Lecture 3: Mixing & $\not{CP}$ in the B-system

Mixing of mesons:

① Def: meson by Quark content

$$B_s \equiv (\bar b s) \qquad \bar B_s \equiv (b \bar s)$$

$B^+ = u\bar b$
$B^0 = d\bar b$
$D^+ = c\bar d$
$D^0 = c\bar u$

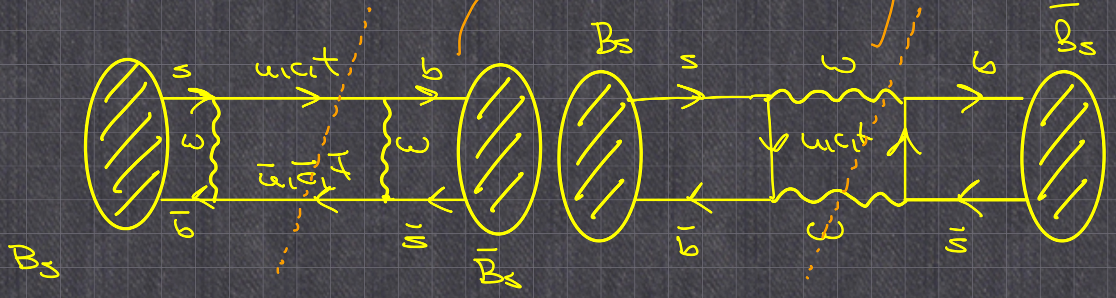
② Naive expectation for time evolution

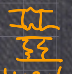
$$\overset{(-)}{B_s}(t) = \overset{(-)}{B_s}(0) \cdot \underbrace{e^{-iM_s t}}_{\text{mass}} \underbrace{e^{-\frac{1}{2}\Gamma_s t}}_{\text{decay const.}}$$

equivalent

$$i\hbar\partial_t \begin{pmatrix} B_s \\ \bar B_s \end{pmatrix} = \begin{pmatrix} M_s - \frac{i}{2}\Gamma_s & \cancel{0} \\ \cancel{0} & M_s - \frac{i}{2}\Gamma_s \end{pmatrix} \begin{pmatrix} B_s \\ \bar B_s \end{pmatrix}$$

③ weak interaction: $B_s \leftrightarrow \bar B_s$

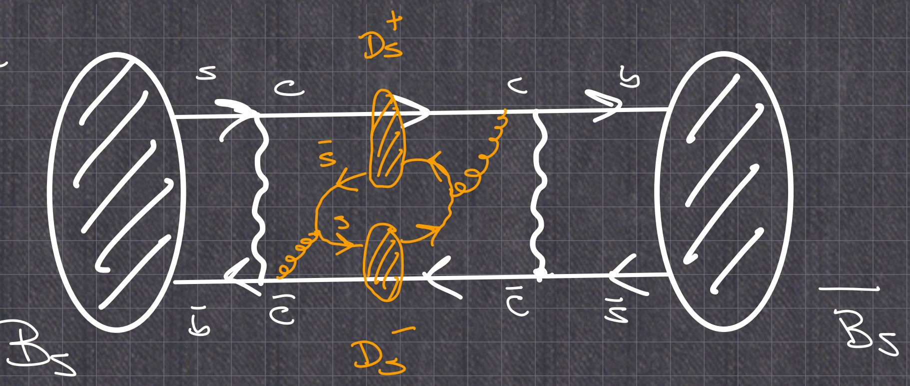


M_{12} = off-shell part of box-diagram : both  u,c,t

→ Γ_{12} = on-shell part — " — : left  u,c

E.g: c-c

Γ_{12}:



④ Most general form of 2x2 mixing matrix

$$\begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{21} - \frac{i}{2}\Gamma_{21} & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix}$$

CPT

$M_{11} = M_{22}$
$\Gamma_{11} = \Gamma_{22}$
$M_{21} = M_{12}^*$
$\Gamma_{21} = \Gamma_{12}^*$

↙

non-diagonal

⟺ B_s, $\bar{B}_s$ are no mass eigenstates

⇒ diagonalise matrix

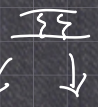
$$(*) \quad \boxed{\begin{aligned} B_{s,H} &= p\, B_s + q\, \bar{B}_s \\ B_{s,L} &= p\, B_s - q\, \bar{B}_s \end{aligned}} \quad \begin{matrix} \text{Heavy} \\ \text{Light} \end{matrix}$$

mass eigenstate

⇒ diagonal matrix

$$\begin{pmatrix} M_{s,H} - \frac{i}{2}\Gamma_{s,H} & 0 \\ 0 & M_{s,L} - \frac{i}{2}\Gamma_{s,L} \end{pmatrix}$$

Physical observables:



$$\Delta M_s = M_{s,H} - M_{s,L} = \Delta M_s(M_{12}, \Gamma_{12})$$
$$\Delta \Gamma_s = \Gamma_{s,H} - \Gamma_{s,L} = \Delta \Gamma_s(M_{12}, \Gamma_{12})$$

