SGWB reconstruction for a non-equilateral and unequal-noise LISA constellation

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Data analysis challenges for SGWB workshop

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Outline

SGWBs detection

The data $\tilde{d}$ (in frequency space) can be expressed as

$$\tilde{d} = \tilde{s} + \tilde{n}$$

For an isotropic SGWB $\longrightarrow$ $\langle h_\lambda(\vec{k})\, h^*_{\lambda'}(\vec{k}')\rangle = P^\lambda_h(k)(2\pi)^3\delta_{\lambda\lambda'}\delta(\vec{k}-\vec{k}')$

Assuming $\langle \tilde{s}\tilde{n}\rangle = 0$ and Gaussian signal and noise

$$\left\langle \tilde{d}^2 \right\rangle = \left\langle \tilde{s}^2 \right\rangle + \left\langle \tilde{n}^2 \right\rangle = \mathcal{R}\, P^\lambda_h + N \equiv \mathcal{R}\left[P^\lambda_h + S_n\right]$$

where we have introduced

- The response function of the instrument $\mathcal{R}$
- The signal power spectrum P^λ_h (in 1/Hz)
- The noise power spectrum N (in 1/Hz)
- The (square of the) Strain sensitivity S_n (in 1/Hz)

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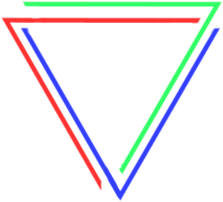
In order to compare with cosmological predictions it's customary to introduce

$$\Omega_{\rm GW} \equiv \frac{1}{3H_0^2 M_p^2}\frac{\partial \rho_{\rm GW}}{\partial \ln f} = \frac{4\pi^2}{3H_0^2} f^3 \sum_\lambda P_h^\lambda \qquad \text{and} \qquad \Omega_n(f) = \frac{4\pi^2}{3H_0^2} f^3 S_n(f)\,,$$

where $H_0 \simeq 3.24 \times 10^{-18}\, h_0\, {\rm Hz}$ is the Hubble constant today.

Some common assumptions...

LISA will have three
(correlated) data streams
dubbed XYZ basis
↓
Diagonalize to get
the (uncorrelated) AET basis



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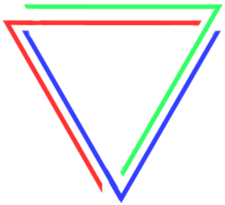
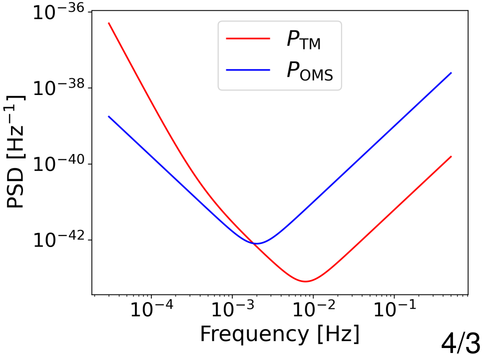


Time Delay Interferometry (TDI) to reduce **huge** laser noise

Low frequencies are dominated by Test Mass (TM) noise large frequencies by Optical Metrology System (OMS) noise:

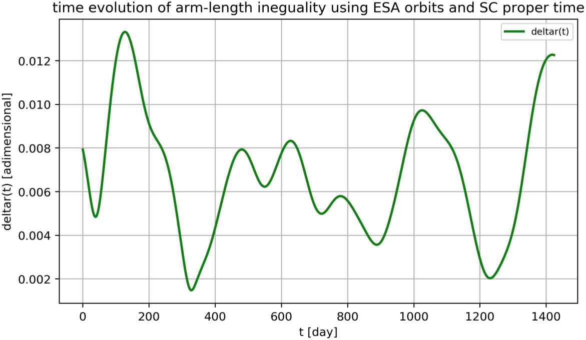
$$P_{TM}(f, A) = A^2 \times 10^{-30} \times F_{TM}(f) ,$$
$$P_{OMS}(f, P) = P^2 \times 10^{-24} \times F_{OMS}(f) ,$$

where $F_{TM}(f)$, $F_{OMS}(f)$ are some functions of frequency.



In reality LISA won’t have equal arms...

Fluctuations in the arm-lengths of order up to 10^{-2} are expected!



Response functions and noise spectra will be modified $\longrightarrow$ Orthogonality of TDI variables might be affected

An accurate description is necessary to avoid biases in the analysis

... nor the same noise levels in all links!

