Modelling of Stochastic Gravitational Wave signals in LISA

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Data analysis challenges for stochastic gravitational wave backgrounds

18 July 2023

ESA | NASA lisa

LISA CONSORTIUM

- ✤ Part I: Data Analysis for stochastic GW signals [Just a recap focusing on SGWB]
- ✤ Part II: Modelling the stochastic signals for the band of LISA

Part I

Data Analysis for stochastic signals

[very briefly, and not about maps]

Assume

$$d(t) = s(t, \vec{\theta}) + n(t)$$

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Then

$$p(n) = C \times \exp\left(-\frac{1}{2}(n|n)\right)$$

where

$$(a|b) = 2\int_0^\infty \mathrm{d}f [\tilde{a}^T C_n^{-1} \tilde{b}^*]$$

Then the likelihood is written as

$$p(d|h) = C \times \exp\left(-\frac{1}{2}(d-h|d-h)\right)$$

- Usually when it comes to stochastic signals we are interested in their power, and not the amplitude at each sample.
- So, if we assume that the amplitude is distributed as a Gaussian variable as

$$p(h|S_h) = C' \times \exp\left(-\frac{h^2}{2S_h}\right)$$

- We can marginalise it over amplitude, which yields

$$p(d|h) = C'' \times \exp\left(-\frac{1}{2}(d|d)\right)$$

- But now, inside the $\int_0^\infty \mathrm{d}f[\tilde{a}^T C_n^{-1} \tilde{b}^*]$, we write:

$$C_n(f) = S_n(f) + R(f)S_h(f)$$

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Challenges!

Those will be discussed *a lot* during this meeting!

J. Gair and N. Cornish gave us a pretty good overview this morning.

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Challenges!

$$C_n(f) = S_n(f) + R(f)S_h(f)$$

- Stationarity (gaps, glitches, astro signals, …)
- Not completely known (many signals in there)
- LPF lessons (unknown noise components)
- Correlations between channels
- Residuals
- […]

Challenges!

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Challenges!

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- Astrophysical & Cosmological
- Non-stationary, anisotropic
- Models with many different spectral shapes
- Parts of response can be similar to noise
- […]
- We will hear a lot about these tomorrow.

Part II

Modelling the stochastic signals for the band of LISA

- Previous speakers gave us a really nice overview on the different sources of stochastic signals.
- We can try now to assign the different models to the various sources.

Cosmological sources

Cosmological sources

- Chiara this morning gave us an overview of the physical processes that might generate cosmological stochastic signals.
- Most processes predict a signal in the LISA band that follows a particular spectral shape.

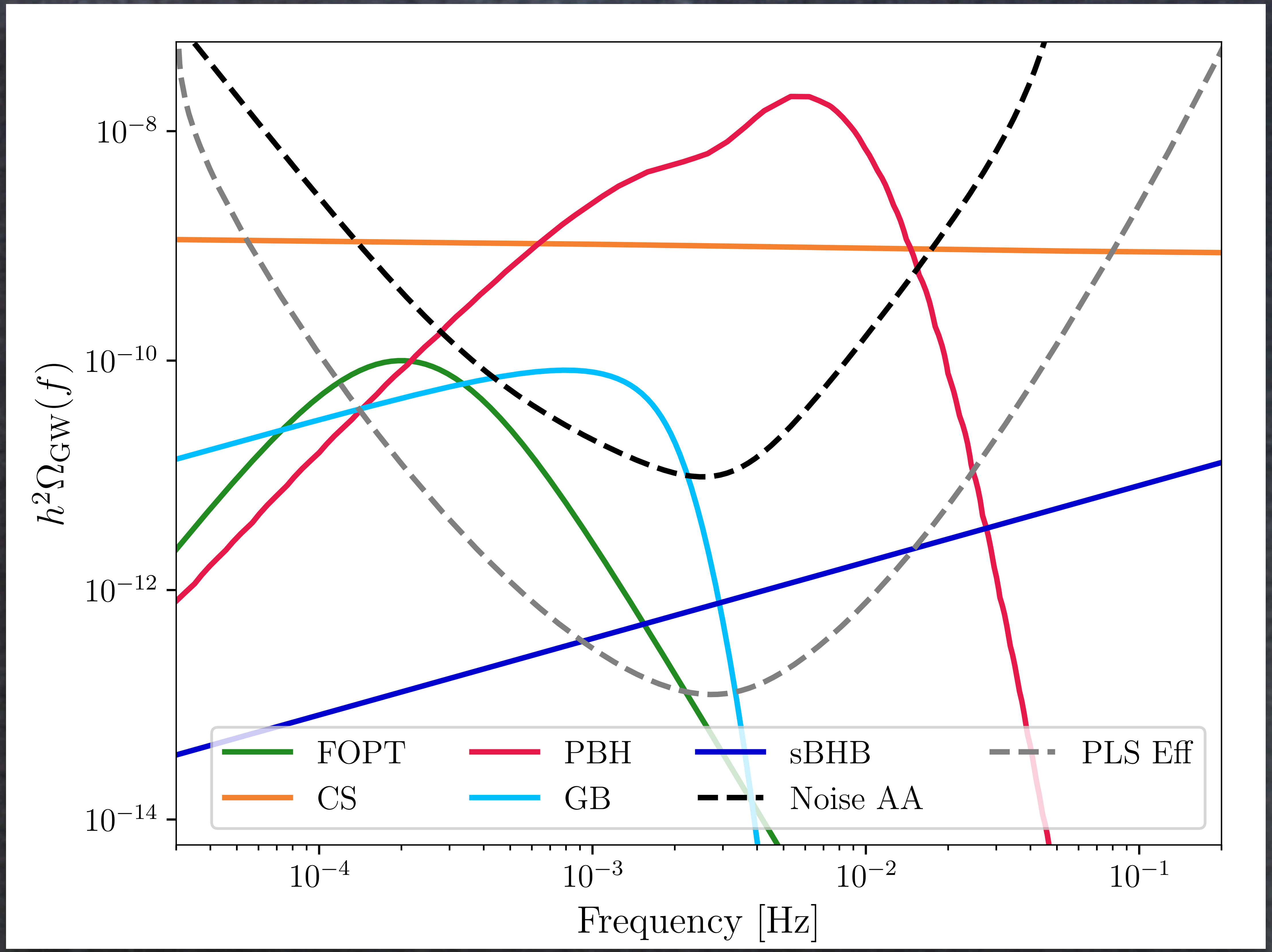


Figure by M. Pieroni, for the Red Book

Stellar Origin Black Hole Binaries

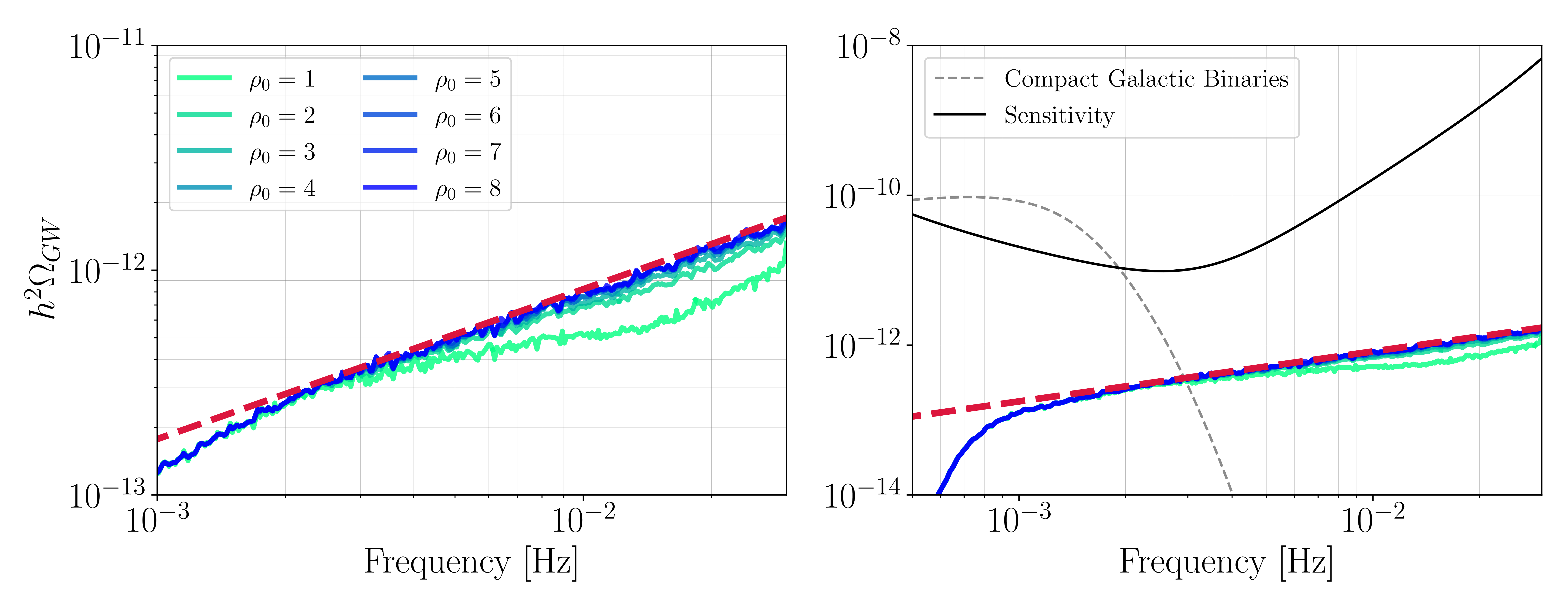
Stellar Origin Black Hole Binaries

- We expect to get

$$h^2\Omega_{\rm GW}(f) = \frac{h^2 8\pi^{5/3} f^{2/3}}{9H_0^2}\int_0^\infty {\rm d}\mathcal{M} p\left(\mathcal{M}(m_1,m_2)\right)\mathcal{M}^{5/3}\int_0^\infty {\rm d}R(z)\frac{(1+z)^{2/3}}{H(z)}$$

- Which means: $h^2\Omega_{\rm GW}(f) \propto f^{2/3}$

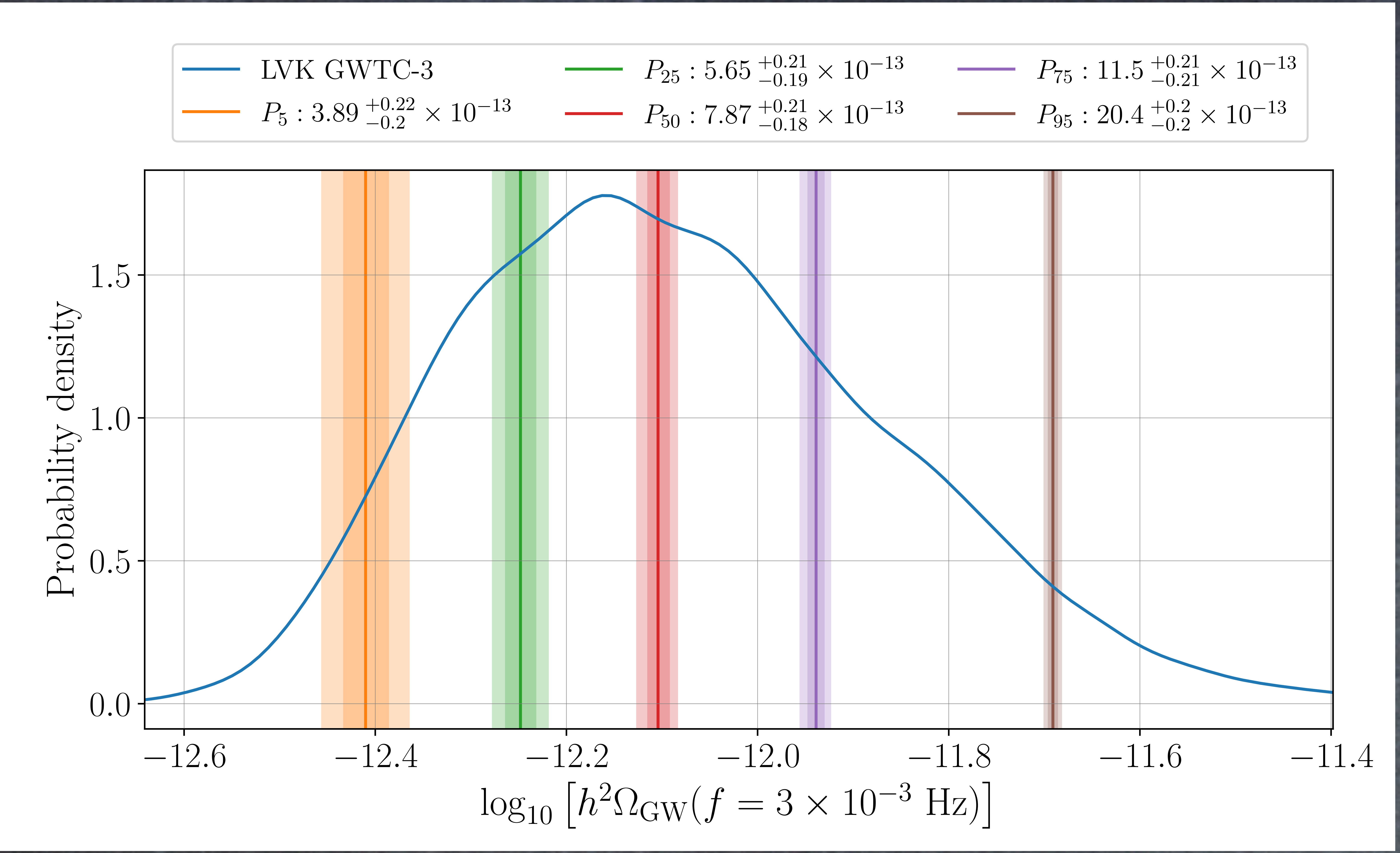
[Babak+. acc. to *JCAP*, 2023]



Stellar Origin Black Hole Binaries

- And how detectable will that signal be?