- in the B-system: $|\Gamma_{12}| << |M_{12}|$
- Taylor expansion in $\left|\frac{\Gamma_{12}}{M_{12}}\right| \simeq 5 \cdot 10^{-3}$

in charmmixing
$|\Gamma_{12}| \simeq |M_{12}|$

$$\Delta M_s = 2\,|M_{12}^s| + \mathcal{O}\left(\left|\frac{\Gamma_{12}}{M_{12}}\right|^2\right)$$
$$\Delta \Gamma_s = 2\,|\Gamma_{12}^s| \cos(\phi_{12}^s) + \mathcal{O}\left(\left|\frac{\Gamma_{12}}{M_{12}}\right|^2\right)$$
$$\phi_{12}^s = \arg\left(-\frac{M_{12}^s}{\Gamma_{12}^s}\right) \approx \frac{1}{250}$$

Time evolution:

mass eigenstates

use (*)

$$\underbrace{|B_{s,\overset{L}{H}}(t)\rangle}_{\text{quark eigenstate}} = e^{-(i\underbrace{M^s_{\overset{}{H}}}_{L} + \frac{1}{2}\underbrace{\Gamma^s_H}_{L})\cdot t}\;\underbrace{|B_{s,H}(0)\rangle}_{\substack{L \\ \text{quark eigenstate}}}$$

result from diag. $\Delta\Gamma_s, \Delta M_s$

$$|B_s(t)\rangle = g_+(t)\,\underbrace{|B_s(0)\rangle}_{} + \frac{q}{p}\,g_-(t)\,\underbrace{|\bar{B}_s(0)\rangle}_{}$$

$$\Longrightarrow g_+(t) = e^{-iM_{B_s}t}\, e^{-\frac{\Gamma_{B_s}t}{2}}\Big[\cosh\frac{\Delta\Gamma_s t}{4}\cdot\cos\frac{\Delta M_s t}{2} - \sinh\frac{\Delta\Gamma_s t}{4}\cdot\sin\frac{\Delta M_s t}{2}\Big]$$

$g_-(t) \equiv$ see e.g. 1511.09466

$$\Gamma \sim \int_{PS} |\langle f|\mathcal{H}_{\text{eff}}|B_s(t)\rangle|^2$$

normalisation factor

$$\boxed{\begin{aligned}\Gamma(\bar{B}_s(t)\to f) = \mathcal{N}_f\, |A_f|^2\, \frac{1+|\lambda_f|^2}{2}\, e^{-\Gamma_s t} \Big\{ &\cosh\frac{\Delta\Gamma_s t}{2} - \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}\cos\Delta M_s t \\ &- \sinh\frac{\Delta\Gamma_s t}{2}\,\frac{2\mathrm{Re}(\lambda_f)}{1+|\lambda_f|^2} + \frac{2\mathrm{Im}(\lambda_f)}{1+|\lambda_f|^2}\sin\Delta M_s t\Big\} \\ &(1+\underline{a_{fs}^s})\end{aligned}}$$

$\mathcal{A}_f = \langle f | \mathcal{H}_{eff} | B_s \rangle$: matrix element for decay (might be super complicated

$\bar{\mathcal{A}}_f = \langle f | \mathcal{H}_{eff} | \bar{B}_s \rangle$

$\lambda_f = \frac{q}{p} \frac{\bar{\mathcal{A}}_f}{\mathcal{A}_f}$

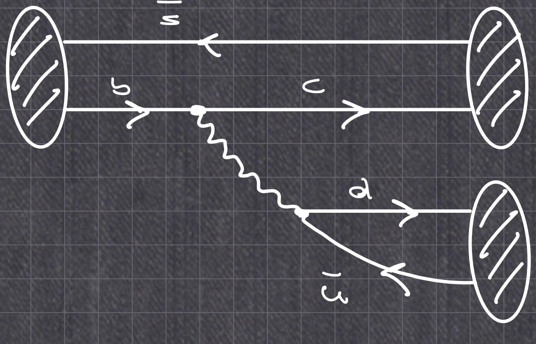
; $a_{fs}^{s} = \text{Im} \frac{\Gamma_{12}}{M_{12}}$ $2 \cdot 10^{-5}$

