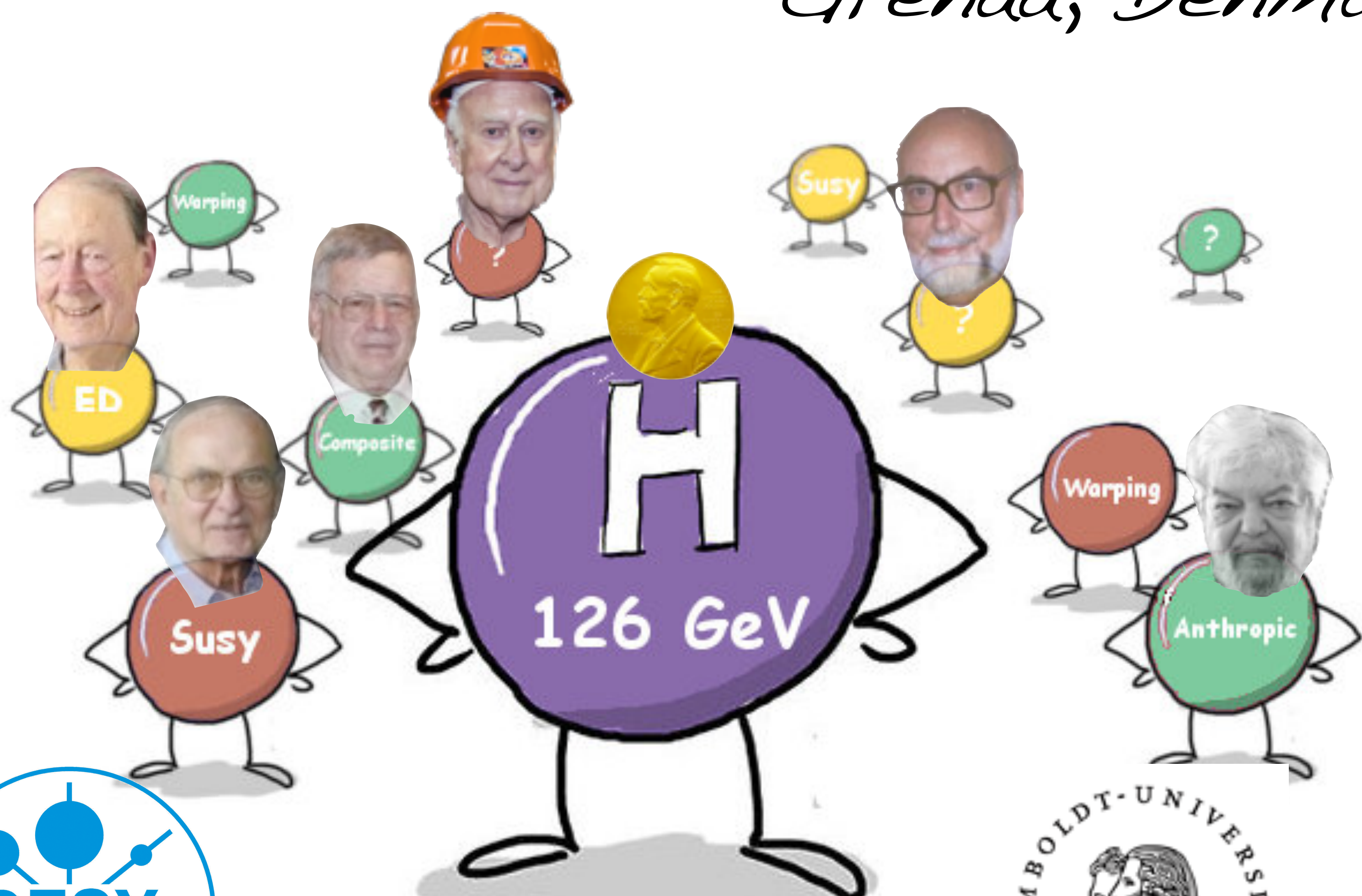
Higgs and Beyond

ESHEP 2023

Grenaa, Denmark



Lecture 1/4

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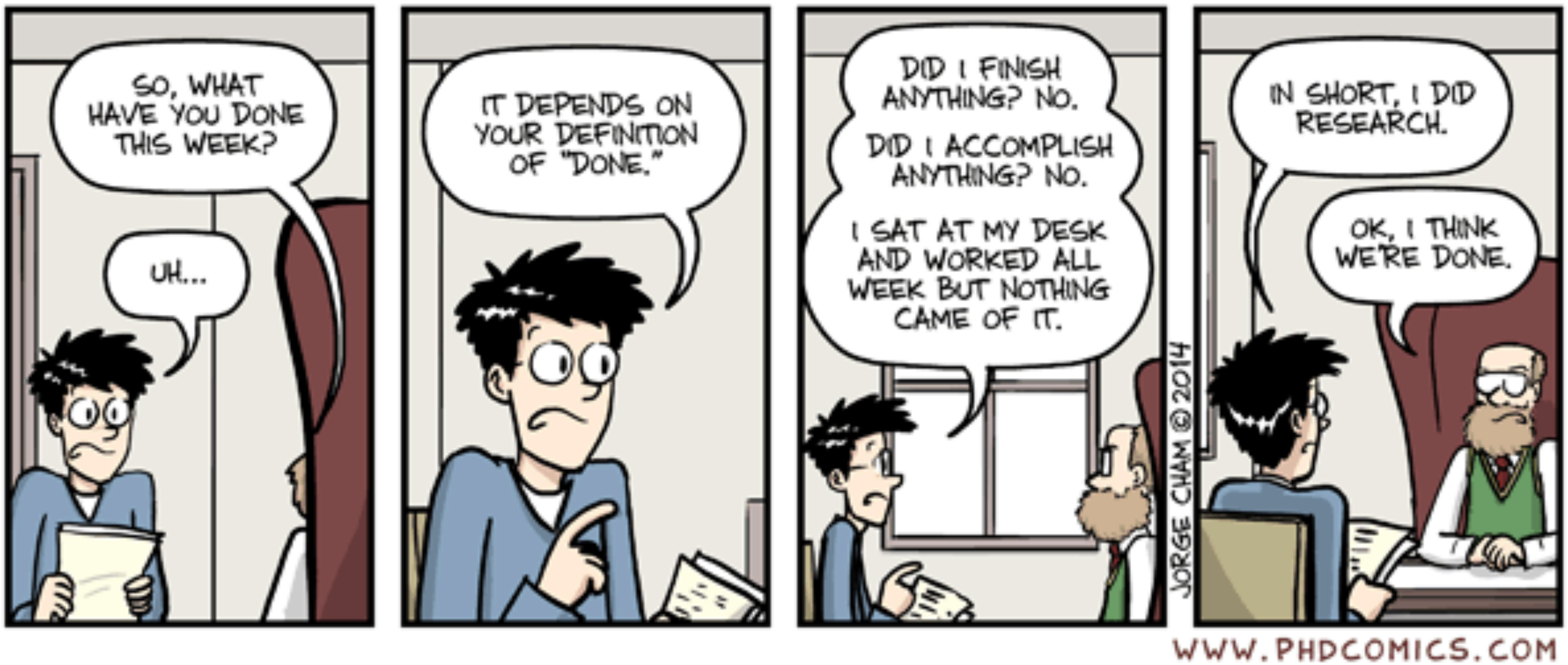
Your work, as students, is to question
all what you are listening during the lectures...



Citius, Altius, Fortius

How high can a human jump with a pole?

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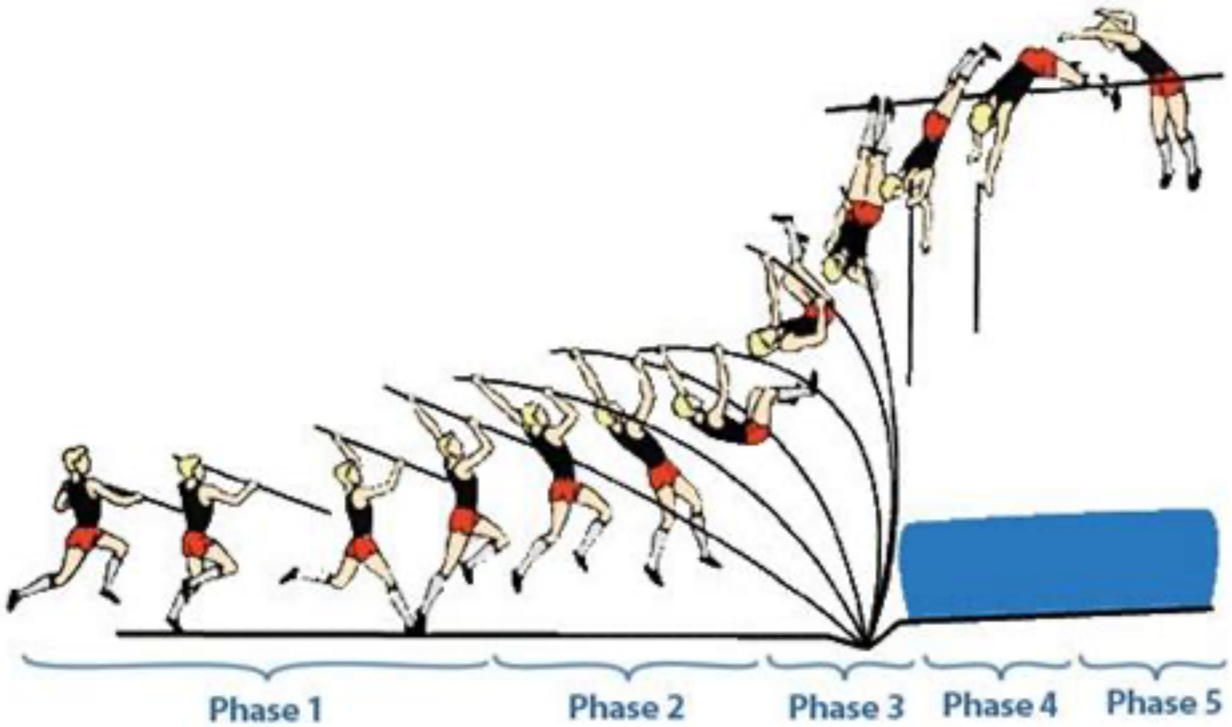

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Citius, Altius, Fortius

How high can a human jump with a pole?

Physics (energy conservation) tells us that longer poles don't help!

$$\Delta h = \frac{v^2}{2g}$$

footspeed: 44.72km/h

(Usain Bolt, Berlin, August 2009, between 60m and 80m)

$$\Delta h = 7.62\,\mathrm{m}$$

Over the years, we have learnt a few other **conservation laws** that tell us what an athlete/a particle can do or cannot do.

