Status and challenges of SGWB search with LISA

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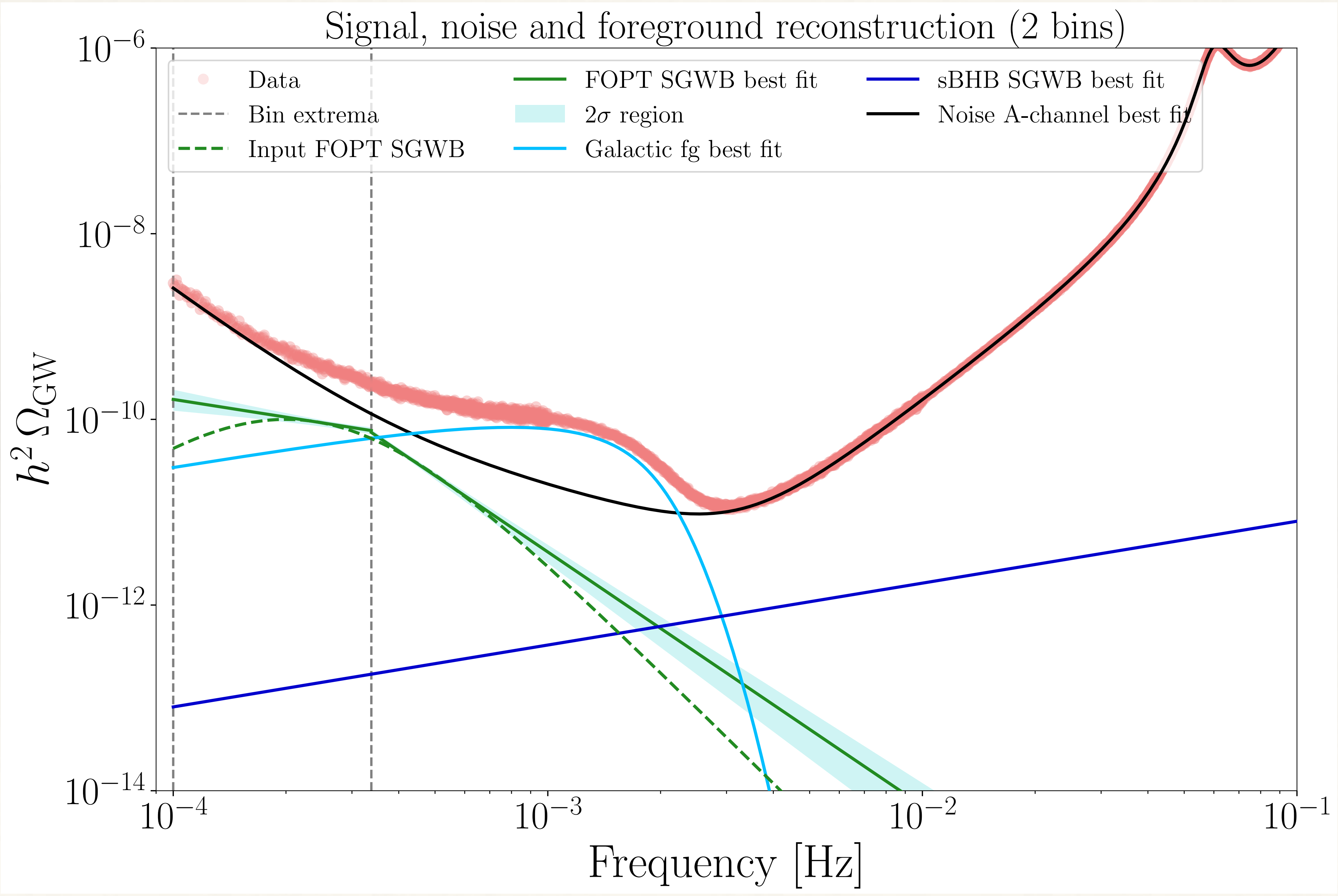


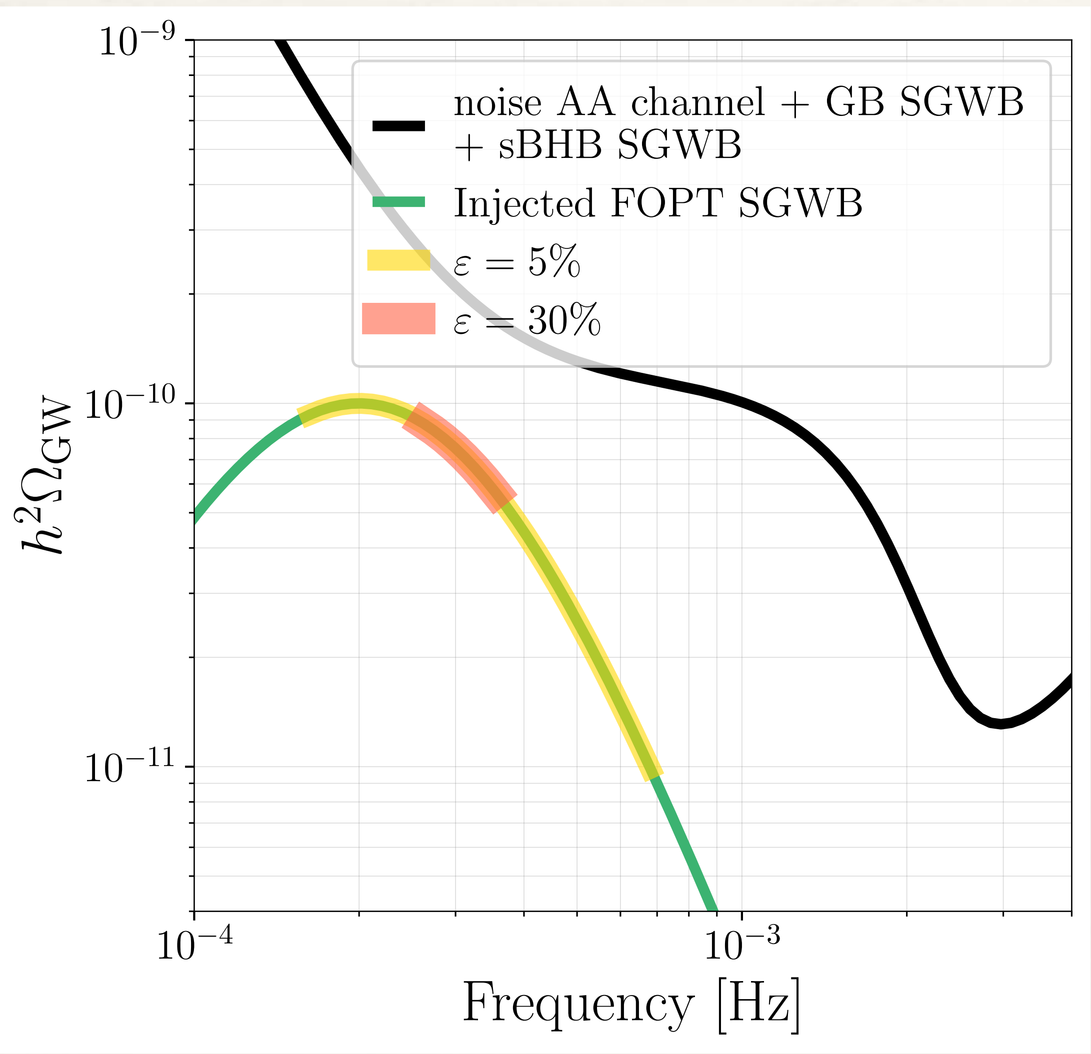
Talk outline

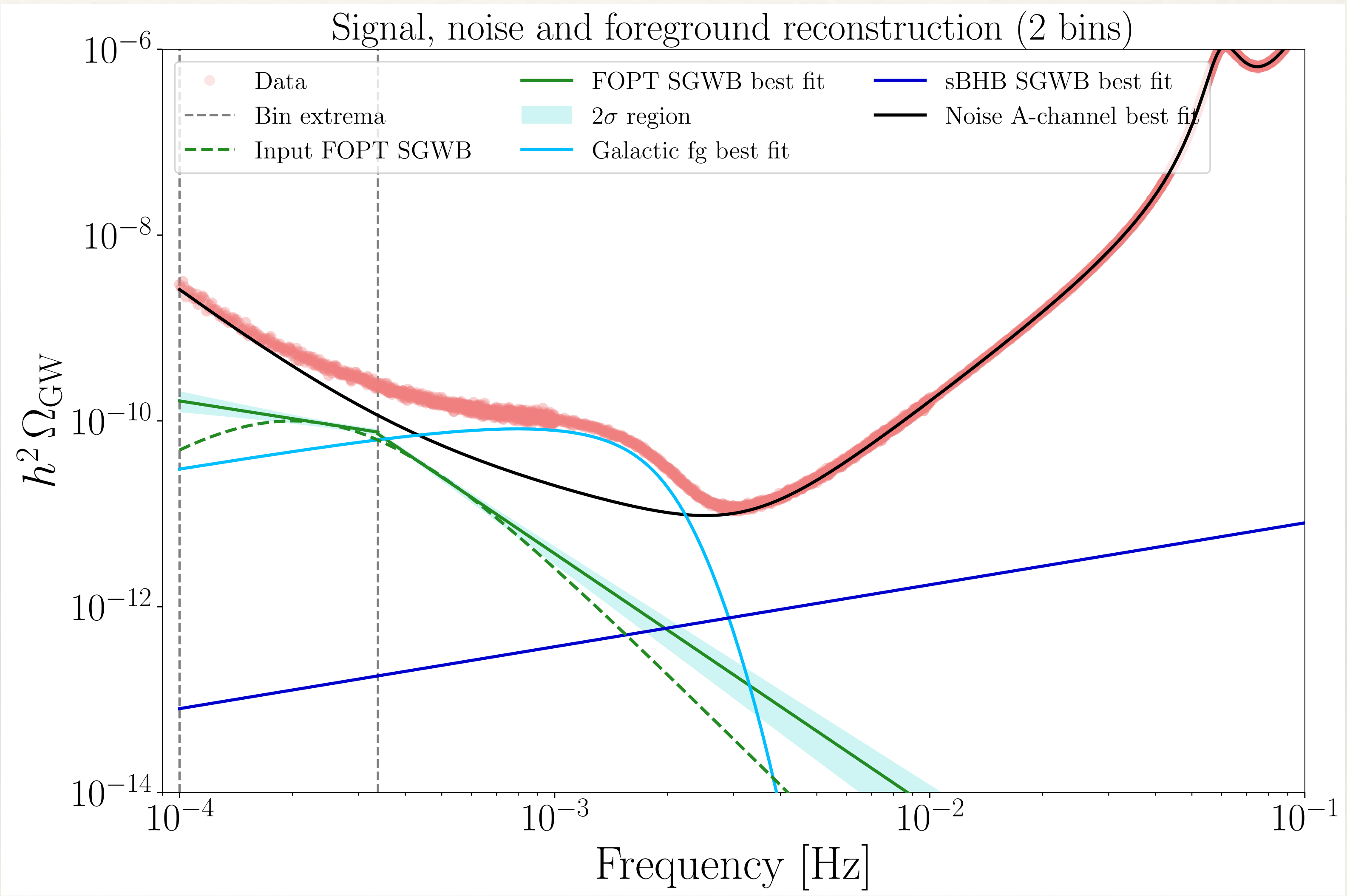
- LISA mission status
- Parameter estimation for SGWBs with LISA
- Status of LISA SGWB characterisation
- Some outstanding challenges
  - Lack of noise knowledge
  - Astrophysical foregrounds
  - Source confusion
  - Instrumental data gaps and glitches

LISA mission status

- LISA expected to be adopted in **January 2024**. Red Book, Science Management Plan and Science Implementation Requirements Document being prepared now.
- Red Book (among other things) describes science objectives of mission. There are three relating to SGWB
  - **Characterise the astrophysical SGWB**
    - What is the amplitude and spectral tilt of the astrophysical SGWB?
    - What does it tell us about its source population?
  - **Measure, or set upper limits on, the spectral shape of the cosmological SGWB**
    - Is there a SGWB of stochastic origin in the LISA data?
    - Can we reconstruct the cosmological SGWB spectral shape, to gather information about the process generating it?
  - **Characterise the large-scale anisotropy of the SGWB**
    - Is the SGWB frame the same as the SMB one?
    - What are the host galaxies of sBHBs?

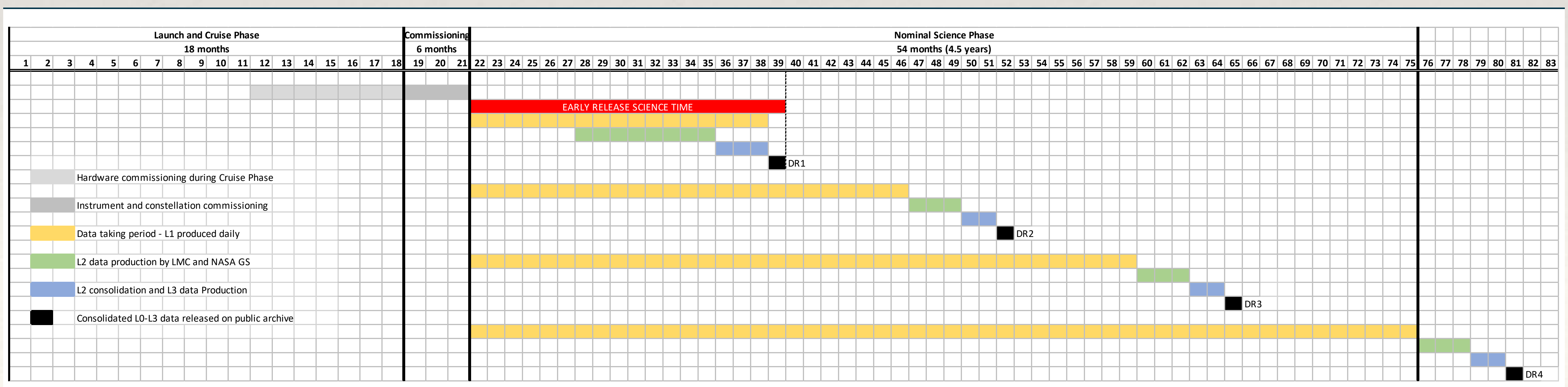
LISA mission status





LISA mission status

- SMP will stipulate (TBC) that there is an *initial closed data period* of ~18 months. After that data will be released regularly (~once per year).
- Science on closed data will be done by *Science Topical Panels* focussing on specific science objectives.
- All data releases will include both TDI data and catalogues. After DR1, these will be accompanied by minimal science interpretation.

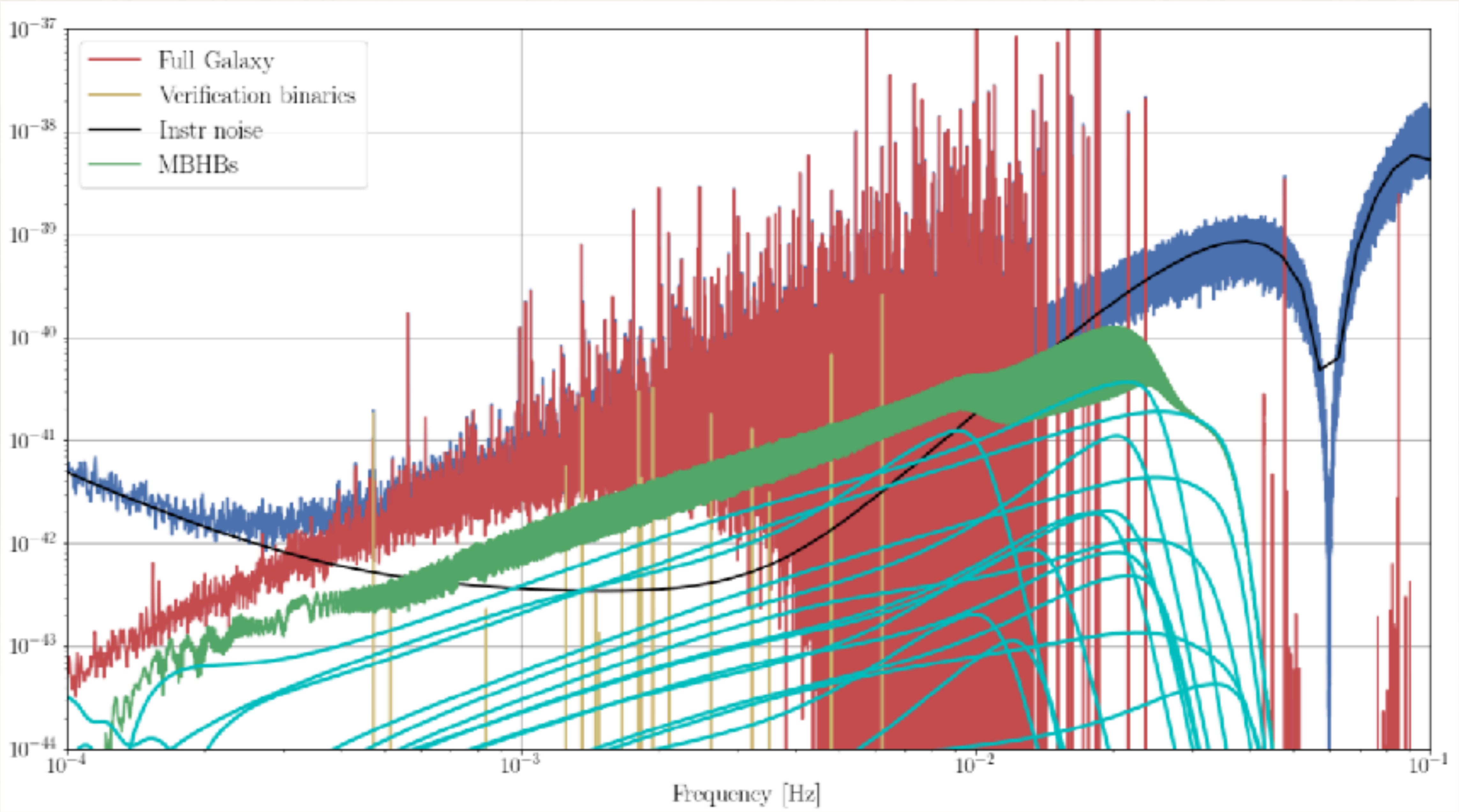


LISA mission status

- Catalogues will include
  - parameter posteriors for all resolvable sources;
  - a description of the stochastic component of the data streams;
  - (at best) minimal separation into instrumental, astrophysical and cosmological components.
- Current work on LISA data processing focuses on building the pipelines that will be used to construct the catalogues.
- SGWB community should
  - prepare for science interpretation of measured backgrounds;
  - lay groundwork for participation in STP(s) and in exploitation of open data;
  - work with data analysts to ensure data products and associated tools produced by the ground segment are suitable for subsequent SGWB analysis.

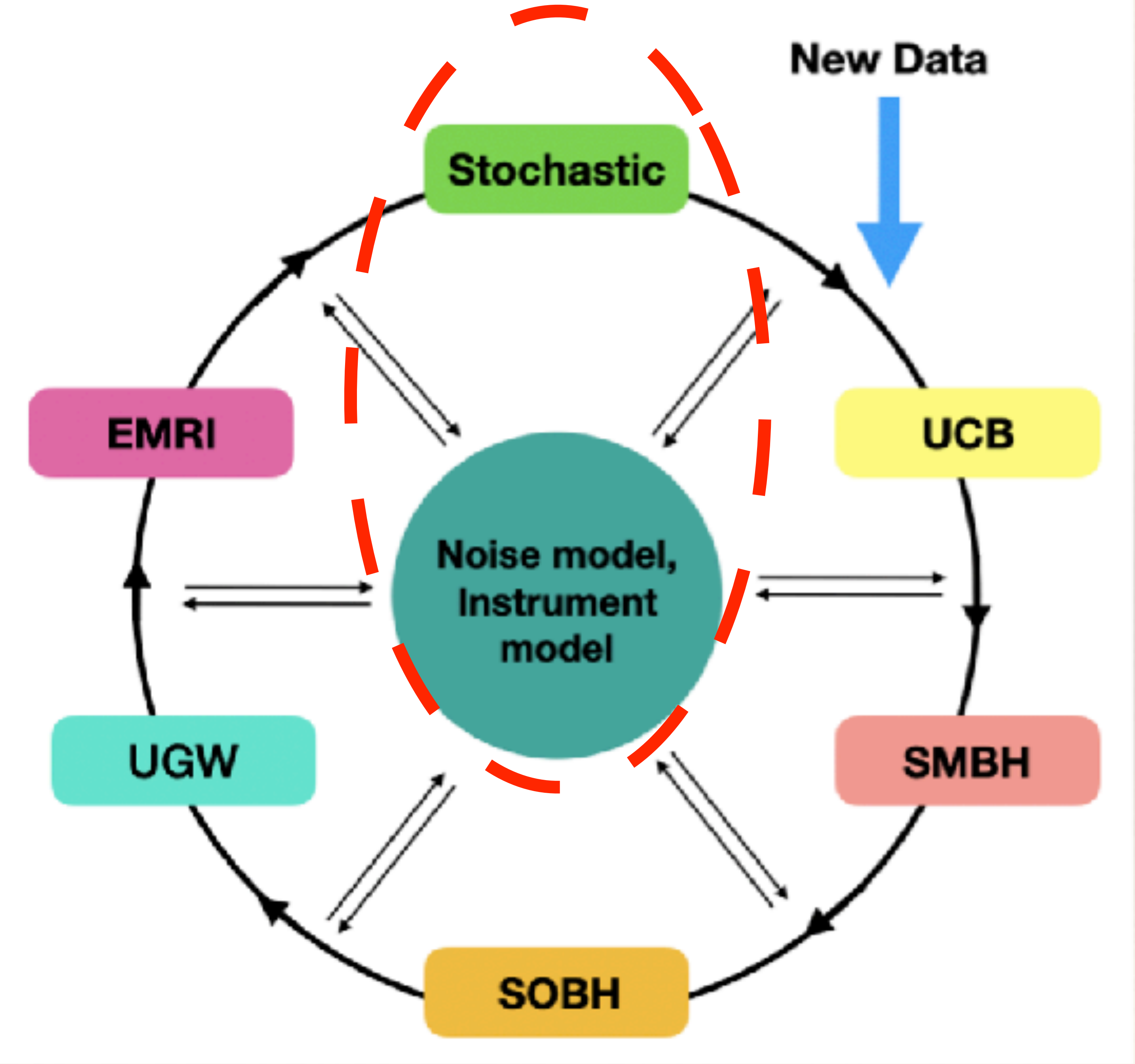
LISA Data Analysis

- LISA Data Analysis requires a *Global Fit* of an unknown number of sources of all of these different types (see Cornish talk).



LISA Data Analysis

- Typical strategy adopted is to iteratively update the solution for one source type and then move to the next.
- Solution will be continuously refined as new data is added.
- A key component of the analysis is *variable dimensionality*.
- SGWB and instrumental noise are components of the global fit.



