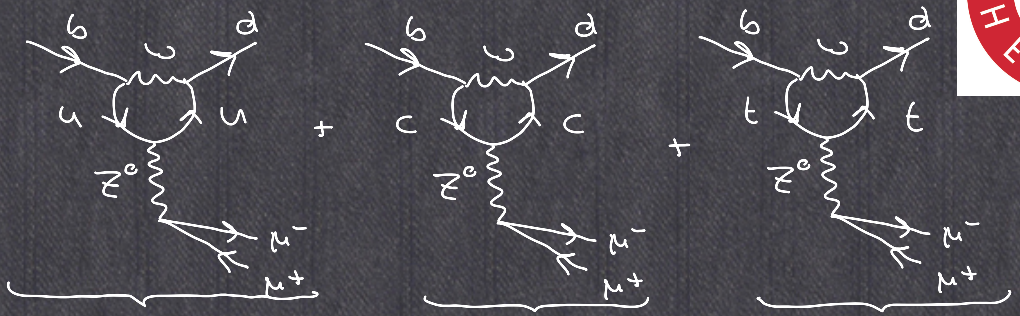
T_c^c

B_d: $\quad 0 = V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^*$



$V_{ub}^* V_{ud}\, f(m_u) + V_{cb}^* V_{cd}\, f(m_c) + V_{tb}^* V_{td}\, f(t)$

* $b \to d \mu \mu$ - Penguin



$V_{ub}^* V_{ud} f(m_u) \quad + V_{cb}^* V_{cd} f(m_c) \quad + V_{tb}^* V_{td} f(m_t)$

Assume: $m_u = m_c = m_t$

$$f(m_u)\{ V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} \} = 0$$

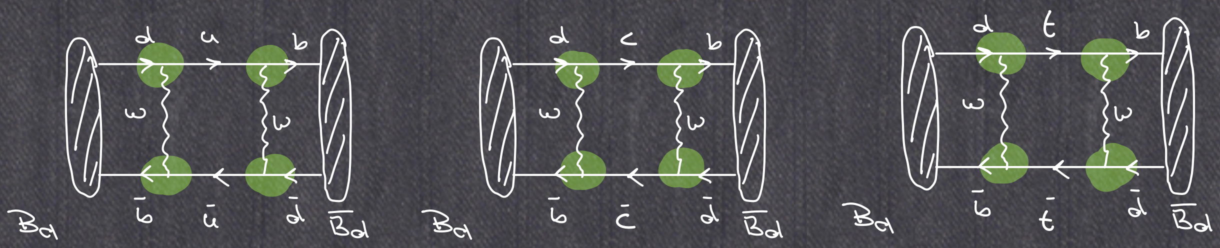
GIM Mechanismus

Glashow Illiopoulos Maiani

Assume: $f(m_q) = f_0 + \tilde{f}(m_q)$

constant terms cancel

* B-mixing: $B_d \leftrightarrow \bar{B}_d$



again: $(V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td})^2$

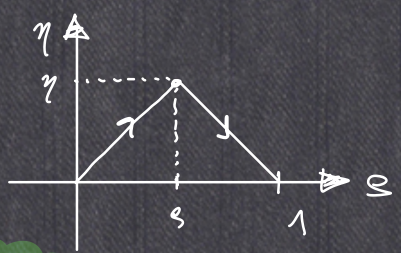
Bd: $V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$

wolfenstein

Σ 3 cplx. numbers $= 0 \implies$

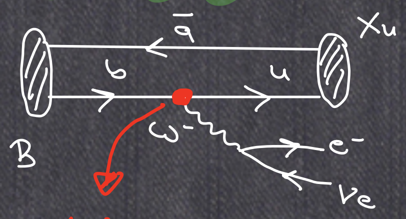
$$A\lambda^3(\rho+i\eta)\left(1-\frac{\lambda^2}{2}\right) + A\lambda^2(-\lambda) + 1\, A\lambda^3(1-\rho-i\eta) = 0 + \mathcal{O}(\lambda^5)$$