Let us have a closer look at the problem of noise characterization
(still stick with TM and OMS noise only with known templates)

Each spacecraft contains two test masses and two lasers $\longrightarrow$ 12 (6 Acc +6 OMS) independent noise components are expected!

$$\begin{pmatrix} A & 0 \\ 0 & P \end{pmatrix} \longrightarrow \begin{pmatrix} \begin{matrix} A_{12} & 0 & 0 & D_{12}A_{21} & 0 & 0 \\ 0 & A_{23} & 0 & 0 & D_{23}A_{32} & 0 \\ 0 & 0 & A_{31} & 0 & 0 & D_{31}A_{13} \\ D_{12}A_{12} & 0 & 0 & A_{21} & 0 & 0 \\ 0 & D_{23}A_{23} & 0 & 0 & A_{32} & 0 \\ 0 & 0 & D_{31}A_{31} & 0 & 0 & A_{13} \end{matrix} & 0 \\ 0 & \begin{matrix} P_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & P_{23} & 0 & 0 & 0 & 0 \\ 0 & 0 & P_{31} & 0 & 0 & 0 \\ 0 & 0 & 0 & P_{21} & 0 & 0 \\ 0 & 0 & 0 & 0 & P_{32} & 0 \\ 0 & 0 & 0 & 0 & 0 & P_{13} \end{matrix} \end{pmatrix}$$

Several complications are added in the problem:

- Noise components propagate differently in different TDI variables
- Higher dimensionality of the parameter space
- Correlations between the noise parameters

Again, an accurate analysis of the scenario is required

Single link signal model

The observable is the difference in the travel time of a photon from j to i:

$$\Delta t_{ij}(t) \simeq \int_0^{L_{ij}} \frac{\hat{l}_{ij}^a \, \hat{l}_{ij}^b}{2} h_{ab}(t(s), \vec{x}(s)) \, \mathrm{d}s \,, \quad \text{or alternatively,} \quad \eta_{ij}^{\mathrm{GW}}(t) \equiv \frac{\mathrm{d}}{\mathrm{d}t} \Delta t_{ij}(t) \,.$$

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To compute this quantity we first expand the signal in plane waves:

$$h_{ab}(\vec{x}, t) = \int_{-\infty}^{\infty} \mathrm{d}f \int \mathrm{d}\Omega_{\hat{k}} \; \mathrm{e}^{2\pi i f(t - \hat{k}\cdot\vec{x})} \sum_A \tilde{h}_A(f, \hat{k}) e_{ab}^A(\hat{k}) \,,$$

and assuming homogeneity, isotropy and no-chirality for the SGWB:

$$\langle \tilde{h}_A(f, \hat{k}) \, \tilde{h}_B^*(f', \hat{k}') \rangle = \delta(f - f') \delta(\hat{k} - \hat{k}') \delta_{AB} \frac{P_h^{AB}(f)}{16\pi} \,, \qquad \langle \tilde{h}_A(f, \hat{k}) \, \tilde{h}_B(f', \hat{k}') \rangle = 0 \,.$$

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The cross-spectral density for the single link measurement thus reads

$$S_{ij,mn}^{\eta,\mathrm{GW}}(f) \equiv \sum_A \mathcal{R}_{ij,mn}^A\, P_h^{AA}(f) = \frac{f^2}{f_{ij} f_{mn}} e^{-2\pi i f(L_{ij}-L_{mn})} \sum_A P_h^{AA}(f)\, \Upsilon_{ij,mn}^A(f)\,,$$

where $\mathcal{R}_{ij,mn}^A$ is the single link response and we introduced:

$$\Upsilon_{ij,mn}^A(f) = \int \frac{\mathrm{d}\Omega_{\hat{k}}}{4\pi}\, \mathrm{e}^{-2\pi i f \hat{k}\cdot(\vec{x}_i-\vec{x}_m)}\, \xi_{ij}^A(f, \hat{k})\, \xi_{mn}^A(f, \hat{k})^*\,.$$

Single link noise model

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Assuming TDI cancels primary noises and only TM and OMS are present:

$$\eta_{ij}^{N} = \mathcal{D}_{ij} n_{ji}^{\mathrm{TM}}(t) + n_{ij}^{\mathrm{TM}}(t) + n_{ij}^{\mathrm{OMS}}(t) \ ,$$

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Assuming stationarity, zero-mean and uncorrelated:

$$\langle \tilde{n}_{ij}^{\mathrm{TM}}(f)\, \tilde{n}_{ij}^{\mathrm{TM}*}(f') \rangle = \frac{1}{2}\, S_{ij}^{\mathrm{TM}}(f)\, \delta(f-f') \,,$$

$$\langle \tilde{n}_{ij}^{\mathrm{OMS}}(f)\, \tilde{n}_{ij}^{\mathrm{OMS}*}(f') \rangle = \frac{1}{2}\, S_{ij}^{\mathrm{OMS}}(f)\, \delta(f-f') \,,$$

the only non-zero cross spectral densities are:

$$S_{ij,ij}^{\eta,\mathrm{N}}(f) = S_{ij}^{\mathrm{TM}}(f) + S_{ji}^{\mathrm{TM}}(f) + S_{ij}^{\mathrm{OMS}}(f) \,,$$