SGWB characterisation: likelihood

- One channel (Whittle) likelihood can be written

$$\ln(p(\mathbf{d}|\vec{\theta},\vec{\lambda})) = -\frac{1}{2}\sum_{k=1}^{n_f}\left[\frac{|\tilde{d}_k - \tilde{h}_k(\vec{\theta})|^2}{S_h(f_k|\vec{\lambda})}\mathrm{d}f + 2\ln S_h(f_k|\vec{\lambda})\right]$$

- With multiple channels (see, e.g., Adams & Cornish 2014)

$$p(\mathbf{d}|\vec{\lambda}) = \prod_k \frac{1}{\sqrt{(2\pi)^{2\times N_c}\det(\Sigma(f_k)|\vec{\lambda})}} e^{-\frac{1}{2}\tilde{\mathbf{d}}(f_k)^T\Sigma^{-1}(f_k|\vec{\lambda})\tilde{\mathbf{d}}(f_k)}$$

- Can do maximum-likelihood estimation, or recover parameters of the background in a Bayesian analysis

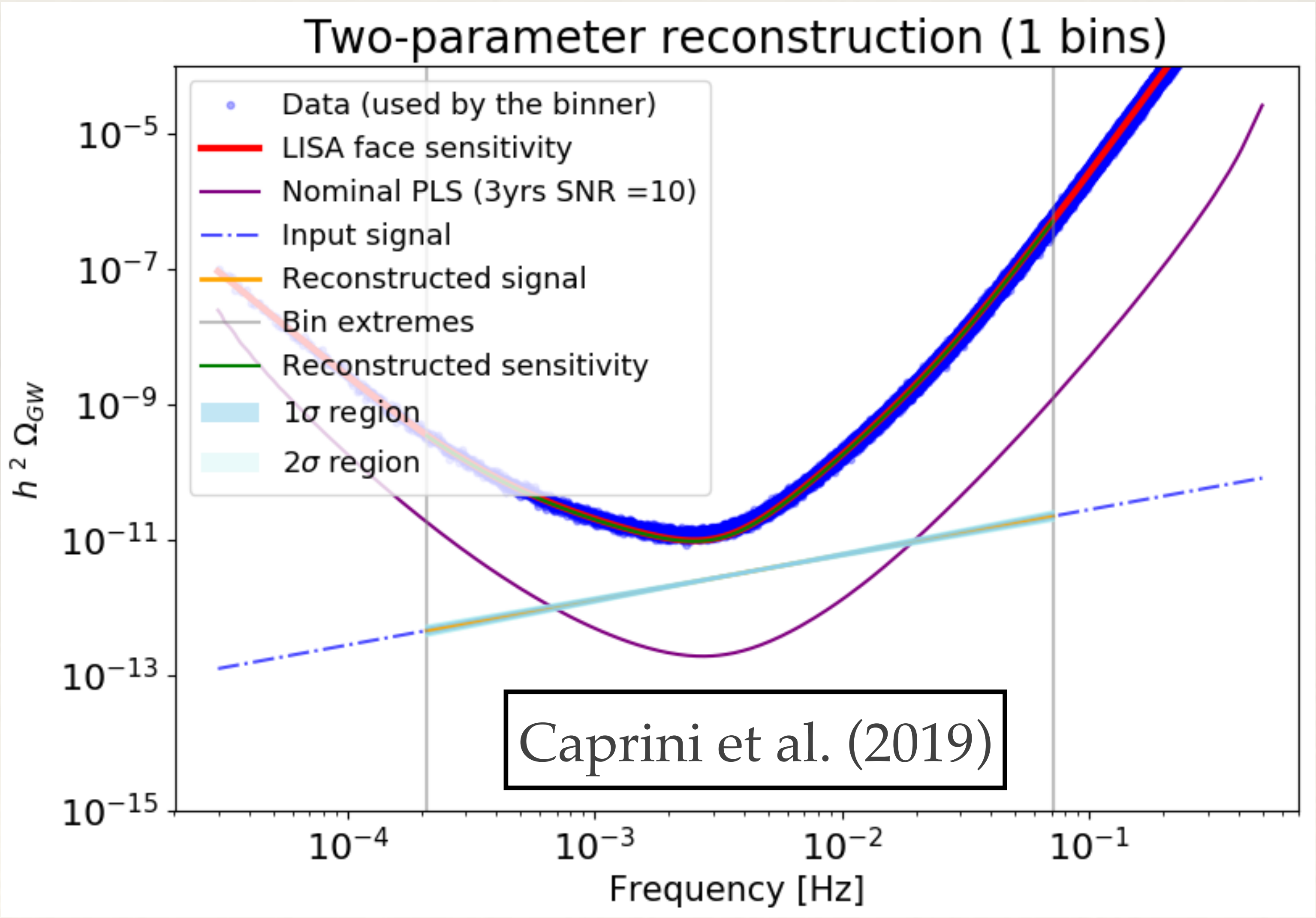
$$p(\vec{\lambda}|\mathbf{d}) = \frac{p(\mathbf{d}|\vec{\lambda})p(\vec{\lambda})}{p(\mathbf{d})}$$

SGWB characterisation: unmodelled

$$\mathcal{L}(\vec{s},\vec{n}) \propto \exp\left(-N_{\rm chunks}\sum_i \frac{1}{2}\left[\frac{\bar{D}_i - h^2\Omega_{\rm GW}(f_i,\vec{s}) - h^2\Omega_s(f_i,\vec{n})}{\sigma_i}\right]^2\right)$$

$$h^2\,\Omega_{\rm GW}(f,\,\vec{s}_i) = 10^{\alpha_i}\left(\frac{f}{\sqrt{f_{\min,i}\,f_{\max,i}}}\right)^{p_i}\theta\,(f-f_{\min,i})\,\theta\,(f_{\max,i}-f)$$

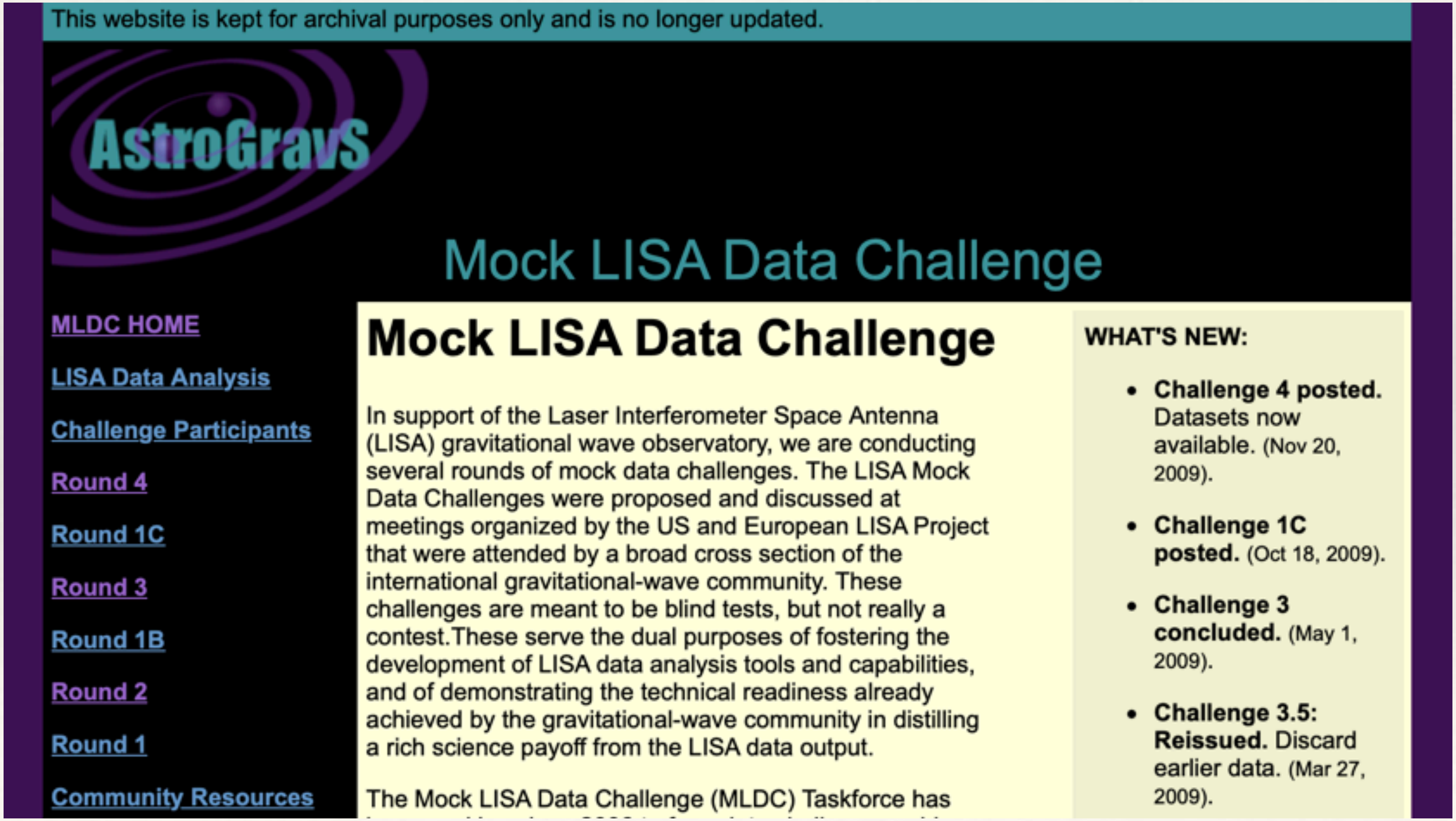
- *SGWBinner* uses a flexible model for the signal component of the PSD, combined with a simple 2-parameter model for the instrumental noise.
- Number of bins used for reconstruction allowed to vary to find optimal fit complexity.
- Successfully able to reconstruct a wide variety of background signals.



Caprini et al. (2019)

Mock LISA Data Challenges

- MLDCs established in 2006 to demonstrate readiness for LISA data processing. Discontinued after Round 4, in 2010.



Mock LISA Data Challenge 3.5

- MLDC data set 3.5 included a stochastic background

$$\langle \tilde{h}^*_A(f,\hat{\Omega})\tilde{h}_{A'}(f',\hat{\Omega}')\rangle = \frac{3H_0^2}{32\pi^3}|f|^{-3}\Omega_{\rm gw}(|f|) \times \delta_{AA'}\delta(f-f')\delta^2(\hat{\Omega},\hat{\Omega}')$$

- with constant energy density and unknown amplitude.
- LISA response approximated by a rigidly-rotating triangle with equal constant arm lengths. Single link optical and test mass acceleration noises were uncertain at +/- 20%.

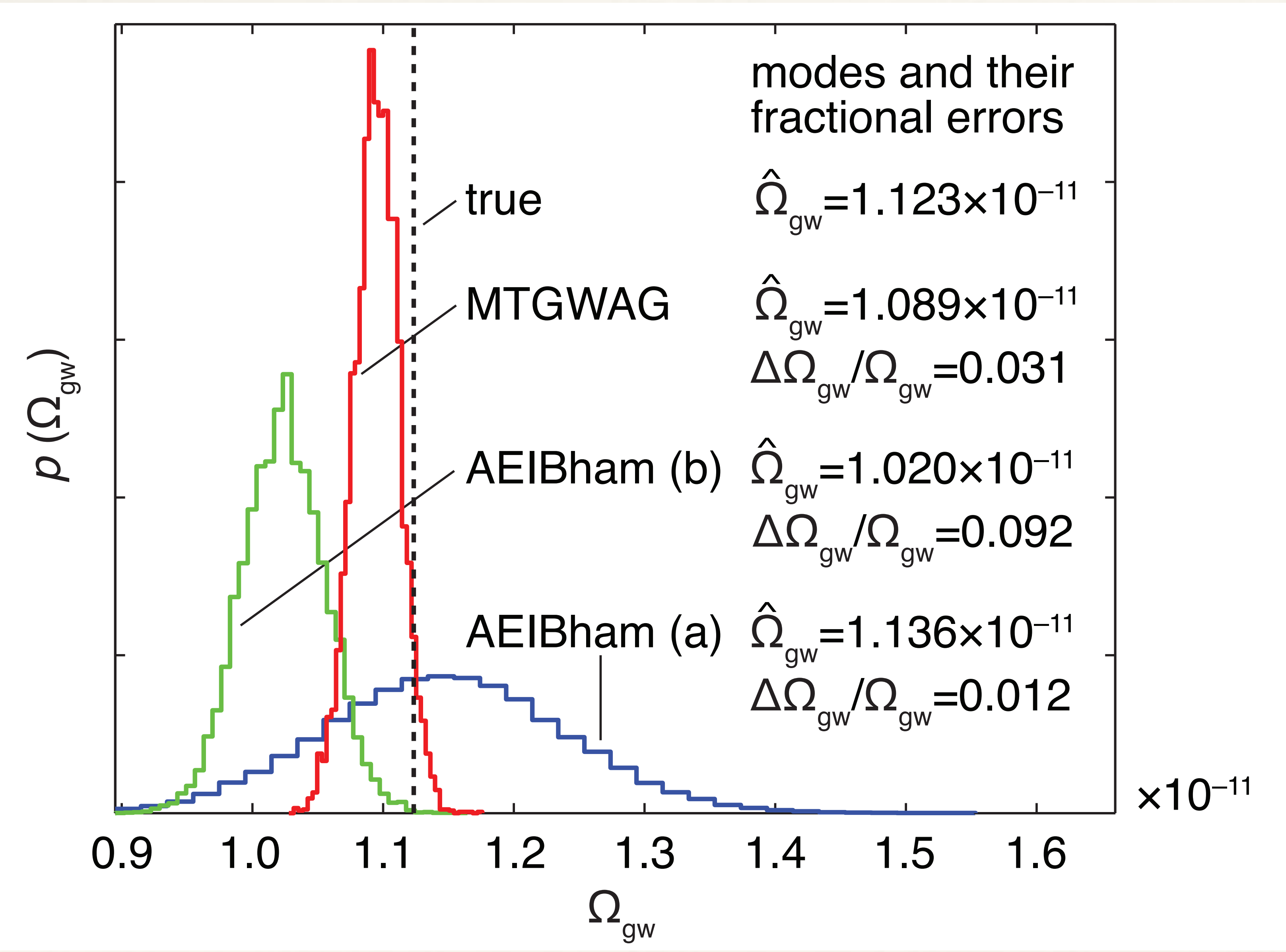
$$\eta^{\rm N}_{ij}(t) = n^{\rm opt}_{ij}(t) + D_{ij} n^{\rm acc}_{ji}(t) + n^{\rm acc}_{ij}(t)$$

$$S_{\rm acc}(f) = 2.5 \times 10^{-48} (f/{\rm Hz})^{-2} [1 + (10^{-4}\,{\rm Hz}/f)^2]\,{\rm Hz}^{-1}$$

$$S_{\rm opt}(f) = 1.8 \times 10^{-37} (f/{\rm Hz})^2\,{\rm Hz}^{-1}.$$

Mock LISA Data Challenge 3.5

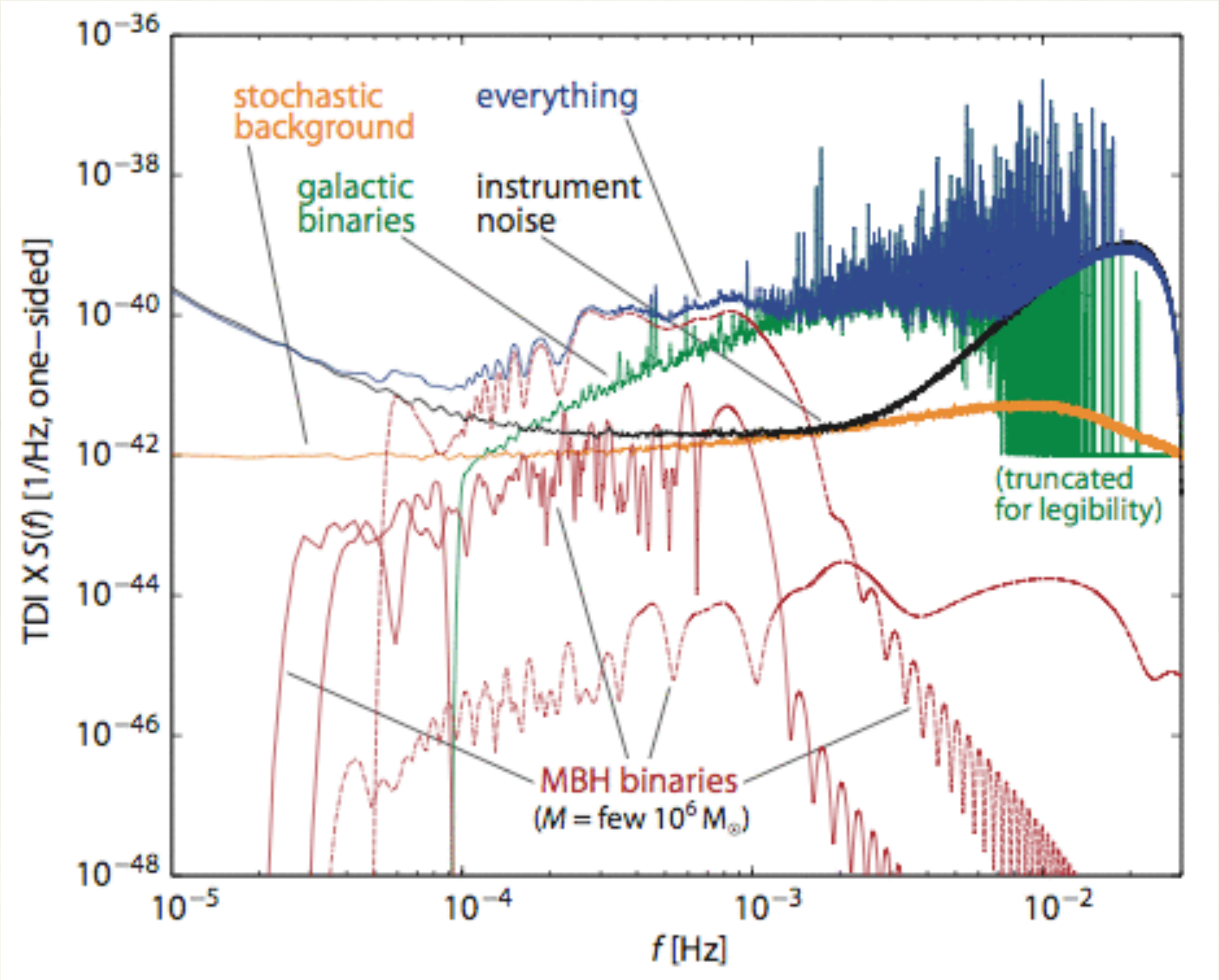
- Two groups analysed the data set correctly.



noise	MTGWAG/true	frac. error
$pm_1 + pm_2^*$	$6.5/4.7 \times 10^{-48}$	0.385
$pm_1^* + pm_3$	$3.2/5.4 \times 10^{-48}$	0.397
$pm_2 + pm_3^*$	$6.9/5.8 \times 10^{-48}$	0.197
$pd_1 + pd_2^*$	$3.733/3.752 \times 10^{-37}$	5.0×10^{-3}
$pd_1^* + pd_3$	$3.568/3.547 \times 10^{-37}$	6.1×10^{-3}
$pd_2 + pd_3^*$	$3.805/3.804 \times 10^{-37}$	3.6×10^{-4}

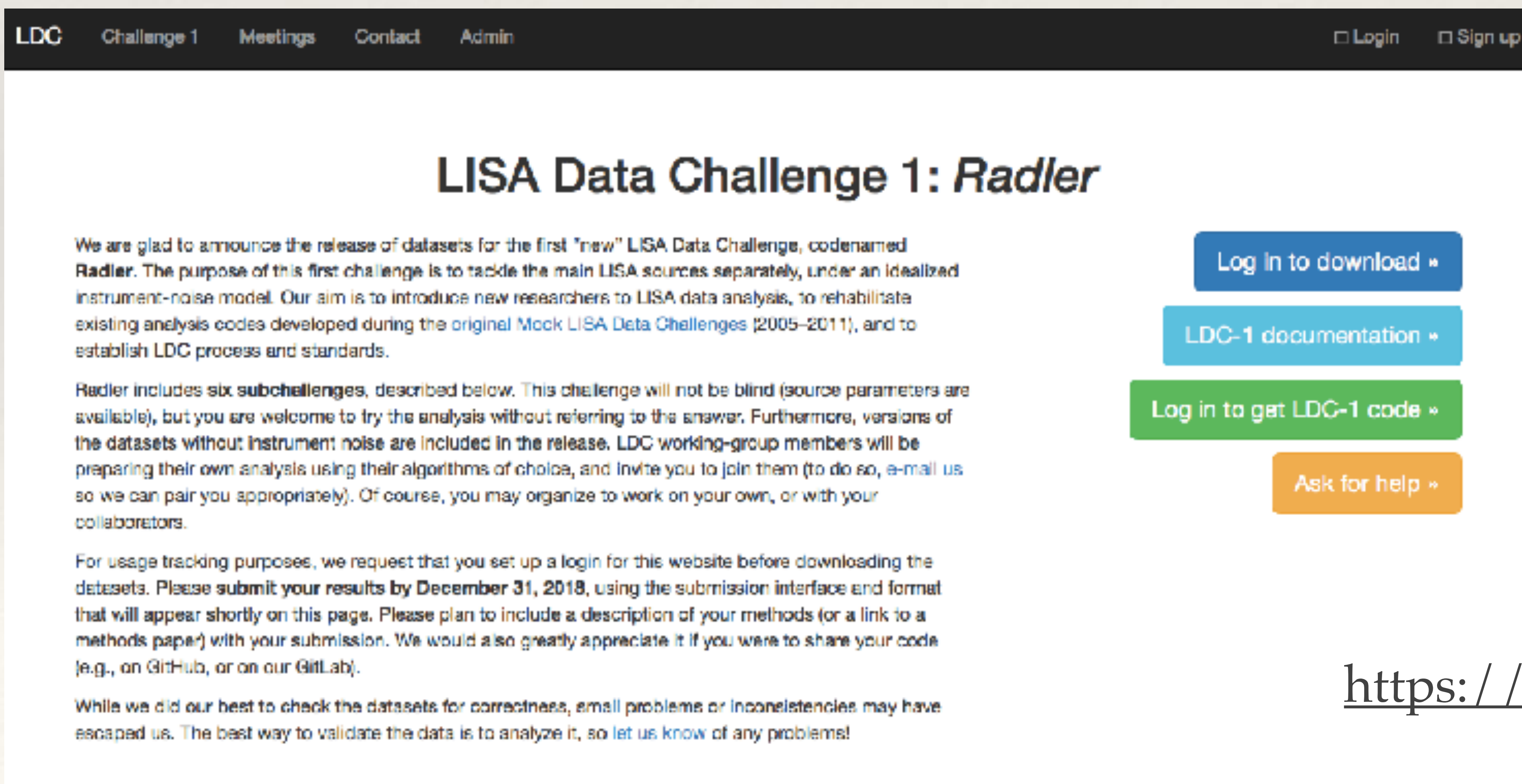
Mock LISA Data Challenge 4

- MLDC round 4 included the “whole enchilada”, but MLDCs were discontinued before submissions were fully finalised / assessed.



LISA Data Challenges

- The LDC group was established in 2018 to resume activities begun by the LISA Mock Data Challenges. There are telecons on Thursdays @ 4pm CEST.
- Ground segment pipeline development is being driven by the Data Challenges. Data sets are being constructed to address specific questions posed by the Science Group.

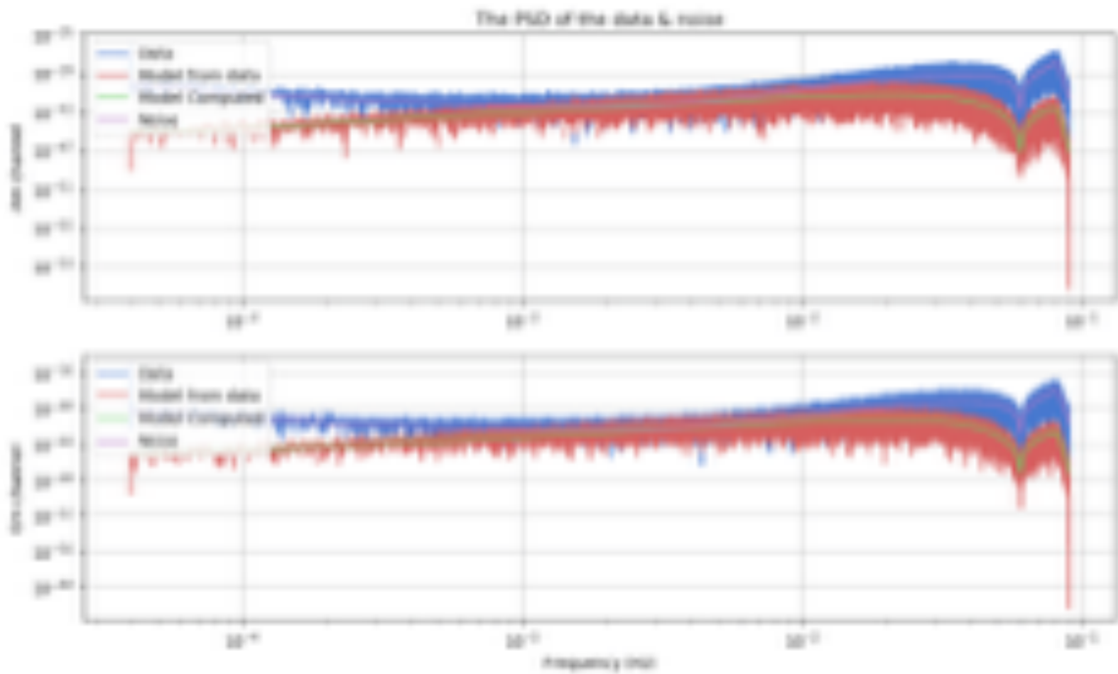


https://lisa-ldc.lal.in2p3.fr/ldc

LISA Data Challenge 1-6

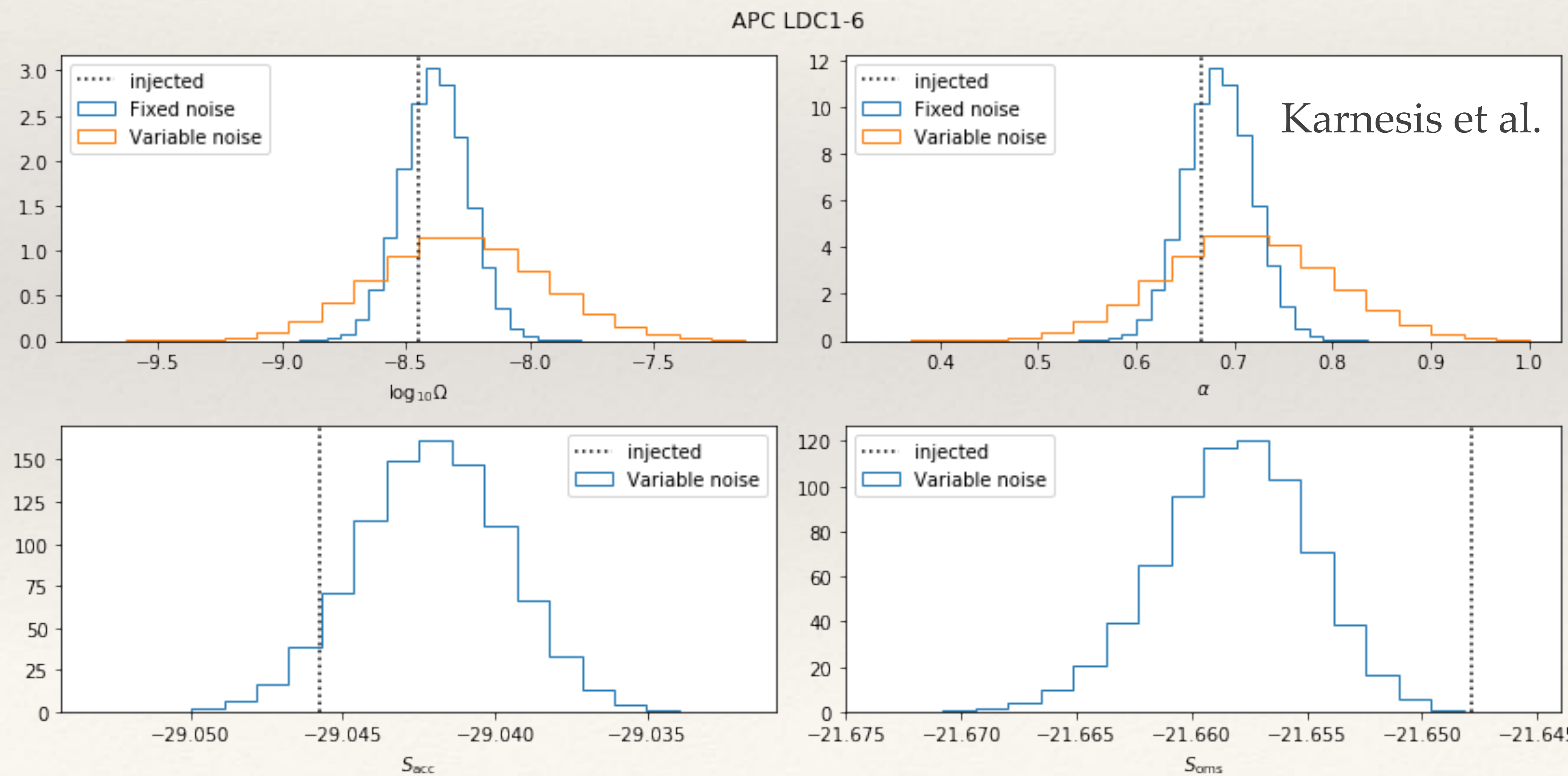
LDC1-6. An isotropic stochastic GW signal of primordial origin.