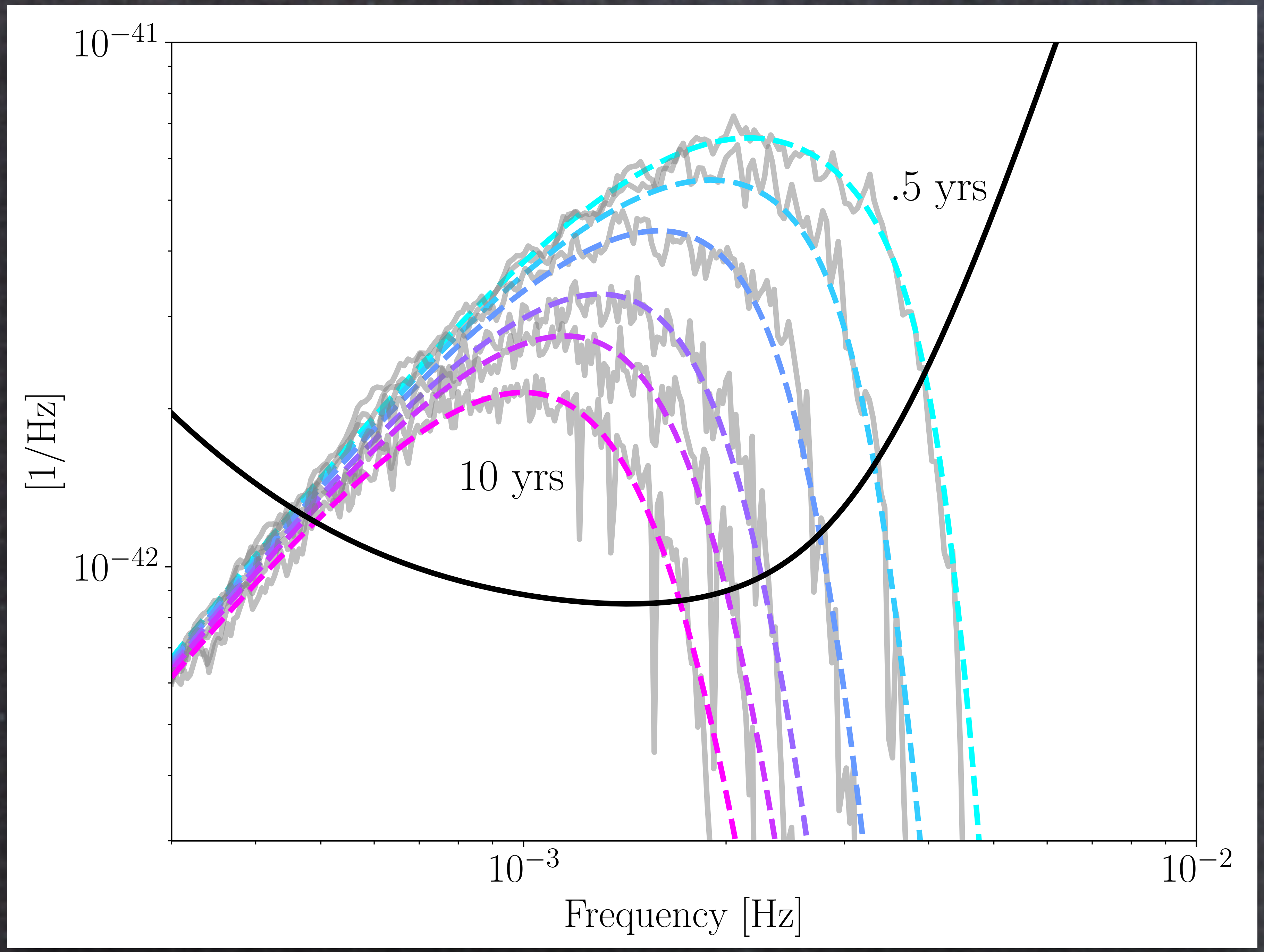
[Babak+, acc. to *JCAP*, 2023]



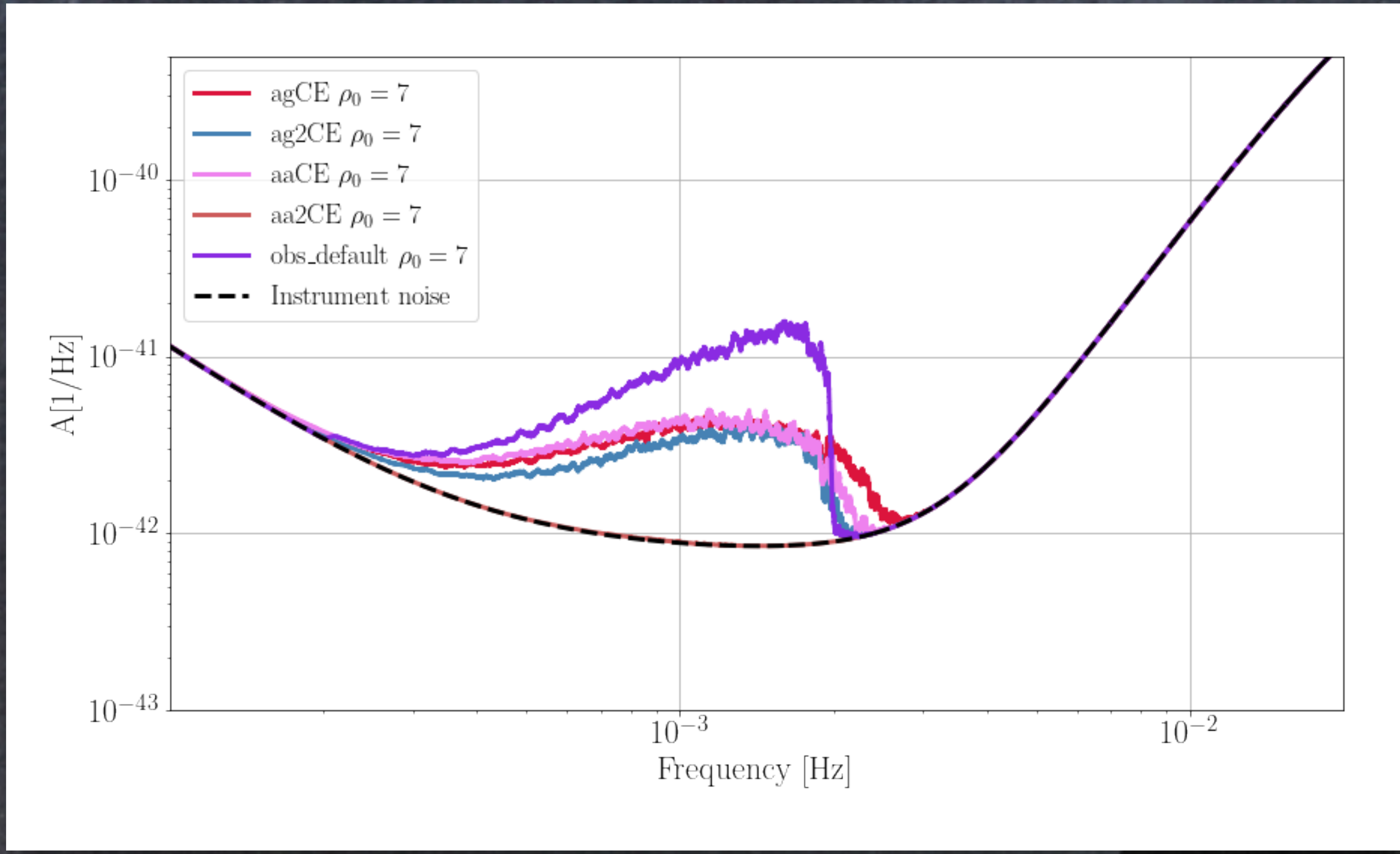
Galactic Binaries

Galactic Binaries

- More complicated stochastic signal that depends on many things
  - Population properties.
  - Our abilities to analyse the data [remember Neils' talk this morning].
  - The measurement length.



[NK+, *PRD* 104 043019, 2021]



[M Georgousi, Msc thesis, AUTh, 2021]
[V Korol+, MNRAS, 511, 4, 2022]

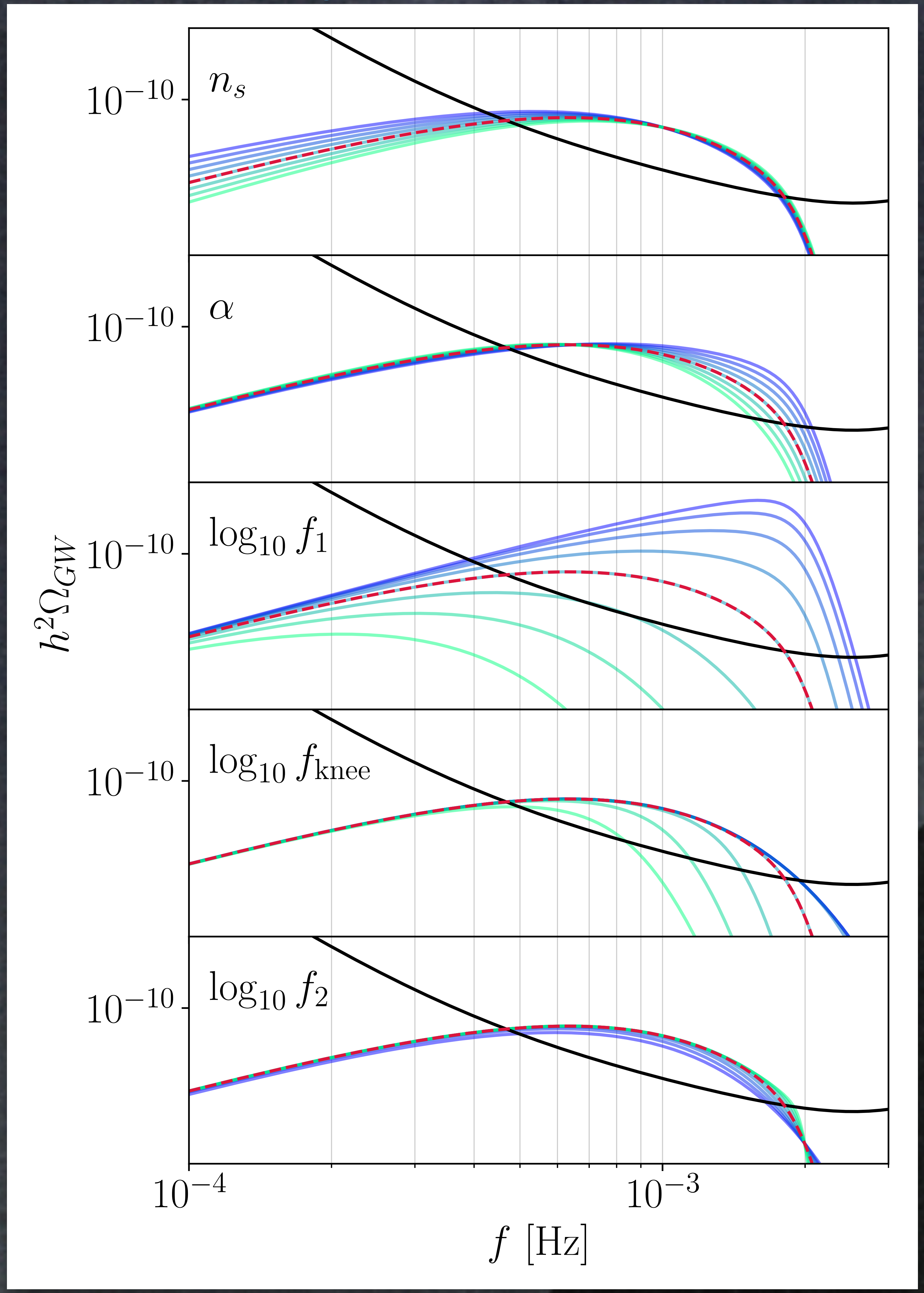
Galactic Binaries

- A great selection of resulting stochastic signals has this particular shape, which can be modelled with an empirical model as:

$$S_{\rm gal} = \frac{A}{2} f^{-n_s} e^{-(f/f_1)^{\alpha}} \left(1 + \tanh\left(\left(f_{\rm knee} - f\right)/f_2\right)\right)$$

- Similar models work equally well…

[M Georgousi+, MNRAS, 519, 2, 2023]

