Double insertions of SMEFT operators in gluon fusion Higgs production

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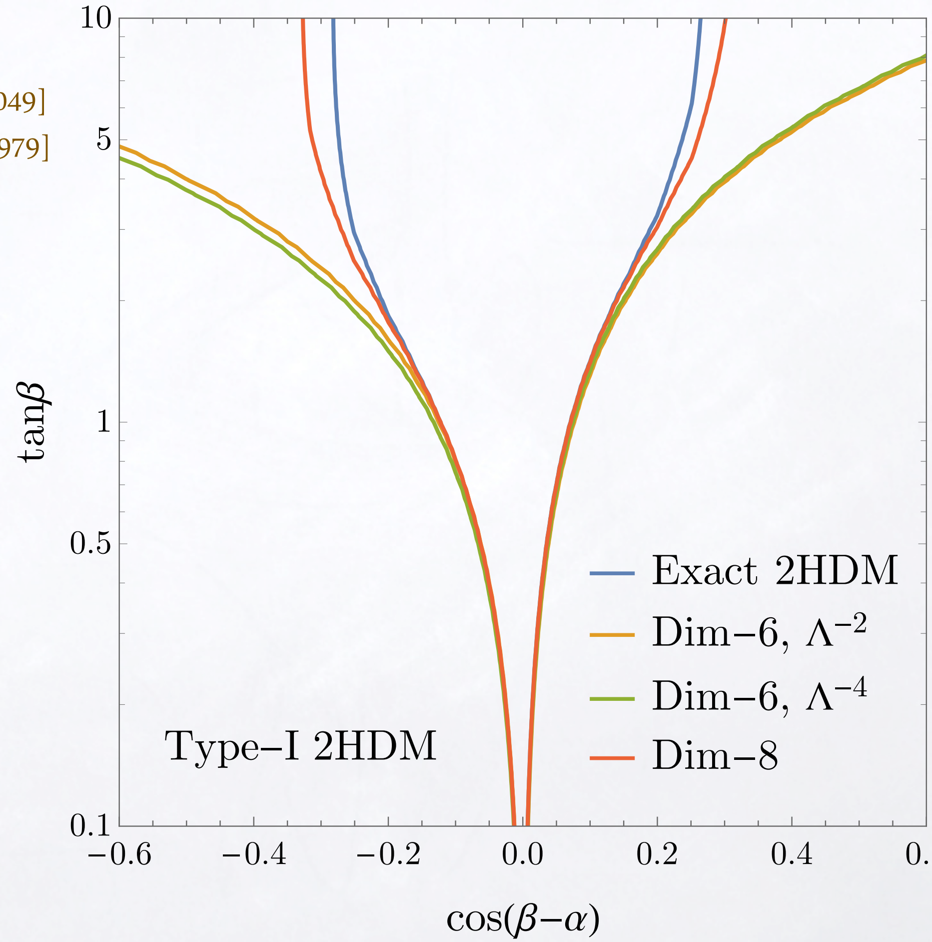
- Although the Standard Model (SM) is extremely powerful, there is physics beyond it (BSM)
- The SM Effective Field Theory – **SMEFT** – is a very convenient description of heavy BSM physics:
  - It configures a consistent and general description of deviations from the SM

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \Sigma_{n,i} \frac{C_i^n O_i^n}{\Lambda^{n-4}}$$

  - Conversion between experimental data and theory has to be done only once
- Assuming lepton and baryon number conservation, n is even. The case n=6 (i.e. dimension-6) has been thoroughly investigated in the last decade [hundreds of references!]
- Squared effects of dimension-6 should be neglected at LO in the EFT expansion (i.e. at $\mathcal{O}(1/\Lambda^2)$)
  - For an amplitude $\mathcal{A} \propto a_0\, g_{\rm SM} + a_1 \frac{C^{(6)}}{\Lambda^2} + \ldots$,

    then $\sigma \propto |\mathcal{A}|^2 \propto |a_0\, g_{\rm SM}|^2 + \frac{2}{\Lambda^2}{\rm Re}[a_0\, a_1\, g_{\rm SM}\, C^{(6)}] + \frac{1}{\Lambda^4}\left\{|a_1\, C^{(6)}|^2\right\} + \ldots$
- But it is time to go beyond $\mathcal{O}(1/\Lambda^2)$ [Altmannshofer et al, 2209.10639]
  - It is known since several years that $\mathcal{O}(1/\Lambda^2)$ may not suffice in some cases [Contino et al, 1604.06444]
    - Light-by-light scattering [Murphy, 2005.00059]
    - Neutral triple gauge couplings [Degrande, 1308.6323] [Ellis et al, 1902.06631, 2008.04298]

- The increasing precision of LHC data is beginning to require the inclusion of $\mathcal{O}(1/\Lambda^4)$ [Alioli et al, 2203.06771]
- Drell-Yan production can be significantly affected by $\mathcal{O}(1/\Lambda^4)$ [Alioli et al, 2203.06771, 2003.11615] [Boughezal et al, 2106.05337, 2207.01703]
- Low-energy phenomena and experiments as well [Mereghetti et al, 1305.7049] [Boughezal et al, 2104.03979]
- $\mathcal{O}(1/\Lambda^4)$ effects can also be crucial in the matching to particular UV models
  - Example: 2HDM Type I [Dawson et al, 2205.01561]