Statistics are Gaussian, but the spectral shape is shrouded in mystery, with parameters chosen for us by the LISA Consortium Cosmology Working Group. The signal is generated using LISACode as a choir of elementary sources uniformly distributed across the sky. To make things easier for you, instrumental noise is Gaussian, uncorrelated, and of the same level in each LISA link.



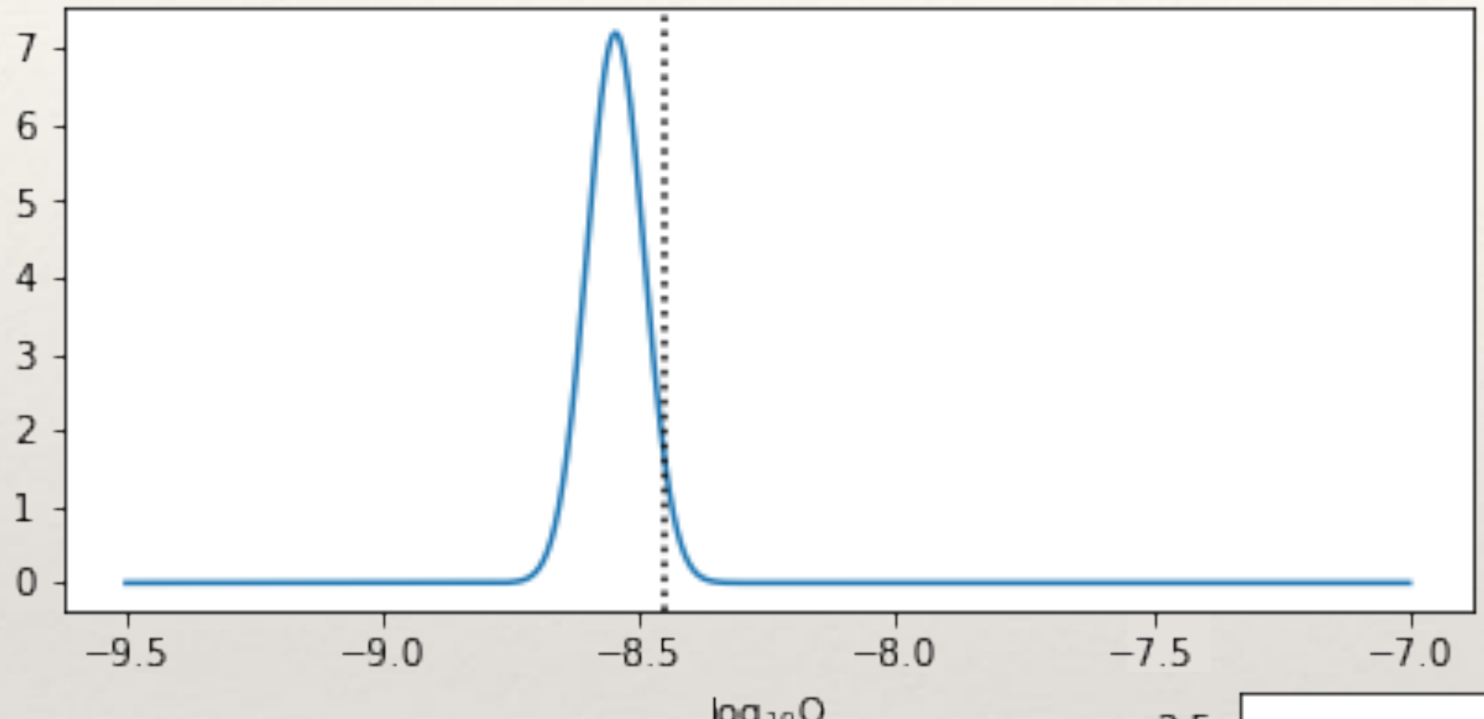
LISA Data Challenge 1-6

- Similar to MLDC 3.5. Data set versions with zero/known/unknown instrumental noise were created. Three groups successfully analysed the data.

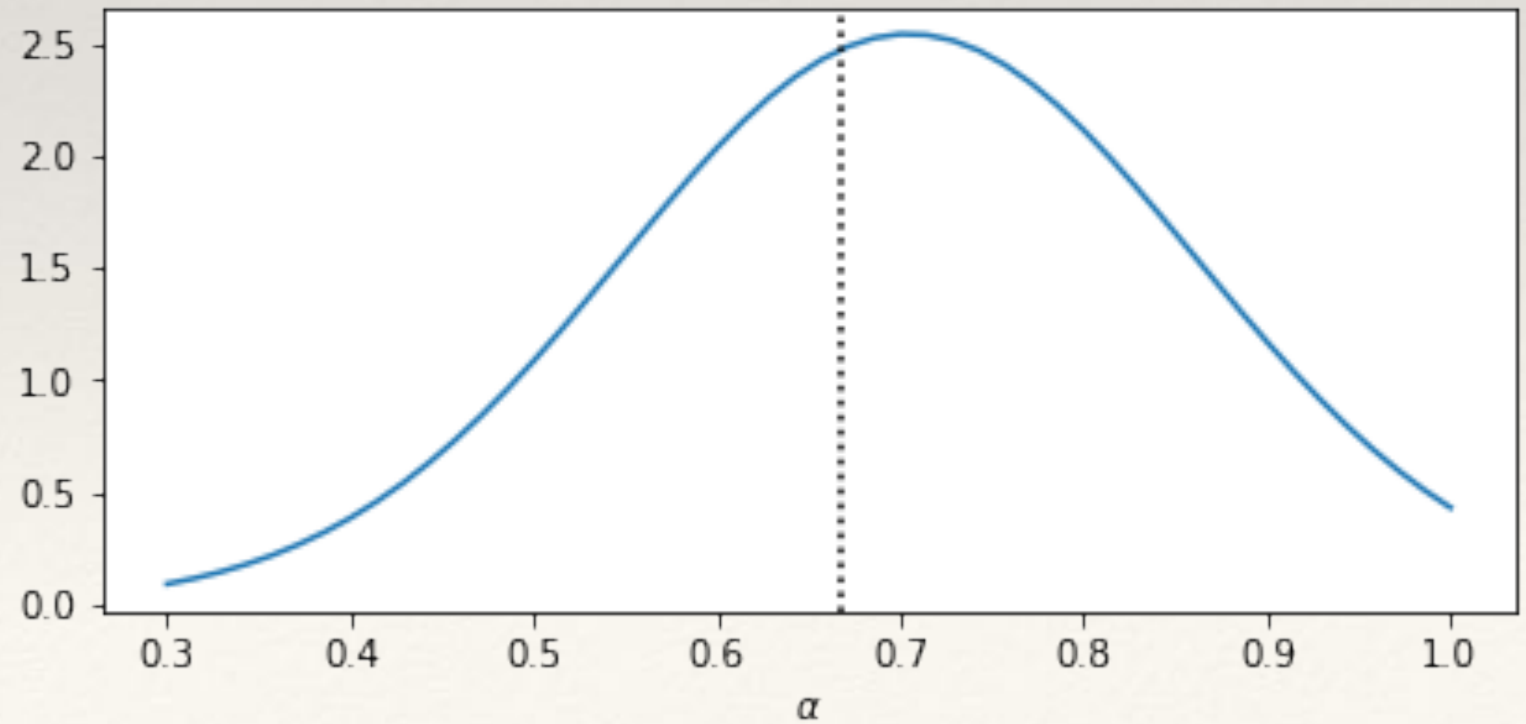


LISA Data Challenge 1-6

- One analysis used SGWBinner (Flauger et al.).

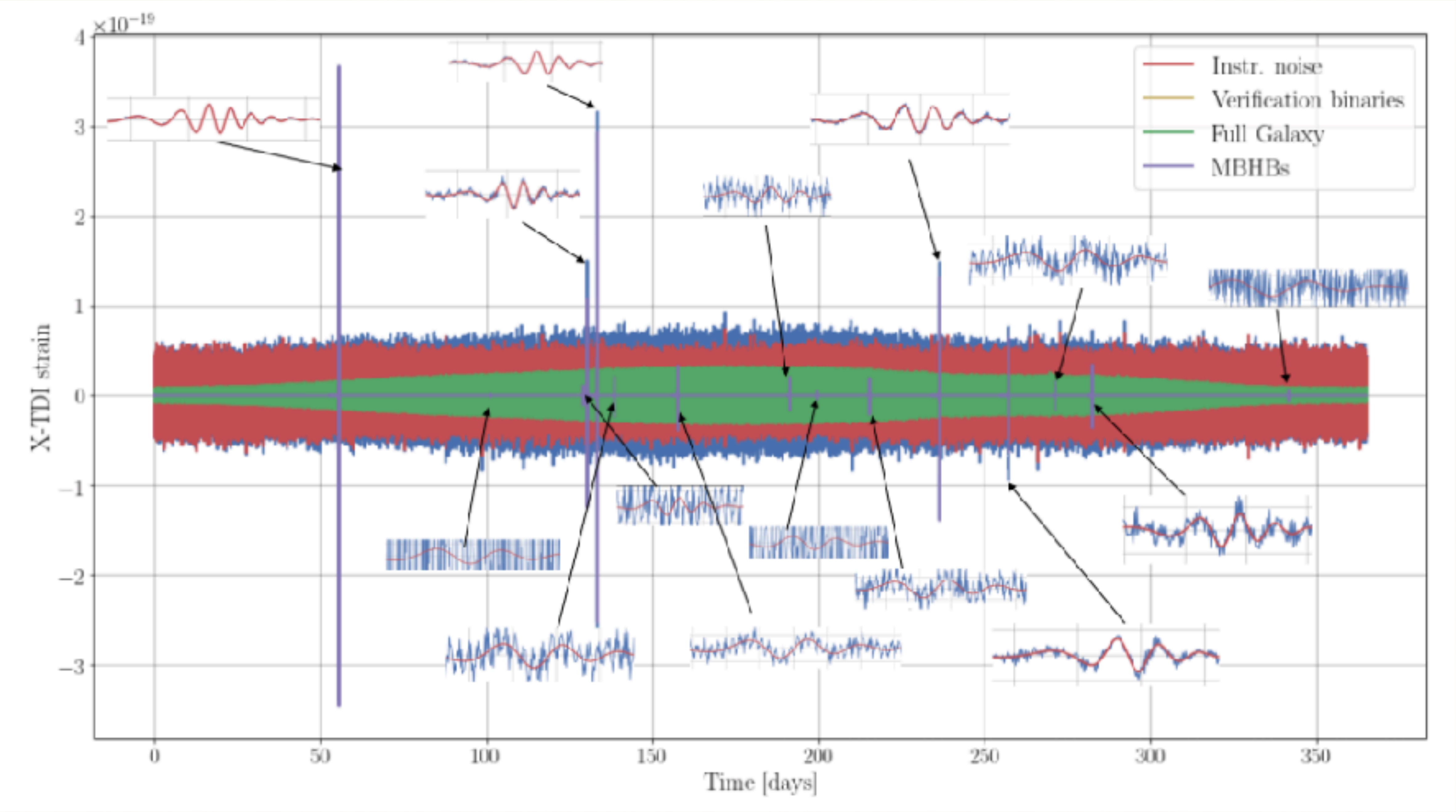


Flauger et al.

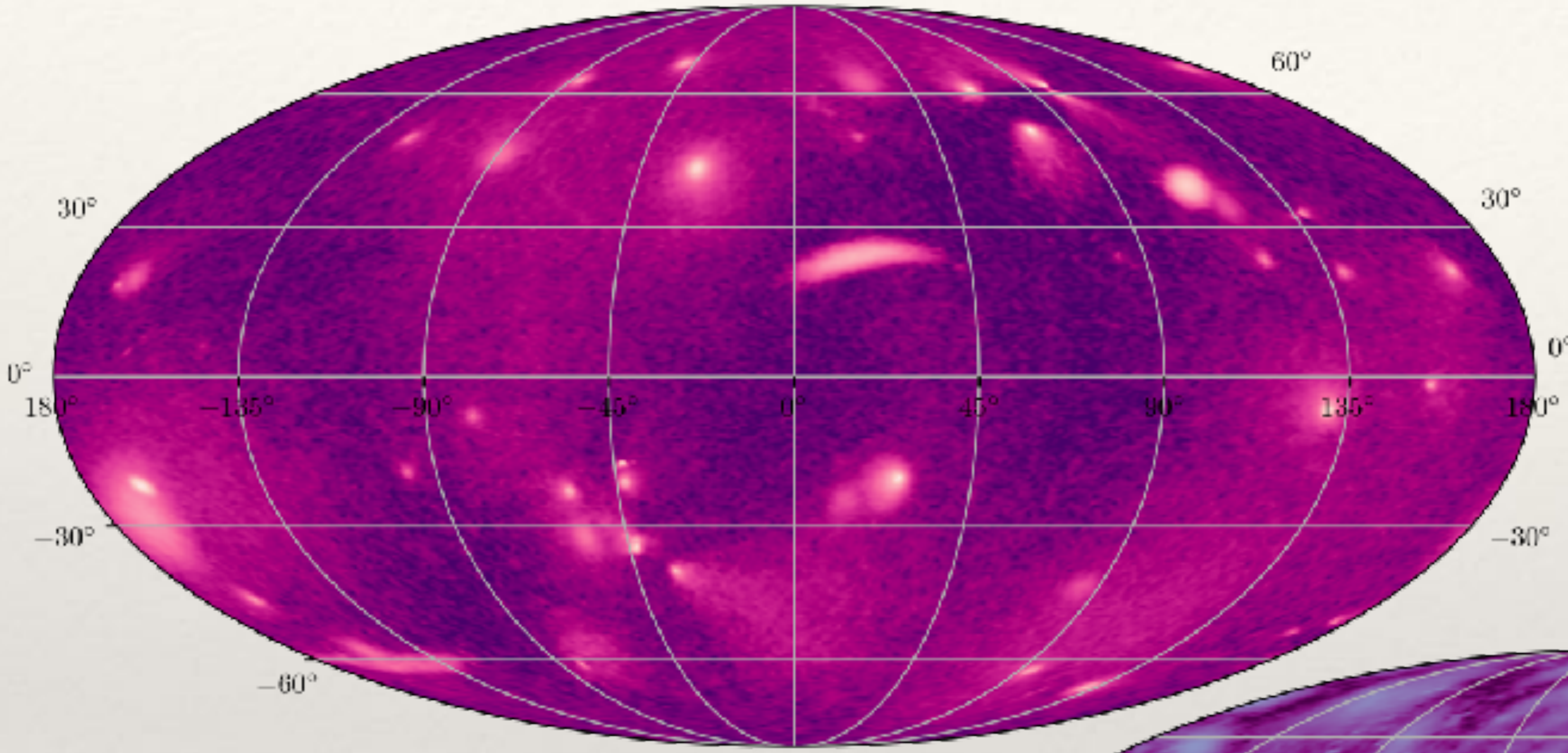


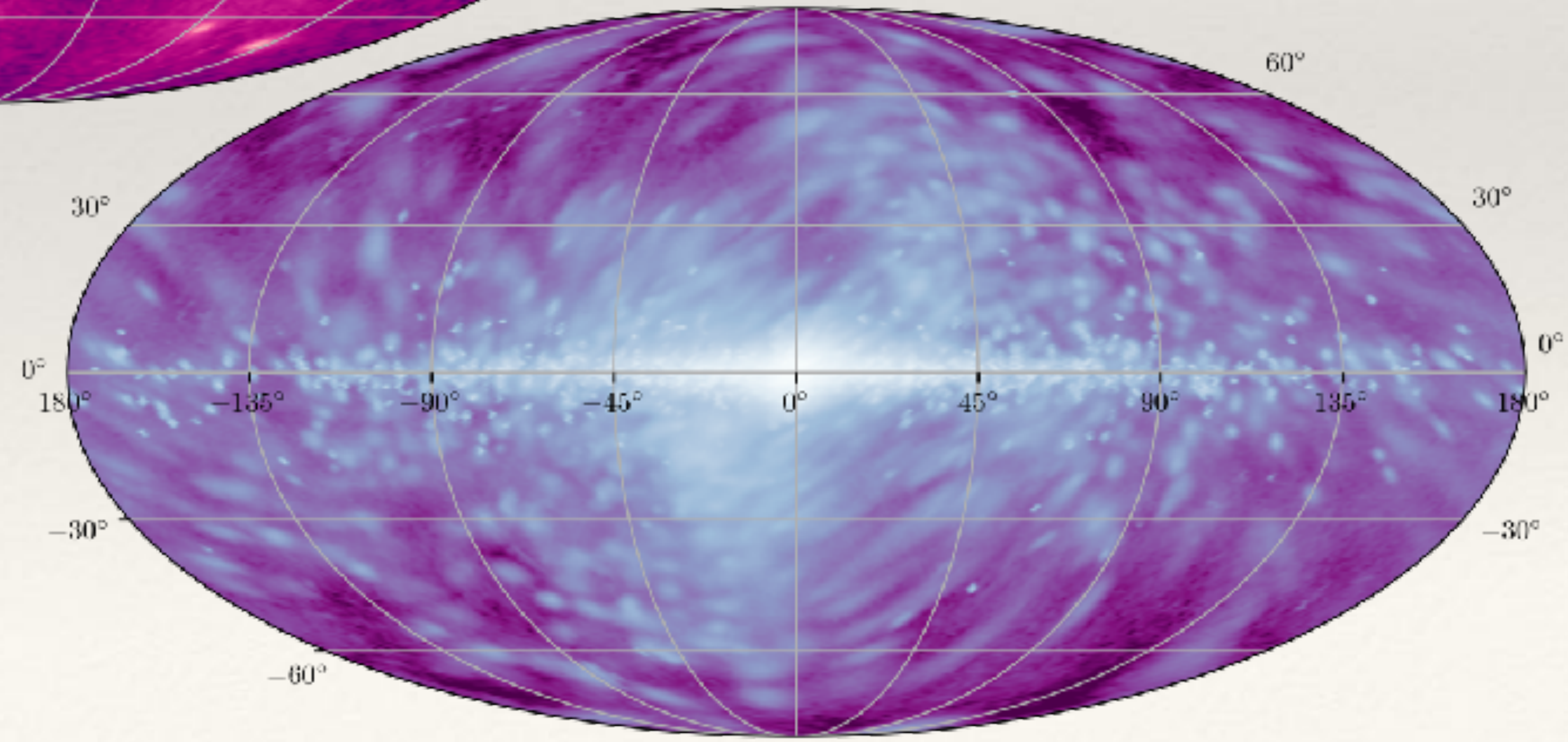
LISA Data Challenge 2a

- The *Sangria* dataset included a full galaxy of GBs, plus massive black hole binaries. The galactic binaries form a cyclo-stationary GW background.



LISA Data Analysis: state of the art

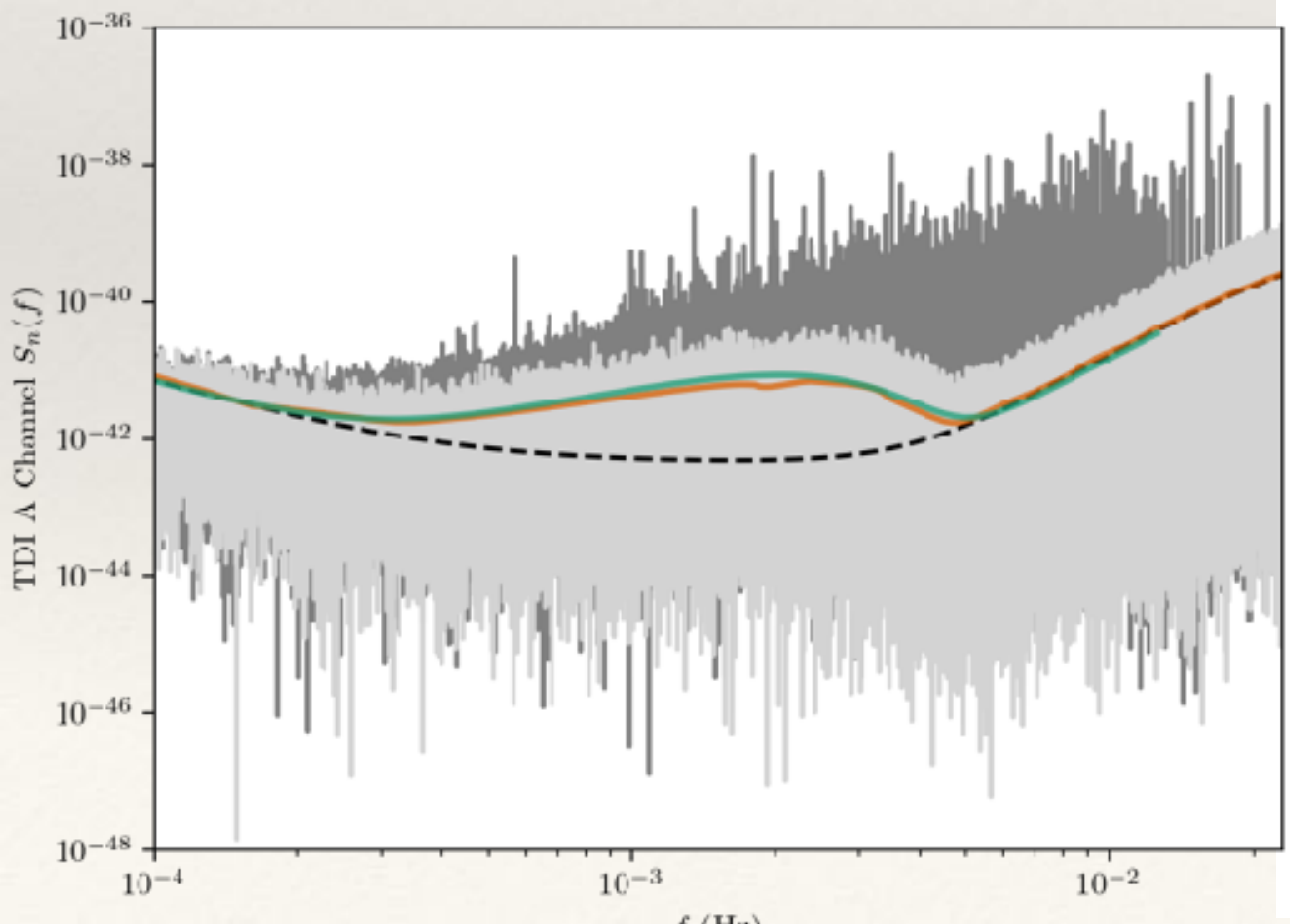


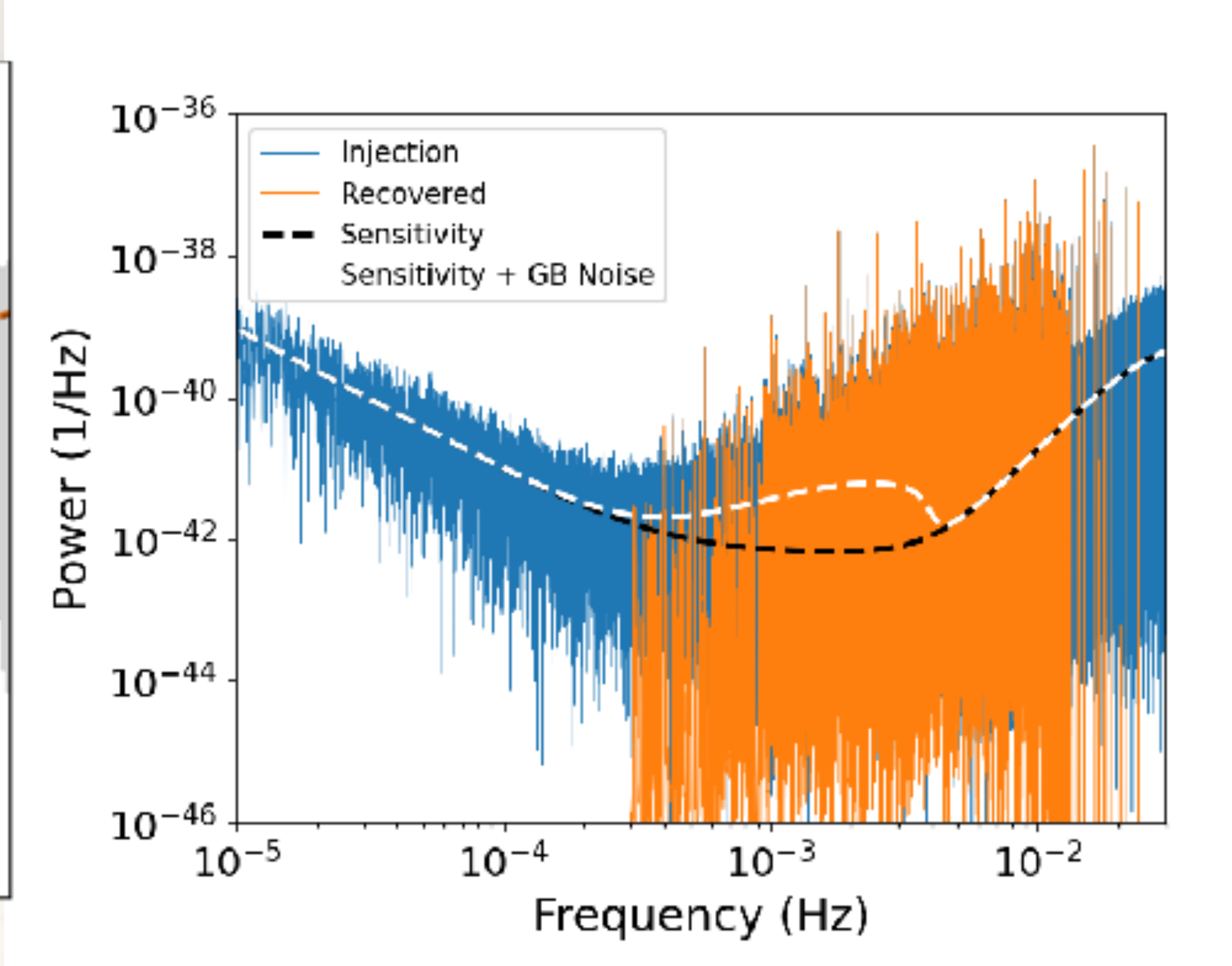


LISA Data Challenge 2a

- Several successful analyses of the LDC 2a data set.
- **Caveat**: all analyses assumed noise was stationary.