— Remarkable breakthrough in the understanding of Nature: —

forces among particles are associated to symmetries

- conservation of E → invariance by (time)-translation
- electro-magnetic forces → (local) invariance by phase rotation of particle wavefunctions

The Standard Model of Particle Physics
Lorentz symmetry + internal SU(3)xSU(2)xU(1) symmetry

Role(s) of Symmetry

— Selection Rules —

- hydrogen atom: energy levels depends on n, but not on l, nor m
  (invariance under rotations as well as another symmetry that leaves the Runge-Lenz vector invariant)
- electric charge conservation: $e^+e^- \overset{\checkmark}{\longrightarrow} \gamma$ but $e^+\gamma \overset{\times}{\longrightarrow}$ e-

— Dynamical Principle —

Demanding that theory describing SM particles is invariant under some (local) symmetries requires the existence of interactions among these particles. And these interactions have a particular structure.

— High Energy Physics —

Particle physics is not about discovering particles or measuring their interactions. It is about understanding the fundamental laws of nature.

Some numerical values used in these lectures...

Fundamental constants

$$c \sim 3 \times 10^8\,\mathrm{m.s^{-1}}$$
$$\hbar \sim 10^{-34}\,\mathrm{J.s}$$
$$e \sim 1.6 \times 10^{-19}\,\mathrm{C}$$
$$G_N \sim 6.67 \times 10^{-11}\,\mathrm{N.kg^{-2}.m^2}$$
$$k_B \sim 1.38 \times 10^{-23}\,\mathrm{J.K^{-1}}$$

Natural units

$$1\,\mathrm{eV} = (6.6 \times 10^{-16}\,\mathrm{s})^{-1} \quad 1\,\mathrm{eV} = (2.0 \times 10^{-7}\,\mathrm{m})^{-1} \quad 1\,\mathrm{eV} = 1.8 \times 10^{-36}\,\mathrm{kg} \quad 1\,\mathrm{eV} = 1.2 \times 10^{4}\,\mathrm{K}$$

Mass spectrum

$$m_p = 938\,\mathrm{MeV} \quad m_n = 939\,\mathrm{MeV} \quad m_{\pi^\pm} = 139\,\mathrm{MeV} \quad m_{\pi^0} = 134\,\mathrm{MeV} \quad m_{K^\pm} = 494\,\mathrm{MeV} \quad m_{K^0} = 498\,\mathrm{MeV}$$
$$m_e = 511\,\mathrm{keV} \quad m_\mu = 106\,\mathrm{MeV} \quad m_\tau = 1.8\,\mathrm{GeV}$$
$$m_u = 2.3\,\mathrm{MeV} \quad m_d = 4.8\,\mathrm{MeV} \quad m_c = 1.3\,\mathrm{GeV} \quad m_s = 100\,\mathrm{MeV} \quad m_t = 173\,\mathrm{GeV} \quad m_b = 4.2\,\mathrm{GeV}$$

Astrophysics

$$M_\odot = 2 \times 10^{30}\,\mathrm{kg} \quad M_\oplus = 6.0 \times 10^{24}\,\mathrm{kg} \quad M_\circ = 7.3 \times 10^{22}\,\mathrm{kg}$$
$$\langle d_{\odot-\oplus} \rangle = 1.5 \times 10^{8}\,\mathrm{km} \quad \langle d_{\oplus-\circ} \rangle = 3.8 \times 10^{5}\,\mathrm{km}$$
$$\langle T_\odot^{\mathrm{surface}} \rangle = 5778\,\mathrm{K}$$

Outline

Lecture #1

- Symmetries, Fields, Lagrangians
- From Fermi theory to the Standard Model
- Chirality and mass problem

Lecture #2

- Spontaneous symmetry breaking, aka Higgs mechanism
- Particles masses, unitarity and the Higgs boson
- Higgs phenomenology (decay and production at colliders)
- Higgs quantum potential (vacuum (meta)stability, naturalness)
- Hierarchy problem

Lecture #3

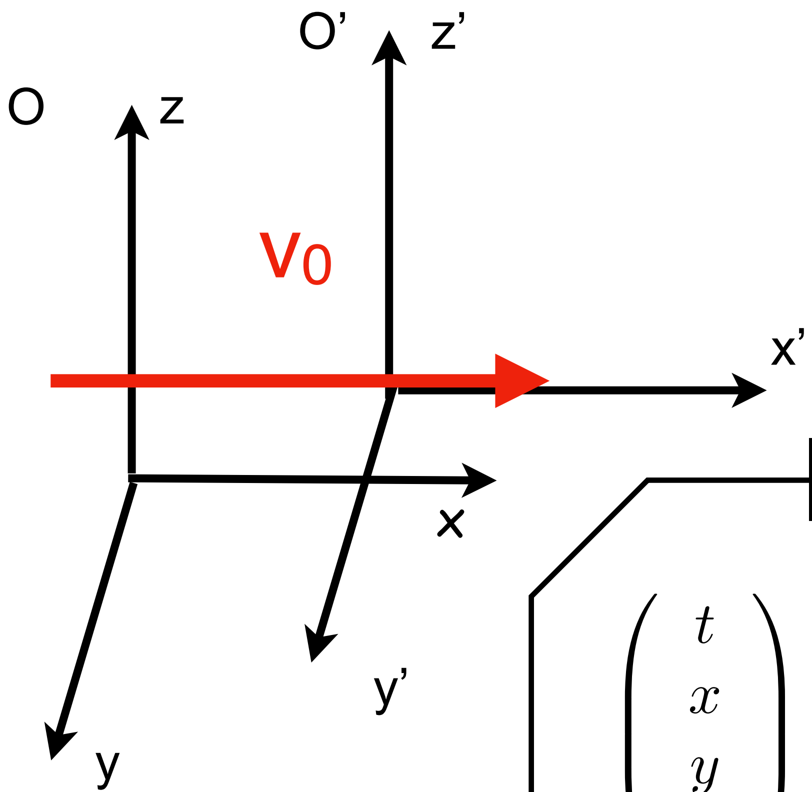
- Supersymmetry
- Composite Higgs
- Extra dimensions

Lecture #4

- Connections particle physics-cosmology
- Quantum gravity: landscape vs swampland
- BSM searches beyond colliders

Symmetries, Fields & Lagrangians

Lorentz Transformations



Consider two observers

in relative motion with a constant speed v_0 along the x-axis

they use their own systems of coordinates (t,x,y,z) and (t',x',y',z')

Galilean transformations

$$\begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} t' = t \\ x' = -\beta_0 ct + x \\ y' = y \\ z' = z \end{pmatrix} \quad \text{with} \quad \beta_0 = \frac{v_0}{c}$$

in particular

$$v' = v - v_0$$

the speed can be arbitrarily large.

Lorentz transformations

$$\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} ct' = \gamma_0 \left(ct - \beta_0 x\right) \\ x' = \gamma_0 \left(-\beta_0 ct + x\right) \\ y' = y \\ z' = z \end{pmatrix} \quad \text{with} \quad \beta_0 = \frac{v_0}{c} \quad \gamma_0 = \frac{1}{\sqrt{1-\beta_0^2}}$$

Note: $\Delta^2 \equiv (ct)^2 - x^2 - y^2 - z^2 = (ct')^2 - x'^2 - y'^2 - z'^2 \equiv \Delta'^2$

in particular

$$v' = \frac{v - v_0}{1 - v \cdot v_0/c^2}$$

The speed of light is

the same for all observers:

if v=c than v'=c too

Equations of Motion of Elementary Particles

Schrödinger Equation (1926):

$$\left(i\hbar\frac{\partial}{\partial t} + \frac{\hbar^2}{2m}\Delta - V\right)\Phi = 0$$

$$E = \frac{p^2}{2m} + V$$

classical ↔ quantum correspondance

$E \to i\hbar\frac{\partial}{\partial t}$ & $p \to i\hbar\frac{\partial}{\partial x}$

Klein-Gordon Equation (1927):

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta + \frac{m^2c^2}{\hbar^2}\right)\Phi = 0$$

$$\frac{E^2}{c^2} = p^2 + m^2c^2$$

Dirac Equation (1928):

$$\left(i\gamma^\mu\partial_\mu - \frac{mc}{\hbar}\right)\Psi = 0$$

$$E = \begin{cases} +\sqrt{p^2c^2 + m^2c^4} & \text{matter} \\ -\sqrt{p^2c^2 + m^2c^4} & \text{antimatter} \end{cases}$$

$$E = \vec{\alpha}\vec{p}\,c + \beta mc^2$$

$$\gamma^0 = \beta,\ \gamma^i = \beta\alpha^i,\ \{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$$

positron (e^+) discovered by C. Anderson in 1932

Scalar Lagrangian

A (real) **scalar** field ϕ
is a real function of space-time coordinates that doesn't change under Lorentz transformations

$$x^\mu \to x'^\mu = \Lambda^\mu{}_\nu x^\nu$$

$$\phi(x) \to \phi'(x') = \phi(x)$$

Lorentz invariant Lagrangian for scalar field?