- To go beyond $\mathcal{O}(1/\Lambda^2)$:
  - V-improved matching: include some effects of dimension-8 operators [Brehmer et al, 1510.03443] [Freitas et al, 1607.08251]
  - GeoSMEFT: SMEFT formulated to all orders in $\mathcal{O}(v/\Lambda)$ for 2- and 3-point functions [Helset et al, 1803.08001] [Corbett et al, 1909.08470] [Helset et al, 2001.01453] [Hays et al, 2007.00565]
  - Dimension-8 operators:

    For $\mathcal{A} \propto a_0\, g_{\rm SM} + a_1 \frac{C^{(6)}}{\Lambda^2} + a_2 \frac{C^{(8)}}{\Lambda^4} + \dots$, it follows:

$$\sigma \propto |a_0\, g_{\rm SM}|^2 + \frac{2}{\Lambda^2} \mathrm{Re}[a_0\, a_1\, g_{\rm SM}\, C^{(6)}] + \frac{1}{\Lambda^4} \Big\{ |a_1\, C^{(6)}|^2 + 2\,\mathrm{Re}[a_0\, a_2\, g_{\rm SM}\, C^{(8)}] \Big\} + \dots$$

    - Complete bases [Li et al, 2005.00008] [Murphy, 2005.00059]
    - Renormalization group equations [Chala et al, 2106.05291, 2112.12724, 2203.06771, 2205.03301] [Helset et al, 2212.03253]
    - Phenomenology [Boughezal et al, 2106.05337, 2207.01703] [Allwicher et al, 2207.10714] [Aoude et al, 2208.04962]
    - Matching to particular models [Hays et al, 1808.00442] [Corbett et al, 2102.02819] [Dawson et al, 2110.06929] [Dawson et al, 2205.01561]

- At $\mathcal{O}(1/\Lambda^4)$, we must consider not only squared dimension-6 and single insertions of dimension-8 operators, but also **double insertions** of dimension-6 operators:
  - For $\mathcal{A} \propto a_0\, g_{\rm SM} + a_1 \frac{C^{(6)}}{\Lambda^2} + a_2 \frac{C^{(8)}}{\Lambda^4} + a_3 \frac{\left(C^{(6)}\right)^2}{\Lambda^4} + \ldots$, we find:

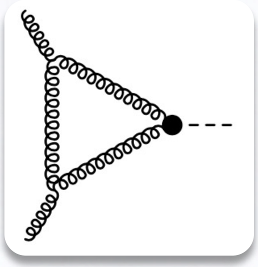
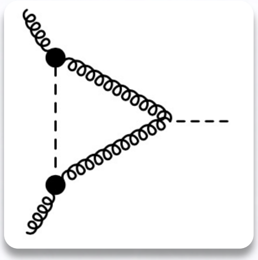
$$\sigma \propto |\mathcal{A}|^2 \propto |a_0\, g_{\rm SM}|^2 + \frac{2}{\Lambda^2}{\rm Re}[a_0\, a_1\, g_{\rm SM}\, C^{(6)}] + \frac{1}{\Lambda^4}\left\{|a_1\, C^{(6)}|^2 + 2\,{\rm Re}\left[a_0\, a_3\, g_{\rm SM} \left(C^{(6)}\right)^2\right] + 2\,{\rm Re}[a_0\, a_2\, g_{\rm SM}\, C^{(8)}]\right\} + \ldots$$

- How relevant are these double insertions of dimension-6 operators?
  - This question has been addressed in some cases at tree-level

[Heinrich et al, 2204.13045] [Allwicher et al, 2207.10714] [Aoude et al, 2208.04962]

- I investigate this question in the context of a process at one-loop level
  - I ignore the contributions of dimension-8 operators
  - I perform the renormalization of the process
  - I numerically compare the impact of double insertions with that of just squared dimension-6

so, it is explicitly *incomplete*; it is not the whole $\mathcal{O}(1/\Lambda^4)$ result, but just an exploration of the importance of double insertions.
Yet, the result is gauge-independent, as dim. 8 terms are not needed to ensure this

- More specifically, I consider **Higgs** production through gluon fusion
  - If ● represents a dim.6 operator connecting Higgs and gluons, a diagram with a single insertion is  , whereas a diagram with a double insertion is 
  - Higgs production via gluon fusion is known in different scenarios:
    - In the SM, up to N3LO [Anastasiou et al, 1602.00695]
    - In the SMEFT, it was calculated at NLO QCD [Deutschmann et al, 1708.00460]
    - In the GeoSMEFT, it was calculated at $\mathcal{O}(1/16\pi^2\Lambda^2)$ and $\mathcal{O}(1/\Lambda^4)$ [Corbett et al, 2107.07470]

- I start by considering the terms in SMEFT relevant for the calculation
  - The dimension-6 Lagrangean has the following relevant terms:

$$\mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{QCD}} + \mathcal{L}_{\text{fermions}}$$

with (using the Warsaw basis): [Grzadkowski et al, 1008.4884]