$$S_{gal}(f) = \frac{Af^{5/3}}{2} e^{-(f/f1)^{\alpha}} (1 + \tanh((f_{knee} - f)/f_2))$$

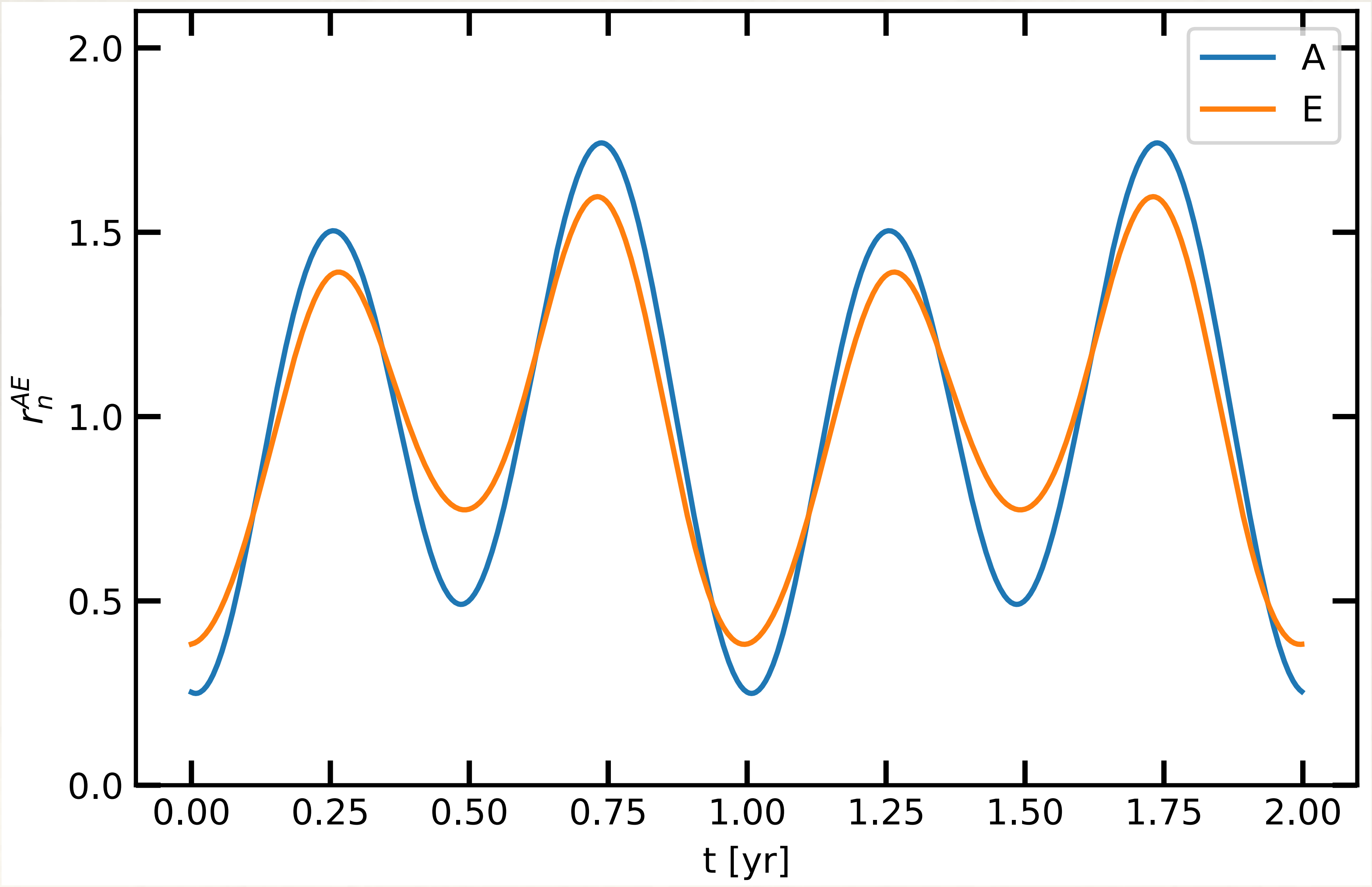




LISA Data Challenge 2a

- Digman and Cornish (2022) have extended the PSD model to allow for cyclostationarity, to incorporate into the global fit pipeline.

$$S^{AE}_{nm,cyclo} = r^{AE}_n \langle S_{m,gal} \rangle + S_{m,inst}$$



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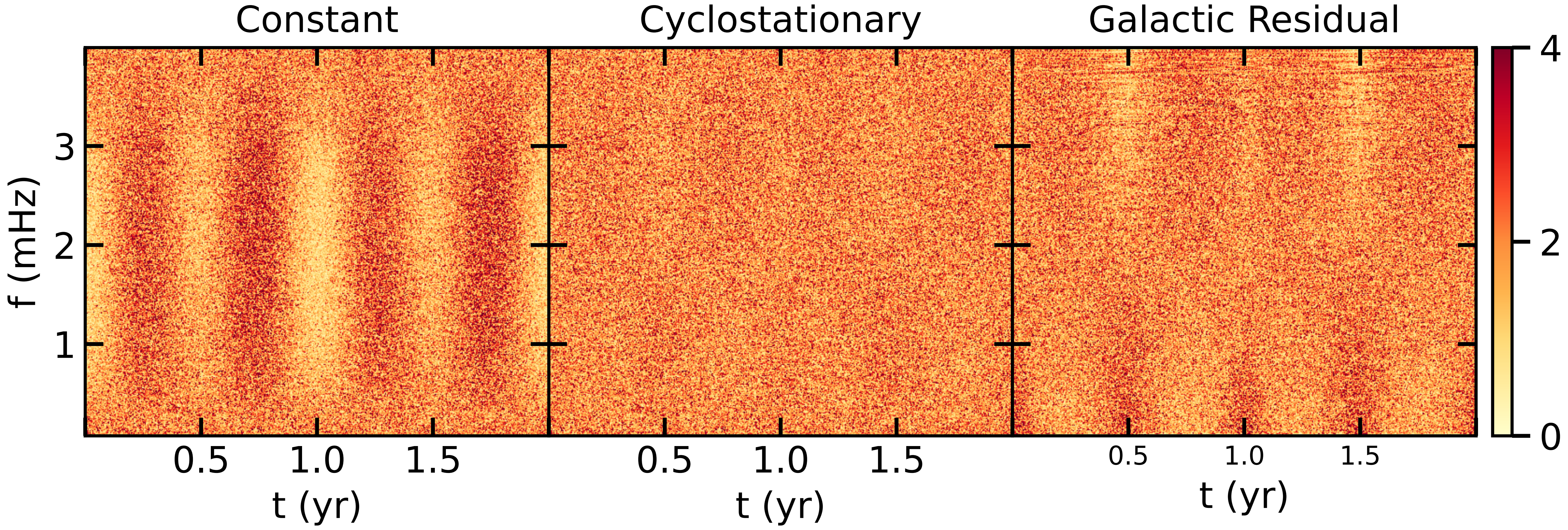
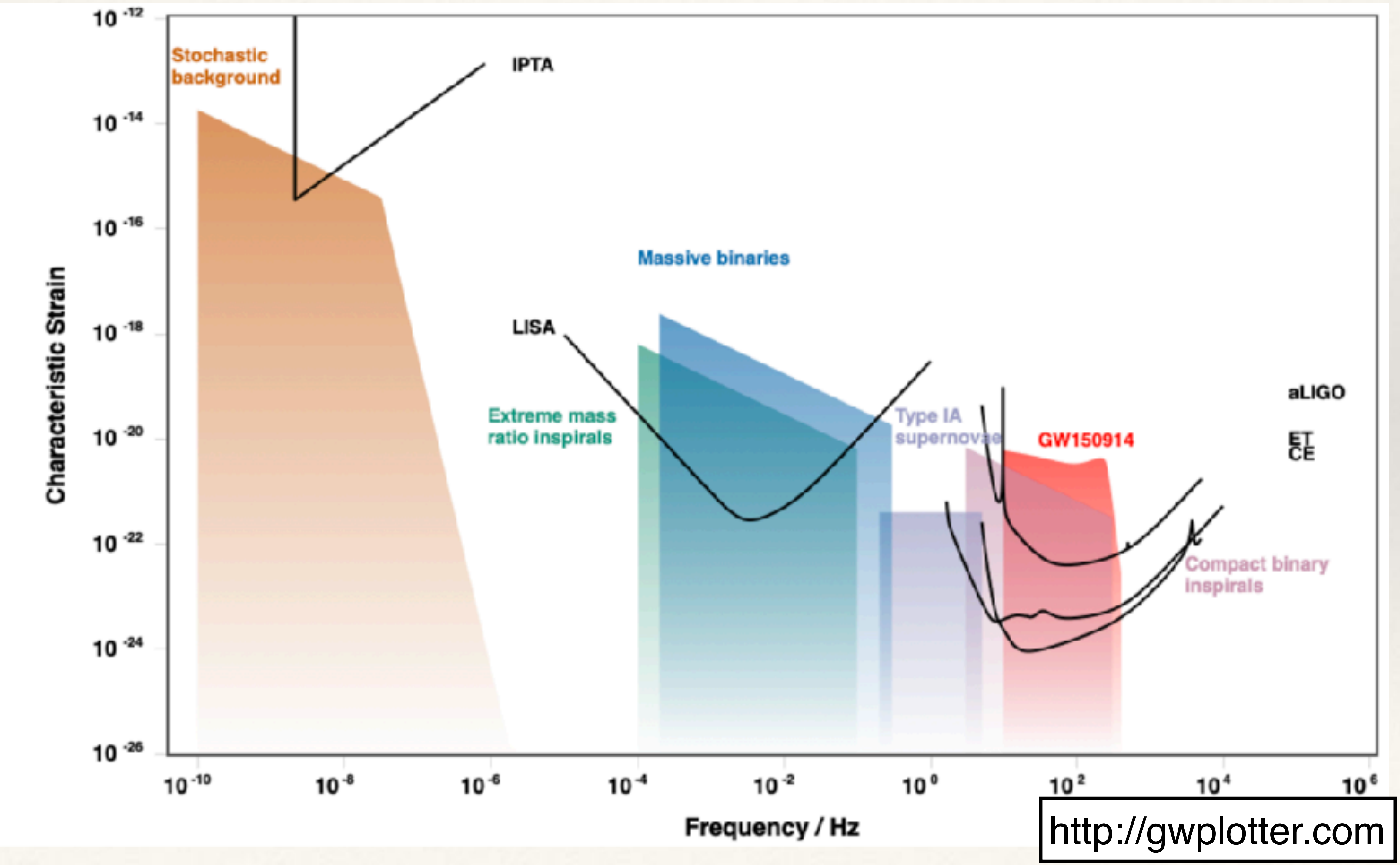


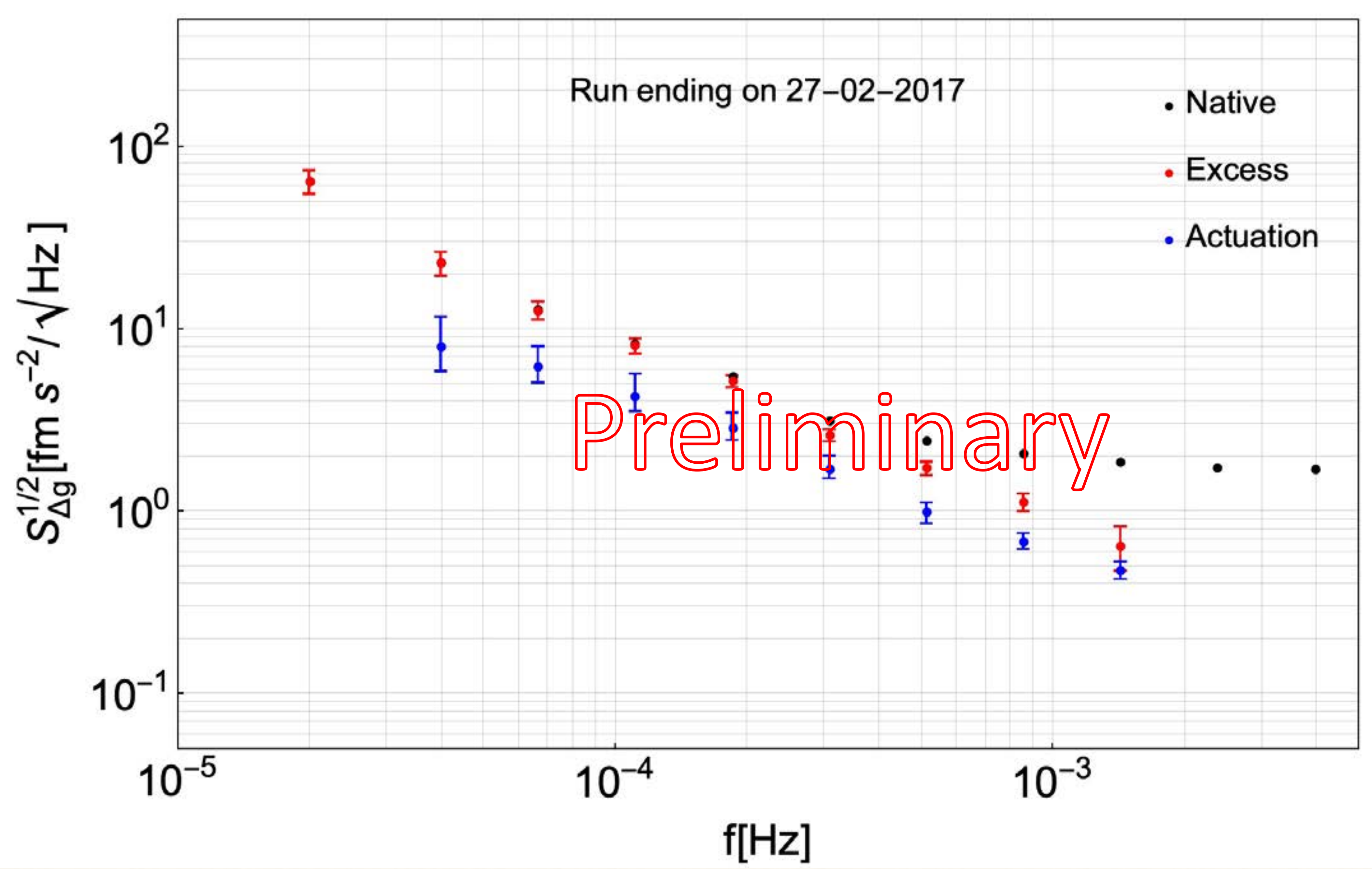
Challenges: lack of noise knowledge

- Typically assume a known sensitivity when assessing mission performance.



Challenges: lack of noise knowledge

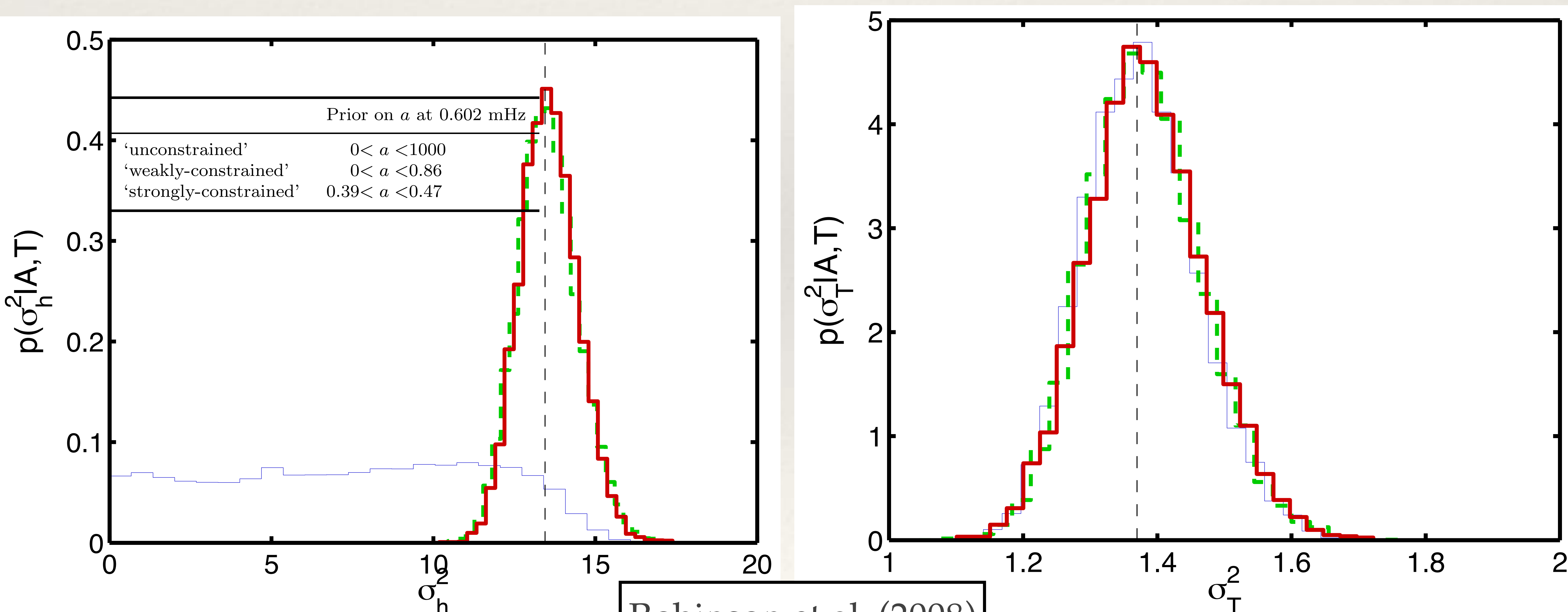
- Reality is different. In LISA Pathfinder only 25% of total noise power was explained by measured noise sources.



Solutions: T channel

- Equal-arm Sagnac channel is insensitive to gravitational waves at low frequency - use this as a noise monitor. $\tilde{A} = \tilde{n}_A + \tilde{h}\,, \qquad \tilde{T} = \tilde{n}_T$

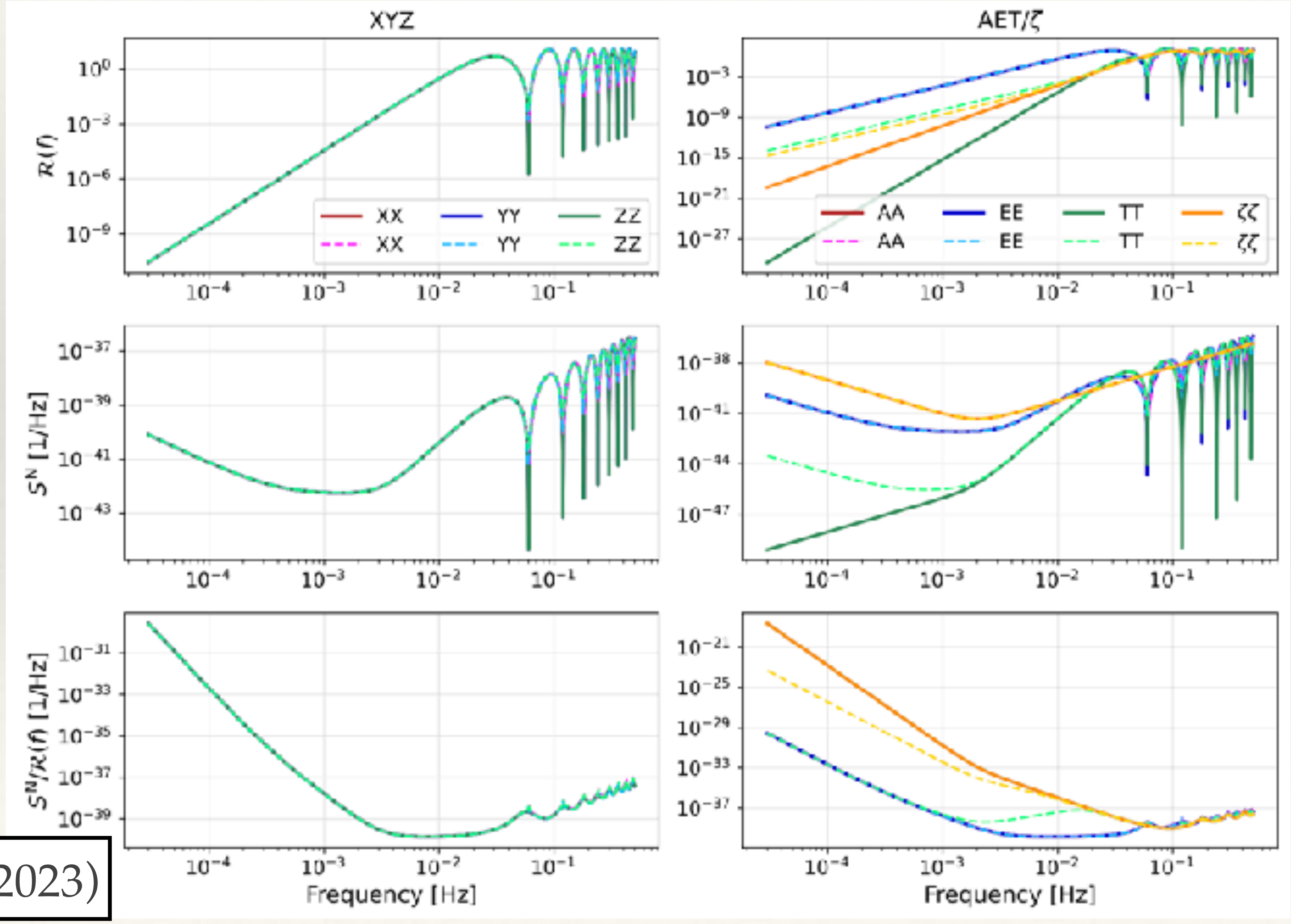
$$\langle|\tilde{n}_A|^2\rangle = \sigma_A^2\,, \qquad \langle|\tilde{n}_T|^2\rangle = \sigma_T^2\,, \qquad \langle|\tilde{h}|^2\rangle = \sigma_h^2 \qquad \sigma_A^2 = a\sigma_T^2$$



Robinson et al. (2008)

Solutions: TDI channels

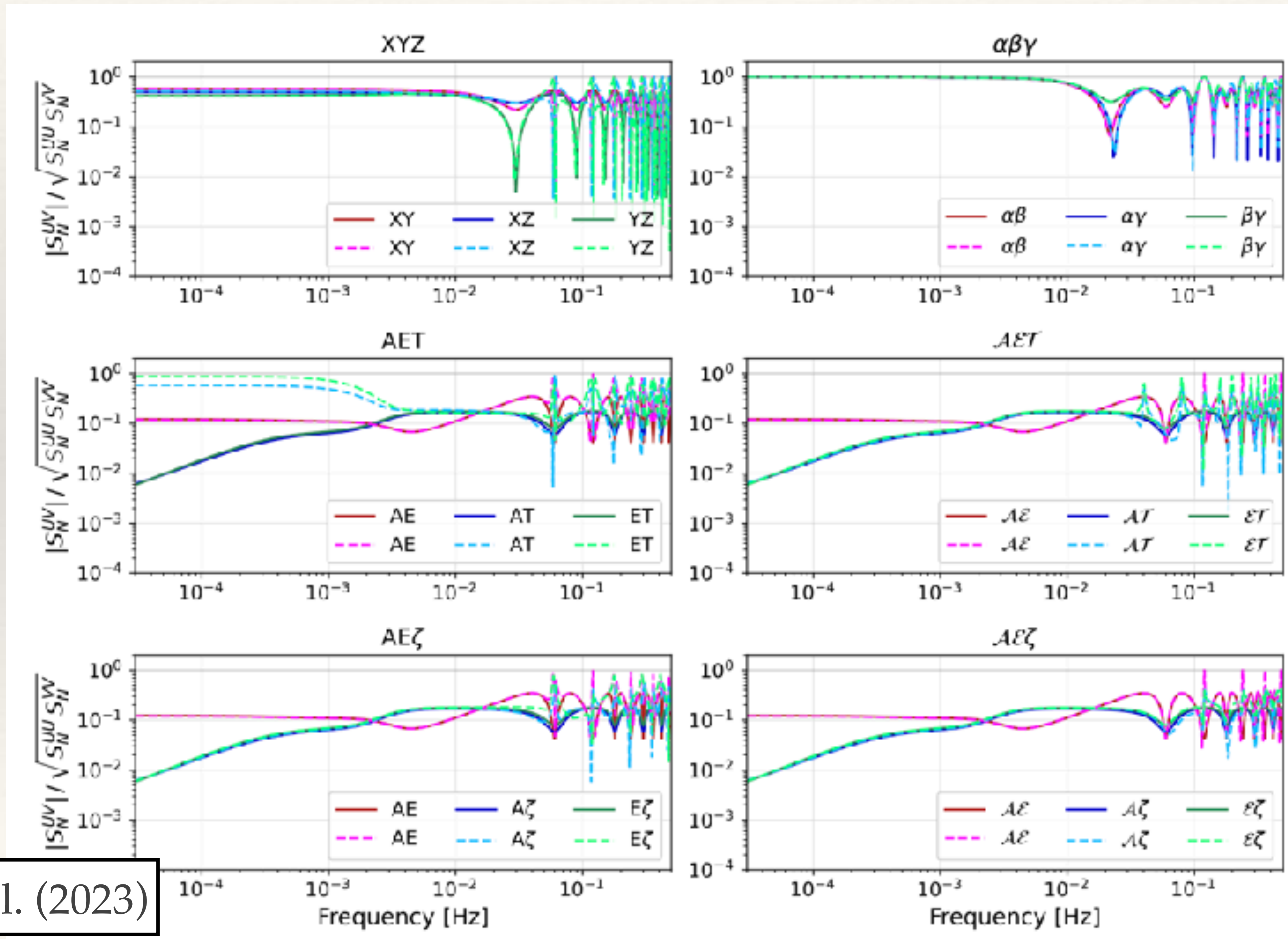
- For an unequal arm length interferometer, T channel is no longer insensitive to gravitational waves. Sagnac ζ channel performs better.