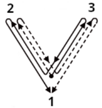
$$S_{ij,ji}^{\eta,\mathrm{N}}(f) = e^{2\pi i f L_{ji}}\, S_{ij}^{\mathrm{TM}}(f) + e^{-2\pi i f L_{ij}}\, S_{ji}^{\mathrm{TM}}(f) \,,$$

where $S_{ji}^{\mathrm{TM}}(f) = A_{ij}^{2} \times F_{TM}(f)$ and $S_{ji}^{\mathrm{OMS}}(f) = P_{ij}^{2} \times F_{OMS}(f)$.

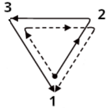
Time Delay Interferometry

Compare measurements at different times (delay operator) $\longrightarrow$ Noise reduction!

Let us start with two possibilities:



- Michelson variables, dubbed XYZ, defined as (YZ are permutations):
  $X \equiv (1 - D_{13}D_{31})(\eta_{12} + D_{12}\eta_{21}) + (D_{12}D_{21} - 1)(\eta_{13} + D_{13}\eta_{31})$



- Sagnac variables, dubbed $\alpha\beta\gamma$, defined as ($\beta\gamma$ permutations):
  $\alpha \equiv \eta_{12} + D_{12}\eta_{23} + D_{12}D_{23}\eta_{31} - (\eta_{13} + D_{13}\eta_{32} + D_{13}D_{32}\eta_{21})$

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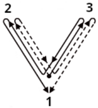

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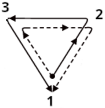

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Both signal and noise are correlated in these variables!

For equal arms, diagonalization via:
(diagonal variables dubbed AET and $\mathcal{AET}$) $\longrightarrow$

$$\begin{pmatrix} -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 1/\sqrt{6} & -2/\sqrt{6} & 1/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{pmatrix}$$

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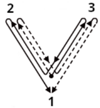

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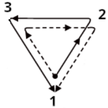

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Let us also introduce the fully symmetric ζ variable:

$$\zeta = D_{12}(\eta_{31} - \eta_{32}) + D_{23}(\eta_{12} - \eta_{13}) + D_{31}(\eta_{23} - \eta_{21}) \,.$$

Single link to TDI in a more formal way

It is convenient to define TDI variables using vector notation:

$$\tilde{V}(f) = \sum_{ij \in \mathcal{I}} c_{ij}^{V}(f)\, \tilde{\eta}_{ij}(f) \ ,$$

i.e., starting from the (FTs of the) 6 single link measurement $\tilde{\eta}_{ij}(f)$, we use a 3×6 matrix $(c_{ij}^{V}(f))$, to project onto any TDI basis.

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In this formalism, the cross spectral densities

$$\langle \tilde{U}(f)\, \tilde{V}^*(f') \rangle = \frac{1}{2} S^{UV}(f)\, \delta(f - f') \,,$$

can be expressed as:

$$\begin{aligned} \langle \tilde{U}(f) \tilde{V}^*(f') \rangle &= \sum_{ij,mn \in \mathcal{I}} c_{ij}^{U}(f)\, c_{mn}^{V*}(f') \langle \tilde{\eta}_{ij}(f)\, \tilde{\eta}_{mn}^{*}(f') \rangle \,, \\ &= \underbrace{\frac{1}{2} \sum_{ij,mn \in \mathcal{I}} \underbrace{c_{ij}^{U}(f)\, c_{mn}^{V*}(f)}_{C_{ij,mn}^{UV}(f)}\, S_{ij,lm}^{\eta}(f)}_{S^{UV}(f)}\, \delta(f - f') \,. \end{aligned}$$

Study the properties of $C_{ij,mn}^{UV}(f)$ $\rightarrow$ make general statements on TDI basis.

Data generation (in frequency domain)

Assuming signal and noise to be Gaussian, stationary, and isotropic

independent realizations for each data segment and frequency are drawn:

$$\tilde{s}_c(f_i) = \frac{\mathcal{N}(0, \sqrt{\Omega_{\rm GW}(f_i)}) + i\,\mathcal{N}(0, \sqrt{\Omega_{\rm GW}(f_i)})}{\sqrt{2}}$$

$$\tilde{n}_c(f_i) = \frac{\mathcal{N}(0, \sqrt{\Omega_n(f_i)}) + i\,\mathcal{N}(0, \sqrt{\Omega_n(f_i)})}{\sqrt{2}}$$

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Given that:

- LISA will be operating for 4yrs (and assume also 75% efficiency)
- We choose data segments of roughly 12 days

in practice we have:

- Roughly 95 independent measurements at each frequency.
- A frequency resolution of around 10^{-6}Hz

Data modeling and likelihood

Which likelihood should we use for our dataset?