- $$\begin{aligned}\mathcal{L}_{\text{Higgs}} &= \left(D^\mu\varphi\right)^\dagger\left(D_\mu\varphi\right) + \mu^2\varphi^\dagger\varphi - \frac{\lambda}{2}\left(\varphi^\dagger\varphi\right)^2 \\ &+\frac{1}{\Lambda^2}\left[C_\varphi\left(\varphi^\dagger\varphi\right)^3 + C_{\varphi\Box}\left(\varphi^\dagger\varphi\right)\Box\left(\varphi^\dagger\varphi\right) + C_{\varphi D}\left(\varphi^\dagger D^\mu\varphi\right)^*\left(\varphi^\dagger D_\mu\varphi\right)\right]\end{aligned}$$

- $$\mathcal{L}_{\text{QCD}} = -\frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} + \frac{C_{\varphi G}}{\Lambda^2}\left(\varphi^\dagger\varphi\right)G^A_{\mu\nu}G^{A\mu\nu} + \frac{C_G}{\Lambda^2}f_{ABC}G^{A\nu}_\mu G^{B\rho}_\nu G^{C\mu}_\rho$$

- $$\begin{aligned}\mathcal{L}_{\text{fermions}} &= -Y_u\,\bar{q}_L\varphi d_R + \left[\frac{C_{t\varphi}}{\Lambda^2}\left(\varphi^\dagger\varphi\right)\left(\bar{q}_L\tilde{\varphi}u_R\right) + \text{h.c.}\right] \\ &+\frac{C_{ll}}{\Lambda^2}(\bar{l}_L\gamma_\mu l_L)(\bar{l}'_L\gamma_\mu l'_L) + \frac{C^{(3)}_{\varphi l}}{\Lambda^2}\varphi^\dagger i\overleftrightarrow{D}^a_\mu\varphi(\bar{l}_L\tau^a\gamma_\mu l_L)\end{aligned}$$

  - We also need to consider a dimension-8 operator, for renormalization purposes:

$$\frac{C_{G^2\varphi^4}}{\Lambda^4}(\varphi^\dagger\varphi)^2 G^A_{\mu\nu}G^{A\mu\nu}$$

- As usual in SMEFT, the inclusion of higher order operators requires a redefinition of fields, so as to obtain canonically normalized propagators. Specifically,

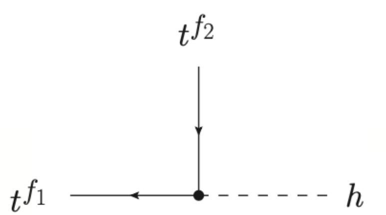
$$h \to h\, R_\phi^{-1}, \qquad \varphi^0 \to \varphi^0\, R_{\varphi^0}^{-1}, \qquad g_\mu^A \to g_\mu^A\, R_g^{-1}$$

- But these redefinitions must now be done up to $\mathcal{O}(1/\Lambda^4)$; we find:

$$\begin{aligned} R_\varphi &= 1 - \frac{v_T^2}{\Lambda^2} X_h - \frac{v_T^4}{2\Lambda^4} X_h^2 + \mathcal{O}(1/\Lambda^6), \\ R_{\varphi^0} &= 1 + \frac{v_T^2}{4\Lambda^2} C_{\varphi D} - \frac{v_T^4}{32\Lambda^4} C_{\varphi D}^2 + \mathcal{O}(1/\Lambda^6), \\ R_g &= 1 - \frac{v_T^2}{\Lambda^2} C_{\varphi G} - \frac{v_T^4}{2\Lambda^4} C_{\varphi G}^2 + \mathcal{O}(1/\Lambda^6), \end{aligned}$$

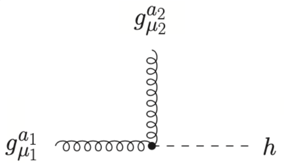
with $X_h \equiv C_{\varphi\Box} - \dfrac{C_{\varphi D}}{4}$ and v_T is the true minimum of the potential

- This also allows us to write the relevant Feynman rules in a compact form; examples:

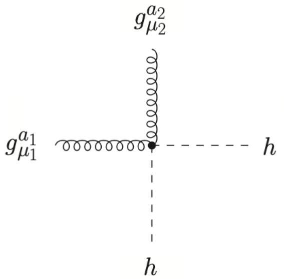


$$-\frac{i}{v_T}\delta_{f_1 f_2} m_u R_\varphi^{-1} + \delta_{f_1 f_2}\frac{i v_T^2 C_{t\varphi}}{\sqrt{2}\Lambda^2} R_\varphi^{-1}$$

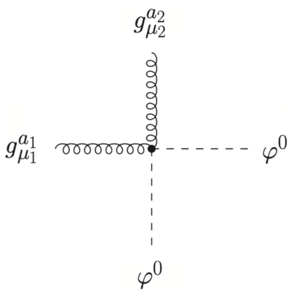
(following the conventions of [Dedes et al, 1704.03888])



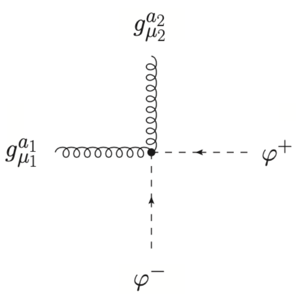
$$+\,4iv_T\delta_{a_1a_2}\frac{C_{\varphi G}}{\Lambda^2}R_g^{-2}R_\varphi^{-1}\left(p_1^{\mu_2}p_2^{\mu_1}-(p_1\cdot p_2)g^{\mu_1\mu_2}\right)$$