Hartwig et al. (2023)

Solutions: TDI channels

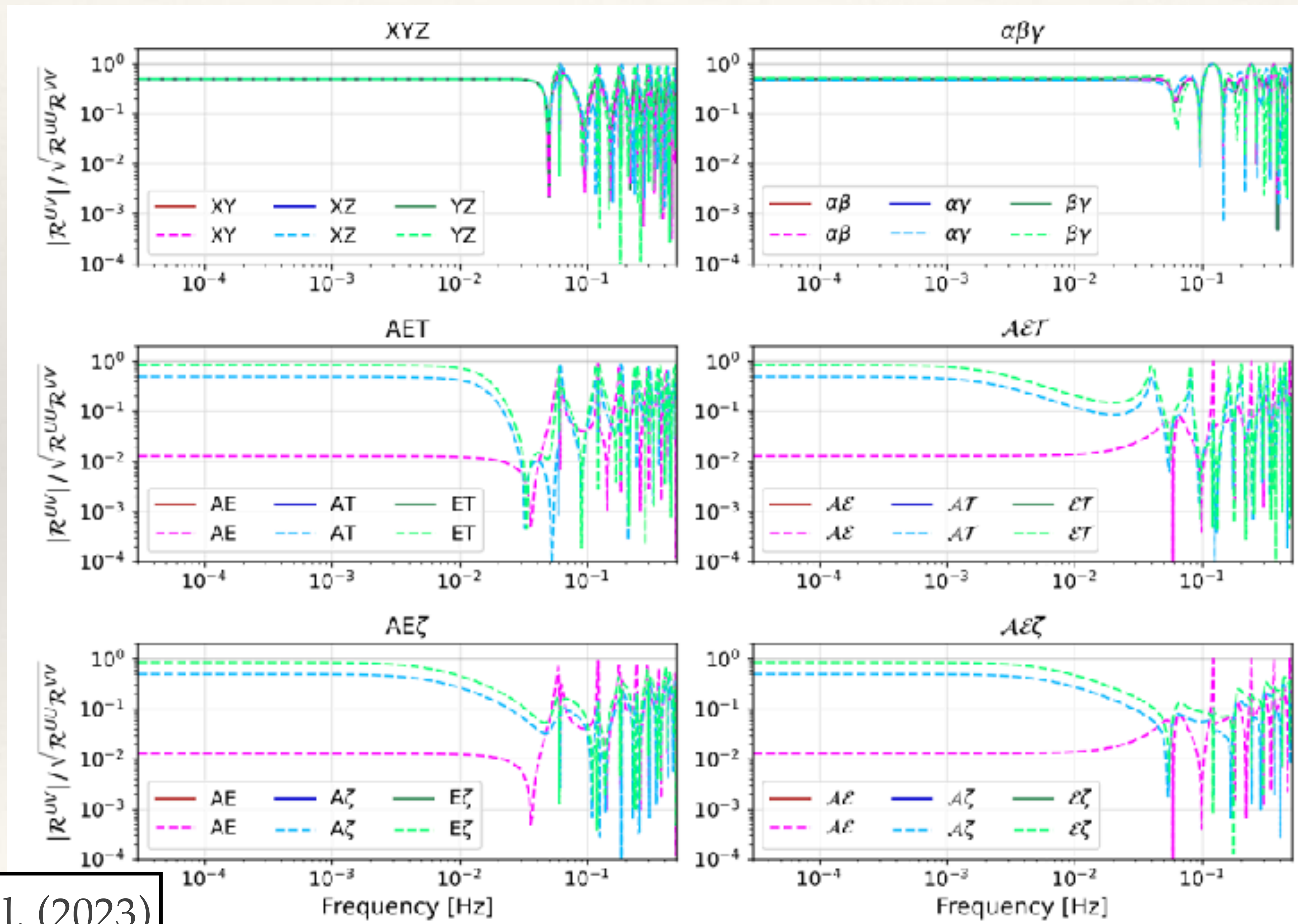
- Channels more strongly correlated for unequal arms or unequal noises.



Hartwig et al. (2023)

Solutions: TDI channels

- Signal is less correlated -> potential to measure noise at high frequency.



Hartwig et al. (2023)

Solutions: simplified uncertainties

- Another approach is to assume a form for the instrumental noise. This was already done in SGWBinner and in the LDCs. SGWBinner uses

$$P_{\rm oms}(f, P) = P^2\,\frac{\rm pm^2}{\rm Hz}\left[1 + \left(\frac{2\,\rm mHz}{f}\right)^4\right]\left(\frac{2\pi f}{c}\right)^2\,,$$

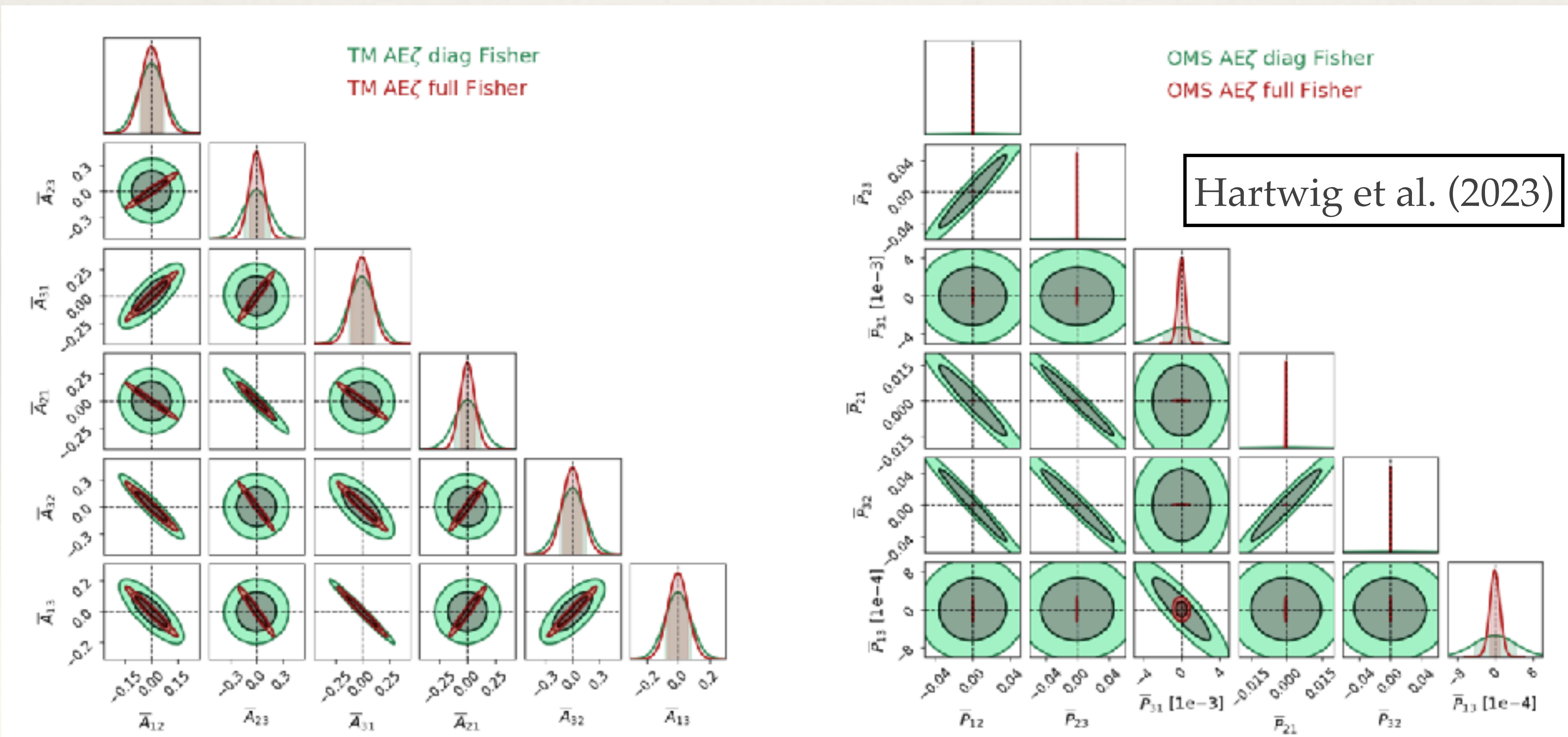
$$P_{\rm acc}(f, A) = A^2\,\frac{\rm fm^2}{\rm s^4\,Hz}\left[1 + \left(\frac{0.4\,\rm mHz}{f}\right)^2\right]\left[1 + \left(\frac{f}{8\,\rm mHz}\right)^4\right]\left(\frac{1}{2\pi f}\right)^4\left(\frac{2\pi f}{c}\right)^2$$

$$P_n^{(X)}(f, P, A) = 16\sin^2\left(\frac{2\pi f L}{c}\right)\left\{P_{\rm oms}(f, P) + \left[3 + \cos\left(\frac{4\pi f L}{c}\right)\right]P_{\rm acc}(f, A)\right\}$$

- MLDC/LDCs employ a similar model, but with the noise levels allowed to vary independently for each optical link.

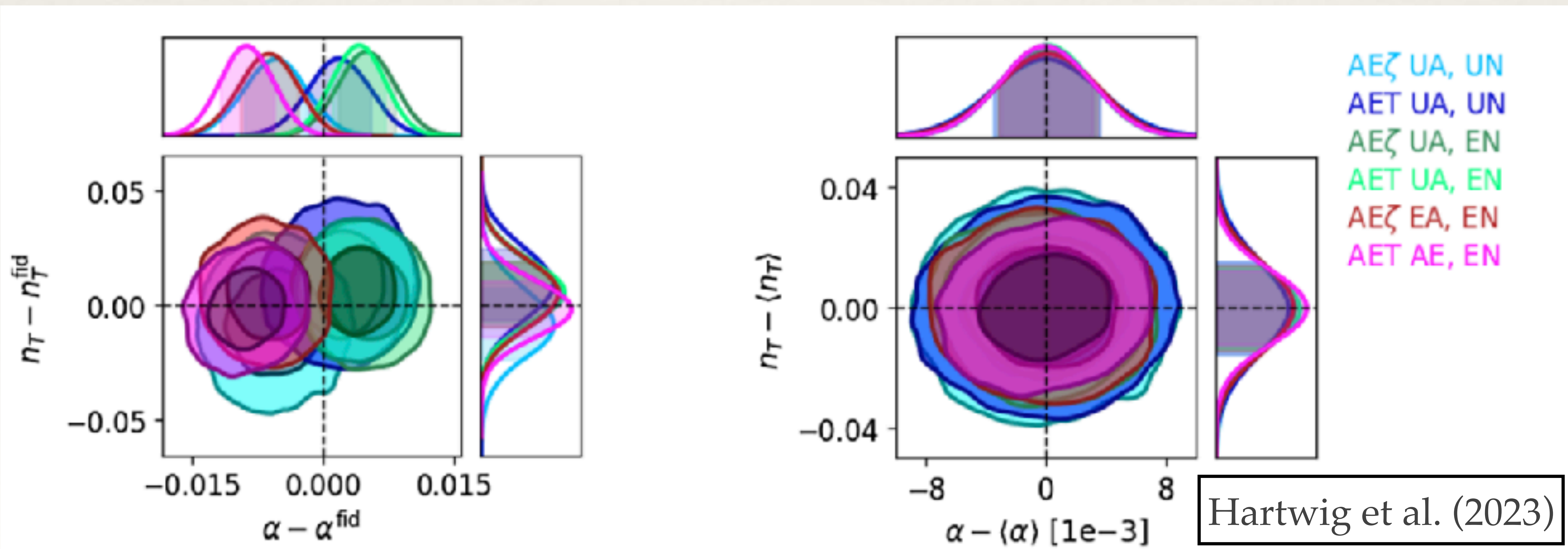
Solutions: simplified uncertainties

- Hartwig et al. (2023) explored the 12 parameter model for an unequal arm interferometer. Correlations between channels help determine noise components.



Solutions: simplified uncertainties

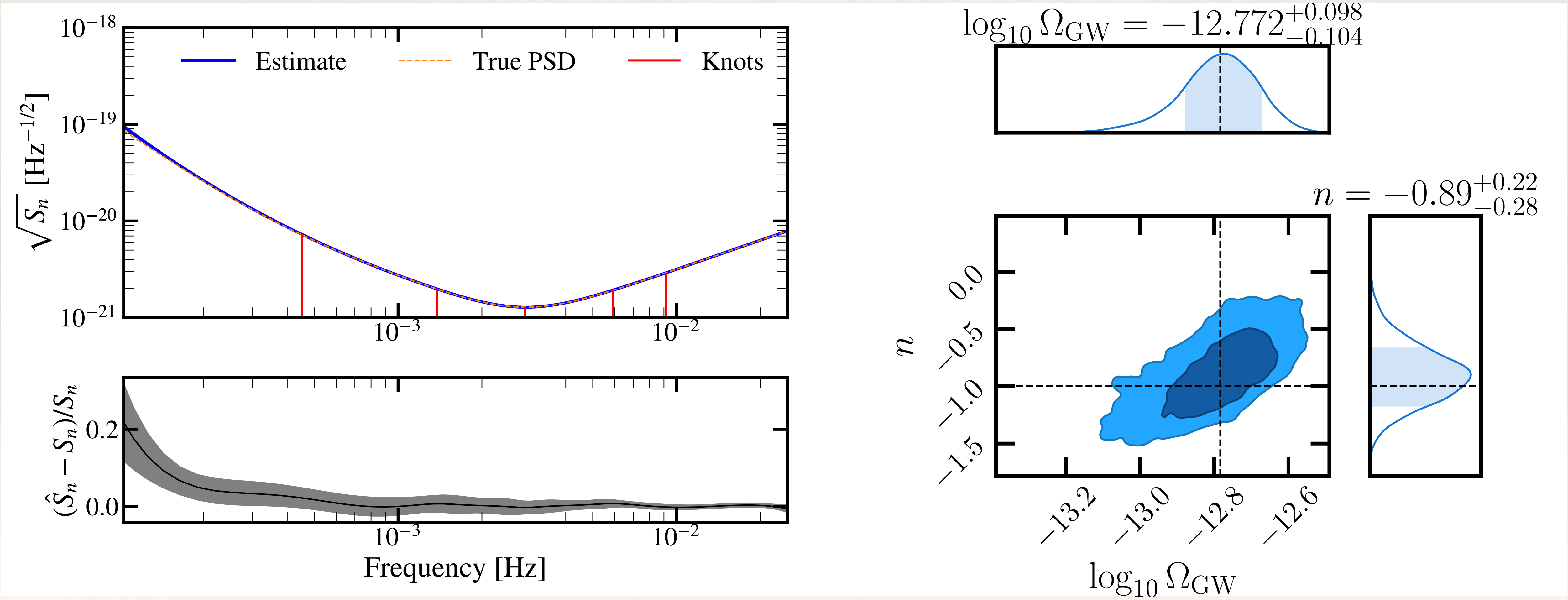
- Background parameters recovered consistently with no bias.



Hartwig et al. (2023)

Challenges: lack of noise knowledge

- Model more generic deviations using splines. Can successfully recover single link PSD and injected background parameters for a power-law SGWB signal.

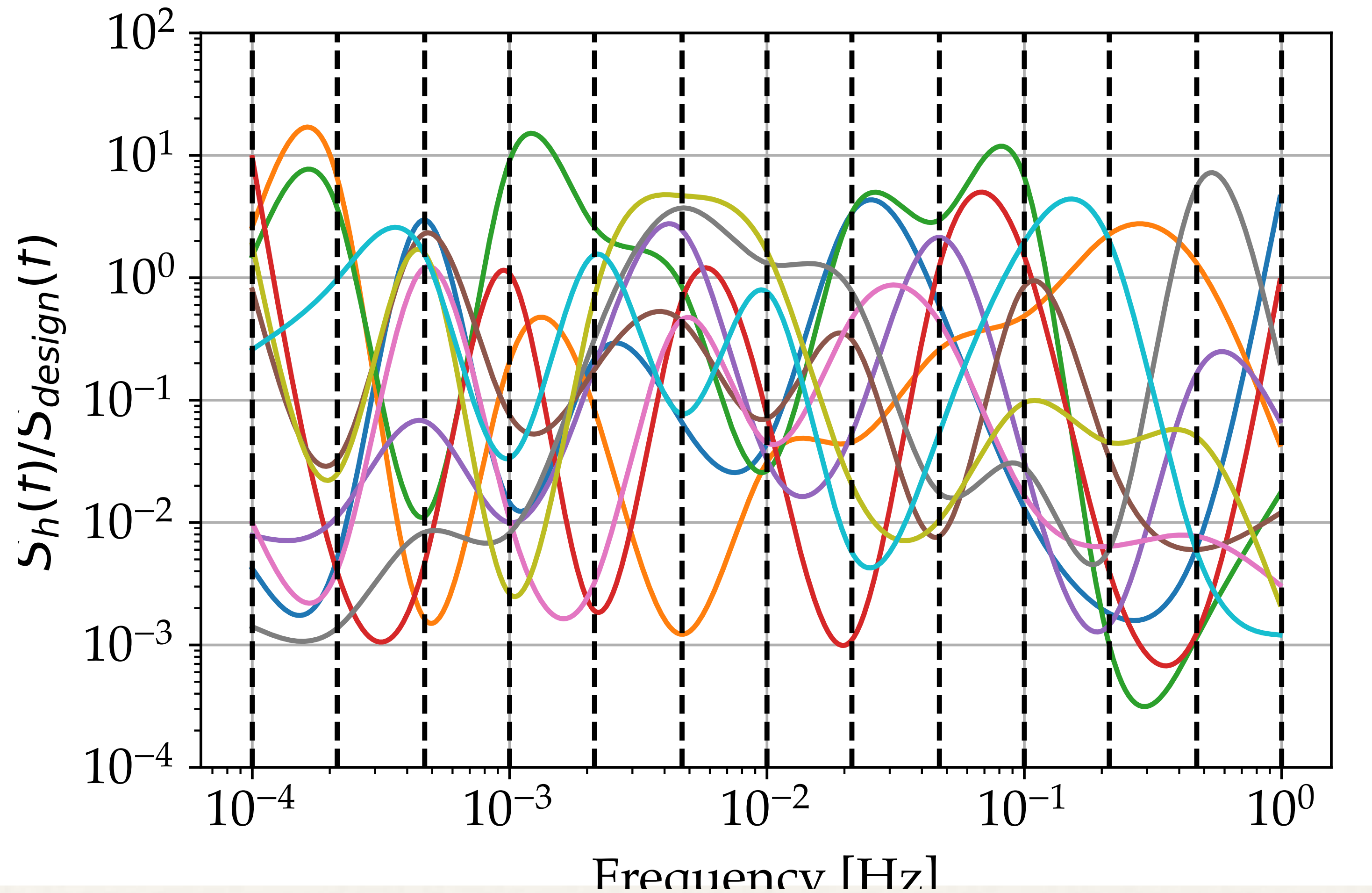


Baghi et al. (2023). See Baghi talk on Thursday

Challenges: lack of noise knowledge

- Include further uncertainties by allowing both PSDs and real/imaginary components of CSD to vary, and assume unequal arm interferometer.

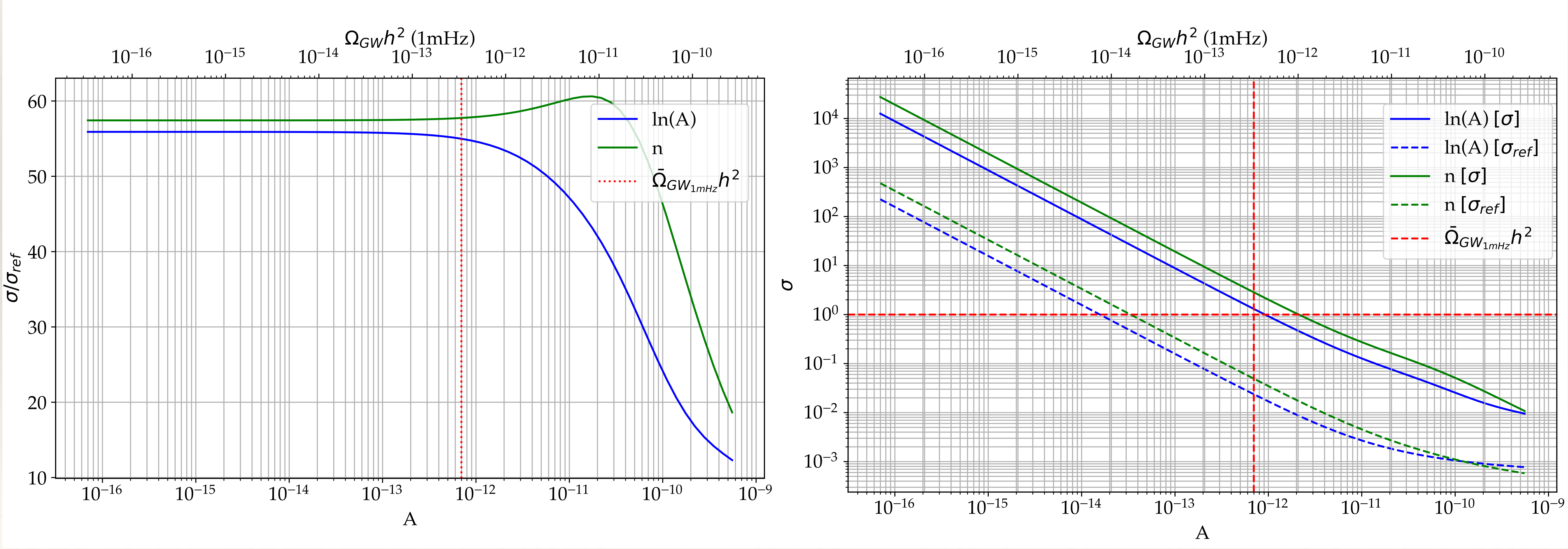
$$S_n(f|\{w_i\}) = S(f)_{\rm des.} 10^{C(f|\{\log_{10}(f_i)\},\{w_i\})}$$



See Muratore talk on Thursday

Challenges: lack of noise knowledge

- Using this model, we find that the amplitude of modelled SGWBs needs to be a factor of a few bigger for confident detection.