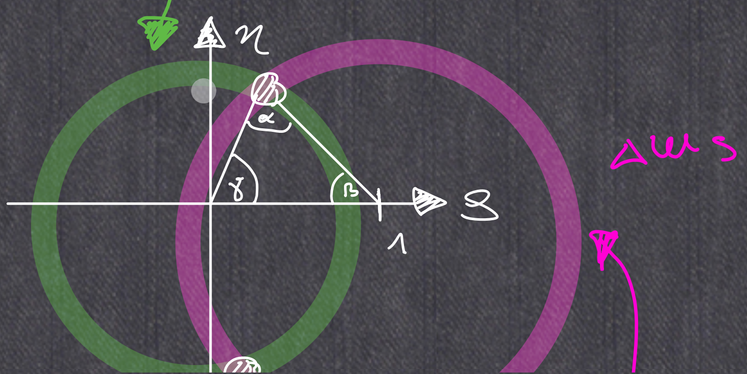
$$= A\lambda^3\left[\rho+i\eta - 1 + 1 - \rho - i\eta\right] + \mathcal{O}(\lambda^5)$$

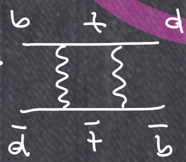


- $\underbrace{Br(B \to X_u e \nu)}_{\text{Exp}} = |V_{ub}|^2 \tilde{f}_{\text{theory}}$ (as μ-decay)

$$= \underbrace{(\rho^2+\eta^2)}_{\text{determined}} \underbrace{A^2\lambda^6 \tilde{f}_{\text{theory}}}_{\text{known}}$$





$\Delta m_d = \Gamma_{B_H} - \Gamma_{B_L} =$ 

$= |V_{tb}V_{td}^*|^2\, \tilde{f}_{\text{theory}}$

$\downarrow$ Exp:

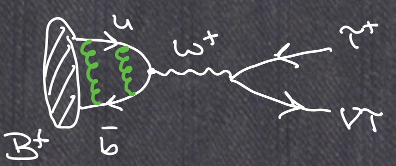
$= \left[(\rho-1)^2 + \eta^2\right] A^2 \lambda^6 \tilde{f}_{\text{theory}}$

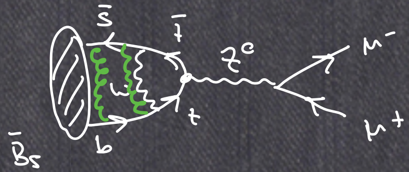
will be explained on Friday

Classification of decays

Tree-level | loop-level

leptonic: $B^+ \to \tau^+ \nu_\tau$ | $\bar{B}_s \to \mu^+\mu^-$





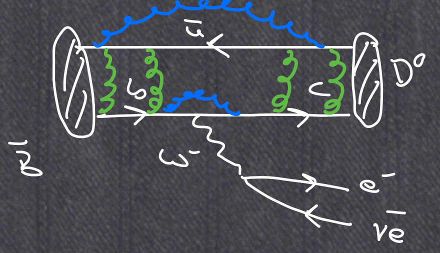
QCD: $\langle 0 | \bar{u}\gamma_5\gamma_\mu b | B^+ \rangle = i q_\mu f_{B^+}$

$\downarrow$ decay constant

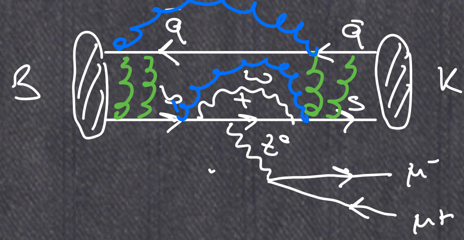
non-pert.: $\boxed{\text{lattice - QCD}}$

google: "Flag lattice"