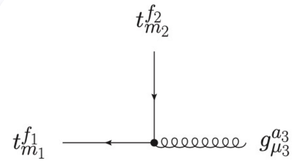
$$+\,4i\delta_{a_1a_2}\frac{C_{\varphi G}}{\Lambda^2}R_g^{-2}R_\varphi^{-2}\left(p_1^{\mu_2}p_2^{\mu_1}-(p_1\cdot p_2)g^{\mu_1\mu_2}\right)$$



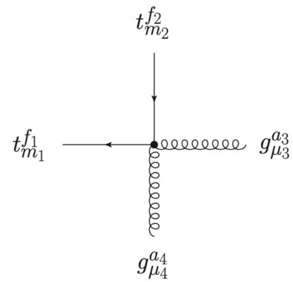
$$+\,4i\delta_{a_1a_2}\frac{C_{\varphi G}}{\Lambda^2}R_g^{-2}R_{\varphi^0}^{-2}\left(p_1^{\mu_2}p_2^{\mu_1}-(p_1\cdot p_2)g^{\mu_1\mu_2}\right)$$



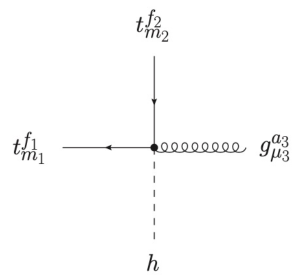
$$+\,4i\delta_{a_1a_2}\frac{C_{\varphi G}}{\Lambda^2}R_g^{-2}\left(p_1^{\mu_2}p_2^{\mu_1}-(p_1\cdot p_2)g^{\mu_1\mu_2}\right)$$



$$-\,i\bar{g}_s\delta_{f_1f_2}\mathcal{T}^{a_3}_{m_1m_2}\gamma^{\mu_3}-\sqrt{2}v_Tp_3^{\nu}\mathcal{T}^{a_3}_{m_1m_2}\sigma^{\mu_3\nu}\frac{C_{tG}}{\Lambda^2}R_g^{-1}$$



$$-\,i\sqrt{2}v_T\bar{g}_sf_{a_3a_4b_1}\mathcal{T}^{b_1}_{m_1m_2}\sigma^{\mu_3\mu_4}\frac{C_{tG}}{\Lambda^2}R_g^{-1}$$



$$-\,\sqrt{2}p_3^{\nu}\mathcal{T}^{a_3}_{m_1m_2}\sigma^{\mu_3\nu}\frac{C_{tG}}{\Lambda^2}R_g^{-1}R_\varphi^{-1}$$

- Effects of $\mathcal{O}(1/\Lambda^4)$ also need to be taken into account when writing the original parameters in terms of independent ones
    - We take as independent parameters: $G_F\,,\ \alpha_s\,,\ M_Z\,,\ M_W\,,\ M_h\,,\ m_t$
    - We need to write the dependent parameters v_T, μ, λ, Y_t, g_s in terms of them. We find:

$$v_T = \frac{1}{(\sqrt{2}G_F)^{\frac{1}{2}}} + \frac{2C_{\varphi l}^{(3)} - C_{ll}}{2(\sqrt{2}G_F)^{\frac{3}{2}}\Lambda^2} + \frac{16(C_{\varphi l}^{(3)})^2 - 12\,C_{\varphi l}^{(3)}\,C_{ll} + 3C_{ll}^2}{8(\sqrt{2}G_F)^{\frac{5}{2}}\Lambda^4}$$

$$\mu^2 = \frac{M_h^2}{2} + \frac{3\,C_\varphi - 4\sqrt{2}X_h G_F\,M_h^2}{8\,G_F^2\,\Lambda^2} + \frac{(2C_{\varphi l}^{(3)} - C_{ll})(3\sqrt{2}C_\varphi - 4X_h G_F M_h^2)}{8\,G_F^3\,\Lambda^4}$$

$$\lambda = G_F M_h^2\sqrt{2} + \frac{3\sqrt{2}C_\varphi + 2(C_{ll} - 2C_{\varphi l}^{(3)} - 2X_h)G_F M_h^2}{2\,G_F\,\Lambda^2} - \frac{3\,C_\varphi\,(C_{ll} - 2\,C_{\varphi l}^{(3)}) + \sqrt{2}\,(C_{\varphi l}^{(3)})^2\,G_F\,M_h^2}{2\,G_F^2\,\Lambda^4}$$

$$Y_t = \sqrt{2}\,(\sqrt{2}G_F)^{\frac{1}{2}}\,m_t\left[1 - \frac{2C_{\varphi l}^{(3)} - C_{ll}}{2(\sqrt{2}G_F)\Lambda^2} - \frac{8(C_{\varphi l}^{(3)})^2 - 4C_{\varphi l}^{(3)}C_{ll} + C_{ll}^2}{8(\sqrt{2}G_F)^2\Lambda^4}\right] + \frac{C_{t\varphi}}{2(\sqrt{2}G_F)\Lambda^2}\left[1 + \frac{2C_{\varphi l}^{(3)} - C_{ll}}{(\sqrt{2}G_F)\Lambda^2}\right]$$