See Muratore talk on Thursday

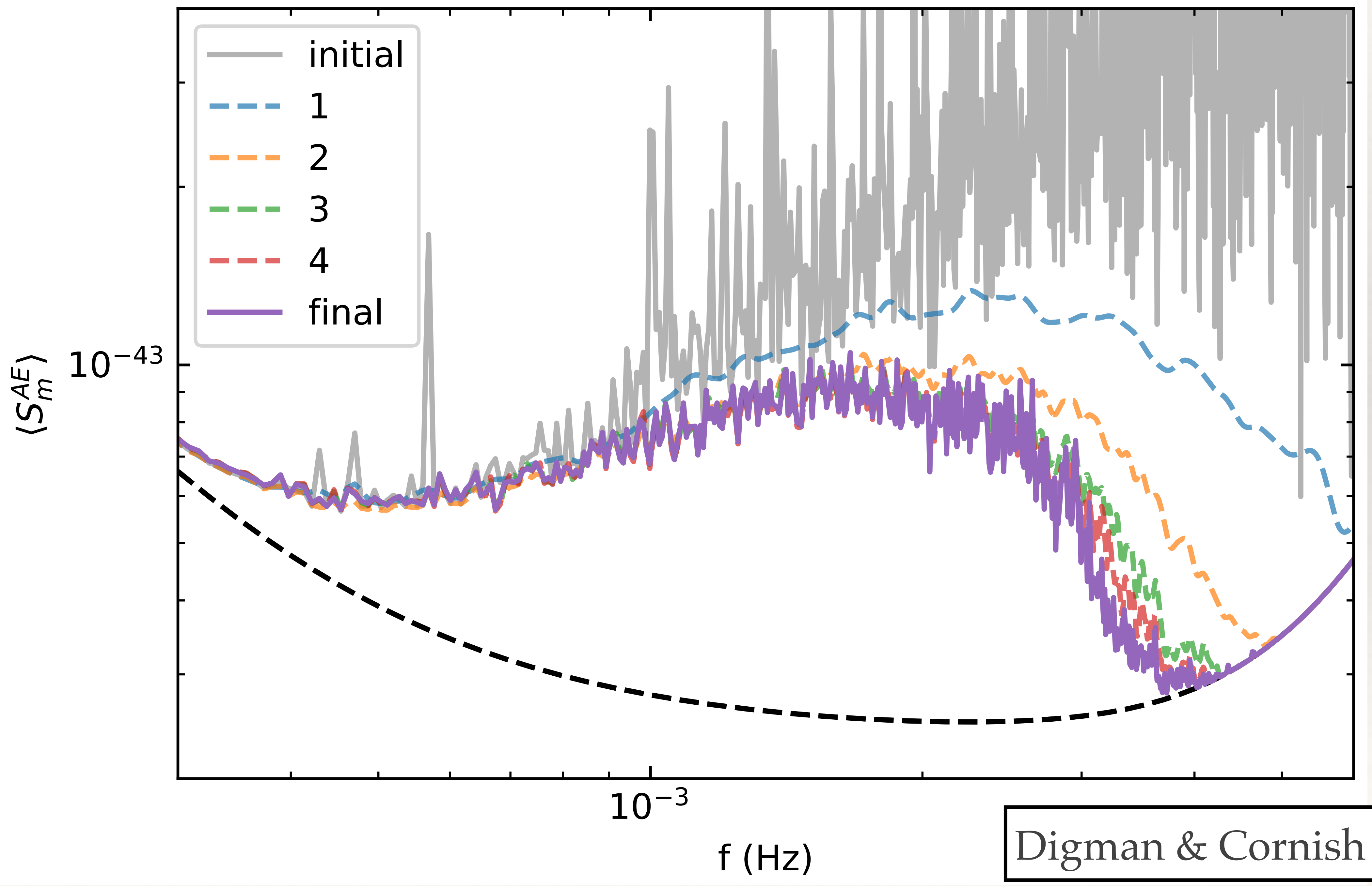
Summary: lack of noise knowledge

- Distinguishing between an SGWB and instrumental noise will be difficult
  - different transfer function offers limited information;
  - using a model for the noise components helps break degeneracies;
  - generic reconstruction of one component only possible if other component is modelled;
  - priors on amplitude of instrumental noise allow reconstruction of loud backgrounds.

See Baptiste-Bayle talk on Thursday

Challenges: astrophysical foregrounds

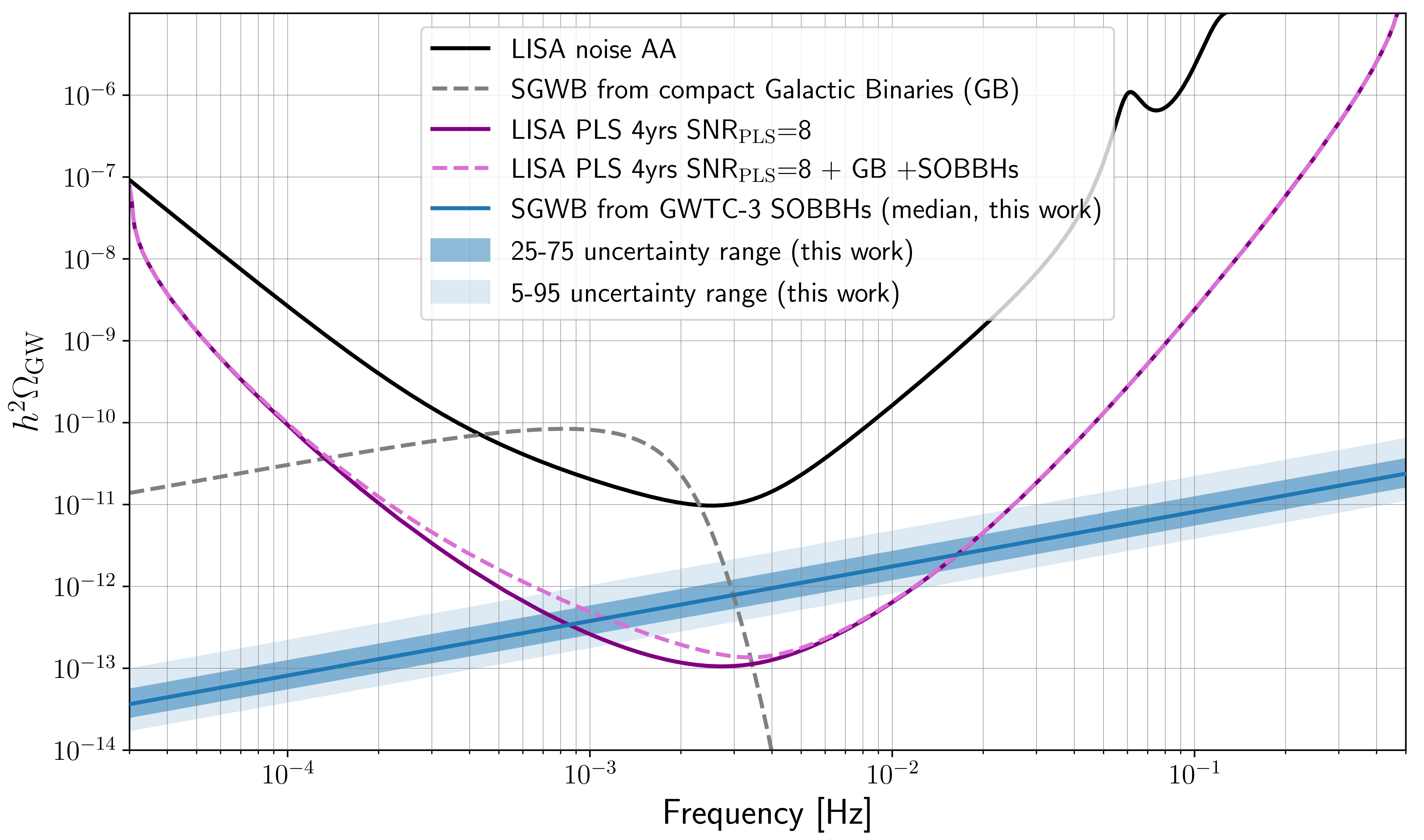
- SGWB will be obscured by astrophysical foregrounds, including galactic white dwarf binaries.



Digman & Cornish (2022)

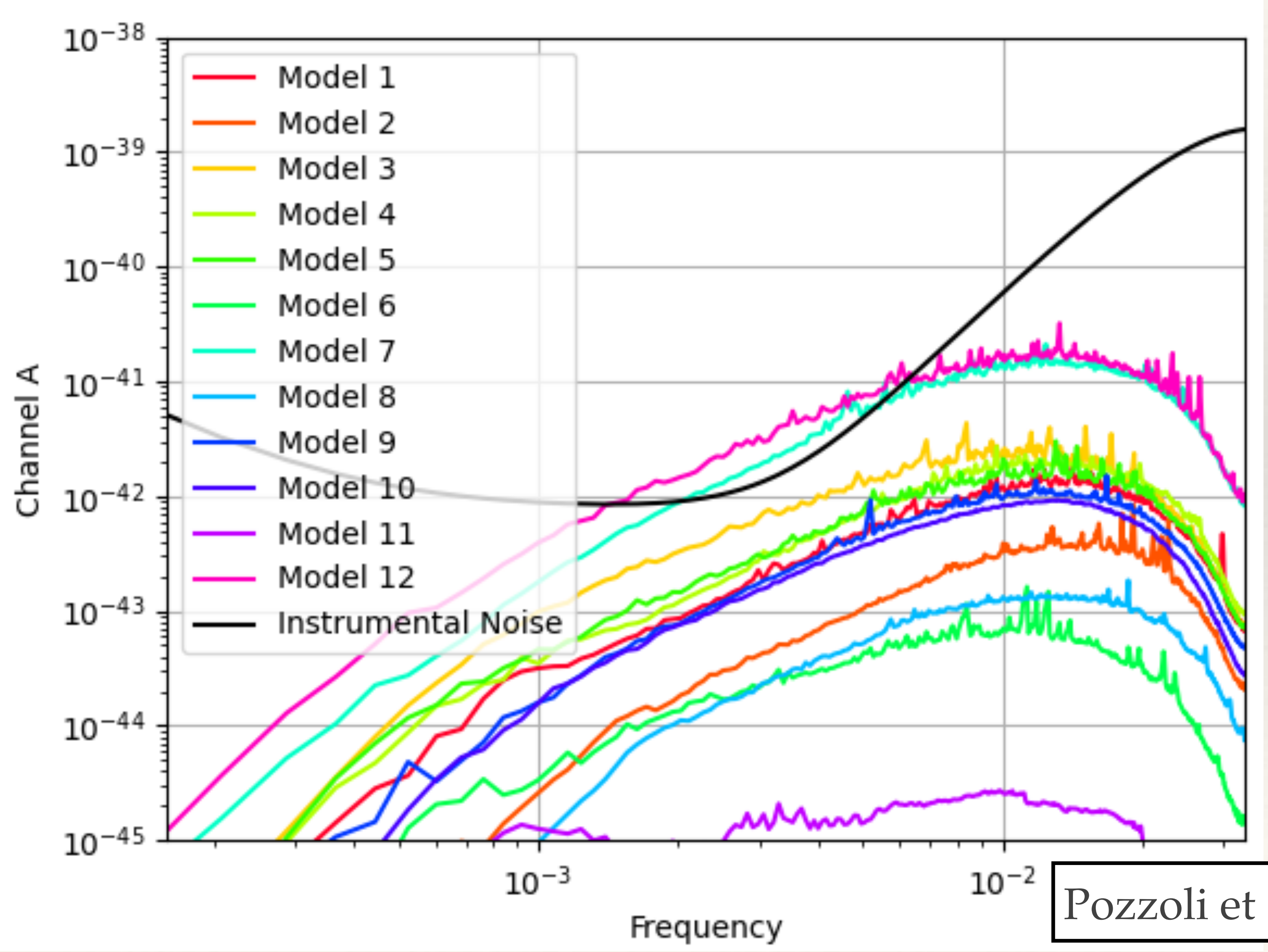
Challenges: astrophysical foregrounds

- There will also be a background from stellar binary black holes…..



Challenges: astrophysical foregrounds

- ….and perhaps from extreme-mass-ratio inspirals.



Pozzoli et al. (2023)

Challenges: source confusion

- Presence of other sources in the data impacts parameter estimates for sources of interest.

- For resolved sources, assess impact using joint Fisher matrix

$$\Gamma = \begin{pmatrix} \Gamma^{(1)} & \Gamma^{\text{mix}} \\ (\Gamma^{\text{mix}})^T & \Gamma^{(2)} \end{pmatrix}; \quad \Gamma^{-1} = \begin{pmatrix} \Gamma^{-1}_{11} & \Gamma^{-1}_{12} \\ (\Gamma^{-1}_{12})^T & \Gamma^{-1}_{22} \end{pmatrix}$$

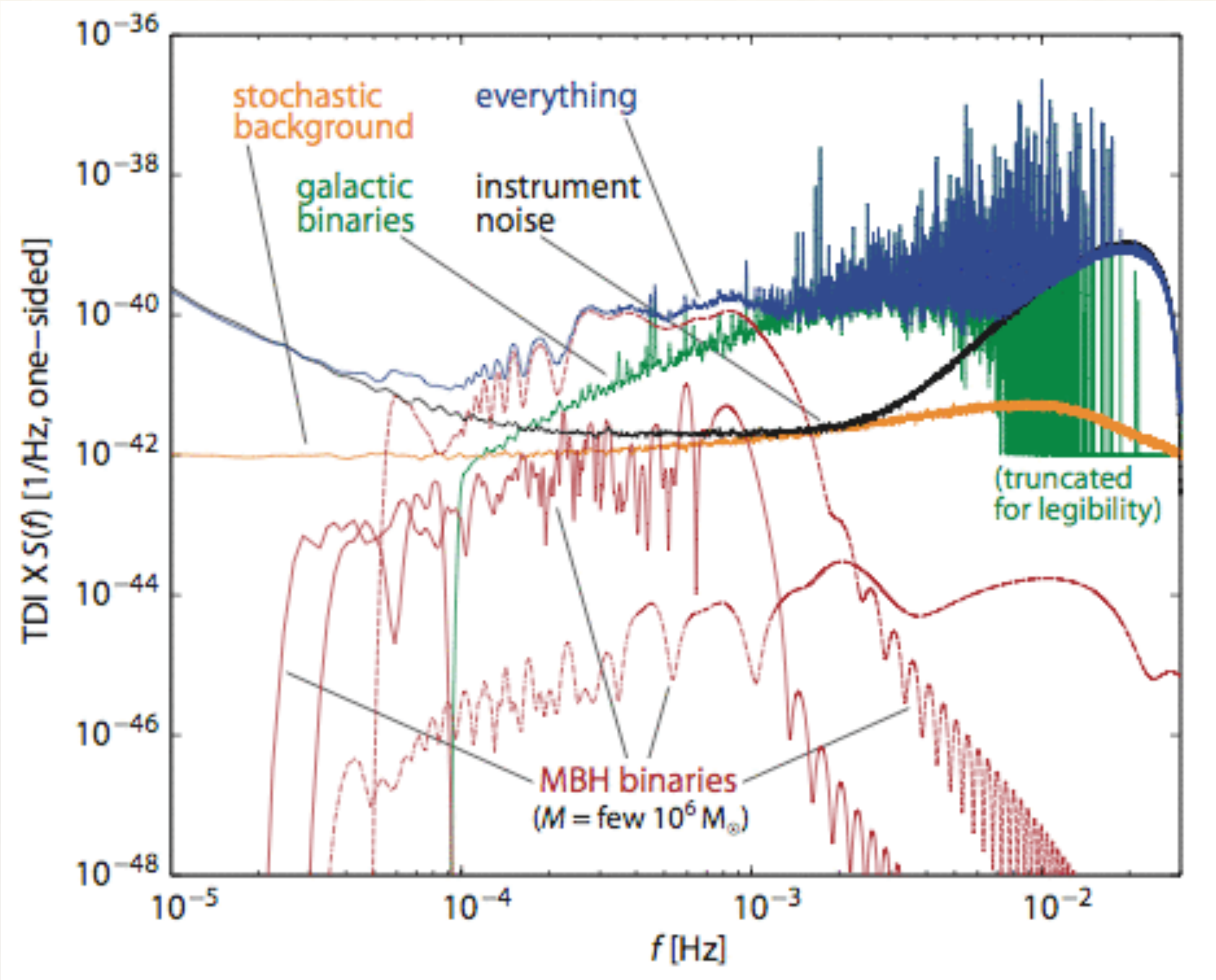
$$\Gamma^{-1}_{11} = \left(\Gamma^{(1)} - \Gamma^{\text{mix}}(\Gamma^{(2)})^{-1}(\Gamma^{\text{mix}})^T\right)^{-1}$$

$$\Gamma^{-1}_{22} = \left(\Gamma^{(2)} - (\Gamma^{\text{mix}})^T(\Gamma^{(1)})^{-1}\Gamma^{\text{mix}}\right)^{-1}$$

$$\Gamma^{-1}_{12} = -\Gamma^{-1}_{11}\,\Gamma^{\text{mix}}(\Gamma^{(2)})^{-1}.$$

- Assuming near-orthogonality

$$\Gamma^{-1}_{11} \approx (\Gamma^{(1)})^{-1} + (\Gamma^{(1)})^{-1}\Gamma^{\text{mix}}(\Gamma^{(2)})^{-1}(\Gamma^{\text{mix}})^T(\Gamma^{(1)})^{-1}$$



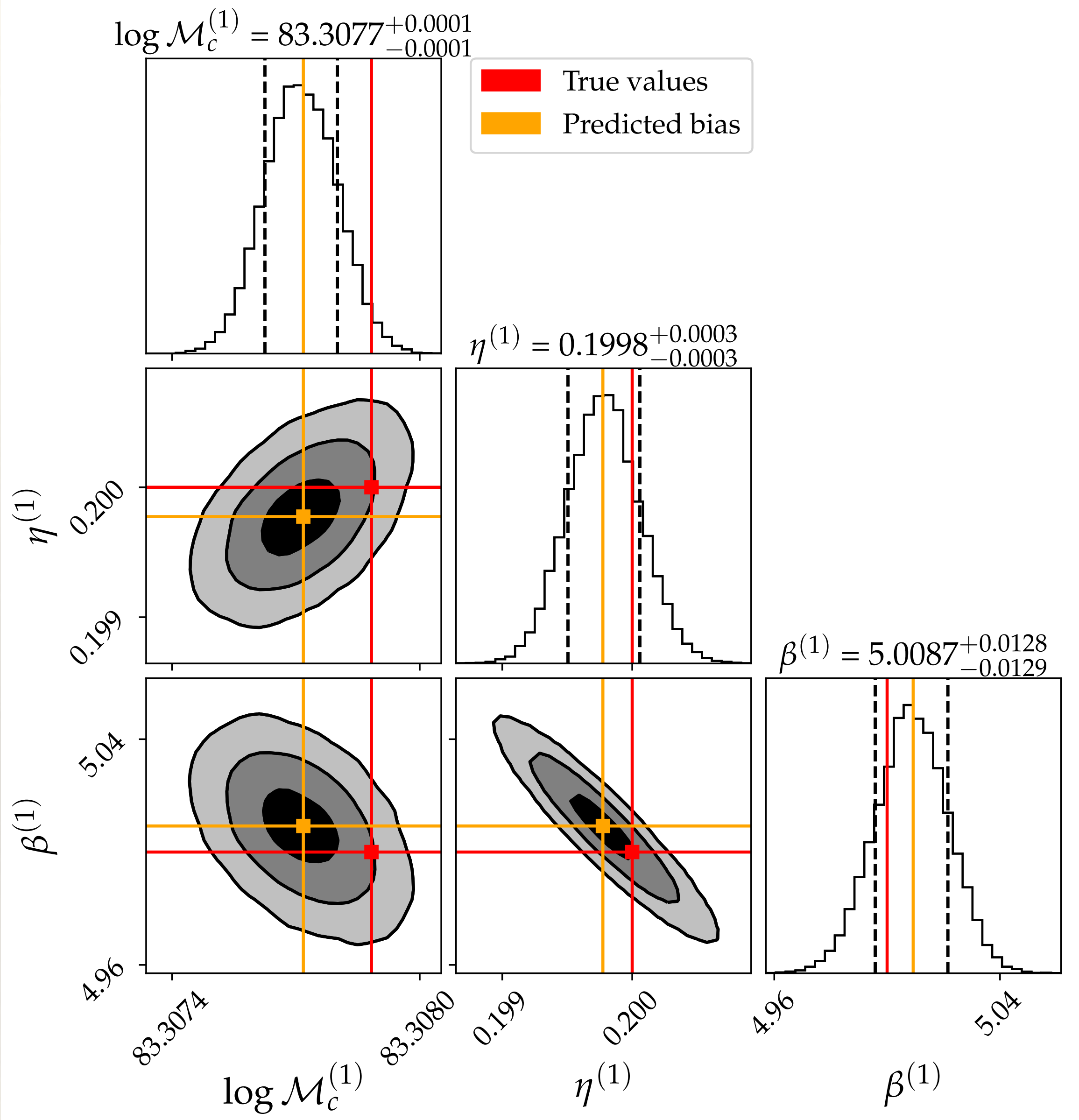
- Can interpret as noise from residuals

$$\Delta\theta^{(1),i}_{\text{sys}} = (\Gamma^{(1)})^{-1}_{ij}(\partial_j h^{(1)}_m | \boldsymbol{h}^{(2)})$$

$$\langle \Delta\theta^{(1),i}_{\text{sys}} \Delta\theta^{(1),j}_{\text{sys}} \rangle = (\Gamma^{(1)})^{-1}_{ik}(\partial_k h^{(1)}_m | \partial_l \boldsymbol{h}^{(2)}) \langle \Delta\theta^l_2 \Delta\theta^m_2 \rangle (\partial_n h^{(1)}_m | \partial_m \boldsymbol{h}^{(2)})(\Gamma^{(1)})^{-1}_{jm}$$

Challenges: source confusion

- Get additional errors from mismodelling of sources, e.g., ignoring environmental effects or GR modifications or waveform errors.
- For example, fitting for a single source of type A in the presence of residuals from a population of type B.
- Residuals create a ~stochastic signal that could be confused with an SGWB.



$$\Delta\theta_i^A = -\sum_{a=1}^{N_B} \left(\Gamma^A\right)^{-1}_{ij} \left[(\partial_j h^A | \delta h^B(\boldsymbol{\theta}_a^B)) + \left(\Gamma^{\rm mix}(\boldsymbol{\theta}_a^B)\right)_{jk} \left(\Gamma^B(\boldsymbol{\theta}_a^B)\right)^{-1}_{kl} \sum_{b=1}^{N_B} (\partial_l h^B(\boldsymbol{\theta}_a^B) | \delta h^B(\boldsymbol{\theta}_b^B)) \right]$$

$$\left(\Gamma^{\rm mix}(\boldsymbol{\theta}_a^B)\right)_{ij} = (\partial_i h^A | \partial_j h^B(\boldsymbol{\theta}_a^B)), \quad \Gamma^A_{ij} = (\partial_i h^A | \partial_j h^A), \quad \left(\Gamma^B(\boldsymbol{\theta}_a^B)\right)_{ij} = (\partial_i h^B(\boldsymbol{\theta}_a^B) | \partial_j h^B(\boldsymbol{\theta}_a^B))$$

Challenges: gaps

- Many possible causes of gaps in the LISA data stream, of both known and unknown origin.

Gap type	Frequency	Duration	Total loss (hr/yr)
Antenna repointing	every 2 weeks	3.3h	1%
PAAM angle adjust	3 per day	100s	0.3%
TM stray pot. est.	2/yr	1 day	0.56%
TTL coupling est.	4/yr	2 days	2.22%
Unplanned: platform	3/yr	2.5 days	2%
Unplanned: payload	4/yr	2.75 days	3%
Unplanned: micro-meteorites	30/yr	1 day	8%

Challenges: gaps

- Various approaches to dealing with gaps: gap filling, noise filtering, time-frequency analysis etc. Results depend critically on assumptions about noise behaviour across gap.
- Treating gap as **missing data**

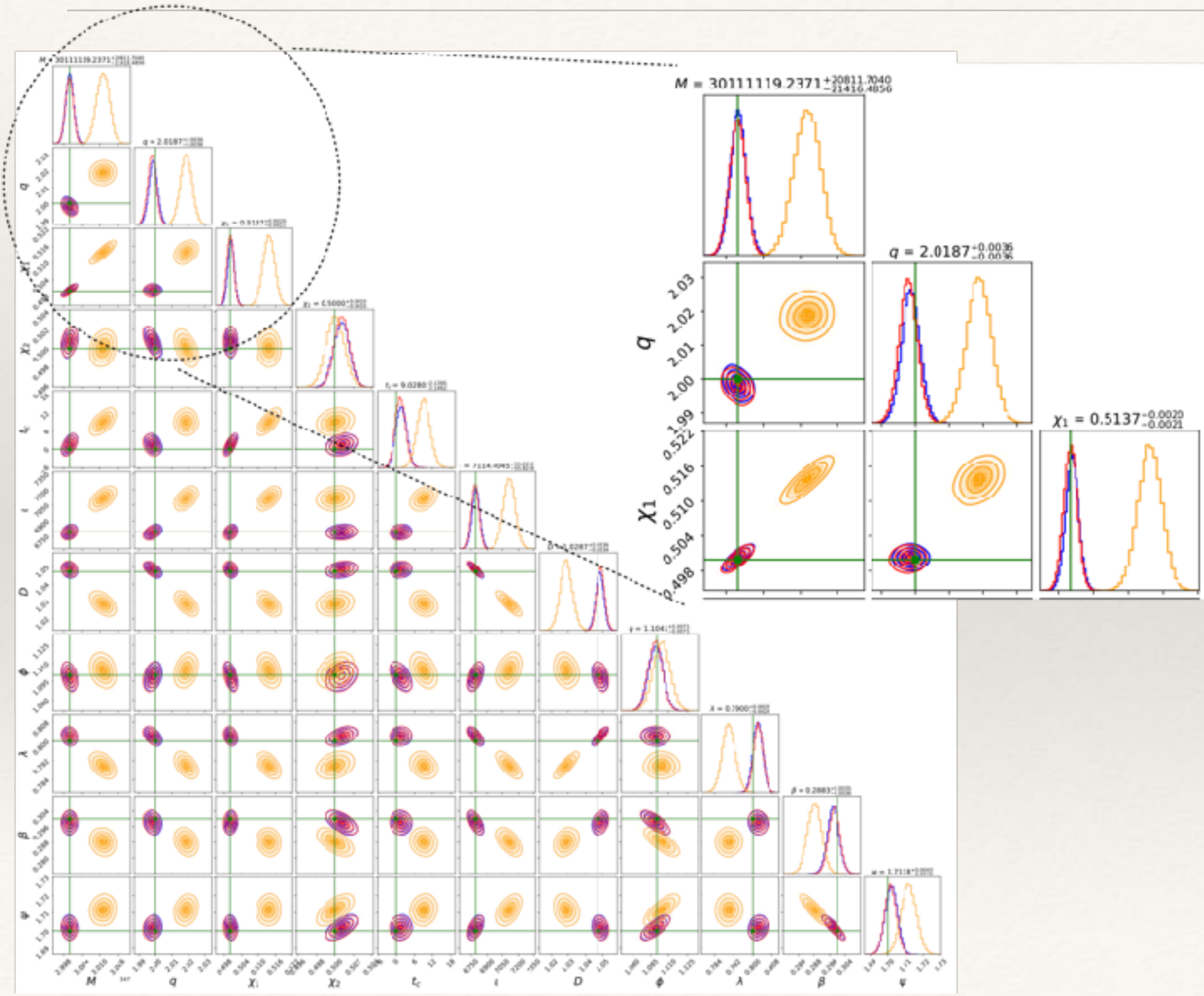
$$D(t) = w(t)h(t;\boldsymbol{\theta}) + w(t)n(t) = H(t;\boldsymbol{\theta}) + N(t)$$

$$\log p(\boldsymbol{D}|\boldsymbol{\theta},\Sigma_N) \propto -\frac{1}{2}(D(t) - H(t;\boldsymbol{\theta})|D(t) - H(t;\boldsymbol{\theta}))_{\Sigma_N} = -(\hat{\boldsymbol{D}} - \hat{\boldsymbol{H}})^{\dagger}\Sigma_N^{-1}(\hat{\boldsymbol{D}} - \hat{\boldsymbol{H}})$$

$$(\Sigma_N)_{ij} \approx \frac{\Delta f}{2}\sum_{p=0}^{\lfloor N/2+1\rfloor} \hat{w}^{\star}(f_i - v_p)\hat{w}(f_j - v_p)S_n(v_p)$$

- Treating noise as **independent** in each between-gap segment: likelihood is product of likelihoods for each segment.