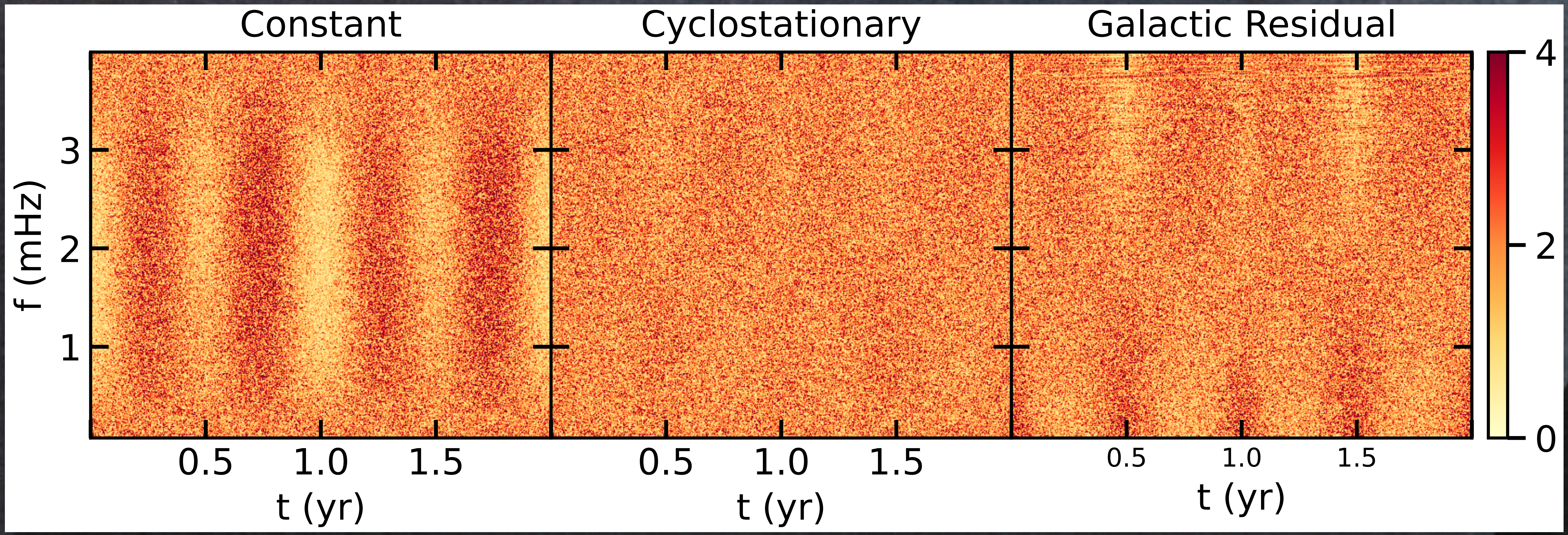


Galactic Binaries

- Cyclo-stationarity can be modelled and taken into account:

$$S_{\rm cyclo}^{AE} = r_{\rm n}^{AE} < \overline{S}_{\rm gal} > + S_{\rm instr}$$

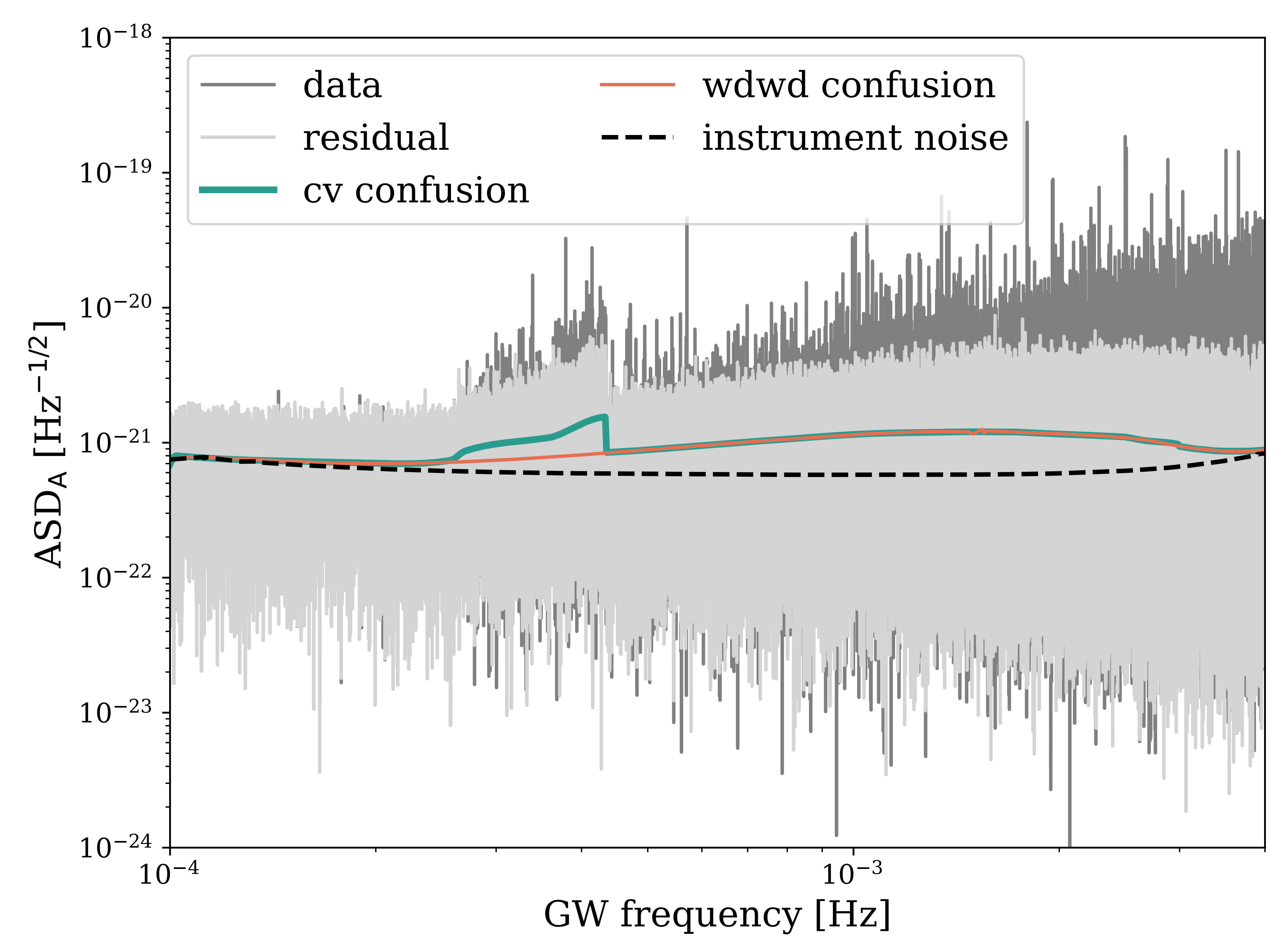
[Digman and Cornish, ApJ 940 10, 2022]



Galactic Binaries

- However, there might be other effects that may “disturb” this smooth shape. For example:

[S. Scaringi+, 2307.02553, 2023]



Generic Spectral shape

- Thus, given the "zoo" of stochastic signals, we might want to take a more *agnostic* route.

Generic Spectral shape

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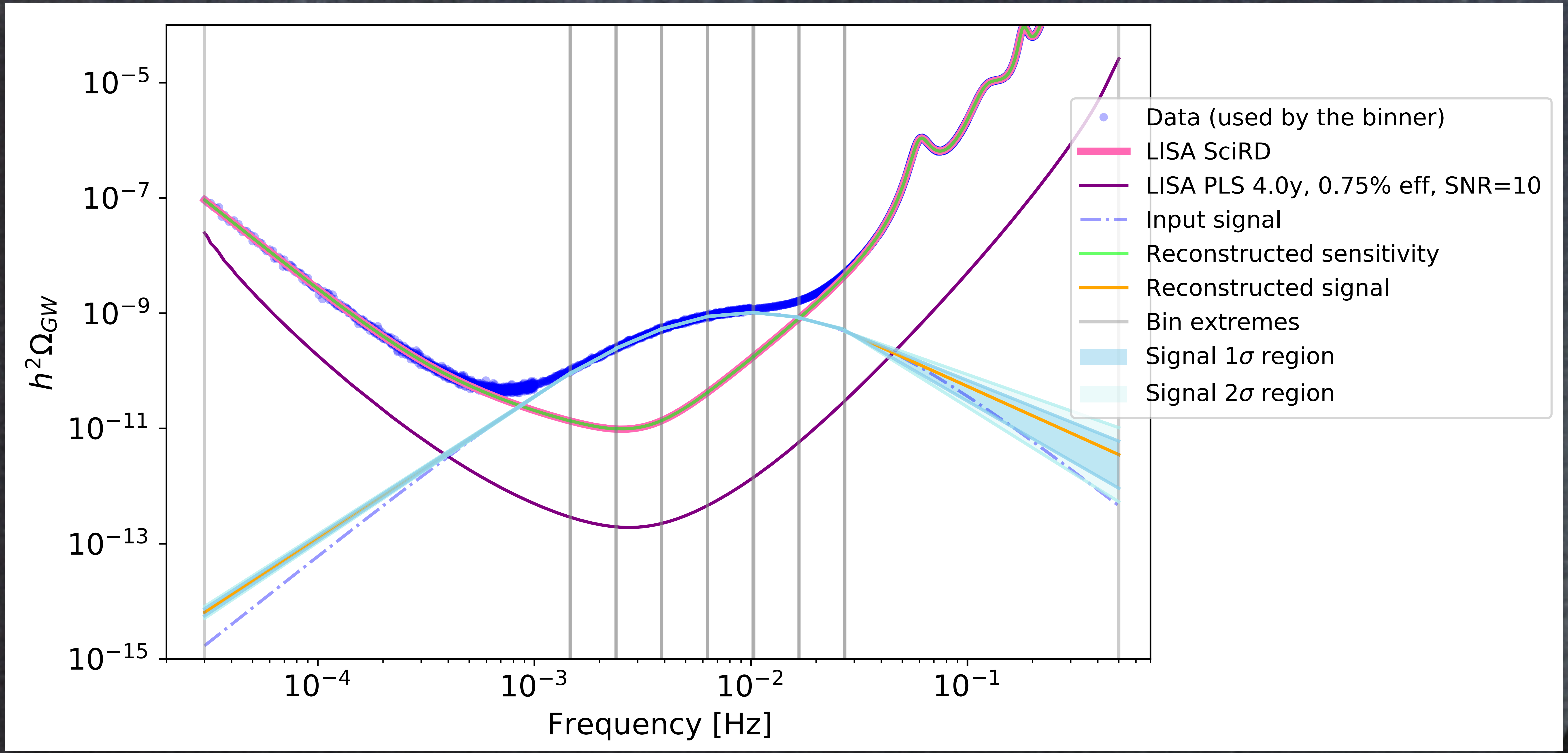
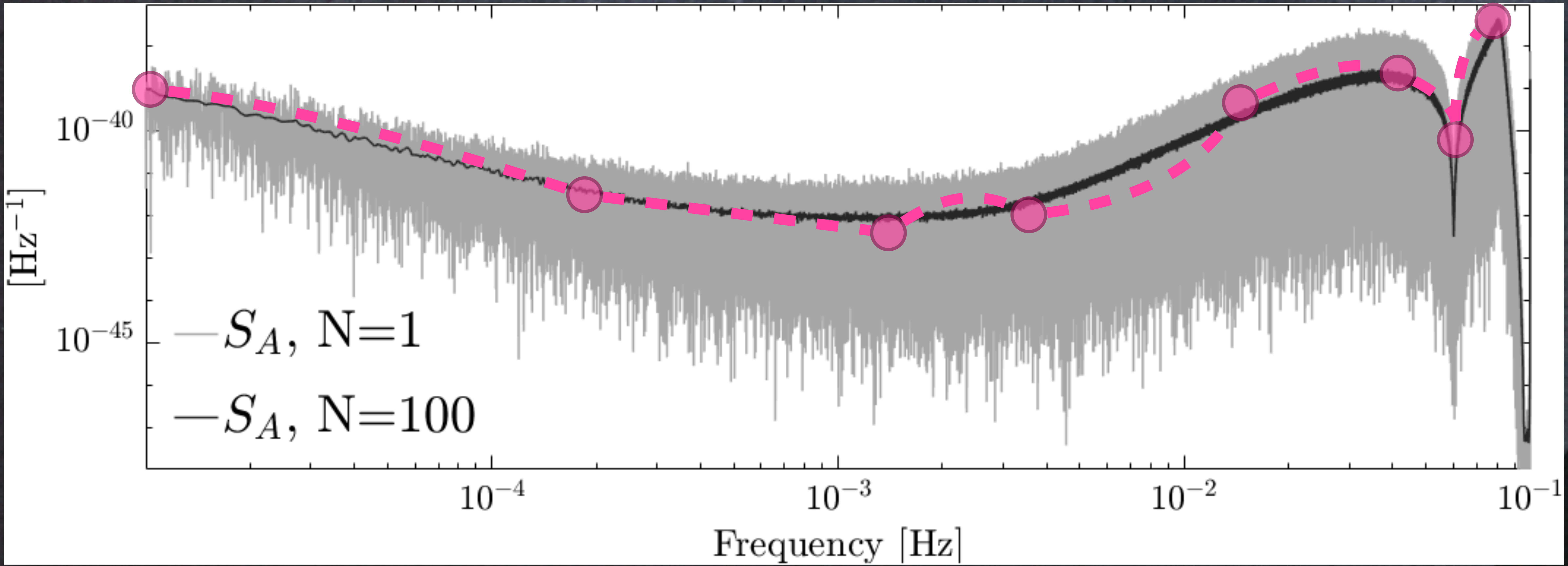


[Flauger+ JCAP01, 059, 2023]

Generic Spectral shape

- Thus, given the "zoo" of stochastic signals, we might want to take a more *agnostic* route. B: Using a spline model.