$$g_s = \bar{g}_s\left[1 - \frac{1}{\sqrt{2}G_F}\frac{C_{\varphi G}}{\Lambda^2} - \frac{1}{4G_F^2}\frac{C_{\varphi G}\,(C_{\varphi G} + 4C_{\varphi l}^{(3)} - 2C_{ll})}{\Lambda^4}\right]$$

with $\quad \bar{g}_s \equiv \sqrt{4\pi\alpha_s}$

- We now focus on the calculation of the UV renormalized $gg \to h$ up to one-loop
- Lorentz- and gauge-invariance imply that the amplitude should have the form:

$$A^{\mu\nu}(p_1,p_2) = i\delta_{AB}\left(p_1^\nu p_2^\mu - p_1\cdot p_2 g^{\mu\nu}\right)\sum_i F_i$$

where, up to one-loop,

$$\sum_i F_i = F_0 + F_{\rm V} + F_{\rm CT}$$

$F_{\rm CT}$ → counterterm
$F_{\rm V}$ → one-loop
F_0 → tree-level

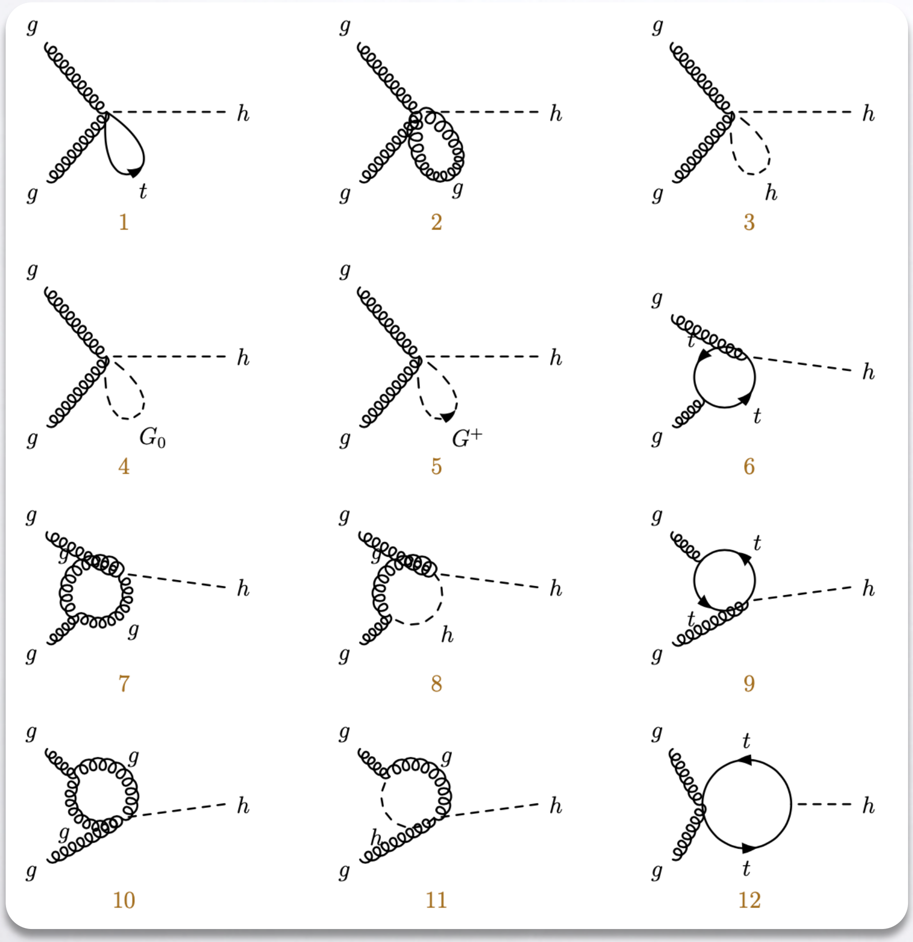
- Each component must be calculated to $\mathcal{O}(1/\Lambda^4)$