Whittle likelihood should work (see any of the existing reviews):

$$-2\ln\mathcal{L}(\tilde{d}|\theta) \propto \sum_f \tilde{d}_i C_{ij}^{-1}(f|\theta)\tilde{d}_j^* + \ln\left[\det C_{ij}(f|\theta)\right] ,$$

where C_{ij} is the covariance matrix for $\langle \tilde{d}_i \tilde{d}_j^* \rangle$ given by:

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For fast estimates of the errors on parameter reconstruction use Fisher:

$$F_{ij} \equiv -\frac{\partial^2 \ln\mathcal{L}(\tilde{d}|\theta)}{\partial\theta_i\partial\theta_j}\bigg|_{\theta_0} = \sum_f = \mathrm{Tr}\left[C^{-1}\frac{\partial C}{\partial\theta_i}C^{-1}\frac{\partial C}{\partial\theta_j}\right] ,$$

However, this cannot tell us anything about possible biases!

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- We average over the (95) data segments:
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  i.e. from the initial linear 10^{-6}Hz spacing ($\sim 5 \times 10^5$ points)
  $\longrightarrow$ we go to some final (and less dense) set of frequencies f_i
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To gain a factor $\sim \mathcal{O}(100 \times 100)$ in computation time!

An accurate likelihood for the compressed data

A Gaussian likelihood would give a systematic low bias!
(astro-ph/9808264, astro-ph/0205387, astro-ph/0302218, 0801.0554)

Consider the Gaussian likelihood:

$$\ln \mathcal{L}_G\left(\vec{\theta}, \vec{n}\right) \propto -\frac{N_{chunks}}{2} \sum_{i,j} \sum_k w_{ij}^{(k)} \left(\frac{D_{ij}^{(k)} - h^2\Omega_{\rm GW}\left(f_{ij}^{(k)}, \vec{\theta}\right) - h^2\Omega_{n,ij}\left(f_{ij}^{(k)}, \vec{n}\right)}{h^2\Omega_{\rm GW}\left(f_{ij}^{(k)}, \vec{\theta}\right) + h^2\Omega_{n,ij}\left(f_{ij}^{(k)}, \vec{n}\right)} \right)^2$$

and the Lognormal likelihood:

$$\ln \mathcal{L}_{LN}\left(\vec{\theta}, \vec{n}\right) \propto -\frac{N_{chunks}}{2} \sum_{i,j} \sum_k w_{ij}^{(k)} \ln^2 \left(\frac{h^2\Omega_{\rm GW}\left(f_{ij}^{(k)}, \vec{\theta}\right) + h^2\Omega_{n,ij}\left(f_{ij}^{(k)}, \vec{n}\right)}{D_{ij}^{(k)}} \right)$$

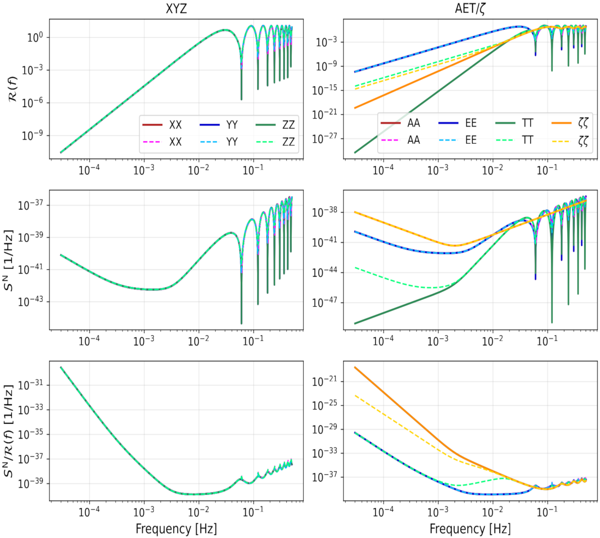
Then we define our likelihood as (astro-ph/0302218, 2009.11845)

$$\ln \mathcal{L} = \frac{1}{3} \ln \mathcal{L}_G + \frac{2}{3} \ln \mathcal{L}_{LN}$$

which removes the skewness contributions and thus is more accurate.

Signal and noise Michelson

- Solid lines → equal arms
- Dashed lines → unequal arms
- Most TDI variables are mildly affected
- Slight displacement of the zeros
- T Michelson looses signal orthogonality!
- ζ is clearly more robust to unequal arm corrections!