- any potential V(Φ) is automatically invariant
- kinetic term?

$$x^\mu \to x'^\mu = \Lambda^\mu{}_\nu x^\nu \quad \phi(x) \to \phi'(x') = \phi(x) \;\Rightarrow\; \partial_\mu \phi = \Lambda^\nu{}_\mu \, \partial'_\nu \phi' \;\Rightarrow\; \partial_\mu \phi \partial^\mu \phi = \underbrace{\eta^{\mu\nu} \Lambda^{\mu'}{}_\mu \, \Lambda^{\nu'}{}_\nu}_{\eta^{\mu'\nu'}} \, \partial'_{\mu'} \phi' \partial'_{\nu'} \phi' = \eta^{\mu'\nu'} \partial'_{\mu'} \phi' \partial'_{\nu'} \phi'$$

(Lorentz transformation)

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi)$$

Eq. of motion: $0 = \delta \mathcal{L} = \left(-\partial_\mu \partial^\mu \phi - \frac{\partial V}{\partial \phi} \right) \delta\phi$ i.e. $\Box \phi = -V'(\phi)$ Klein-Gordon equation

Fermion Lagrangian

$$\mathcal{L} = \psi^\dagger \gamma^0 \left(i\gamma^\mu \partial_\mu - m \right) \psi$$

ψ 4-component Dirac spinor describes a spin-1/2 particle when quantised

$\gamma^\mu \; (\mu = 0,1,2,3)$ are four 4x4 matrices

- **Equation of motion:**

$$0 = \delta\mathcal{L} = \psi^\dagger \gamma^0 \left(i\gamma^\mu \partial_\mu - m \right) \delta\psi$$

Dirac equation $\left(i\gamma^\mu \partial_\mu - m \right) \psi = 0$

- **Lorentz invariance:** (see technical slides at the end of the lecture)

$$x^\mu \to x'^\mu = (\delta^\mu{}_\nu + \omega^\mu{}_\nu) x^\nu \quad \text{with} \quad \omega_{\mu\nu} + \omega_{\nu\mu} = 0$$

$$\psi(x) \to \psi'(x') = \left(1_4 + \frac{1}{8}\omega_{\mu\nu}[\gamma^\mu, \gamma^\nu] \right) \psi(x)$$

- **Dirac algebra:**

For this equation to be consistent with Einstein equation ($m^2=E^2-p^2$) or Klein-Gordon eq., the γ^μ matrices have to obey the Clifford algebra

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$$

- **Dirac matrices:** One particular realisation of the Dirac algebra (not unique)

$$\gamma^0 = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} & & & 1 \\ & & 1 & \\ & -1 & & \\ -1 & & & \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} & & & -i \\ & & i & \\ & i & & \\ -i & & & \end{pmatrix}, \quad \gamma^3 = \begin{pmatrix} & & 1 & \\ & & & -1 \\ -1 & & & \\ & 1 & & \end{pmatrix}$$

U(1) Gauge Symmetry — QED

Quantum ElectroDynamics : the phase of an electron is not physical and can be rotated away
(internal symmetry, same transformation in all Dirac components)

$$\psi \to e^{i\theta}\psi$$

If the phrase transformation is **local**, i.e., depends on space-time coordinate, then

$$\partial_\mu \psi \to e^{i\theta}\left(\partial_\mu \psi + i(\partial_\mu \theta)\psi\right)$$

and the kinetic term is no-longer invariant due to the presence of the non-homogenous piece

To make the theory invariant under **local** transformation, one needs to introduce a **gauge field** that keeps track/memory of how the phase of the electron changes from one point to another. For that, we build a **covariant derivative** that has nice homogeneous transformations

$$D_\mu \psi = \partial_\mu \psi + ieA_\mu \psi \to e^{i\theta} D_\mu \psi \quad \text{iff} \quad A_\mu \to A_\mu - \frac{1}{e}\partial_\mu \theta$$

$$\mathcal{L} = \psi^\dagger \gamma^0 \left(i\gamma^\mu D_\mu - m\right)\psi$$

invariant under

- Lorentz transformation
- local phase rotation

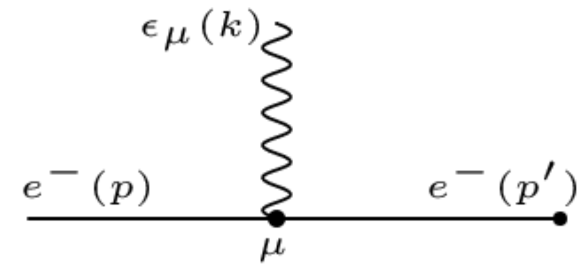
Dynamical Principle

$$\mathcal{L} = \psi^\dagger \gamma^0 \left(i\gamma^\mu D_\mu - m \right) \psi$$

interaction between

gauge field (aka photon) and electron

$$e A_\mu \psi^\dagger \gamma^0 \gamma^\mu \psi$$

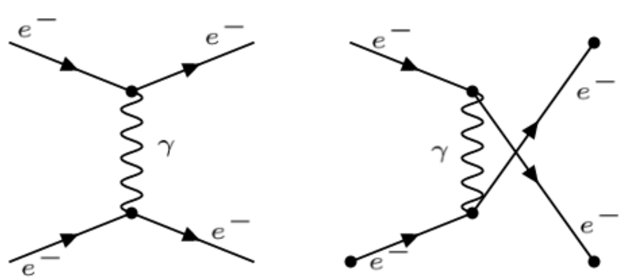


Gauge invariance is a dynamical principle: it predicts some interactions among particles.

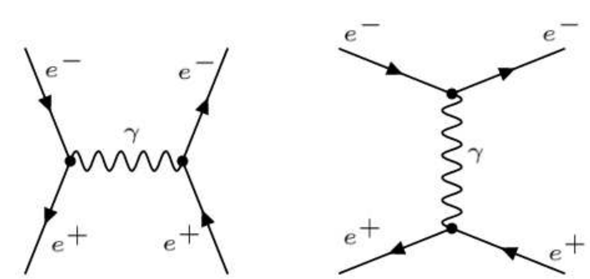
It also explains why the QED interactions are universal
(an electron interacts with a photon in the same way on Earth, on the Moon and at the outskirts of the Universe)

— Some examples of QED processes —

- **Moeller scattering** : $e^- + e^- \to e^- + e^-$

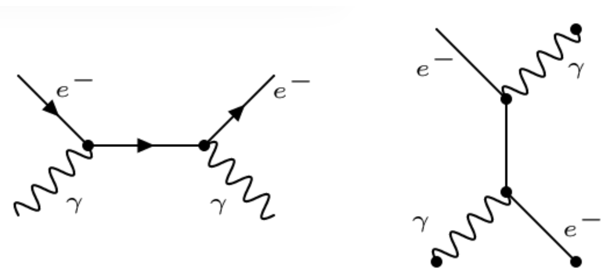


- **Bhabha scattering** : $e^- + e^+ \to e^- + e^+$

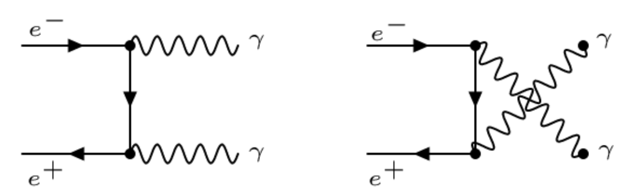


$$F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}$$

- **Compton scattering** : $e^- + \gamma \to e^- + \gamma$



- **Pair annihilation** : $e^- + e^+ \to \gamma + \gamma$



Gauge Field Kinetic Term

To build the QED Lagrangian, we had to introduce a new field A_μ

it is propagating degree of freedom we need to add a kinetic term in the Lagrangian.

Tensor field strength: $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

- **Lorentz transformations:** $x^\mu \to x'^\mu = \Lambda^\mu{}_\nu x^\nu$, $A^\mu \to A'^\mu = \Lambda^\mu{}_\nu A^\nu$ ➡ $F^{\mu\nu} \to F'^{\mu\nu} = \Lambda^\mu{}_\rho \Lambda^\mu{}_\sigma F^{\rho\sigma}$
- **U(1) gauge transformations:** $A_\mu \to A_\mu - \frac{1}{e}\partial_\mu \theta$ ➡ $F_{\mu\nu} \to F_{\mu\nu}$

$$\mathcal{L}_{\text{kin}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

invariant under
- Lorentz transformation
- local phase rotation

equations of motion ↔ **Maxwell equations** of electromagnetism

A^0=EM scalar potential, $A^{i=1,2,3}$ = EM vector potential

$$\vec{E} = -\vec{\nabla}\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}, \quad \vec{B} = \vec{\nabla} \wedge \vec{A}$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & -\vec{E}_x & -\vec{E}_y & -\vec{E}_z \\ \vec{E}_x & 0 & -\vec{B}_z & \vec{B}_y \\ \vec{E}_y & \vec{B}_z & 0 & -\vec{B}_x \\ \vec{E}_z & -\vec{B}_y & \vec{B}_x & 0 \end{pmatrix}$$

➡ $\partial_\mu F^{\mu\nu} = J^\nu$

Remark: no interaction among photons (photons only interact with electrically charged fields)

Natural & Planck Units

- $[G_N]$=mass^{-1} L^3 T^{-2}
- $[\hbar]$=mass L^2 T^{-1}
- [c]=L T^{-1}

- Planck mass: $M_{\rm Pl} = \sqrt{\frac{\hbar c}{G_{\rm N}}} \sim 10^{19}\,{\rm GeV/c^2} \sim 2\times 10^{-5}\,{\rm g}$
- Planck length: $l_{\rm Pl} = \sqrt{\frac{\hbar G_{\rm N}}{c^3}} \sim 10^{-33}\,{\rm cm}$
- Planck time: $\tau_{\rm Pl} = \sqrt{\frac{\hbar G_{\rm N}}{c^5}} \sim 10^{-44}\,{\rm s}$

In High Energy Physics, it is a current practise to use a system of units for which ħ=1 and c=1

energy~ mass ~ distance^{-1} ~ time^{-1}

Unit conversion: SI ↔ HEP

E	T	L
1eV	10^{-16}s	10^{-7}m
10^{-16}eV	1s	10^{9}m
10^{-7}eV	10^{-9}s	1m

- The string theorists will remember:

$$M_{\rm Pl} \sim 10^{19}\,{\rm GeV} \quad \leftrightarrow \quad \tau_{\rm Pl} \sim 10^{-44}\,{\rm s} \quad \leftrightarrow \quad l_{\rm Pl} \sim 10^{-33}\,{\rm cm}$$

- The nuclear physicists will remember:

$$\hbar c \sim 200\,{\rm MeV}\cdot{\rm fm}$$
$$10^{8}\,{\rm eV} \quad \leftrightarrow \quad 10^{-15}\,{\rm m} \quad \leftrightarrow \quad 10^{-24}\,{\rm s}$$

- The others will remember:

average mosquito m~10^{-3}g=100M_{Pl}
Compton wavelength 0.01L_{Pl}=10^{-35}cm, Schwarzschild radius 100L_{Pl}=10^{-31}cm
(much smaller than its physical size, so a mosquito is not a Black Hole)

Dimensional Analysis

$$[S]_m = 0 \quad \Rightarrow \quad [\mathcal{L}]_m = 4$$

$$S = \int d^4x\,\mathcal{L}$$

Scalar field $\mathcal{L} = \partial_\mu\phi\partial^\mu\phi + \ldots$ ⇒ $[\phi]_m = 1$

Spin-1/2 field $\mathcal{L} = \psi^\dagger\gamma^0\gamma^\mu\partial_\mu\psi$ ⇒ $[\psi]_m = 3/2$

Spin-1 field $\mathcal{L} = F_{\mu\nu}F^{\mu\nu} + \ldots$ with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \ldots$ ⇒ $[A_\mu]_m = 1$

Particle lifetime of a (decaying) particle: $[\tau]_m = -1$ Width: $[\Gamma = 1/\tau]_m = 1$