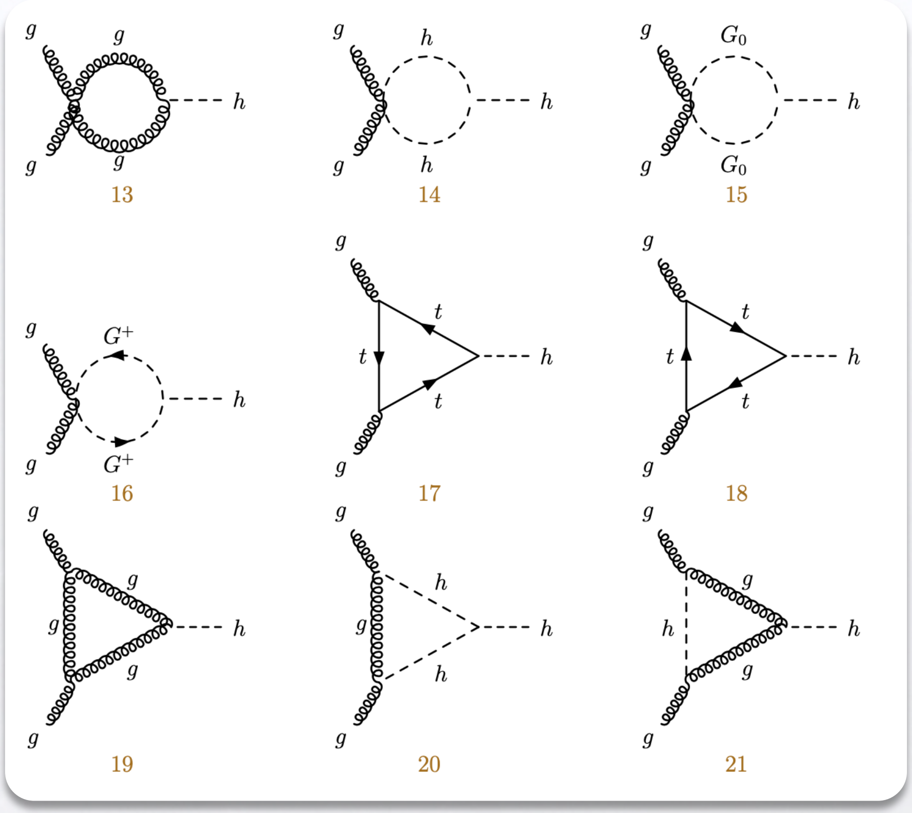
since $\sigma \propto \left|\sum F_i\right|^2$, then σ calculated to $\mathcal{O}(1/\Lambda^4)$ requires interefere F_i of $\mathcal{O}(1/\Lambda^4)$ with F_i of $\mathcal{O}(1/\Lambda^0)$

- The tree-level component is simply given by:

$$F_0 = \frac{4C_{\varphi G}}{(\sqrt{2}G_F)^{\frac{1}{2}}\Lambda^2} + \frac{C_{\varphi G}}{(\sqrt{2}G_F)^{\frac{3}{2}}\Lambda^4}\left[8\,C_{\varphi G} + 4X_h + 4C_{\varphi l}^{(3)} - 2C_{ll}\right]$$

- F_V and F_{CT} were calculated with FeynMaster and Package-X
  [DF, Romão, 1909.05876, 2103.06281] [Patel, 1503.01469]
- We use the Parameter-Renormalized tadpole scheme → one-loop tadpoles are not included in the diagrams
  [Denner, 0709.1075]
- The Feynman diagrams contributing to F_V in this tadpole scheme are:





- F_{CT} is obtained by identifying the original parameters and fields as bare and expanding them into renormalized and counterterm:

$$h_{(0)} = \left(1+\frac{1}{2}\delta Z_h\right)h\,, \quad g^{A,\mu}_{(0)} = \left(1+\frac{1}{2}\delta Z_g\right)g^{A,\mu}\,, \quad G_{F(0)} = (1+\delta G_F)\,G_F\,, \quad C_{X(0)} = C_X + \delta C_X$$

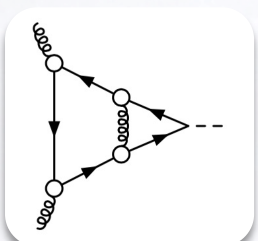
leading to:

$$F_{\mathrm{CT}} = \frac{2}{(\sqrt{2}G_F)^{\frac{1}{2}}}\frac{2\,\delta C^6_{\varphi G} + C_{\varphi G}\,(\delta Z_h + 2\,\delta Z_g - \delta G_F)}{\Lambda^2} + \frac{4}{(\sqrt{2}G_F)^{\frac{3}{2}}}\frac{\delta C^8_{\varphi G}}{\Lambda^4}$$
$$+ \frac{1}{2(\sqrt{2}G_F)^{\frac{3}{2}}\,\Lambda^4}\Big[8\,\delta C_{G^2\varphi^4} + 8\,C_{\varphi\Box}\,\delta C^6_{\varphi G} - 2\,C_{\varphi D}\,\delta C^6_{\varphi G} + 32\,C_{\varphi G}\,\delta C^6_{\varphi G} + 8\,C^{(3)}_{\varphi l}\,\delta C^6_{\varphi G} - 4\,C_{ll}\,\delta C^6_{\varphi G} + 8\,C_{\varphi G}\,\delta C^{(3)}_{\varphi l}$$
$$- 4\,C_{\varphi G}\,\delta C_{ll} + 3\,C_{\varphi D}\,C_{\varphi G}\,\delta G_F - 24\,C^2_{\varphi G}\,\delta G_F - 12\,C_{\varphi G}\,C^{(3)}_{\varphi l}\,\delta G_F + 6\,C_{\varphi G}\,C_{ll}\,\delta G_F - 2\,C_{\varphi D}\,C_{\varphi G}\,\delta Z_g$$
$$+ 16\,C^2_{\varphi G}\,\delta Z_g + 8\,C_{\varphi G}\,C^{(3)}_{\varphi l}\,\delta Z_g - 4\,C_{\varphi G}\,C_{ll}\,\delta Z_g - C_{\varphi D}\,C_{\varphi G}\,\delta Z_h + 8\,C^2_{\varphi G}\,\delta Z_h + 4\,C_{\varphi G}\,C^{(3)}_{\varphi l}\,\delta Z_h$$
$$- 2\,C_{\varphi G}\,C_{ll}\,\delta Z_h + 4\,C_{\varphi\Box}\,C_{\varphi G}\,(-3\,\delta G_F + 2\,\delta Z_g + \delta Z_h)\Big]$$