semi-leptonic: $\bar{B} \to D^0 e^- \bar{\nu}_e$



$B \to K \mu^+ \mu^-$



QCD: $\langle D^0 | \bar{c} \gamma_\mu b | B \rangle$

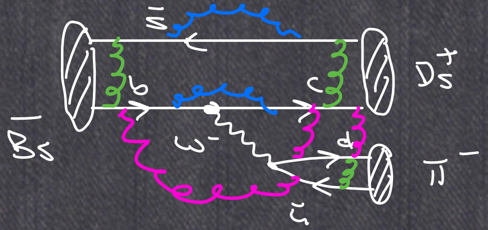
$\sim (p_B^\mu, p_D^\mu) \; f(q^2)$

↳ form-factor

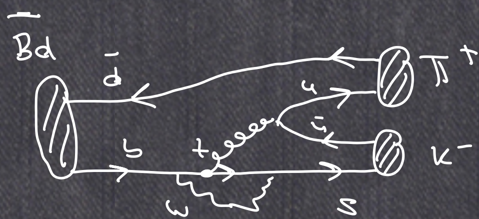
non-pert.: $\longrightarrow$ lattice

$\longrightarrow$ light-cone-sum-rule

non-leptonic: $\bar{B}_s \to D_s^+ \pi^-$



$\bar{B}_d \to \pi^+ K^-$



QCD: $\langle D_s^+ \pi^- | (\bar{c} b)(\bar{d} u) | \bar{B}_s \rangle$

↳ QCD factorisation

$$= \underbrace{\langle D_s^+ | (\bar{c} b) | \bar{B}_s \rangle}_{\text{form-factor}} \underbrace{\langle \pi^- | \bar{d} u | 0 \rangle}_{\text{decay constant}} + O\left(\frac{\Lambda_{QCD}}{m_b}\right)$$

↳ can be proven for $m_b \to \infty$

- currently not clear: how large can be $\mathcal{O}\left(\frac{\Lambda_{QCD}}{m_b}\right)$

- set $\mathcal{O}\left(\frac{\Lambda_{QCD}}{m_b}\right)$ to zero

$\Rightarrow$ $Br(\bar{B}_s \to D_s^+ \pi^-)$ differs by more than 5σ from QCDf prediction

$\to$ new physics ?

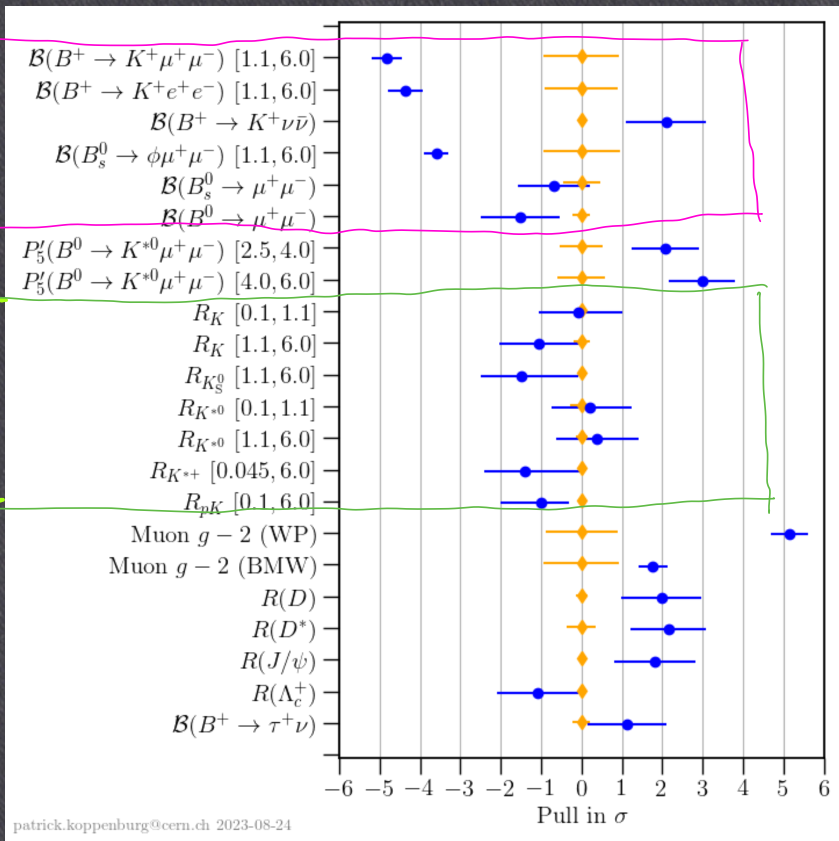
$\to$ hadronic physics ?

B anomalies:

Branching fractions

~ form factors

=> hadronic uncertainties



$R_K = \frac{Br(B \to K\mu\mu)}{Br(B \to K ee)}$

form factor cancels almost exactly

=> theoretically super-clean

before Dez'22

also some $\simeq 3\sigma$ discrepancies

Dez'22: this was an experimental issue!