$\downarrow$

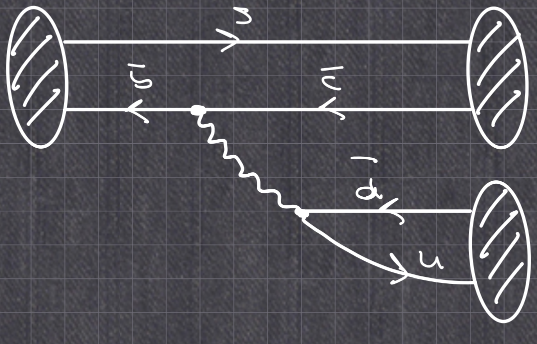
flavour specific decays

- $B \to f$; $\cancel{B \to \bar{f}}$ $\cancel{\bar{B} \to f}$
- $\lambda_f = 0$

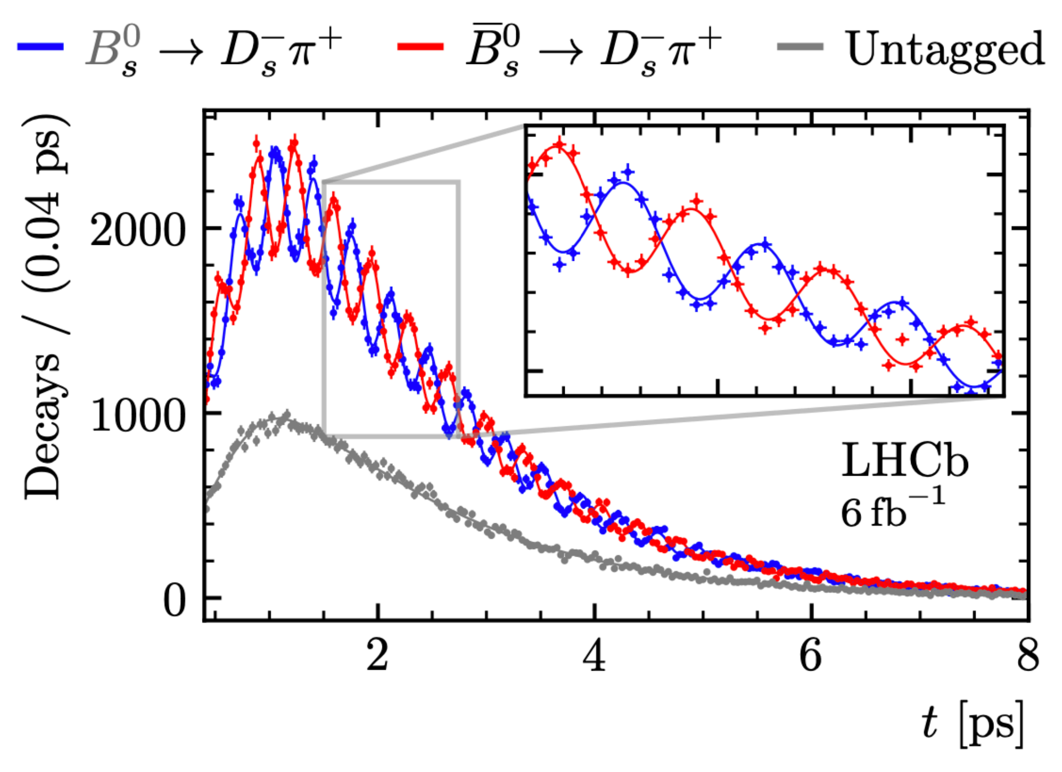
$\bar{B}_s \to D_s^+ \pi^-$



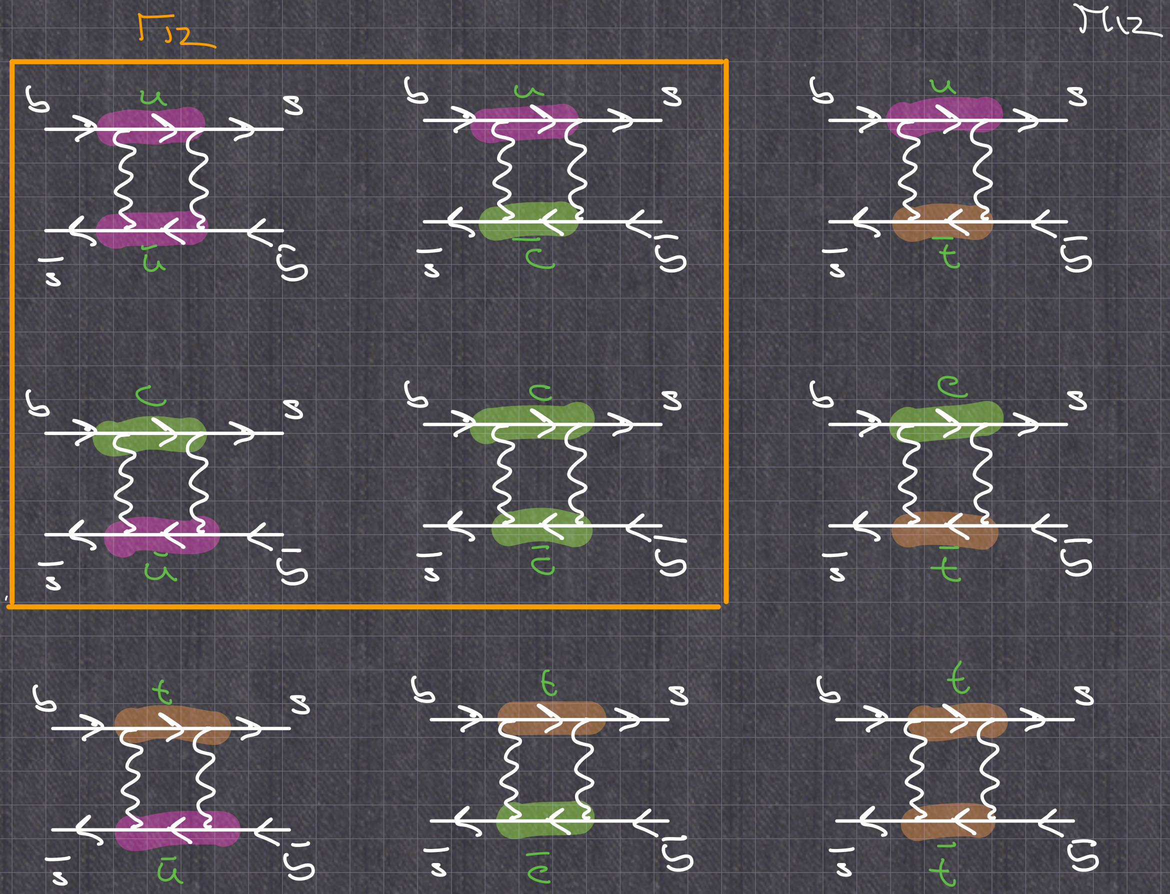
$B_s \to D_s^- \pi^+$

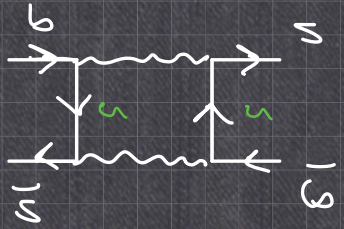


How to measure ΔM?

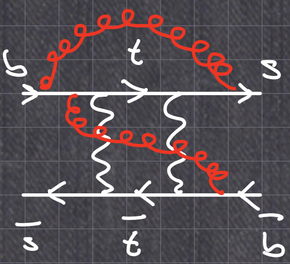


How to calculate ΔM?





...



$\lambda_u = V_{ub}V_{us}^*$ $\lambda_c = V_{cb}V_{cs}^*$ $\lambda_t = V_{tb}V_{ts}^*$

$$\underline{M_{12}} = \lambda_u F(u,u) + \lambda_u \lambda_c F(u,c) + \lambda_u \lambda_t F(u,t)$$

$$+ \lambda_c \lambda_u F(c,u) + \lambda_c^2 F(c,c) + \lambda_c \lambda_t F(c,t)$$

$$+ \lambda_t \lambda_u F(\ ,u) + \lambda_t \lambda_c F(t,c) + \lambda_t^2 F(t,t)$$

wolfenstein

$$\text{CKM:} \quad \lambda_u = V_{us}^* V_{ub} \sim \lambda^{4.8}$$

$$\lambda_c = V_{cs}^* V_{cb} \sim \lambda^2$$

$$\lambda_t = V_{ts}^* V_{tb} \sim \lambda^2$$

V_{CKM} is unitary $\Rightarrow \lambda_u + \lambda_c + \lambda_t = 0$

$$\lambda_c = -\lambda_u - \lambda_t$$

$$\Rightarrow M_{12} = \lambda_u^2 \left[F(c,c) - 2F(u,c) + F(u,u)\right]_1$$

$$+ 2\lambda_u \lambda_t \left[F(c,c) - F(u,c) + F(u,t) - F(c,t)\right]_2$$

$$+ \lambda_t^2 \left[F(c,c) - 2F(c,t) + F(t,t)\right]_3$$

CKM leading

- if $m_u = m_c = m_t \Rightarrow [\ldots]_1 = [\ldots]_2 = [\ldots]_3 = 0$