Cross-section ("area" of the target): $[\sigma]_m = -2$

From Fermi to the Standard Model

Beta decay

$${}^{40}_{19}\mathrm{K} \to {}^{40}_{20}\mathrm{Ca}^+ + e^- \qquad {}^{64}_{29}\mathrm{Cu} \to {}^{64}_{30}\mathrm{Zn}^+ + e^- \qquad {}^{3}_{1}\mathrm{H} \to {}^{3}_{2}\mathrm{He}^+ + e^-$$

• Two body decays: A→B+C

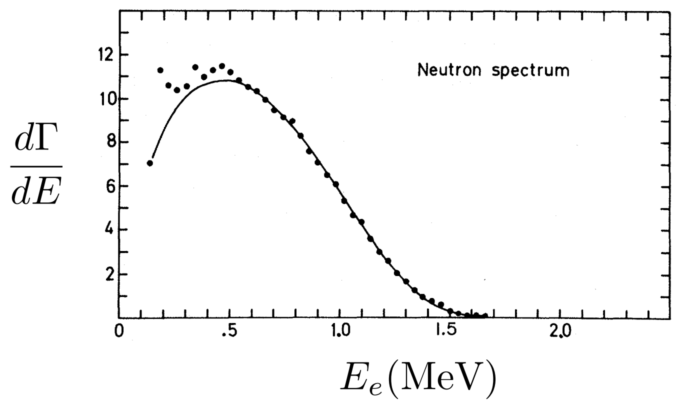
$$E_B = \frac{m_A^2 + m_B^2 - m_C^2}{2m_A} c^2 \qquad p = \frac{\sqrt{\lambda(m_A, m_B, m_C)}}{2m_A} c$$

$$\lambda(m_A, m_B, m_C) = (m_A + m_B + m_C)(m_A + m_B - m_C)(m_A - m_B + m_C)(m_A - m_B - m_C)$$

fixed energy of daughter particles

(pure SR kinematics, independent of the dynamics)

TH prediction



EXP measurements

⇛ non-conservation of energy?

Pauli '30: ∃ **neutrino**, very light since end-point of spectrum is close to 2-body decay limit

ν first observed in '53 by Cowan and Reines

• N-body decays: A→B$_1$+B$_2$+...+B$_N$

$$E_{B_1}^{\min} = m_{B_1} c^2 \qquad E_{B_1}^{\max} = \frac{m_A^2 + m_{B_1}^2 - (m_{B_2} + \ldots + m_{B_N})^2}{2m_A} c^2$$

— How are neutrinos produced? —

$\pi \to \mu\bar{\nu}$ (more about pion decay later later)

$\mu \to e\bar{\nu}_e\nu_\mu$ need 2 neutrino flavours and flavour conservation since $\mu \not\to e\gamma$

Lederman, Schwartz, Steinberger '62: $p\,\bar{\nu}_\mu \to n\,\mu^+$ but $p\,\bar{\nu}_\mu \not\to n\,e^+$

Fermi theory '33

(paper rejected by Nature: declared too speculative !)

$$\mathcal{L} = G_{\mathcal{F}}(\bar{n}p)(\bar{\nu}_e e)$$

exp: $G_F = 1.166\times10^{-5}$ GeV^{-2}

We'll see later that the structure is a bit more complicated

Lifetime "Computations"

muon and neutron are unstable particles

$$\mu \to e \nu_\mu \bar{\nu}_e$$

$$n \to p\, e\, \bar{\nu}_e$$

We'll see that the interactions responsible for the decay of muon and neutron are of the form

$$\mathcal{L} = G_F\, \psi^4 \quad \Rightarrow \quad \Gamma \propto G_F^2 m^5$$

$\mathcal{L}$: $[\text{mass}]^4$; G_F: $[\text{mass}]^{-2}$; ψ^4: $[\text{mass}]^{3/2\times 4}$; m: $[\text{mass}]$

G$_F$ = Fermi constant: $G_F \sim \dfrac{10^{-5}}{m_{\text{proton}}} \sim 10^{-5}\,\text{GeV}^{-2}$

$1 = \hbar c \sim 200\,\text{MeV} \cdot \text{fm}$

E	T	L
1eV	10^{-16}s	10^{-7}m

For the **muon**, the relevant mass scale is the muon mass m_μ=105MeV:

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} \sim 10^{-19}\,\text{GeV} \quad \text{i.e.} \quad \tau_\mu \sim 10^{-6}\,\text{s}$$

For the **neutron**, the relevant mass scale is (m_n-m_p)≈1.29MeV:

$$\Gamma_n = \mathcal{O}(1)\frac{G_F^2 \Delta m^5}{\pi^3} \sim 10^{-28}\,\text{GeV} \quad \text{i.e.} \quad \tau_\text{n} \sim 10^3\,\text{s}$$

What if particles were spin-0?

$$\mathcal{L} = G_F \phi^4$$

$[\text{mass}]^4$, $[\text{mass}]^0$, $[\text{mass}]^{1\times 4}$

$$\Gamma \propto G_F^2 m$$

$[\text{mass}]$

$$\Gamma_\mu = \left(10^{-6\prime\prime}\right)^{-1} = \frac{G_F^2 m_\mu}{192\pi^2}$$

$$\Gamma_n = \left(15^\prime\right)^{-1} = \frac{G_F^2 (m_n - m_p)}{192\pi^2}$$

$$10^{11} = \frac{\Gamma_\mu}{\Gamma_n} \overset{?}{=} \frac{m_\mu}{m_n - m_p} = 10^2$$

TH prediction

It could still have been true but we would need to give up universality of the Fermi interactions.
Remember theorists like to connect phenomena are are seemingly different.
Even more true when they follow from simple assumptions.

Universality of Weak Interactions

$$\mu \to e \nu_\mu \bar{\nu}_e \qquad\qquad n \to p\, e\, \bar{\nu}_e$$

$\tau_\mu \approx 10^{-6}$s $\qquad$ $\tau_n \approx 900$s

$$\mathcal{L} = G_F\, \psi^4$$

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} \sim 1/10^{-6\prime\prime} \qquad\qquad \Gamma_n = \frac{G_F^2 \Delta m^5}{192\pi^3} \sim 1/15^\prime$$

[factor 192 not exactly correct because n and p are not elementary particles: form factors are involved]

$$\mathcal{L} \stackrel{?}{=} G_F\ (\bar{n} p \bar{e} \nu_e + \bar{\mu} \nu_\mu \bar{e} \nu_e)$$

By analogy with electromagnetism, one can see the Fermi force as a current-current interaction (vector-vector interaction instead of scalar-scalar interaction)

$$\mathcal{L} = G_F\, J_\mu^* J^\mu \qquad \text{with} \qquad J^\mu \stackrel{?}{=} (\bar{n}\gamma^\mu p) + (\bar{e}\gamma^\mu \nu_e) + (\bar{\mu}\gamma^\mu \nu_\mu) + \ldots$$

it can be shown (thanks to the transformation law of spin-1/2 field given before) that this Lagrangian is invariant under Lorentz transformation

The cross-terms generate both neutron decay and muon decay.
The life-times of the neutron and muon tell us that the relative factor between the e and the μ in the current is of order one: the weak force has the **same strength for e and μ**.

Pion decay(s)

What about $\pi^{\pm}$ decay $\tau_{\pi} \approx 10^{-8}$s?

$$\pi^- \to \mu \bar{\nu}_\mu \qquad\qquad \pi^- \to e^- \bar{\nu}_e$$

experimentally the pions decay dominantly into muons and not electrons.

Why $\left.\dfrac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)}\right|_{\text{EXP}} \sim 10^{-4}$? And not $\dfrac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} \sim \left.\dfrac{(m_\pi - m_e)^5}{(m_\pi - m_\mu)^5}\right|_{\text{TH}} \sim 500$?

Does it mean that our way to compute decay rate is wrong?
Is pion decay mediated by another interaction?
The pion is a composite particle: does is mean that the form factors drastically change our estimates?
Is the weak interaction non universal, i.e. is the value of G_F processus dependent?

Pathology at High Energy

What about weak scattering process, e.g . $e\nu_e \to e\nu_e$?

$$\mathcal{L} = G_F\, J^*_\mu J^\mu \qquad \text{with} \qquad J^\mu = (\bar{n}\gamma^\mu p) + (\bar{e}\gamma^\mu \nu_e) + (\bar{\mu}\gamma^\mu \nu_\mu) + \ldots$$

The same Fermi Lagrangian will thus also contain a term

$$G_F\, (\bar{e}\gamma^\mu \nu_e)(\bar{\nu}_e \gamma^\mu e)$$

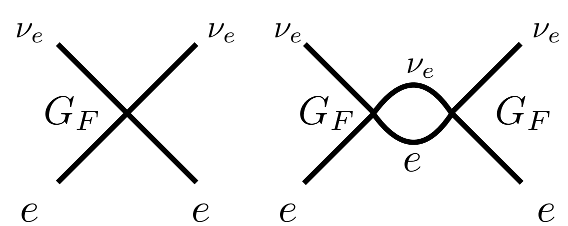
that will generate e-ν_e scattering whose cross-section can be guessed by dimensional arguments

$$\sigma \propto G_F^2 E^2$$

$[\text{mass}]^{-2}$ → σ; $[\text{mass}]^{-2\times 2}$ → G_F^2; $[\text{mass}]^2$ → E^2

non conservation of probability
(non-unitary theory)
inconsistent at high energy

It means that, at high-energy, the quantum corrections to the classical contribution can be sizeable:

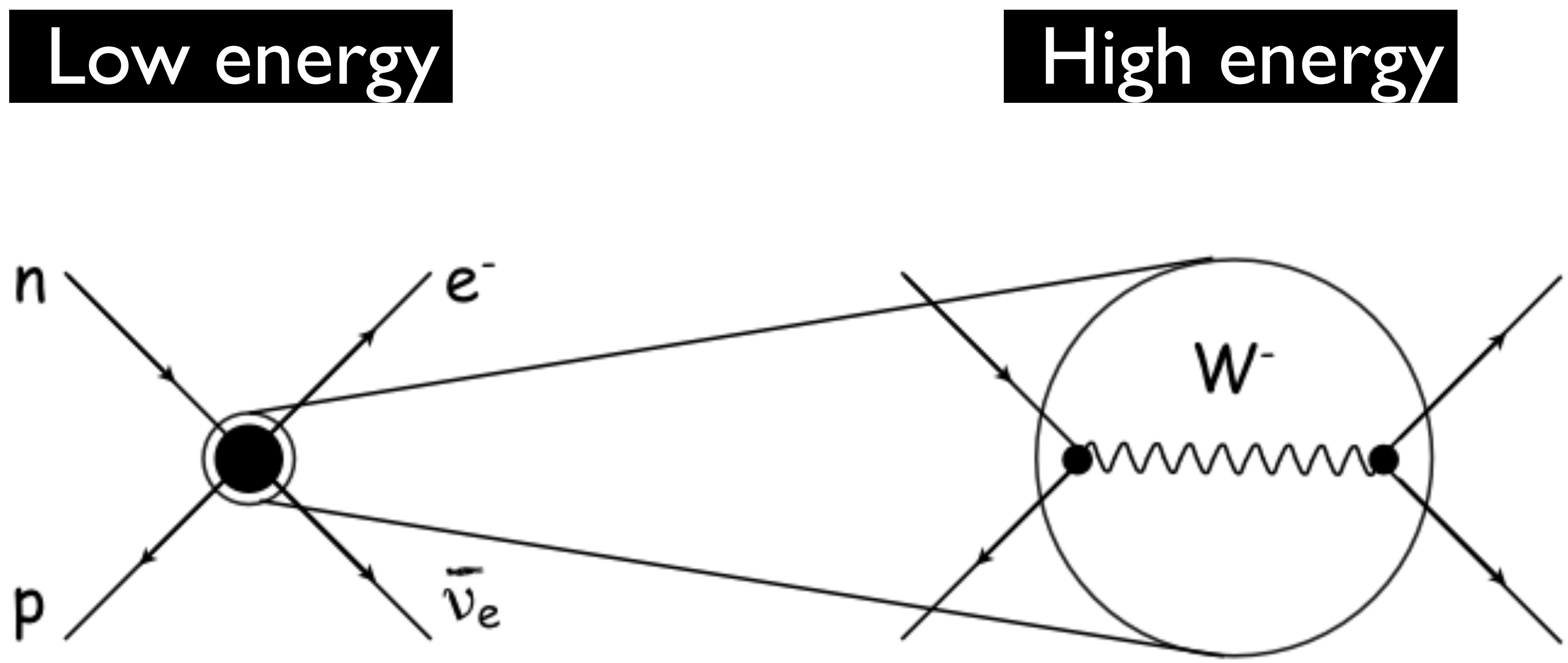


$$\sigma \propto G_F^2 E^2 + \frac{1}{16\pi^2} G_F^4 E^6 + \ldots$$

The theory becomes non-perturbative at an energy $E_{\text{max}} = \frac{2\sqrt{\pi}}{\sqrt{G_F}} \sim 100\,\text{GeV–}1\,\text{TeV}$

unless new degrees of freedom appear before to change the behaviour of the scattering

Electroweak Interactions



$$\sigma \propto G_F^2 E^2$$

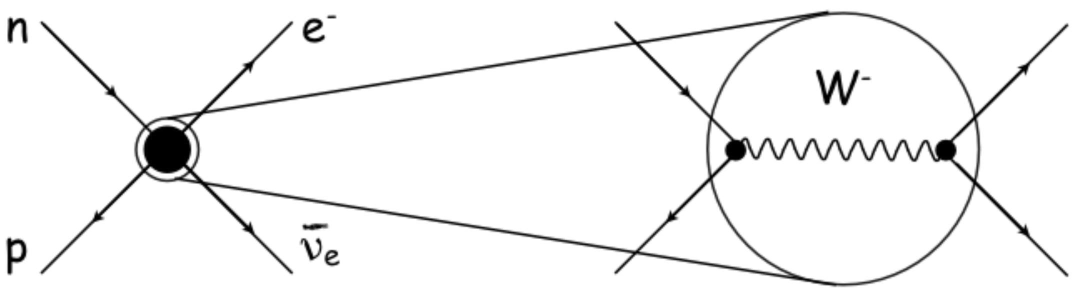
$$\sigma \propto g^4 \frac{E^2}{m_W^2\,(E^2 + m_W^2)}$$

— matching —

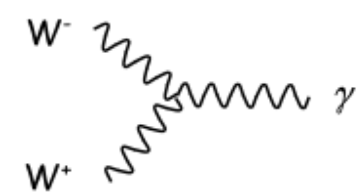
$$G_F \propto \frac{g^2}{m_W^2}$$

The Fermi interaction is not a fundamental interaction of Nature.
It is a low energy effective interaction.

Electroweak Interactions



charged W ⇛ must couple to photon:



⇛ non-abelian gauge symmetry [Q,T±]=±T±

1. No additional “force” (Georgi, Glashow ’72) mathematical consistency ⇛ **extra matter**

$SU(2)$	$SU(1,1)$	E_2
$[T^a, T^b] = i\epsilon^{abc}T^c$ $[T^+, T^-] = Q \quad [Q, T^\pm] = +\pm T^\pm$ $T^\pm = \frac{1}{\sqrt{2}}(T^1 \pm iT^2)$ $\mathrm{Tr}_{\mathrm{irrep}} T^3 = 0$ ⇛ extra matter $\begin{pmatrix} X_L \\ \nu_L \\ e_L \end{pmatrix} \begin{pmatrix} X_R \\ \nu_R \\ e_R \end{pmatrix}$	$[T^+, T^-] = -Q$ $[Q, T^\pm] = +\pm T^\pm$ non-compact unitary rep. has dim ∞	2D Euclidean group $[T^+, T^-] = 0$ $[Q, T^\pm] = +\pm T^\pm$ only one unitary rep. of finite dim. = trivial rep.

2. No additional “matter” (Glashow ’61, Weinberg ’67, Salam ’68): SU(2)xU(1)

⇛ **extra force**

$Q = T^3?$

as Georgi-Glashow

⇛ extra matter

$Q = Y?$

$Q(e_L) = Q(\nu_L)$

$Q = T^3 + Y!$

Gell-Mann ’56, Nishijima-Nakano ’53

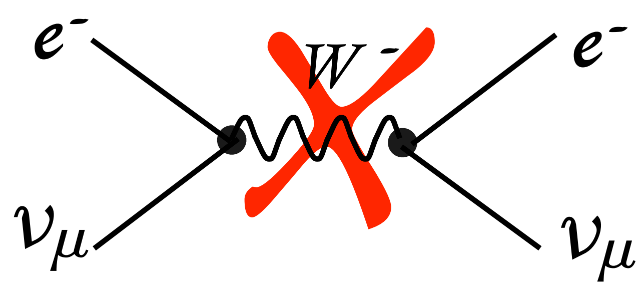
Electroweak Interactions

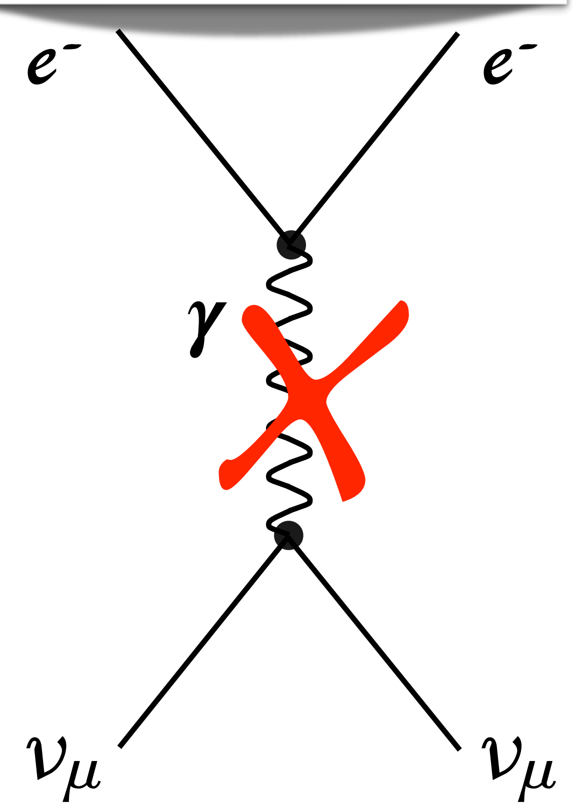
Gargamelle experiment '73 first established the SU(2)xU(1) structure

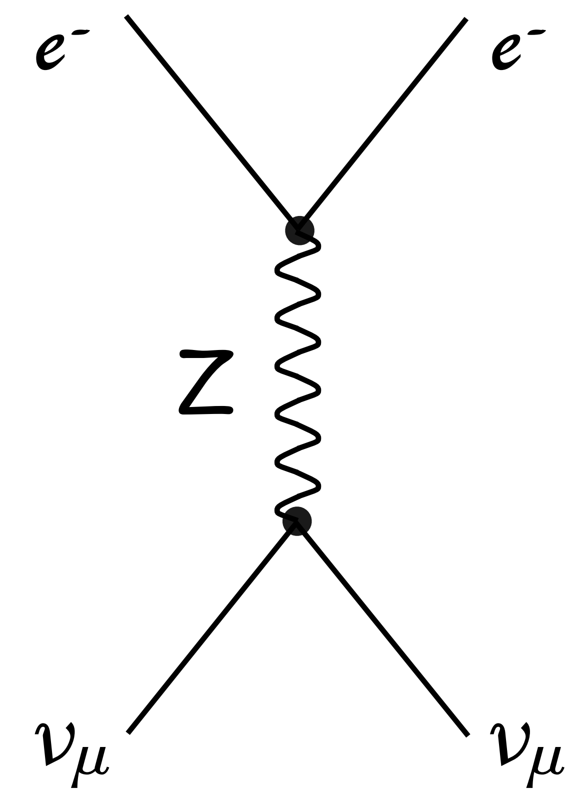
How?

rely on a particle that doesn't interact with photon to prove the existence a new neutral current process!