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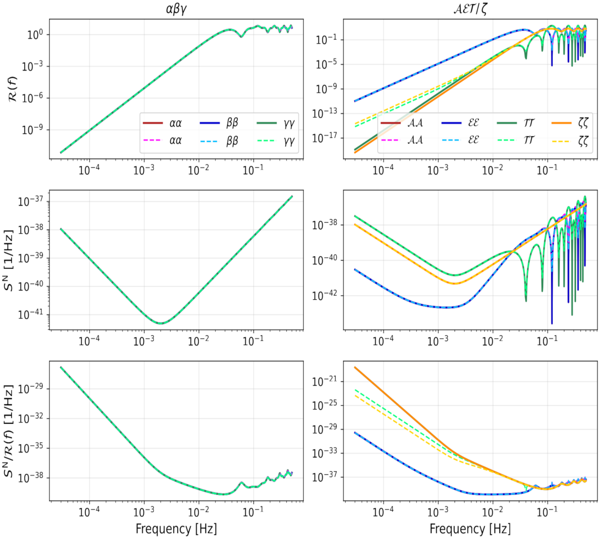
Signal and noise Sagnac

- Solid lines → equal arms
- Dashed lines → unequal arms
- Sensitivity is similar to Michelson variables (but less zeros)
- $\mathcal{T}$ is way more robust than T!
- $\mathcal{T}$ and ζ's signal orthogonality are comparable!



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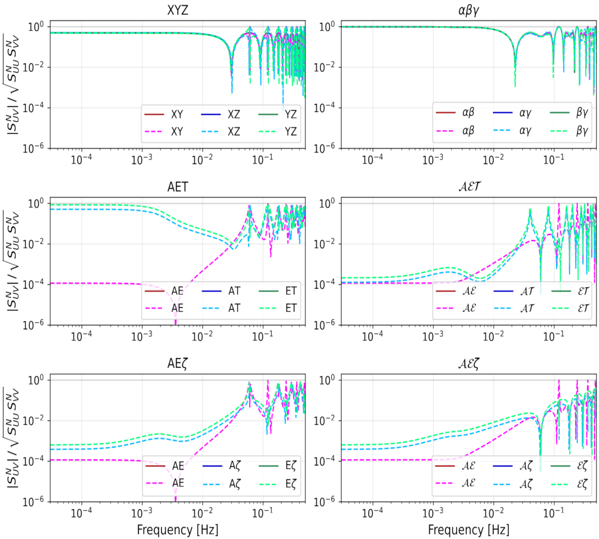
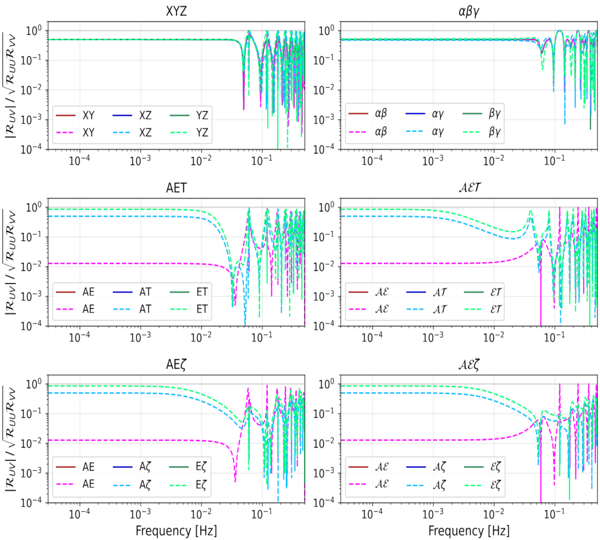
Noise correlations

- Solid lines → equal arms
- Dashed lines → unequal arms
- Unequal arms break orthogonality
- Correlations involving T are most affected
- $\mathcal{AET}$ is again more robust
- Replacing T with ζ might help
- No real benefit in trading $\mathcal{T}$ for ζ



Signal correlations

- Solid lines → equal arms
- Dashed lines → unequal arms
- All bases are affected (∼ in the same way)!
- Leakage into T, $\mathcal{T}$, ζ induces correlations
- Signal-sensitive channels stay orthogonal



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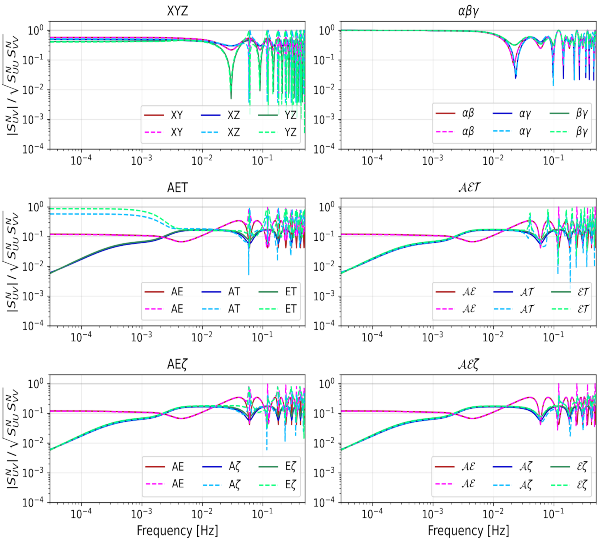
Noise correlations v2 (unequal noise levels)