GIM - mechanism

Glashow Illiopoulos Maiani

- Loop-integration

$$x = \frac{m^2}{M_W^2}$$

$$F(p,q) = f_0 + f(x_q, x_p)$$

$$\begin{aligned}
x_u &= \frac{m_u^2}{M_W^2} \simeq \overset{\text{cancels due to GIM}}{0} \\
x_c &= \frac{m_c^2}{M_W^2} \simeq 2.5 \cdot 10^{-4} \simeq 0 \\
x_t &= \frac{m_t^2}{M_W^2} \simeq 4.5
\end{aligned}$$

$$\Rightarrow M_{12} = \lambda_u^2 \cdot 0 + 2\lambda_u\lambda_t \cdot 0 + \lambda_t^2 \underbrace{\left[f(0,0) - 2f(0,x_t) + f(x_t,x_t) \right]}_{\substack{\text{Inami-Lim} \\ \text{function} \\ S(x_t)}}$$

$$\left[S(x) = \frac{4x - 11x^2 + x^3}{4(1-x)^2} - \frac{3x \ln x}{2(1-x)^2} \right]$$

The full SM value of M_{12} reads:-

$$\boxed{M_{12}^s = \frac{G_F^2 M_W^2}{12\pi^2} \lambda_t^2 S(x_t) \underbrace{f_{B_s}^2}_{\substack{\text{decay} \\ \text{constant}}} \cdot \underbrace{B_{B_s}}_{\substack{\text{Bag} \\ \text{parameter}}} \cdot \underbrace{\hat{\eta}_B}_{\substack{\text{pert. QCD} \\ \text{corrections}}}}$$

$M_{12} \sim \langle B_s^0 |$  $| \bar{B}_s^0 \rangle$

$$< B_s^0 | (\bar{s}b)_{V-A} (\bar{s}b)_{V-A} | \bar{B}_s^0 >$$

$$\equiv \frac{8}{3} f_{B_s}^2 B_{B_s} M_{B_s}^2$$

$$\sum_n < B_s | (\bar{s}b)_{V-A} \underline{|n><n|} (\bar{s}b)_{V-A} | \bar{B}_s >$$

$$= B \underbrace{< B_s | (\bar{s}b)_{V-A} | 0 >}_{f_{B_s} p_B^\mu} \underbrace{< 0 | (\bar{s}b)_{V-A} | \bar{B}_s >}_{f_{B_s} p_B^\mu}$$