  - Because they enter already at $\mathcal{O}(1/\Lambda^2)$, we only need to calculate the counterterms at $\mathcal{O}(1/\Lambda^2)$
  - We are only interested in the UV poles
  - δZ_h and δZ_g can be calculated from the Higgs and gluon self-energy, respectively
  - δG_F can be calculated from the muon decay

- For the numerical analysis, we assume $C_{\varphi G}$ to be zero after renormalization
    - This is the same as assuming this WC to be generated only at loop level
    - That is a realistic scenario from a model-building perspective [de Blas et al, 1711.10391]
    - Otherwise, real corrections and 2-loop virtual amplitudes would need to be considered

For example, letting ○ represent g_s and ● represent $C_{\varphi G}$, then:



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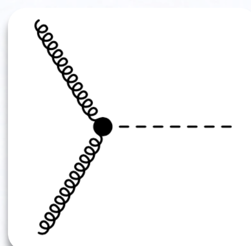
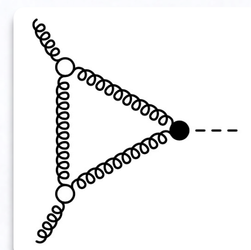


would be of the same order as:



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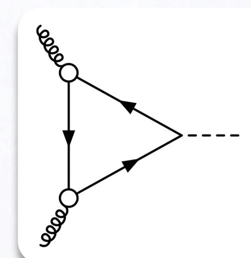


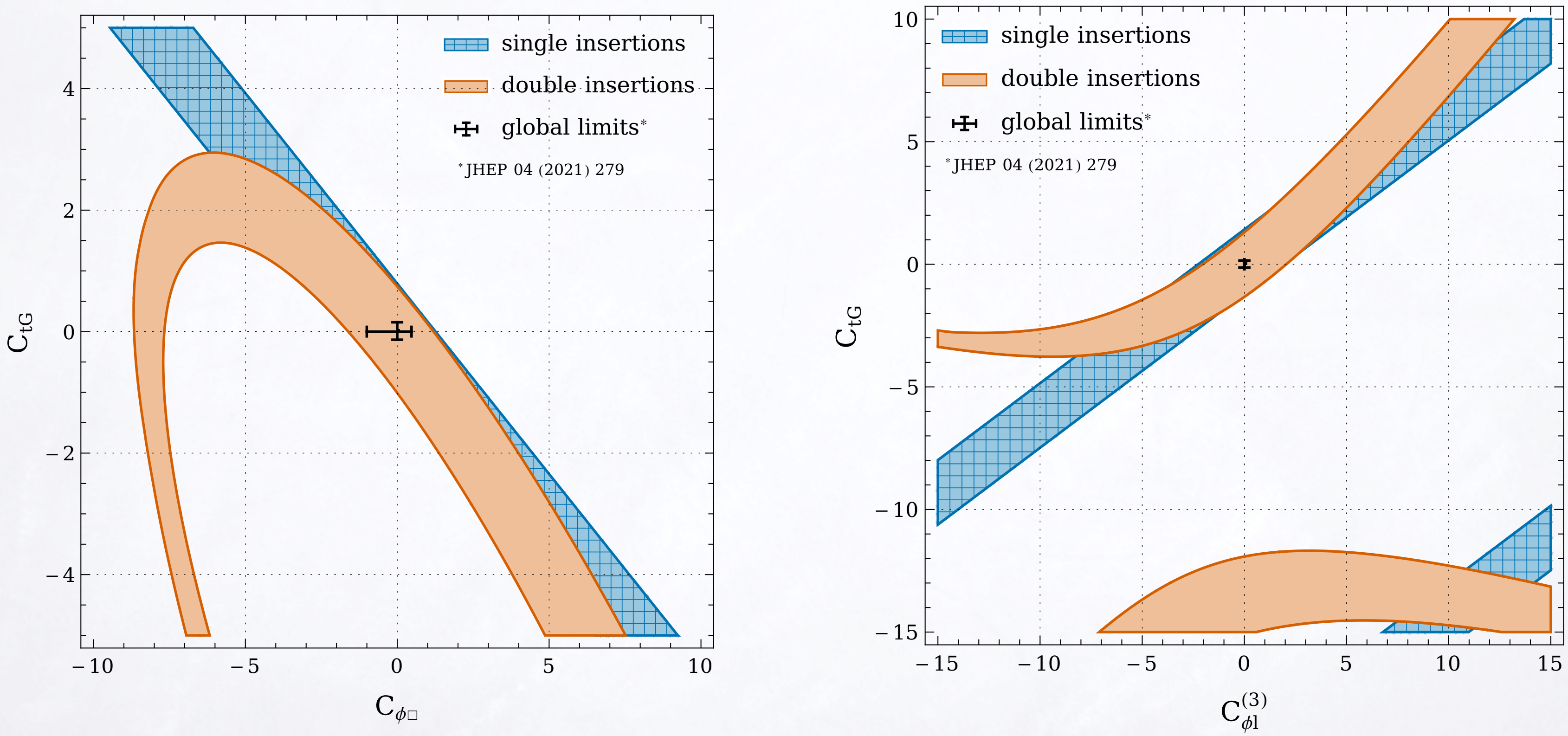
- With $C_{\varphi G} = 0$, then, we have:

$$\mu_{ggh} \equiv \frac{\sigma(gg \to h)}{\sigma(gg \to h)|_{\text{SM}}} = \left| \frac{\sum_i F_i}{F_{\text{SM}}} \right|^2 \equiv 1 + \sum_i a_i \frac{C_i}{\Lambda^2} + \sum_{i,\, j \leq i} b_{ij} \frac{C_i C_j}{\Lambda^4}$$