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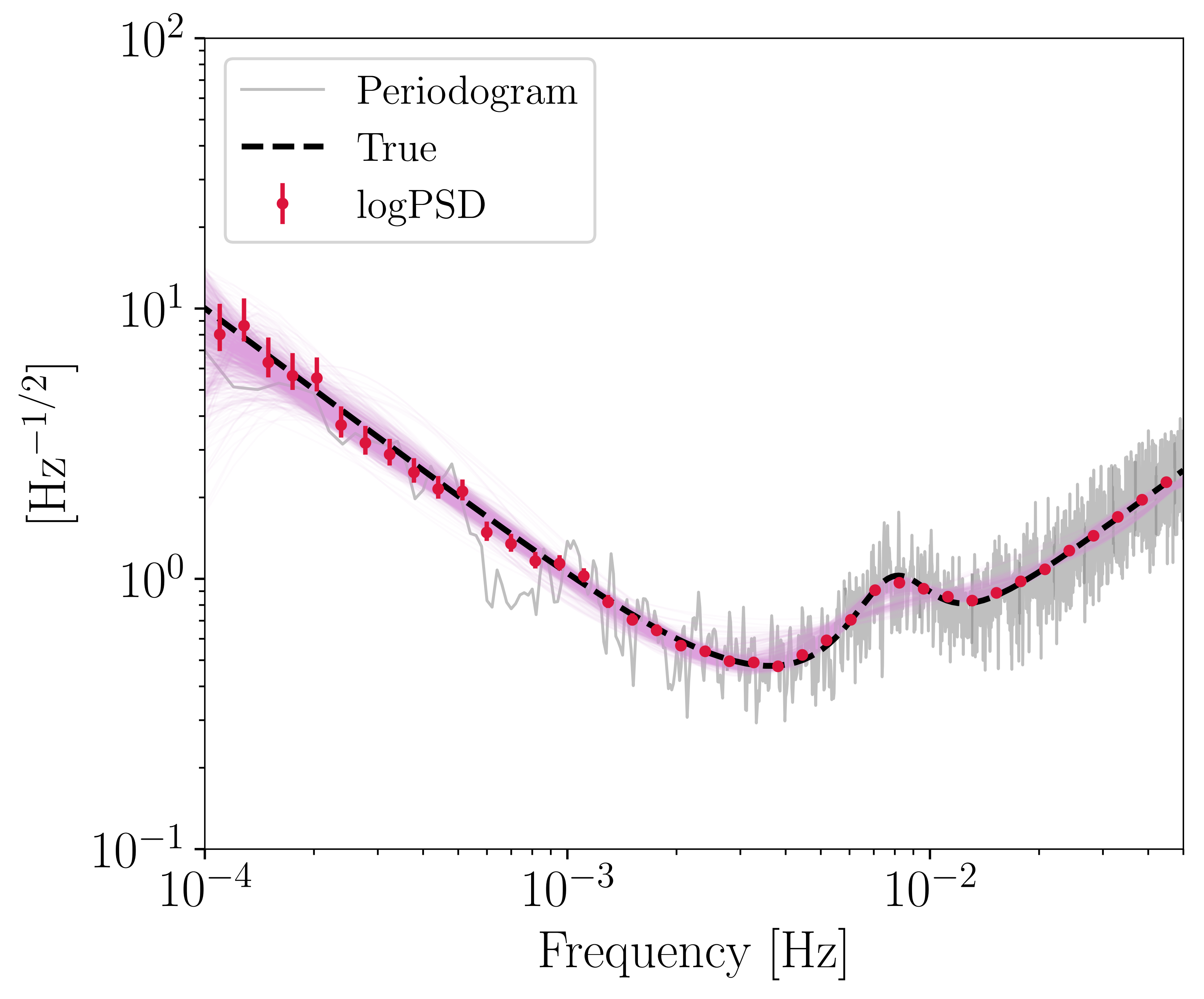

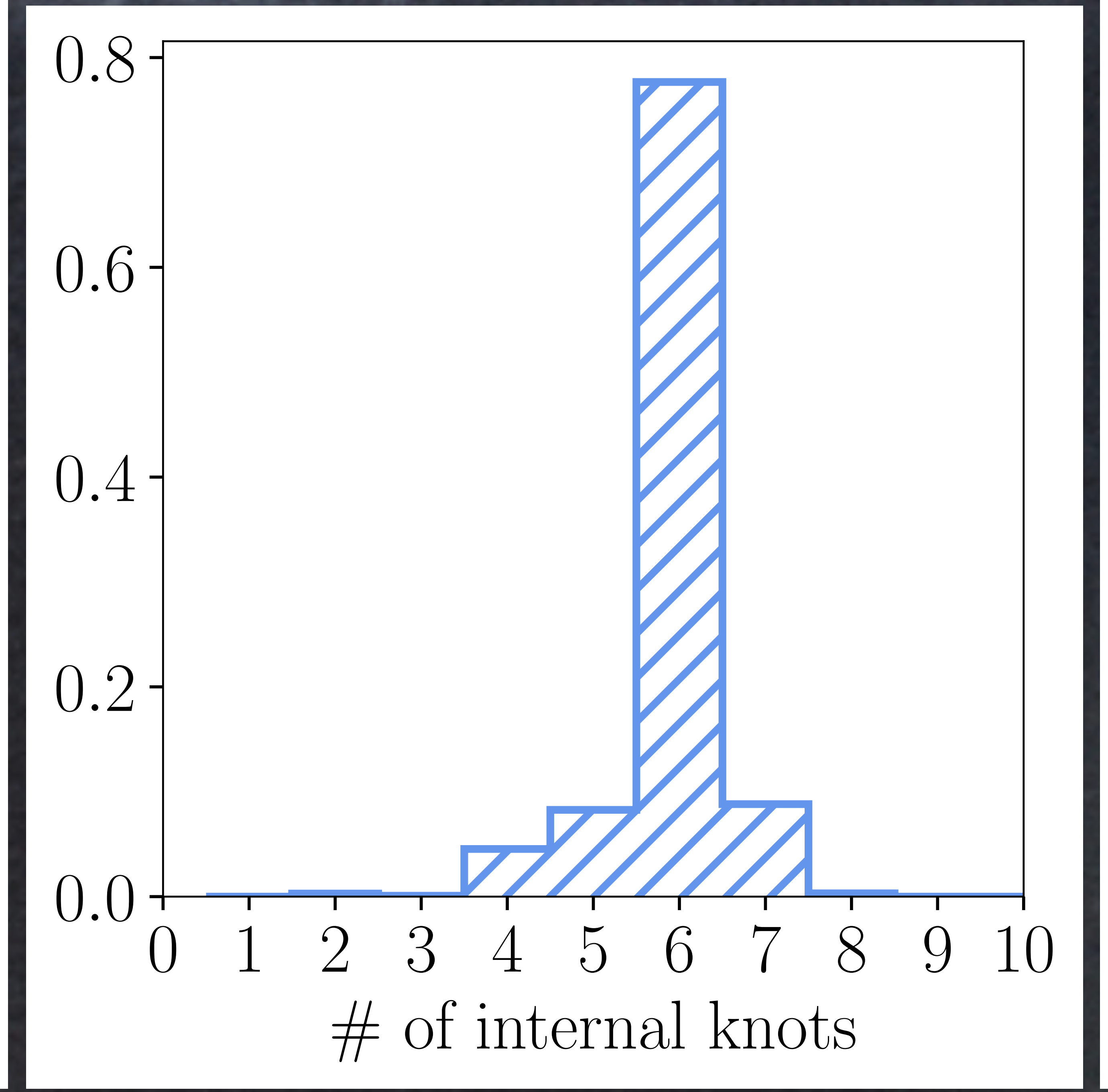

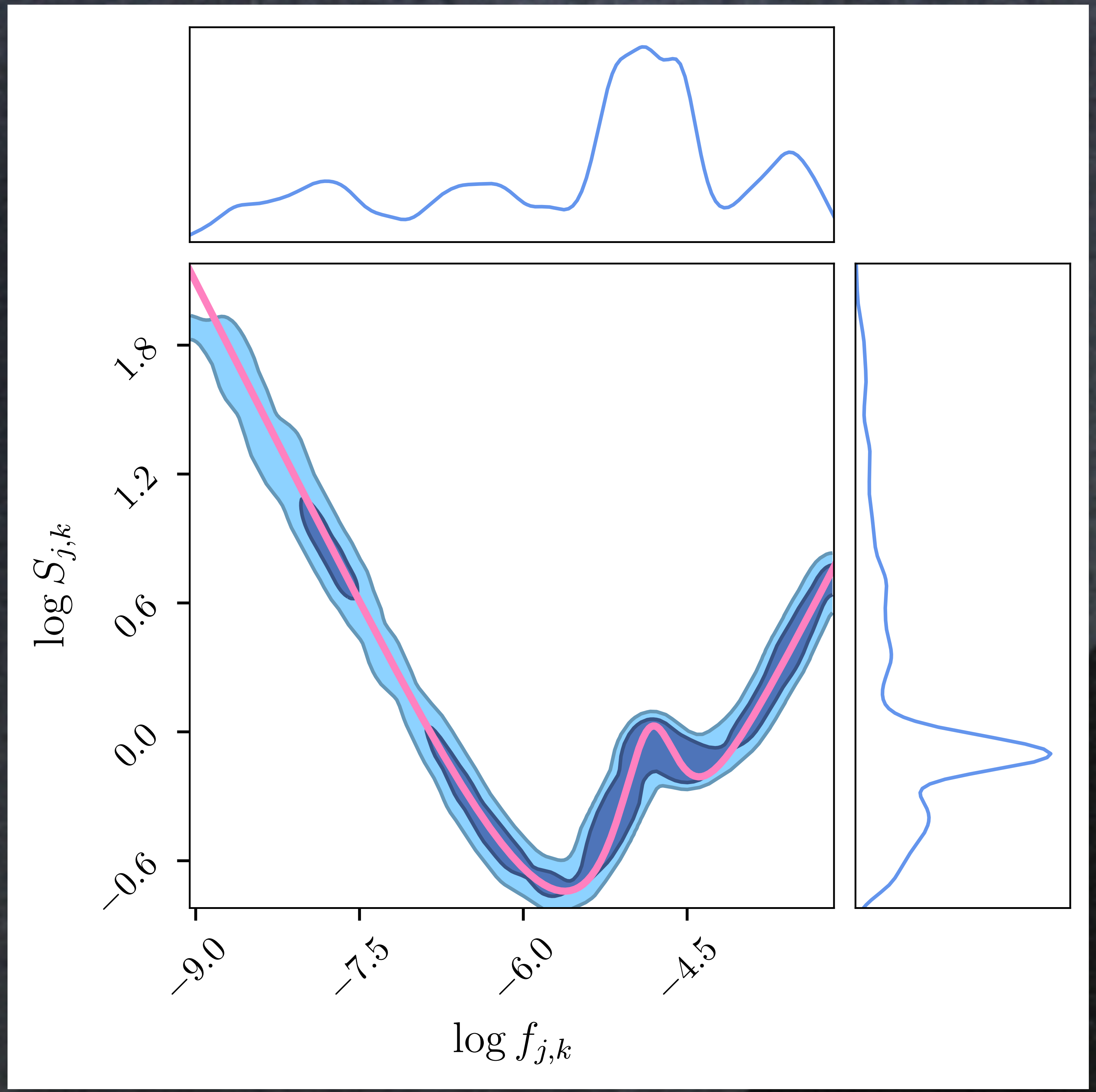

[NK+ arXiv:2303.02164, 2023]

