Higgs & Beyond  Homework 1  Date: 15.09.2023

**Exercice 1: Natural units**

a) Show that \( [\hbar] = M \cdot L^2 \cdot T^{-1} \) and \([c] = L \cdot T^{-1}\).

b) Check the consistency of the classical/quantum correspondence at the dimensional level:

\[
E \to i\hbar \frac{\partial}{\partial t} \quad \& \quad p \to i\hbar \frac{\partial}{\partial x}
\]

c) Show that

\[
1 \text{ s} = 1.52 \cdot 10^{27} \hbar/\text{TeV}, \quad 1 \text{ m} = 5.1 \cdot 10^{18} \hbar c/\text{TeV}, \quad 1 \text{ kg} = 5.61 \cdot 10^{23} \text{ TeV}/c^2
\]

d) Using the Newton constant, \( \hbar \) and \( c \), construct a mass scale, a length scale and a time scale. They are defining the Planck scales. Compute the matter density of the universe today \((10^{-29} \text{ g/cm}^3)\) in Planck units.

e) The Schwarzschild radius of an object of mass \( m \) is the measure of its mass in Planck units. The Compton wavelength is defined as \( \hbar/(mc) \). Compute the Schwarzschild radius of the Earth, the Sun, a neutron star, a stellar black-hole, a super-massive BH, a micro-BH (you’ll check on Wikipedia the characteristic mass of these objects). What do you conclude? Compute the Schwarzschild radius of a micro-BH assuming that the Planck scale has been reduced to 1 TeV. What do you conclude?

f) Using \( e, m_e \) and \( c \), construct a length scale. This is the classical radius of the electron.

Using \( e, m_e \) and \( \hbar \), construct a length scale. This is the Bohr radius of the electron.

g) The pion Compton wavelength in natural units is \( \lambda_\pi = \hbar/(m_\pi c) \). A typical hadronic cross section is of order \( \sigma \approx \lambda_\pi^2 \approx 1/(140 \text{ MeV})^2 \). Express this quantity in units of barns \((1 \text{ barn} = 10^{-28} \text{ m}^2)\).

**Exercice 2: Value of \( e \) in HEP units**

The electromagnetic fine-structure constant was defined by A. Sommerfeld in 1916. It is given by

\[
\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c},
\]

where \( e = 1.6 \times 10^{-19} \text{ C} \) is the unit electric charge, \( \varepsilon_0 = 8.8 \times 10^{-12} \text{ F} \cdot \text{m}^{-1} \) is the vacuum permittivity.

a) Compute the value of \( \alpha \). Check that it is a dimensionless quantity (we remind that \( 1 \text{ F} = 1 \text{ C}^2 \cdot \text{J}^{-1} \))

b) Deduce the value of the electric charge \( e \) in the HEP units \((\hbar = c = \varepsilon_0 = 1)\).
Exercice 3: Order of magnitude estimates

a) Estimate the energy of the cosmic rays given that the lifetime of a muon is about 1 μs.

Exercice 4: Average temperatures on the planets of the Solar system

We’ll assume that the sun and the solar system planets are perfect black-bodies, and we’ll neglect any effects of the planet atmospheres.

Exercice 5: Hawking Black Hole radiation