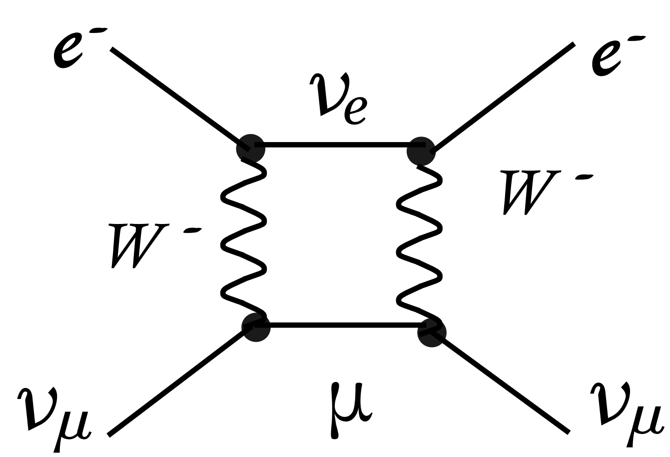
$$\nu_\mu e^- \to \nu_\mu e^-$$







loop-suppressed contribution from W:



From Gauge Theory to Fermi Theory

We can derive the Fermi current-current contact interactions by "integrating out" the gauge bosons, i.e., by replacing in the Lagrangian the W's by their equation of motion. Here is a simple derivation:
(a better one should take taking into account the gauge kinetic term and the proper form of the fermionic current that we'll figure out tomorrow, for the moment, take it as a heuristic derivation)

$$\mathcal{L} = -m_W^2 W_\mu^+ W_\nu^- \eta^{\mu\nu} + g W_\mu^+ J_\nu^- \eta^{\mu\nu} + g W_\nu^- J_\nu^+ \eta^{\mu\nu}$$

$$J^{+\mu} = \bar{n}\gamma^\mu p + \bar{e}\gamma^\mu \nu_e + \bar{\mu}\gamma^\mu \nu_\mu + \dots \qquad \text{and} \qquad J^{-\mu} = \left(J^{+\mu}\right)^*$$

The equation of motion for the gauge fields: $\dfrac{\partial \mathcal{L}}{\partial W_\mu^+} = 0 \quad \Rightarrow \quad W_\mu^- = \dfrac{g}{m_W^2} J_\mu^-$

Plugging back in the original Lagrangian, we obtain an *effective Lagrangian* (valid below the mass of the gauge bosons):

$$\mathcal{L} = \frac{g^2}{m_W^2} J_\mu^+ J_\nu^- \eta^{\mu\nu}$$

which is the Fermi current-current interaction. The Fermi constant is given by
(the correct expression involves a different normalisation factor)

$$G_F = \frac{g^2}{m_W^2}$$

But what is the origin of the W mass?
By the way, it is not invariant under SU(2) gauge transformation…
That's what the Higgs mechanism will take care of!

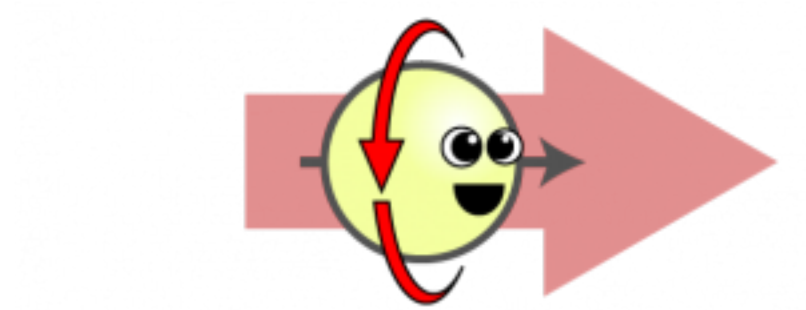
Chirality and the Mass Problem

Chirality & Masslessness

Quantum Mechanics 1.0.1

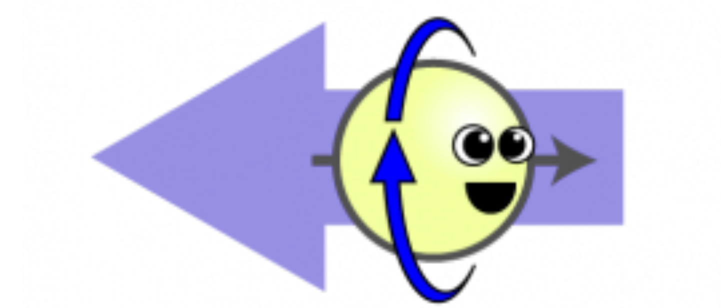
Particle of spin s has 2s+1 polarisation states

Particle spinning anticlockwise wrt its direction of motion



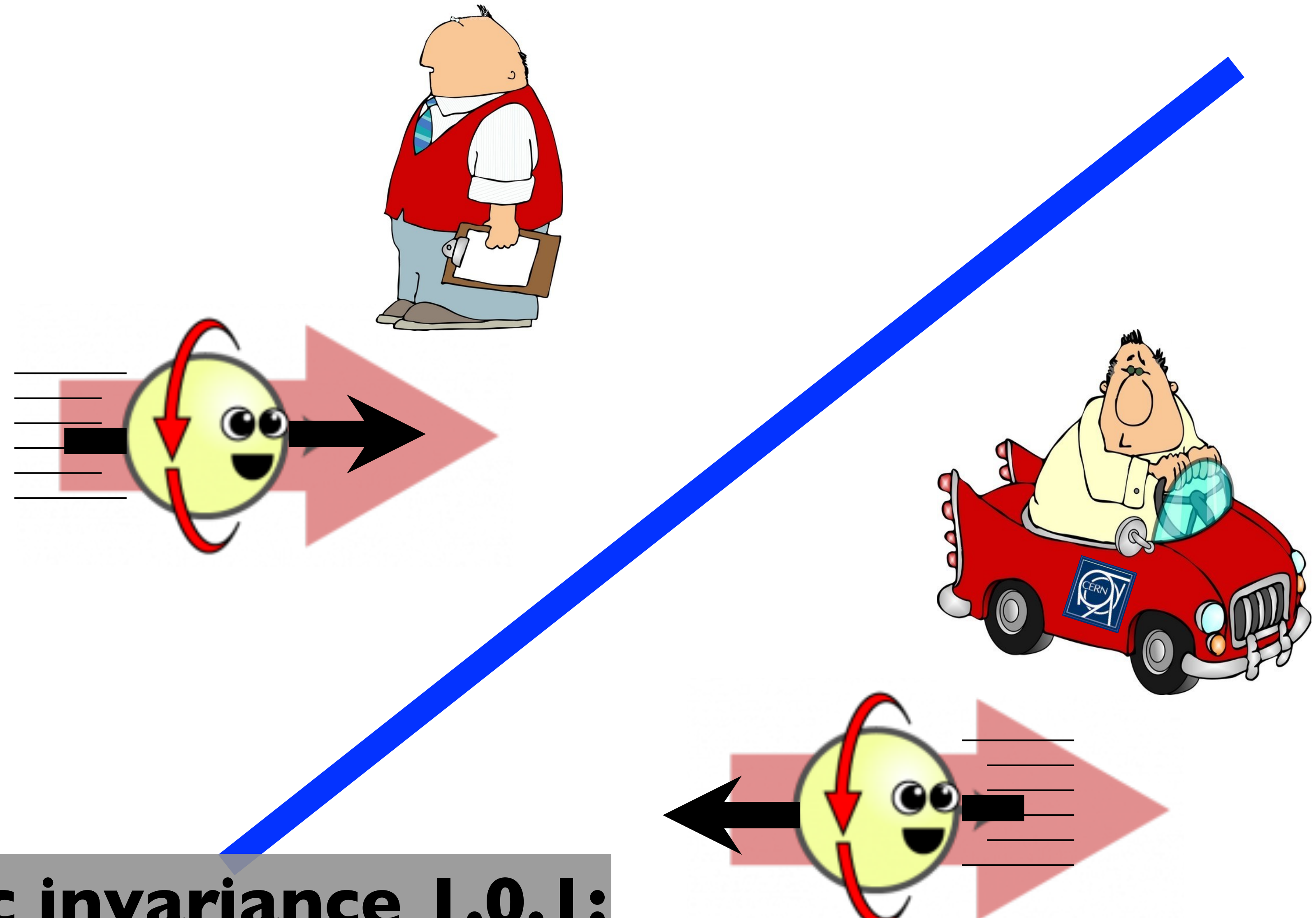
electron
has 2 polarisation

Particle spinning clockwise wrt its direction of motion



Picture courtesy to G. Giudice

Chirality & Masslessness



Picture courtesy to G. Giudice

Relativistic invariance 1.0.1:

there must be no distinction for massive particles between particles spinning clockwise or anti-clockwise

[chirality operator doesn't commute with the Hamiltonian]

Theorem ⇒ **If your theory sees a difference between e_L and e_R, either your theory is wrong or m_e=0**

Chirality of SM & Mass problem

Weak interaction
(force responsible for
neutron decay)
is chiral!

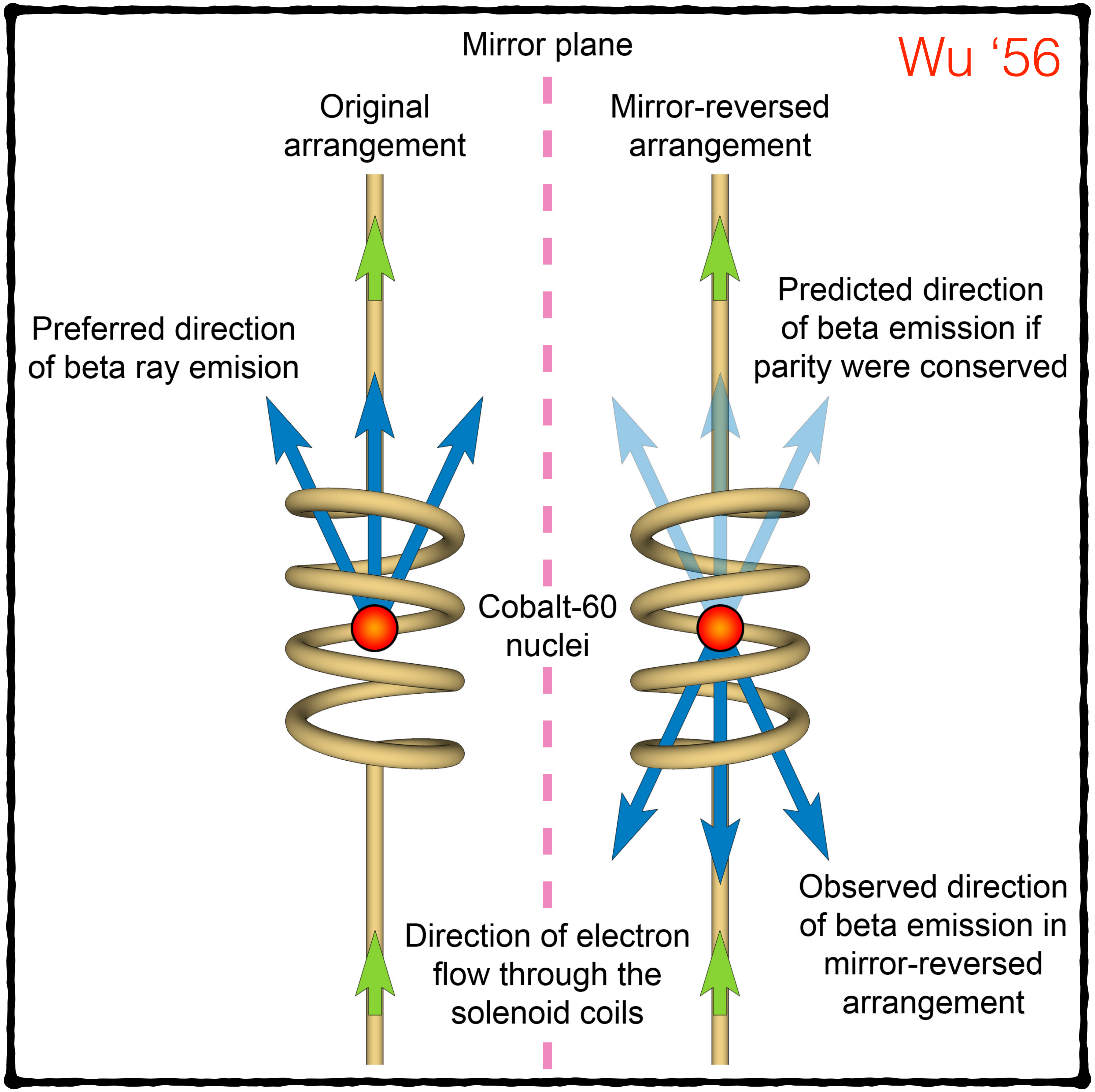
[e_L and e_R are fundamentally two different particles
Only an accident of the history of physics that they are both called electron]