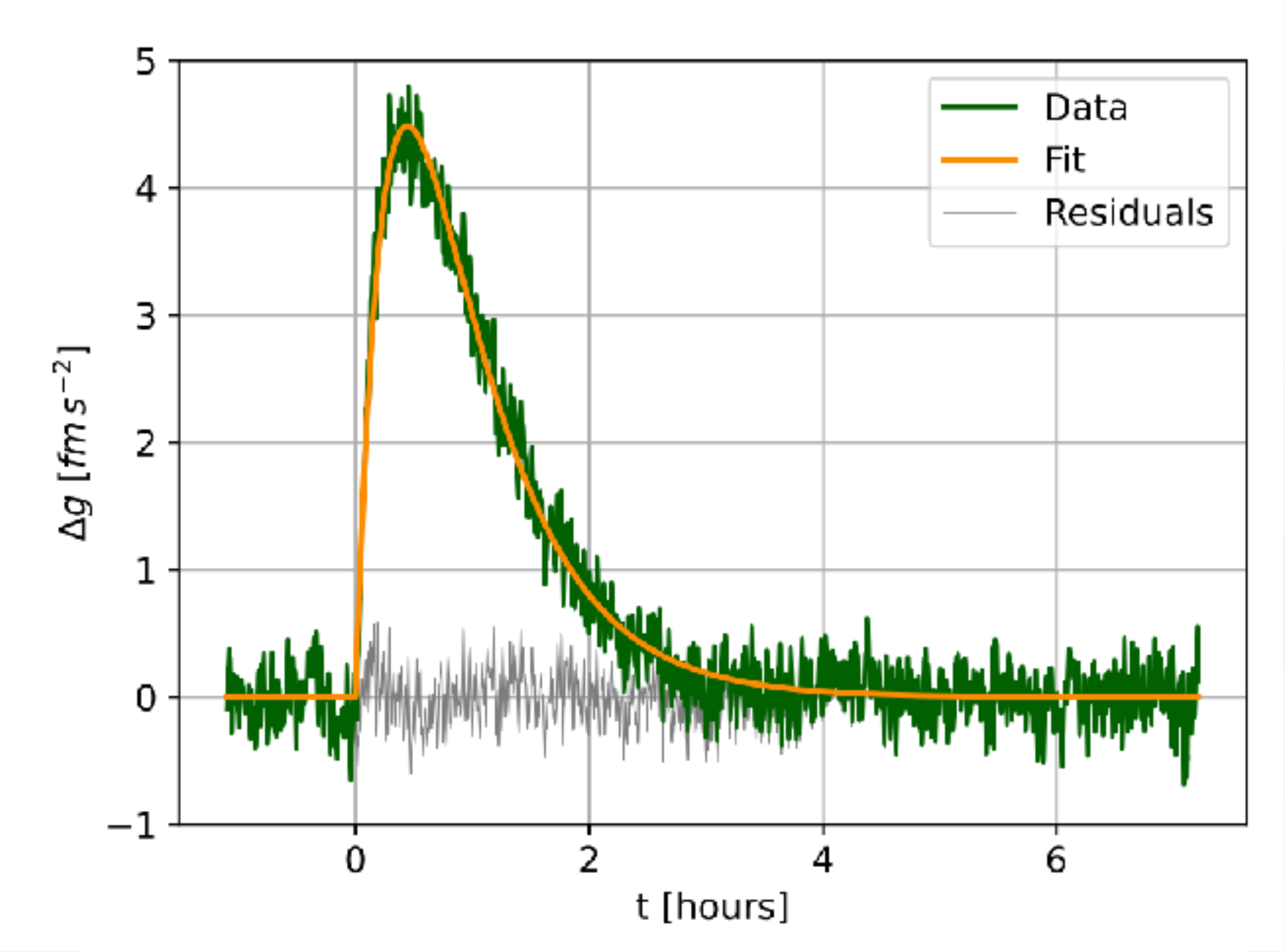
Challenges: gaps



- Using the wrong model leads to biases for resolved sources.
- For SGWBs, it is less clear. Gaps may provide natural segment breaks for stochastic analyses.
- Analysis may ultimately be done in time-frequency domain.

Challenges: glitches

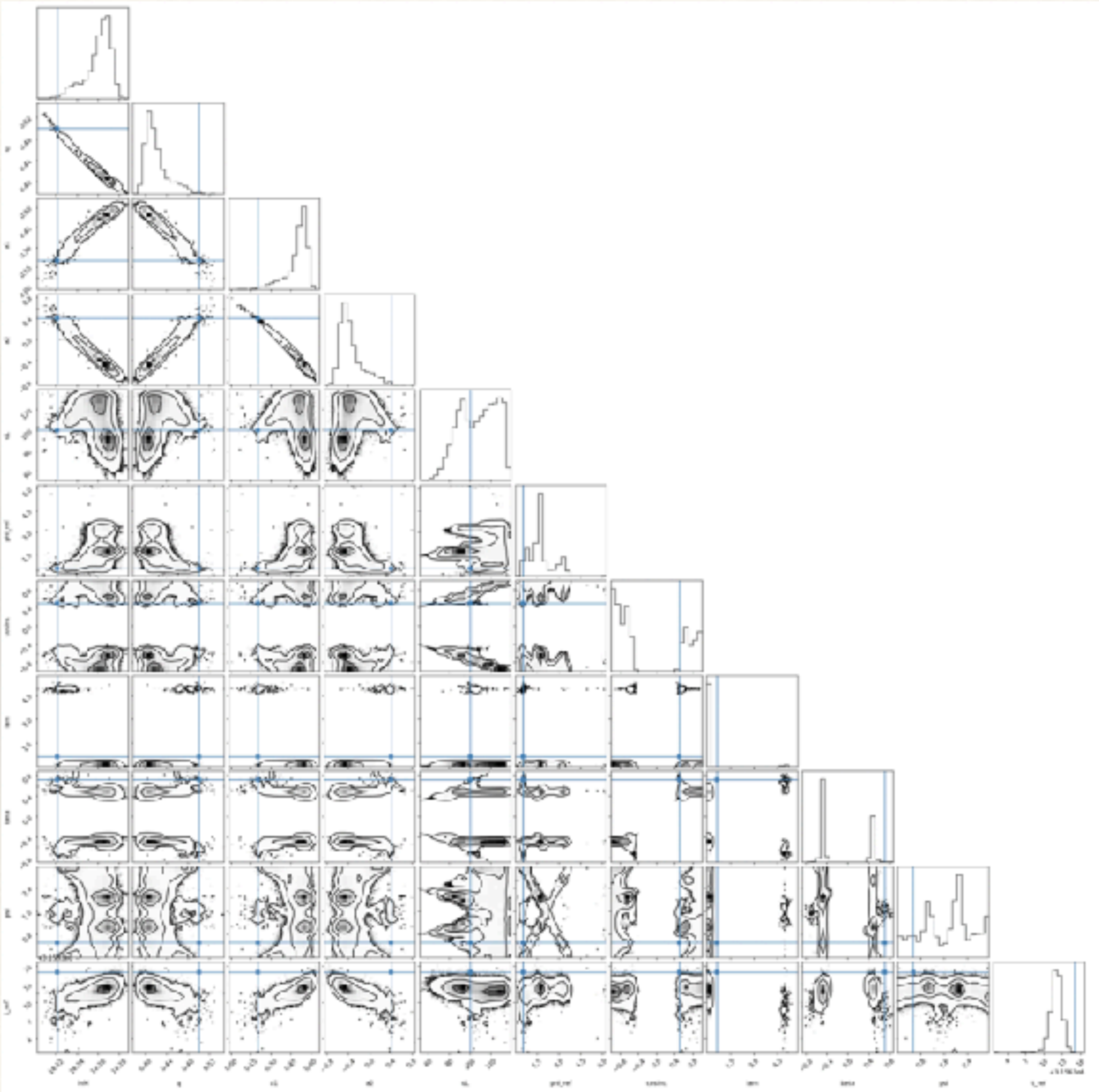
- LISA Pathfinder observed glitches at a rate of 1/day. Expect glitches in LISA too.
- Pathfinder glitches well described by a single exponential



$$h_1(t) = \frac{\Delta v}{\tau^2}\, t'\, e^{-t'/\tau}\, \Theta(t'), \quad t' = t - t_0$$

Challenges: glitches

- If glitch overlaps merger, can get biases for individual resolvable sources.
- Avoid biases by fitting for glitch simultaneously with signal parameters.
- Need reliable glitch model.
- Ignoring glitches or fitting them poorly could lead to residual noise that is confused with SGWB.



Summary

- SGWB detection and characterisation is an important part of the LISA science case described in the Red Book.
- Development of data analysis strategies is underway, partly drive by the ongoing LISA Data Challenges.
- State of the art: characterisation of galactic binary foreground within global fit to *Sangria* data set, unmodelled / modelled recovery of SGWB in isolated data sets.
- Many challenges still to overcome: lack of noise knowledge, astrophysical foregrounds, confusion noise, modelling errors, data gaps, glitches etc.
- Work on these topics needed now to inform construction of global fits and prepare the way for LISA science exploitation within the STPs / on open data.

Extra slides

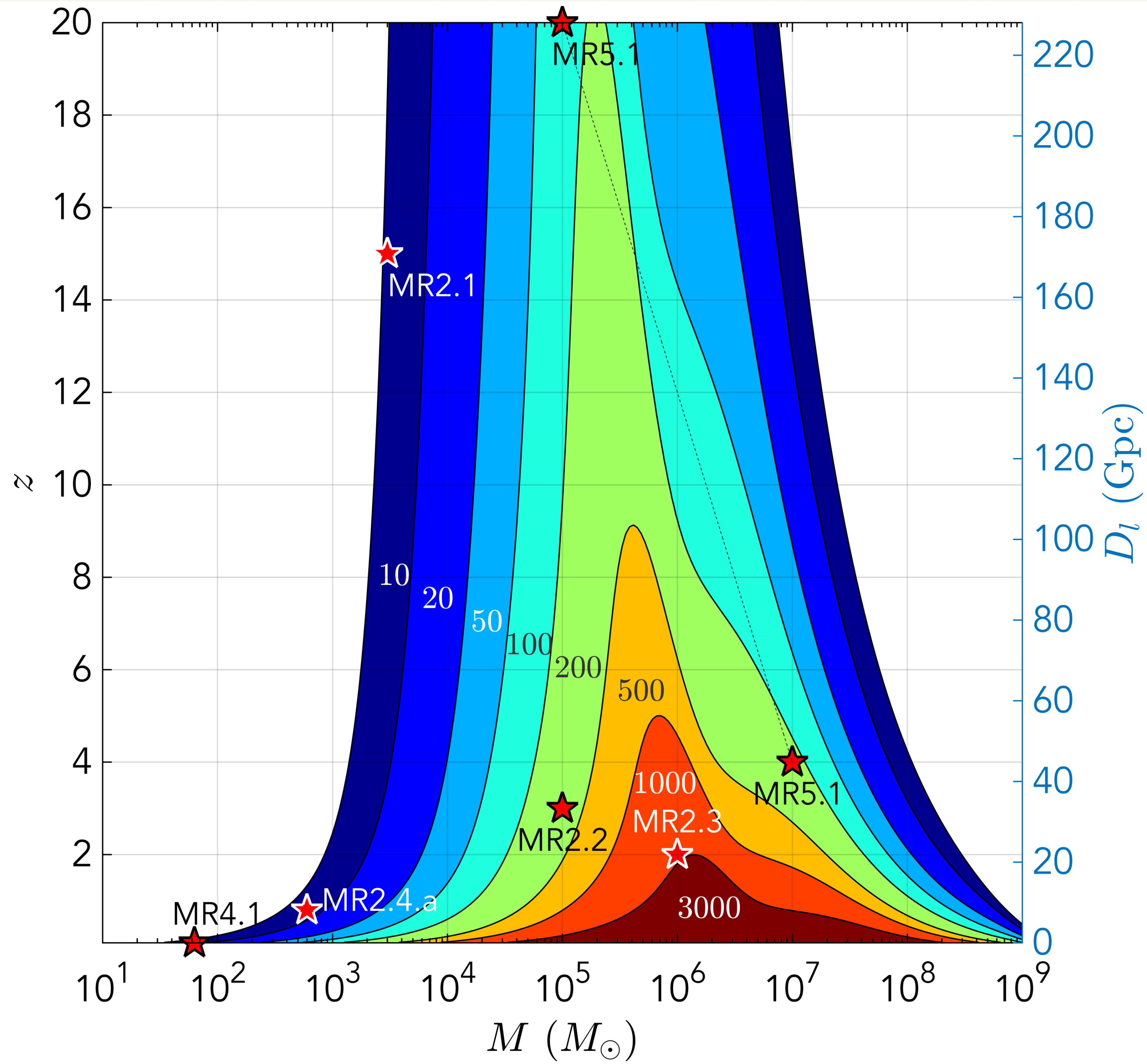
LISA mission status

- Project must build a mission that **can** address Red Book science objectives. This is encoded in *observing requirements*.
  - **OR7.1**: Characterise the stochastic GW background from SOBH binaries with energy density normalised to the critical energy density in the Universe today, Ω, based on the inferred rates from the LIGO detections, i.e., at the lowest $\Omega = 2 \times 10^{-10} (f / 25 \text{ Hz})^{2/3}$. This requires the ability to verify the spectral shape of this stochastic background, and to measure its amplitude in the frequency ranges 0.8 mHz < f < 4 mHz and 4mHz < f < 20mHz.
  - **OR7.2**: Probe a broken power-law stochastic back- ground from the early Universe as predicted, for example, by first order phase transitions (other spectral shapes are expected, for example, for cosmic strings and inflation). Therefore, we need the ability to measure $\Omega = 1.3 \times 10^{-11} (f / 10^{-4} \text{ Hz})^{-1}$ in the frequency ranges 0.1mHz < f < 2mHz and 2mHz < f < 20mHz, and $\Omega = 4.5 \times 10^{-12} (f/10^{-2} \text{ Hz})^{3}$ in the frequency ranges 2mHz < f < 20mHz and 0.02 < f < 0.2 Hz.
- Project not required to necessarily deliver the Red Book science.

Sources: massive black hole mergers

- Expected to occur following mergers of the host galaxies. LISA can observe gravitational waves from these with very high signal-to-noise ratio.

Sources: massive black hole mergers

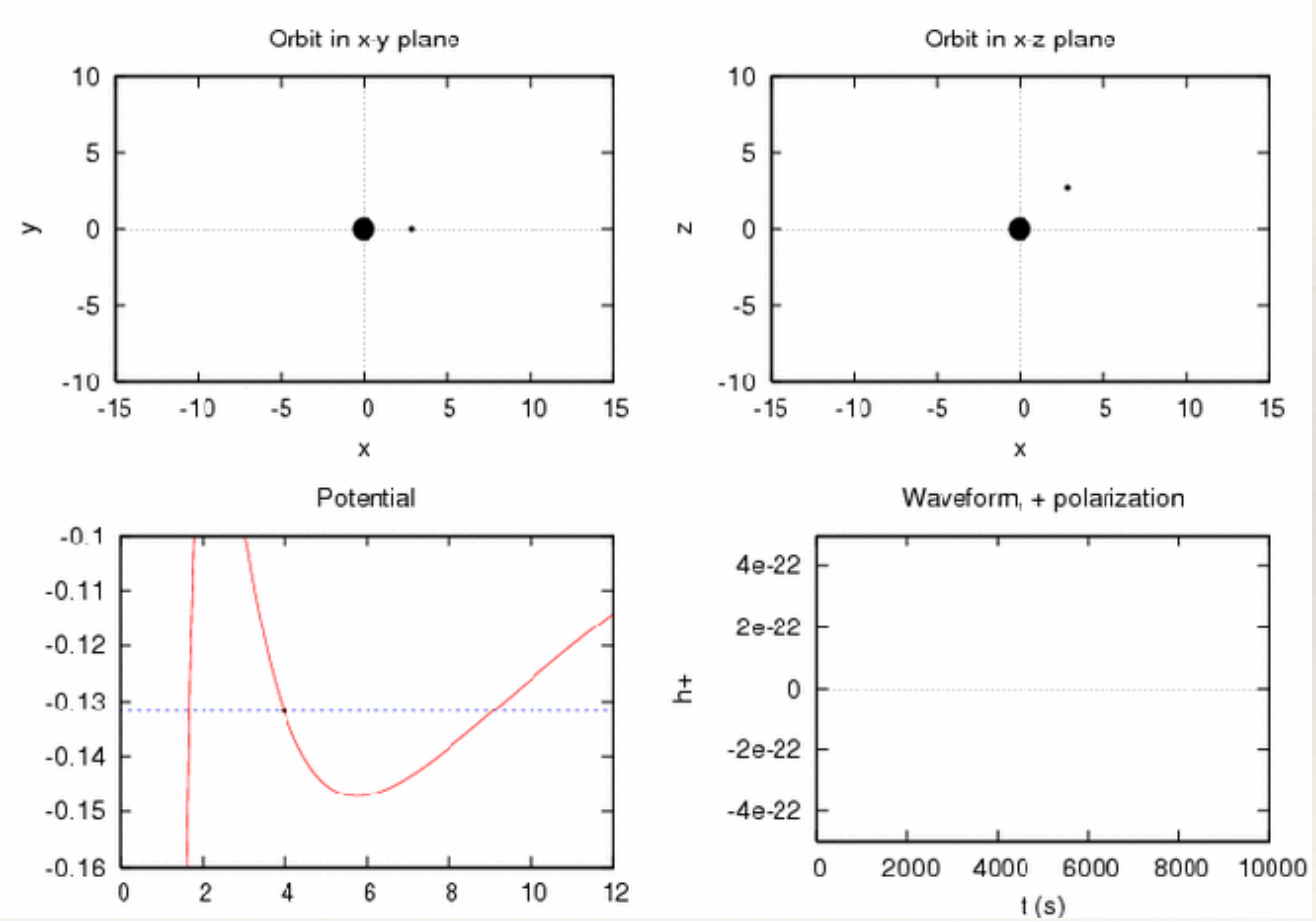


Sources: massive black hole mergers

- Expected to occur following mergers of the host galaxies. LISA can observe gravitational waves from these with very high signal-to-noise ratio.
- Expected event rate depends on assumptions about black hole population (Klein+, 2016)
  - Light pop-III seed model: expect to see ~350 events.
  - Heavy seed model, no delay in binary formation: ~550 events.
  - Heavy seed model, with delays: ~50 events.
- LISA observations expected too provide mass measurements to ~ 0.1-1%, spin measurements to 1-10%, sky location to ~tens of square degrees and luminosity distance to ~10%.

Sources: extreme-mass-ratio inspirals

- The inspiral of a compact object into a massive black hole in the centre of a galaxy.
- Form as a result of scattering in dense galacto-centric stellar clusters.
- Orbits are expected to be both eccentric and inclined - rich waveform structure.



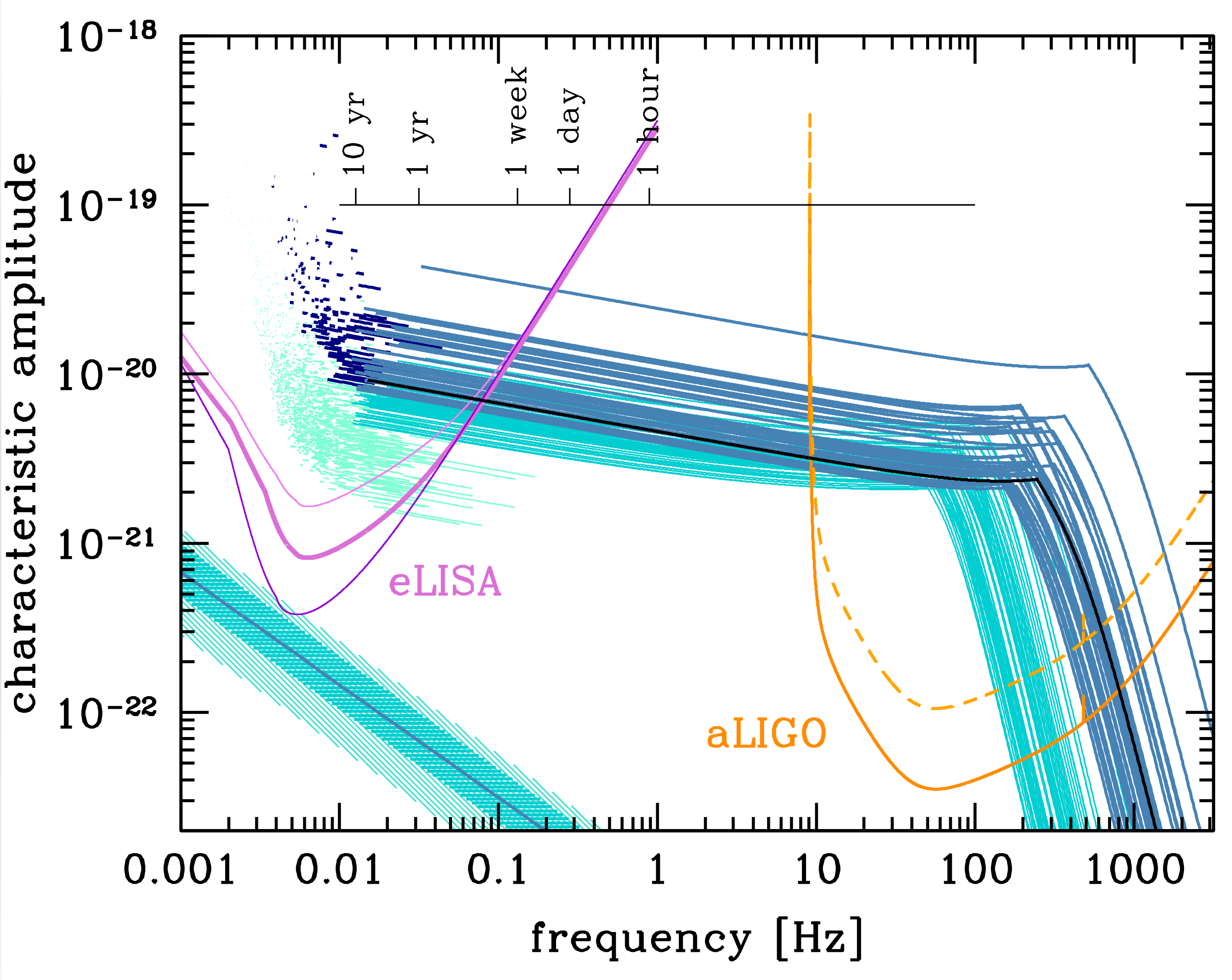
Sources: extreme-mass-ratio inspirals

- There are large astrophysical uncertainties, but expect to see between a few tens and a few hundreds of events.

Model	Mass function	MBH spin	Cusp erosion	M–σ relation	N_{p}	CO mass $[M_\odot]$	EMRI rate $[\mathrm{yr}^{-1}]$ Total	Detected (AKK)	Detected (AKS)
M1	Barausse12	a98	yes	Gultekin09	10	10	1600	294	189
M2	Barausse12	a98	yes	KormendyHo13	10	10	1400	220	146
M3	Barausse12	a98	yes	GrahamScott13	10	10	2770	809	440
M4	Barausse12	a98	yes	Gultekin09	10	30	520 (620)	260	221
M5	Gair10	a98	no	Gultekin09	10	10	140	47	15
M6	Barausse12	a98	no	Gultekin09	10	10	2080	479	261
M7	Barausse12	a98	yes	Gultekin09	0	10	15800	2712	1765
M8	Barausse12	a98	yes	Gultekin09	100	10	180	35	24
M9	Barausse12	aflat	yes	Gultekin09	10	10	1530	217	177
M10	Barausse12	a0	yes	Gultekin09	10	10	1520	188	188
M11	Gair10	a0	no	Gultekin09	100	10	13	1	1
M12	Barausse12	a98	no	Gultekin09	0	10	20000	4219	2279

Stellar-origin black hole binaries

- GW150914 would have been observable by LISA ~5 years before being observed by LIGO, with S/N~10 in a 5yr observation. (Sesana 2016)
- LISA provides sky location to ~0.few square degrees and time of coalescence to ~few s.
- Number of events could be high (several tens) but there are significant uncertainties.