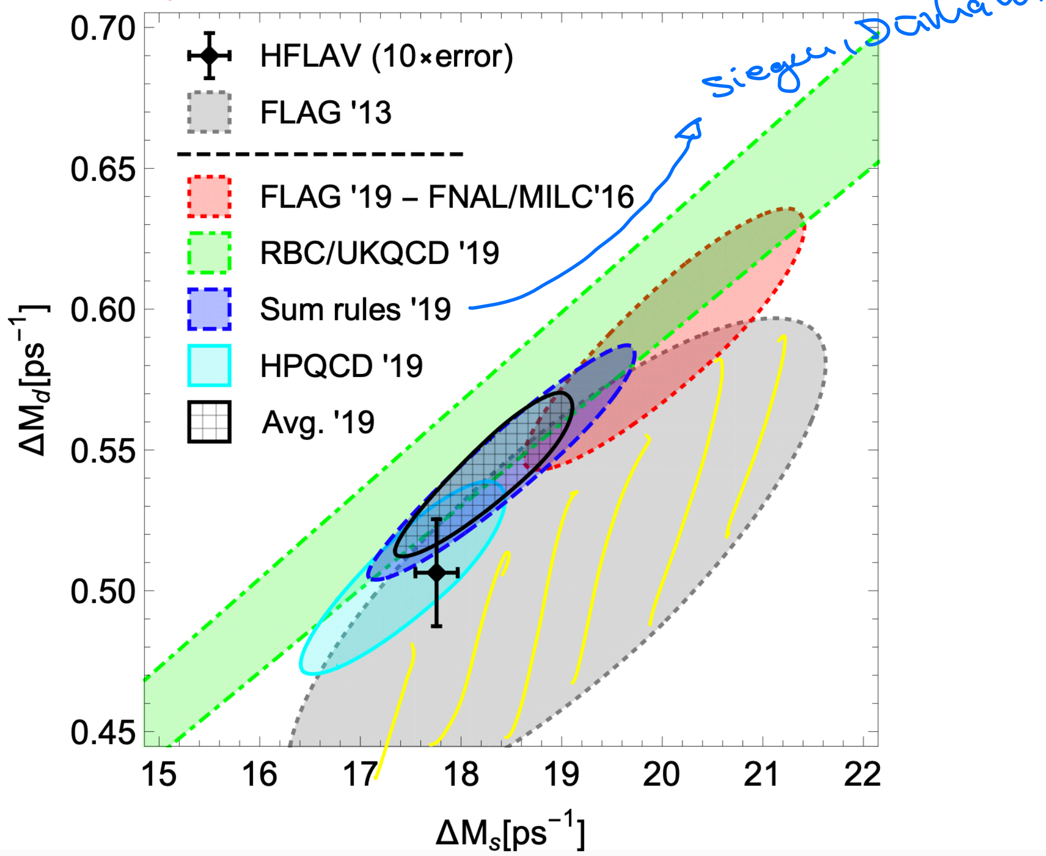
$$= f_{B_s}^2 M_{B_s}^2 B$$

$$\downarrow$$

Lattice , sum rules

$$B - 1 \sim 0.02 \pm 0.02$$

Average lattice & sum rules : 1909.11087



$$\Delta M_s^{\text{theory}} = \left(18.4 {}^{+0.7}_{-1.2}\right) \text{ps}^{-1}$$

1909.11087

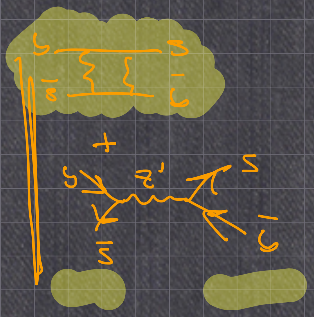
$$\Delta M_s^{\text{Exp}} = (17.7 \ldots\ldots)$$