- Solid lines → equal arms
- Dashed lines → unequal arms
- Equal arms also have non-zero correlations!
- Correlations in all TDI bases
- Again, Michelson performs worse than other bases



Case study

Let's test the impact of including these effects on parameter reconstruction.

Let us consider the following scenario:

- Constant but unequal armlenghts
- Noise described by a template (controlled by some parameters)
- Constant (check both equal and unequal) noise parameters
- Signal described by a power law $h^2\Omega_{GW} = 10^{alpha}(f/f_*)^{n_\mathrm{T}}$
- Injection with quite large SNR ($\alpha = -11.5$, $n_\mathrm{T} = 0 \rightarrow$ SNR ~ 270)
- Fisher matrix to get uncertainties and assess the impact of CSDs
- MCMC to validate Fisher results and look for biases
- Compressed data only uses the diagonal (still can check biases)

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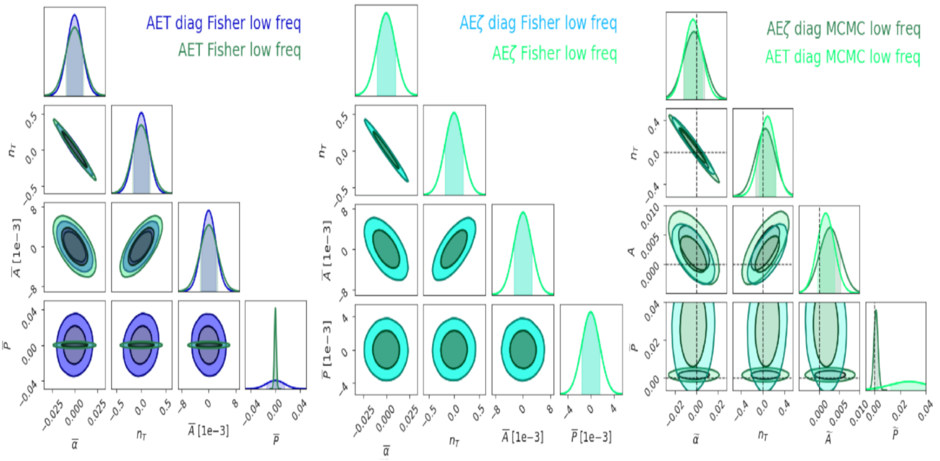
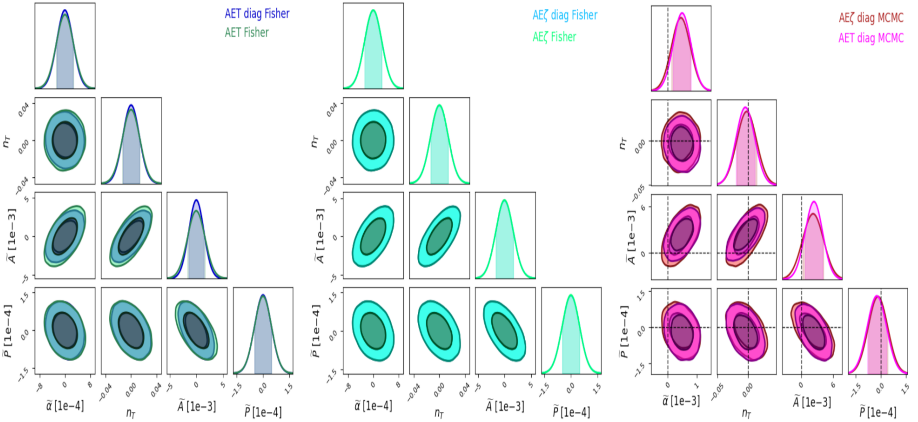
Low frequency (equal noise) analysis



- MCMC (diagonal only): Fisher approximation works quite well
- Fisher: Neglecting correlations is dangerous for AET, but not much for AEζ!
- MCMC/Fisher: AET performs much worse than AEζ

Full frequency (equal noise) analysis



- MCMC (diagonal only): Fisher approximation works quite well
- Fisher: Neglecting correlations only marginally affects the reconstruction
- MCMC/Fisher: AET slightly underestimates errors (wrt AEζ)