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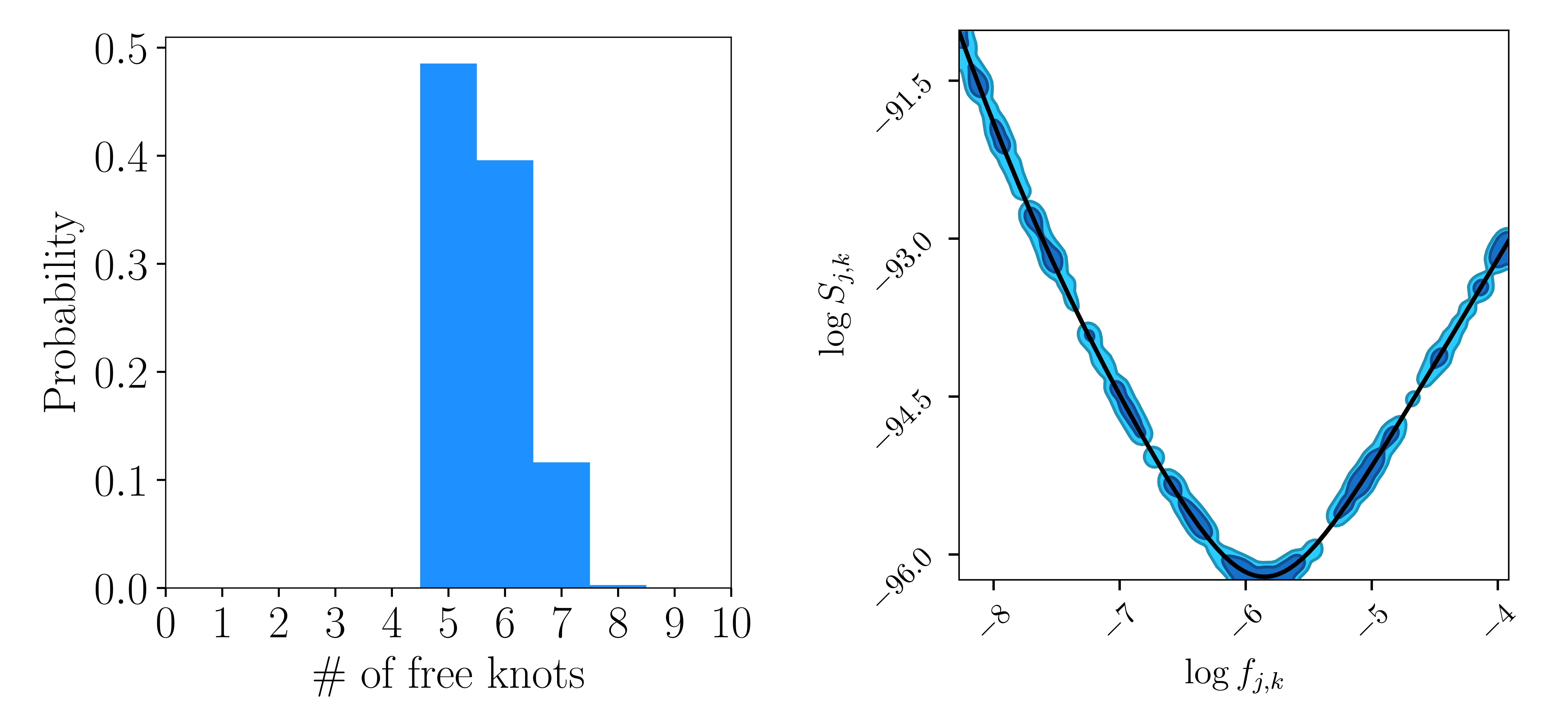
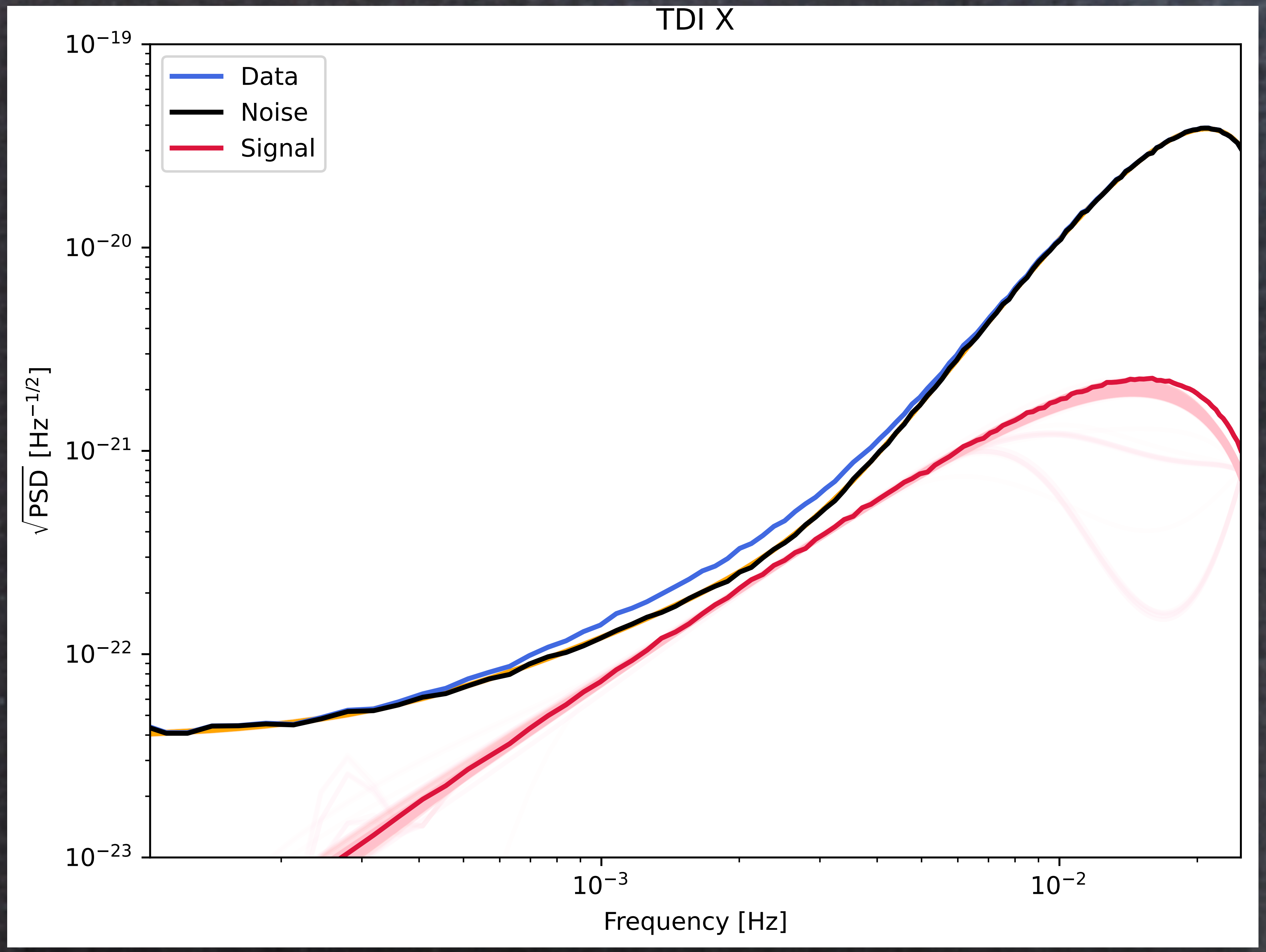
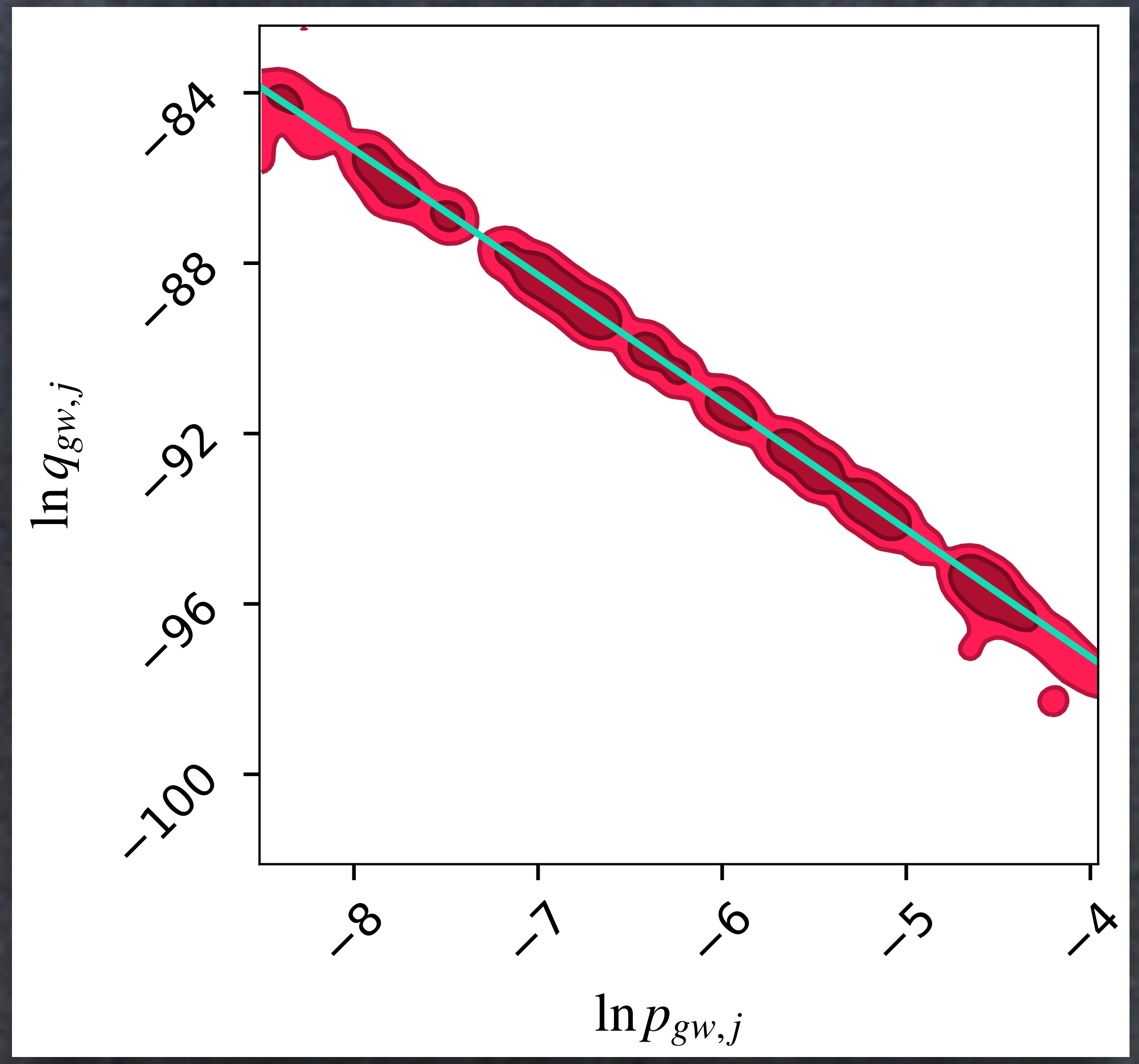
[Q Baghi+ *JCAP* 04, 066, 2023]

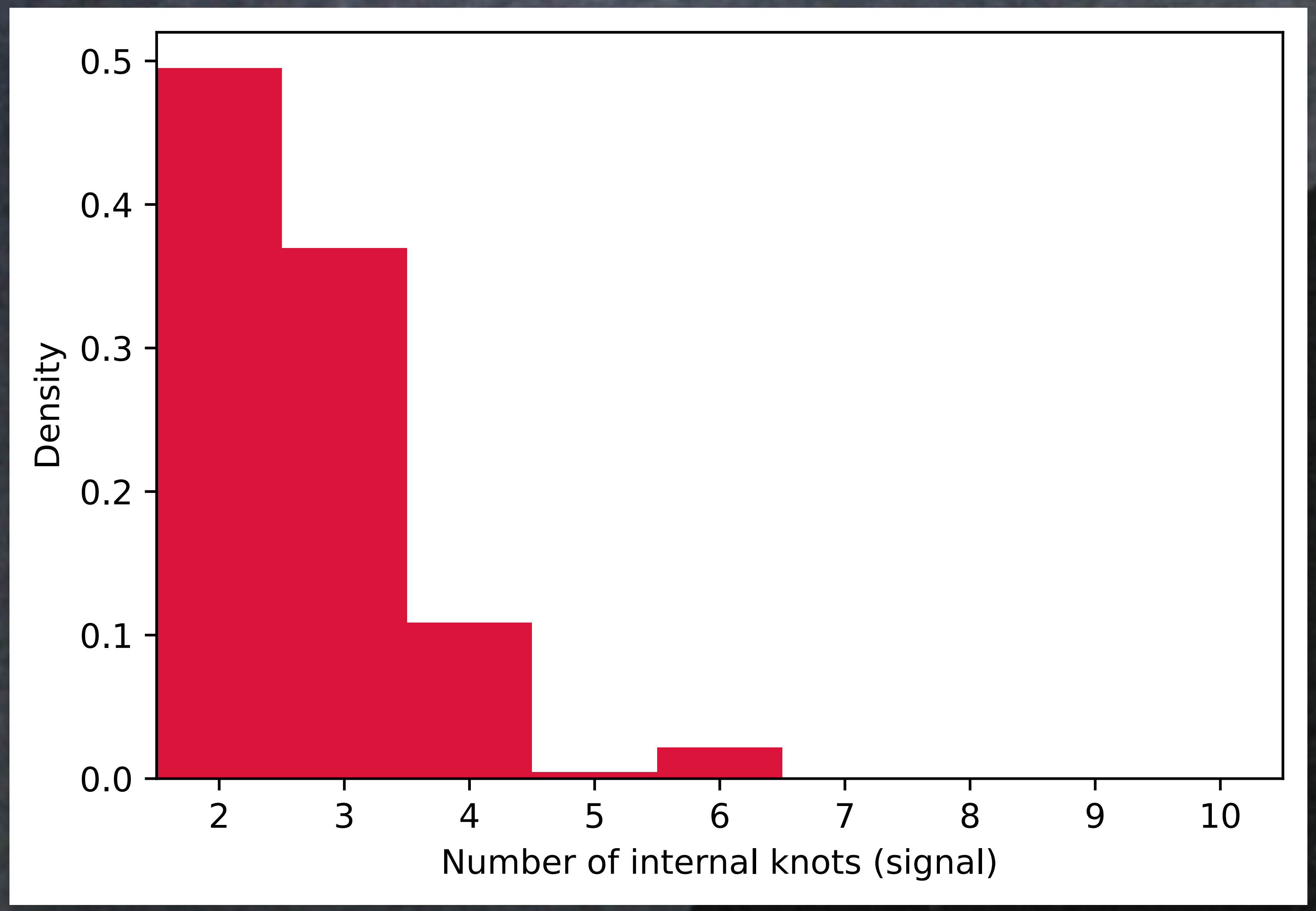


Generic Spectral shape

- Thus, given the "zoo" of stochastic signals, we might want to take a more *agnostic* route. B: Using a spline model.
- Too much freedom is causing degeneracies though. It's very hard to assume a shape-agnostic model for the noise *and* the signal…







[N Galanis, *BSc thesis*, AUTh, 2023]

Generic Spectral shape

- More complicated models might be needed.
- For example a comparison of models based on B-splines and more shape-specific models.
- For example we can check the `Bayesline` work.