$m_e=0$

but since we know it is not true, we

need a new phenomena to generate mass: Higgs mechanism

TH: Yang&Lee '56. EXP: Wu '57



Chirality of SM & Mass problem

TH: Yang&Lee '56. EXP: Wu '57

Weak interaction
(force responsible for
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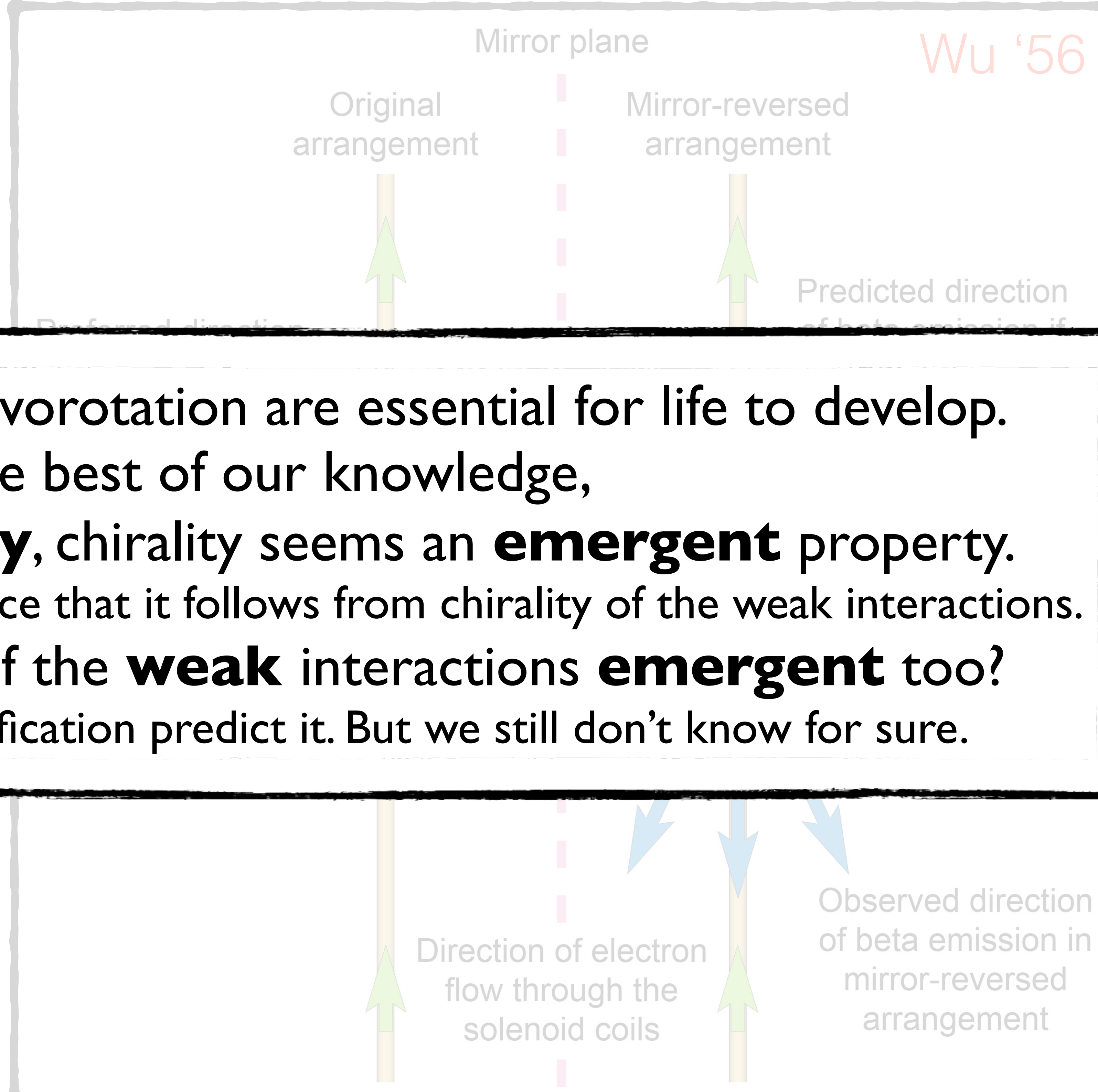
is chiral!

[e_L and e_R are fundamentally



Dextrorotation and Levorotation are essential for life to develop.
To the best of our knowledge,
in **molecular biology**, chirality seems an **emergent** property.
At least, there is no clear evidence that it follows from chirality of the weak interactions.
Are the chiral nature of the **weak** interactions **emergent** too?
Some models of grand unification predict it. But we still don't know for sure.

But since we know it is not true, we

need a new phenomena to generate mass: Higgs mechanism

SM is a Chiral Theory

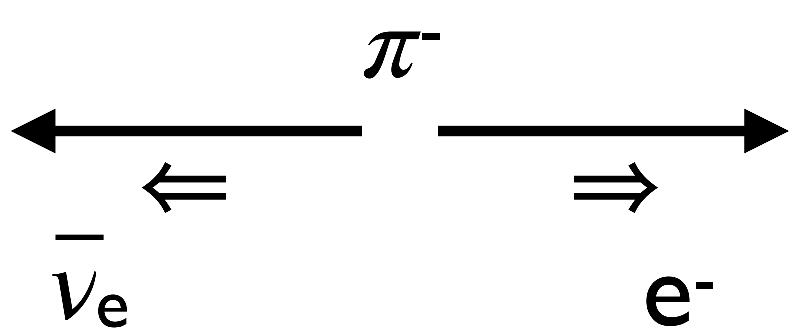
Weak interactions maximally violates P

$^{60}_{27}\mathrm{Co} \to ^{60}_{28}\mathrm{Ni} + e^- + \bar{\nu}_e$ only left-handed (LH) e^- produced

Weak interactions act only on LH particles (and RH anti-particles)

this property has an important consequence (aka selection rule) for pion decay

$\rightarrow$ direction of momentum $\Rightarrow$ direction of spin



Conservation of momentum and spin imposes to have a RH e^-

Weak decays proceed only w/ LH e^-
So the amplitude is prop. to m_e

$$\mathcal{L}_{\mathrm{Dirac}} = \bar{\psi}_L \gamma^\mu \partial_\mu \psi_L + \bar{\psi}_R \gamma^\mu \partial_\mu \psi_R + m\left(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L\right)$$

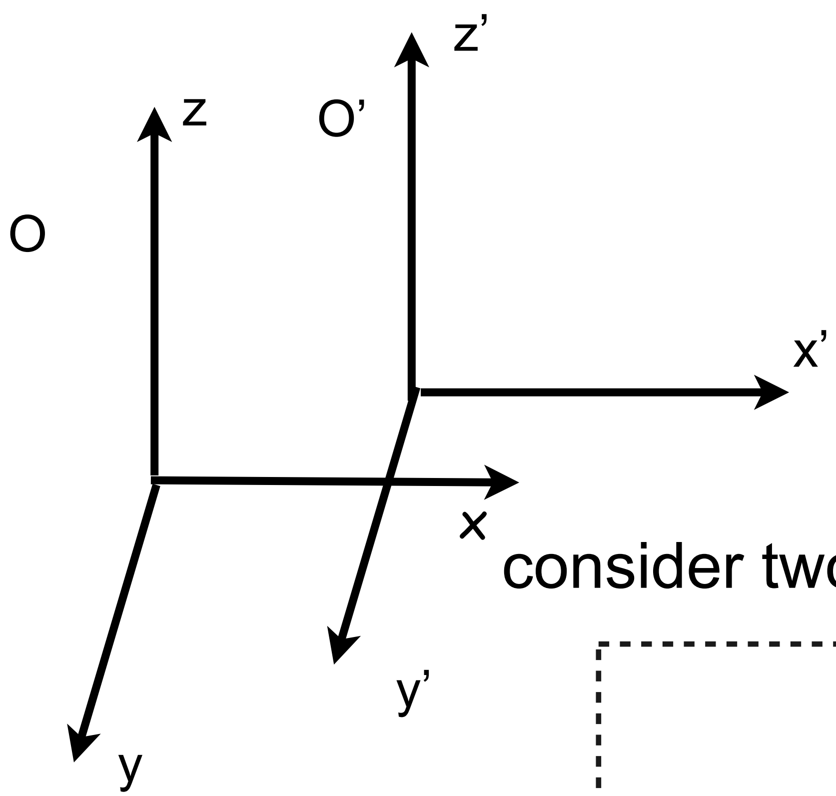
Lorentz structure of fermion mass

$$\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} \propto \frac{m_e^2}{m_\mu^2} \sim 2 \times 10^{-5} \sim 10^{-4}_{\mathrm{obs}}$$

Extra phase-space factor

Technical Details
for Advanced Students

Time-ordering ≠ Causality



$$\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} ct' = \gamma_0 \left(ct - \beta_0 x\right) \\ x' = \gamma_0 \left(-\beta_0 ct + x\right) \\ y' = y \\ z' = z \end{pmatrix}$$