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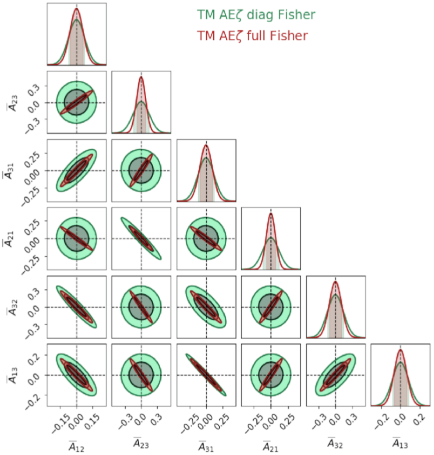
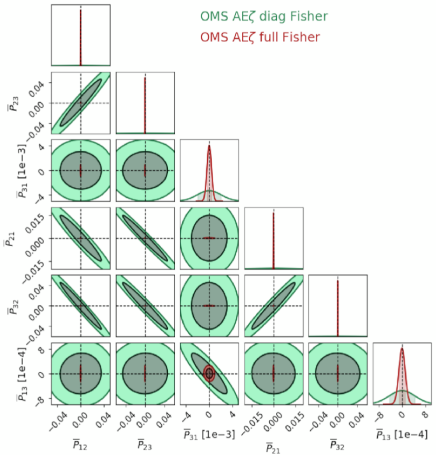
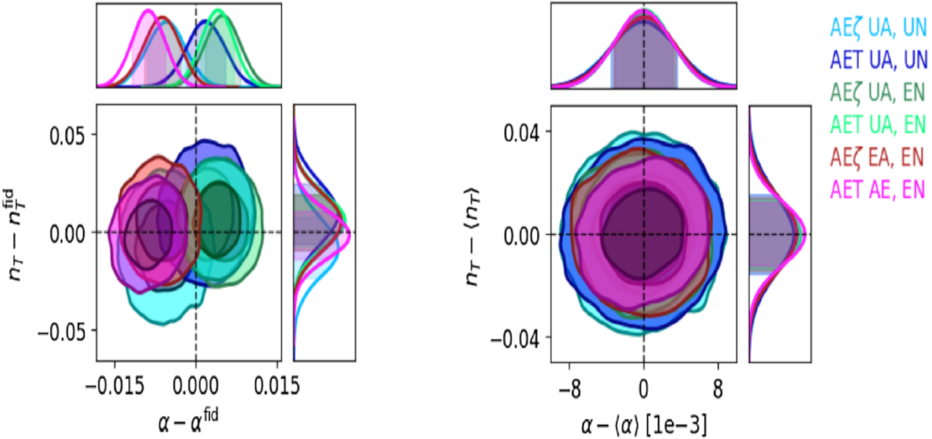
Full frequency (unequal noise) analysis





- MCMC (diagonal only): (not shown here but) Fisher approximation works quite well
- Fisher: Neglecting correlations might be dangerous for both bases (here only AEζ)
- MCMC/Fisher: more impact on worsley constrained (subdominant) parameters!

Signal parameters



Good news for theorists in the room:

Signal parameters don't care much about all this!

(remember the caveats though)

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ML for SGWB data analysis

Traditional methods (MCMC, nested sampling, whatever) are quite efficient and guaranteed to converge (in some cases)

but

scale poorly with number of parameters and require explicit likelihoods

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Normally, with Bayesian inference, we try to study the posterior probability:

$$p(\theta|d) = \frac{p(d|\theta)\pi(\theta)}{p(d)} \equiv r(d,\theta)\pi(\theta) \, ,$$

where we have introduced:

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i.e. $r(d,\theta)$ is the ratio between joint probability and marginal probability.

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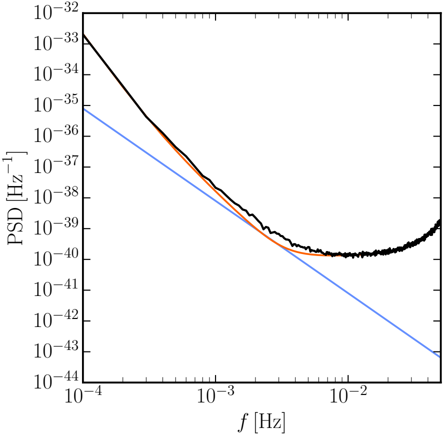
i.e. $r(d,\theta)$ is the ratio between joint probability and marginal probability.

Given a pair (θ, d), $r(d,\theta)$ can be used to assess whether θ can generate d!

This can be cast in a minimization problem that can be solved with ML the approach is typically referred to as Neural Ratio Estimation.

Recover previous results I ...

Assume we inject a power law signal:
Can we recover it with the same level of accuracy?