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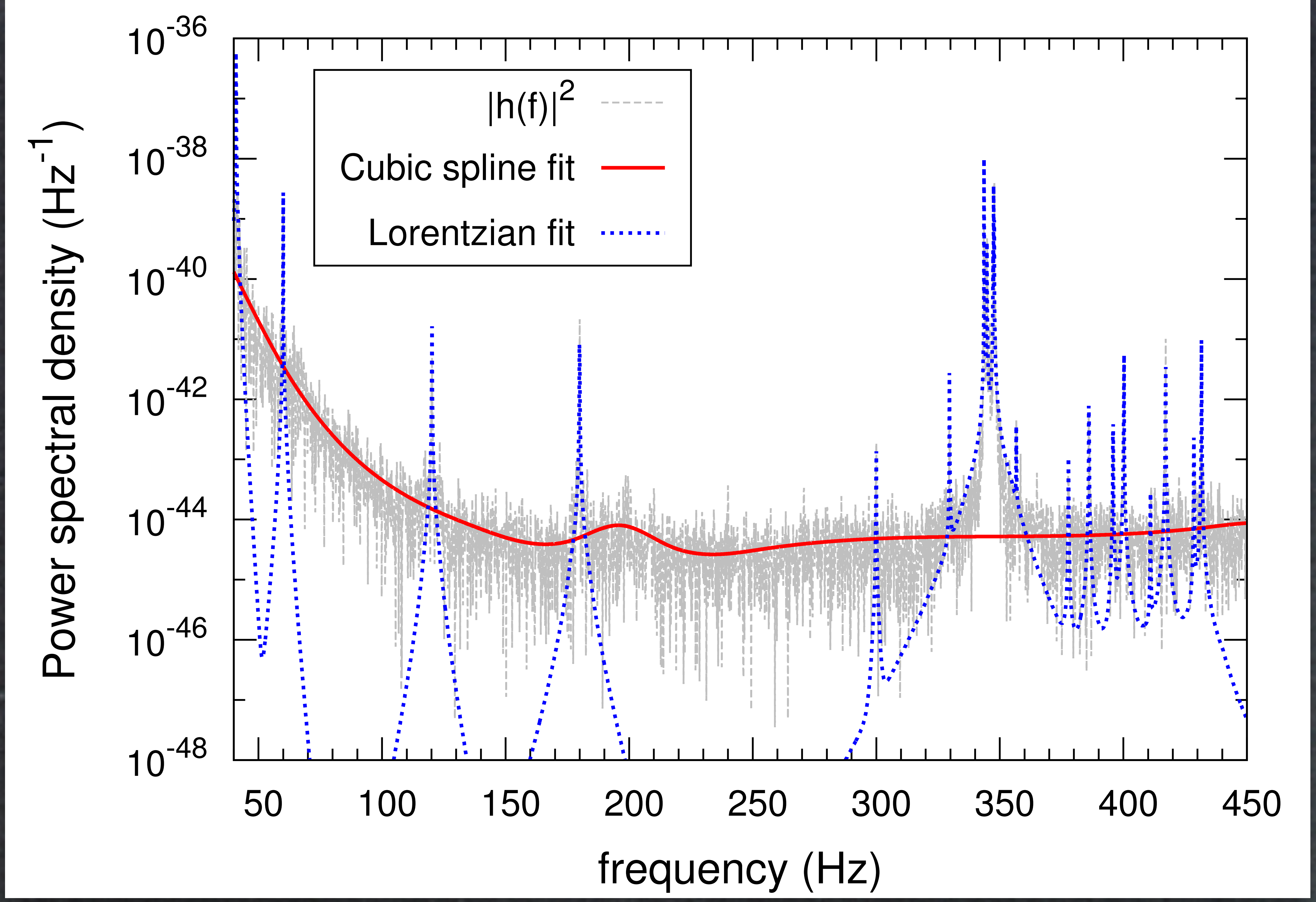
- For example a comparison of models based on B-splines and more shape-specific models.
- An example is the `Bayesline` pipeline. [Littenberg+, PRD, 91, 084034, 2015]



Generic Spectral shape

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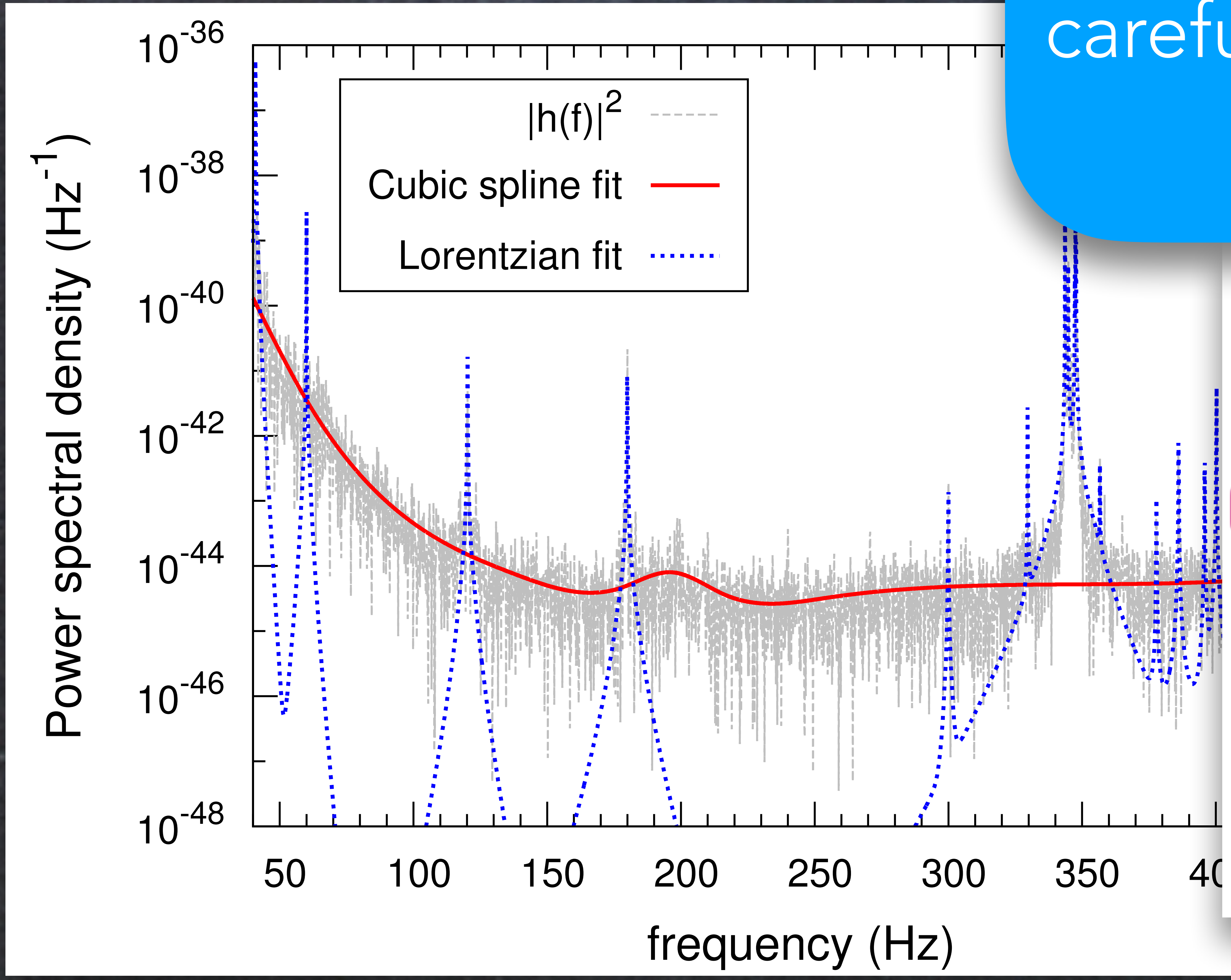
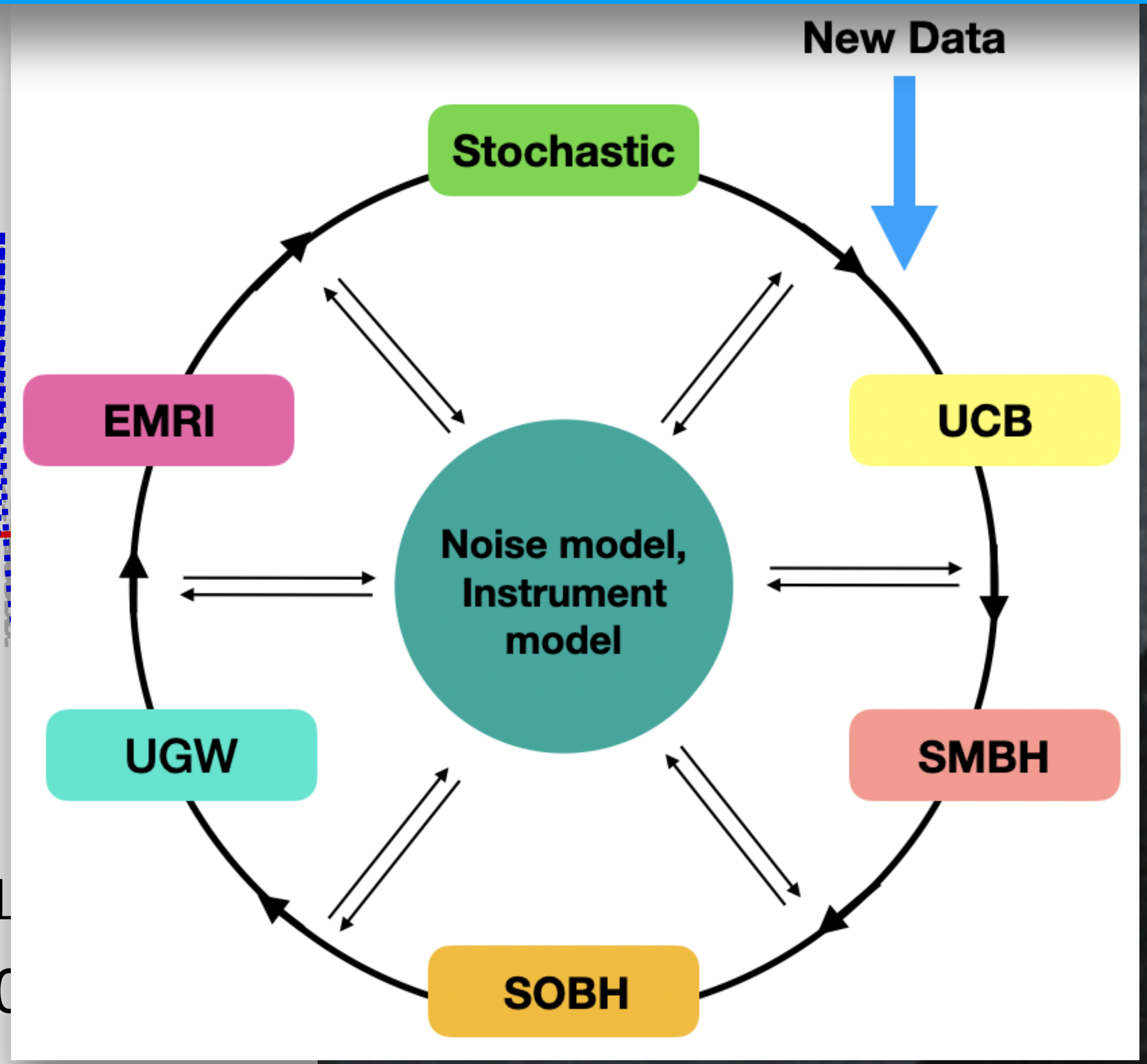
- An example is the `Bayesline` pipeline. [Littenb

This needs to go into the Global-Fit, so we need to be careful to not "eat" signal from different boxes





[Littenberg+, 101, 123021, 2020]

Modelling non-Gaussianities

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- Example: mixture of Gaussians.

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- We can use the Generalized Hyperbolic model, which is basically the "mother" of exponential distributions.