- The terms with b_{ij} are the interesting ones; they can be obtained in two ways:
    - with *single insertions* only: here, F_i are calculated neglecting $1/\Lambda^4$ terms
    - with *double insertions*: here, F_i are calculated including $1/\Lambda^4$ terms

- Let us consider regions with $|\mu_{ggh} - 1| < 5\%$ to study single vs. double approaches

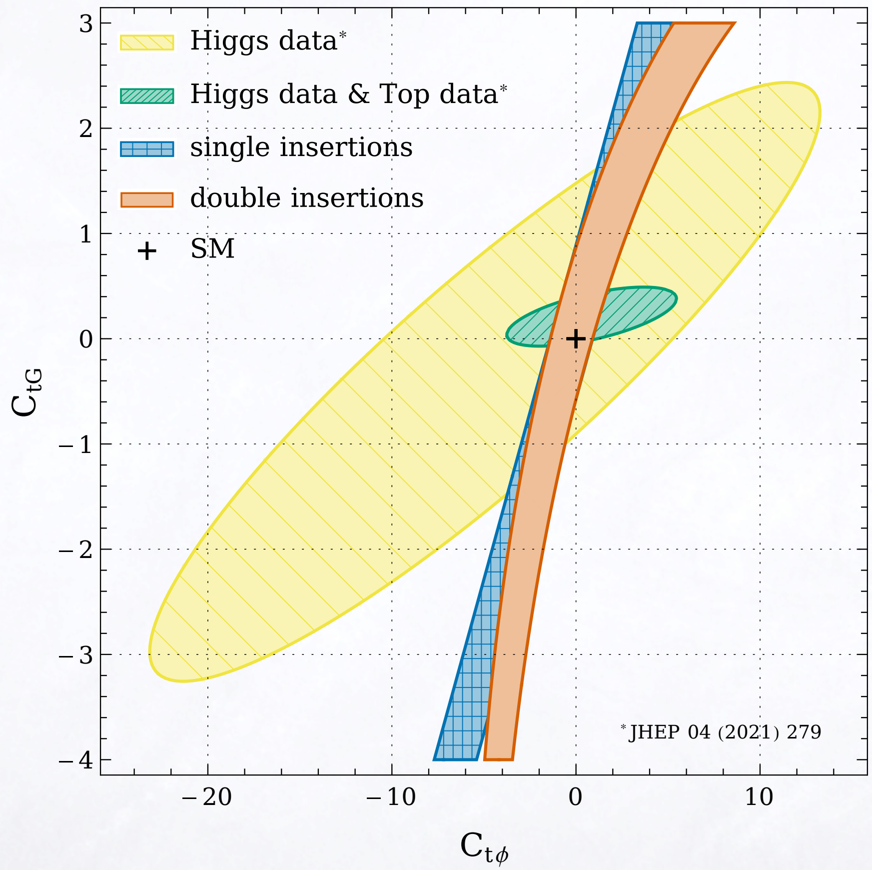
- Let us consider two WCs at a time (varying all others within 95% CL fits to individual coefficients)



- The black cross shows limits from global fits to individual operators at 95% CL [Ellis et al, 2012.02779]
- Although differences between single and double approaches can in general be found, they are phenomenologically negligible

- The parameter space allowed by global fits in the case of C_{tG} vs $C_{t\varphi}$ is richer:

[Ellis et al, 2012.02779]



- If only Higgs data is considered, the double insertions could have a certain impact
- The inclusion of top data, however, renders that impact negligible

- In SMEFT, it is time to ascertain the importance of effects beyond $\mathcal{O}(1/\Lambda^2)$
- Besides dimension-8 operators, (and squared single insertions of dimension-6 operators) double insertions of dimension-6 operators contribute to $\mathcal{O}(1/\Lambda^4)$
- I discussed the role of double insertions in gluon-gluon Higgs production at one-loop:
  - I derived the relevant Feynman rules, as well as the UV counterterm
  - For the numerical analysis, I set the WC $C_{\varphi G}$ to zero, which is well motivated
  - The analysis is not a complete result at $\mathcal{O}(1/\Lambda^4)$, but just an illustration
  - Given current constraints from global fits, the impact of double insertions is neglegible
- For the future:
  - Considering $C_{\varphi G}$ as non-zero (requires two-loop virtual amplitudes and real corrections)
  - Include dimension-8 operators, so as to have a complete result at $\mathcal{O}(1/\Lambda^4)$
  - Perform similar analyses for other LHC Higgs processes, such as $h \to \gamma\gamma$