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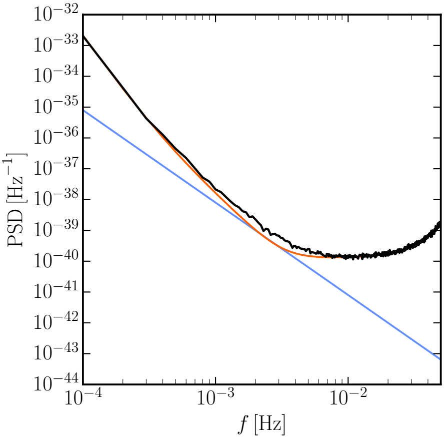

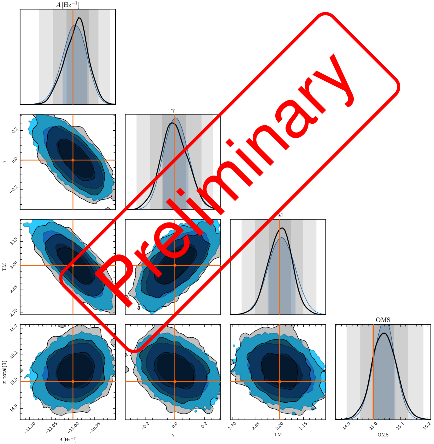

Good news!

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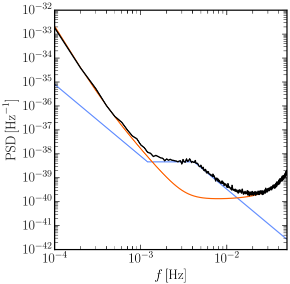
Recover previous results II...

Can we also do something ”template-free”?



Recover previous results II...

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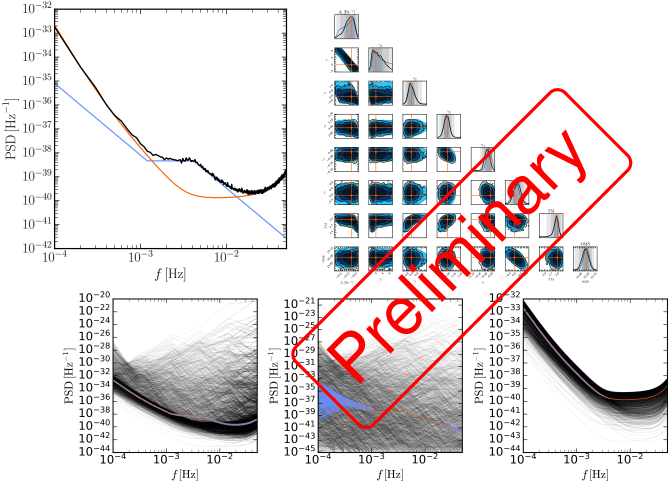

Good news again!

In collaboration with James Alvey, Uddipta Bhardwaj, Christoph Weniger and Valerie Domcke

... plus something completely new!

What if there's something else beyond SGWB and noise?

For example, assume some sources slightly below the threshold for detection are randomly injected.

Would this still work??

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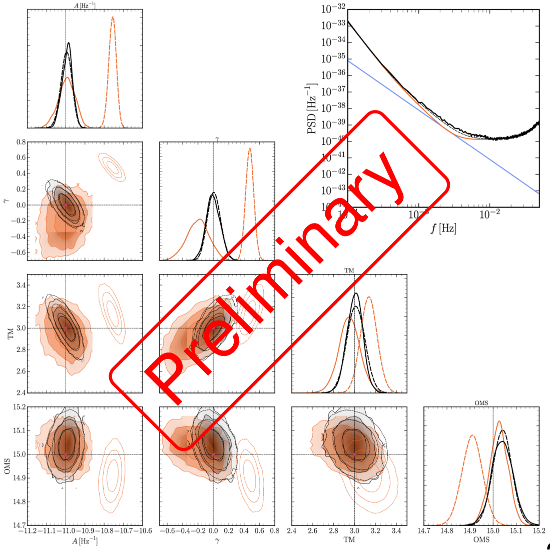

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Conclusions and future perspectives

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- Some TDI variables are more robust to unequal arm corrections
- Similarly, unequal noise levels also introduce correlations
- Signal reconstruction looks mildly affected (with caveats...)
- ML techniques for SGWB look quite promising...

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- Signal reconstruction looks mildly affected (with caveats...)
- ML techniques for SGWB look quite promising...

Future perspectives

- Application to concrete cases (inflation, phase transitions, ...)
- Keep improving on detector modeling
- More realistic noise model and data generation procedure
- Drop stationarity (both for signal and noise)
- Include anisotropies
- New techniques?

Last Slide

The End

Thank you