[Sasli+, arXiv:2305.04709, 2023]

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$$\Lambda_{\mathrm{hyp}}(\alpha, \delta; f_i) = n \sum_i^{N_f} \left[\left(\frac{d+1}{2} \log\left(\frac{\alpha}{\delta}\right) + \frac{1-d}{2} \log(2\pi) - \log(2\alpha) - \log\left(K_{(d+1)/2}(\delta\alpha)\right) \right) - \alpha\sqrt{\delta^2 + (\tilde{d}_i - \tilde{h}_i)^2 / S_{n,i}} \right]$$

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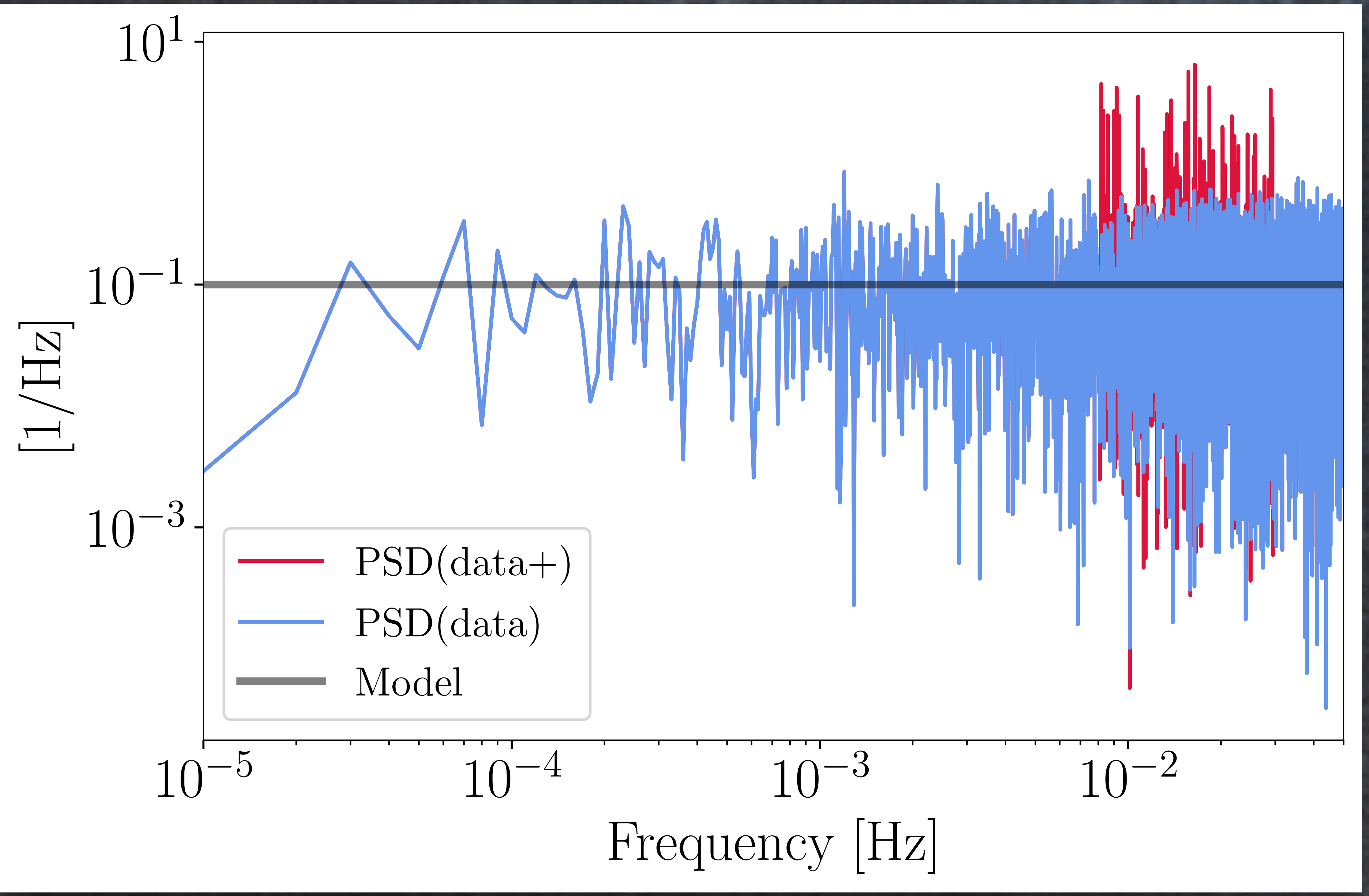
$\mathcal{N}$ if $\delta/\alpha \to S_n$ as $\alpha,\delta\to\infty$

Student-t, Log-normal, Normal-Inverse-Gaussian, Variance Gamma, [...], for other combinations

[Sasli+, arXiv:2305.04709, 2023]

Modelling non-Gaussianities

- A toy example as demonstration:



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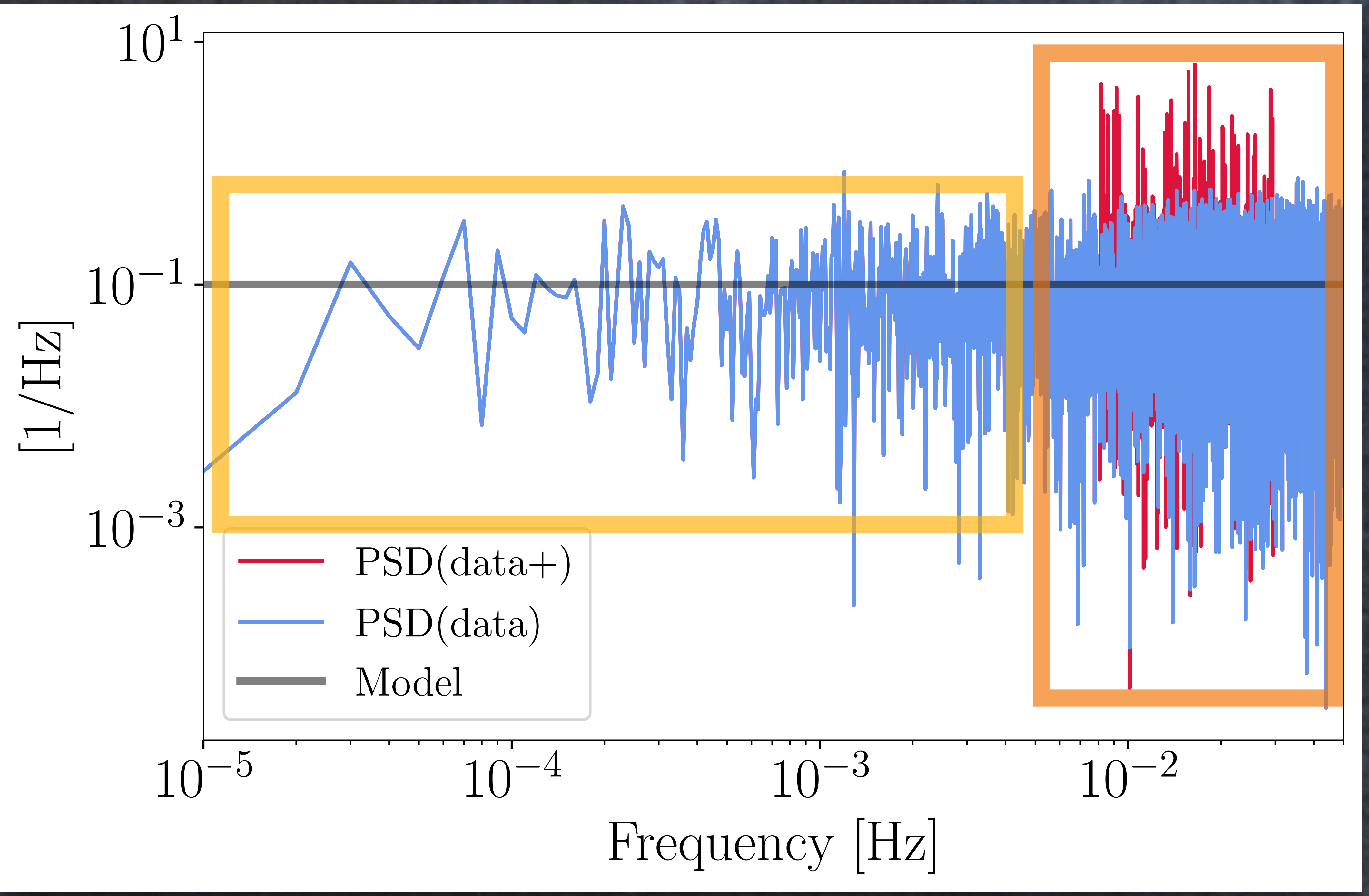

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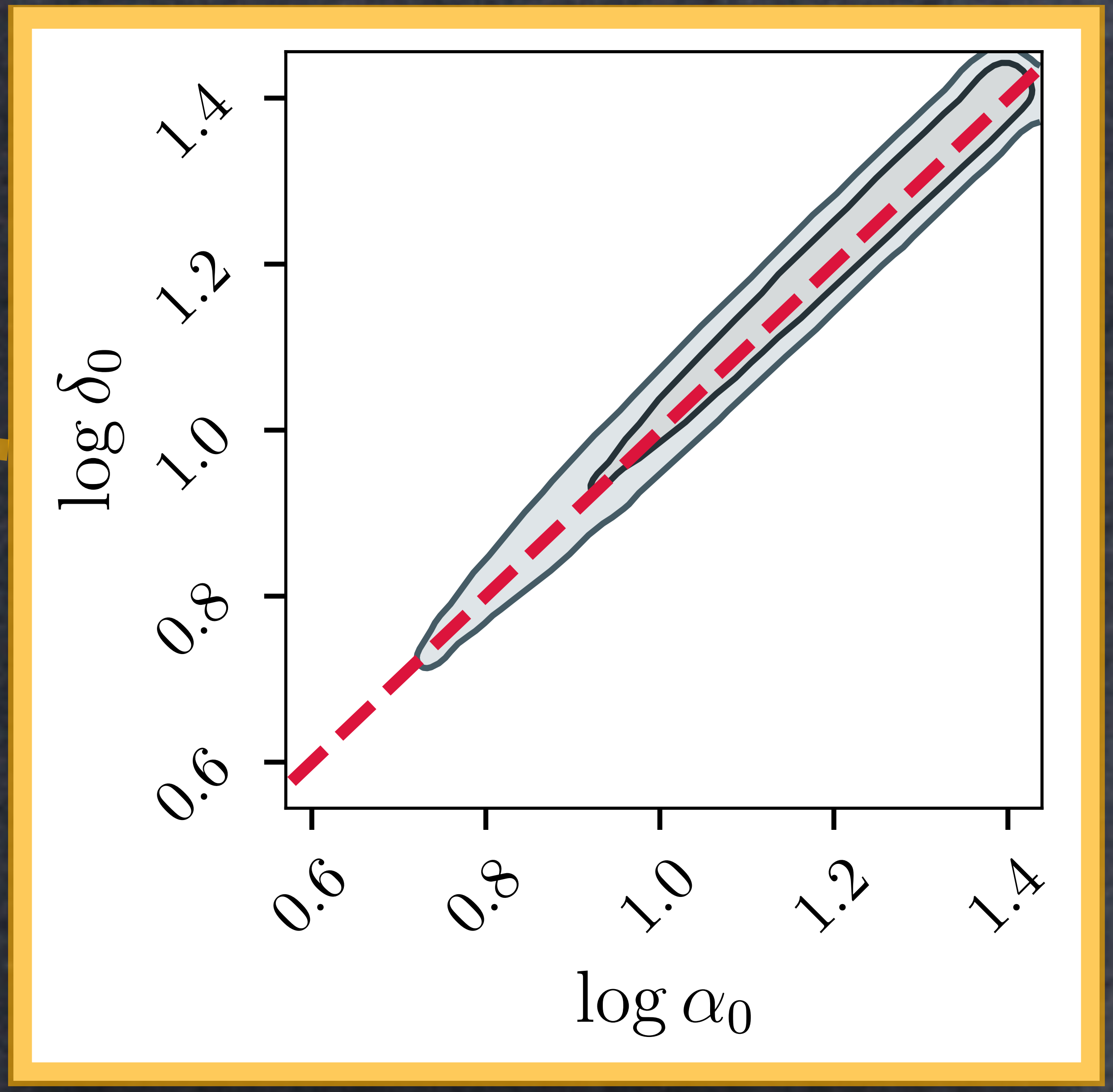