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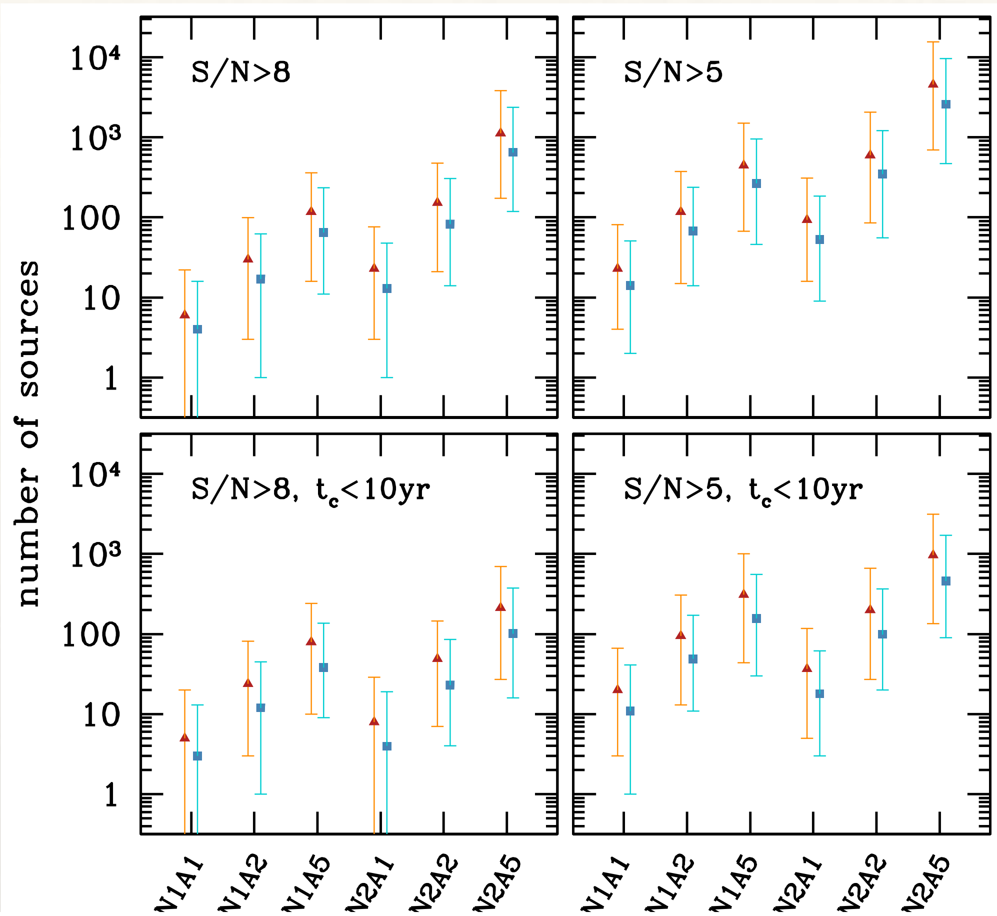

Mass distribution	$R/(\mathrm{Gpc}^{-3}\mathrm{yr}^{-1})$		
	PyCBC	GstLAL	Combined
Event based			
GW150914	$3.2^{+8.3}_{-2.7}$	$3.6^{+9.1}_{-3.0}$	$3.4^{+8.6}_{-2.8}$
LVT151012	$9.2^{+30.3}_{-8.5}$	$9.2^{+31.4}_{-8.5}$	$9.4^{+30.4}_{-8.7}$
GW151226	35^{+92}_{-29}	37^{+94}_{-31}	37^{+92}_{-31}
All	53^{+100}_{-40}	56^{+105}_{-42}	55^{+99}_{-41}
Astrophysical			
Flat in log mass	31^{+43}_{-21}	30^{+43}_{-21}	30^{+43}_{-21}
Power Law (−2.35)	100^{+136}_{-69}	95^{+138}_{-67}	99^{+138}_{-70}

Other sources

- Cosmological sources
  - Processes occurring at the TeV scale in the early Universe
  - Cosmic string networks.

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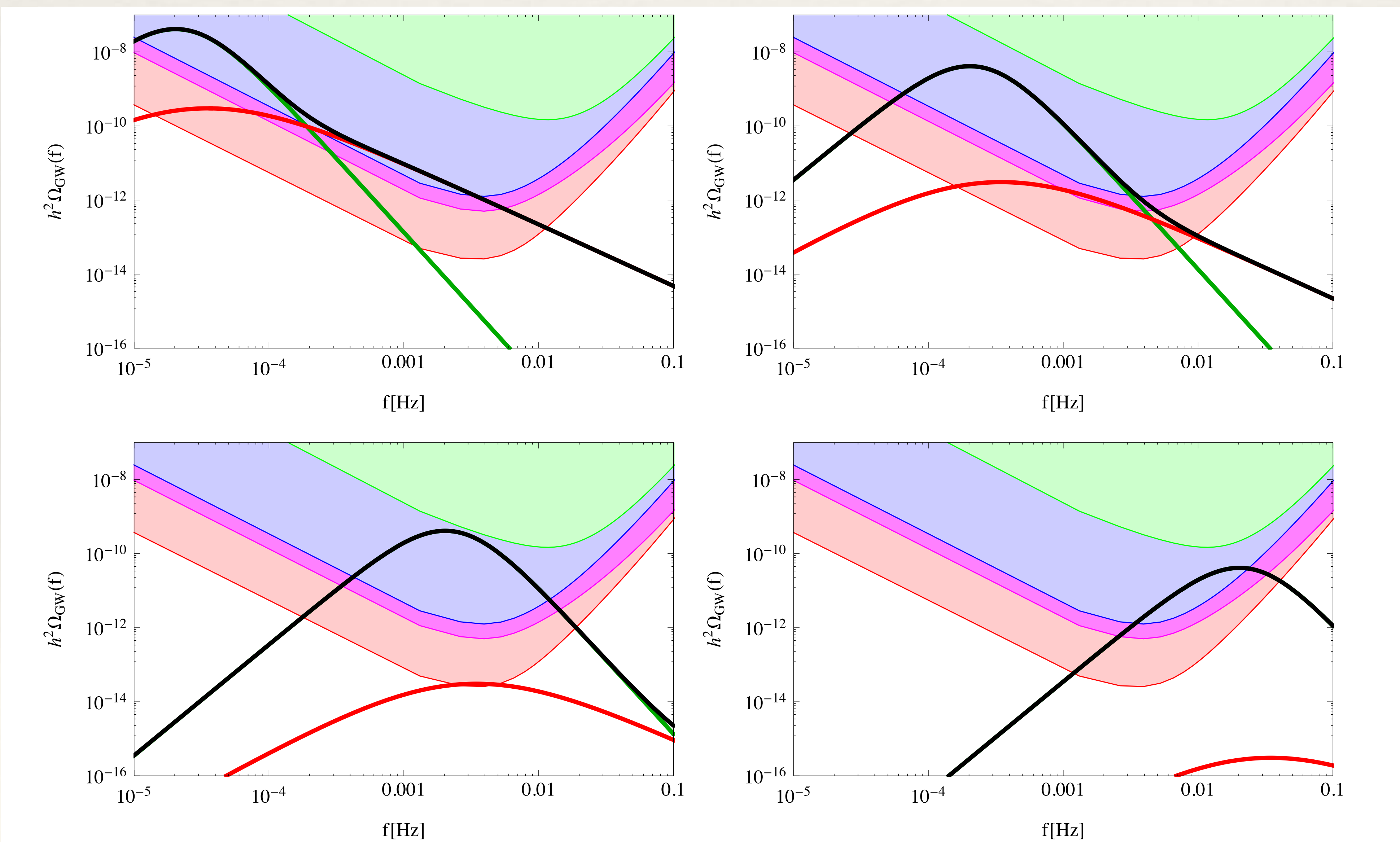


Caprini et al. (2016)

SGWB detectability: correlation

- Assuming two data streams of the form $d_I = n_I + h_I$, satisfying

$$\langle \tilde{h}_I(f)\tilde{h}_J^*(f')\rangle = \frac{1}{2}\delta(f-f')\Gamma_{IJ}(f)S_h(f) \qquad \langle \tilde{n}_I(f)\tilde{n}_J^*(f')\rangle = \frac{1}{2}\delta(f-f')\delta_{IJ}S_I(f)$$

- Filter data using a Kernel function of the form

$$C_{IJ}(K) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d_I(t)K(t-t')d_J(t')\,\mathrm{d}t\,\mathrm{d}t' = \int_{-\infty}^{\infty} \tilde{d}_I(f)\tilde{K}(f)\tilde{d}_J^*(f)\,\mathrm{d}f$$

- Optimal filter maximises the response in the presence of the signal to the RMS response to the noise. Desired filter is of the form

$$\tilde{K}_{IJ}(f) \propto \Gamma_{IJ}(f)S_h(f)/S_I^2(f)$$

- This is a matched filter for cross-correlation and yields optimal SNR

$$\rho = \sqrt{2T}\left[\int_{f_{\min}}^{f_{\max}} df\, \frac{\Gamma_{IJ}^2(f)S_h^2(f)}{P_{nI}(f)P_{nJ}(f)}\right]^{1/2}$$

SGWB detectability: Fisher matrix

- Fisher matrix provides Gaussian approximation to precision of parameter estimation. Increasingly valid as SNR increases.
- Computed using

$$\Gamma_{ij} = \mathbb{E}_p \left[\frac{\partial \ln p}{\partial \theta^i} \frac{\partial \ln p}{\partial \theta^j} \right]$$

- For estimation of SGWB parameters, assuming stationary-Gaussian noise, expression becomes

$$\Gamma_{ij} = T \int_0^\infty \frac{1}{S_n(f)^2} \frac{\partial S_n(f)}{\partial \lambda^i} \frac{\partial S_n(f)}{\partial \lambda^j} \mathrm{df}$$

- For multiple detectors:

$$\Gamma_{ij} = T \int_0^\infty (\Sigma_k^{-1})_{lr} \frac{\partial \Sigma_k^{rp}}{\partial \lambda^i} (\Sigma_k^{-1})_{pm} \frac{\partial \Sigma_k^{ml}}{\partial \lambda^j} \,\mathrm{df}$$

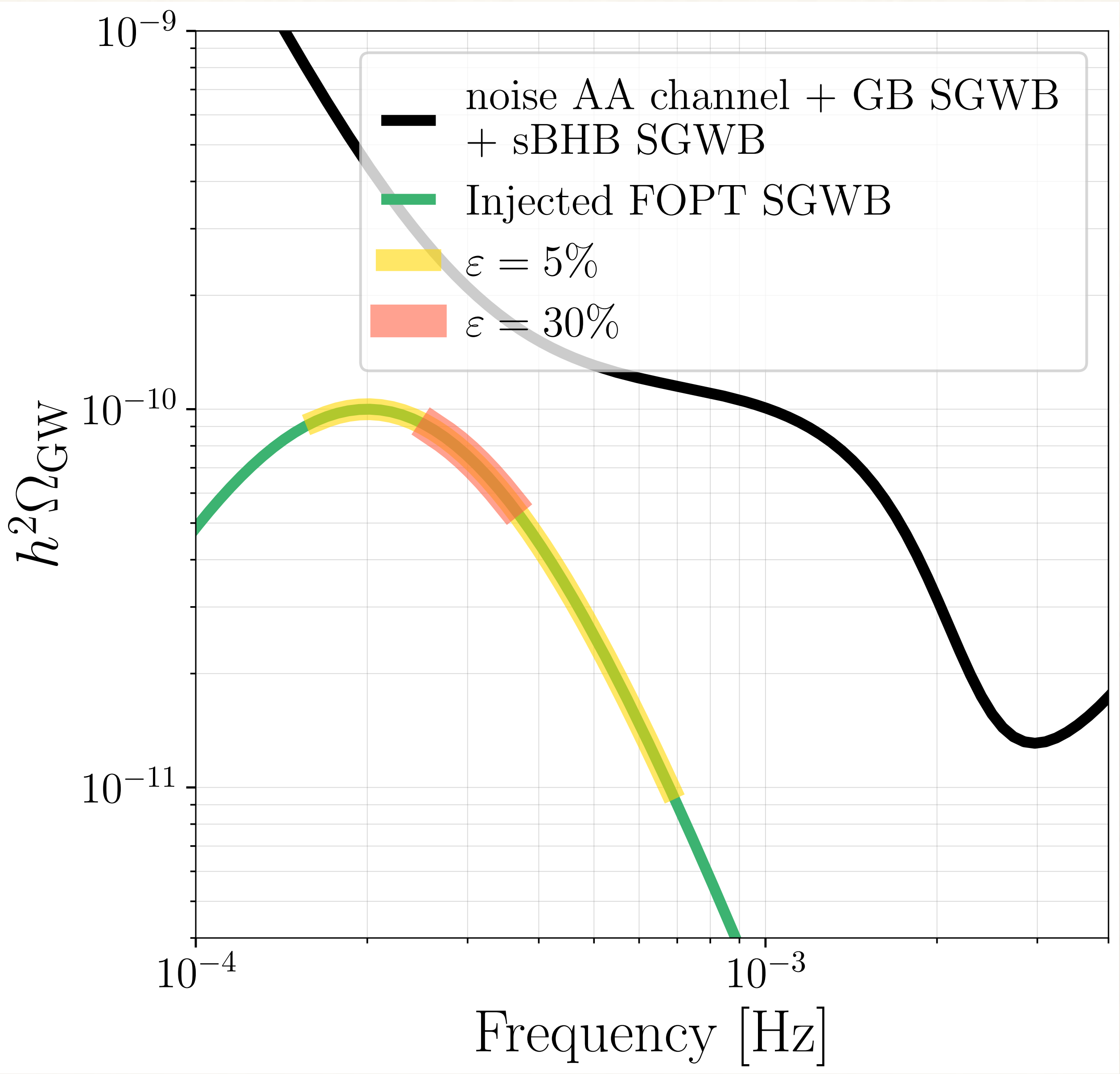
SGWB detectability: frequency dependence

- Karnesis, Lilley & Petiteau (2020) suggested a simple approach to characterising SGWB detectability.
- Use piecewise constant model for spectrum with instrumental noise uncertain by a specified amount.

$$p(\overline{D}|S_\mathrm{n}, S_\mathrm{o}) = \int_{\bar{S}_\mathrm{n}-\epsilon^-}^{\bar{S}_\mathrm{n}+\epsilon^+} \frac{e^{-N\frac{\overline{D}}{S_\mathrm{o}+S_\mathrm{n}}}}{\left(S_\mathrm{o}+S_\mathrm{n}\right)^N} \mathrm{d}S_\mathrm{n}$$

$$p(S_\mathrm{o}|\overline{D}, S_\mathrm{n}) = C\left(\Gamma_{N-1}\left(A^+\right) - \Gamma_{N-1}\left(A^-\right)\right)$$

$$A^\pm = \frac{N\overline{D}}{\overline{S}_\mathrm{n} + S_\mathrm{o} \pm \epsilon^\pm} \qquad C = \frac{1}{\left(\epsilon^+ + \epsilon^-\right)\left(N\overline{D}\right)^{N-1}}$$



- Can compute analytic Bayes factor for signal versus noise hypothesis and hence assess detectability vs f.

Single link signal model

- Decompose signal into Fourier modes

$$h_{ab}(\vec{x},t) = \int_{-\infty}^{\infty} \mathrm{d}f \int \mathrm{d}\Omega_{\hat{k}}\, \mathrm{e}^{2\pi i f(t-\hat{k}\cdot\vec{x})} \sum_A \tilde{h}_A(f,\hat{k}) e^A_{ab}(\hat{k})$$

- Assume statistics for <u>homogeneous, isotropic and non-chiral SGWB</u>:

$$\langle \tilde{h}_A(f,\hat{k})\, \tilde{h}^*_B(f',\hat{k}')\rangle = \delta(f-f')\delta(\hat{k}-\hat{k}')\delta_{AB}\frac{P_h^{AB}(f)}{16\pi} \qquad \langle \tilde{h}_A(f,\hat{k})\, \tilde{h}_B(f',\hat{k}')\rangle = 0$$

- Compute the cross-spectral densities for all <u>single link</u> measurements

$$S^{\eta,\mathrm{GW}}_{ij,mn}(f) \equiv \sum_A \mathcal{R}^A_{ij,mn}\, P_h^{AA}(f) = \frac{f^2}{f_{ij} f_{mn}} e^{-2\pi i f(L_{ij}-L_{mn})} \sum_A P_h^{AA}(f)\, \Upsilon^A_{ij,mn}(f)$$

$$\Upsilon^A_{ij,mn}(f) = \int \frac{\mathrm{d}\Omega_{\hat{k}}}{4\pi}\, \mathrm{e}^{-2\pi i f \hat{k}\cdot(\vec{x}_i - \vec{x}_m)}\, \xi^A_{ij}(f,\hat{k})\, \xi^A_{mn}(f,\hat{k})^*$$

Single link signal model

- Consider just 2 main noise sources in each single link

$$\eta_{ij}^{\mathrm{N}}(t) = n_{ij}^{\mathrm{OMS}}(t) + D_{ij} n_{ji}^{\mathrm{TM}}(t) + n_{ij}^{\mathrm{TM}}(t)$$

- Assume fixed and perfectly known noise shape

$$S_{ij}^{\mathrm{TM}}(f) = A_{ij}^2 \times 10^{-30} \times \left(1 + \left(\frac{0.4\,\mathrm{mHz}}{f}\right)^2\right)\left(1 + \left(\frac{f}{8\,\mathrm{mHz}}\right)^4\right) \times \left(\frac{1}{2\pi f c}\right)^2 \times (\mathrm{m}^2/\mathrm{s}^3)\,, \qquad (2.22\mathrm{a})$$

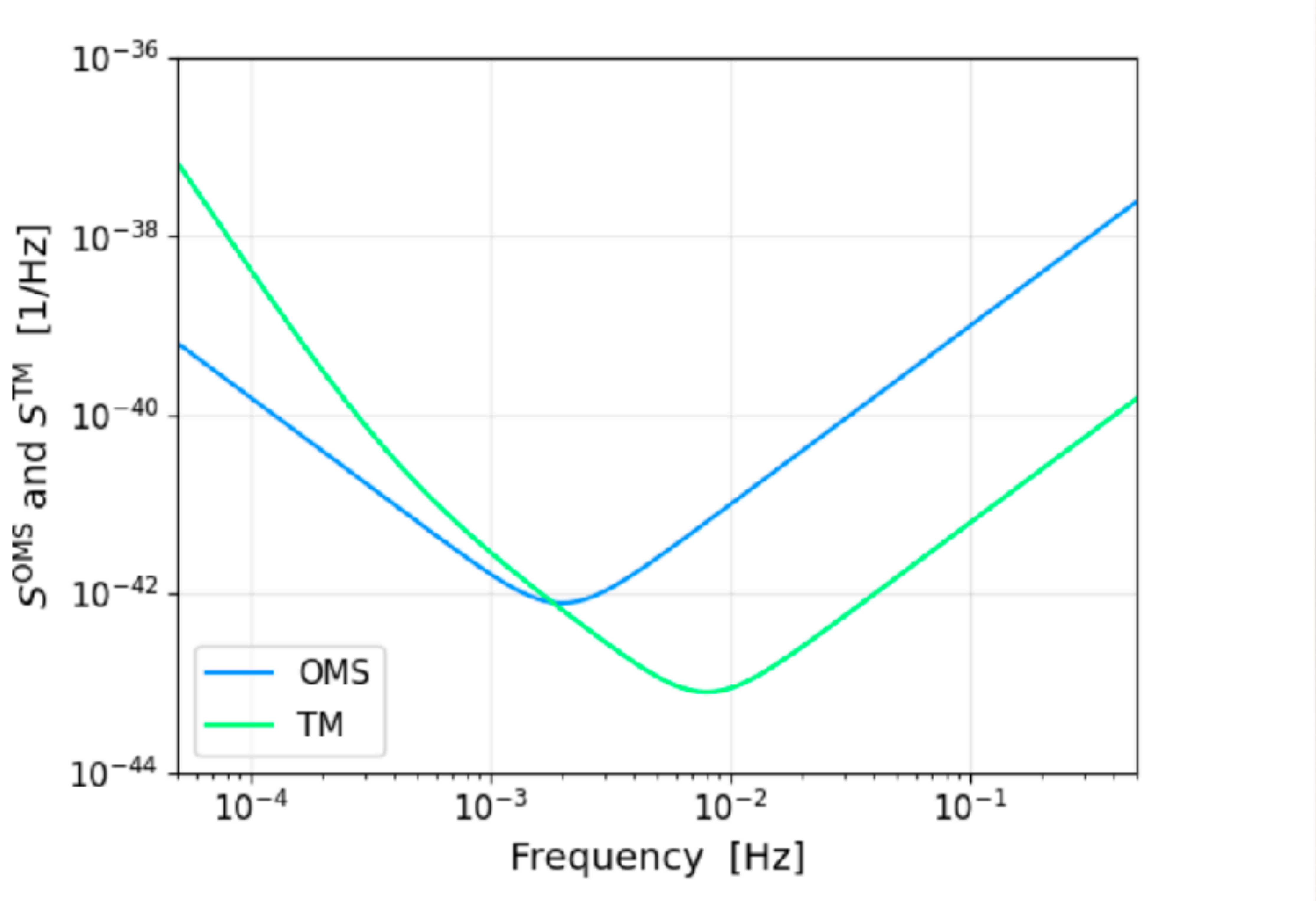
$$S_{ij}^{\mathrm{OMS}}(f) = P_{ij}^2 \times 10^{-24} \times \left(1 + \left(\frac{2 \times 10^{-3}\,\mathrm{Hz}}{f}\right)^4\right) \times \left(\frac{2\pi f}{c}\right)^2 \times (\mathrm{m}^2/\mathrm{Hz})\,, \qquad (2.22\mathrm{b})$$

- Consider either equal noise levels: $A_{ij} = A = 3 \text{ and } P_{ij} = P = 15$

- or unequal noise levels (20% std): $A_{ij} = \{3.61, 3.02, 2.87, 3.43, 2.65, 3.45\}\,,$

$$P_{ij} = \{14.00, 16.93, 9.43, 21.55, 17.04, 20.83\}$$

Single link signal model



Single link signal model

- Consider 1st generation TDI for simplicity

- Compare Michelson X,Y,Z,

$$\mathrm{X} = (1 - D_{13}D_{31})(\eta_{12} + D_{12}\eta_{21}) + (D_{12}D_{21} - 1)(\eta_{13} + D_{13}\eta_{31}) \ ,$$

- and Sagnac

$$\alpha = \eta_{12} + D_{12}\eta_{23} + D_{12}D_{23}\eta_{31} - (\eta_{13} + D_{13}\eta_{32} + D_{13}D_{32}\eta_{21}) \ ,$$

$$\zeta = D_{12}(\eta_{31} - \eta_{32}) + D_{23}(\eta_{12} - \eta_{13}) + D_{31}(\eta_{23} - \eta_{21}) \ .$$

- Construct quasi-orthogonal variables

$$\mathcal{A} = \frac{\gamma - \alpha}{\sqrt{2}} \ , \qquad \mathcal{E} = \frac{\alpha - 2\beta + \gamma}{\sqrt{6}} \ , \qquad \mathcal{T} = \frac{\alpha + \beta + \gamma}{\sqrt{3}} \ ,$$

$$\mathrm{A} = \frac{\mathrm{Z} - \mathrm{X}}{\sqrt{2}} \ , \qquad \mathrm{E} = \frac{\mathrm{X} - 2\mathrm{Y} + \mathrm{Z}}{\sqrt{6}} \ , \qquad \mathrm{T} = \frac{\mathrm{X} + \mathrm{Y} + \mathrm{Z}}{\sqrt{3}} \ .$$

Single link signal model

- Define Fourier trafo of TDI variables as vector of Fourier coefficients applied to single link measurements:

$$\tilde{V}(f) = \sum_{ij\in\mathcal{I}} c^{V}_{ij}(f)\, \tilde{\eta}_{ij}(f)$$

Formally define CSD via expectation value of FT

$$\langle \tilde{U}(f)\, \tilde{V}^*(f') \rangle = \frac{1}{2}\, S^{UV}(f)\, \delta(f - f')\,,$$

- Write result as contraction of single-link CSD matrix and TDI coefficient matrix

$$\begin{aligned}
\langle \tilde{U}(f)\tilde{V}^*(f') \rangle &= \sum_{ij,mn\in\mathcal{I}} c^{U}_{ij}(f)\, c^{V*}_{mn}(f') \langle \tilde{\eta}_{ij}(f)\, \tilde{\eta}^*_{mn}(f') \rangle\,, \\
&= \underbrace{\frac{1}{2} \sum_{ij,mn\in\mathcal{I}} \underbrace{c^{U}_{ij}(f)\, c^{V*}_{mn}(f)}_{C^{UV}_{ij,mn}(f)}\, S^{\eta}_{ij,mn}(f)}_{S^{UV}(f)}\, \delta(f - f')\,.
\end{aligned}$$