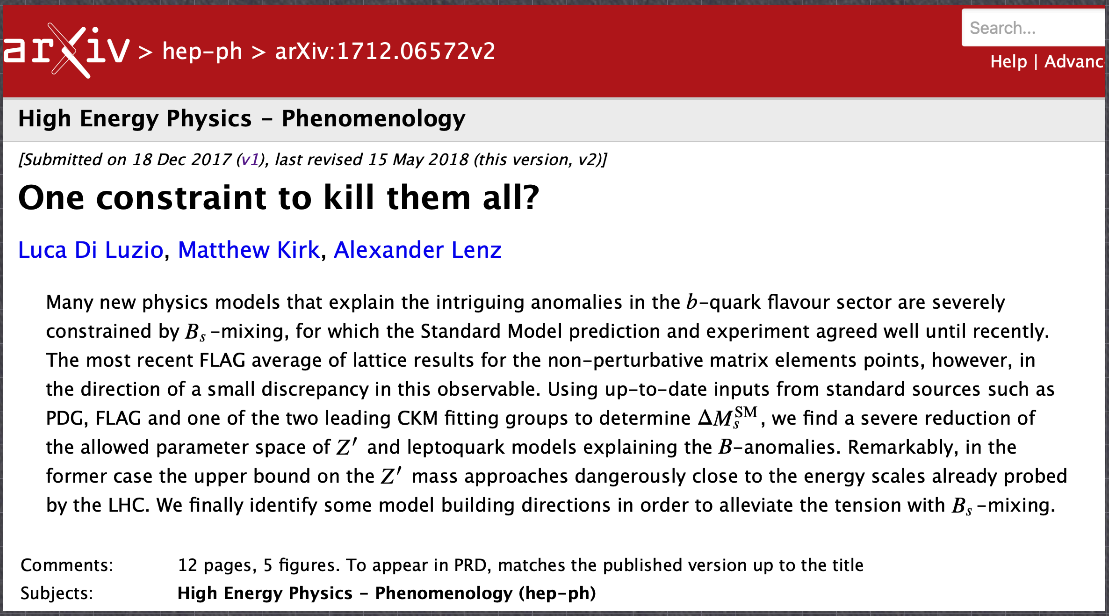
B-anomalies

$b \to s \mu^+ \mu^-$

+

≡ Exp.





High Energy Physics – Phenomenology

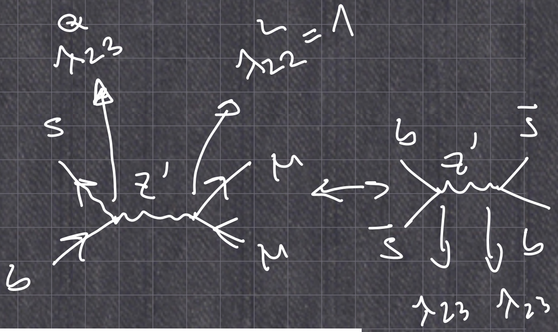
[Submitted on 18 Dec 2017 (v1), last revised 15 May 2018 (this version, v2)]

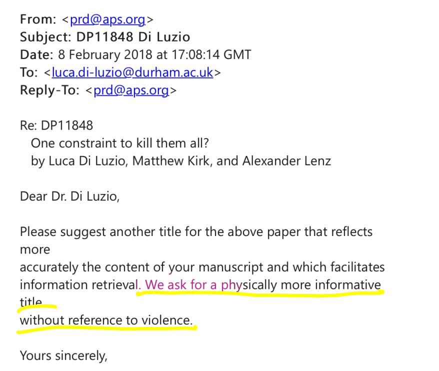
One constraint to kill them all?

Luca Di Luzio, Matthew Kirk, Alexander Lenz

Many new physics models that explain the intriguing anomalies in the b-quark flavour sector are severely constrained by B_s-mixing, for which the Standard Model prediction and experiment agreed well until recently. The most recent FLAG average of lattice results for the non-perturbative matrix elements points, however, in the direction of a small discrepancy in this observable. Using up-to-date inputs from standard sources such as PDG, FLAG and one of the two leading CKM fitting groups to determine $\Delta M_s^{\rm SM}$, we find a severe reduction of the allowed parameter space of Z' and leptoquark models explaining the B-anomalies. Remarkably, in the former case the upper bound on the Z' mass approaches dangerously close to the energy scales already probed by the LHC. We finally identify some model building directions in order to alleviate the tension with B_s-mixing.

Comments:	12 pages, 5 figures. To appear in PRD, matches the published version up to the title
Subjects:	**High Energy Physics – Phenomenology (hep-ph)**





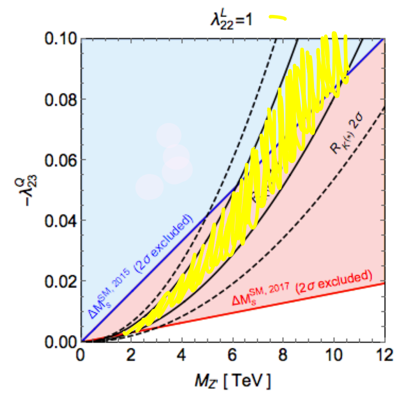
From: <prd@aps.org>
Subject: DP11848 Di Luzio
Date: 8 February 2018 at 17:08:14 GMT
To: <luca.di-luzio@durham.ac.uk>
Reply-To: <prd@aps.org>

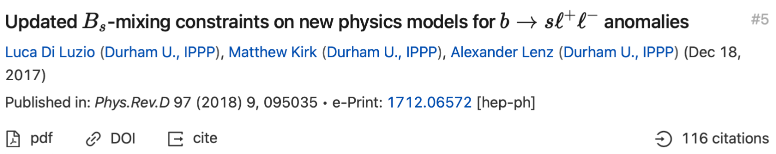
Re: DP11848
One constraint to kill them all?
by Luca Di Luzio, Matthew Kirk, and Alexander Lenz

Dear Dr. Di Luzio,

Please suggest another title for the above paper that reflects more accurately the content of your manuscript and which facilitates information retrieval. We ask for a physically more informative title without reference to violence.