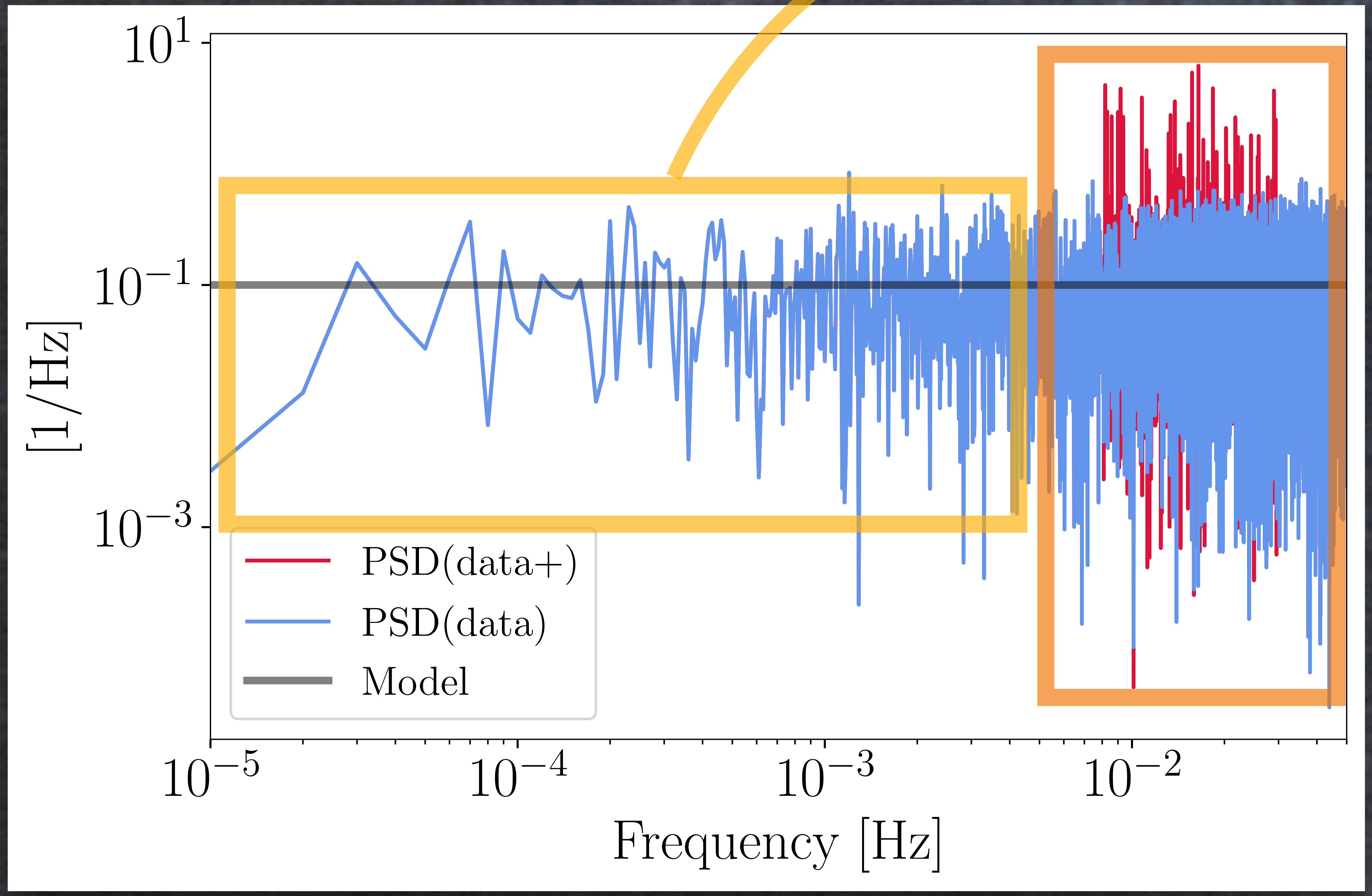

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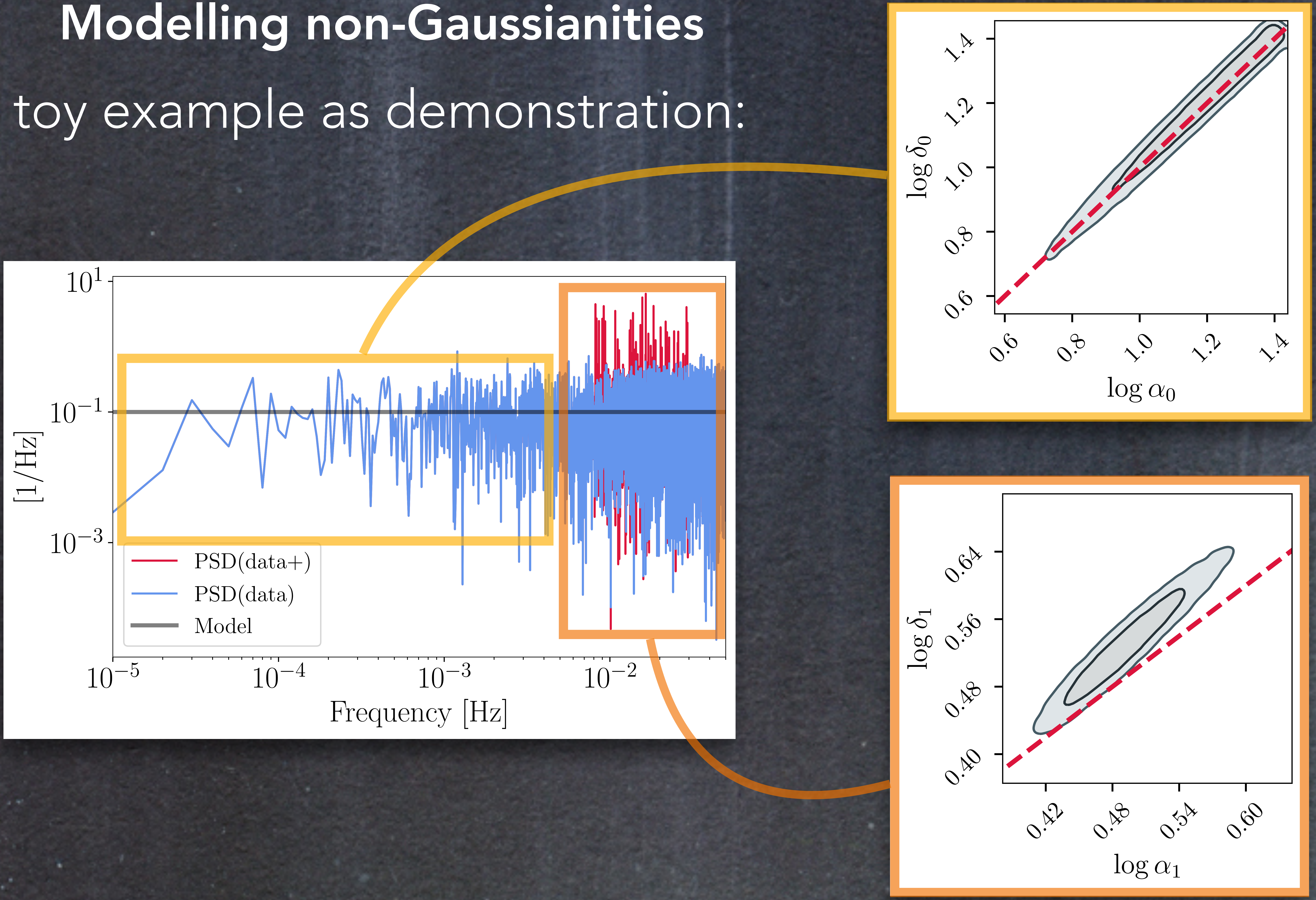

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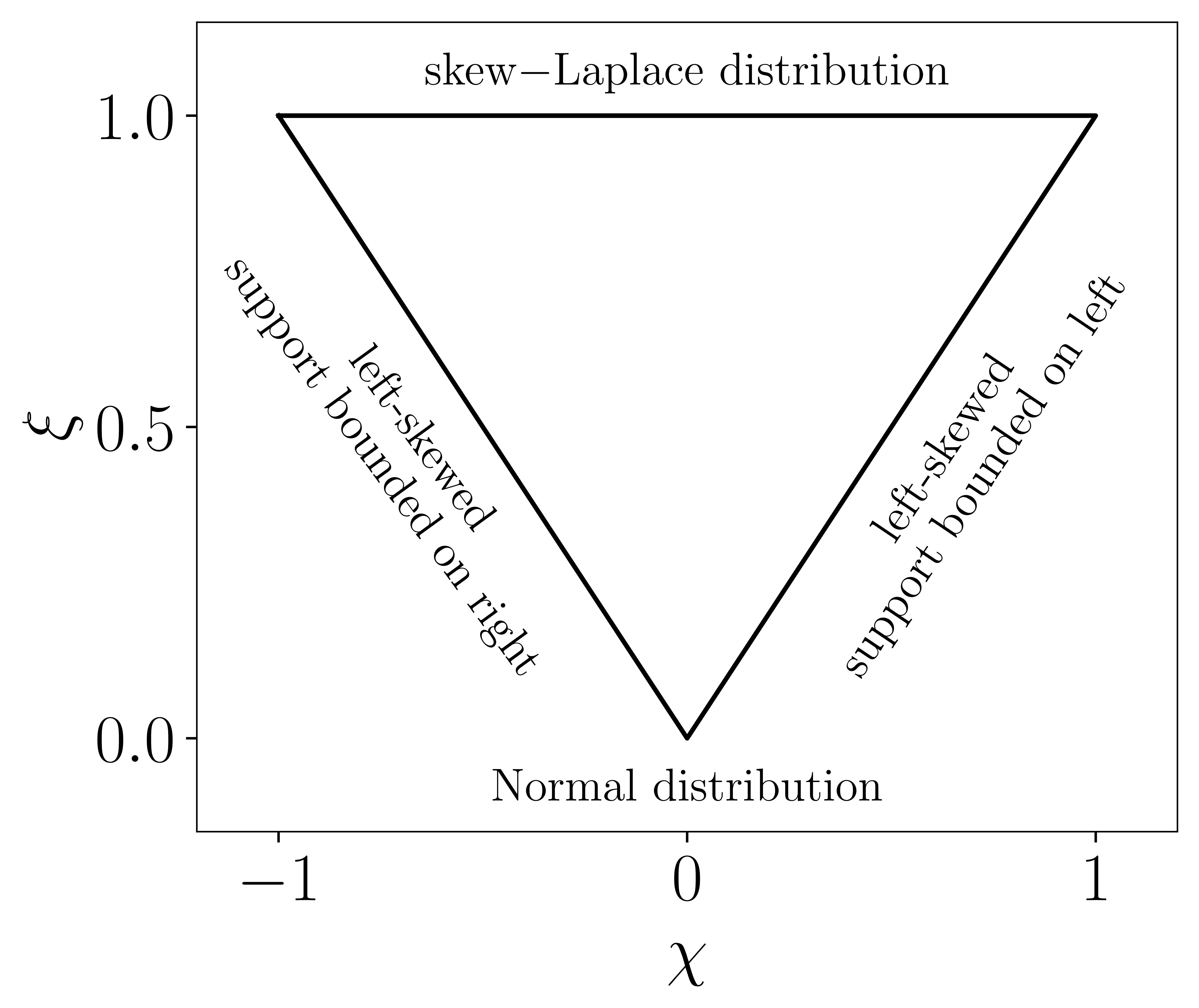

$$\zeta = \delta\sqrt{\alpha^2 - \beta^2}, \quad \varrho = \beta/\alpha,$$

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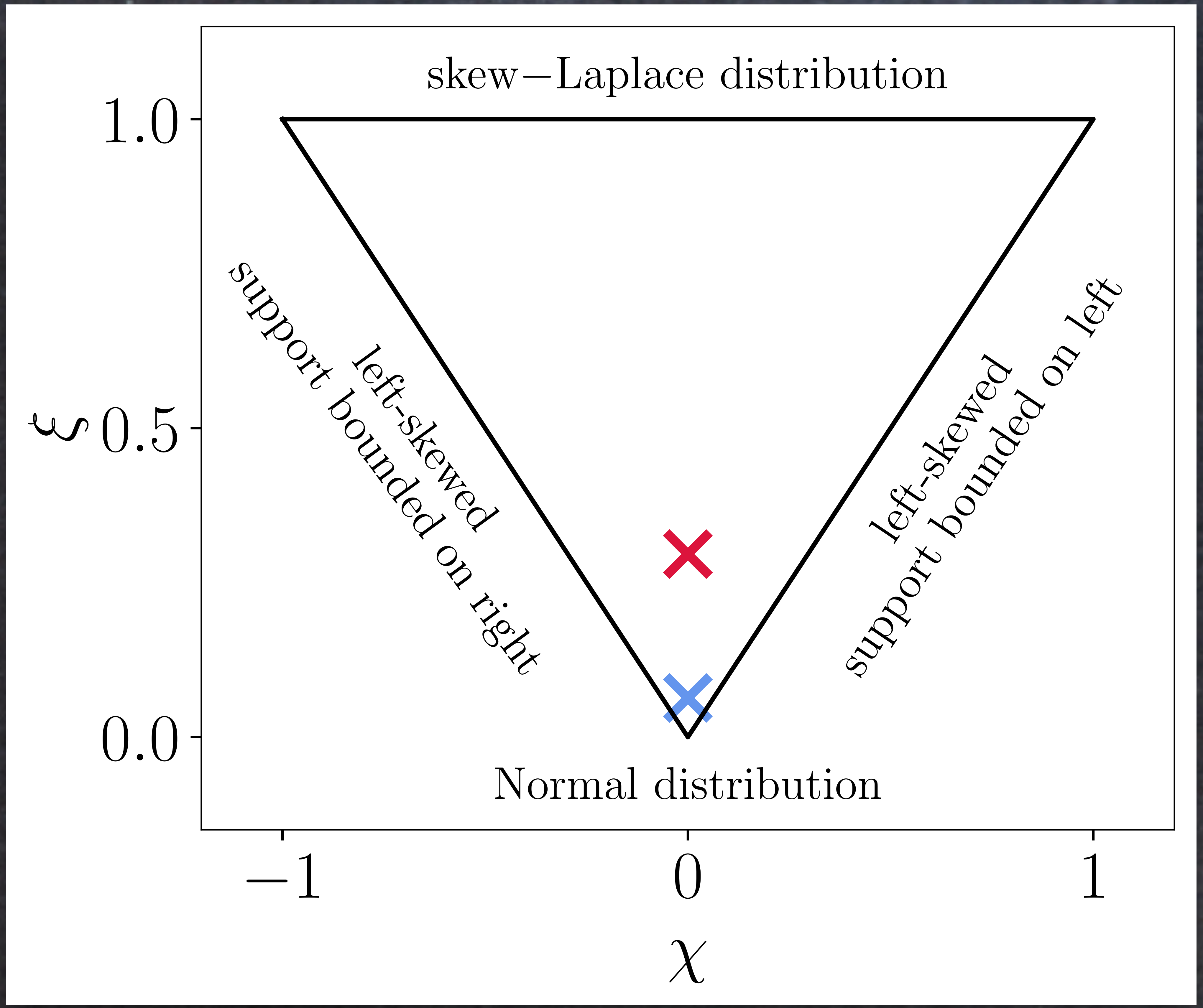

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[Sasli+, arXiv:2305.04709, 2023]

- In terms of modelling the stochastic signals, there is a huge library of spectral models.
- We have shape-agnostic models that are very useful for data analysis.
- We need to make use of the different responses of the instrument.
- *We need*: work with more realistic data scenarios, where components of the noise are not fully known.
- Put all the pieces together.