"time dilation + space contraction"

consider two events E_1 and E_2 characterised by their space-time coordinates

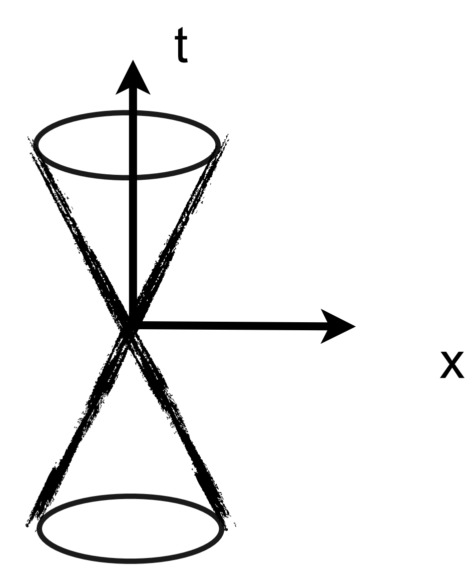
E_1	
$t_1 = 0$	$t'_1 = 0$
$x_1 = 0$	$x'_1 = 0$

E_2	
$t_2 > 0$	$ct'_2 = \gamma\left(ct_2 - \beta x_2\right)$
$x_2 > 0$	$x'_2 = \gamma\left(-\beta ct_2 + x_2\right)$

t'_2 can be positive or negative
causality ≠ time ordering

Proper space-time distance Δ is independent of the observer:

$$\Delta'^2 = \left(ct'_2\right)^2 - \left(x'_2\right)^2 = \left(ct_2\right)^2 - x_2^2 = \Delta^2$$



Only events inside the past/future light cones are causally connected
The light cones are invariant under Lorentz transformations

Spinor Transformation

Transformation law: $\psi(x) \to \psi'(x') = S(\Lambda)\psi(x)$

We want the Dirac equation to take the same form in the two systems of coordinates x and x'

$$(i\gamma^\mu \partial_\mu - m)\psi = 0 \qquad\qquad (i\gamma^\mu \partial'_\mu - m)\psi' = 0$$

This implies the condition: $S\gamma^\mu \Lambda^\nu{}_\mu S^{-1} = \gamma^\nu$

We consider small Lorentz transformations: $\Lambda_\mu{}^\nu = \delta^\mu_\nu + \omega^\mu{}_\nu \qquad S = 1 - \frac{i}{4}\sigma^{\mu\nu}\omega_{\mu\nu}$

The covariance of the Dirac equation then implies that the matrices $\sigma_{\mu\nu}$ have to satisfy the relation

$$[\gamma^\nu, \sigma^{\rho\sigma}] = 2i(\eta^{\nu\rho}\gamma^\sigma - \eta^{\nu\sigma}\gamma^\rho)$$

It is easy to check that the following matrices fit the bill: $\sigma^{\rho\sigma} = \frac{i}{2}[\gamma^\rho, \gamma^\sigma]$

$$x^\mu \to x'^\mu = (\delta^\mu{}_\nu + \omega^\mu{}_\nu)x^\nu \quad \text{with} \quad \omega_{\mu\nu} + \omega_{\nu\mu} = 0$$

$$\psi(x) \to \psi'(x') = \left(1_4 + \frac{1}{8}\omega_{\mu\nu}[\gamma^\mu, \gamma^\nu]\right)\psi(x)$$

Lorentz-invariant Lagrangian

$\mathcal{L} = \psi^\dagger M \,(i\gamma^\mu\partial_\mu - m)\,\psi$ is Lorentz-invariant iff $\gamma^0[\gamma^\nu, \gamma^\mu]\gamma^0 M + M[\gamma^\mu, \gamma^\nu] = 0$

$M = \gamma^0$ is a solution and it defines the Dirac Lagrangian. $\boxed{\bar\psi \equiv \psi^\dagger\gamma^0}$

SU(N) non-Abelian Gauge Symmetry

We generalise the QED construction by considering general transformation of a N-vector

$$\phi \to U\phi$$

We build a **covariant derivative** that again has nice homogeneous transformations

$$D_\mu \phi = \partial_\mu \phi + igA_\mu \phi \to UD_\mu \phi \qquad \text{iff} \qquad A_\mu \to UA_\mu U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1}$$

g is the gauge coupling and defines the strength of the interactions

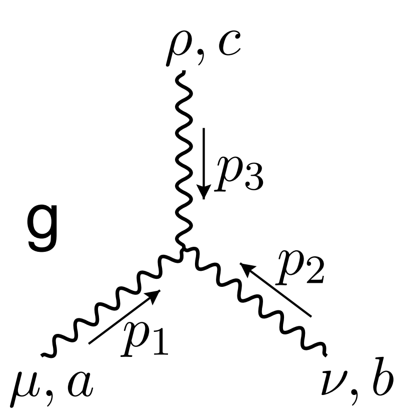
For the field strength to transform homogeneously, one needs to add a non-Abelian piece

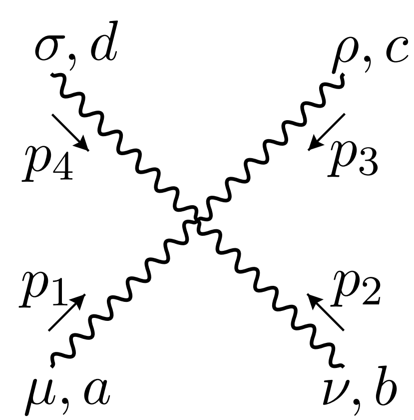
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu] \to UF_{\mu\nu}U^{-1}$$

Contrary to the Abelian case, the gauge fields are now charged and interact with themselves

$$\mathcal{L}_{\text{kin}} = \text{Tr}F_{\mu\nu}F^{\mu\nu} \supset g\partial AAA + g^2 AAAA$$

g





g^2

∃ gauge boson self-interactions

Symmetries and invariants

SU(N)

the transformations among the components of a complex N-vector that leaves its norm invariant

$$|\phi|^2 = \phi_1^*\phi_1 + \dots \phi_N^*\phi_N \to |\phi'|^2 = |\phi|^2$$

SU(N,M)

the transformations among the components of a complex (N+M)-vector that leaves its (N,M) norm invariant

$$|\phi|^2 = \phi_1^*\phi_1 + \dots \phi_N^*\phi_N + \phi_{N+1}^*\phi_{N+1} - \dots - \phi_{N+M}^*\phi_{N+M} \to |\phi'|^2 = |\phi|^2$$

SO(N)

the transformations among the components of a real N-vector that leaves its norm invariant

$$|\phi|^2 = \phi_1^2 + \dots \phi_N^2 \to |\phi'|^2 = |\phi|^2$$

SO(N,M)

the transformations among the components of a real (N+M)-vector that leaves its (N,M) norm invariant

$$|\phi|^2 = \phi_1^2 + \dots \phi_N^2 + \phi_{N+1}^2 - \dots - \phi_{N+M}^2 \to |\phi'|^2 = |\phi|^2$$

The Lorentz group is thus SO(1,3)

Lorentz transformation

SO(1,3)

The elements of SO(1,3) satisfy $U^t \eta U = \eta$ where =diag(1,-1,-,1,-1)

The infinitesimal transformations are $U = e^{\theta^a T^a} \approx 1 + \theta^a T^a + \dots$

The generators satisfy the constraints: $T^{a\,t}\eta + \eta T^a = 0$

One particular generator is $T = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

We obtain $e^{\theta T} = \begin{pmatrix} \cosh\theta & \sinh\theta & 0 & 0 \\ \sinh\theta & \cosh\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

We indeed recover the usual Lorentz transformation with the identification

$$\gamma = \cosh\theta \quad \text{and} \quad \beta\gamma = \sinh\theta$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \quad \Leftrightarrow \quad \cosh^2\theta - \sinh^2\theta = 1$$

Chirality

Chirality matrix

$$\boxed{\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3}$$

A few remarkable properties

$$\left(\gamma^5\right)^2 = 1_4$$

$$\{\gamma^5, \gamma^\mu\} = 0$$

$$\gamma^{5\dagger} = \gamma^5 = -\gamma^0\gamma^5\gamma^0$$

Chiral/Weyl spinor

A **chiral/Weyl** spinor is an eigenvector of the chirality matrix $\psi_{L,R} = \pm\gamma^5\psi_{L,R}$

From the Lorentz-transformation law of a spinor, it is obvious that the chirality condition is frame-independent

A Dirac spinor can also be written as a sum of two chiral spinors

$$\psi = \frac{1}{2}\left(1_4 + \gamma^5\right)\psi + \frac{1}{2}\left(1_4 - \gamma^5\right)\psi \equiv \psi_L + \psi_R$$

Charge conjugation

In general, ψ and ψ^* do not transform in the same way under Lorentz transformations and the naive reality condition $\psi = \psi^*$ is frame dependent

But it is possible to find a matrix C, called charge conjugation matrix, such that

$$\psi \quad \text{and} \quad \psi_C = C\,\psi^*$$

transform in the same way under Lorentz transformations

The matrix C needs to satisfy $C\gamma^* = -\gamma^\mu C$

In the Dirac and Weyl representations, $C = i\gamma^2$

In the Majorana representation, $C = 1_4$

Basic properties of the charge conjugation matrix: $C^2 = 1_4,\ C^\dagger = C,\ C^* = C$

The charge conjugated spinor, ψ_C, satisfies the same Dirac equation as ψ, with the same mass but opposite electric charge (when the spinor is minimally coupled to a U(1) gauge field)

A **Majorana** spinor satisfies the (Lorentz invariant!) condition $\psi = \psi_C$

Note that in 4D, a spinor cannot be simultaneously chiral and Majorana

Dirac and Majorana Masses

By construction, the following two mass terms in the Lagrangian are Lorentz-invariant

Dirac mass: $\mathcal{L}_{\text{Dirac}} = m\bar{\psi}\psi$ (conserves fermion number)

Majorana mass: $\mathcal{L}_{\text{Majorana}} = m\bar{\psi}_C\,\psi$ (changes fermion number by 2)

These two mass terms have different a chirality structure

$$\mathcal{L}_{\text{Dirac}} = m\left(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\right)$$

$$\mathcal{L}_{\text{Majorana}} = m\left(\bar{\psi}_{L_C}\psi_L + \bar{\psi}_{R_C}\psi_R\right)$$

A chiral fermion can have a Majorana mass
A Dirac mass requires spinors of opposite chirality

Whether or not a Dirac or a Majorana mass can be included in the Lagrangian depends on transformation laws of the spinors under the gauge transformations

Within the SM (with the Higgs field), a Dirac mass can written for the charged leptons and the quarks while a Majorana mass can be written for the neutrinos.