Yours sincerely,





Updated B_s-mixing constraints on new physics models for $b \to s\ell^+\ell^-$ anomalies

Luca Di Luzio (Durham U., IPPP), Matthew Kirk (Durham U., IPPP), Alexander Lenz (Durham U., IPPP) (Dec 18, 2017)

Published in: *Phys.Rev.D* 97 (2018) 9, 095035 • e-Print: 1712.06572 [hep-ph]

116 citations

$\not{C}\not{P}$ in the B-system:

∃ 3 different kinds of $\not{C}\not{P}$

① $\not{C}\not{P}$ in mixing

Consider: • flavour specific decay

$$B \to f \ni \cancel{B \to \bar f}$$

$$\cancel{\bar B \to f}$$

• no direct $\not{C}\not{P}$ (see below)

$$B \to f \equiv \bar B \to \bar f$$

$$A_f = \bar A_{\bar f}$$

e.g.

$\bar B_s \to D_s^+ \pi^-$

$B \to D \ell \nu$

$$\lambda_f = \frac{q}{p}\frac{\bar A_f}{A_f} = 0 \, !$$

$\bar A_f = 0$

⇓

$\Gamma(\bar B_s(t) \to f)$ easy!

$$a_{CP}^{①} := \frac{\Gamma(\bar B_s(t) \to f) - \Gamma(B_s(t) \to \bar f)}{\ldots + \ldots}$$

all non perturbative quantities cancel ($A_f = \bar A_{\bar f}$) or vanish ($\lambda_f = 0$)

$$= a_{fs} = \mathrm{Im}\, \frac{\Gamma_{12}}{M_{12}} = 2 \cdot 10^{-5}$$

↑ SM

↳ tiny!

⇒ null test of the SM

Exp. looked for, but no yet measured

$$a_{fs}^{Exp} = (-60 \pm 280) \cdot 10^{-5}$$

② Indirect $\cancel{CP}$ or $\cancel{CP}$ in interference of mixing & decay

History: · $\cancel{CP}$ was discovered in the K-system in 1964

- as a tiny effect $\mathcal{O}(‰)$
- 1973 Kobayashi, Maskawa: $\cancel{CP}$ in V_{CKM}
- 1977 discovery of B-quark

⟹ Expect $\cancel{CP}$ of the order of 50% in B decay

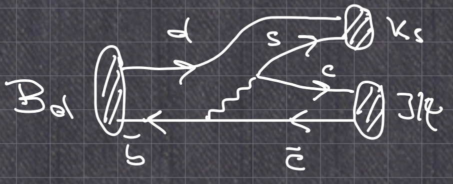
⟹ Build dedicated experiments to create as many B's as possible and measure $\cancel{CP}$ in the gold plated mode $B_d \to J/\psi K_s$

B-factories → SLAC, US Babar
→ KEK, Japan Belle

$$\overset{②}{a_{CP}} := \frac{\Gamma(B_d(t) \to f) - \Gamma(\bar B_d(t) \to \bar f)}{\ldots + \ldots}$$

$B_d \to J/\psi K_s$

Remarks:
→ $\bar B_d \to J/\psi K_s$
→ $\overline{(J/\psi K_s)} = J/\psi K_s$
i.e. $f = \bar f$



most general form

$$A_f = a \cdot \underbrace{e^{i\varphi}}_{\text{strong phase}} \cdot \underbrace{e^{i\vartheta}}_{\text{weak phase}}$$

non-perturbative

$\frac{q}{p} \simeq 1 + \mathcal{O}(a_{fs})$

$$\overset{CP}{\Longrightarrow} \bar A_{\bar f} = a e^{i\varphi} e^{-i\vartheta} = \bar A_f$$

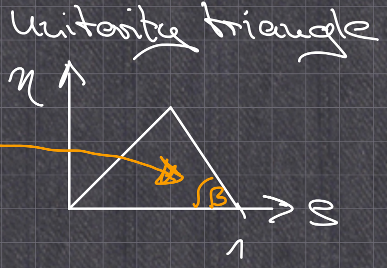
$$\Longrightarrow \lambda_f = \frac{q \bar A_f}{p A_f} = \frac{a e^{i\varphi} e^{-i\vartheta}}{a e^{i\varphi} e^{+i\vartheta}} = e^{-2i\vartheta}$$

$\|$

⇓

all complicated
non-pert. objects have
cancelled !

$$\Rightarrow a_{cp}^{②} = \dots = \sin 2\beta$$

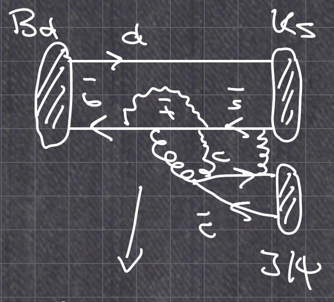


2001
$\text{Exp} \simeq 50\%$

⟹ huge $\cancel{CP}$

⟹ Nobel prize for
Kobayashi, Maskawa

But: ∃ 2nd decay amplitude !



Penguin

$$\Rightarrow A_f = a e^{i\varphi} e^{i\vartheta} + \underbrace{b e^{i\varphi'} e^{i\vartheta'}}_{\text{penguin}}$$

$$\Rightarrow \lambda_f = \frac{A_{\bar f}}{A_f} = \frac{a e^{i\varphi} e^{i\vartheta} + b e^{i\varphi'} e^{i\vartheta'}}{a e^{i\varphi} e^{-i\vartheta} + b e^{i\varphi'} e^{-i\vartheta}} \sim e^{-2i\vartheta}\left(1 + \frac{b}{a} \dots\right)$$

$$\Rightarrow a_{cp}^{②} = \sin 2\beta \left[1 + \frac{b}{a} \dots \right]$$

↳ penguin pollution
estimated to be ±1°

! since 2023 experimental
precision in β smaller than
1° ⟹ penguins are now crucial !

③ Direct $\not{C}\not{P}$:

$$\overset{\textcircled{3}}{a}_{cp} = \frac{\Gamma(B\to f) - \Gamma(\bar{B}\to\bar{f})}{\text{— '' —} + \text{— '' —}}$$

↳ also for charged B-mesons

assume: $A_f = a\, e^{i\varphi} e^{i\vartheta} + b\, e^{i\varphi'} e^{i\vartheta'}$

$$\Rightarrow \overset{\textcircled{3}}{a}_{cp} \sim \frac{b}{a} \sin(\varphi-\varphi') \sin(\vartheta-\vartheta')$$

Remarks:

(i) Direct $\not{C}\not{P}$ requires at least 2 two different decay topologies with different strong phases & different weak phases

e.g. $\bar{B}_s \to D_s^+ \pi^-$

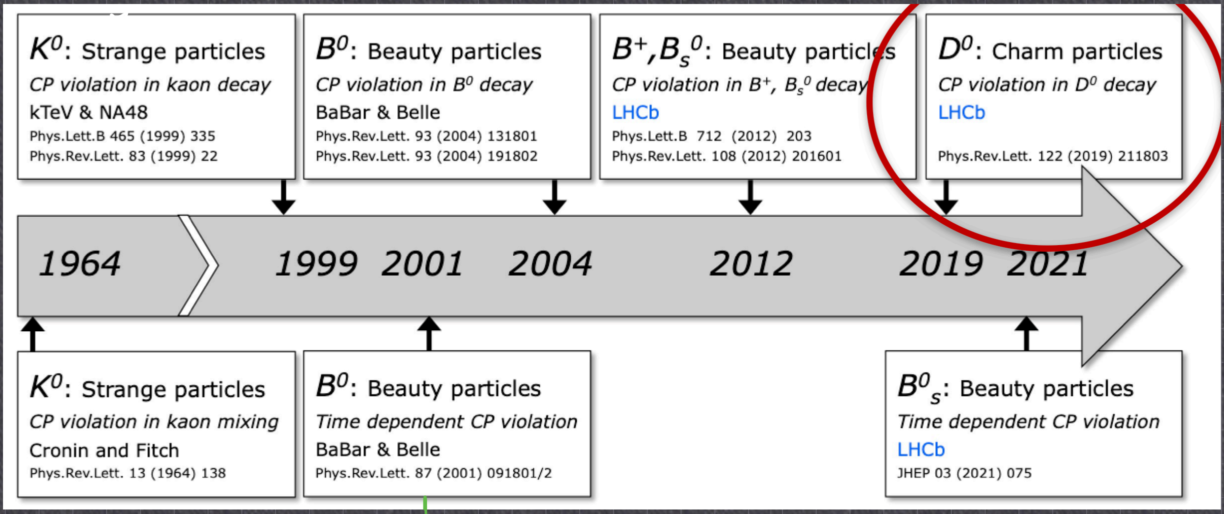
has only 1 decay topology

$\Rightarrow \overset{\textcircled{3}}{a}_{cp}(\bar{B}_s \to D_s^+ \pi^-)^{SM} = 0$

↳ exact !

(ii) $\overset{\textcircled{3}}{a}_{cp}$ is directly proportional to the non-pert. ratio b/a

$\Rightarrow$ very complicated to make precise predictions ! (except $b=0$ ☺)



K^0: Strange particles
CP violation in kaon decay
kTeV & NA48
Phys.Lett.B 465 (1999) 335
Phys.Rev.Lett. 83 (1999) 22

B^0: Beauty particles
CP violation in B^0 decay
BaBar & Belle
Phys.Rev.Lett. 93 (2004) 131801
Phys.Rev.Lett. 93 (2004) 191802

B^+,B_s^0: Beauty particles
CP violation in B^+, B_s^0 decay
LHCb
Phys.Lett.B 712 (2012) 203
Phys.Rev.Lett. 108 (2012) 201601

D^0: Charm particles
CP violation in D^0 decay
LHCb
Phys.Rev.Lett. 122 (2019) 211803

K^0: Strange particles
CP violation in kaon mixing
Cronin and Fitch
Phys.Rev.Lett. 13 (1964) 138

B^0: Beauty particles
Time dependent CP violation
BaBar & Belle
Phys.Rev.Lett. 87 (2001) 091801/2

B^0_s: Beauty particles
Time dependent CP violation
LHCb
JHEP 03 (2021) 075

$B_d \rightarrow J/\psi K_s$